

Search ...



SKIP LIST

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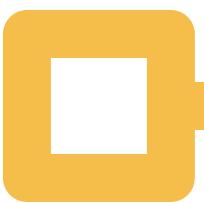
INTRODUCTION





WHAT IS A

SKIP



LIST

1

420

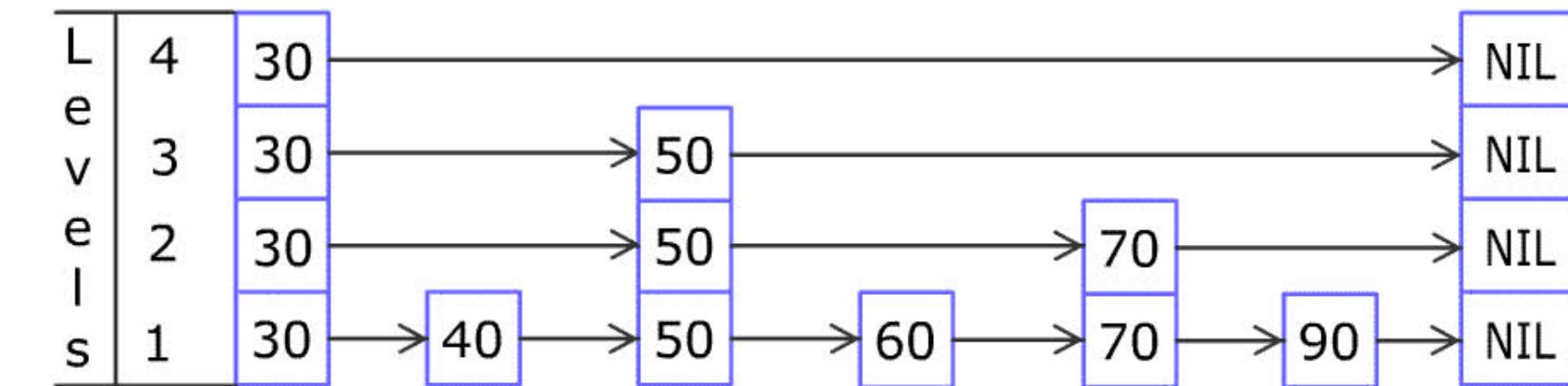


WHAT IS A SKIP LIST ?

DESCRIPTION

“Skip lists are a probabilistic data structure that seem likely to supplant balanced trees as the implementation method of choice for many applications. Skip list algorithms have the same asymptotic expected time bounds as balanced trees and are simpler, and faster.”

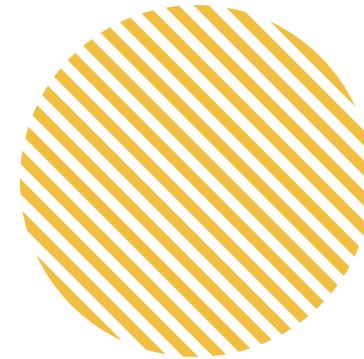
— William Pugh, Concurrent Maintenance of Skip Lists (1989)





ADDITIONAL INFORMATION

In computer science, a skip list is a probabilistic data structure that allows $O(\log n)$ search complexity as well as $O(\log n)$ insertion complexity within an ordered sequence of n elements. Thus, it can get the best features of a sorted array (for searching) while maintaining a linked list-like structure that allows insertion, which is not possible with a static array.



Skip lists are used for efficient statistical computations of running medians (also known as moving medians).

Skip lists are also used in distributed applications (where the nodes represent physical computers, and pointers represent network connections) and for implementing highly scalable concurrent priority queues with less lock contention, or even without locking, as well as lock-free concurrent dictionaries.



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WHO IS WILLIAM PUGH





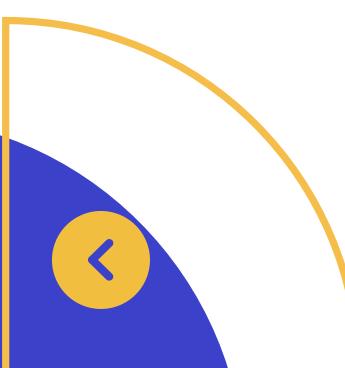
“ WILLIAM
WORTHINGTON
PUGH JR.

An American Computer Scientist who invented the Skip List in 1989 at 29 years of age.



Also known as

“BILL” PUGH ”



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TIME AND SPACE

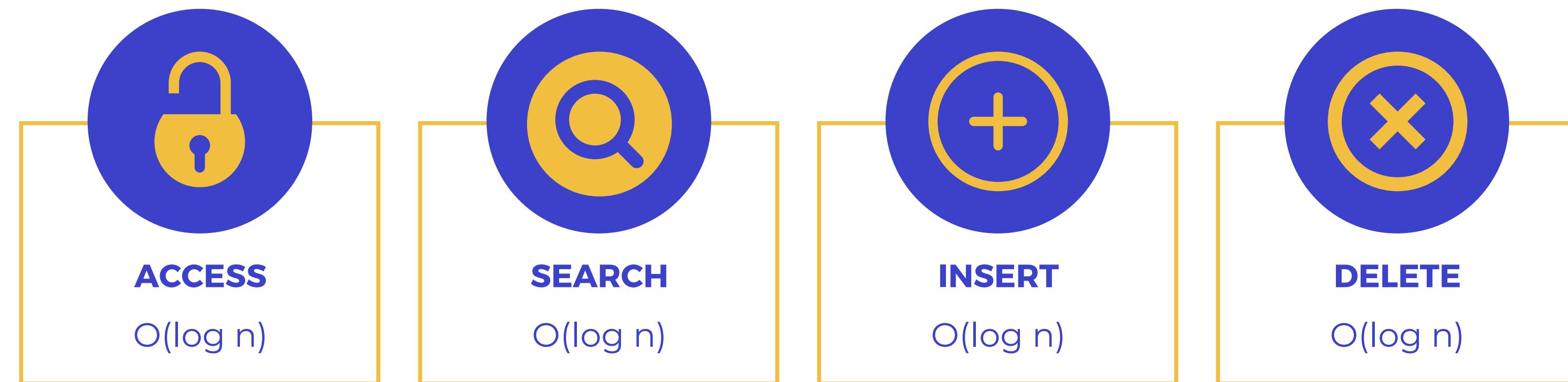
COMPLEXITY





TIME COMPLEXITY

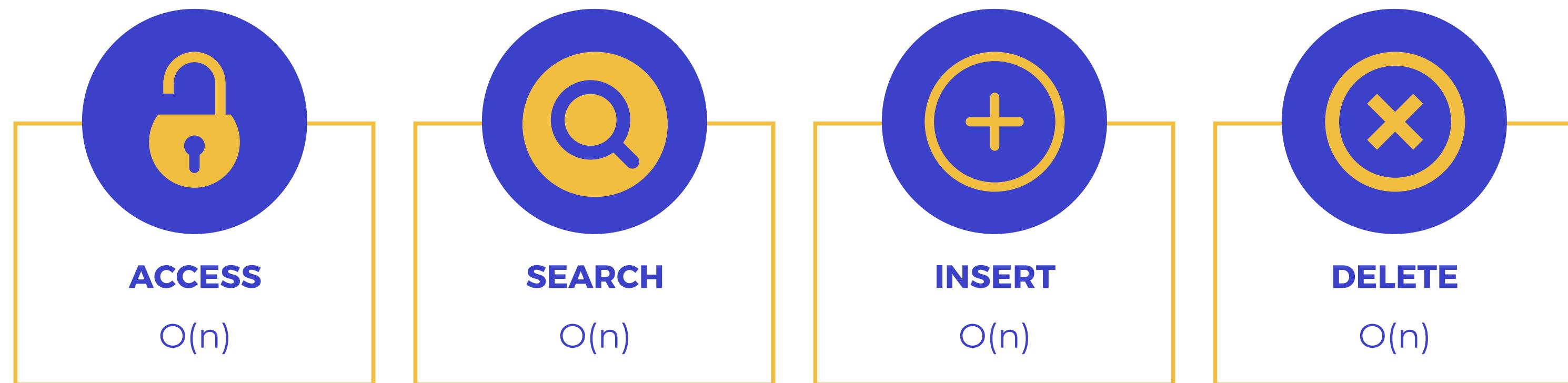
AVERAGE





TIME COMPLEXITY

WORST





SPACE COMPLEXITY

WORST

$$O(n \log n)$$




VARIATIONS OF A SKIP LIST





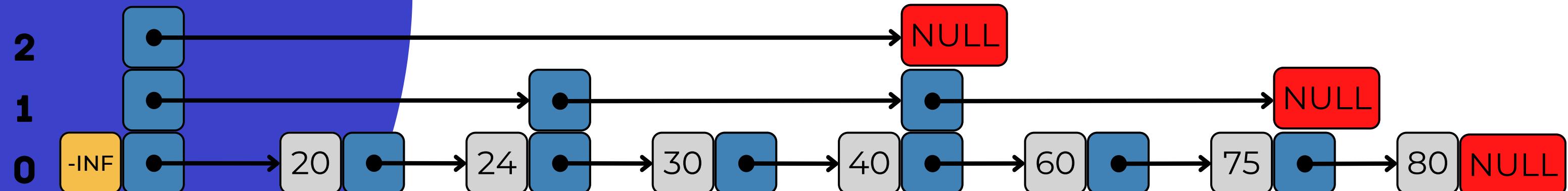
01

PERFECT SKIP LIST

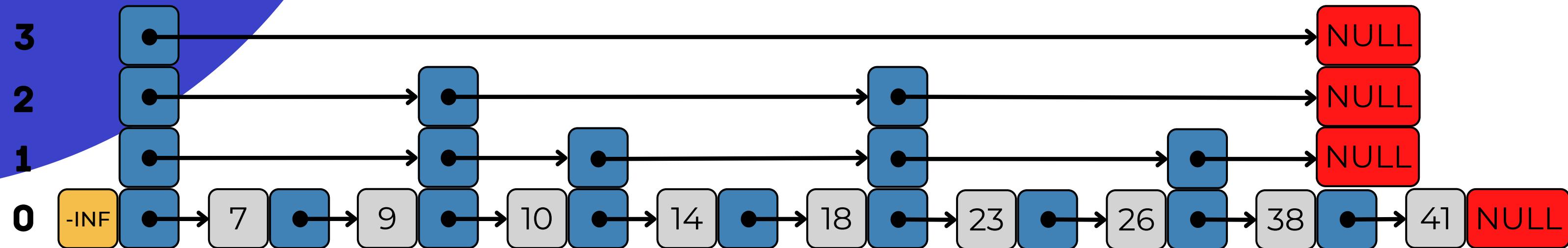
02

RANDOMIZED SKIP LIST

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PERFECT SKIP LIST



RANDOMIZED SKIP LIST



PERFECT SKIP LIST

- Keys are in sorted order
- $O(\log n)$ levels
- Header node and element pointing to **NULL** are in every level
- Each higher level contains 1 / 2 the elements of the level below it
- Too structured to support **efficient** updates such as deletion and insertion

RANDOMIZED SKIP LIST

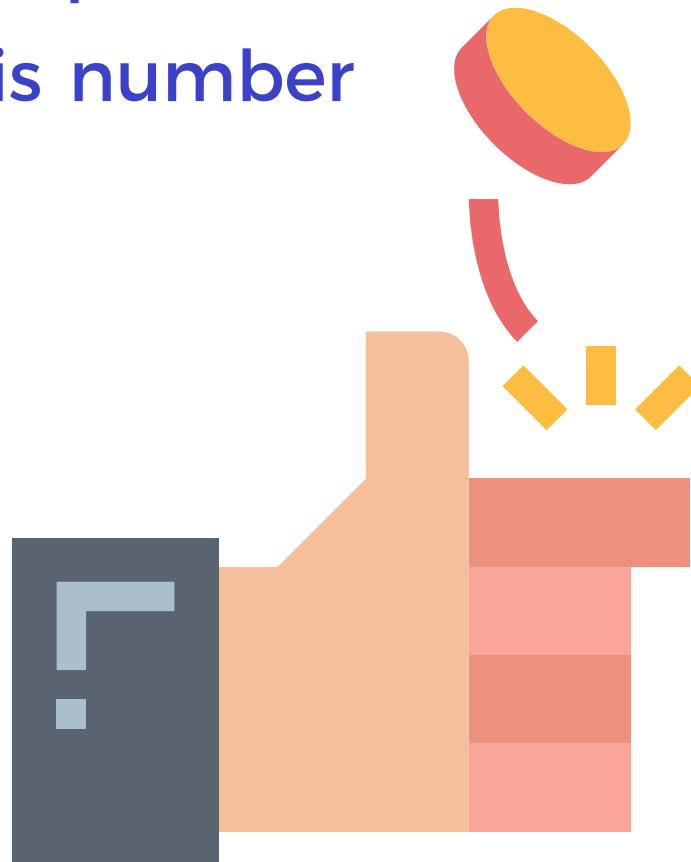
- Keys are in sorted order
- $O(\log n)$ levels
- Header node and element pointing to **NULL** are in every level
- Allows for some **imbalance** like in AVL trees
- randomized to support **efficient** updates (as it should be)



WHY IS IT RANDOMIZED?



The level of an element is chosen in essence by flipping a coin. For example, we could think of flipping a coin until it comes up tails. We count the number of times the coin came up heads before coming up tails and add 1 to this number. This number represents the level of the element.





DATA STRUCTURE OF SKIP LIST



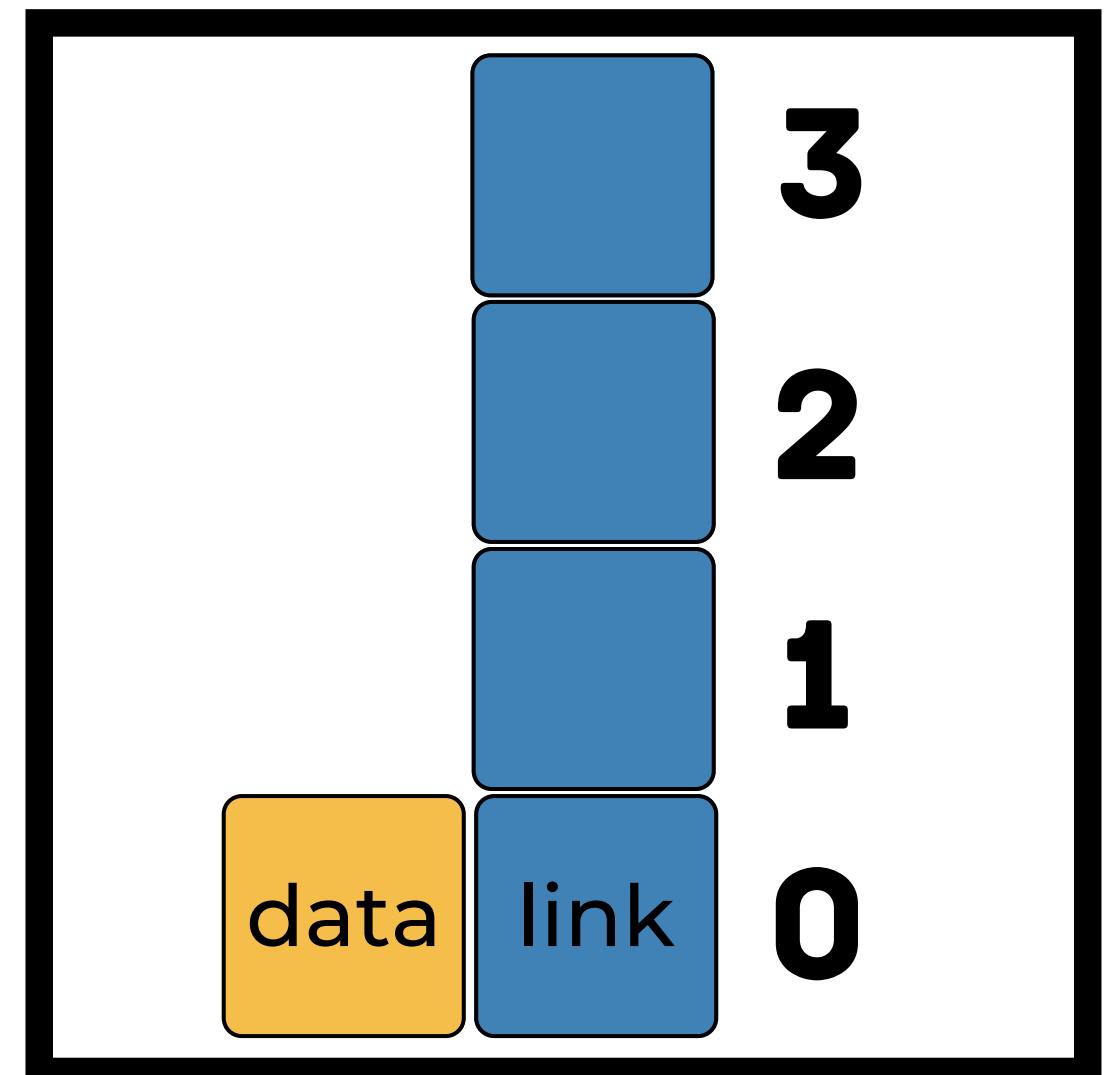
DATA STRUCTURE

```
#define MAXLEVEL 3
```

```
+  
typedef struct node{  
    struct node **link;  
    int data;  
}*Node;
```

```
typedef struct {  
    Node head;  
    int level;  
}SkipList;
```

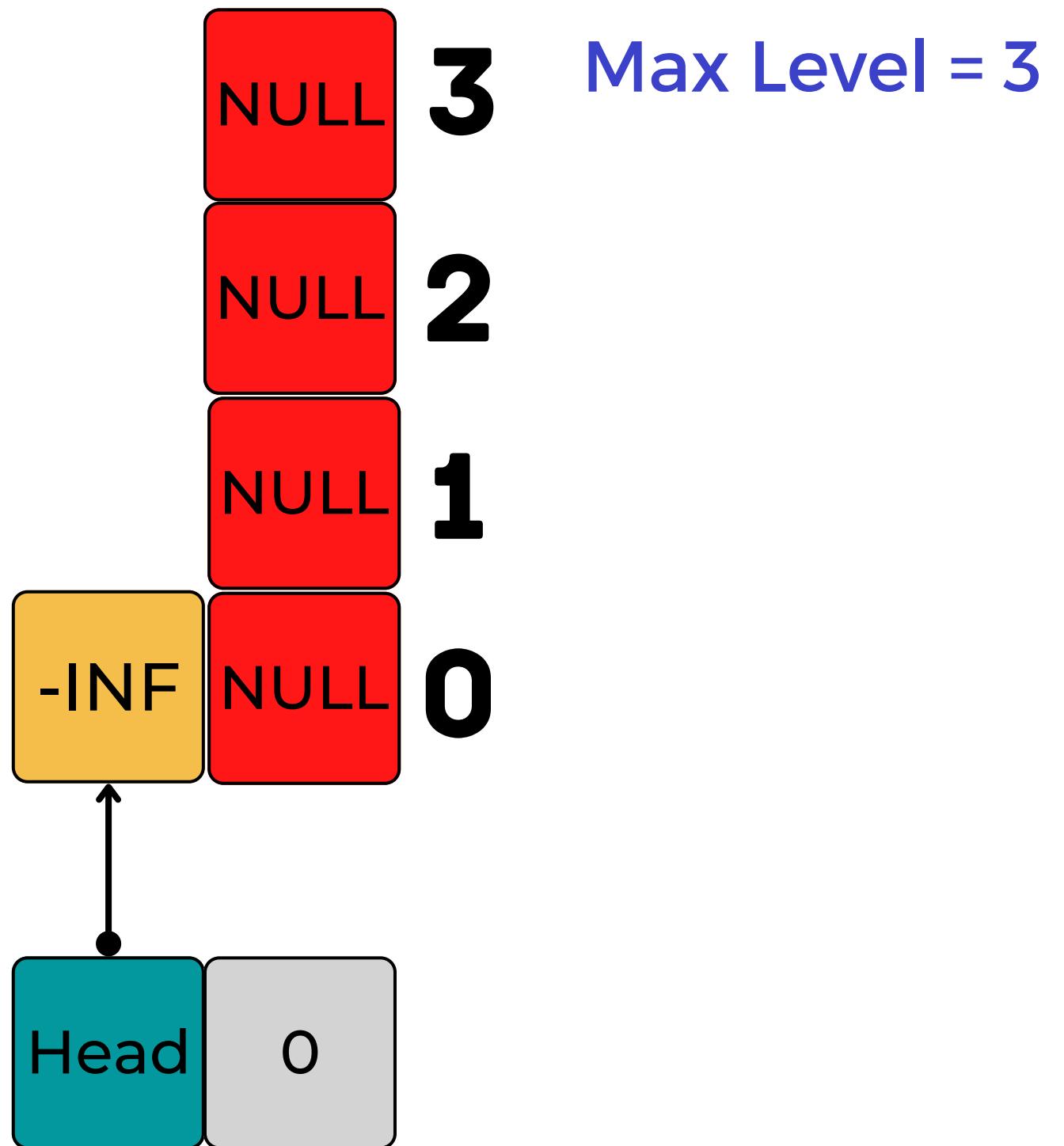
node



SkipList

INITIALIZATION

- + Set Level to 0;
- + Allocate memory to Head;
- + Allocate memory to Link array with the size based on max level;
- + Set data to -INF;
- + Set all value of Link to NULL;



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SIMULATION OF CODE

Randomized SkipList

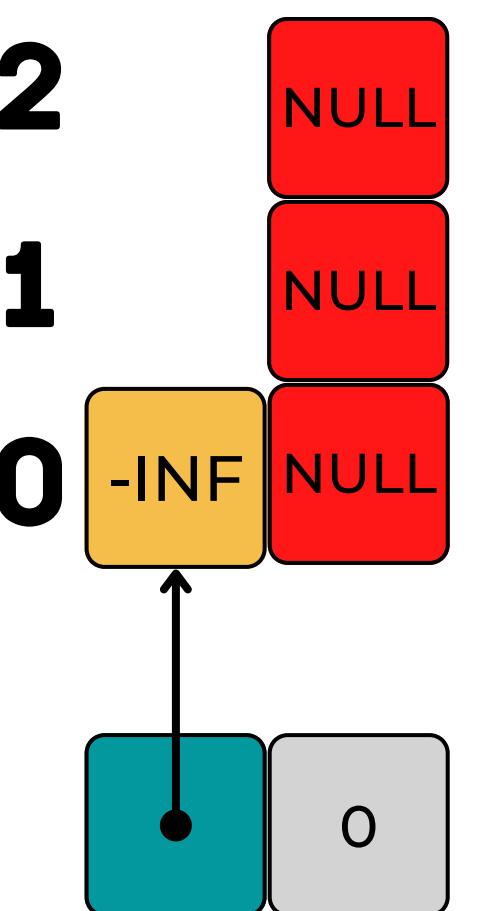




INSERTION

Max Level = 2

Level = 0



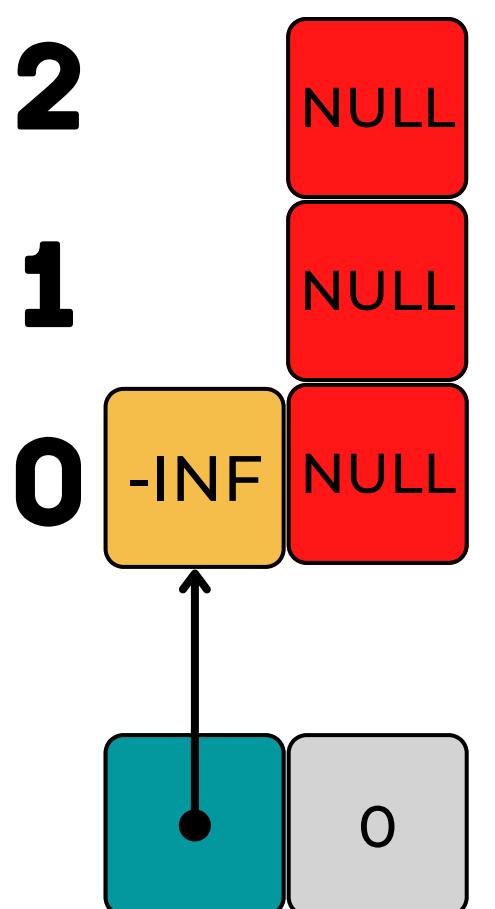


INSERTION

Max Level = 2

Level = 0

Update =



INSERT(30)

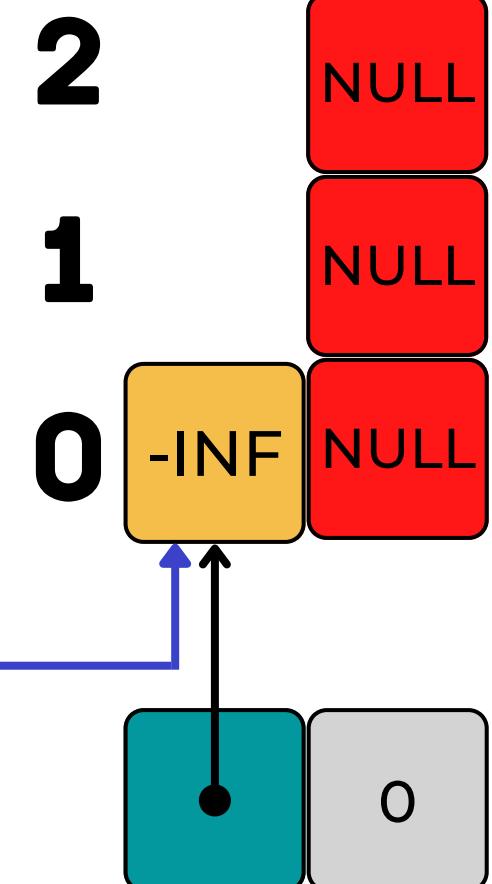


INSERTION

Max Level = 2

Level = 0

Update =



since link is NULL? Move down a level

INSERT(30)

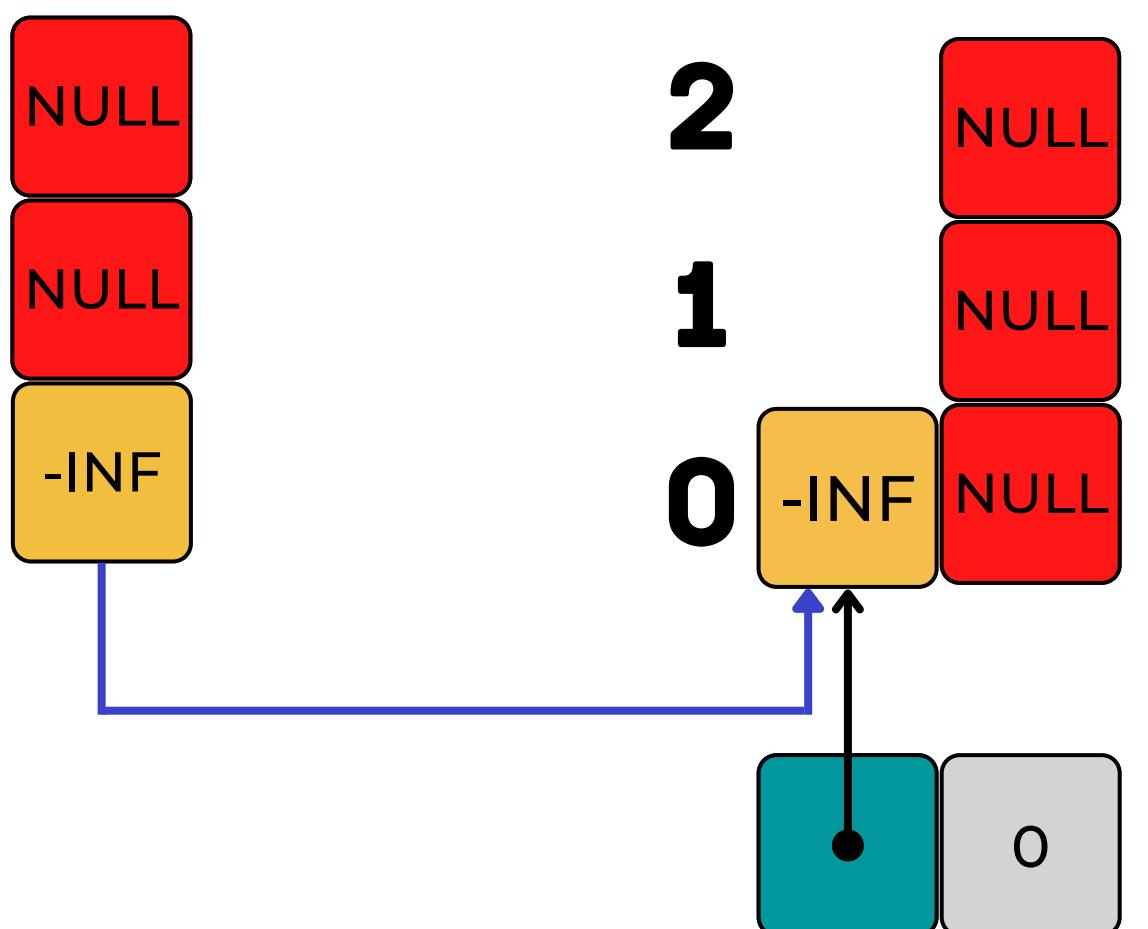


INSERTION

Max Level = 2

Level = -1

Update =

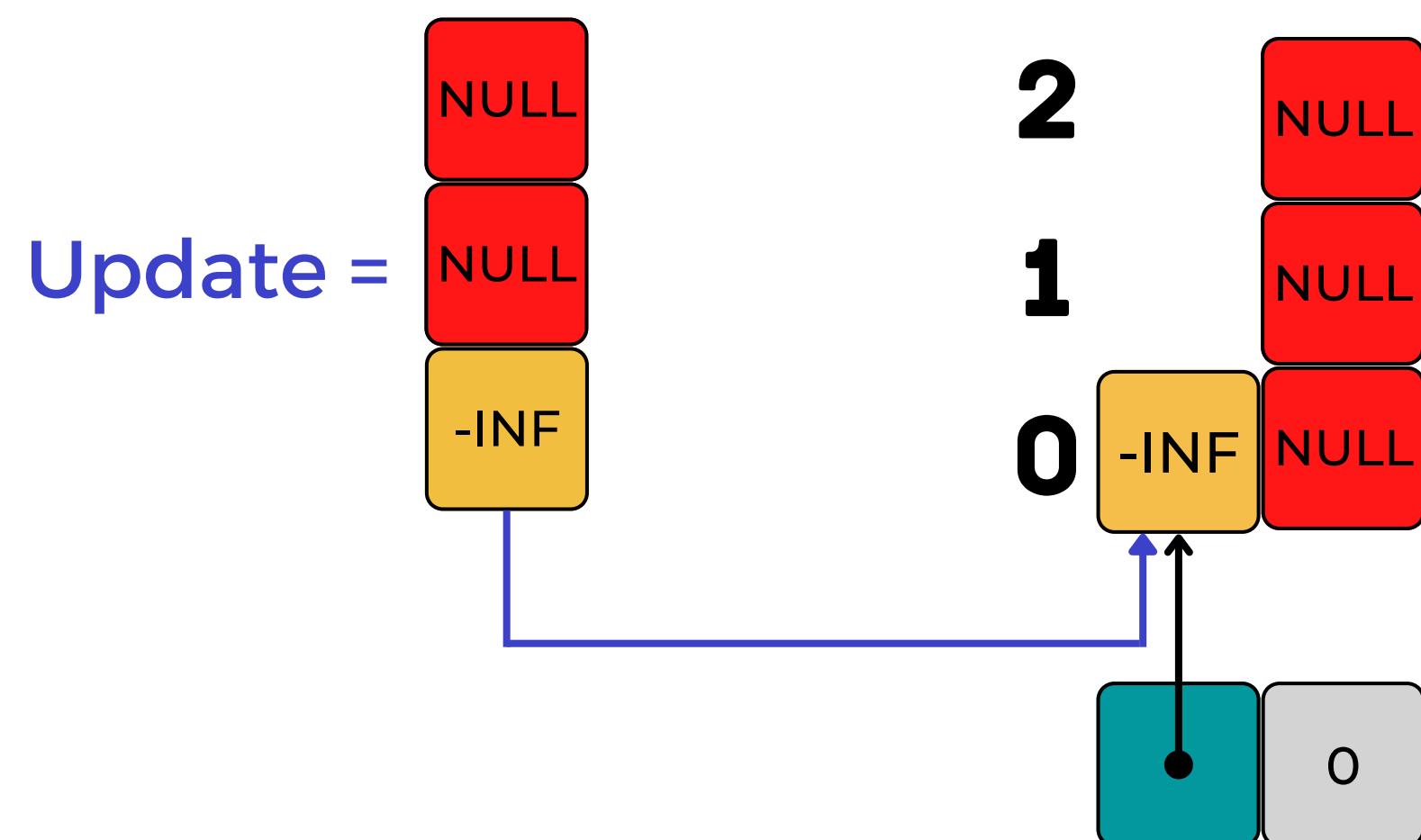


INSERT(30)



INSERTION

Max Level = 2



do coin flip;
while coin is heads;
increment new level;
stop when coin is tails
return new level;

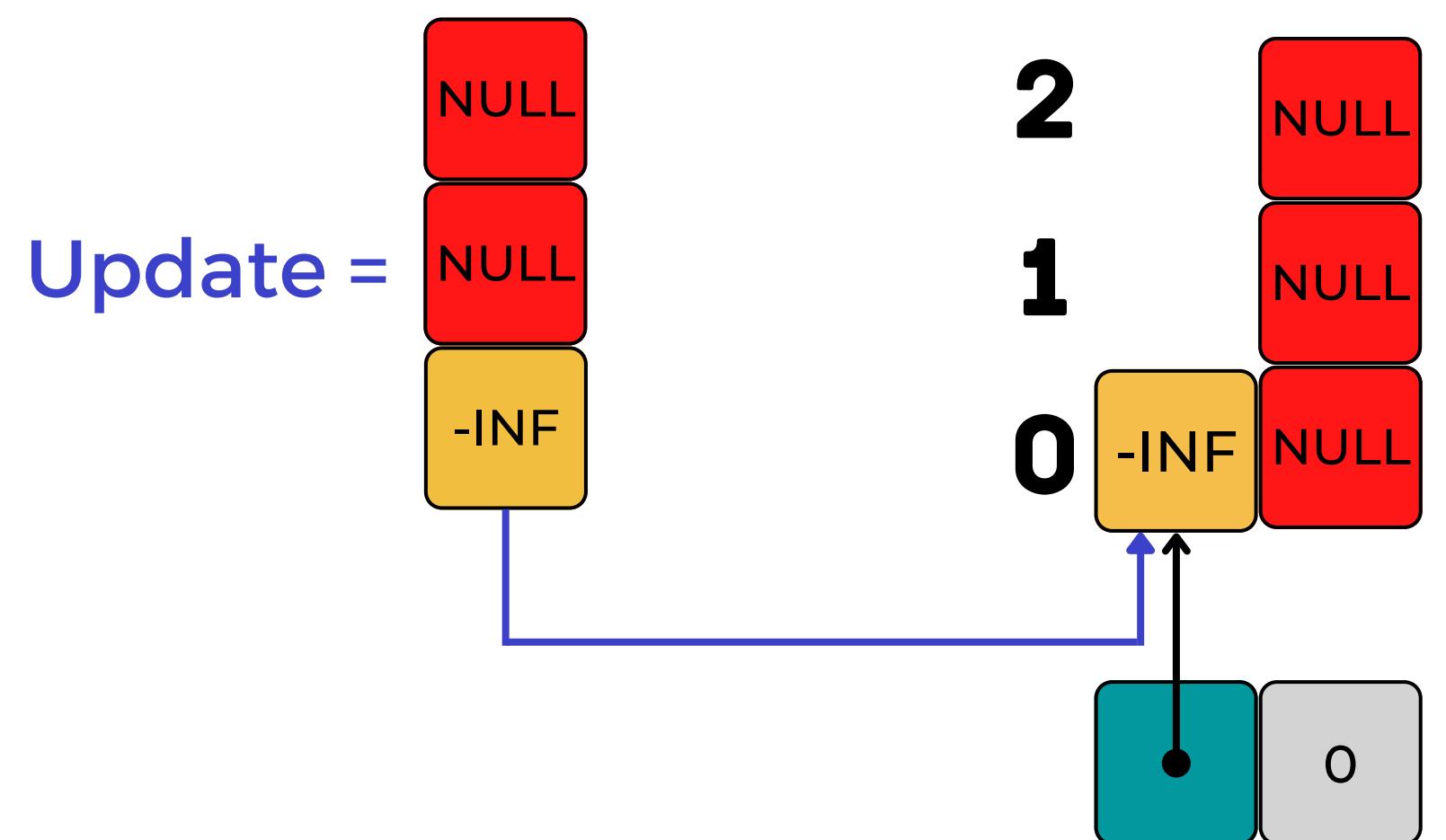
INSERT(30)





INSERTION

New Level = 2

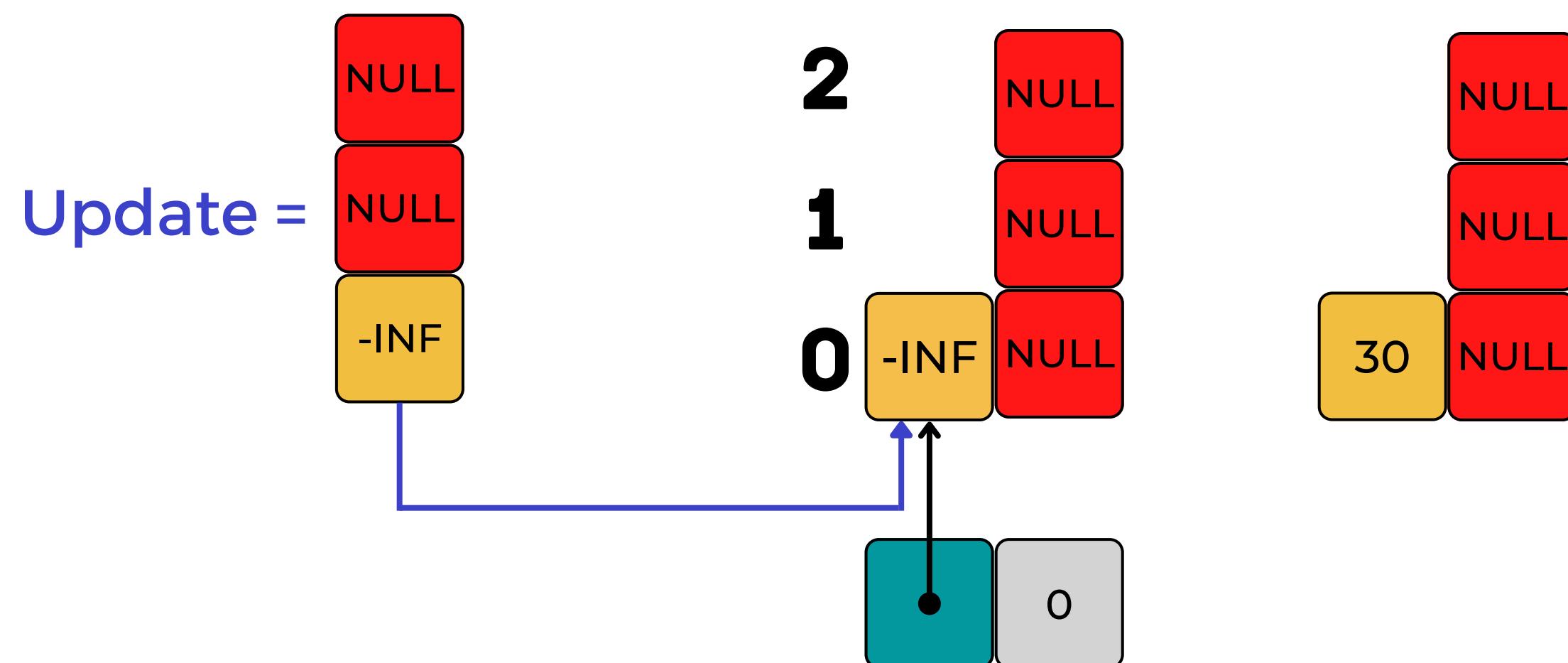


INSERT(30)



