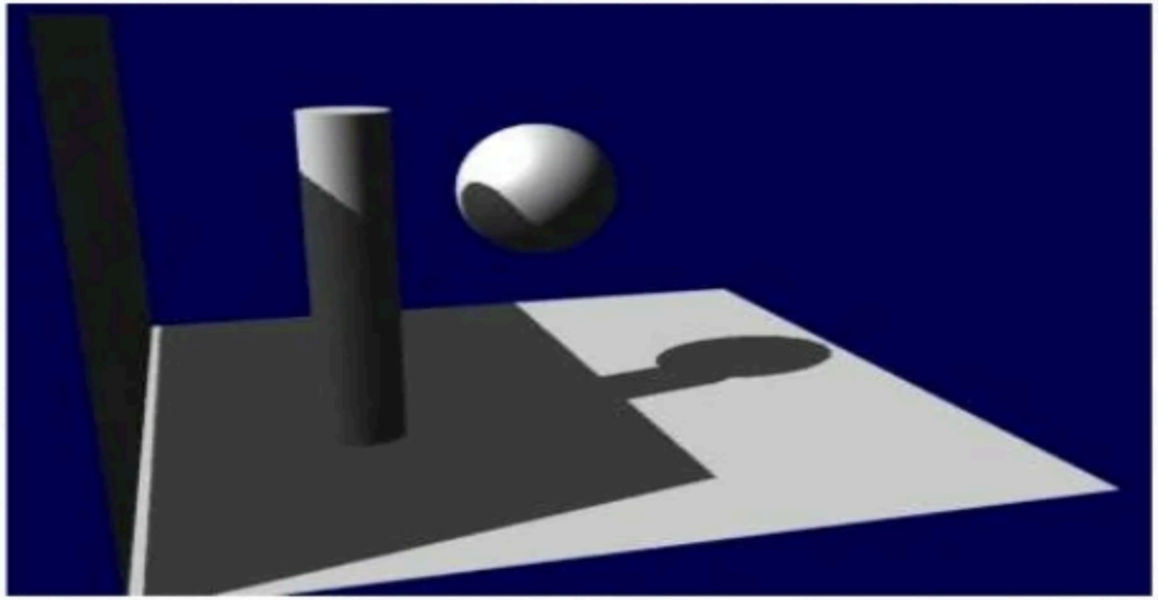
INTRO TO MATRICES

Why Matrices?

- Simply put, matrices are a simple way of displaying data, with extraneous information removed
- Other applications:
  - Graphic design (like reflections and shadows)
  - Solving Equations



Linear System of Matrices

Linear system

$$\begin{cases} 2x - y = 5 \\ x + 4y = 7 \end{cases}$$

Equation 1

Equation 2

Augmented matrix

$$\begin{bmatrix} 2 & -1 & 5 \\ 1 & 4 & 7 \end{bmatrix}$$

$$\begin{array}{ccc} x & y & \end{array}$$

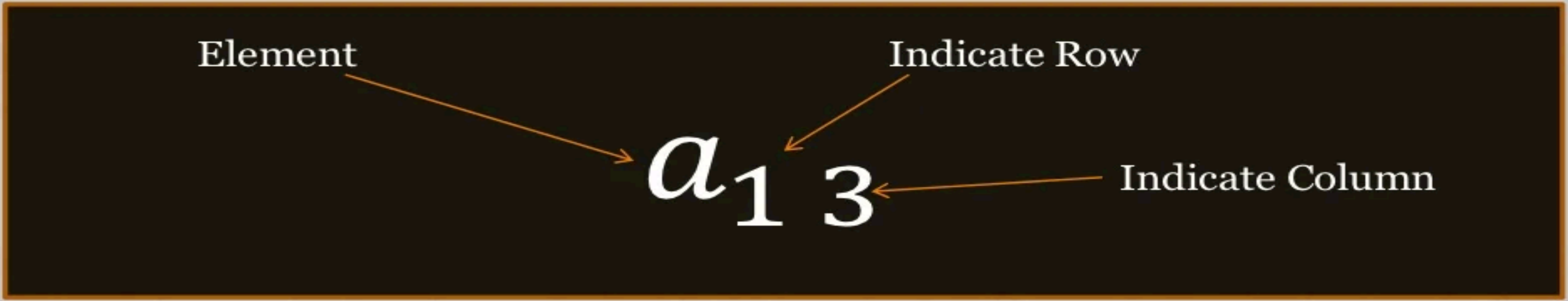
- This matrix is called the augmented matrix of the system.
- The augmented matrix contains the same information as the system, but in a simpler form.
- The operations we learned for solving systems of equations can now be performed on the augmented matrix.