INSERTION

New Level = 2

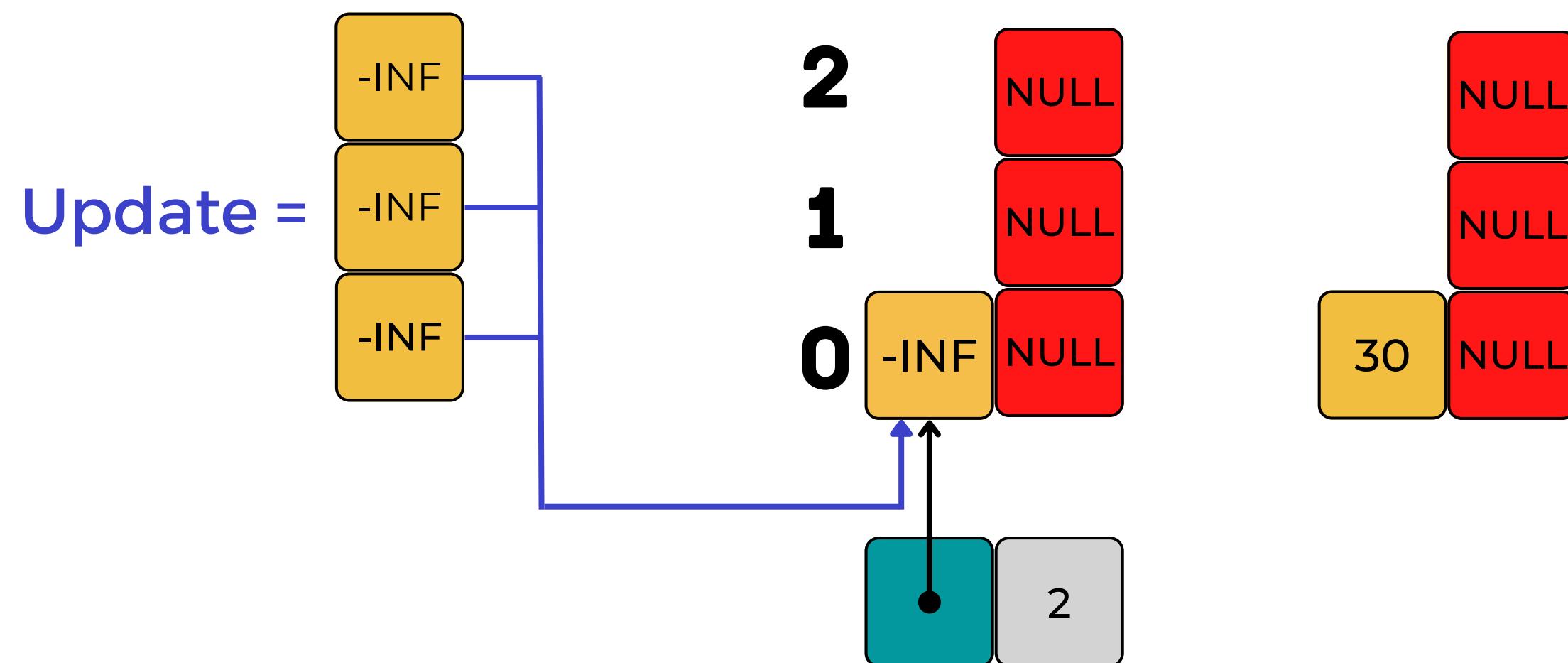


INSERT(30)



INSERTION

New Level = 2

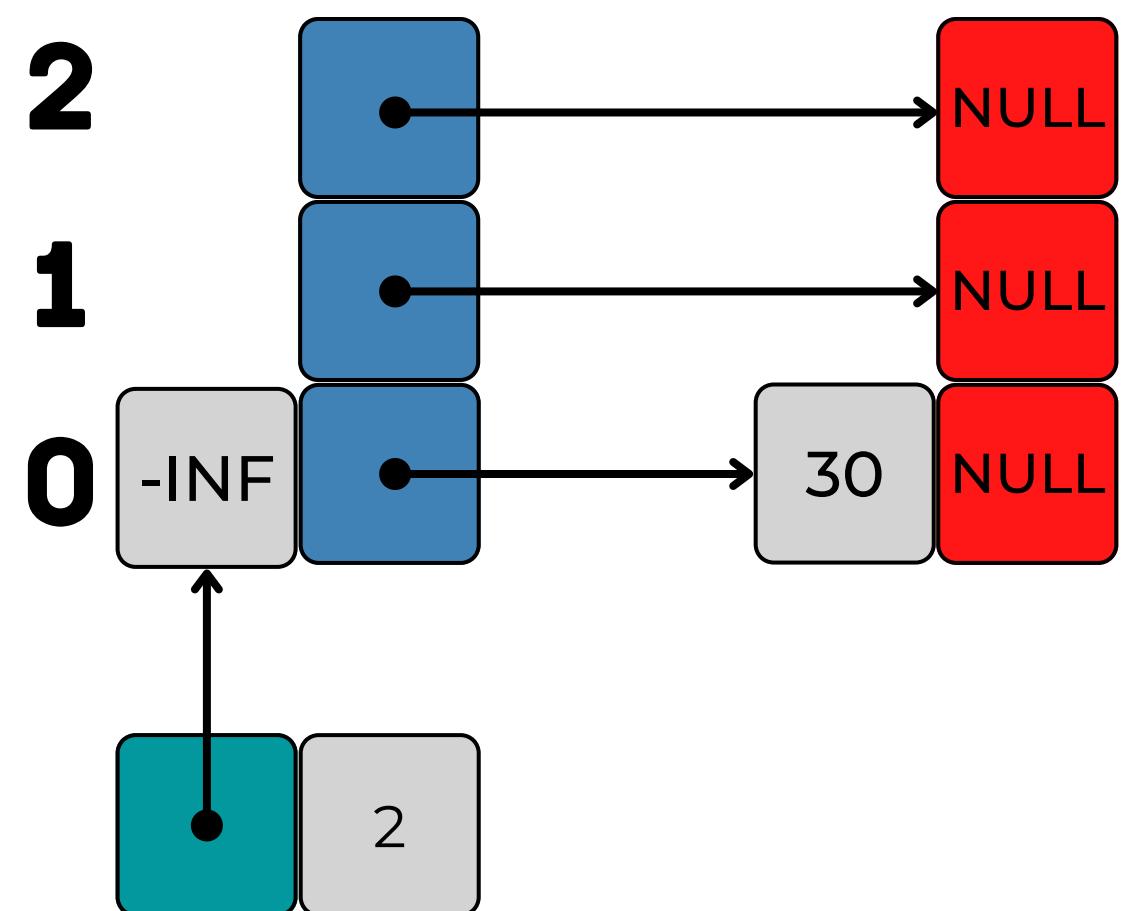


INSERT(30)



INSERTION

New Level = 0



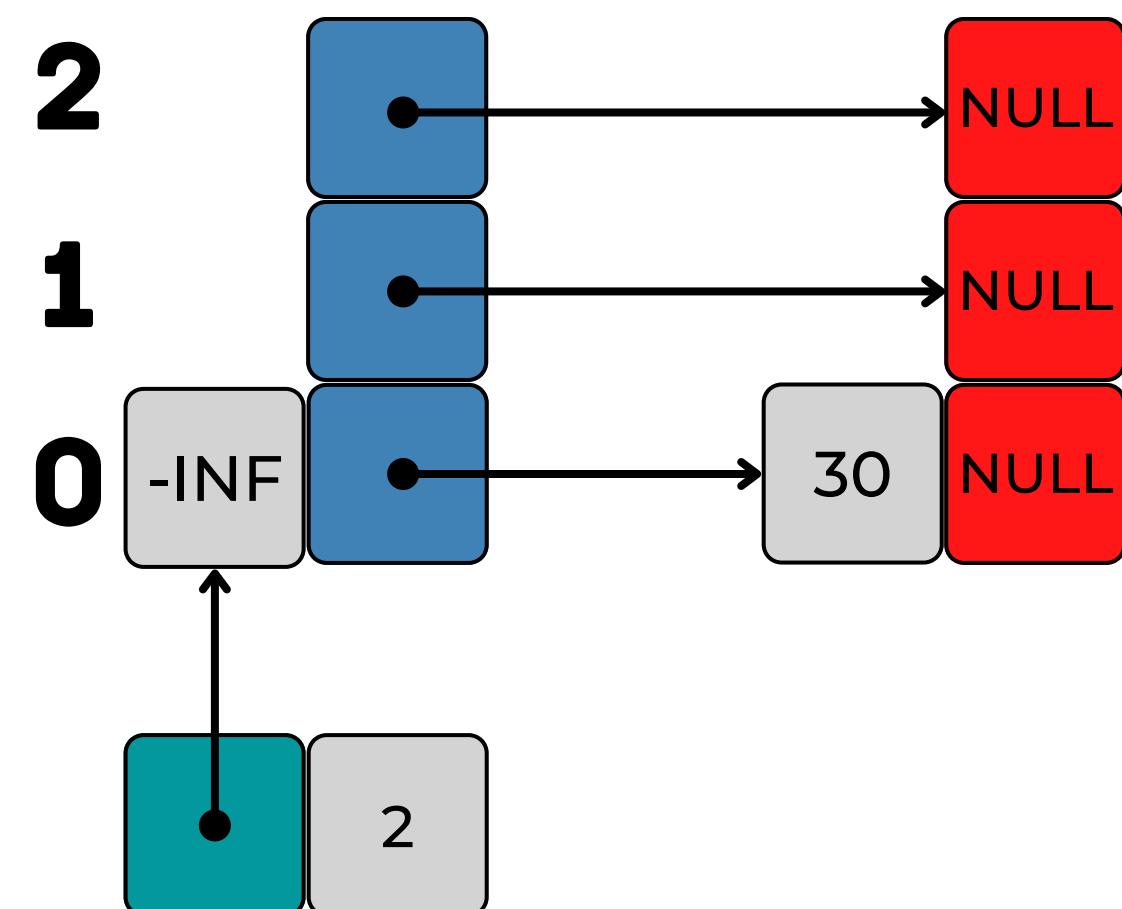
INSERT(30)



INSERTION

Max Level = 2

Level = 0



INSERT(75)

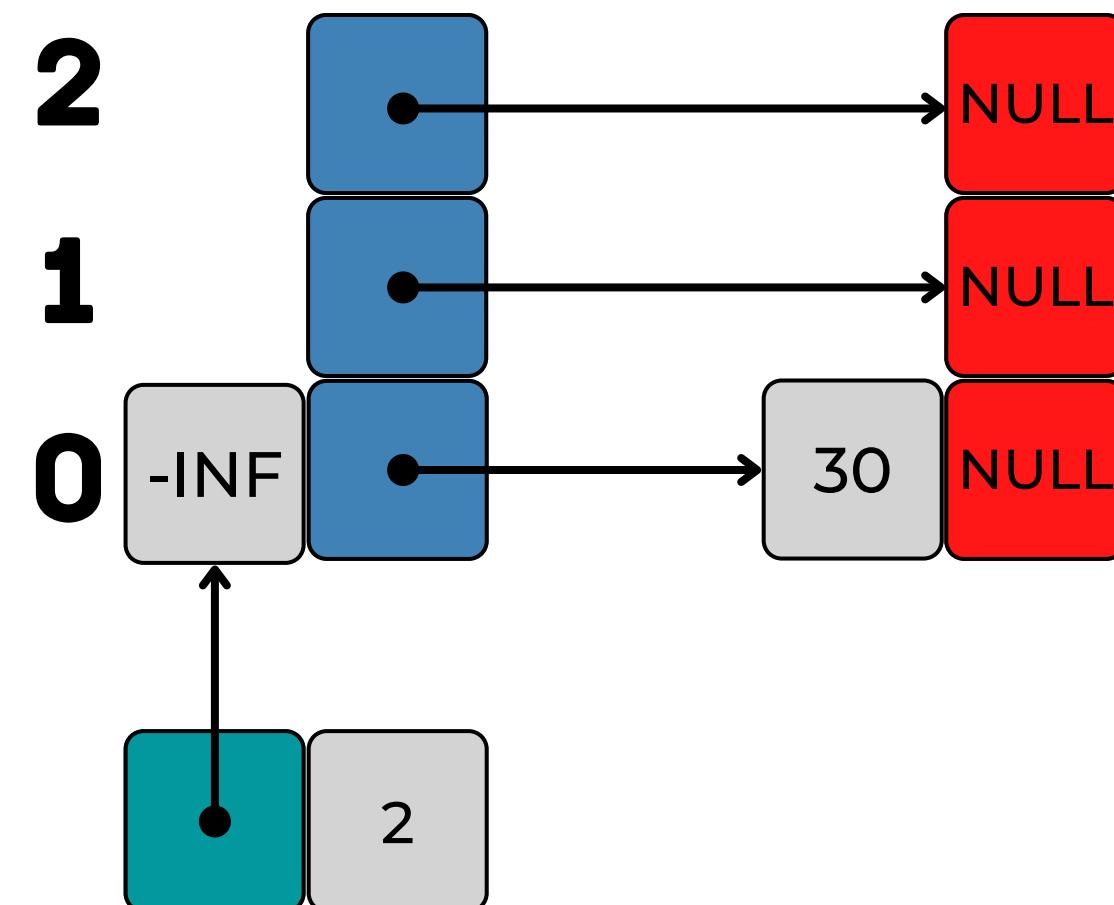


INSERTION

Max Level = 2

Level = 0

Update =



INSERT(75)



INSERTION

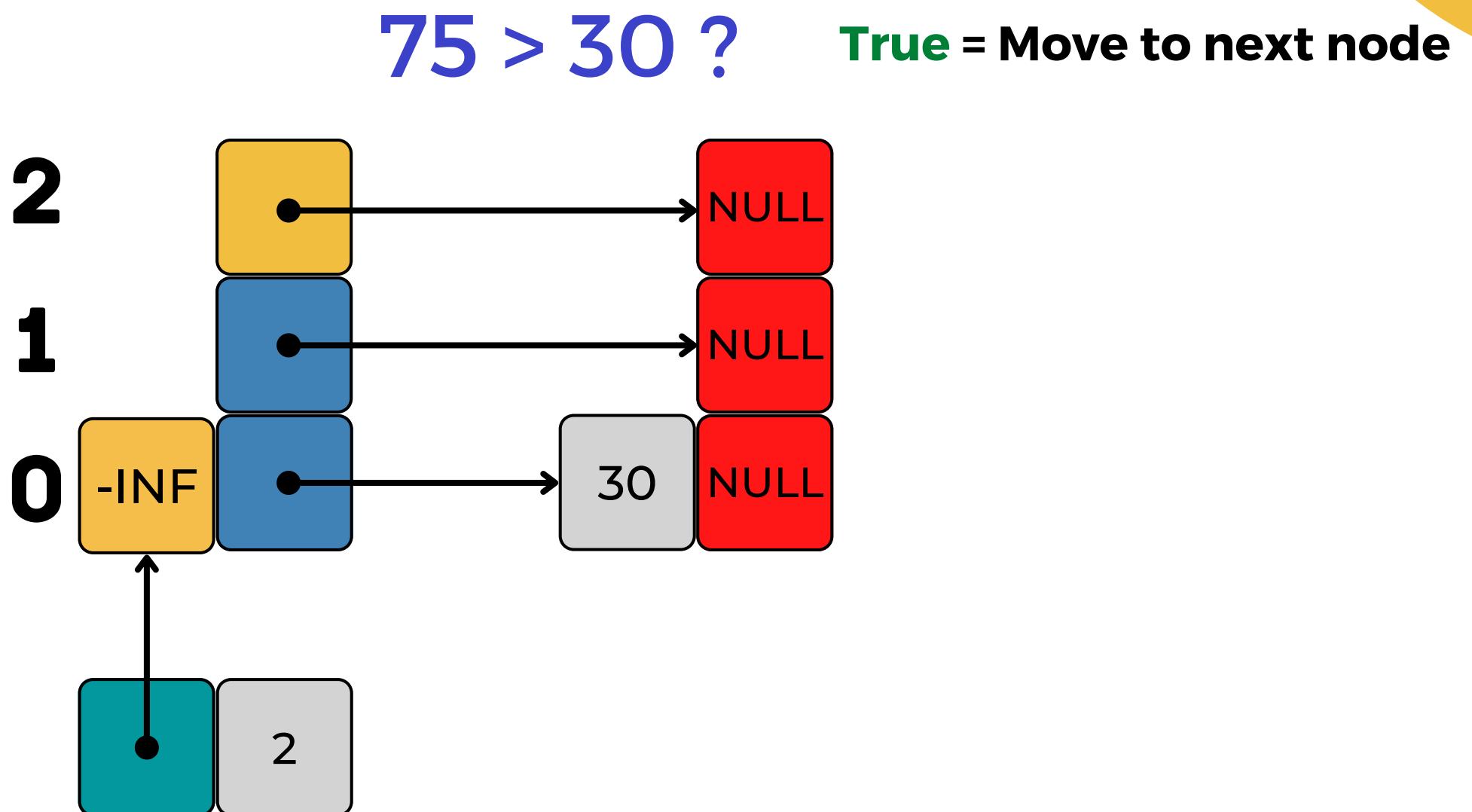
Max Level = 2

Level = 2

Update =



INSERT(75)



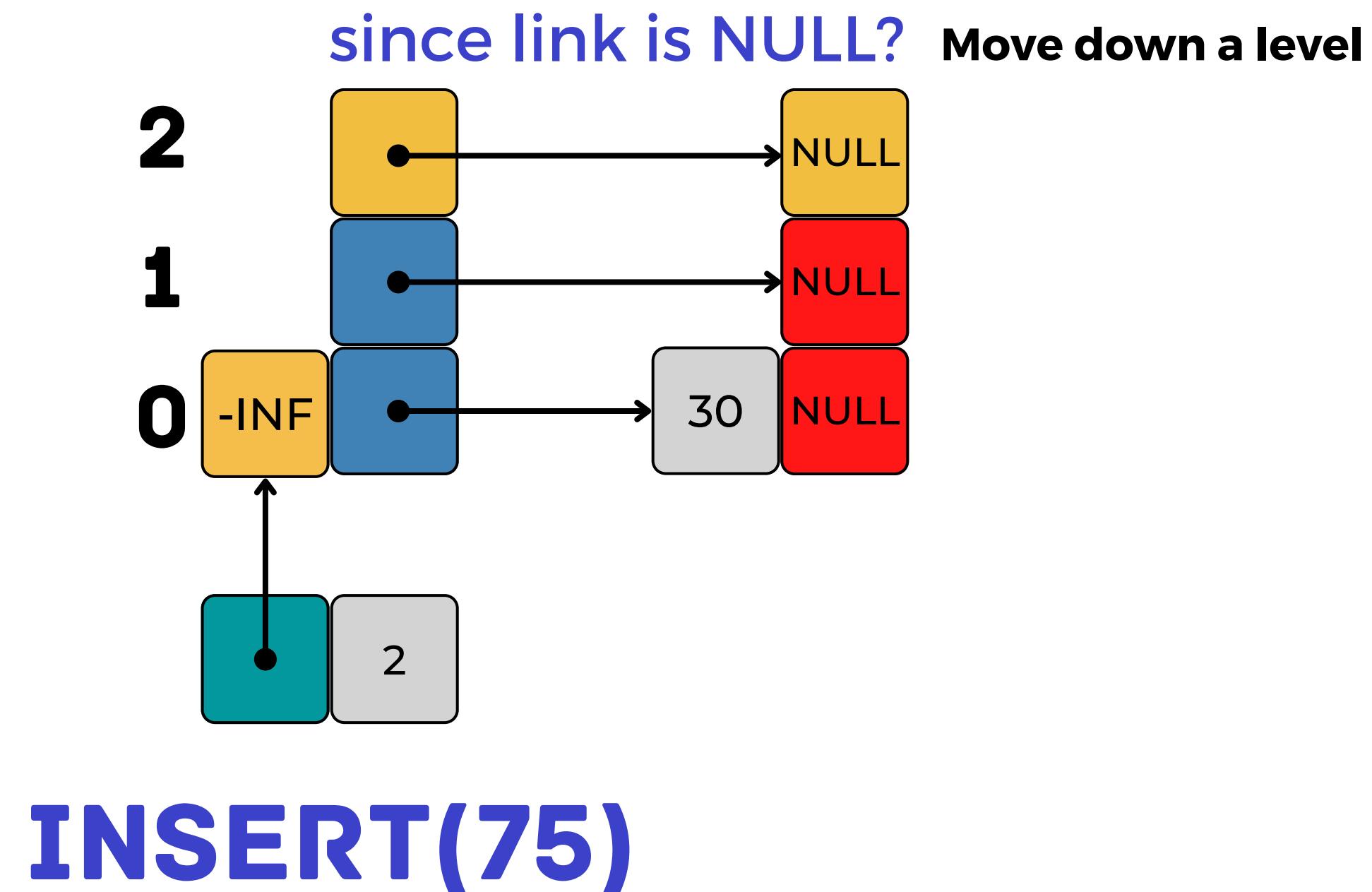


INSERTION

Max Level = 2

Level = 2

Update =



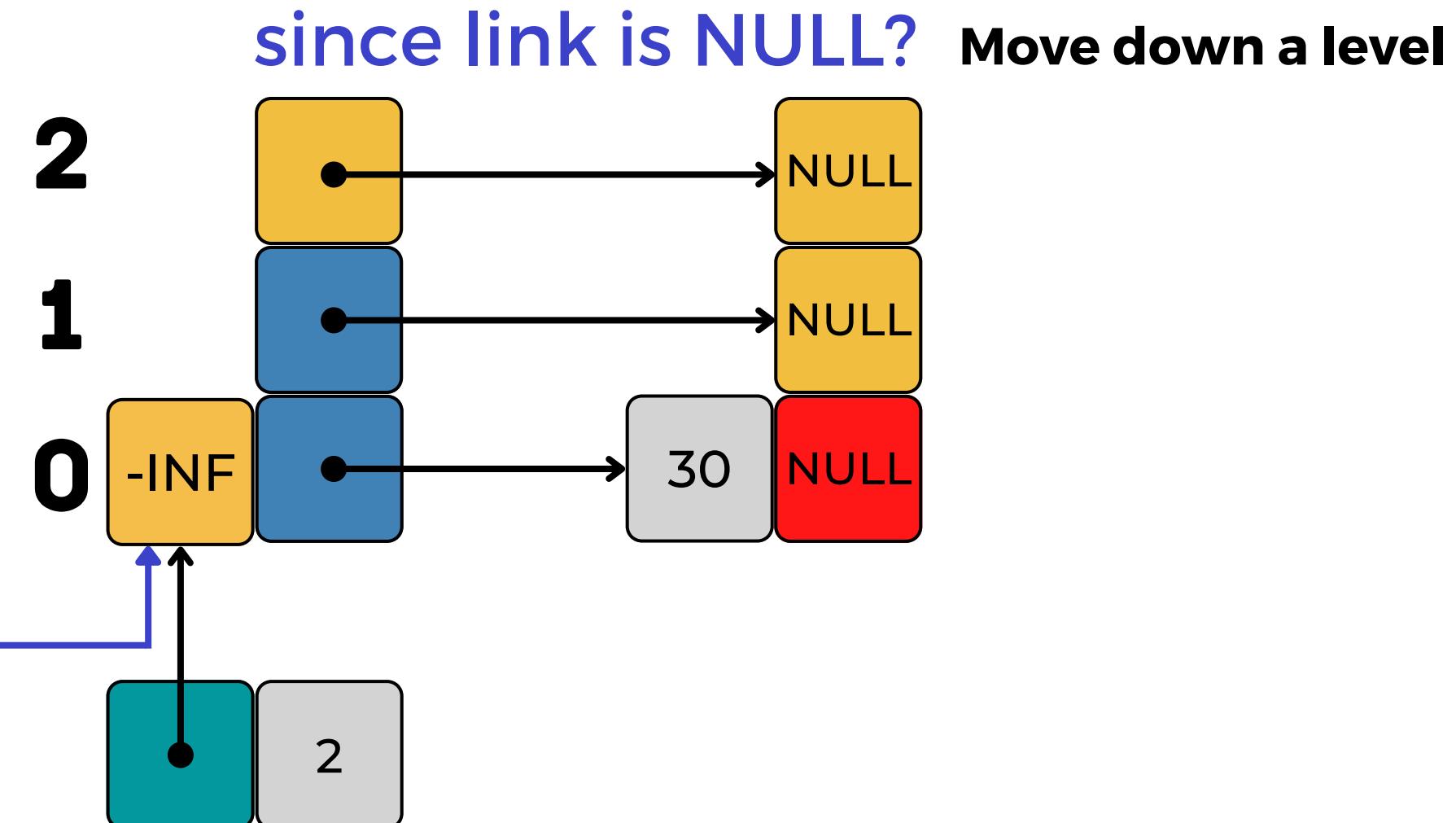
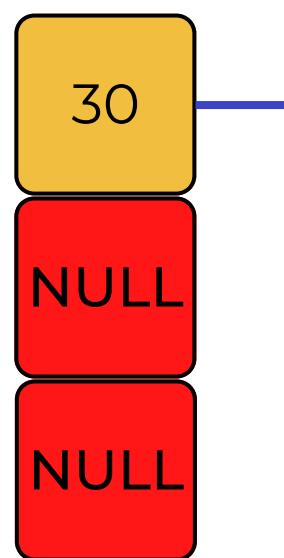


INSERTION

Max Level = 2

Level = 1

Update =



INSERT(75)

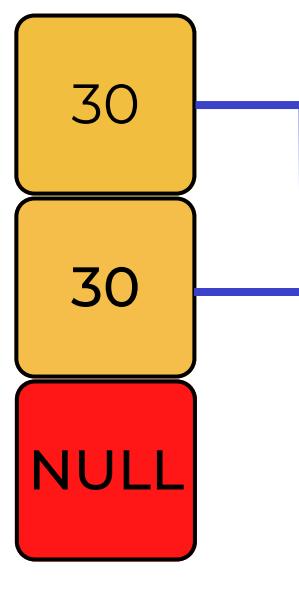


INSERTION

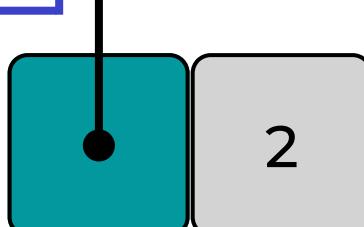
Max Level = 2

Level = 0

Update =



2
1
0



since link is NULL?

Move down a level

INSERT(75)

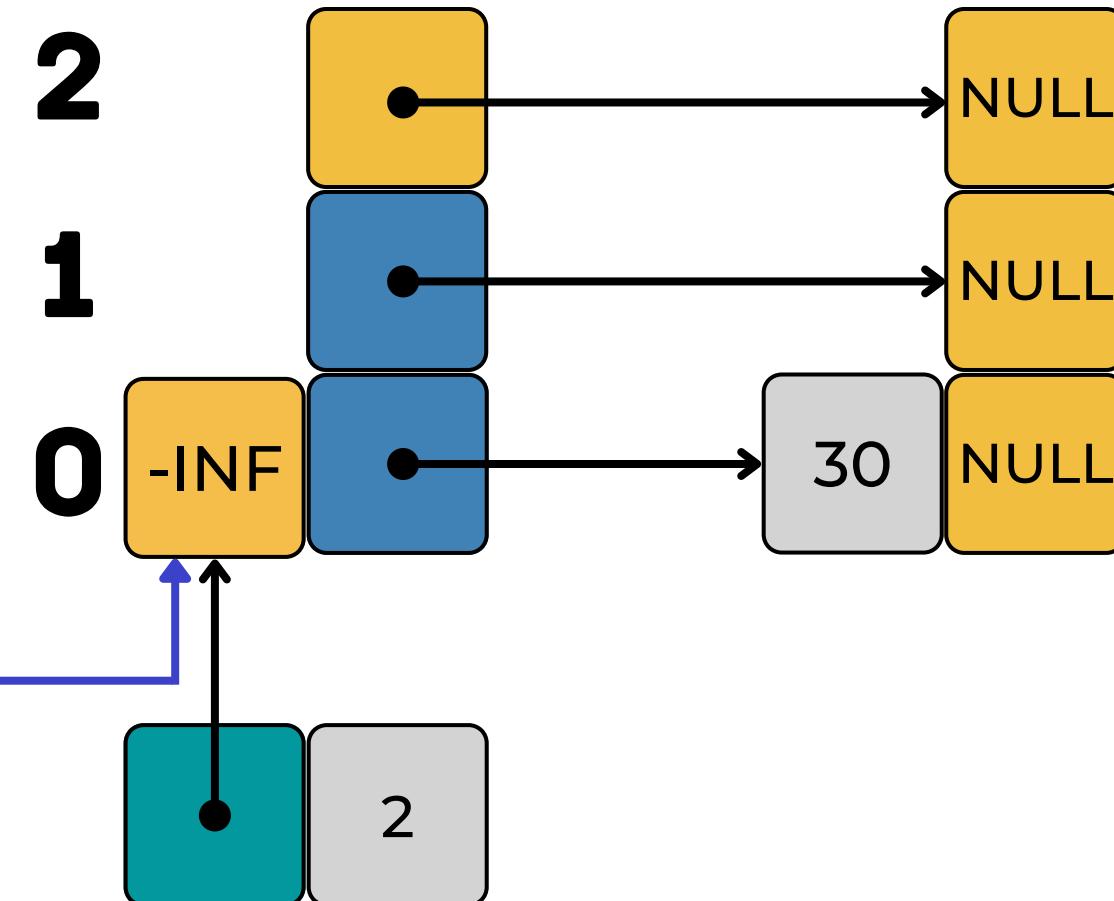
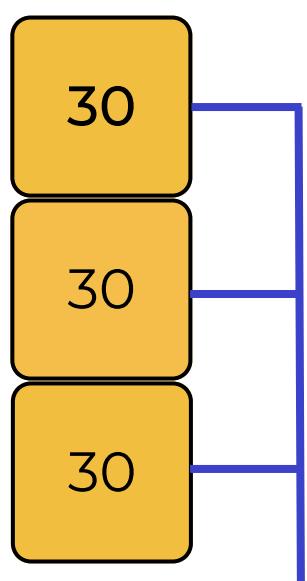


INSERTION

Max Level = 2

Level = -1

Update =



INSERT(75)

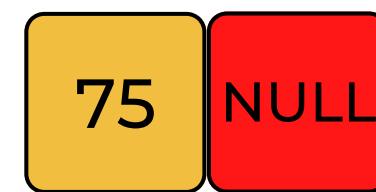
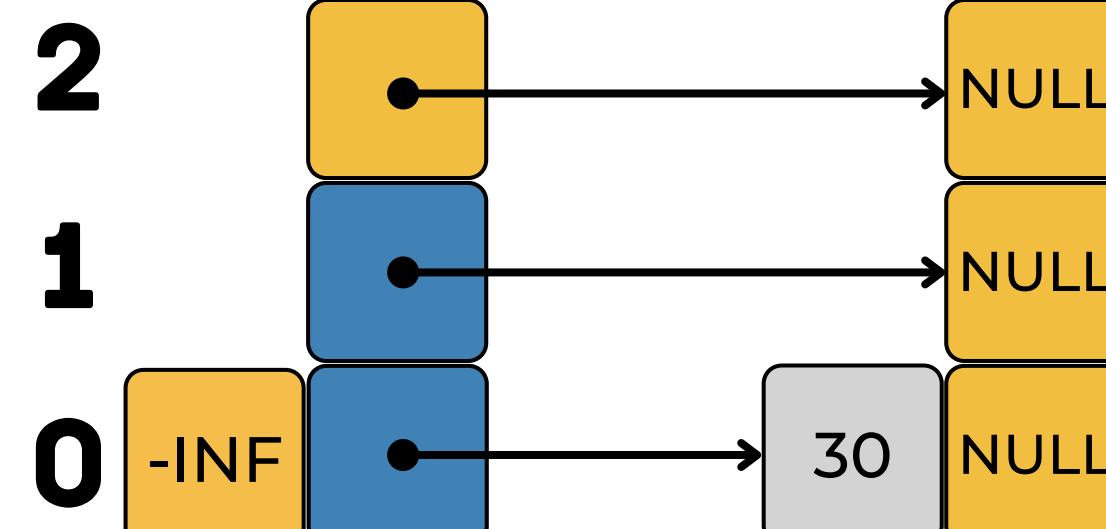
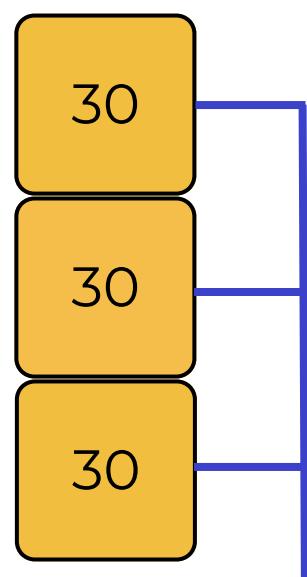




INSERTION

New Level = 0

Update =

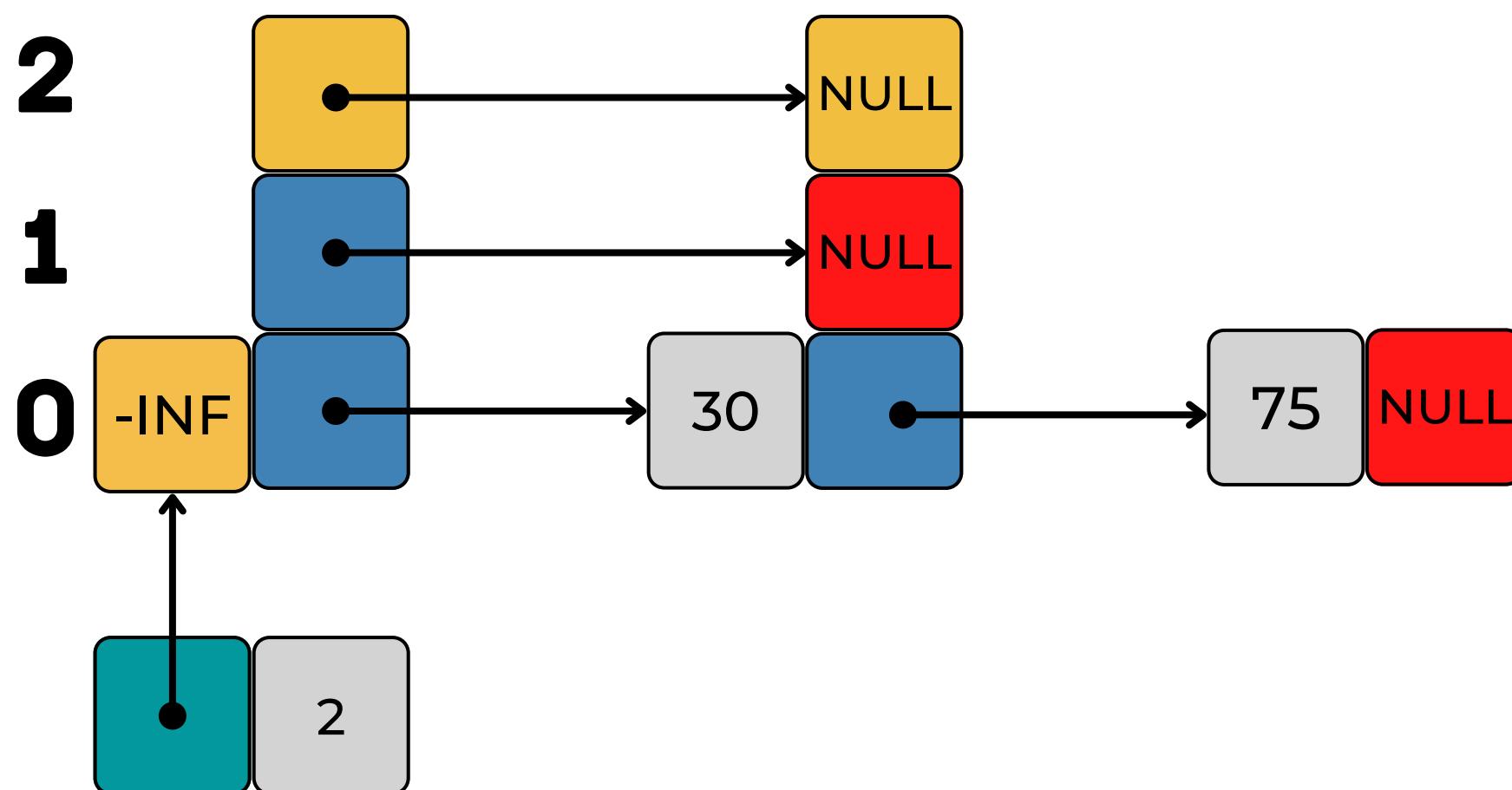


INSERT(75)



INSERTION

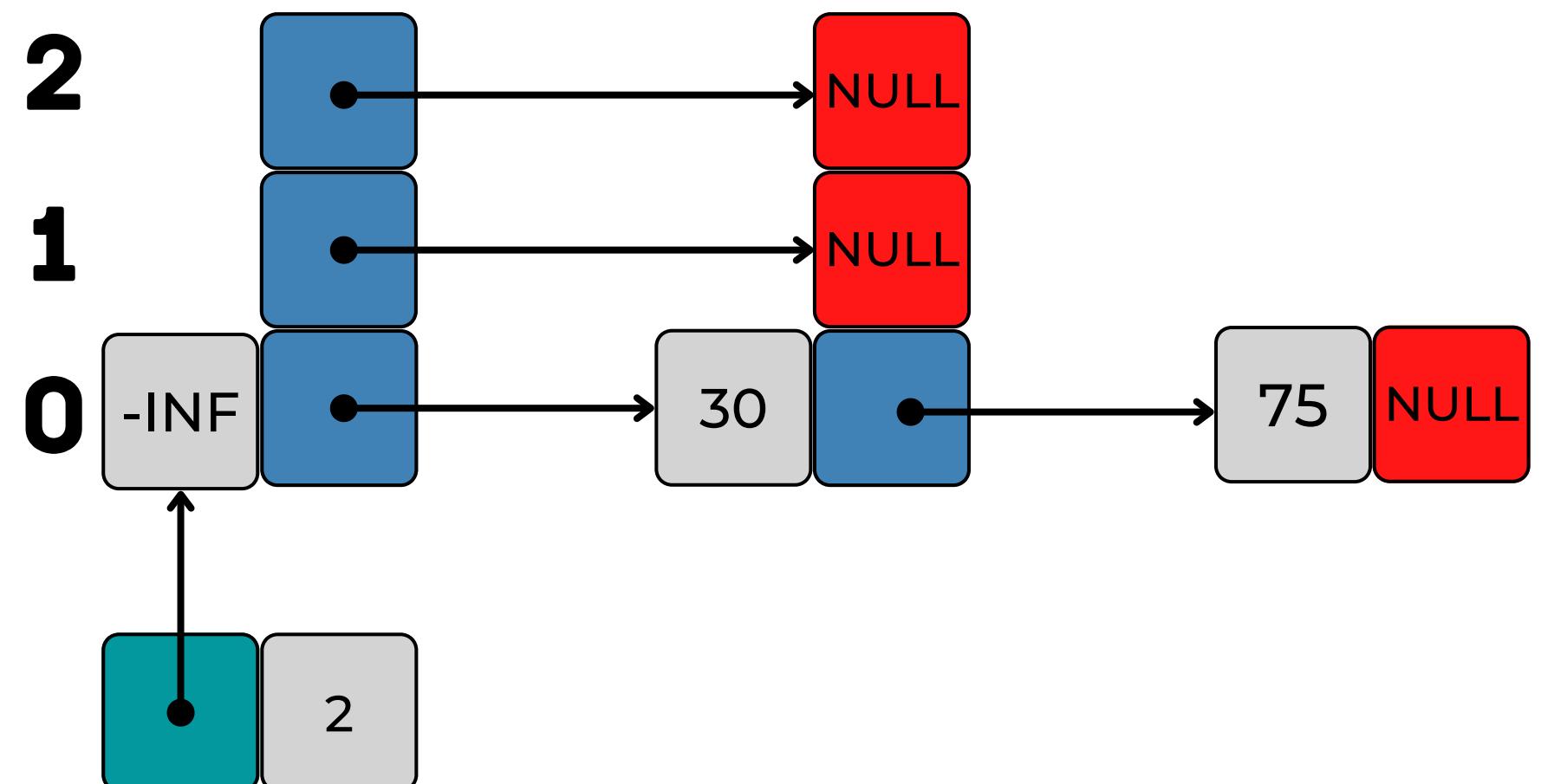
New Level = 0



INSERT(75)