What is a matrix?

- A Matrix is just rectangular arrays of items
- A typical matrix is a rectangular array of numbers arranged in rows and columns.

$$\underset{3x4}{A} = \begin{bmatrix} 21 & 62 & 33 & 93 \\ 44 & 95 & 66 & 13 \\ 77 & 38 & 79 & 33 \end{bmatrix}$$

- Matrices are denoted by capital letters A, B, C or X , Y , Z etc.
- Its elements are denoted by small letters a, b, c ..... etc.
- The elements of the matrix are enclosed by any of the brackets i.e. [ ], ( ), { } .
- The position of the elements of a Matrix is indicated by the subscripts attached to the element. e.g. a_{13} indicates that element 'a' lies in first row and third column i.e. first subscript denotes row and second subscript denote column.



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Sizing a matrix

- By convention matrices are “sized” using the number of rows (m) by number of columns (n).

$$\underset{3x4}{A} = \begin{bmatrix} 21 & 62 & 33 & 93 \\ 44 & 95 & 66 & 13 \\ 77 & 38 & 79 & 33 \end{bmatrix} \qquad \underset{3x3}{B} = \begin{bmatrix} 7 & 3 & 2 \\ 8 & 4 & 1 \\ 6 & 5 & 9 \end{bmatrix}$$

$$\underset{4x2}{C} = \begin{bmatrix} 11 & 4 \\ 14 & 7 \\ 16 & 8 \\ 22 & 3 \end{bmatrix} \qquad \underset{1x1}{D} = \begin{bmatrix} 17 \end{bmatrix}$$

- The number of rows and columns of a matrix determines the order of the matrix.
- Hence , a matrix , having m rows and n columns is said to be of the order $m \times n$ (read as 'm' by 'n').
- In particular, a matrix having 3 rows and 4 columns is of the order 3×4 and it is called a 3×4 matrix e.g.

❖ $A= \begin{bmatrix} 2 & 3 & 5 \\ 4 & 6 & 7 \end{bmatrix}$ is a matrix of order 2×3 since there are two rows and three columns.

❖ $A = \begin{bmatrix} 3 & 5 & 7 \end{bmatrix}$ is a matrix of order 1×3 since there are one row and three columns.

❖ $A = \begin{bmatrix} 5 \\ 6 \\ 8 \end{bmatrix}$ is a matrix of order 3×1 since there are three rows and one column.

“Special” Matrices

- Square matrix: a square matrix is an mxn matrix in which m = n.

$$\underset{3x3}{B} = \begin{bmatrix} 7 & 3 & 2 \\ 8 & 4 & 1 \\ 6 & 5 & 9 \end{bmatrix}$$

- Vector: a vector is an mxn matrix where either m OR n = 1 (but not both).

$$\underset{4x1}{X} = \begin{bmatrix} 12 \\ 9 \\ -4 \\ 0 \end{bmatrix} \qquad \underset{1x3}{Y} = \begin{bmatrix} 7 & -22 & 14 \end{bmatrix}$$

"Special" Matrices

- Scalar: a scalar is an mxn matrix where BOTH m and n = 1.

$$\underset{1x1}{D} = [17]$$

- Zero matrix: an mxn matrix of zeros.

$$\underset{3x2}{0} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

- Identity Matrix: a square (mxm) matrix with 1s on the diagonal and zeros everywhere else.

$$\underset{3x3}{I} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- **Rectangular Matrix :** A matrix in which the number of rows and columns are not equal is called a rectangular matrix e.g. ,

$$A=\begin{bmatrix}2 & 4 & 5\\ 4 & 6 & 7\end{bmatrix}$$ 2 × 3 is rectangular matrix of order 2×3

- **Square Matrix :** A matrix in which the number of rows is equal to the number of columns is called a square matrix e.g.

$$A=\begin{bmatrix}2 & 3 & 4\\ 3 & 2 & 5\\ 7 & 8 & 5\end{bmatrix}$$

Principal Diagonal

Note : The elements 2, 2 , 5 in the above matrix are called diagonal elements and the line along which they lie is called the principal diagonal

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Diagonal Matrix : A square matrix in which all diagonal elements are non- zero and all non-diagonal elements are zeros is called a diagonal matrix e.g.