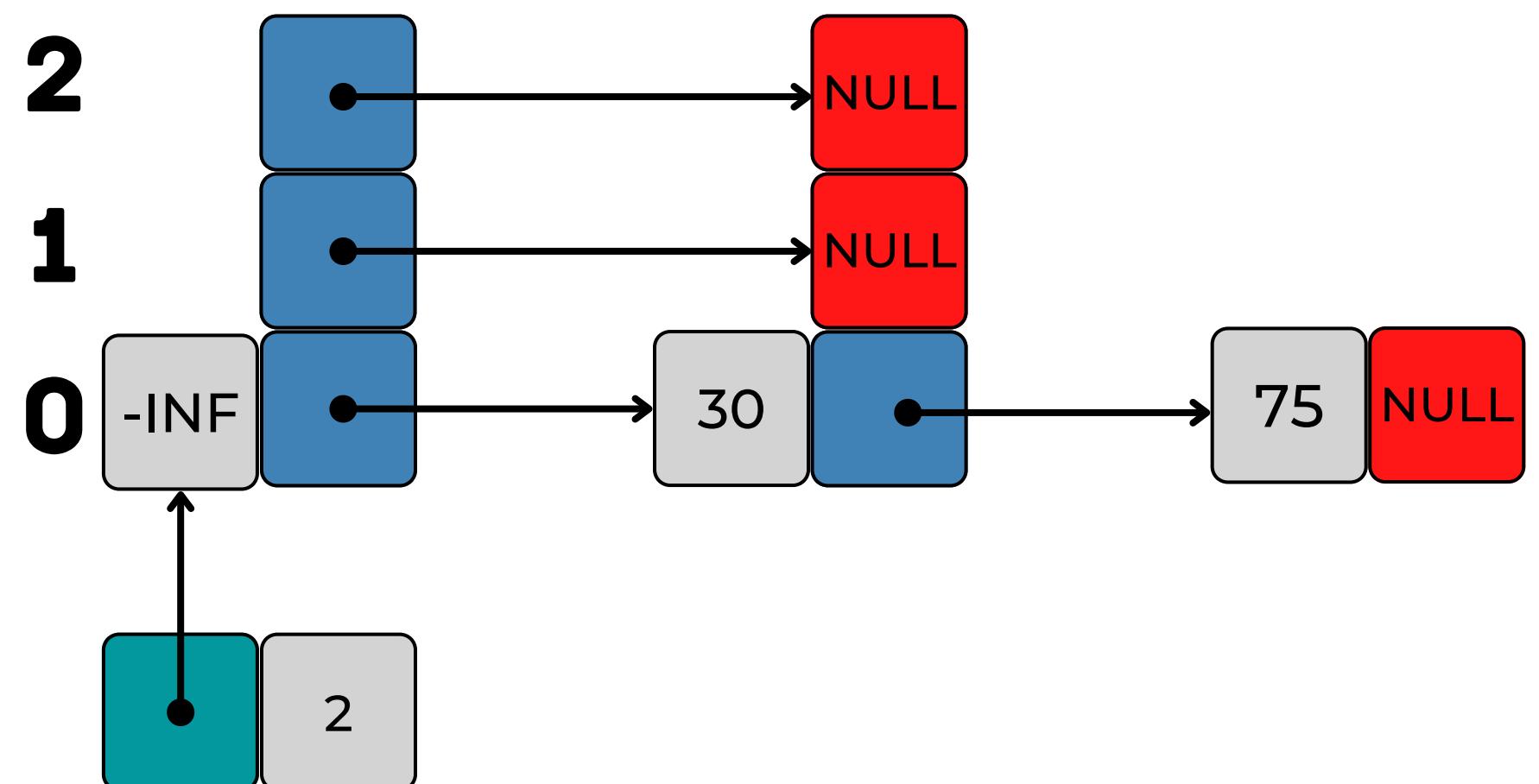
INSERTION



INSERT(20)



INSERTION



Update



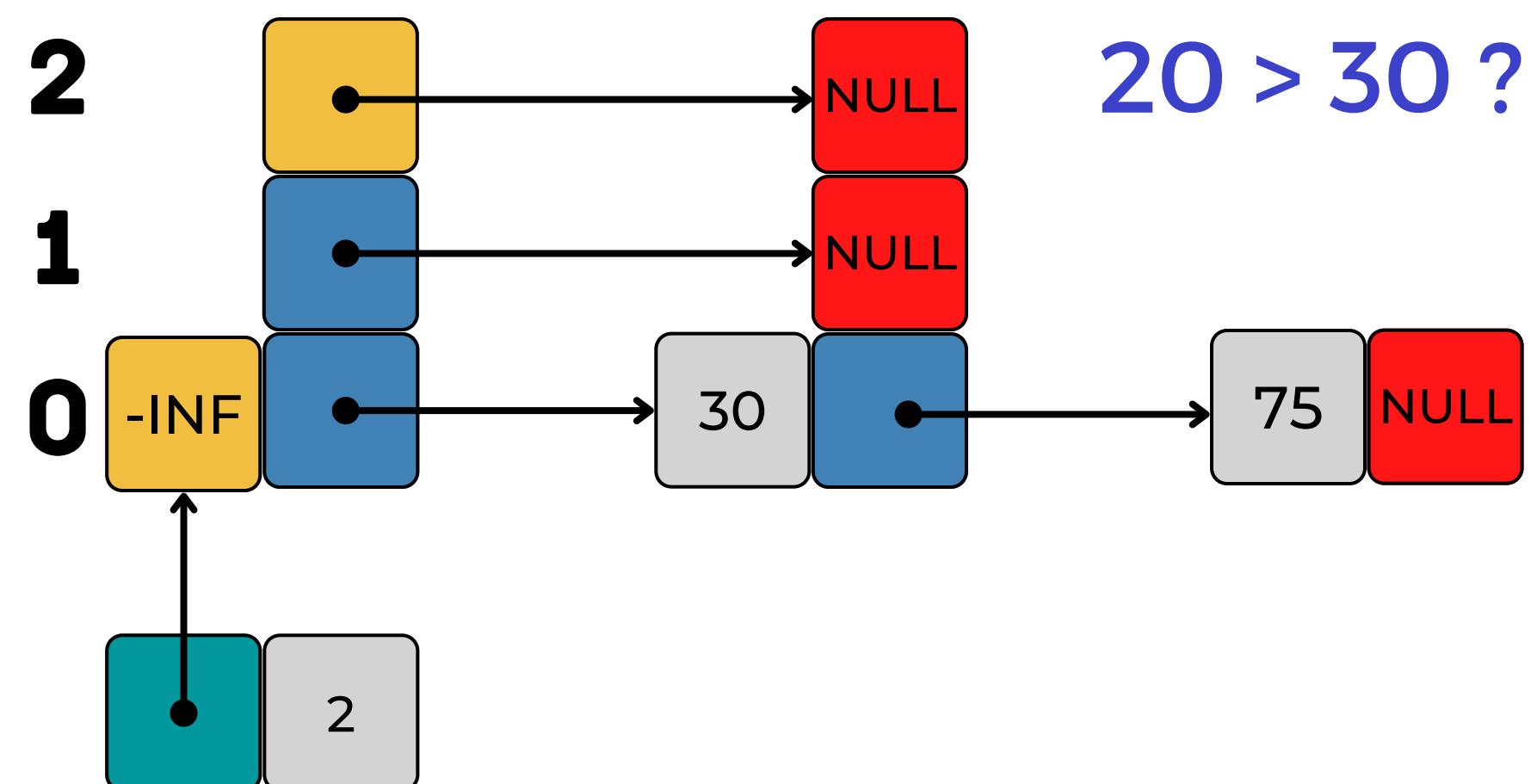
INSERT(20)



INSERTION

Max Level = 2

Level = 2



INSERT(20)

Update

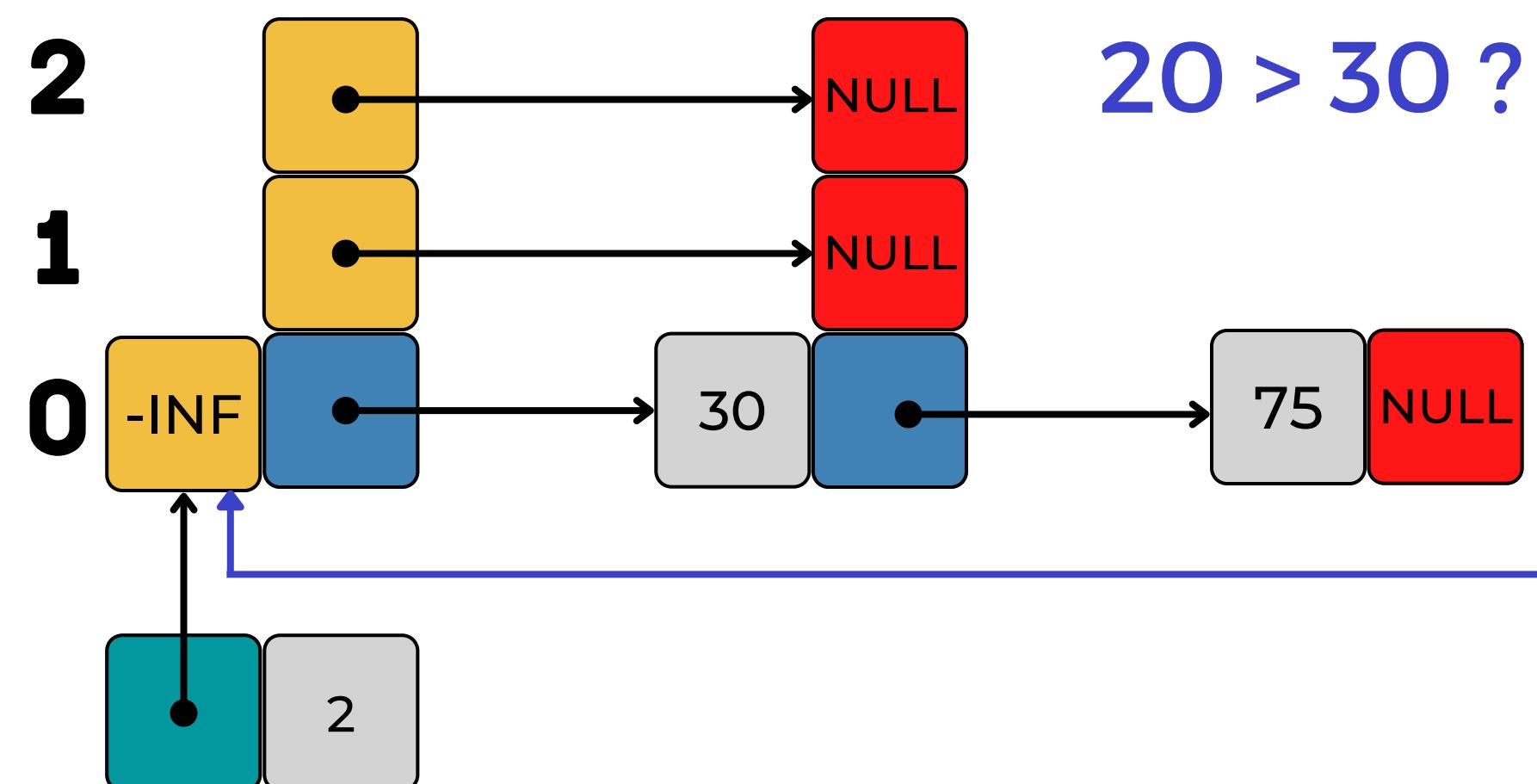




INSERTION

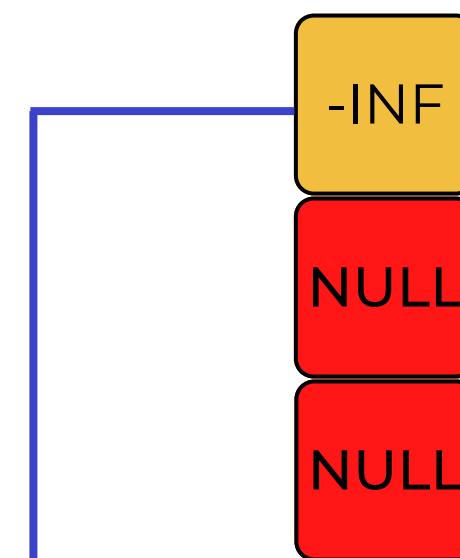
Max Level = 2

Level = 1



INSERT(20)

Update

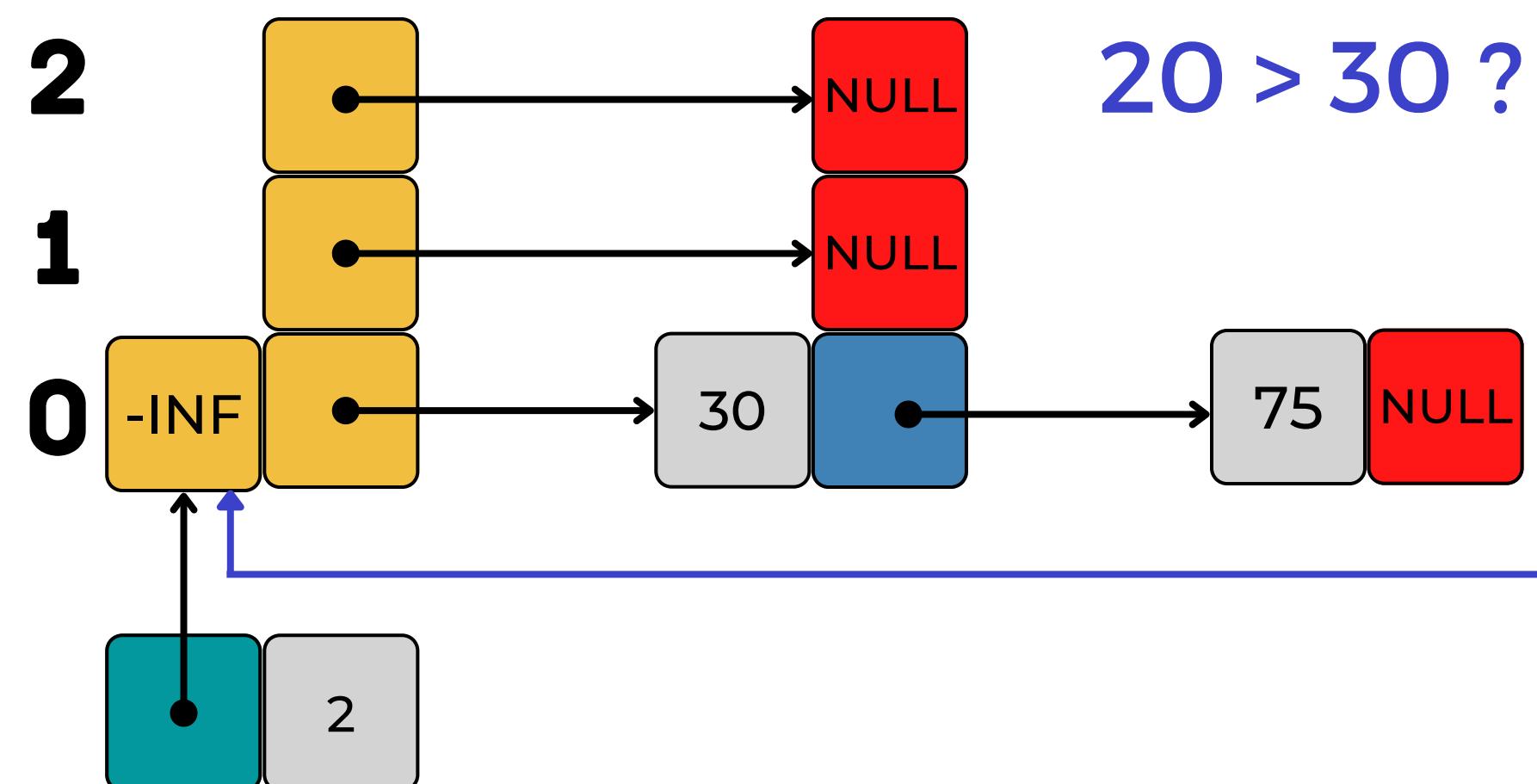




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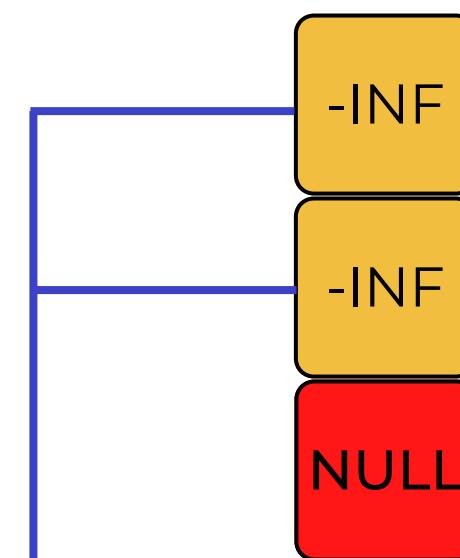
Max Level = 2

Level = 0



INSERT(20)

Update

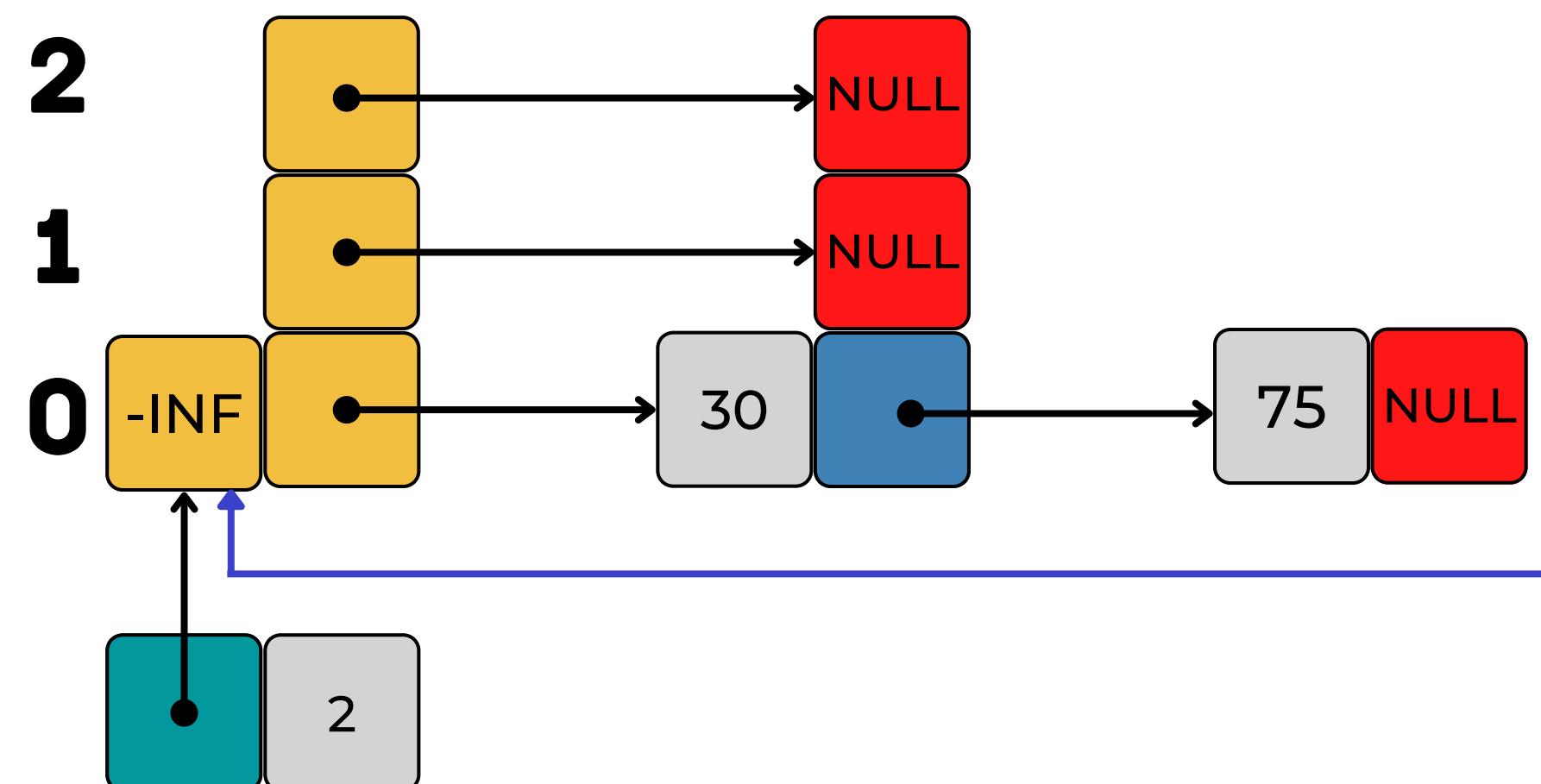




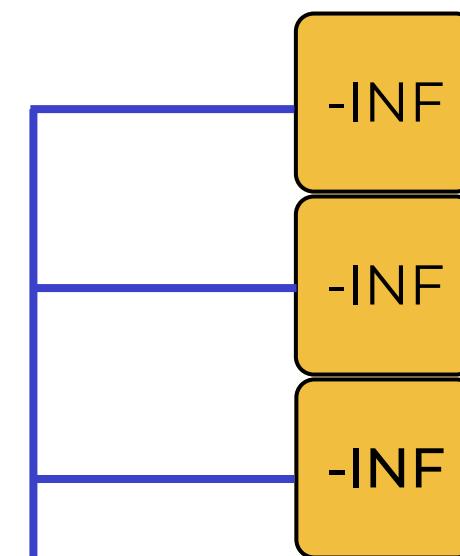
INSERTION

Max Level = 2

Level = -1



Update



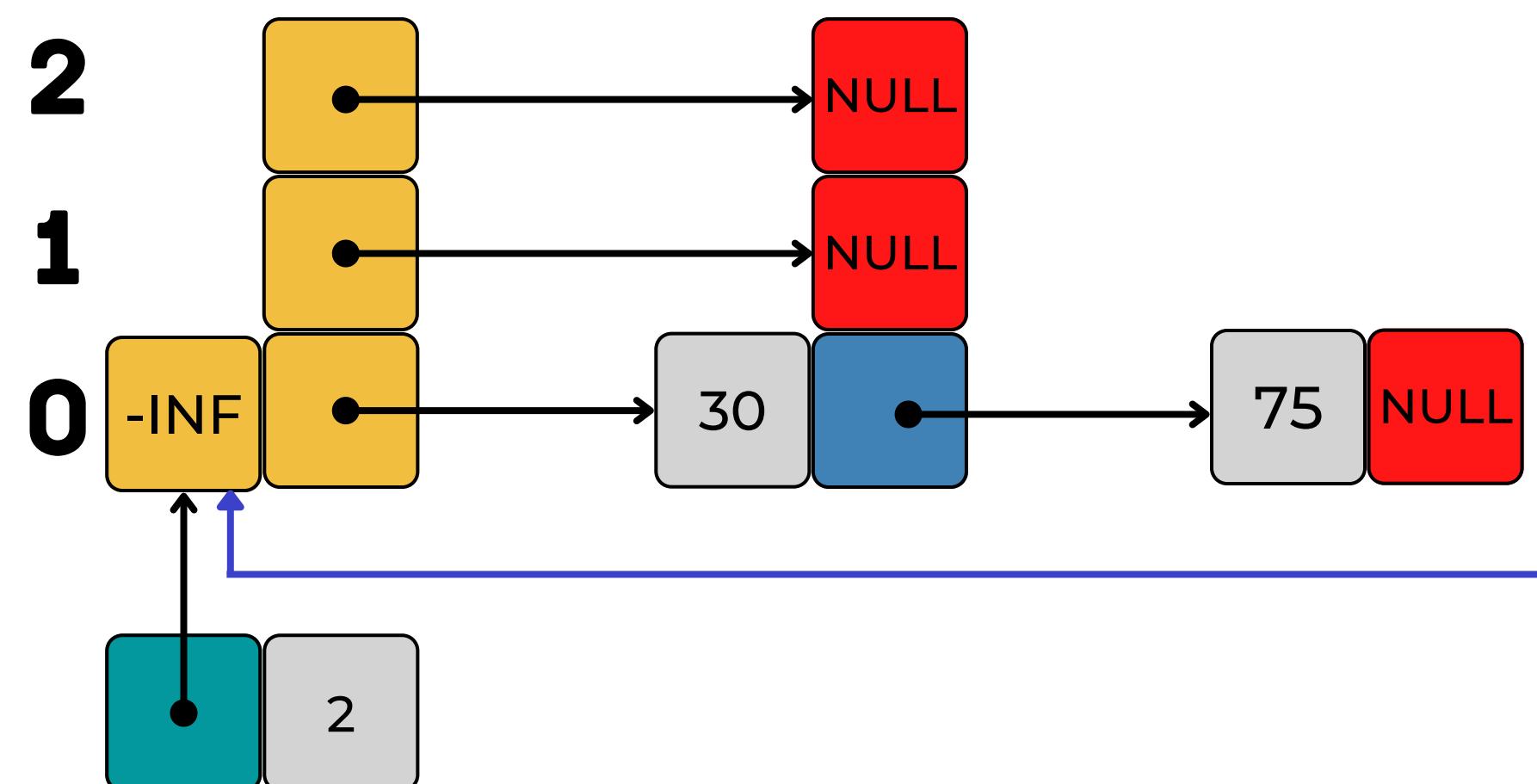
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INSERTION

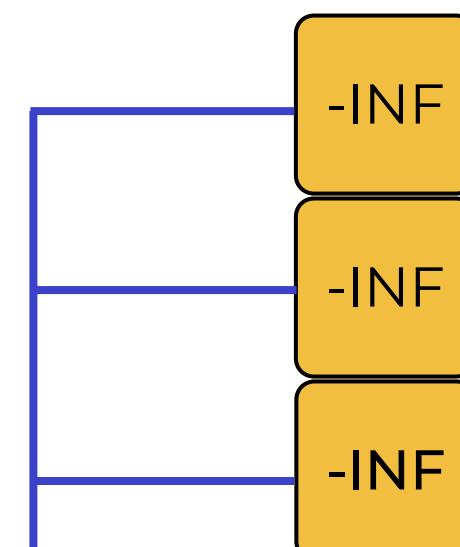
Max Level = 2

Level = -1



INSERT(20)

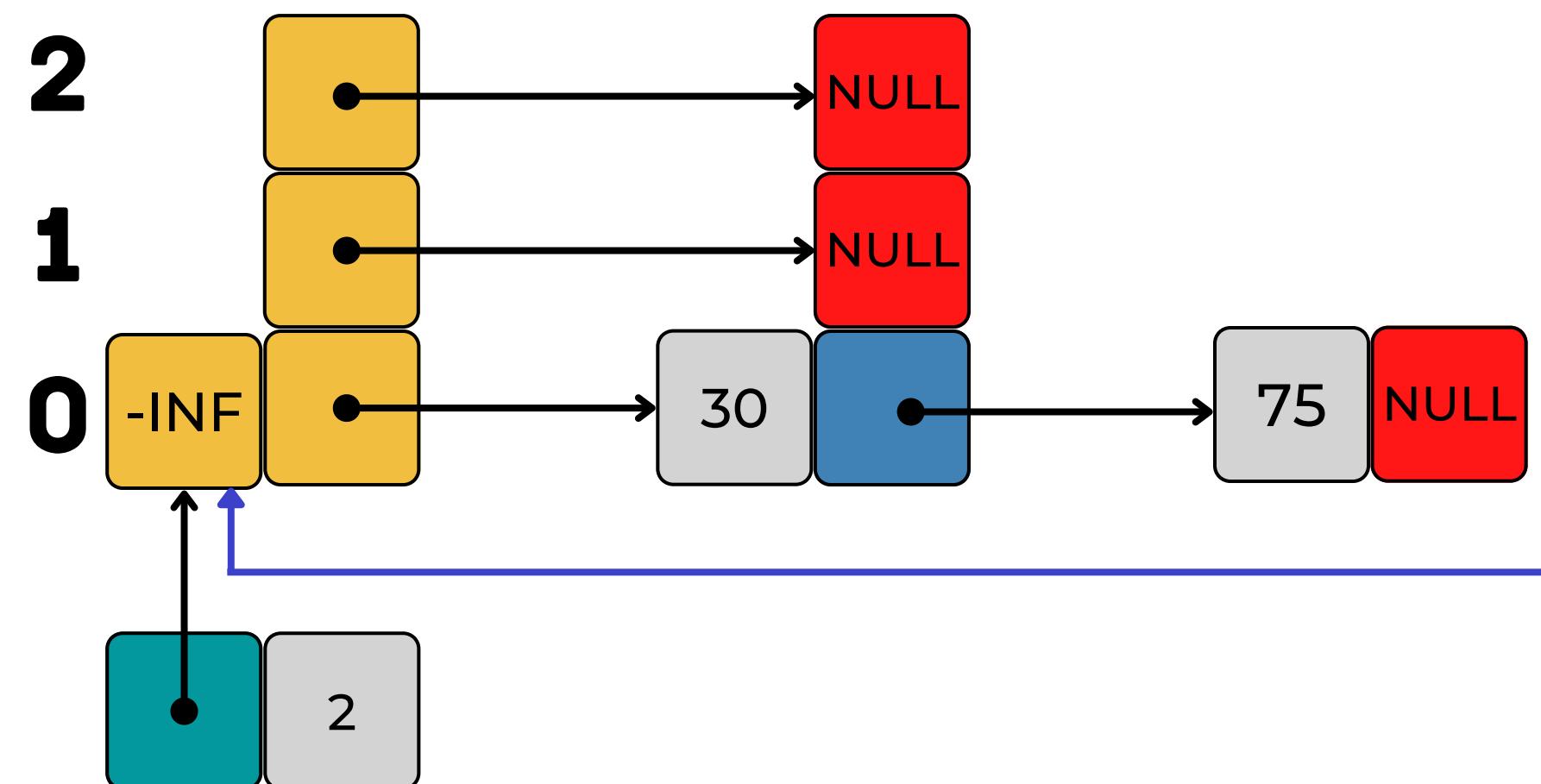
Update



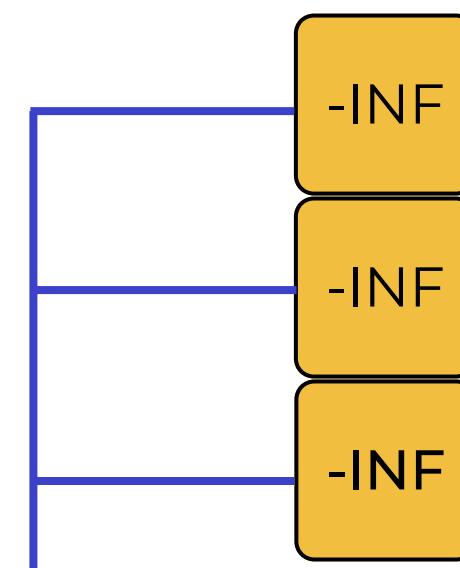


INSERTION

New Level = 1



Update

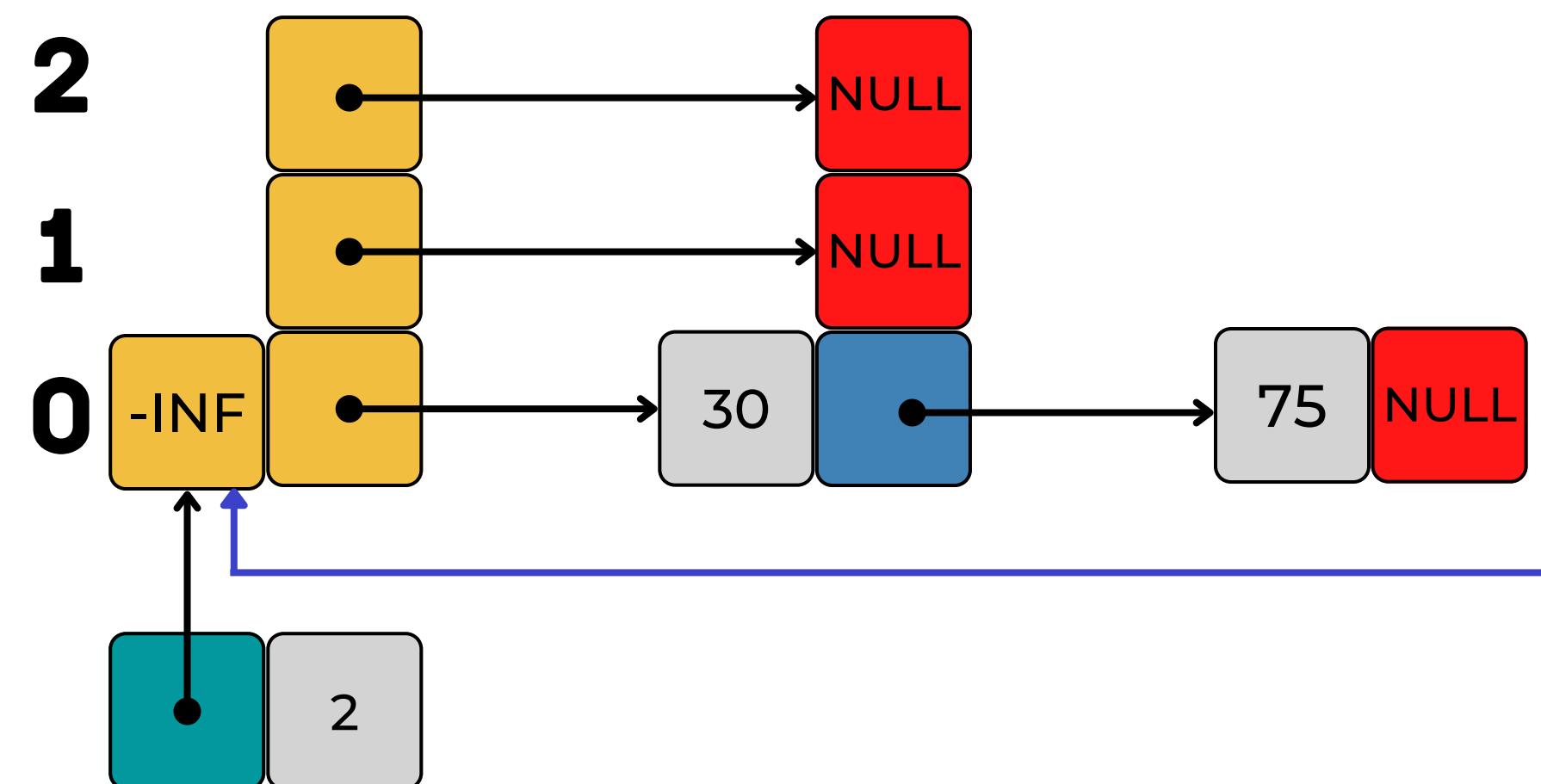


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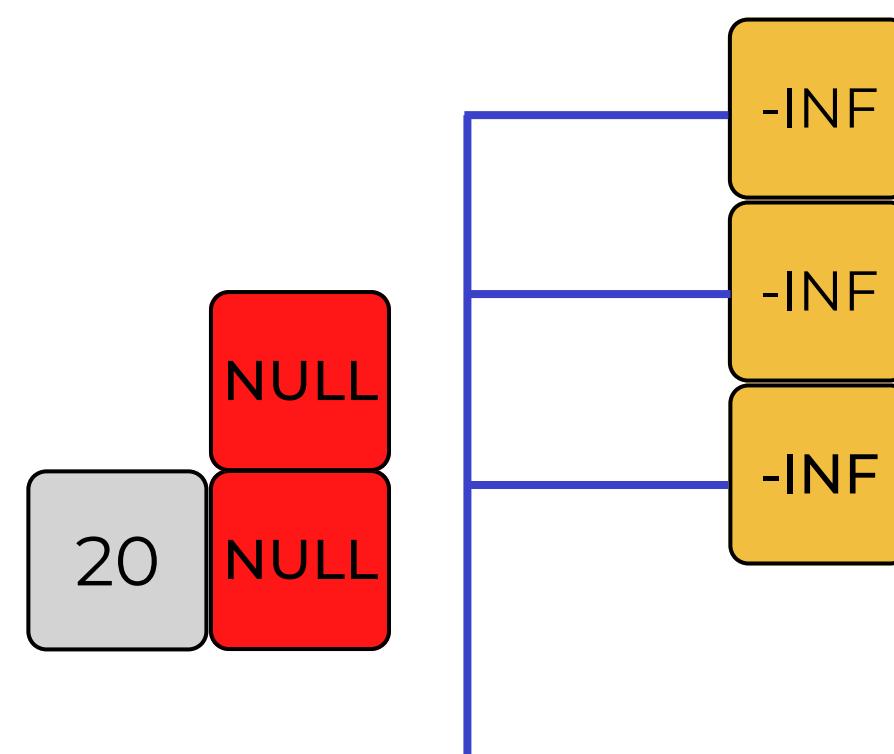


INSERTION

New Level = 1



Update

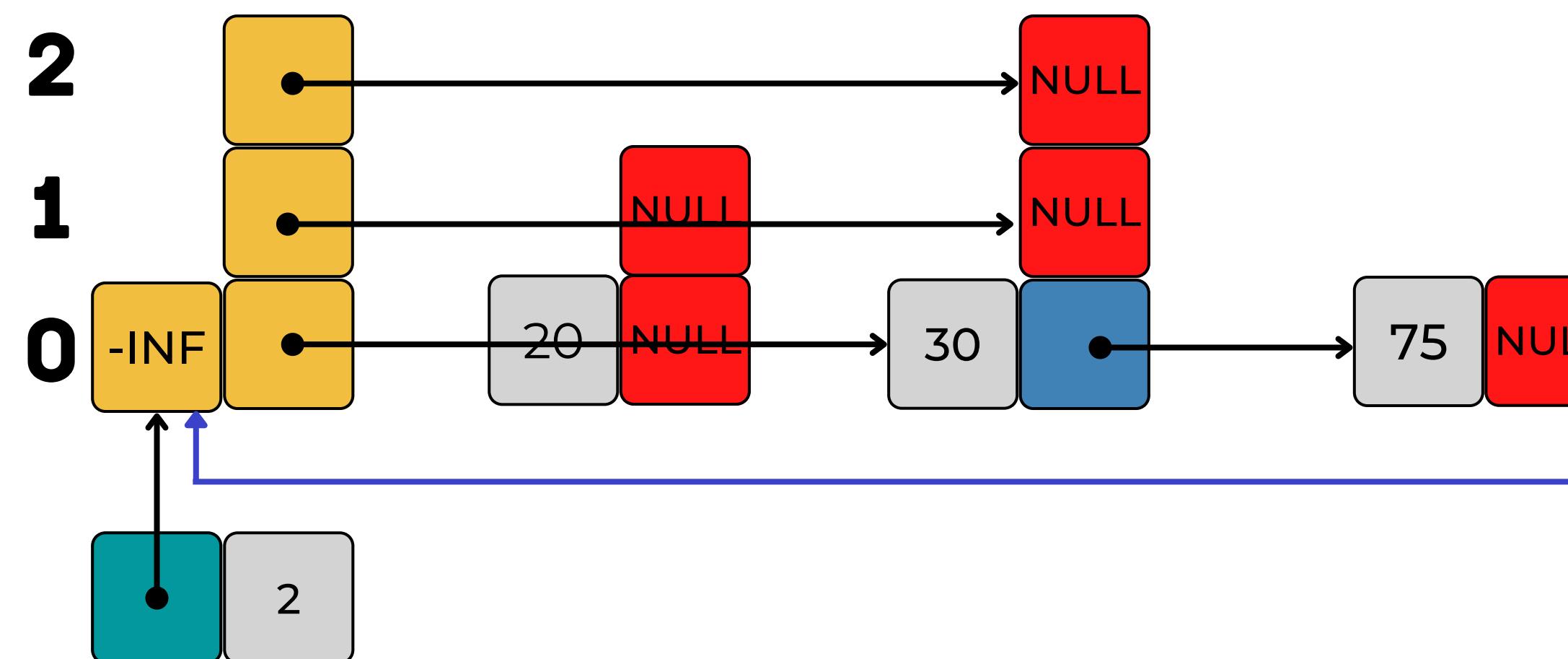


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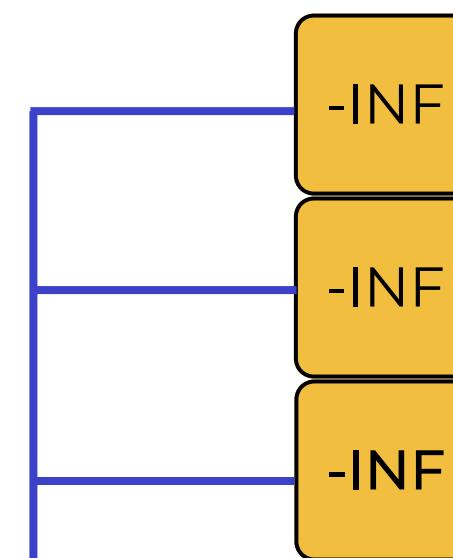


INSERTION

New Level = 1



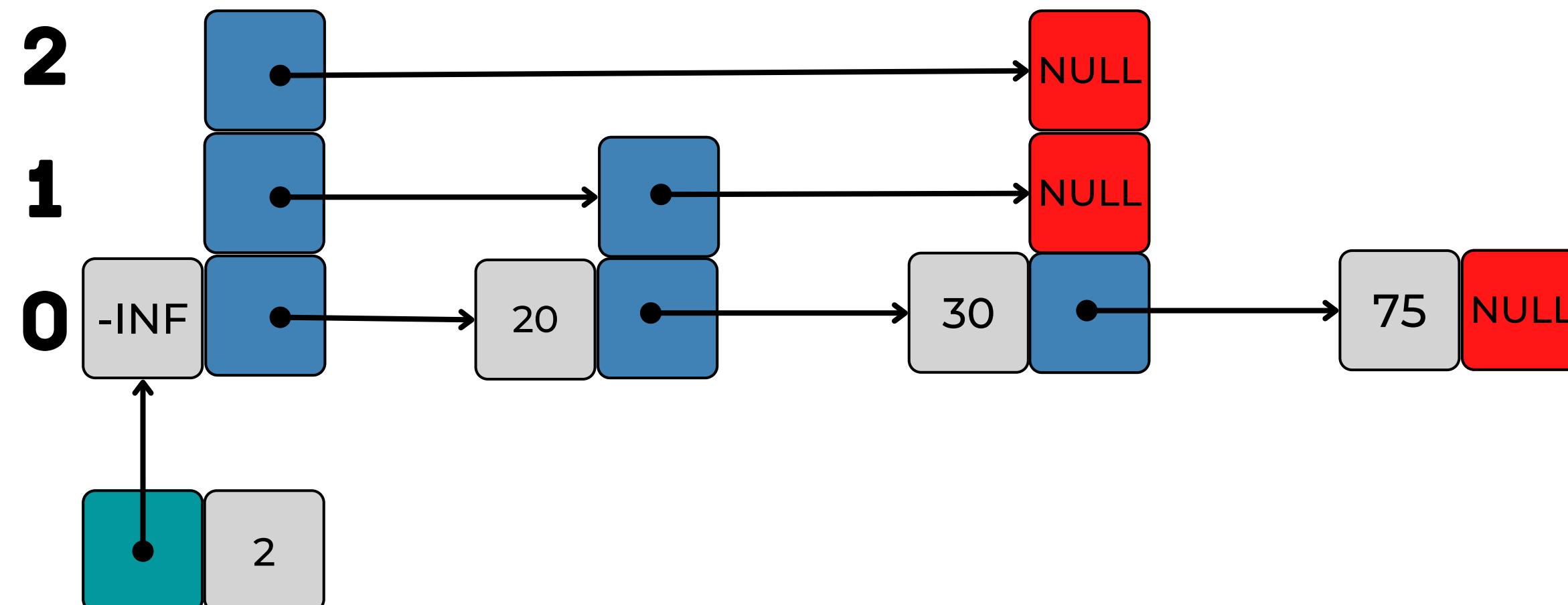
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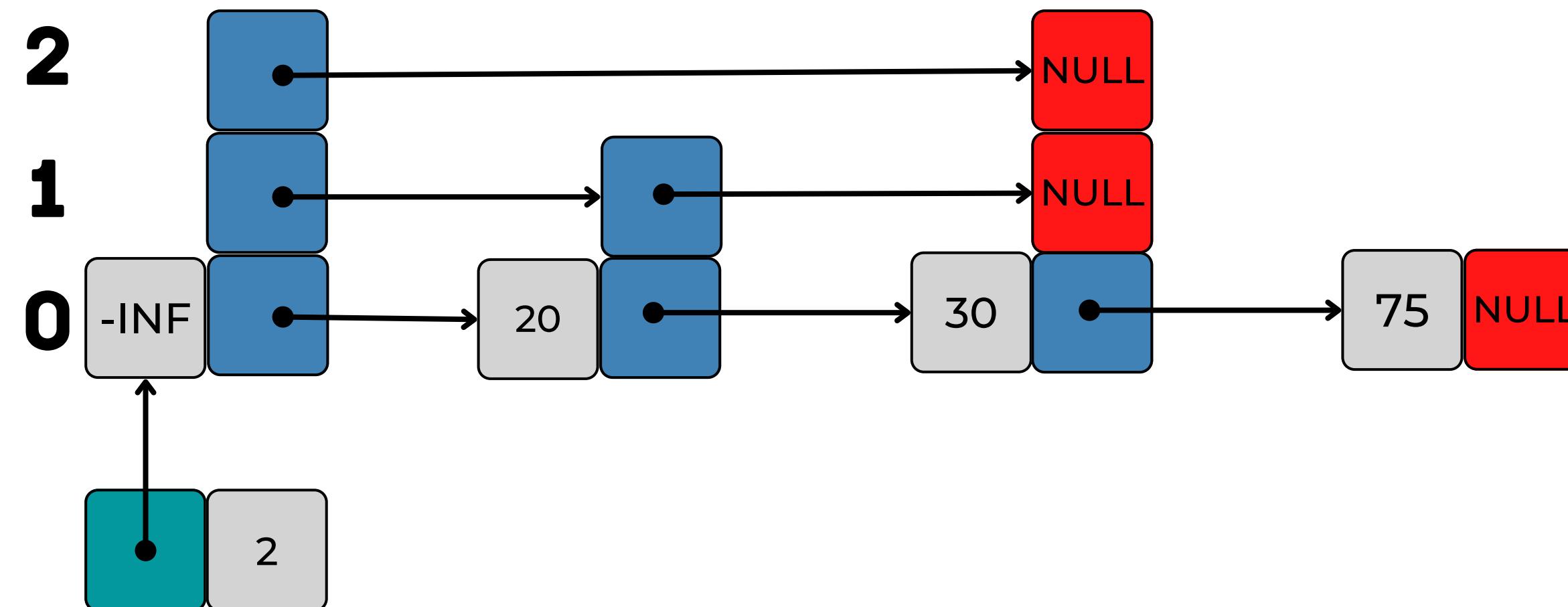


INSERTION





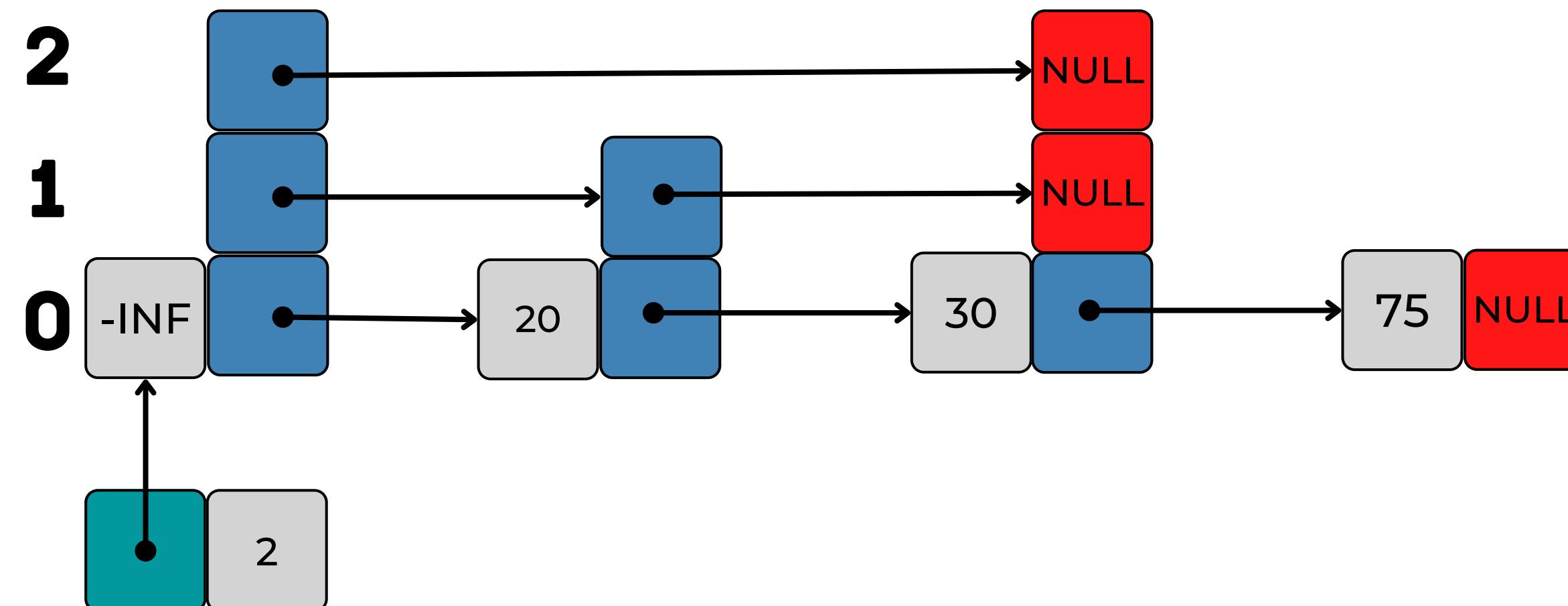
INSERTION



INSERT(50)



INSERTION



INSERT(50)

Update =



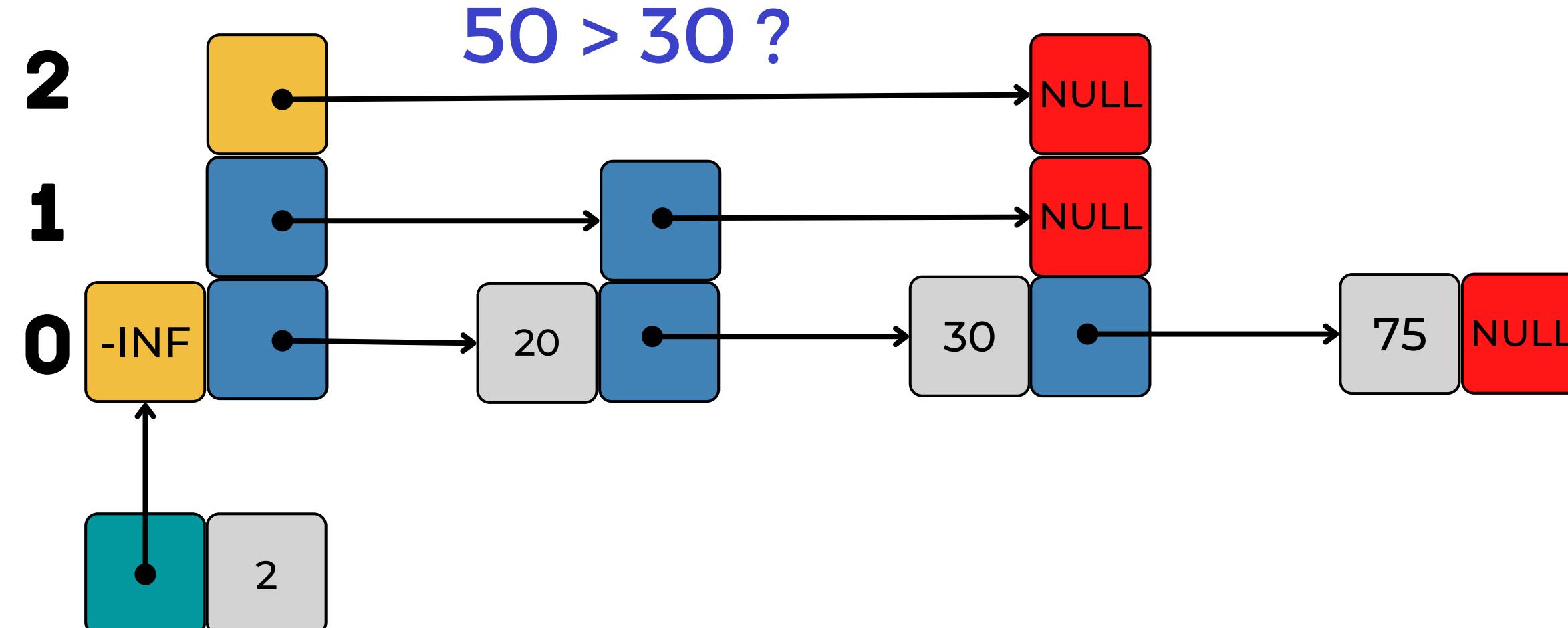


INSERTION

Max Level = 2

Level = 2

True = Move to next node



INSERT(50)

Update =





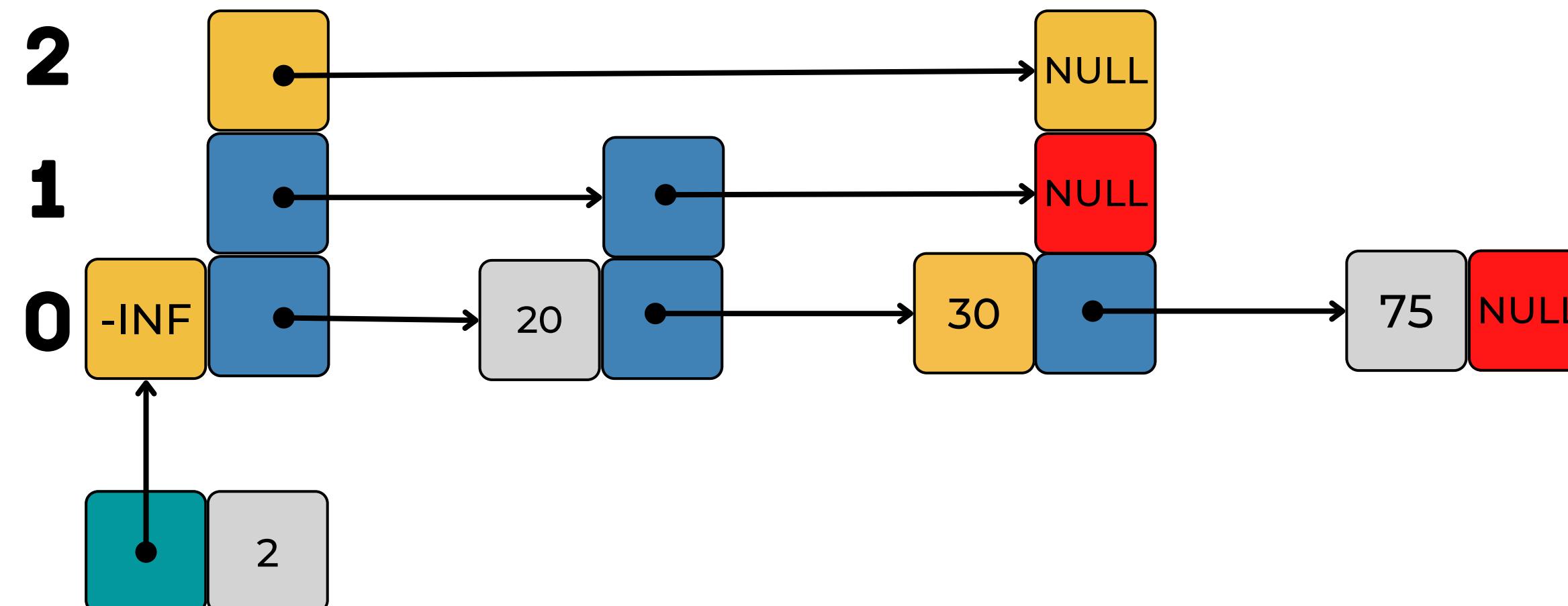
INSERTION

Max Level = 2

Level = 2

since link is NULL? Move down a level

Update



INSERT(50)





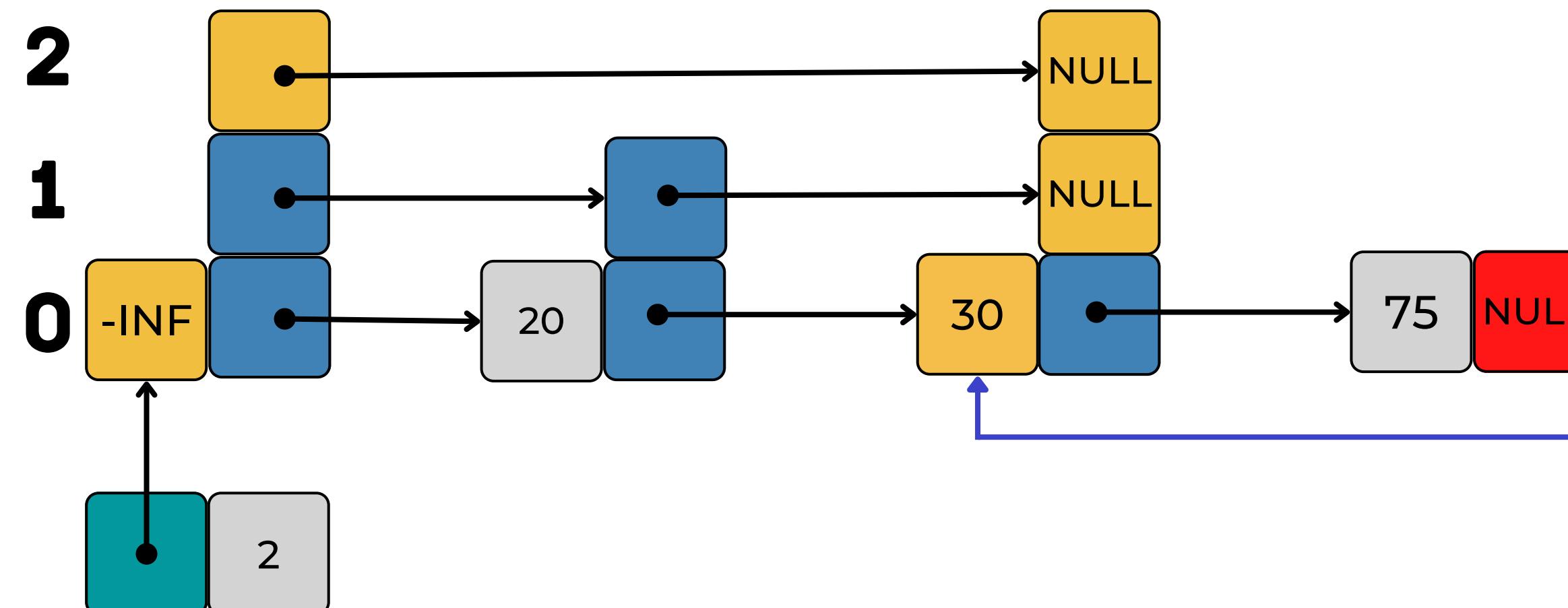
INSERTION

Max Level = 2

Level = 1

since link is NULL? Move down a level

Update



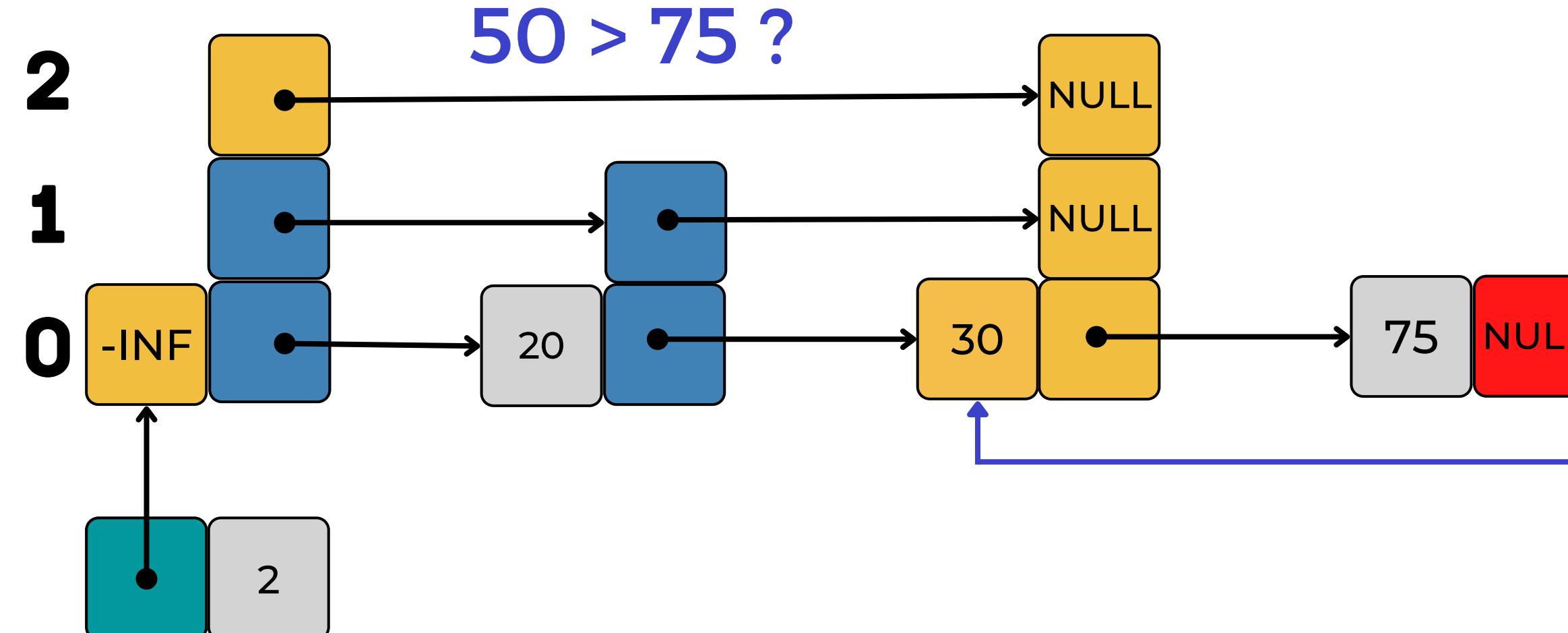
INSERT(50)



INSERTION

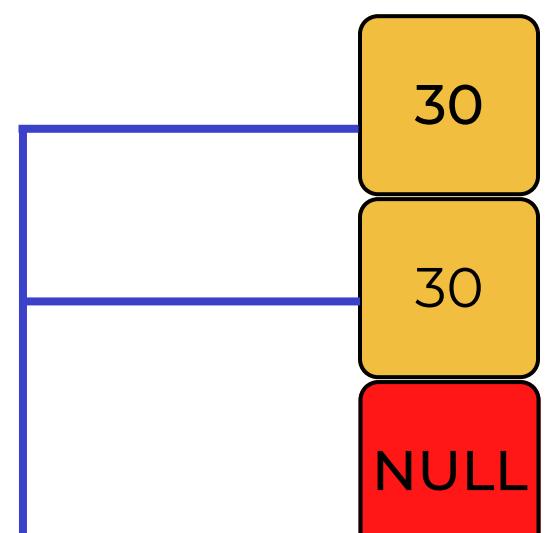
Max Level = 2

Level = 0



INSERT(50)

Update



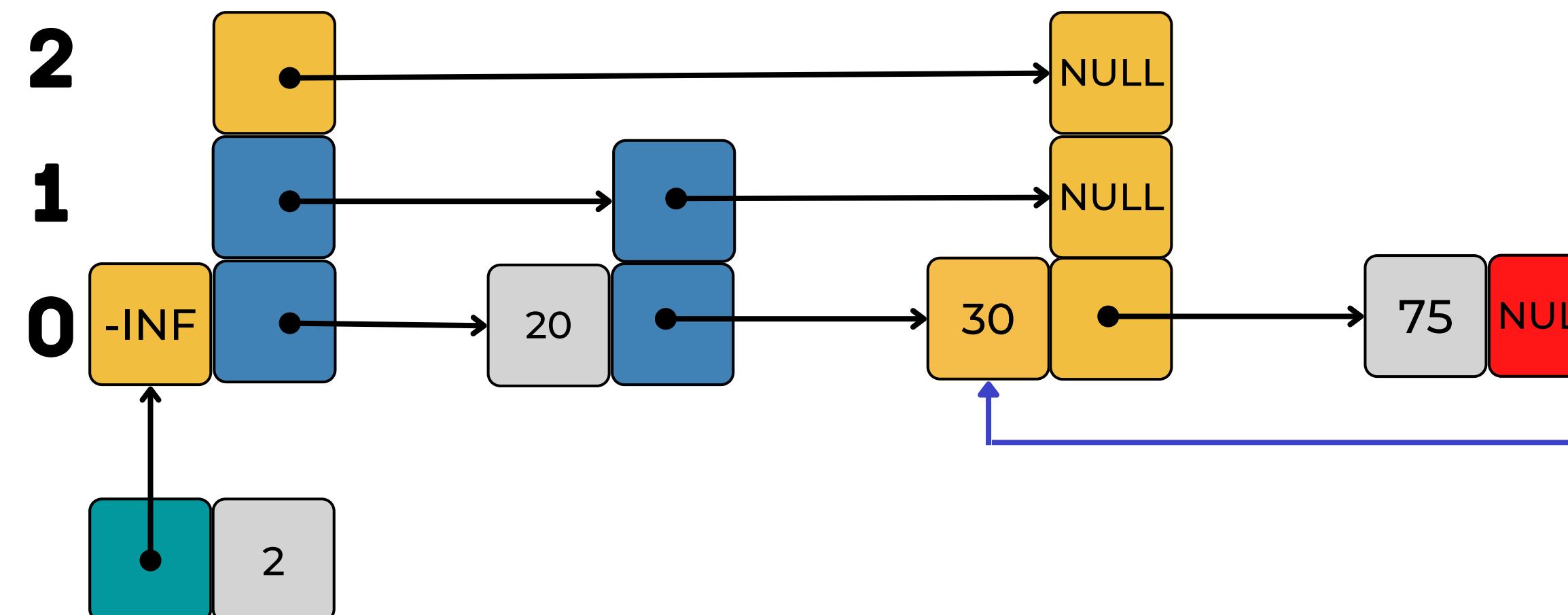


INSERTION

Max Level = 2

Level = -1

Update



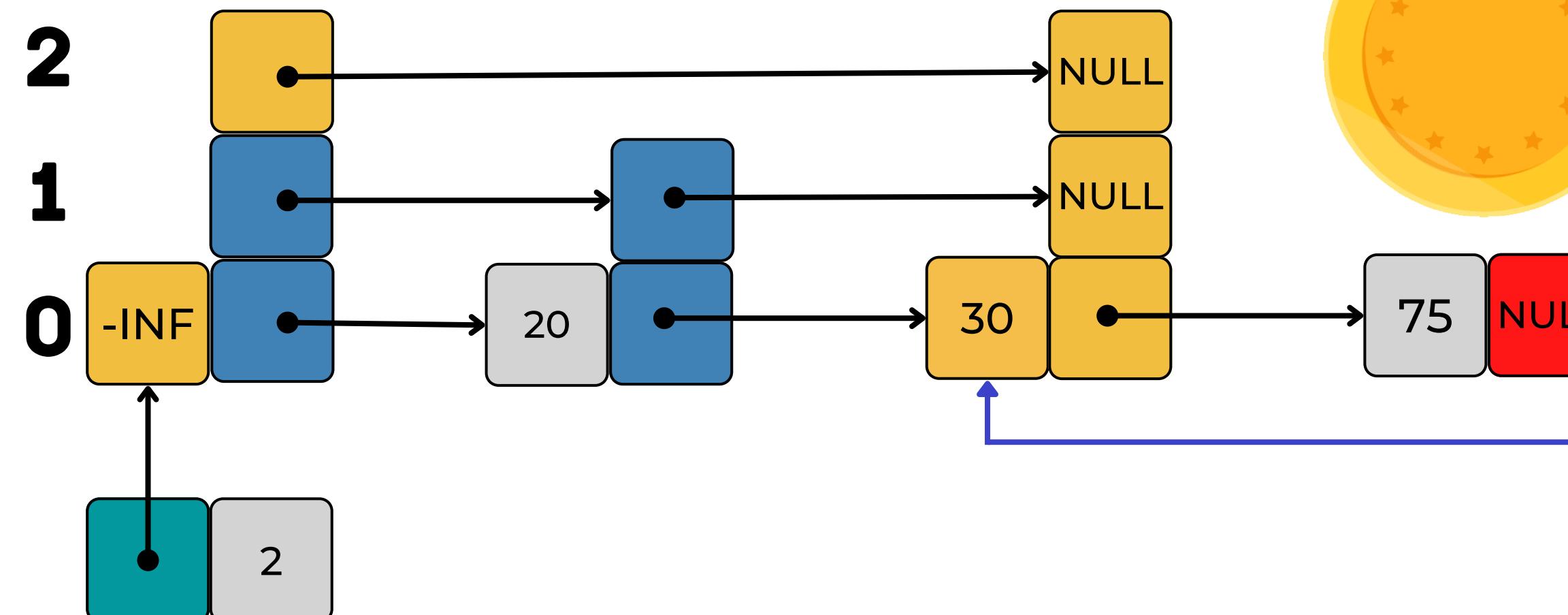
INSERT(50)



INSERTION

Max Level = 2

Level = -1



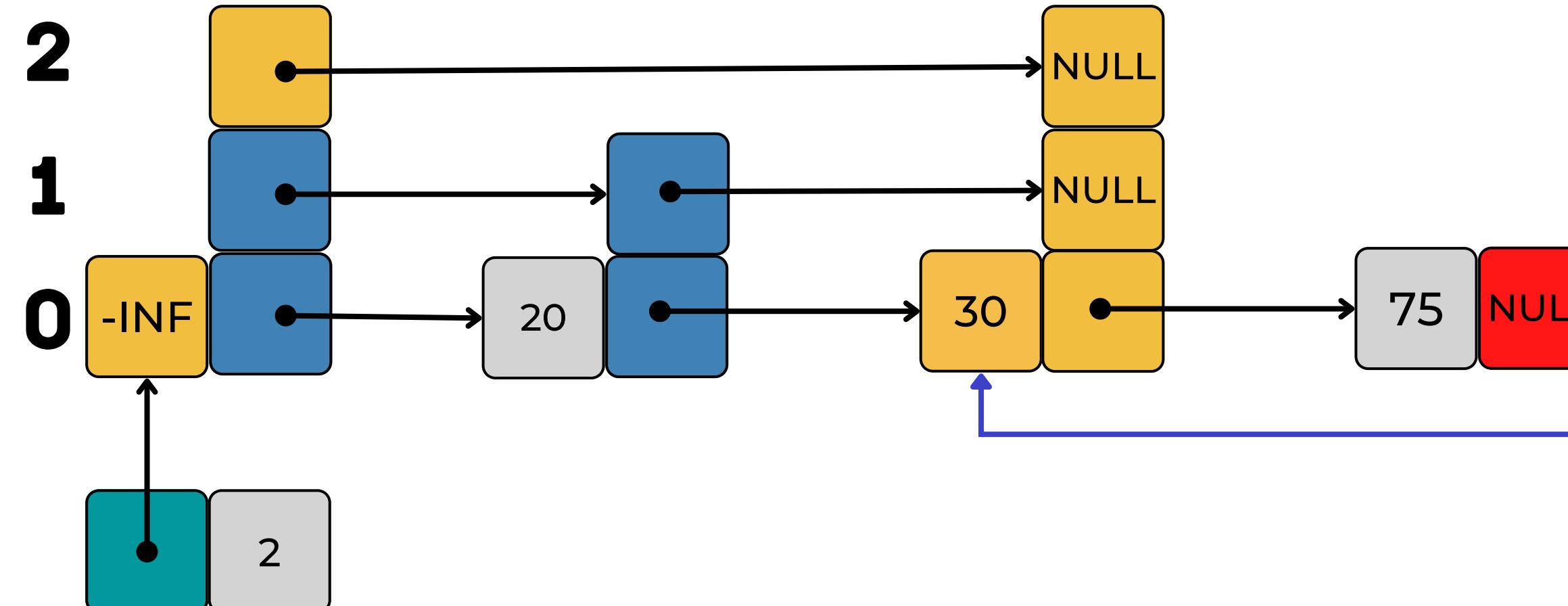
INSERT(50)

Update



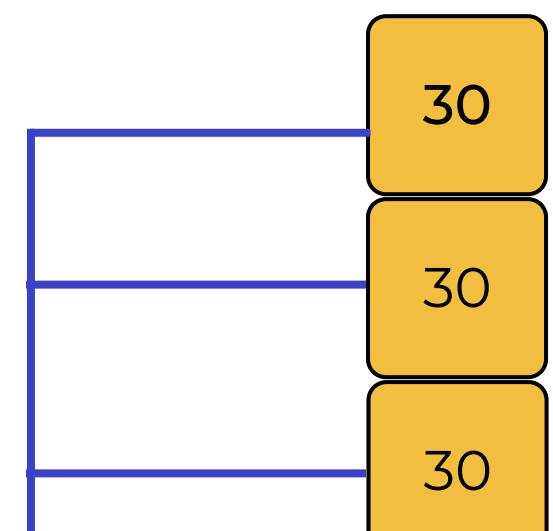
INSERTION

New Level = 1



INSERT(50)

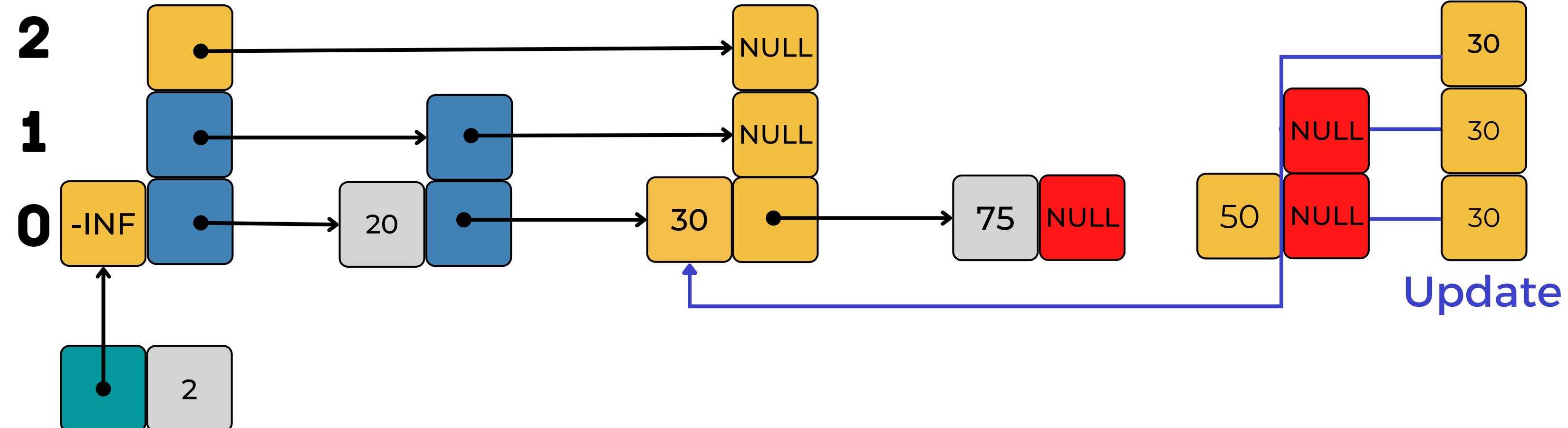
Update





INSERTION

New Level = 1

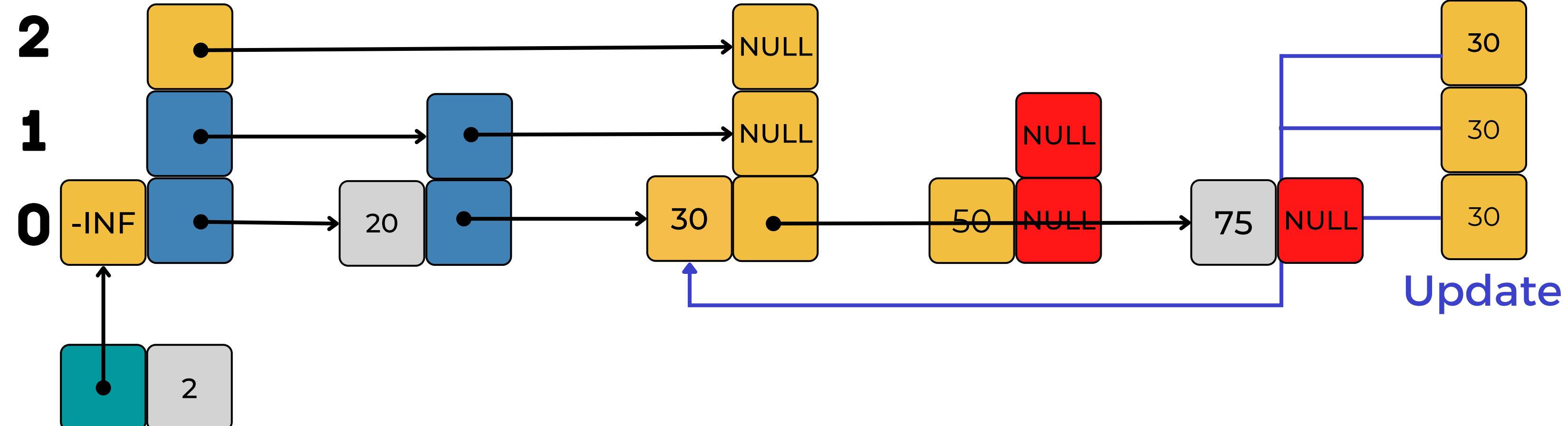


INSERT(50)



INSERTION

New Level = 1



INSERT(50)

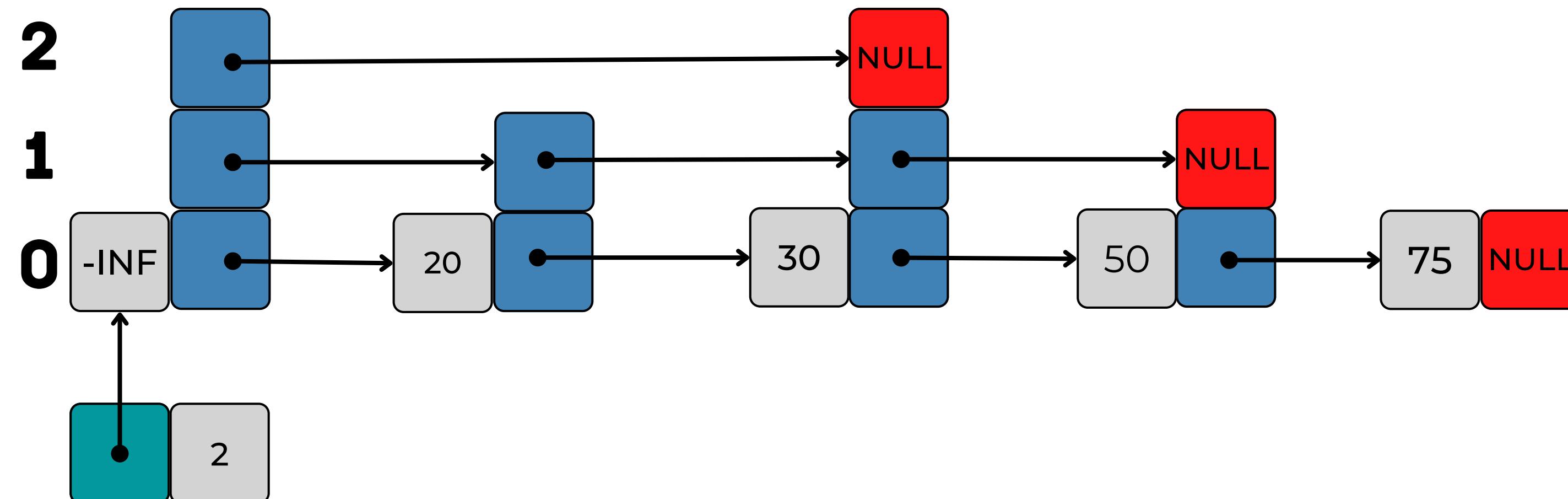
TEAM JC

Search ...