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 5 \end{bmatrix} \text{ is a diagonal matrix of } 3 \times 3$$

Scalar Matrix : A diagonal matrix in which diagonal elements are equal (but not equal to 1), is called a scalar matrix e.g.

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \text{ is a scalar matrix of } 3 \times 3$$

Identity (or Unit) Matrix : A square matrix whose each diagonal element is unity and all other elements are zero is called and Identity (or Unit) Matrix. An Identity matrix of order 3 is denoted by I_3 or simply by I.

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e.g.

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ is a unit matrix of order 3}$$

- **<u>Null (Zero) Matrix :</u>** A matrix of any order (rectangular or square) whose each of its element is zero is called a null matrix (or a Zero matrix) and is denoted by O. e.g.

$O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ and $O = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ are null matrices of order 2×2 and 2×3 respectively.

- **<u>Row Matrix :</u>** A matrix having only one row and any number of columns is called a row matrix (or a row vector) e.g.

$$A = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \text{ is a row matrix of order } 1 \times 3$$

- **<u>Column Matrix :</u>** A matrix having only one column and any number of rows is called a column matrix (or a column vector) e.g.

$$A = \begin{bmatrix} 1 \\ 3 \\ 6 \end{bmatrix} \text{ is a column matrix or order } 3 \times 1$$

- **<u>Upper Triangular and Lower Triangular Matrix</u>:** A square matrix is called an upper triangular matrix if all the elements below the principal diagonal are zero and it is said to be lower triangular matrix if all the elements above the principal diagonal are zero e.g.

UTM $A = \begin{bmatrix} 2 & 3 & 4 \\ 0 & 1 & 5 \\ 0 & 0 & 7 \end{bmatrix}$, LTM $B = \begin{bmatrix} 5 & 0 & 0 \\ 8 & 7 & 0 \\ 4 & 3 & 1 \end{bmatrix}$

- **<u>Sub Matrix :</u>** A matrix obtained by deleting some rows or column or both of a given matrix is called its sub matrix. e.g.

$A = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 1 & 5 \\ 3 & 9 & 7 \end{bmatrix}$, Now $\begin{bmatrix} 2 & 3 \\ 3 & 9 \end{bmatrix}$ is a sub matrix of given matrix A. The sub matrix obtained by deleting 2^{nd} row and 3^{rd} column of matrix A.

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Matrix Rank

- Matrix Rank: the rank of a matrix is the maximum number of linearly independent vectors (either row or column) in a matrix
- Full Rank: A matrix is considered full rank when all vectors are linearly independent

Transposing a Matrix

- Matrix Transpose: is the mxn matrix obtained by interchanging the rows and columns of a matrix (converting it to an nxm matrix)

$$\underset{4x1}{X} = \begin{bmatrix} 12 \\ 9 \\ -4 \\ 0 \end{bmatrix} \qquad \underset{1x4}{X}' = \begin{bmatrix} 12 & 9 & -4 & 0 \end{bmatrix}$$

$$\underset{3x4}{A} = \begin{bmatrix} 21 & 62 & 33 & 93 \\ 44 & 95 & 66 & 13 \\ 77 & 38 & 79 & 33 \end{bmatrix} \qquad \underset{4x3}{A}' = \begin{bmatrix} 21 & 44 & 77 \\ 62 & 95 & 38 \\ 33 & 66 & 79 \\ 93 & 13 & 33 \end{bmatrix}$$

The sum of all the elements on the principal diagonal of a square matrix is called the trace of the matrix. It is denoted by **tr. A**.

If $A=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ then

trace of A = tr.A = $a_{11} + a_{22} + a_{33}$

e.g. $A = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 1 & 5 \\ 3 & 9 & 7 \end{bmatrix}$, then

trace of A = tr. A =2+1+7=10

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Two matrices ‘A’ and ‘B’ are said to be equal if only if they are of the same order and each element of ‘A’ is equal to the corresponding element of ‘B’. Given

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13}\ldots\ldots \\ a_{21} & a_{22} & a_{23}\ldots\ldots \\ a_{31} & a_{32} & a_{33}\ldots\ldots \end{bmatrix}_{m\times n} \text{ and } B = \begin{bmatrix} b_{11} & b_{12} & b_{13}\ldots\ldots \\ b_{21} & b_{22} & b_{23}\ldots\ldots \\ b_{31} & b_{32} & b_{33}\ldots\ldots \end{bmatrix}_{x\times y}$$