SEARCH

Level = 2



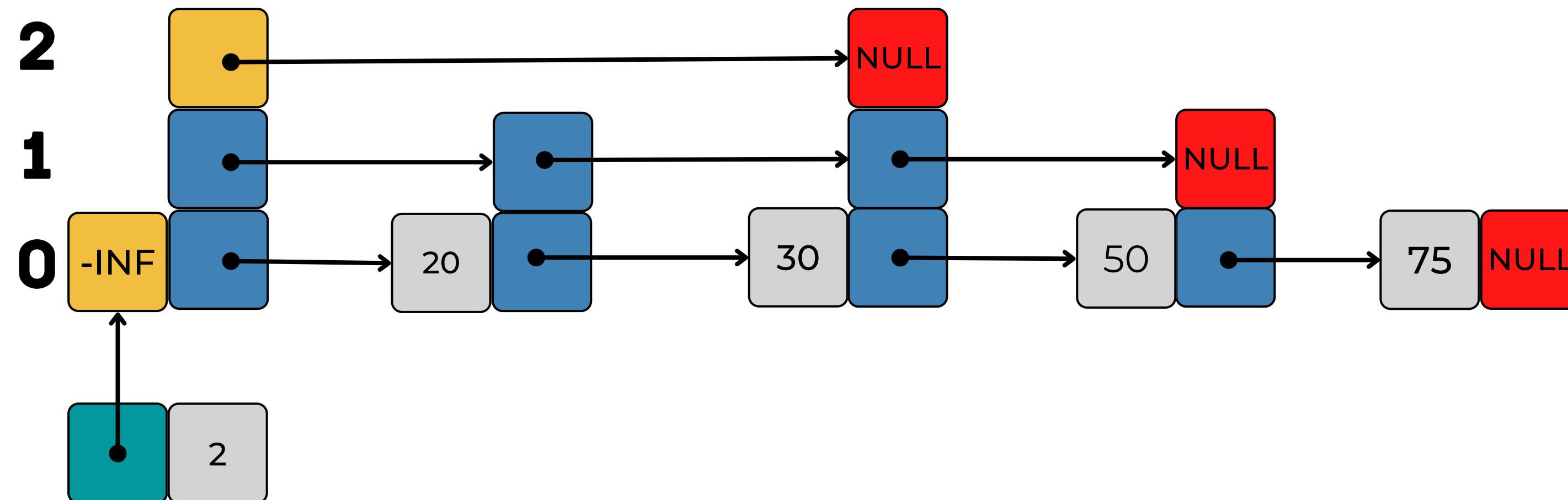
SEARCH(75)



SEARCH

Level = 2

75>30? True = Move to next node



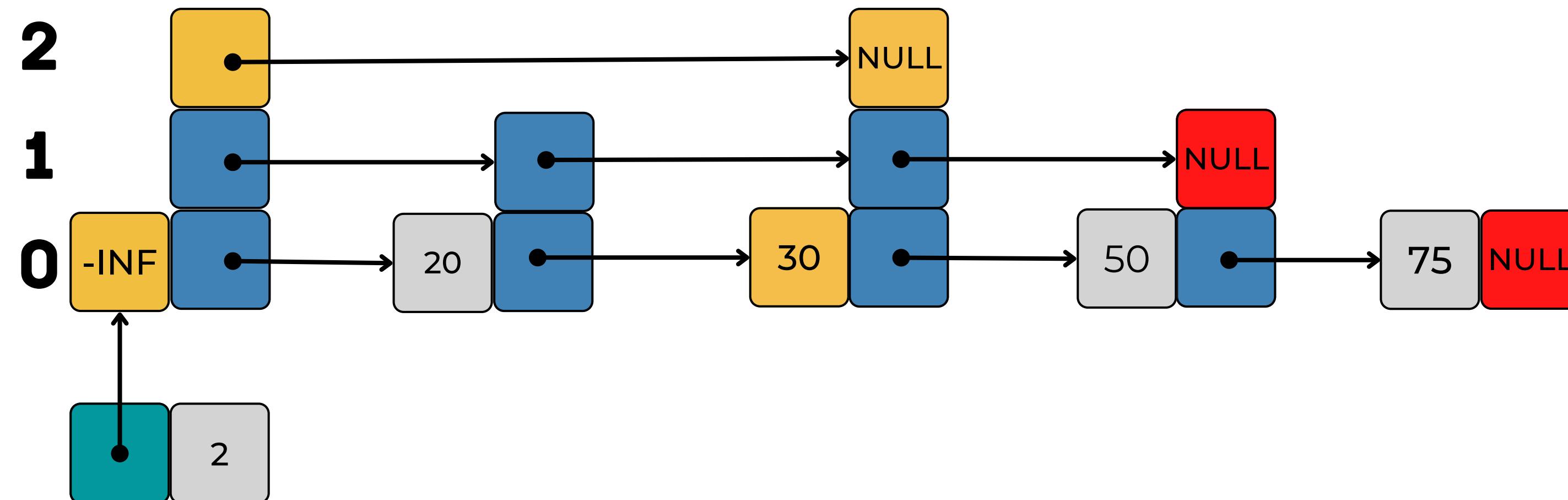
SEARCH(75)



SEARCH

Level = 2

since link is NULL? **Move down a level**



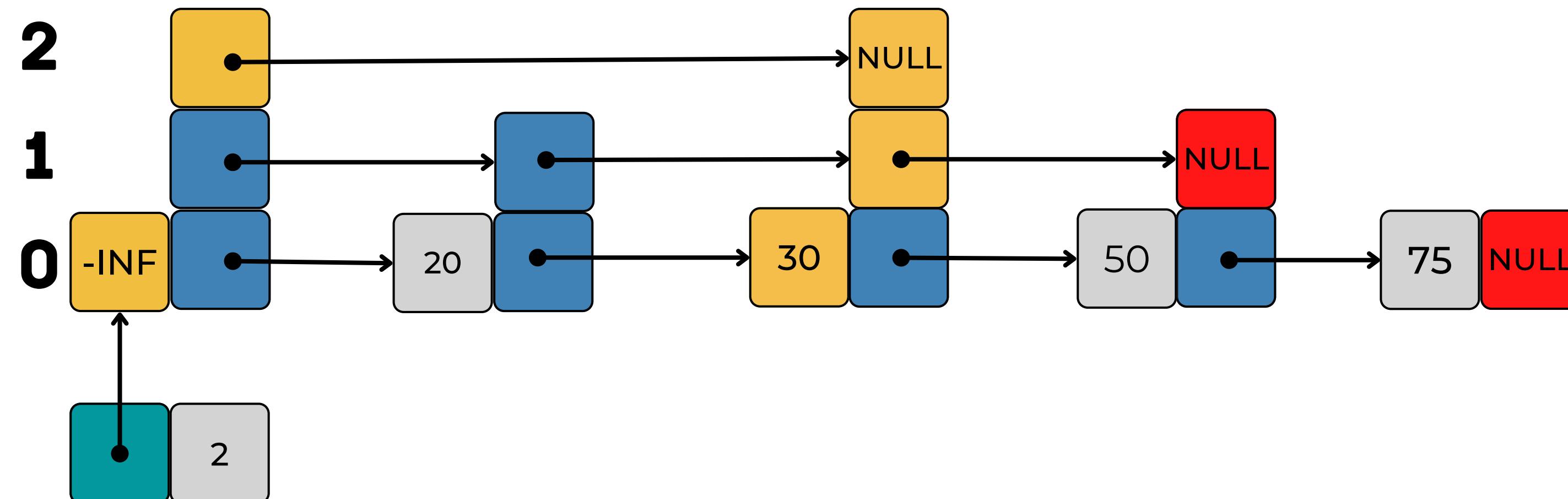
SEARCH(75)



SEARCH

Level = 1

75>50? **True** = Move to next node



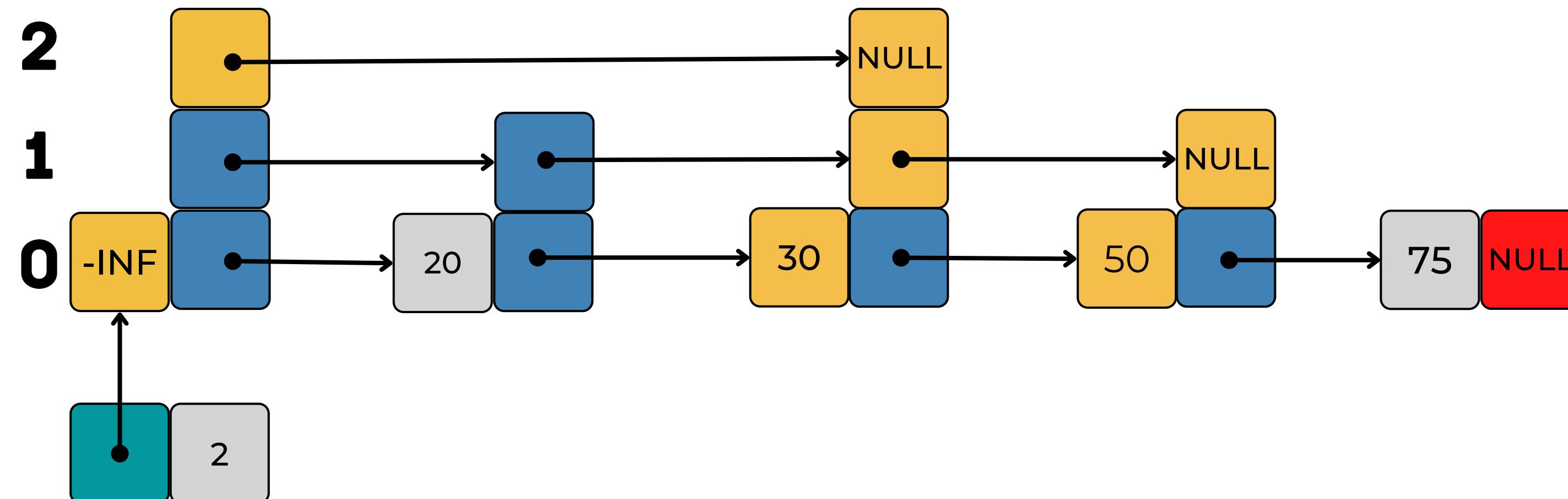
SEARCH(75)



SEARCH

Level = 1

since link is NULL? Move down a level



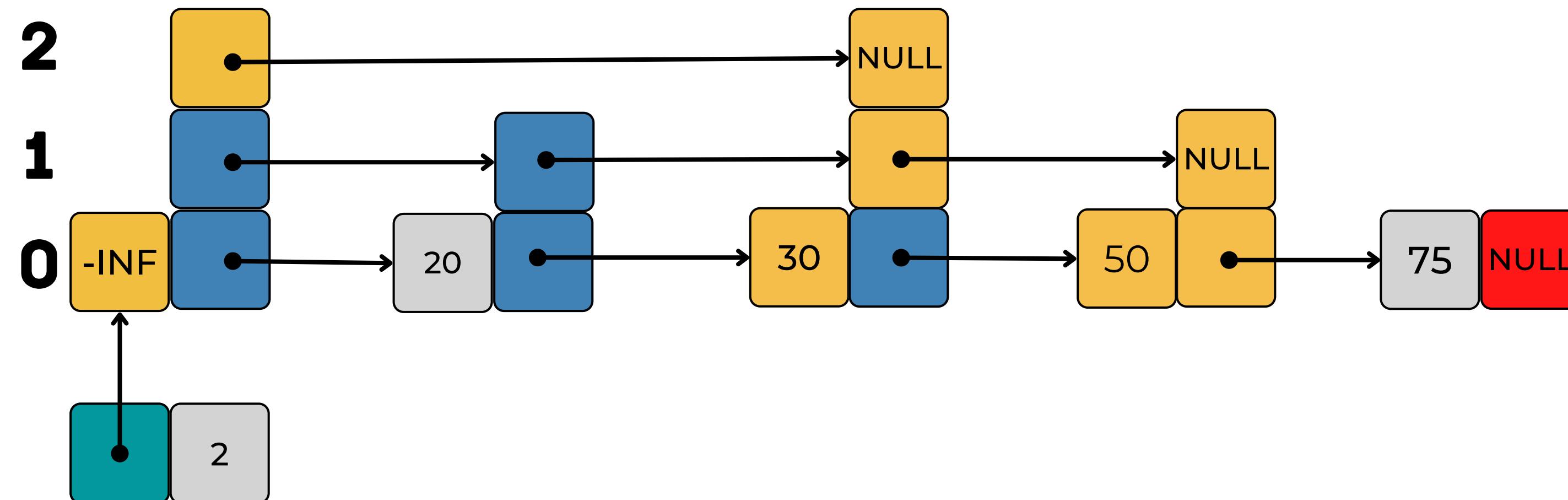
SEARCH(75)



SEARCH

Level = 0

75>75? **False** = Move down a level



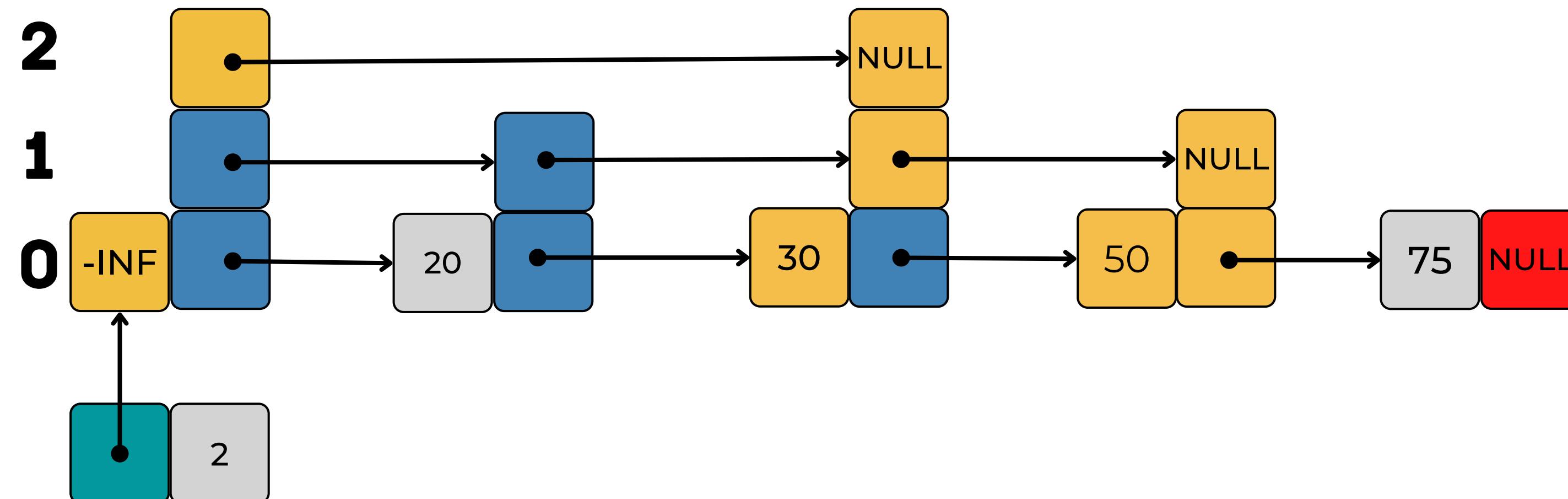
SEARCH(75)



SEARCH

Level = -1

since **level < 0** then return the next node



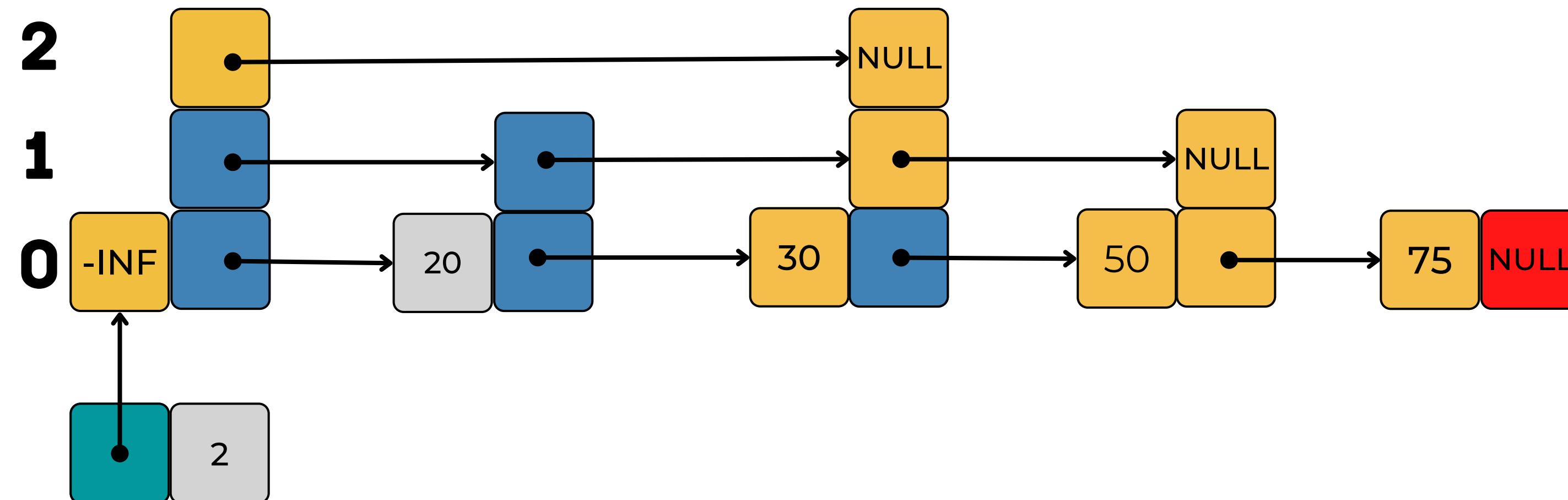
SEARCH(75)



SEARCH

Level = -1

75 == 75? **True** then 75 is an element
from the skip list



SEARCH(75)

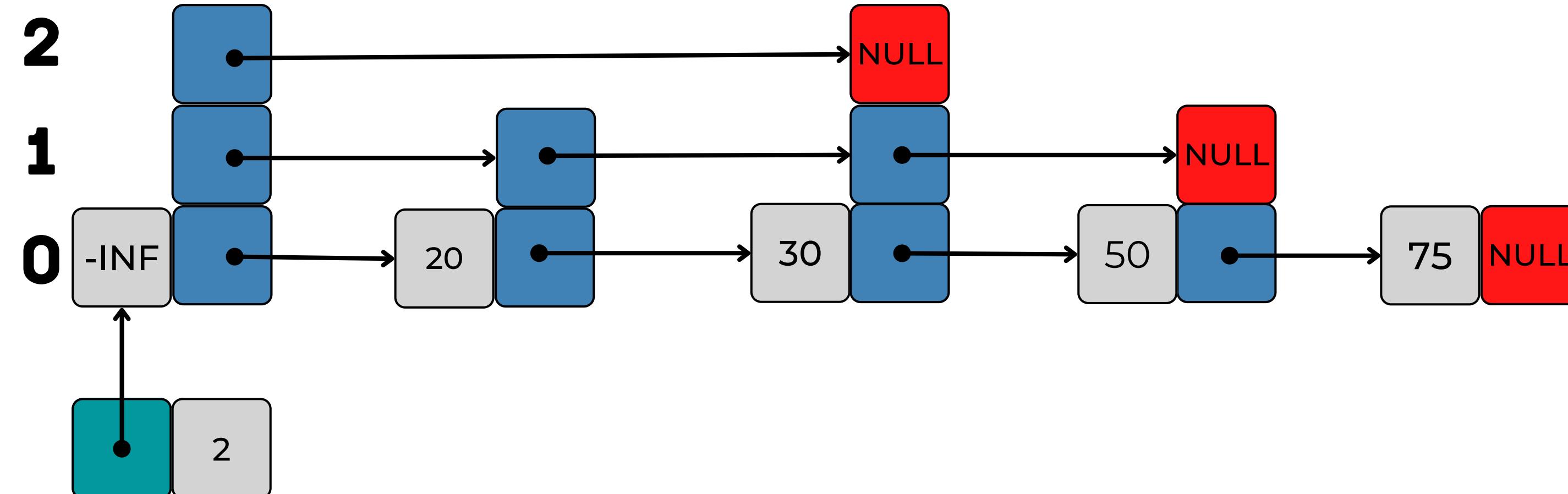
TEAM JC

Search ...



SEARCH

Level = 2



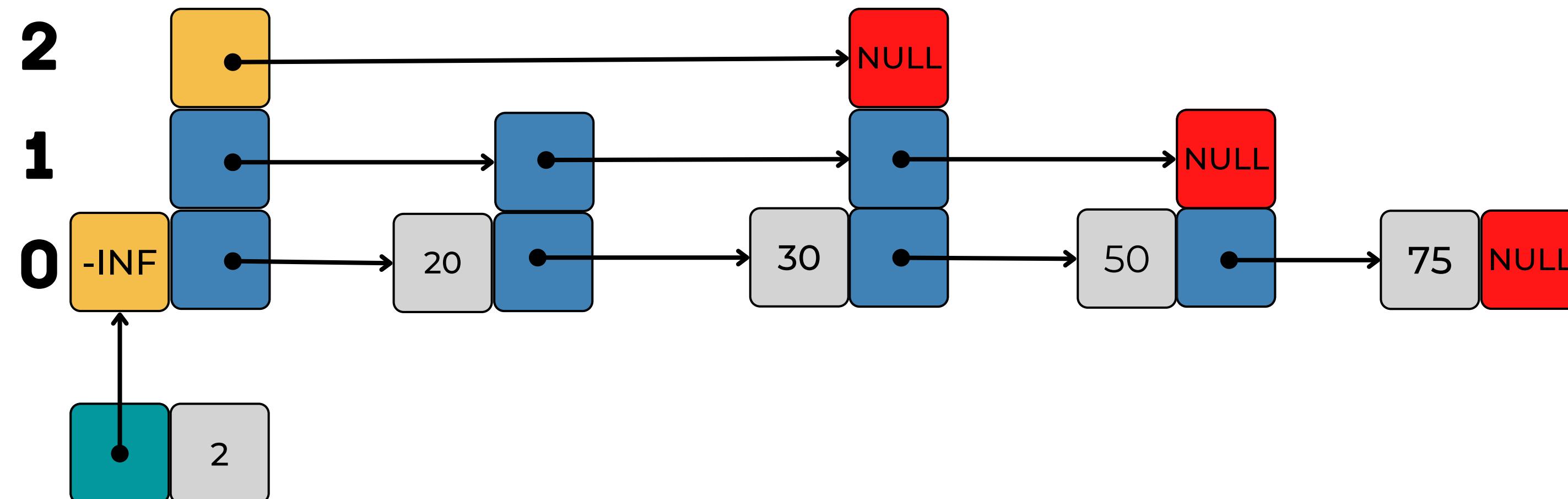
SEARCH(40)



SEARCH

Level = 2

40>30? **True** = Move to next node



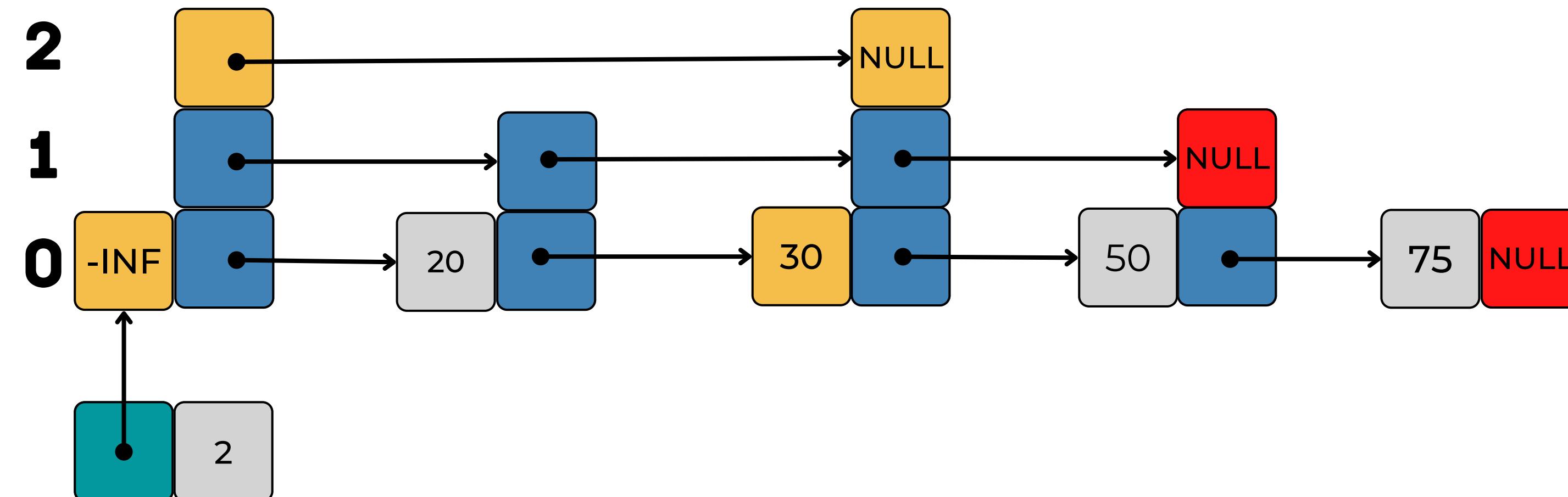
SEARCH(40)



SEARCH

Level = 2

since link is NULL? **Move down a level**



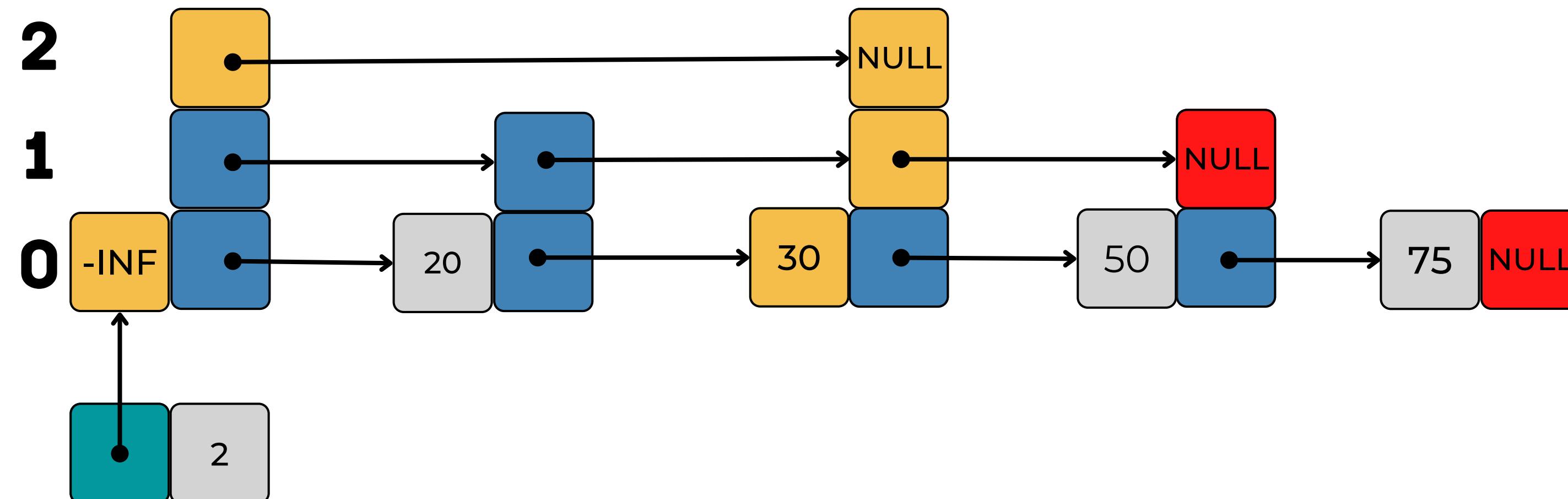
SEARCH(40)



SEARCH

Level = 1

40>50? **False** = Move down a level



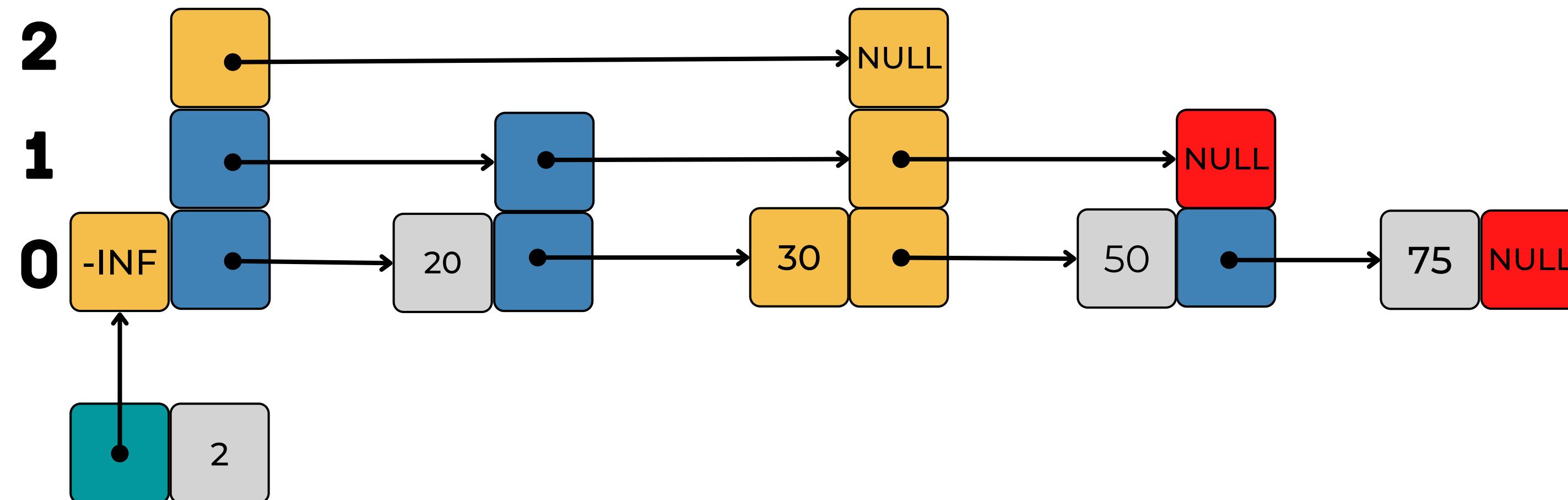
SEARCH(40)



SEARCH

Level = 0

40>50? **False** = Move down a level



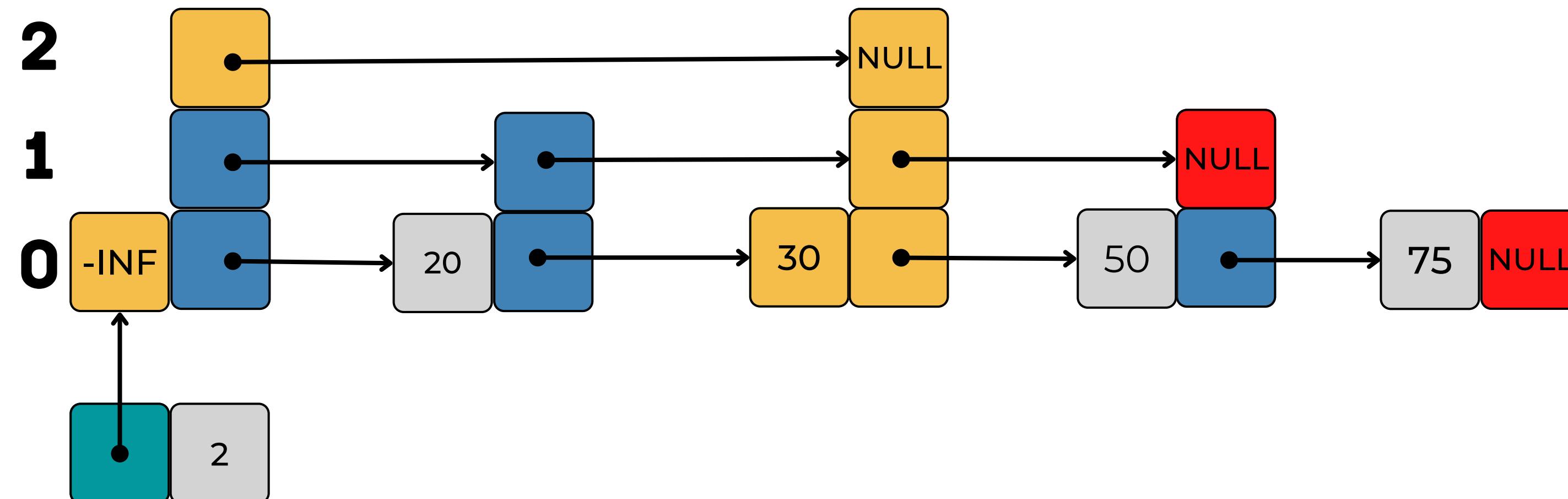
SEARCH(40)



SEARCH

Level = -1

since **level < 0** then return the next node



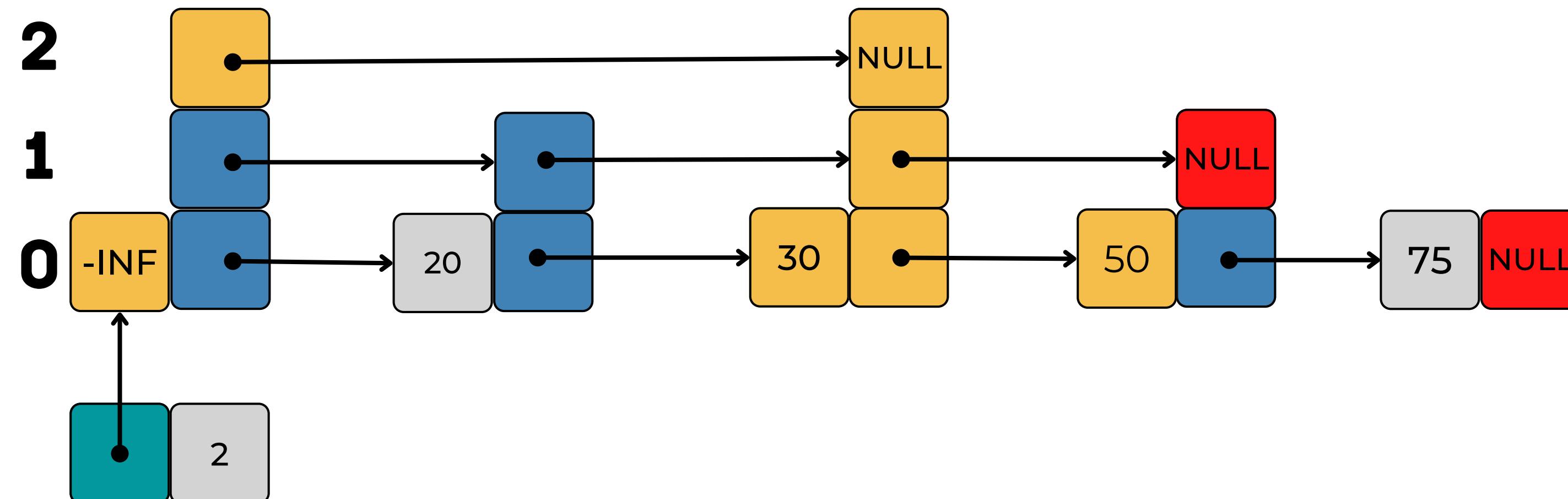
SEARCH(40)



SEARCH

Level = -1

40 == 50? **False** then **40 is not an element from the skiplist**

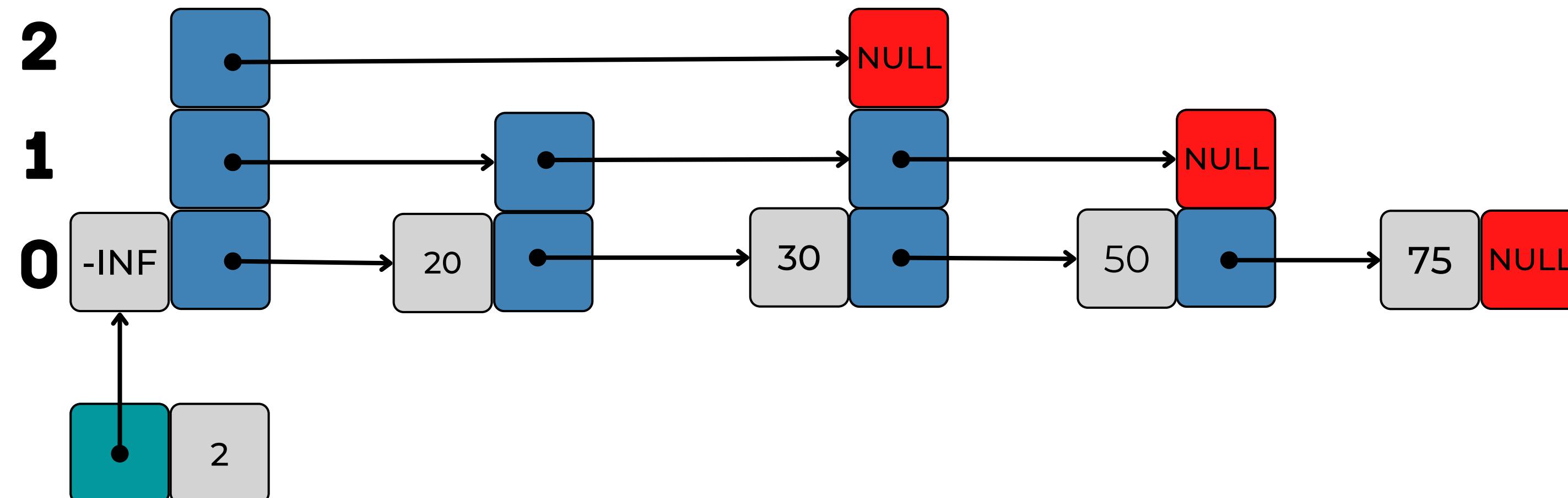


SEARCH(40)



DELETION

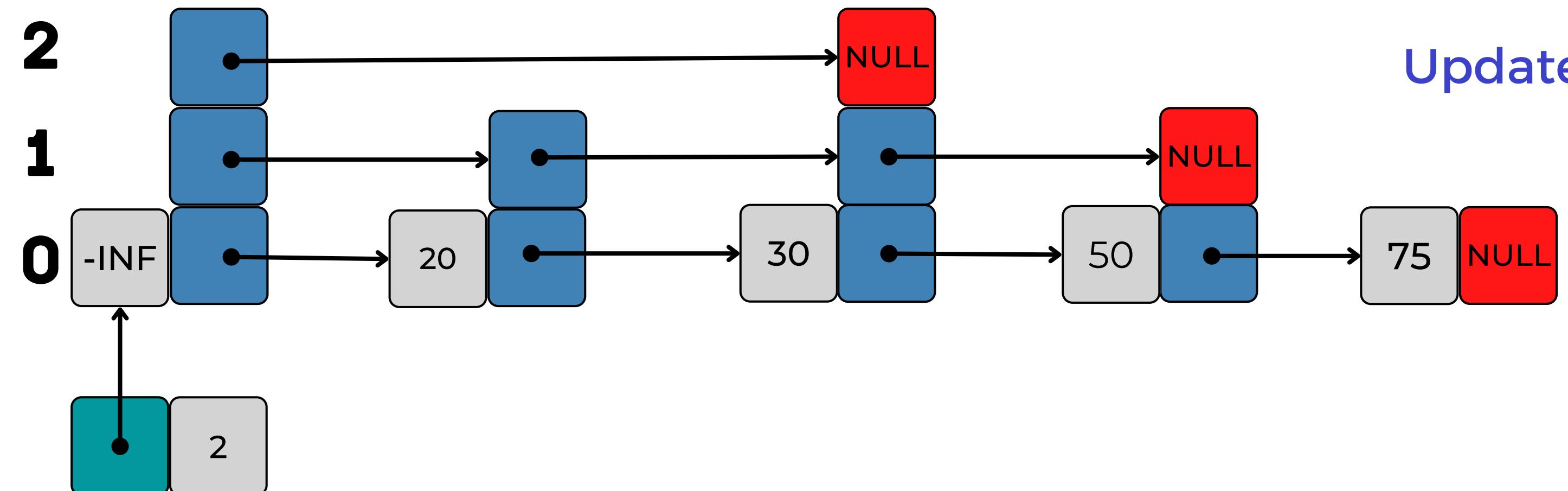
Level = 2





DELETION

Level = 2

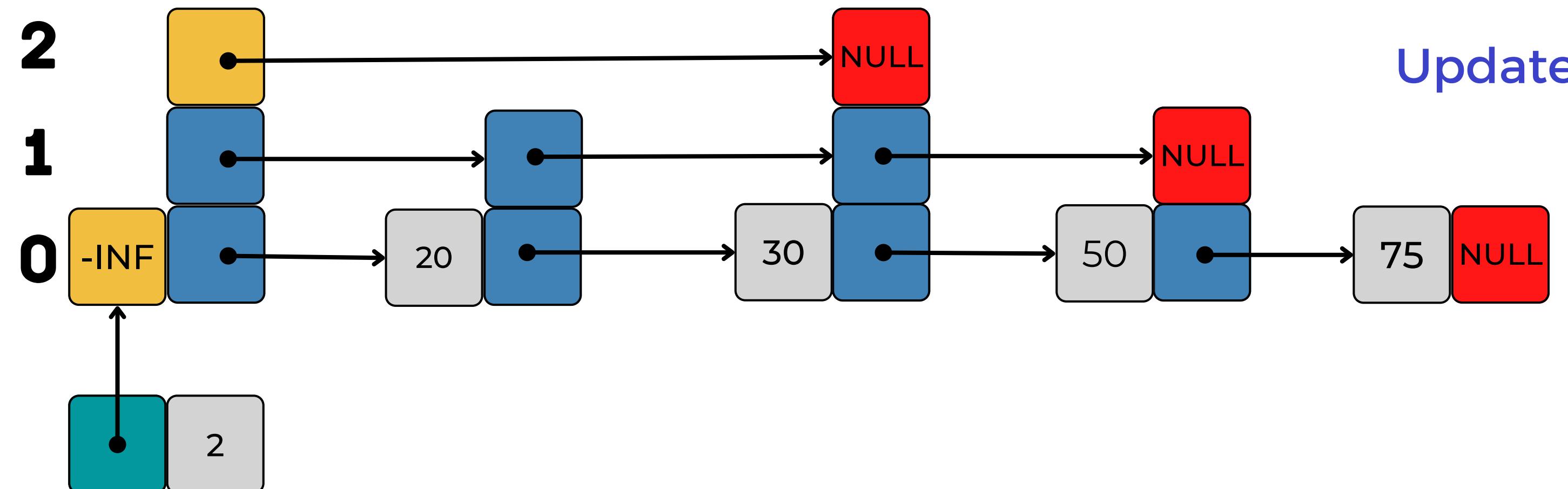




DELETION

Level = 2

50>30?



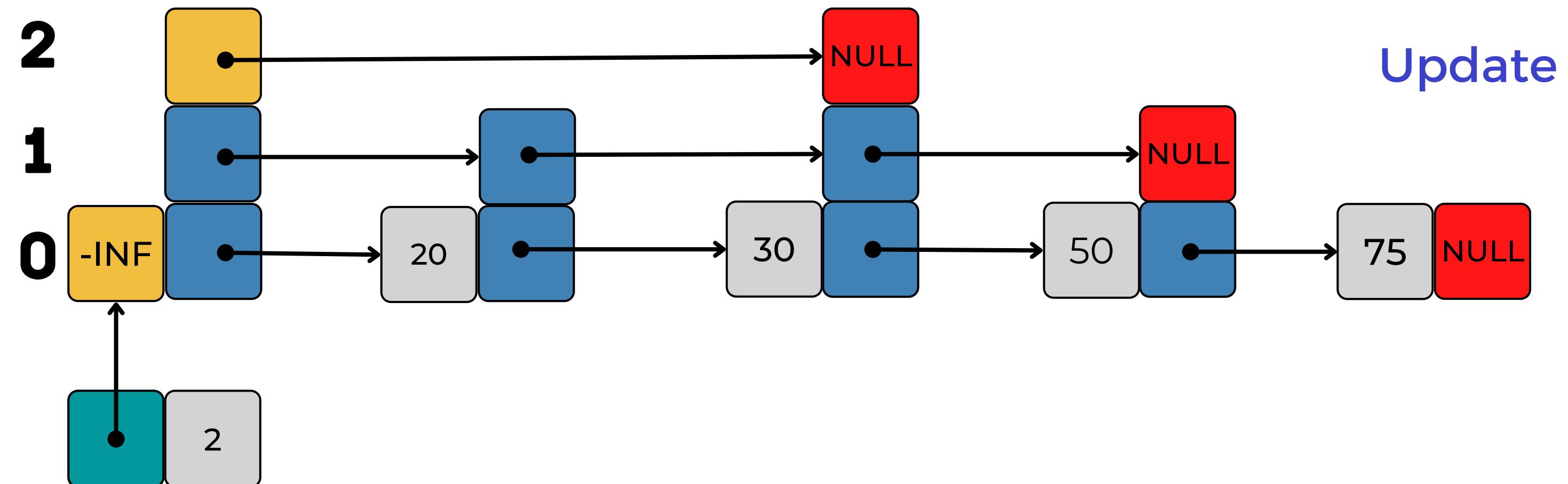
DELETE(50)



DELETION

Level = 2

50>30? **True** = Move to next node



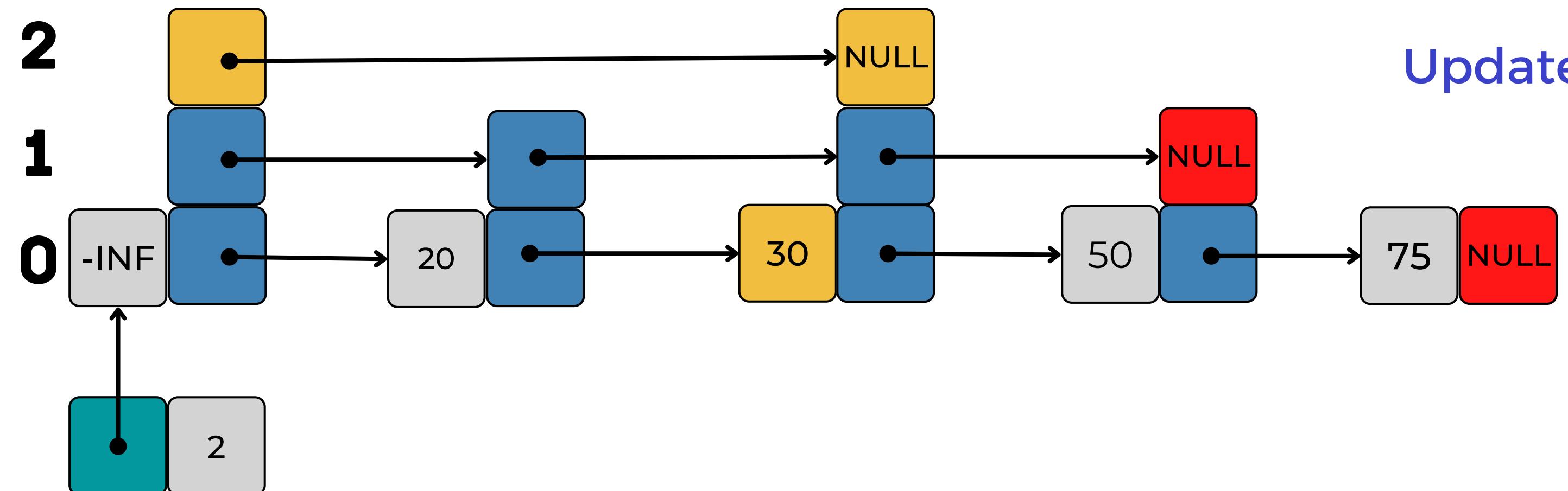
DELETE(50)



DELETION

Level = 2

NULL?

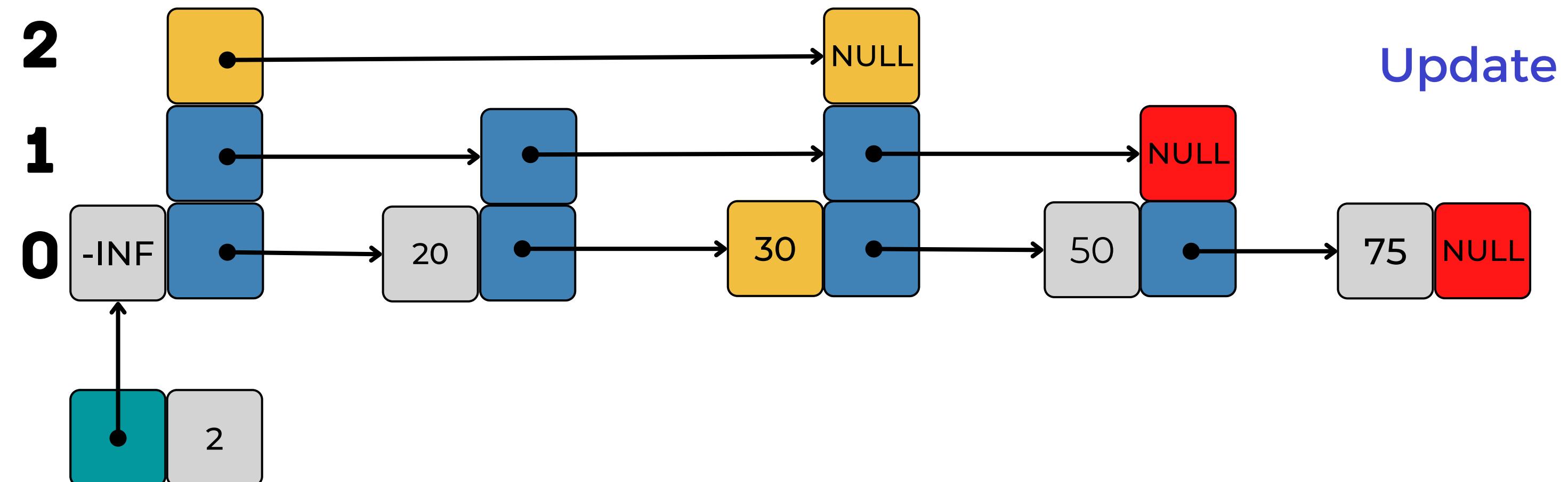




DELETION

Level = 2

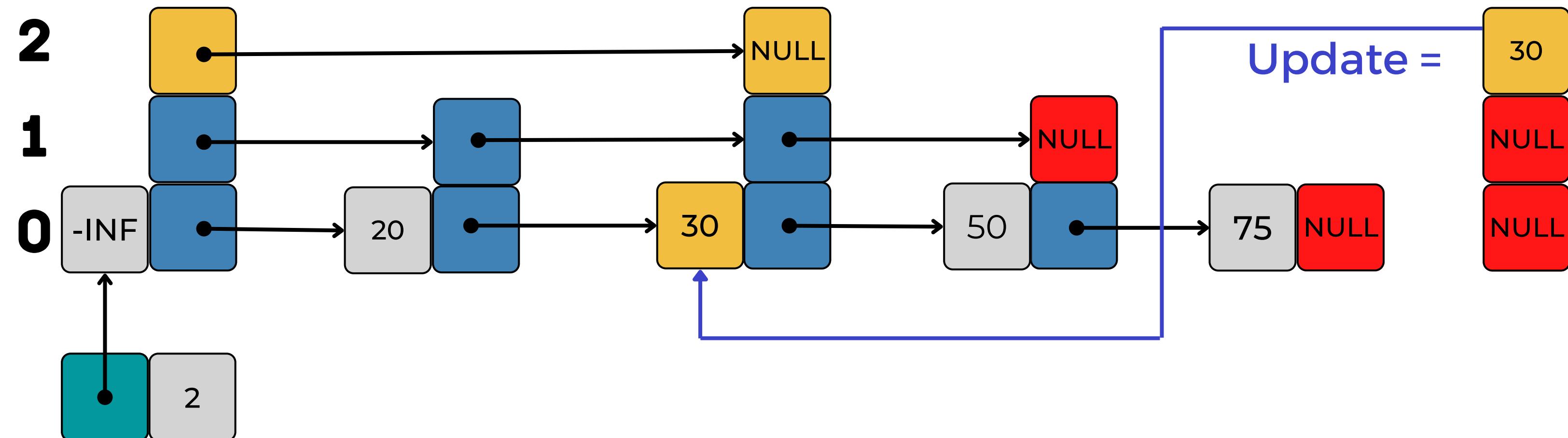
NULL? **True** = Move down a level





DELETION

Level = 2

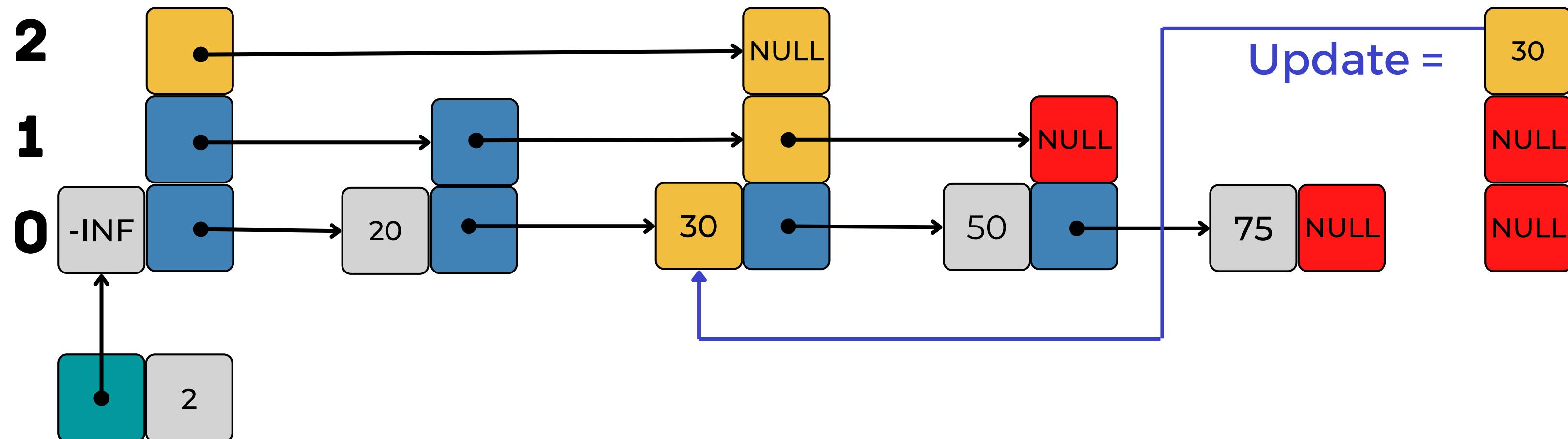




DELETION

Level = 1

50>50?



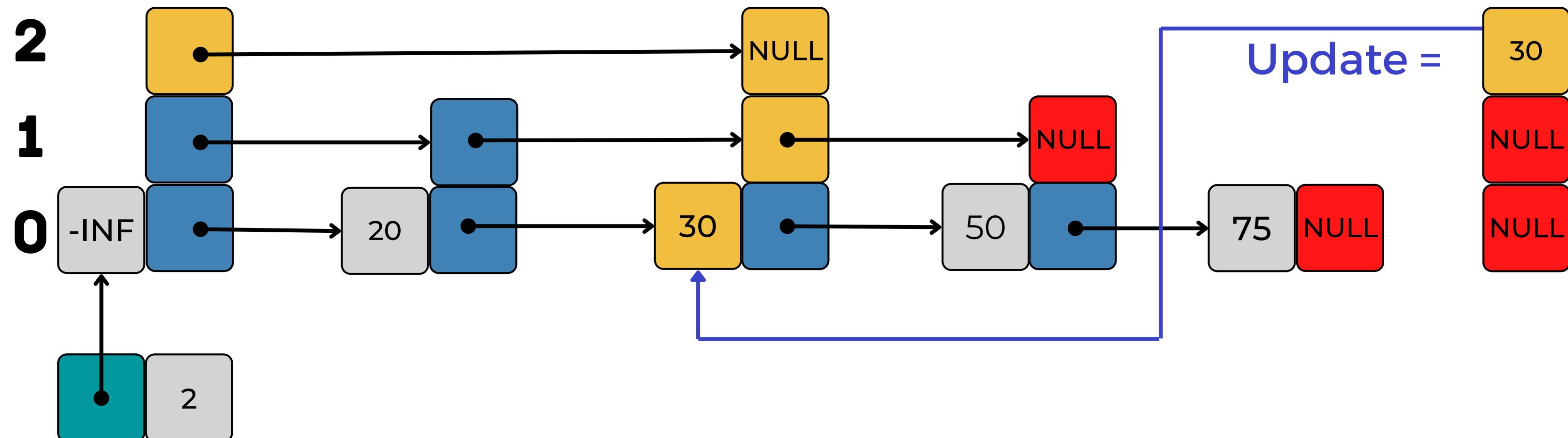
DELETE(50)



DELETION

Level = 1

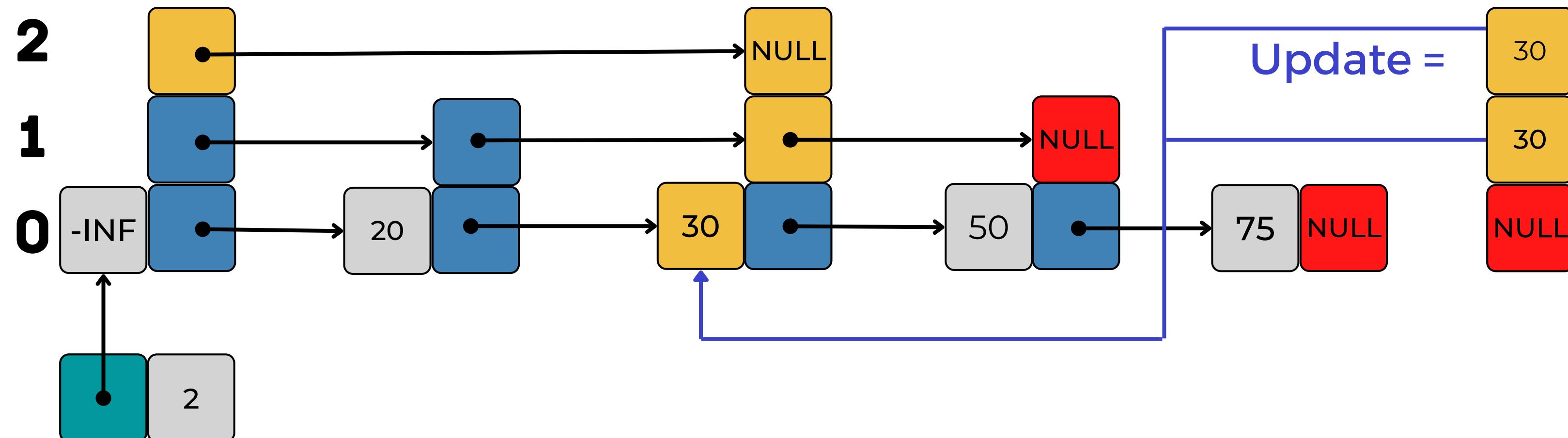
50>50? **False** = Move down a level





DELETION

Level = 1

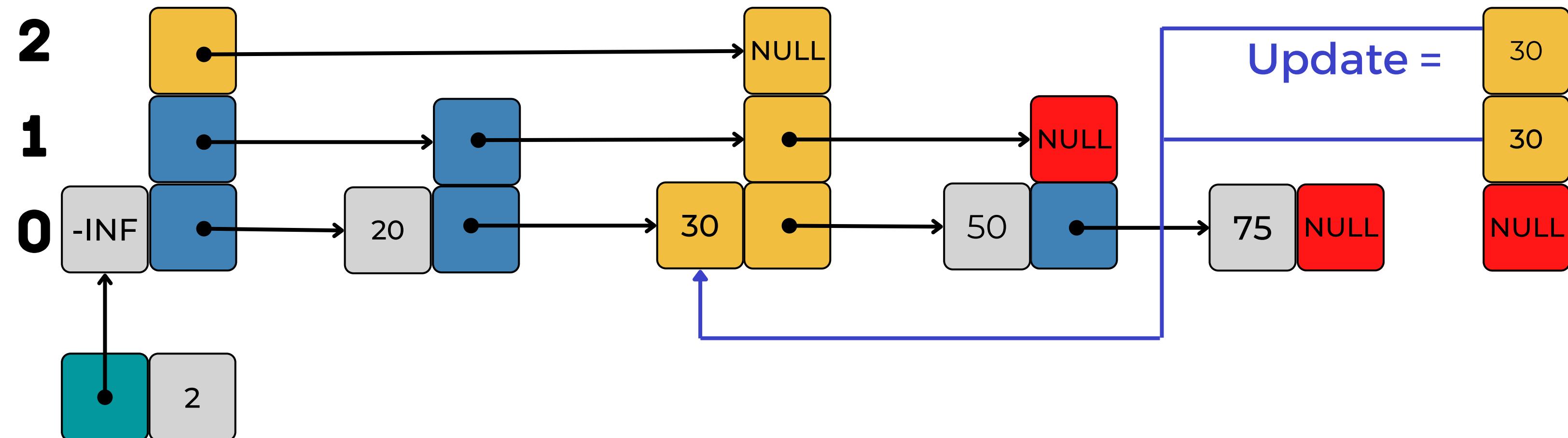


DELETE(50)

DELETION

Level = 0

50>50?



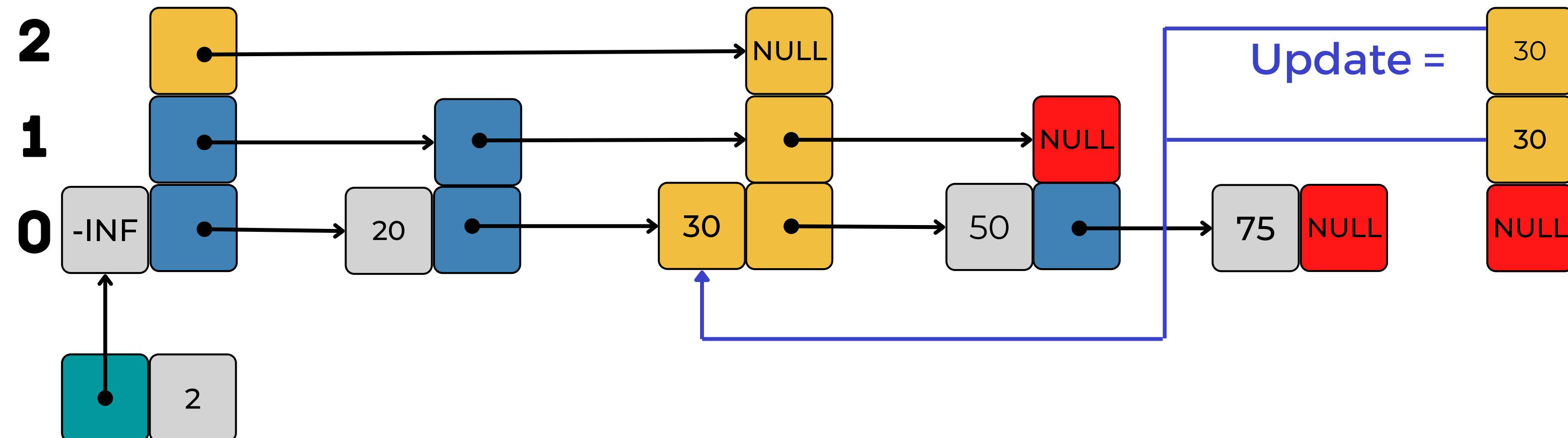
DELETE(50)



DELETION

Level = 0

50>50? **False** = Move down a level

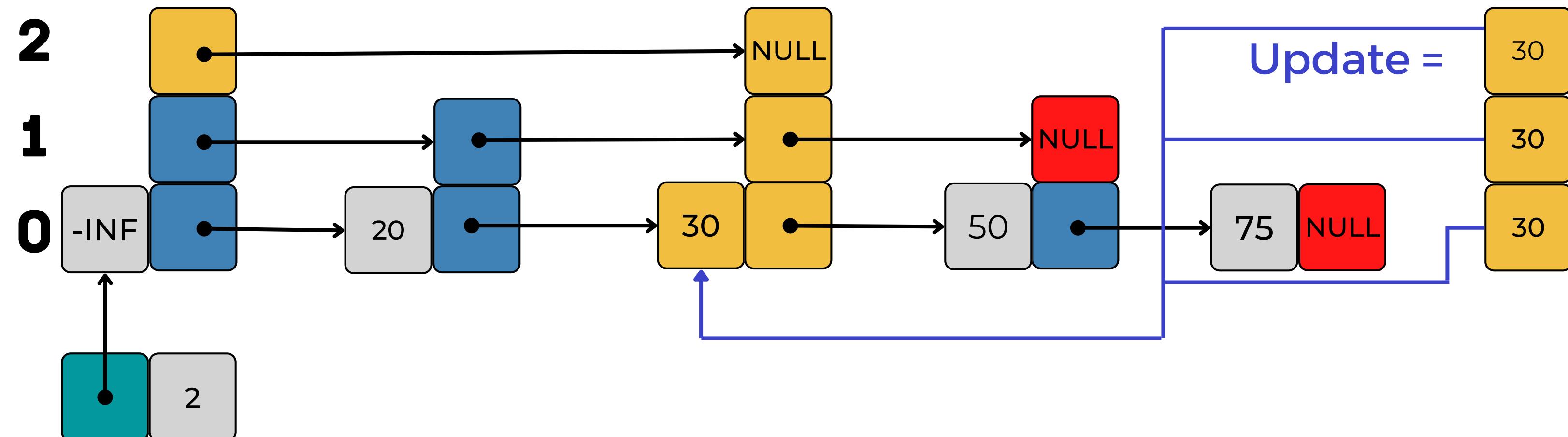


DELETE(50)



DELETION

Level = -1

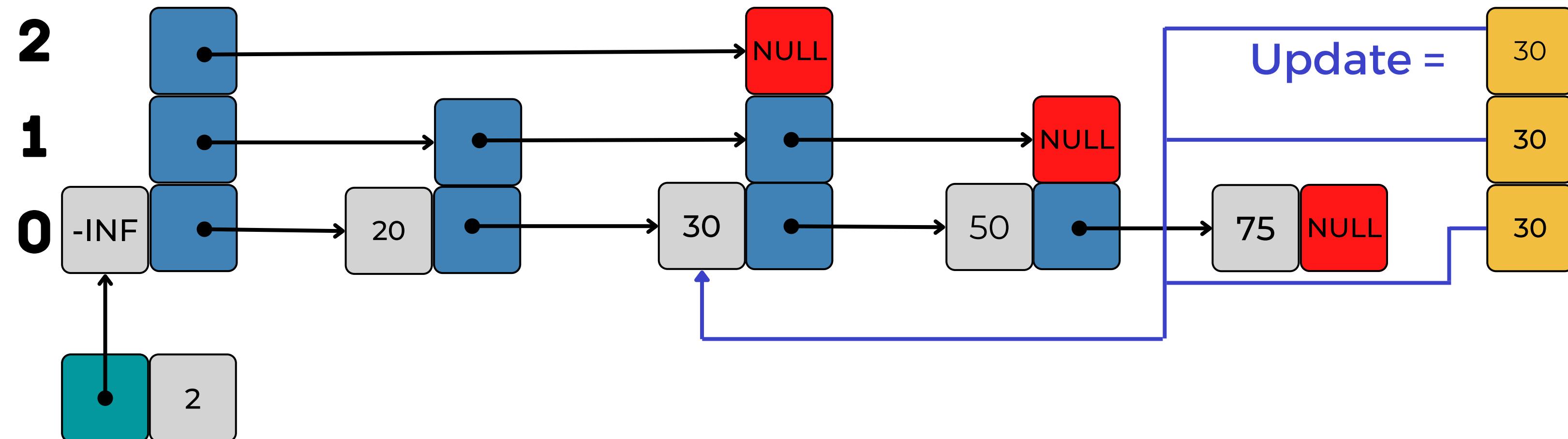




DELETION

Level = -1

Level -1 meaning end of traversal



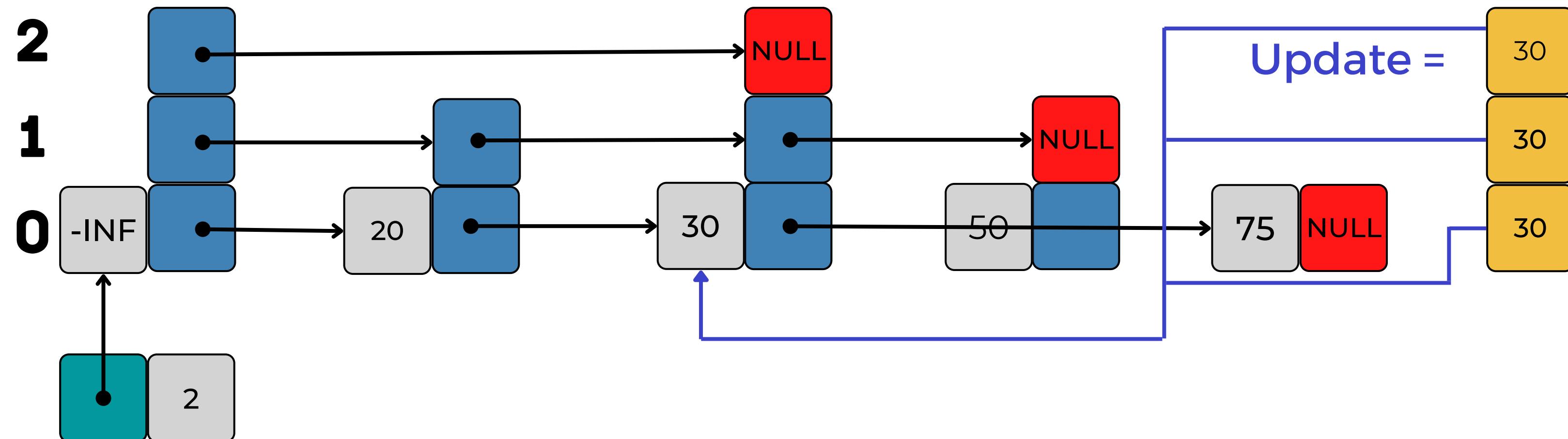
DELETE(50)



DELETION

Level = -1

Connect update array node to node after



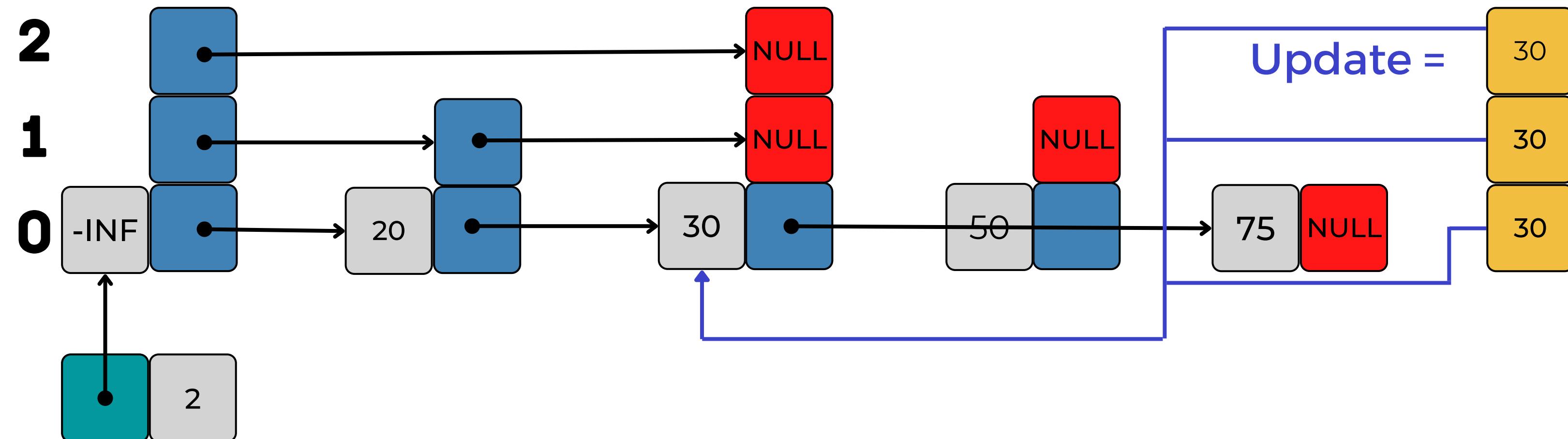
DELETE(50)



DELETION

Level = -1

Connect update array node to node after



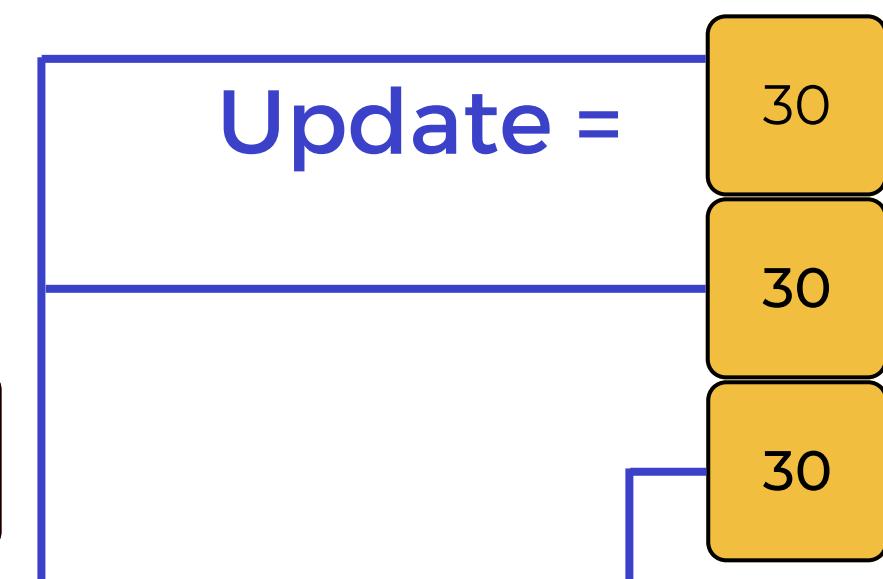
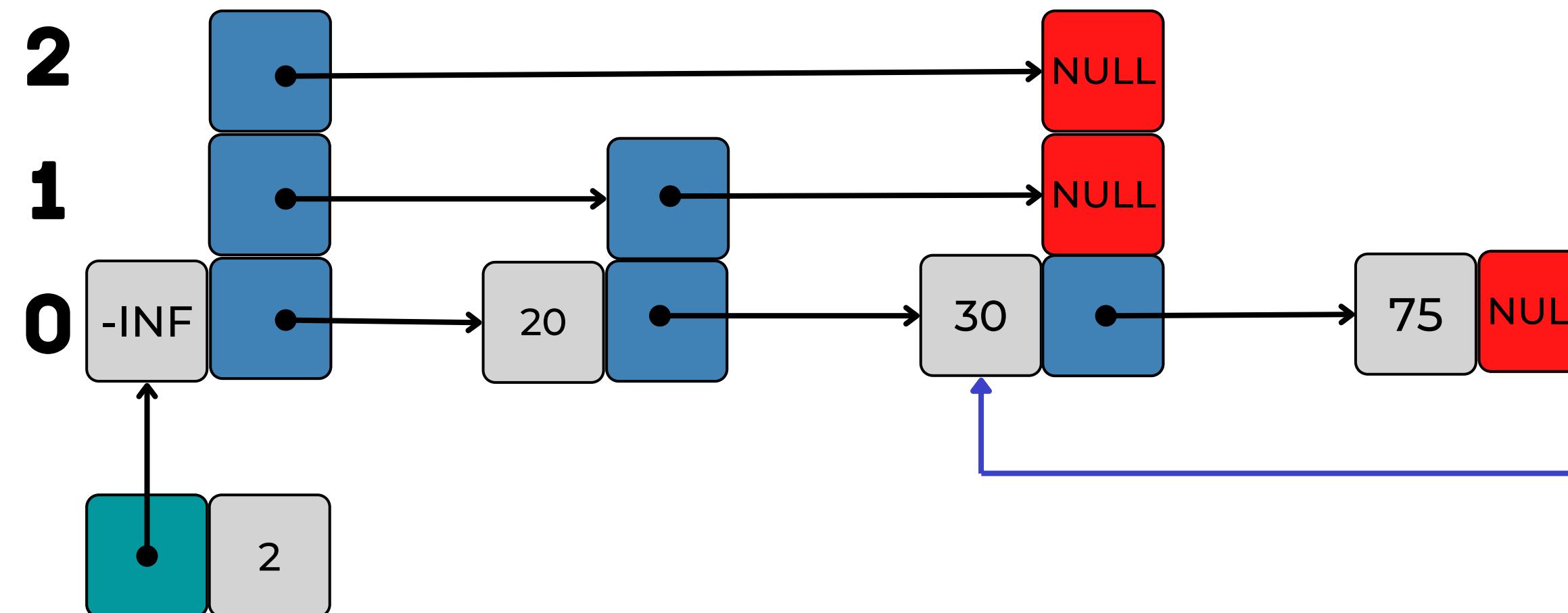
DELETE(50)



DELETION

Level = -1

node 50 is now deleted



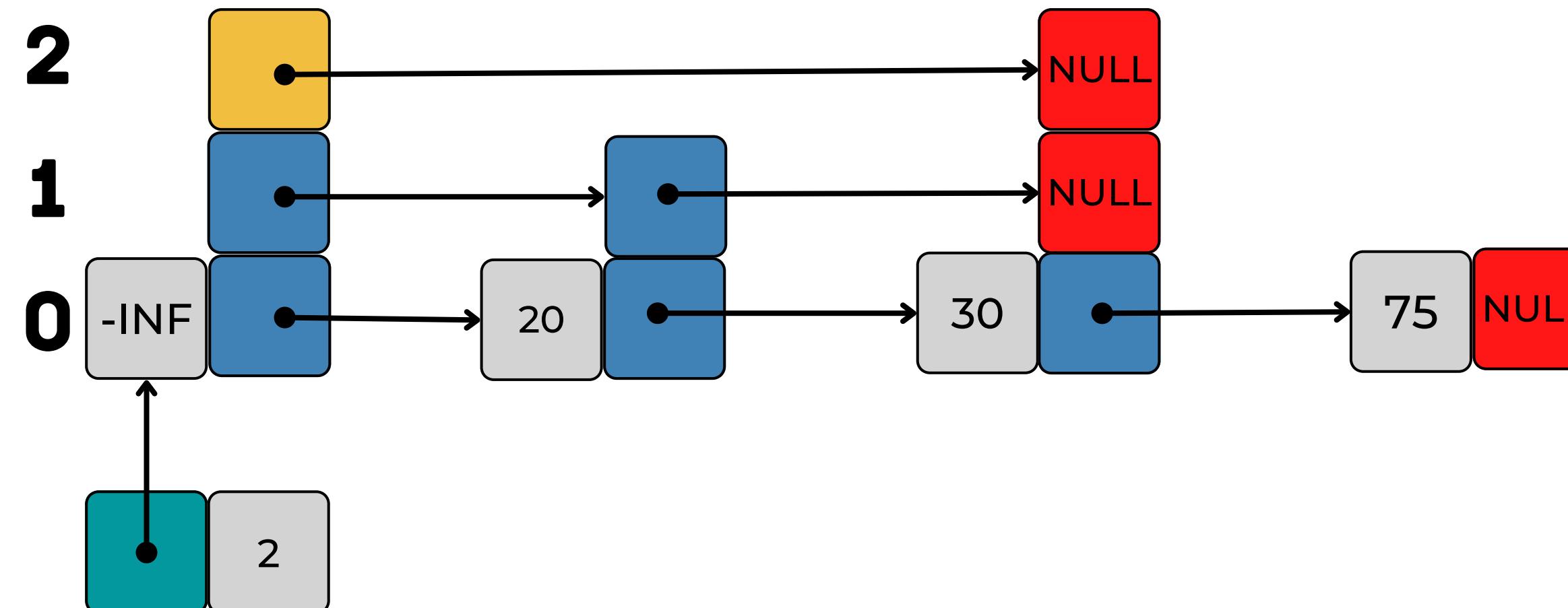
DELETE(50)



DELETION

Level = -1

Is head link (lvl 2) null?