The above two matrices will be equal if and only if

- ✓ No. of rows and column of A and B are equal i.e. m=x and n=y.
- ✓ Each element of A is equal to the corresponding element of B i.e. $a_{11}=b_{11}$, $a_{12} = b_{12}, a_{13} = b_{13}$ ……… and so on

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}$, according to the condition of equality of matrices both matrices have same order i.e. 2×2 but the 2nd necessary condition is a=3, b=4, c=5 , d= 6

Matrix Addition

- Matrices can be added (or subtracted) as long as the 2 matrices are the same size
  - Simply add or subtract the corresponding components of each matrix.

$$\underset{2x3}{A} = \begin{bmatrix} 1 & 2 & 3 \\ 7 & 8 & 9 \end{bmatrix} \quad \underset{2x3}{B} = \begin{bmatrix} 5 & 6 & 7 \\ 3 & 4 & 5 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 1 & 2 & 3 \\ 7 & 8 & 9 \end{bmatrix} + \begin{bmatrix} 5 & 6 & 7 \\ 3 & 4 & 5 \end{bmatrix} = \begin{bmatrix} 1+5 & 2+6 & 3+7 \\ 7+3 & 8+4 & 9+5 \end{bmatrix} = \begin{bmatrix} 6 & 8 & 10 \\ 10 & 12 & 14 \end{bmatrix}$$

$$A + B = B + A$$

$$A - B = \begin{bmatrix} 1 & 2 & 3 \\ 7 & 8 & 9 \end{bmatrix} - \begin{bmatrix} 5 & 6 & 7 \\ 3 & 4 & 5 \end{bmatrix} = \begin{bmatrix} 1-5 & 2-6 & 3-7 \\ 7-3 & 8-4 & 9-5 \end{bmatrix} = \begin{bmatrix} -4 & -4 & -4 \\ 4 & 4 & 4 \end{bmatrix}$$

Matrix Multiplication

- Multiplying a matrix by a scalar: each element in the matrix is multiplied by the scalar.

$$\underset{2x3}{A} = \begin{bmatrix} 1 & 2 & 3 \\ 7 & 8 & 9 \end{bmatrix} \text{ and } \underset{1x1}{x} = 5; \text{ then}$$

$$xA = \begin{bmatrix} 5*1 & 5*2 & 5*3 \\ 5*7 & 5*8 & 5*9 \end{bmatrix} = \begin{bmatrix} 5 & 10 & 15 \\ 35 & 40 & 45 \end{bmatrix}$$

Matrix Multiplication

- Multiplying a matrix by a matrix:
  - the product of matrices A and B (AB) is defined if the number of columns in A equals the number of rows in B.
  - Assuming A has ixj dimensions and B has jxk dimensions, the resulting matrix, C, will have dimensions ixk
  - In other words, in order to multiply them the inner dimensions must match and the result is the outer dimensions.
  - Each element in C can by computed by:

$$C_{ik} = \Sigma_j A_{ij} B_{jk}$$

Matrix Multiplication

- Multiplying a matrix by a matrix:

$$\underset{2x3}{A} = \begin{bmatrix} 1 & 2 & 3 \\ 7 & 8 & 9 \end{bmatrix} \qquad \underset{3x2}{B'} = \begin{bmatrix} 5 & 3 \\ 6 & 4 \\ 7 & 5 \end{bmatrix}$$

$$\underset{2x3}{A}\,\underset{3x2}{B'} = \underset{2x2}{C} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}$$

Matching inner dimensions!!

Resulting matrix has outer dimensions!!!

ROW-ECHELON FORM

- A matrix is in row-echelon form if it satisfies the following conditions.
  1. The first nonzero number in each row (reading from left to right) is 1. This is called the leading entry.
  2. The leading entry in each row is to the right of the leading entry in the row immediately above it.
  3. All rows consisting entirely of zeros are at the bottom of the matrix.

Echelon Form

- A matrix is in row **echelon form** when it satisfies the following conditions. The first non-zero element in each row, called the leading entry, is 1. Each leading entry is in a column to the right of the leading entry in the previous row.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 4 \\ 0 & 0 & 0 \end{bmatrix}$$

Elementary Row Operations

1. Add a multiple of one row to another.
2. Multiply a row by a nonzero constant.
3. Interchange two rows.

- Note that performing any of these operations on the augmented matrix of a system does not change its solution.

take note

Key Concept Row Operations

Switch any two rows. $\begin{bmatrix} 2 & -1 & 3 \\ 3 & 2 & 5 \end{bmatrix}$ becomes $\begin{bmatrix} 3 & 2 & 5 \\ 2 & -1 & 3 \end{bmatrix}$