DELETE(50)

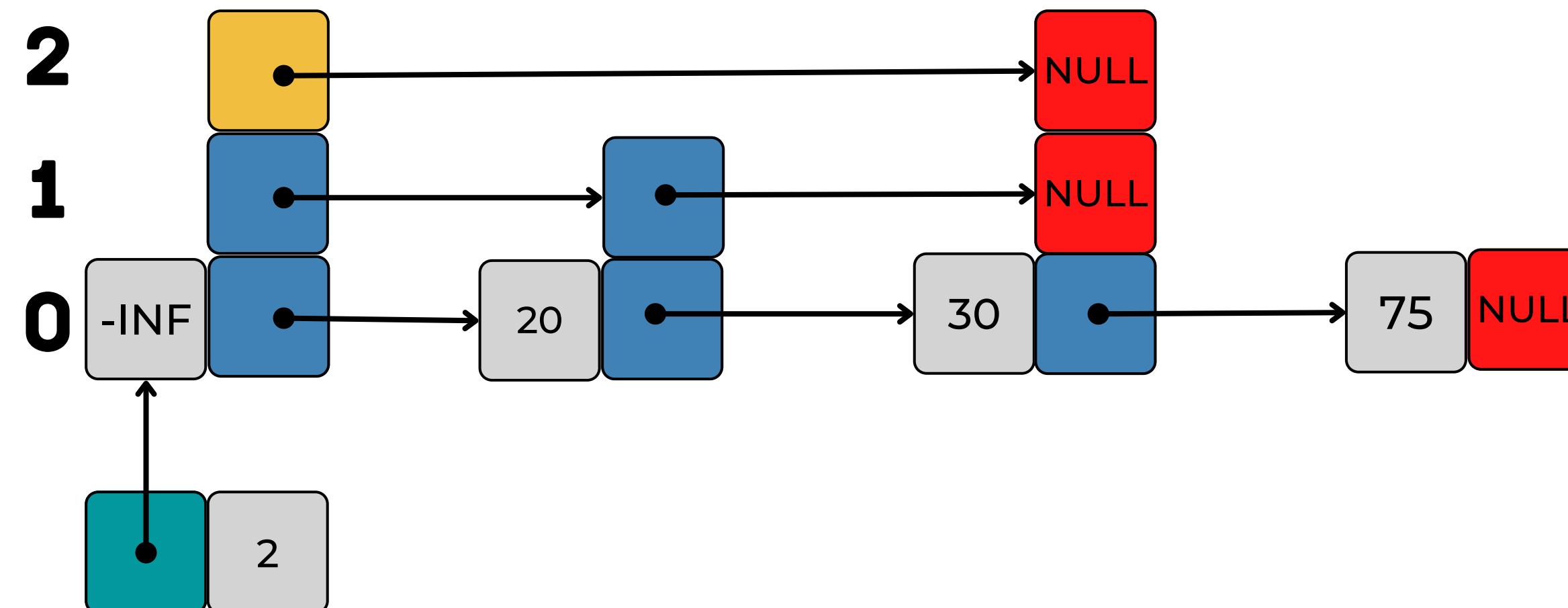


DELETION

Level = -1

Is head link (lvl 2) null?

False = End traversal

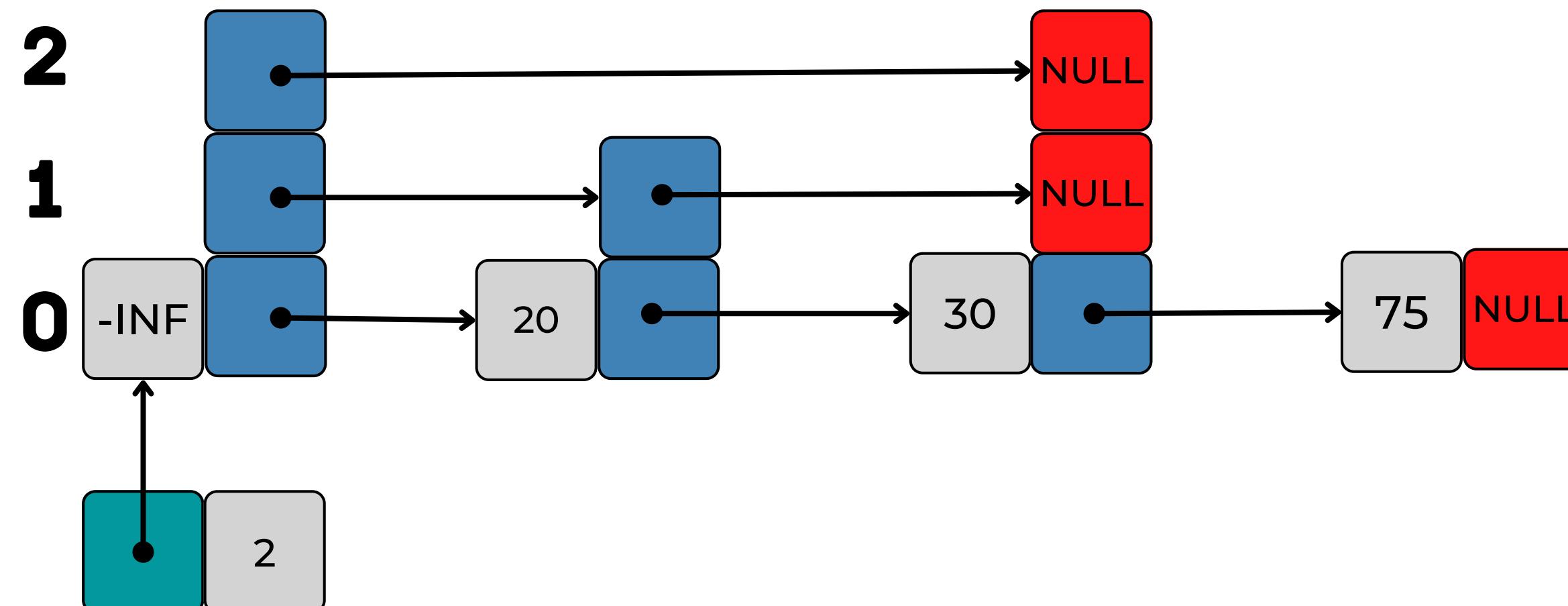


DELETE(50)



DELETION

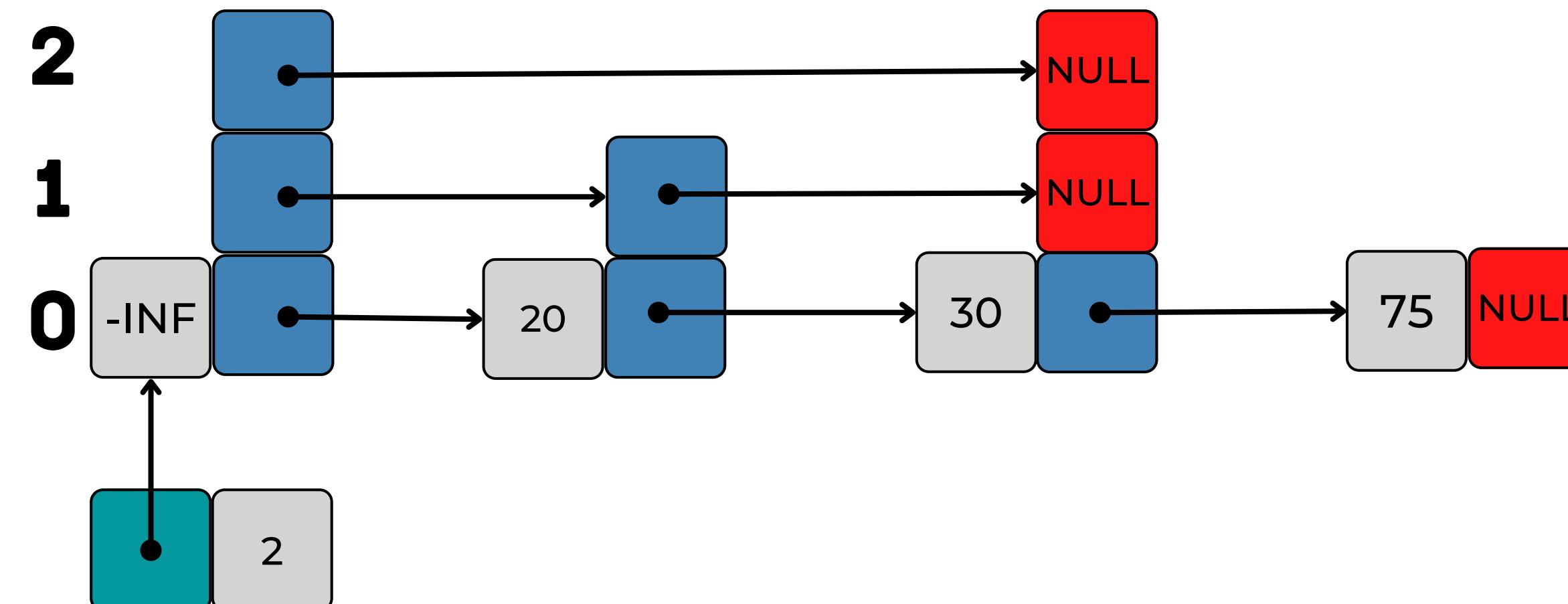
Level = 2





DELETION

Level = 2



DELETE(30)

Update =

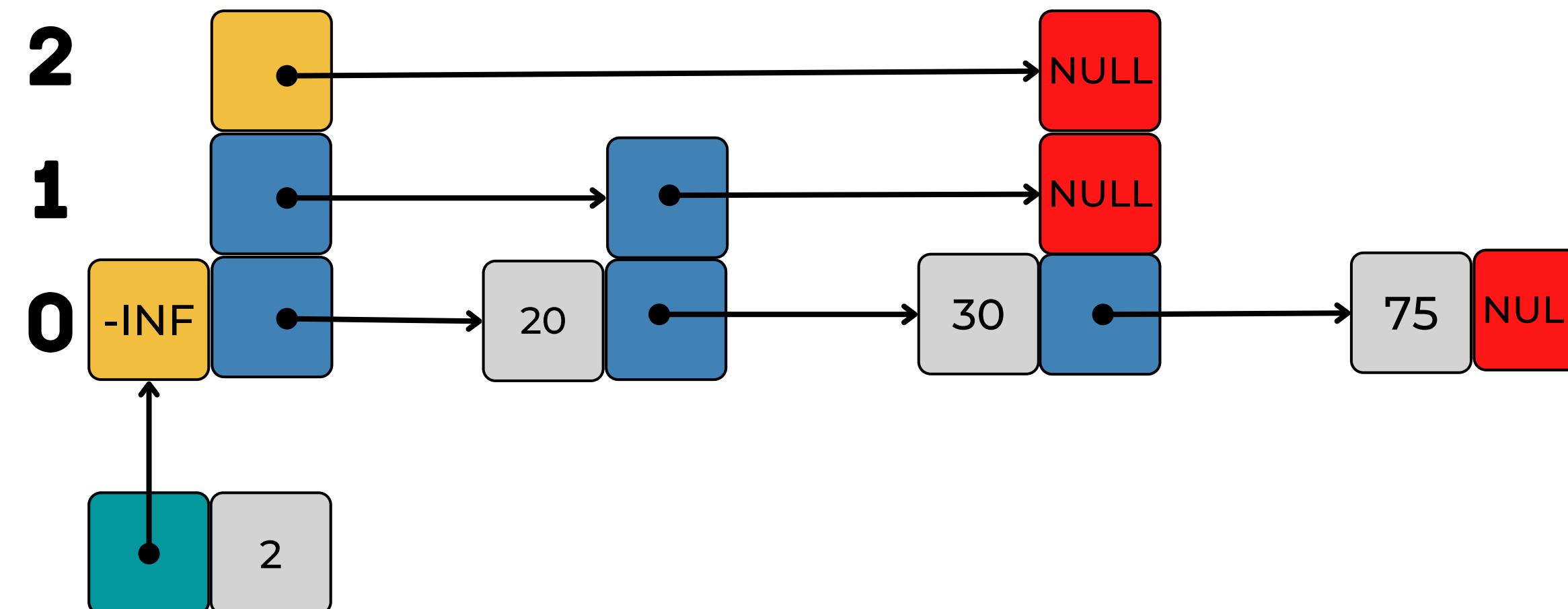




DELETION

Level = 2

30>30?



DELETE(30)

Update =

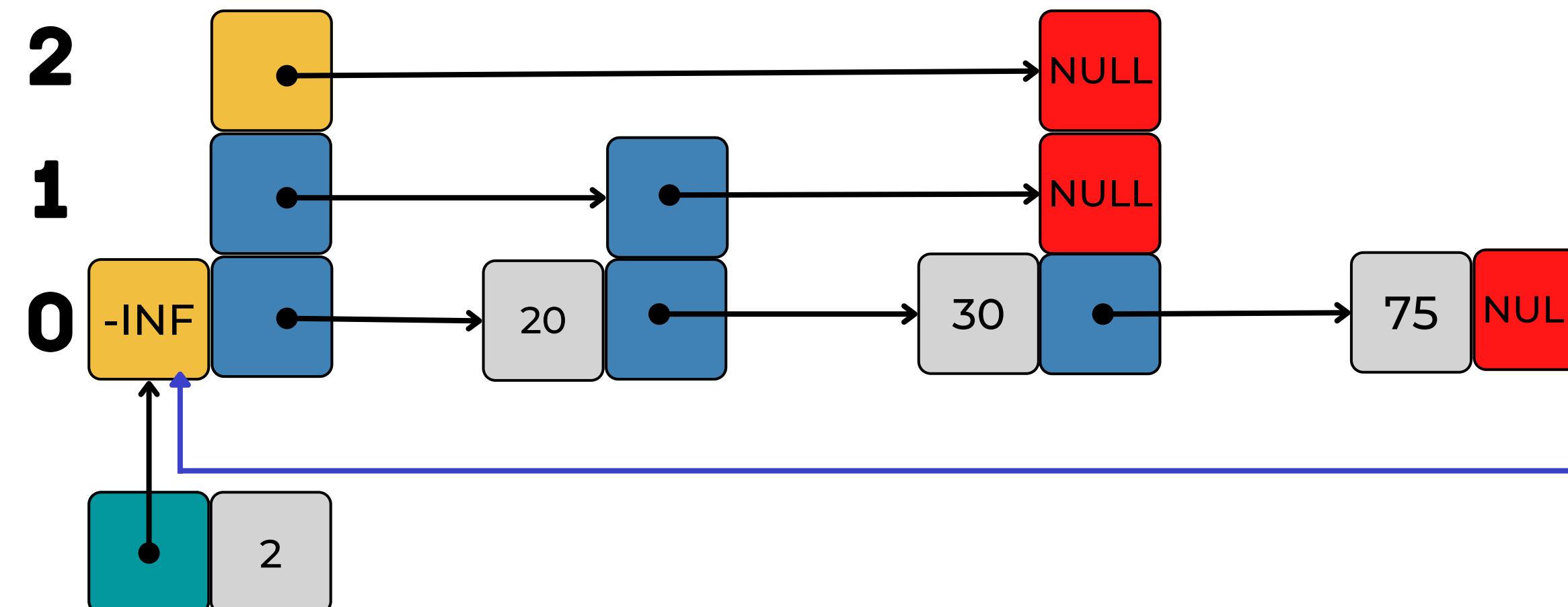




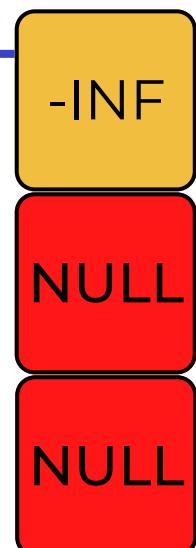
DELETION

Level = 2

30>30? **True** = Move to next node



Update =

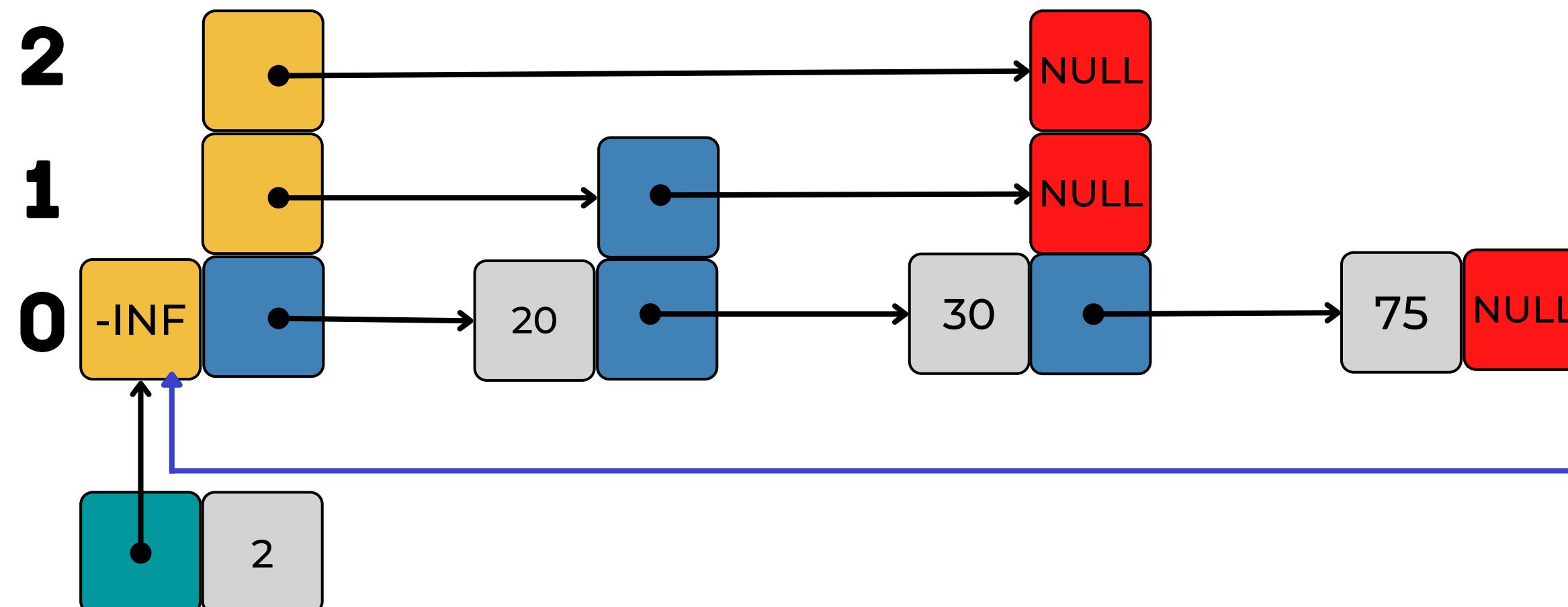


DELETE(30)

DELETION

Level = 1

30>20?



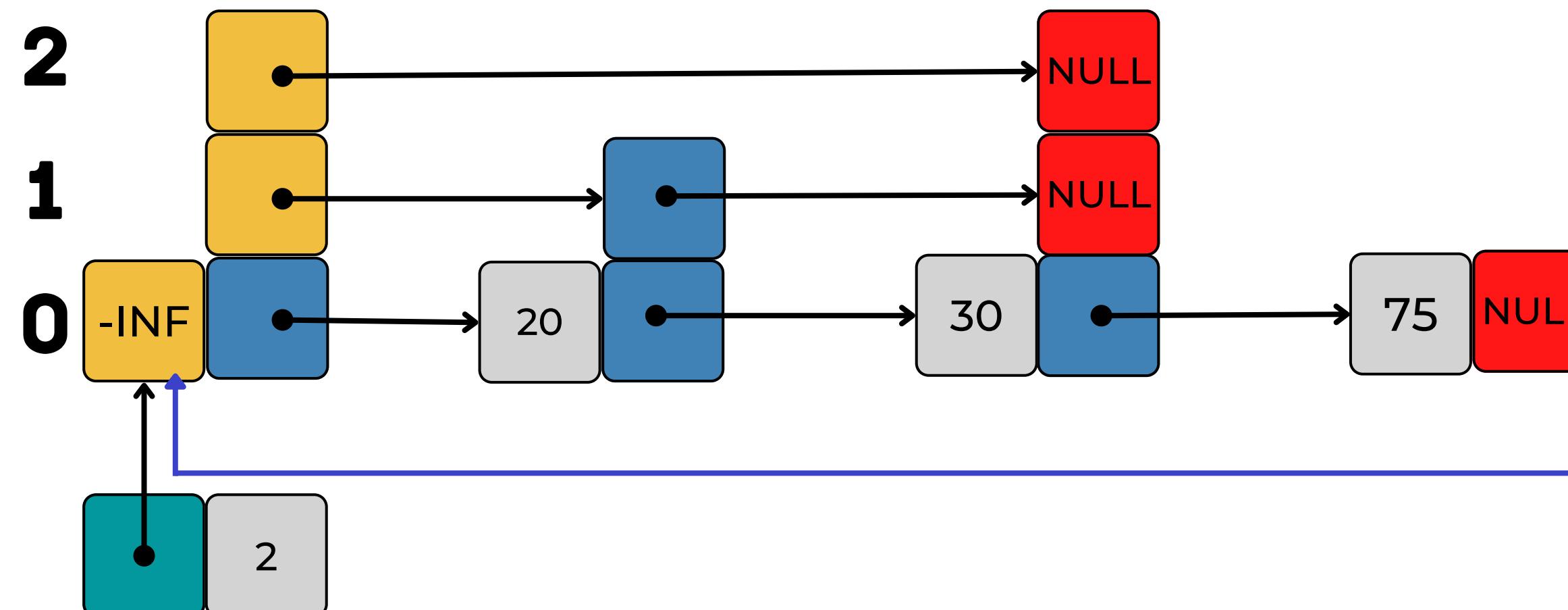
DELETE(30)



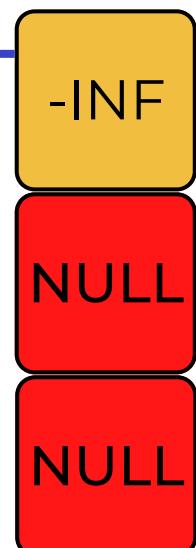
DELETION

Level = 1

30>20? **True** = Move to next node



Update =



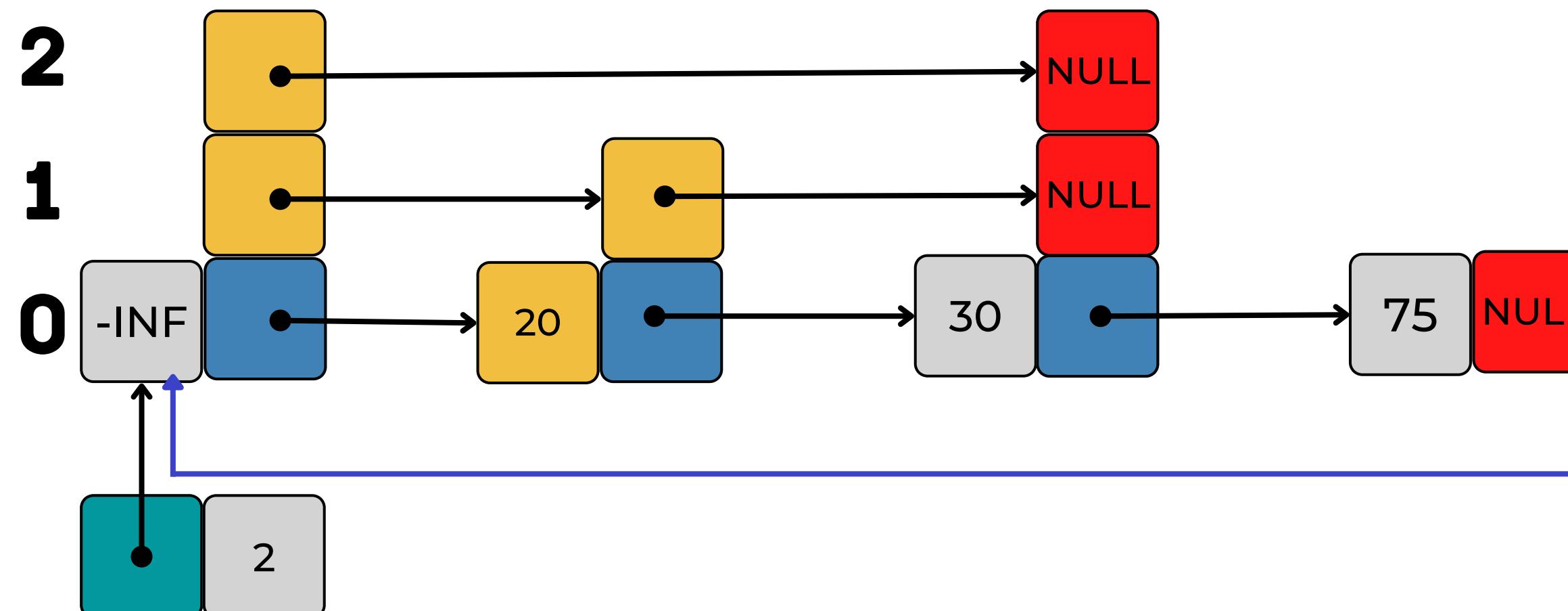
DELETE(30)



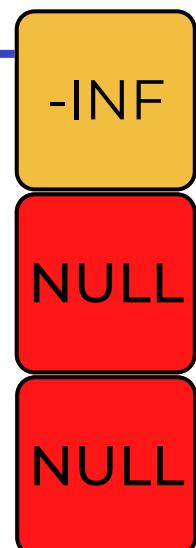
DELETION

Level = 1

30>30?



Update =



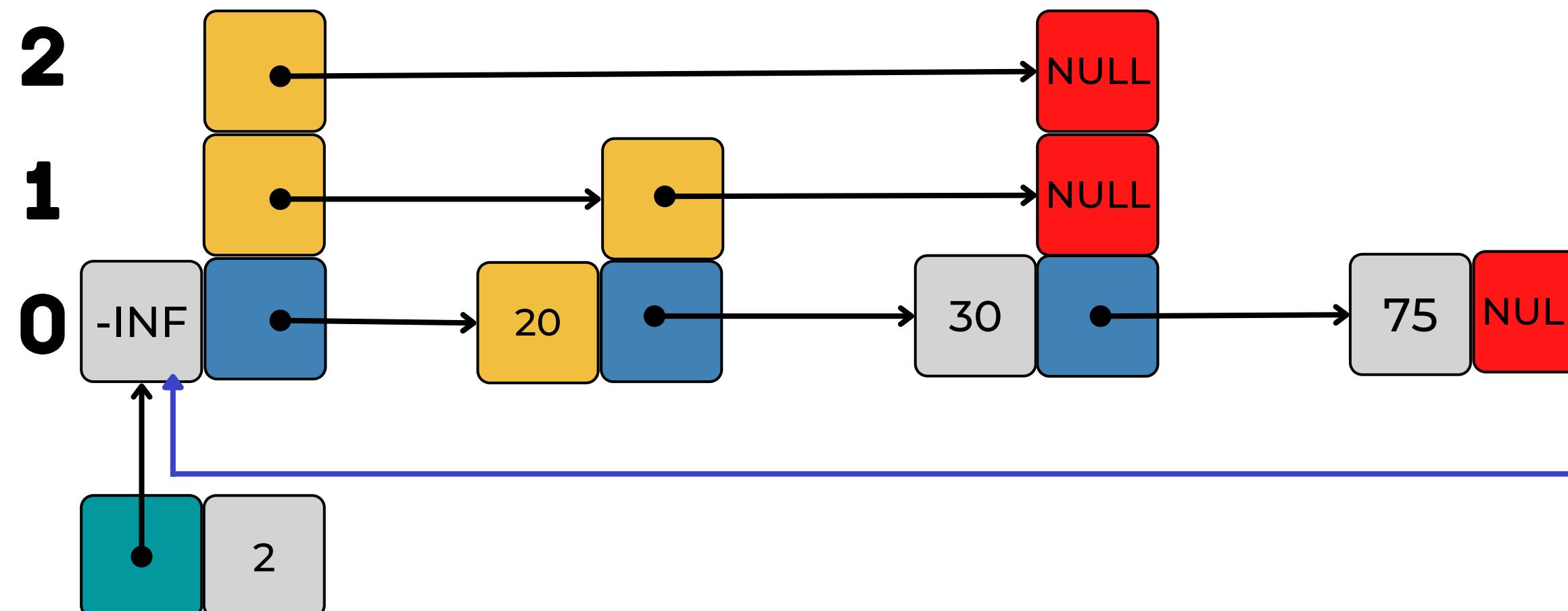
DELETE(30)



DELETION

Level = 1

30>30? False = Move down a level



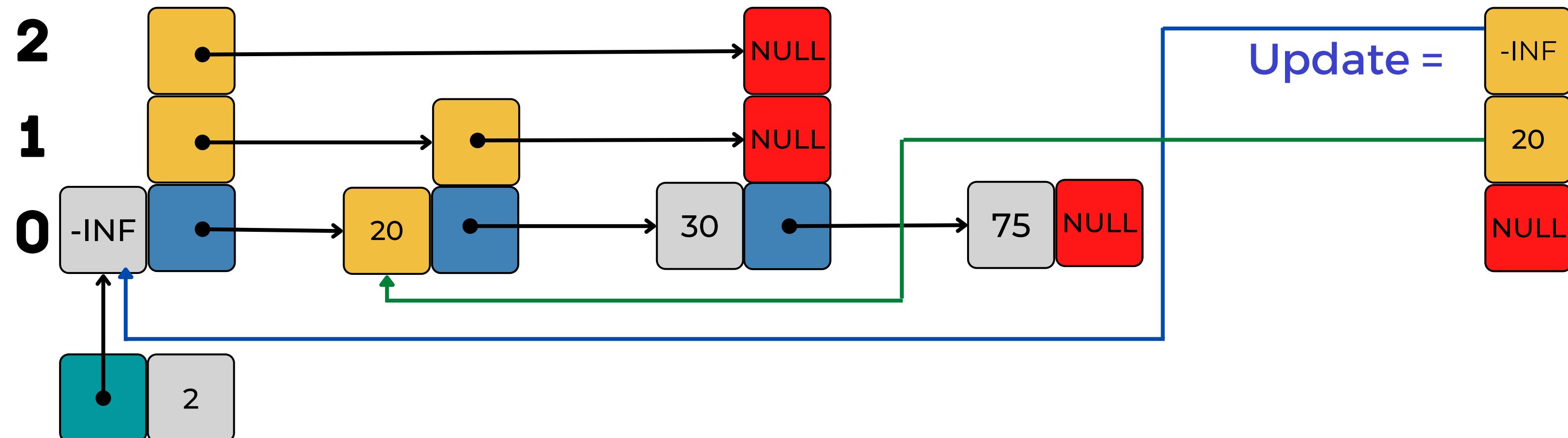
Update =





DELETION

Level = 1



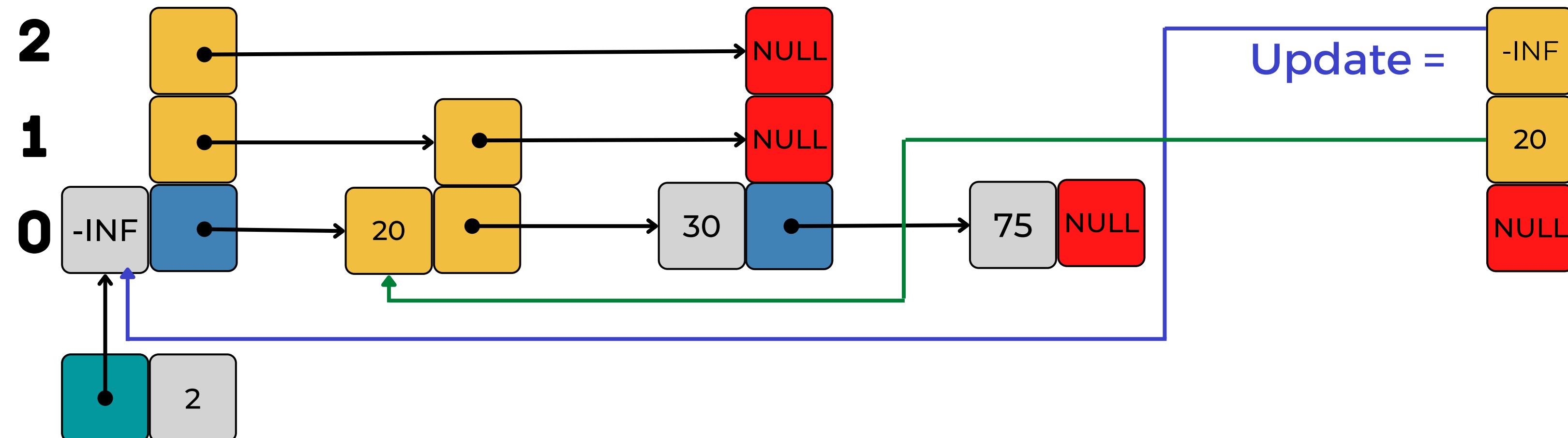
DELETE(30)



DELETION

Level = 0

30>30?