Multiply a row by a constant. $\begin{bmatrix} 3 & 2 & 5 \\ 2 & -1 & 3 \end{bmatrix}$ becomes $\begin{bmatrix} 3 & 2 & 5 \\ 2 \cdot 2 & -1 \cdot 2 & 3 \cdot 2 \end{bmatrix} = \begin{bmatrix} 3 & 2 & 5 \\ 4 & -2 & 6 \end{bmatrix}$

Add one row to another. $\begin{bmatrix} 3 & 2 & 5 \\ 4 & -2 & 6 \end{bmatrix}$ becomes $\begin{bmatrix} 3+4 & 2-2 & 5+6 \\ 4 & -2 & 6 \end{bmatrix} = \begin{bmatrix} 7 & 0 & 11 \\ 4 & -2 & 6 \end{bmatrix}$

Combine any of these steps.

Example

$$\begin{cases} x - y + z = -4 \\ 2x - 3y + 4z = -15 \\ 5x + y - 2z = 12 \end{cases}$$

$$\left.\begin{array}{ccc|c} 1 & -1 & 1 & -4 \\ 2 & -3 & 4 & -15 \\ 5 & 1 & -2 & 12 \end{array}\right. \quad R_2 \to -2r_1 + r_2$$

$$\left.\begin{array}{ccc|c} 1 & -1 & 1 & -4 \\ 0 & -1 & 2 & -7 \\ 5 & 1 & -2 & 12 \end{array}\right. \quad R_3 \to -5r_1 + r_3 \quad \left.\begin{array}{ccc|c} 1 & -1 & 1 & -4 \\ 0 & -1 & 2 & -7 \\ 0 & 6 & -7 & 32 \end{array}\right. \quad R_2 \to -r_2$$

$$\left.\begin{array}{ccc|c} 1 & -1 & 1 & -4 \\ 0 & 1 & -2 & 7 \\ 0 & 6 & -7 & 32 \end{array}\right. \quad R_3 \to -6r_2 + r_3 \quad \left.\begin{array}{ccc|c} 1 & -1 & 1 & -4 \\ 0 & 1 & -2 & 7 \\ 0 & 0 & 5 & -10 \end{array}\right. \quad R_3 \to \tfrac{1}{5}r_3$$

$$\left.\begin{array}{ccc|c} 1 & -1 & 1 & -4 \\ 0 & 1 & -2 & 7 \\ 0 & 0 & 1 & -2 \end{array}\right.$$

row echelon form

$$x - y + z = -4$$
$$y - 2z = 7$$
$$z = -2$$

$$y - 2(-2) = 7$$
$$y = 3$$
$$x - (3) + (-2) = -4$$
$$x = 1$$

Solution:

(x, y, z)

$(1, 3, -2)$

CONSISTENT OR INCONSISTENT?

$$\left.\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 2 \end{array}\right. \qquad \begin{array}{l} x = 1 \\ y = 5 \\ z = 2 \end{array}$$

One solution: Consistent system

$$\left.\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 0 & 2 \end{array}\right. \qquad \begin{array}{l} x = 4 \\ y = 7 \\ 0 = 2 \end{array}$$

No solution: Inconsistent system

$$\left.\begin{array}{ccc|c} 1 & 0 & 5 & 2 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right. \qquad \begin{array}{l} x + 5z = 2 \\ y + 3z = 2 \\ z = \text{any real number} \end{array}$$

Infinite solutions: Consistent system

$$(x = 2 - 5z, y = 2 - 3z, z = \text{any real number})$$

or

$$\{(x, y, z) | x = 2 - 5z, y = 2 - 3z, z = \text{any real number}\}$$

Example for no solution

$$\begin{cases} x + 2y - z = 3 \\ 2x - y + 2z = 6 \\ x - 3y + 3z = 4 \end{cases}$$

Augmented matrix

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 2 & -1 & 2 & 6 \\ 1 & -3 & 3 & 4 \end{array}\right] \quad R_2 \to -2r_1 + r_2$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & -5 & 4 & 0 \\ 1 & -3 & 3 & 4 \end{array}\right] \quad R_3 \to -r_1 + r_3$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & -5 & 4 & 0 \\ 0 & -5 & 4 & 1 \end{array}\right] \quad R_2 \to -\tfrac{1}{5}r_2$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & 1 & -4/5 & 0 \\ 0 & -5 & 4 & 1 \end{array}\right] \quad R_3 \to 5r_2 + r_3$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & 1 & -4/5 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right] \quad 0 = 1$$

No solution: inconsistent system

Example for Infinitely many solutions

$$\begin{cases} x + 2y - 3z - 4w = 10 \\ x + 3y - 3z - 4w = 15 \\ 2x + 2y - 6z - 8w = 10 \end{cases}$$

Thus, the complete solution is:

$$x = 3s + 4t$$
$$y = 5$$
$$z = s$$
$$w = t$$

where s and t are any real numbers.

$$\begin{bmatrix} 1 & 2 & -3 & -4 & 10 \\ 1 & 3 & -3 & -4 & 15 \\ 2 & 3 & -6 & -8 & 10 \end{bmatrix} \xrightarrow[R_3 - 2R_1 \to R_3]{R_2 - R_1 \to R_2} \begin{bmatrix} 1 & 2 & -3 & -4 & 10 \\ 0 & 1 & 0 & 0 & 5 \\ 0 & -2 & 0 & 0 & -10 \end{bmatrix}$$

$$\xrightarrow{R_3 + 2R_2 \to R_3} \begin{bmatrix} 1 & 2 & -3 & -4 & 10 \\ 0 & 1 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_1 - 2R_2 \to R_1} \begin{bmatrix} 1 & 0 & -3 & -4 & 0 \\ 0 & 1 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Reducing Square Matrices

- Determinant:
  - The determinant of a matrix is a scalar representation of matrix; considered the “volume” of the matrix or in the case of a VCV matrix it is the generalized variance.
  - Only square matrices have determinants.
  - Determinants are also useful because they tell us whether or not a matrix can be inverted (next).
  - Not all square matrices can be inverted (must be full rank, non-singular matrix)

Reducing Square Matrices

- Determinant:

$$\underset{1x1}{C} = [4] \quad |C| = 4$$

$$\underset{2x2}{C} = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} \quad |C| = (a_1 * b_2) - (b_1 * a_2)$$

$$\underset{2x2}{C} = \begin{bmatrix} 3 & 2 \\ 5 & 1 \end{bmatrix} \quad |C| = (3*1) - (2*5) = 3 - 10 = -7$$

Reducing Square Matrices

- Determinant:

$$\underset{3x3}{C} = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \qquad |C| = a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$$

$$\underset{3x3}{C} = \begin{bmatrix} 2 & -2 & 0 \\ -1 & 5 & 1 \\ 3 & 4 & 5 \end{bmatrix}$$

$$|C| = [2(5*5-1*4)] - [-2(-1*5-1*3)] + [0(-1*5-5*3]$$

$$|C| = [2(25-5)] - [-2(-5-3)] + [0(-5-15]$$

$$|C| = [40] - [16] + [0] = 24$$

Matrix Inverse

- Matrix Inverse: Needed to perform the “division” of 2 square matrices
  - In scalar terms A/B is the same as A * 1/B
  - When we want to divide matrix A by matrix B we simply multiply by A by the inverse of B
  - An inverse matrix is defined as

$$\underset{nxn}{A}^{-1} \xrightarrow{Defined} \underset{nxn}{A}\,\underset{nxn}{A}^{-1} = \underset{nxn}{I} \; AND \; \underset{nxn}{A}^{-1}\,\underset{nxn}{A} = \underset{nxn}{I}$$

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Matrix Inverse

- Matrix Inverse:
  - For a 2x2 matrix the inverse is relatively simple

$$\underset{2x2}{C} = \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \end{bmatrix} \qquad \underset{2x2}{C^{-1}} = \frac{1}{|C|}\begin{bmatrix} a_1 & -b_1 \\ -a_2 & b_2 \end{bmatrix}$$

$$\underset{2x2}{C} = \begin{bmatrix} 3 & 2 \\ 5 & 1 \end{bmatrix} \qquad |C| = -7$$

$$\underset{2x2}{C^{-1}} = \frac{1}{-7}\begin{bmatrix} 3 & -2 \\ -5 & 1 \end{bmatrix} = \begin{bmatrix} -3/7 & 2/7 \\ 5/7 & -1/7 \end{bmatrix}$$

Singular Matrix

- Singular Matrix: A matrix is considered singular if the determinant of the matrix is zero
  - The matrix cannot be inverted
  - Usually caused by linear dependencies between vectors
  - When a matrix is not full rank

$$\underset{2x2}{A} = \begin{bmatrix} 2 & 6 \\ 1 & 3 \end{bmatrix} \qquad |A| = (2*3) - (1*6) = 0$$