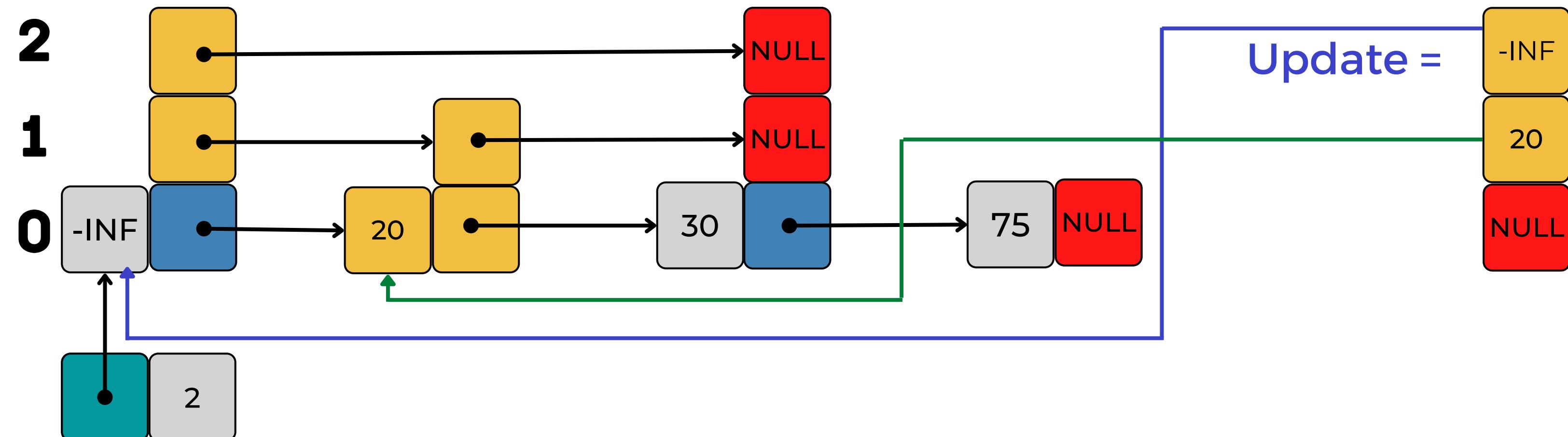
DELETE(30)



DELETION

Level = 0

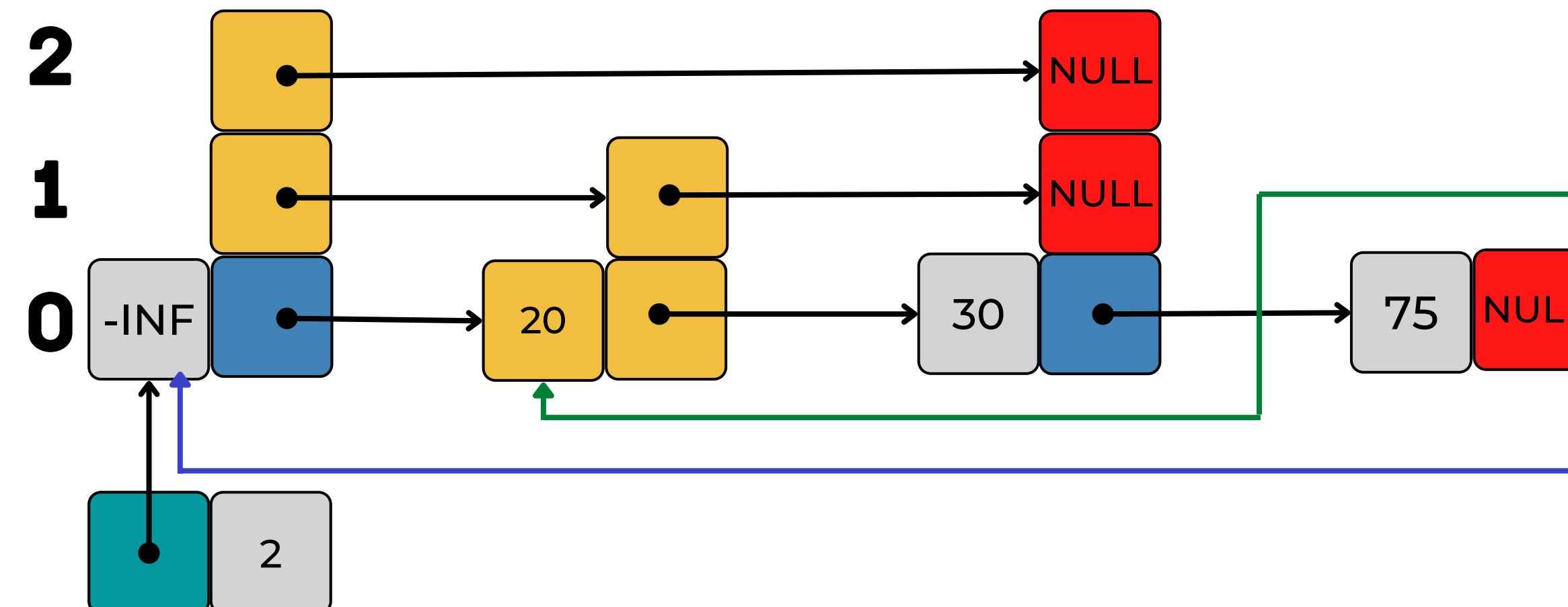
$30 > 30?$ False = Move down a level





DELETION

Level = 0

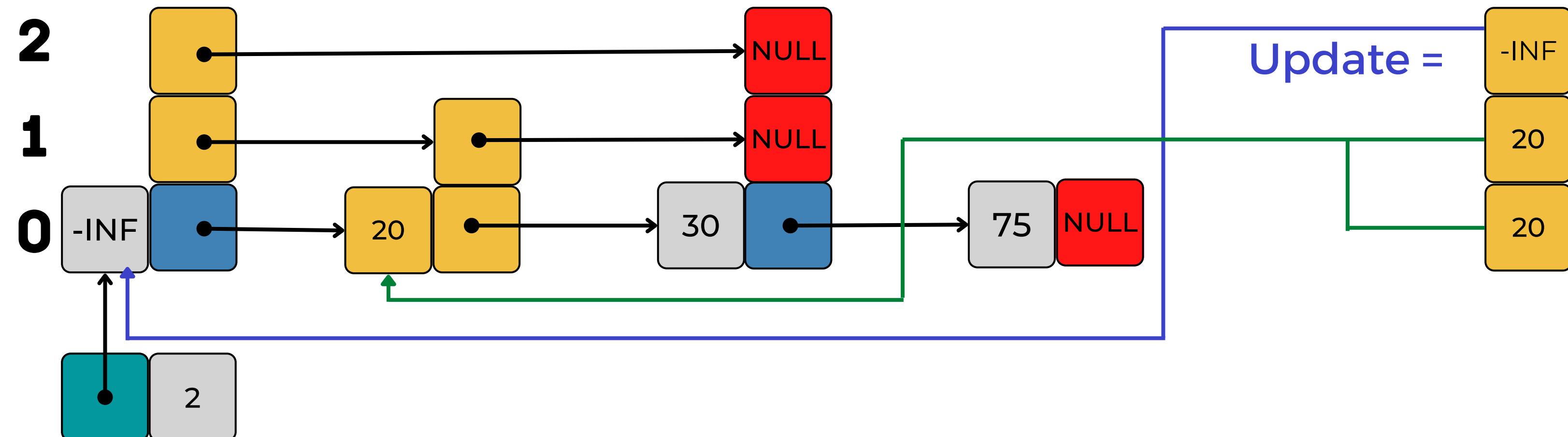


DELETE(30)

DELETION

Level = -1

Level -1 meaning end of traversal

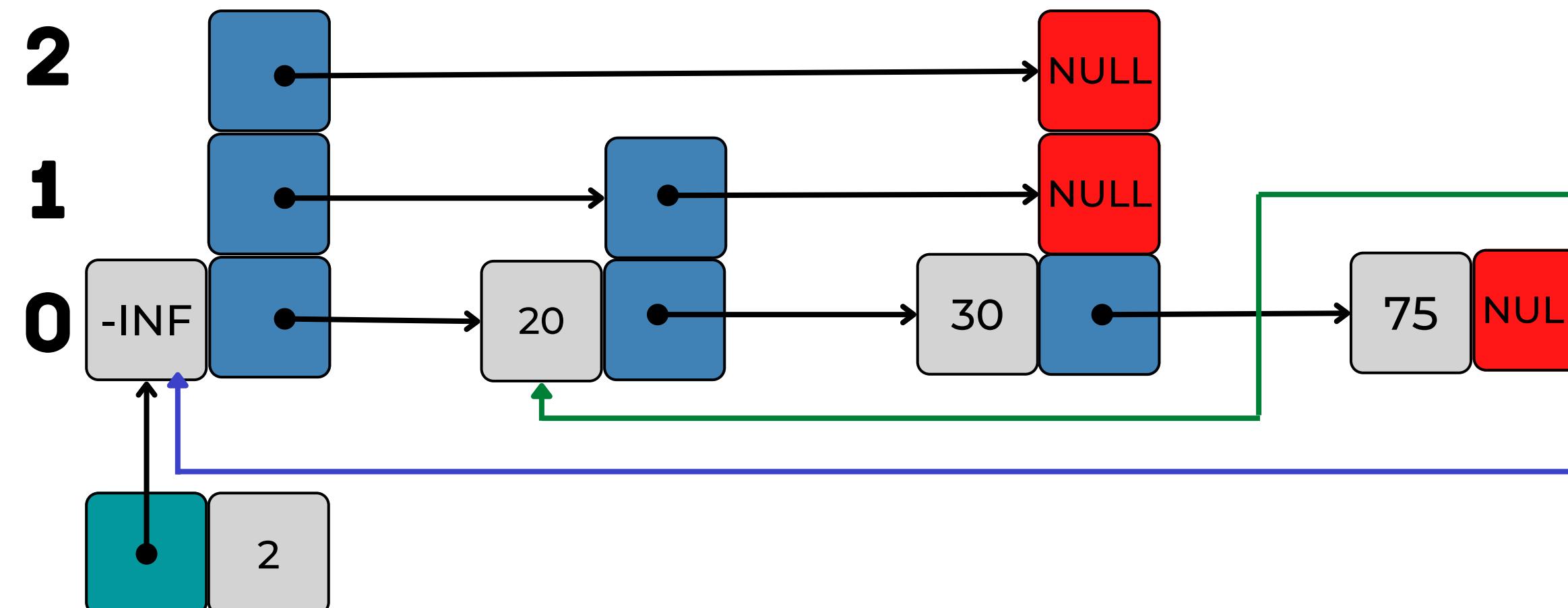


DELETE(30)



DELETION

Level = -1



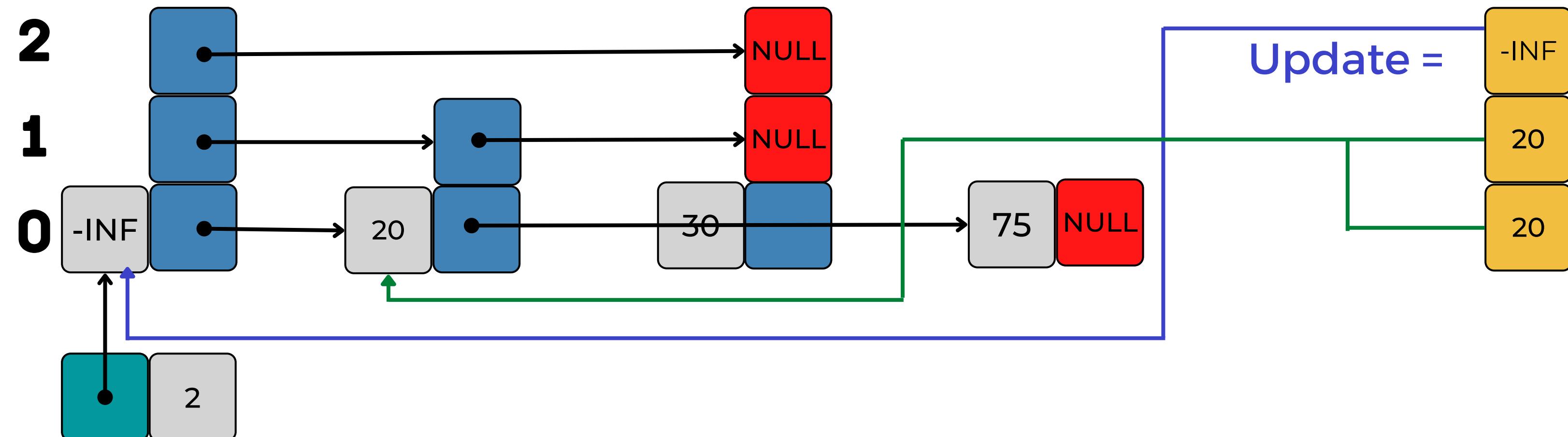
DELETE(30)



DELETION

Level = -1

Connect update array node to node after



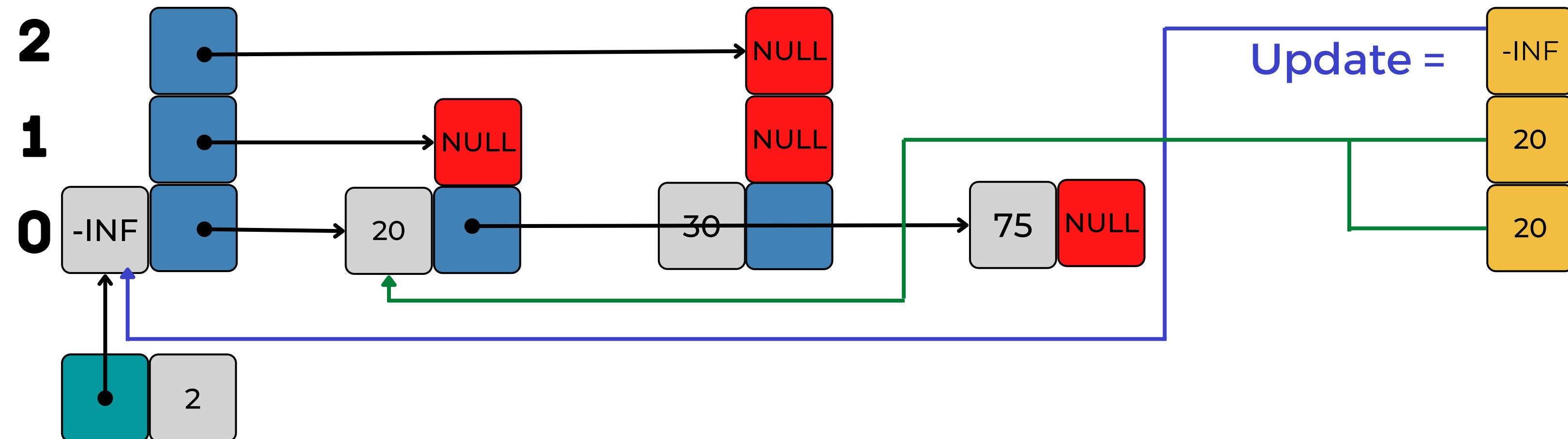
DELETE(30)



DELETION

Level = -1

Connect update array node to node after

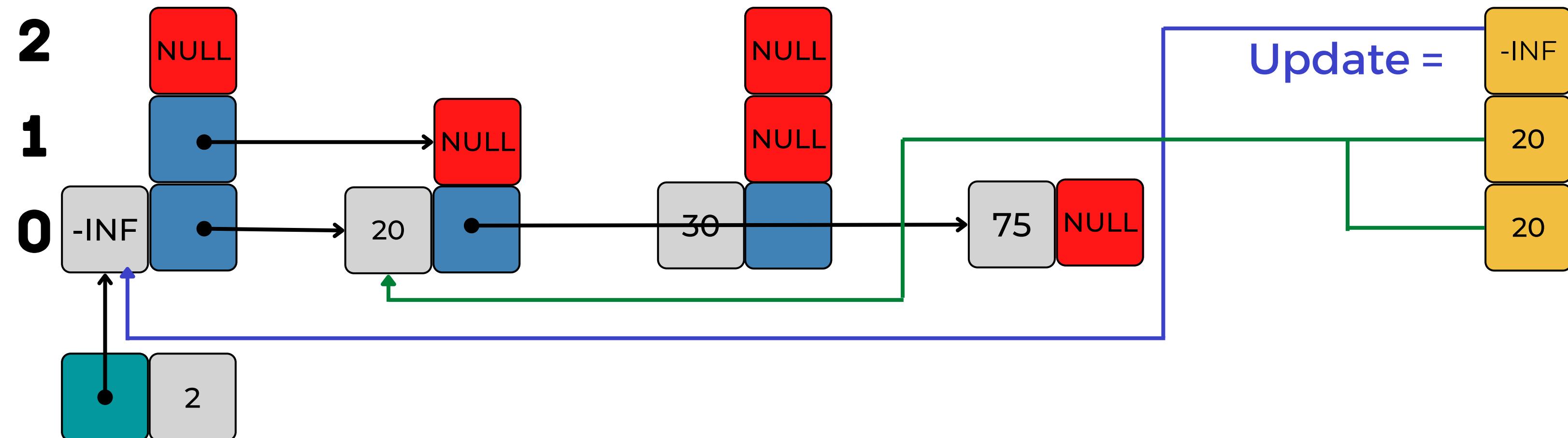




DELETION

Level = -1

Connect update array node to node after



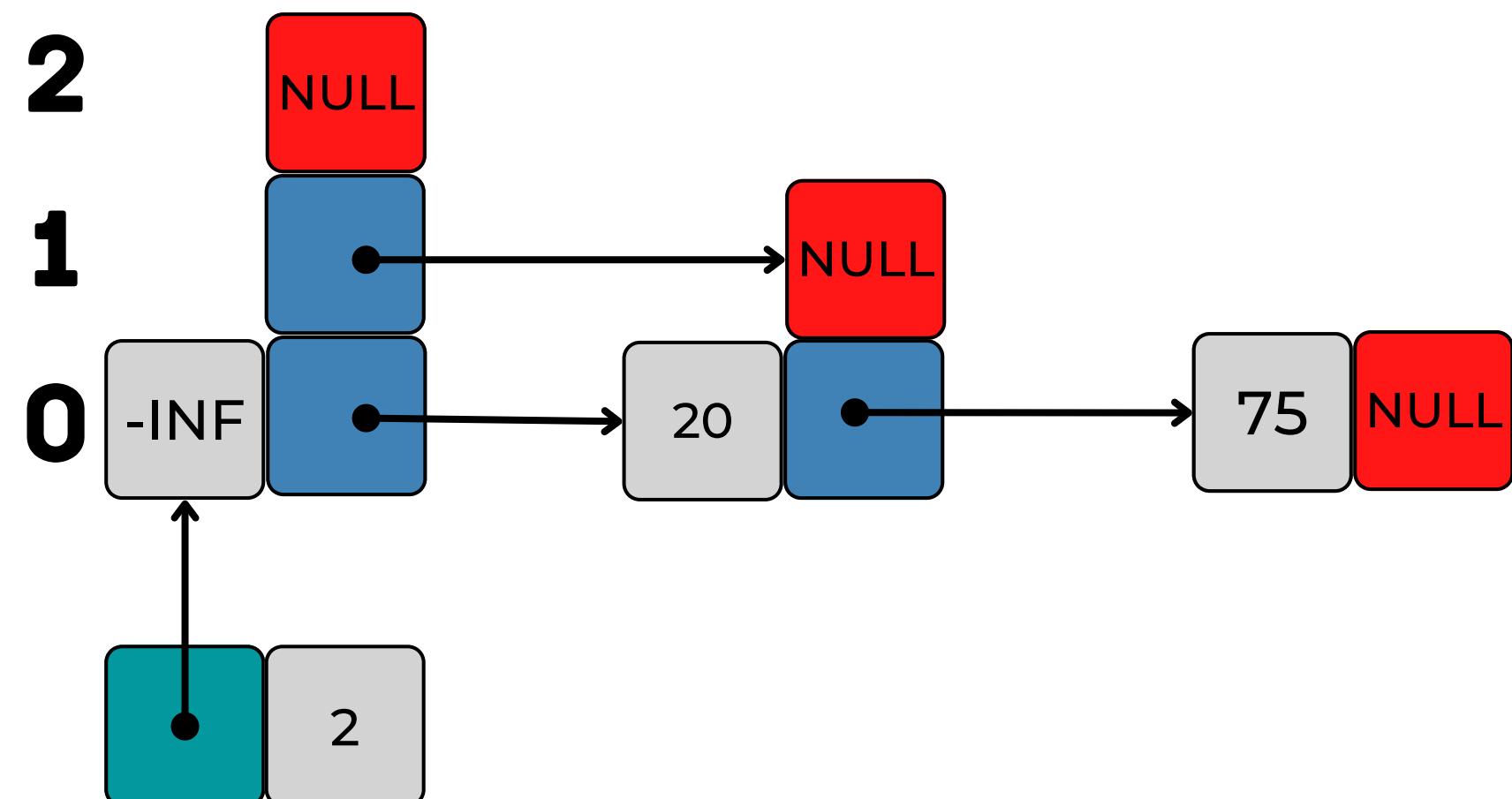
DELETE(30)



DELETION

Level = -1

node 30 is now deleted



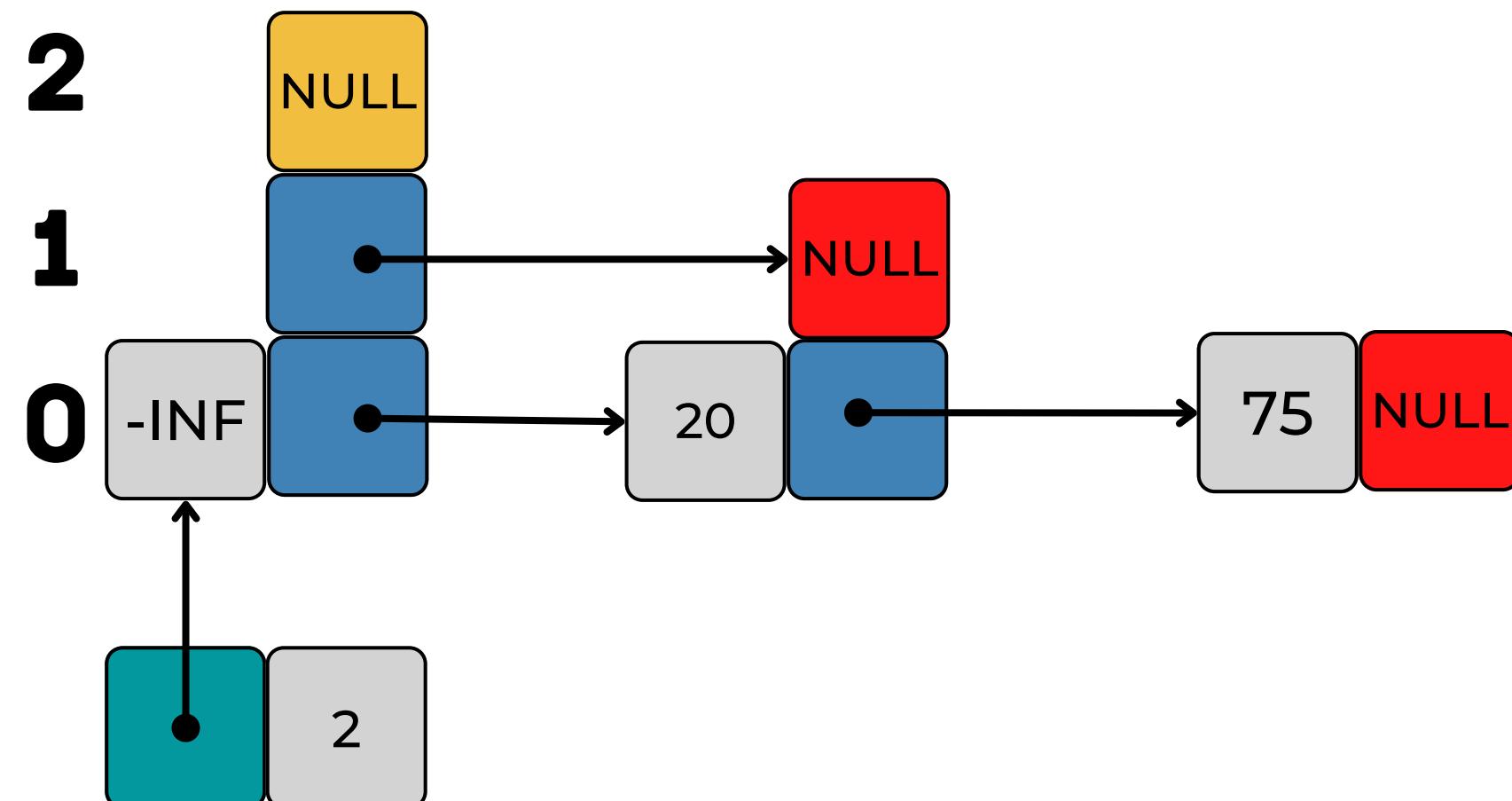
DELETE(30)



DELETION

Level = -1

Is head link (lvl 2) null?



DELETE(30)

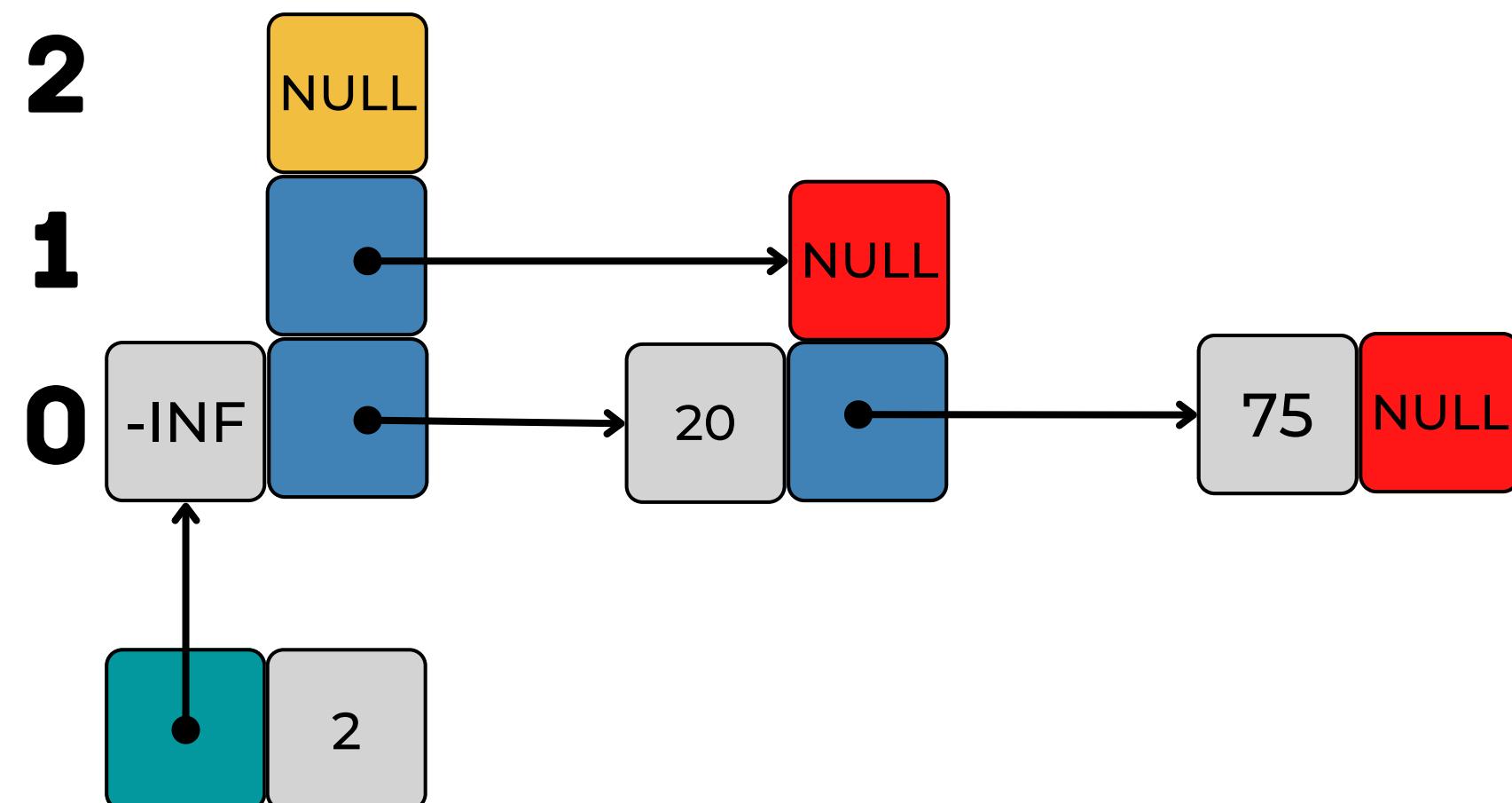


DELETION

Level = -1

Is head link (lvl 2) null?

True = Subtract level count



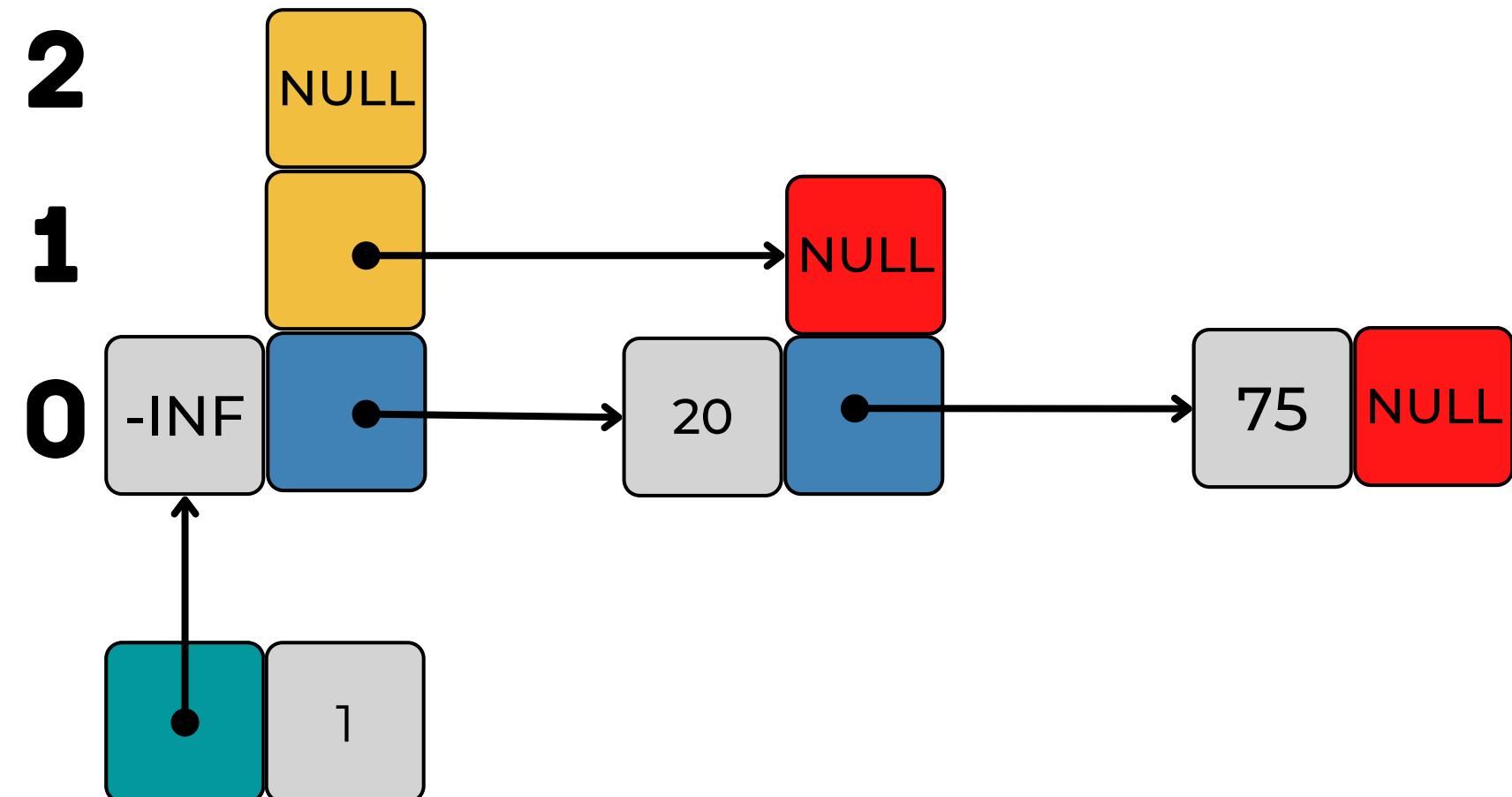
DELETE(30)



DELETION

Level = -1

Is head link (lvl 1) null?



DELETE(30)

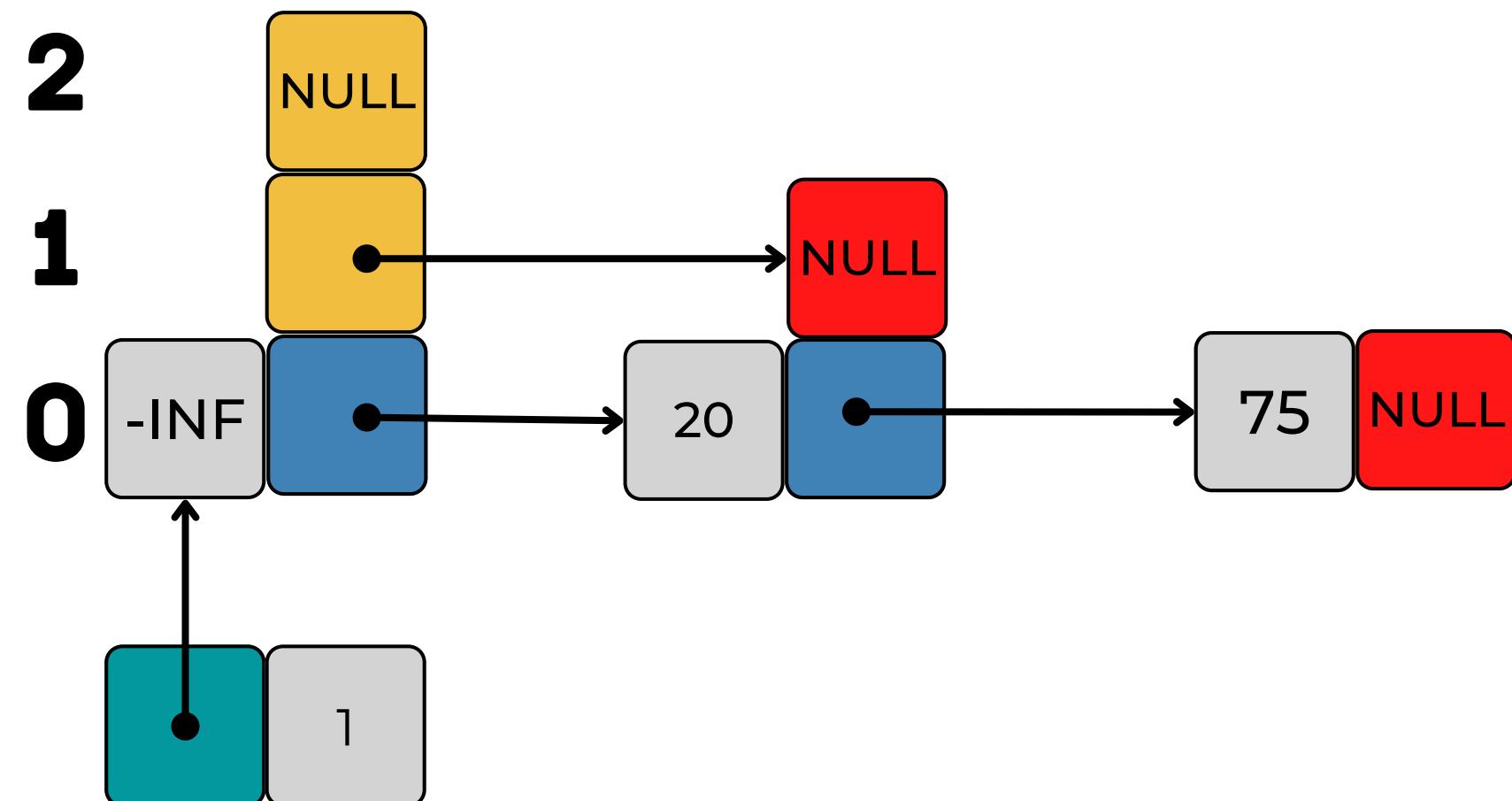


DELETION

Level = -1

Is head link (lvl 1) null?

False = End traversal



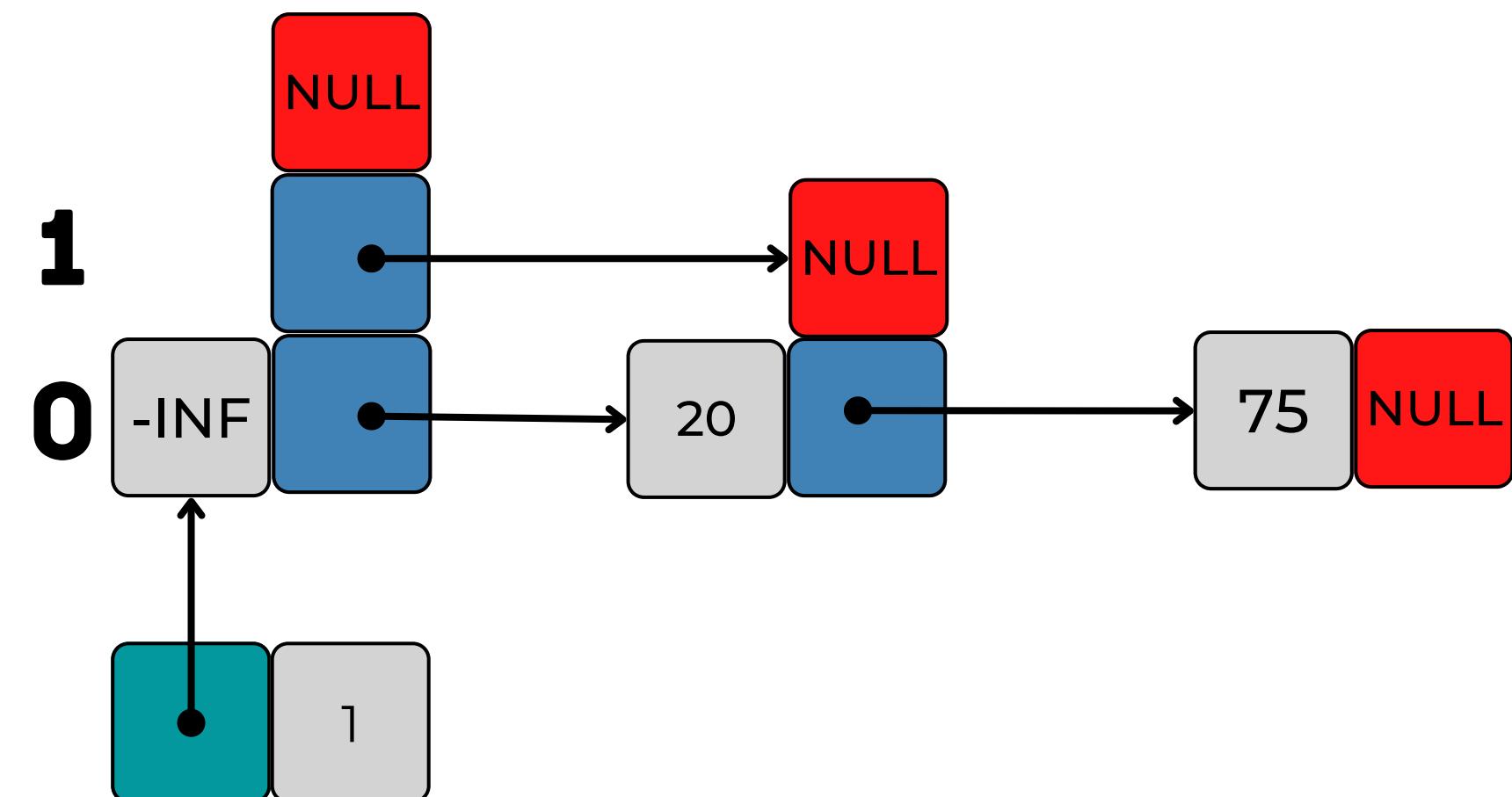
DELETE(30)



DELETION

Level = 1

node 30 is now deleted



DELETE(30)



TEAM JC



Search ...



CODE COMPARISON



STRUCTURE

```
class Node
{
public:
    int key;
    Node **forward;
    Node(int, int);
};

class SkipList
{
    int MAXLVL;

    float P;

    int level;

    Node *header;
public:
    SkipList(int, float);
    int randomLevel();
    Node* createNode(int, int);
    void insertElement(int);
    void displayList();
};
```



```
#define MAXLEVEL 3

typedef struct node{
    struct node **link;
    int data;
}*Node;

typedef struct {
    Node head;
    int level;
}SkipList;
```



INTERNET INITIALIZATION

```
SkipList::SkipList(int MAXLVL, float P)
{
    this->MAXLVL = MAXLVL;
    this->P = P;
    level = 0;

    // create header node and initialize key to -1
    header = new Node(-1, MAXLVL);
};

Node::Node(int key, int level)
{
    this->key = key;
    // Allocate memory to forward
    forward = new Node*[level+1];
    // Fill forward array with 0(NULL)
    memset(forward, 0, sizeof(Node*)*(level+1));
};
```



STREAMLINED

```
void initList(SkipList *list)
{
    list->level = 0;

    // create head
    list->head = (struct node*)calloc(sizeof(struct
node),1);
    // initialize data to INT_MIN
    list->head->data = -2147483647;
    // initialize pointer of arrays
    list->head->link = (struct
node**)calloc(sizeof(struct node*),MAXLEVEL+1);
}
```



INSERTION

```
void SkipList::insertElement(int key)
{
    Node *current = header;
    // create update array and initialize it
    Node *update[MAXLVL+1];
    memset(update, 0, sizeof(Node*)*(MAXLVL+1));
    for (int i = level; i >= 0; i--){
        while (current->forward[i] != NULL &&
               current->forward[i]->key < key)
            current = current->forward[i];
        update[i] = current;
    }
    current = current->forward[0];
    if (current == NULL || current->key != key)
    {
        // Generate a random level for node
        int rlevel = randomLevel();
        if (rlevel > level){
            for (int i=level+1;i<rlevel+1;i++)
                update[i] = header;
            level = rlevel;
        }
        // create new node with random level generated
        Node* n = createNode(key, rlevel);
        // insert node by rearranging pointers
        for (int i=0;i<=rlevel;i++){
            n->forward[i] = update[i]->forward[i];
            update[i]->forward[i] = n;
        }
    }
};
```

STREAMLINED

```
void insertData(SkipList *list, int data)
{
    // initialize update array
    Node *update = (struct node**)calloc(sizeof(struct
node*),MAXLEVEL+1);
    Node node = updateInsert(list->head, data, update,
list->level);
    if(node == NULL || node->data != data) {
        // Generate random level
        int nlevel = coinFlipLevel();
        // Create new node
        Node newNode = (struct node*)calloc(sizeof(struct
node),1);
        newNode->data = data;
        newNode->link = (struct
node**)calloc(sizeof(struct node*),nlevel+1);
        int i;
        for(i = list->level+1; i <= nlevel; i++) {
            update[i] = list->head;
            list->level = i;
        }
        // insert node by rearranging pointers
        for(i = 0; i <= nlevel; i++) {
            newNode->link[i] = update[i]->link[i];
            update[i]->link[i] = newNode;
        }
    }
    free(update);
}
```



INSERTION

```
void SkipList::insertElement(int key)
{
    Node *current = header;
    // create update array and initialize it
    Node *update[MAXLVL+1];
    memset(update, 0, sizeof(Node*)*(MAXLVL+1));
    for (int i = level; i >= 0; i--){
        while (current->forward[i] != NULL &&
               current->forward[i]->key < key)
            current = current->forward[i];
        update[i] = current;
    }
    current = current->forward[0];
    if (current == NULL || current->key != key)
    {
        // Generate a random level for node
        int rlevel = randomLevel();
        if (rlevel > level){
            for (int i=level+1;i<rlevel+1;i++)
                update[i] = header;
            level = rlevel;
        }
        // create new node with random level generated
        Node* n = createNode(key, rlevel);
        // insert node by rearranging pointers
        for (int i=0;i<=rlevel;i++){
            n->forward[i] = update[i]->forward[i];
            update[i]->forward[i] = n;
        }
    }
};
```

STREAMLINED

```
Node updateInsert(Node head, int data, Node
*update, int level)
{
    if(head == NULL || head->data >= data )
        return head;
    update[level] = head;
    if (head->link[level] != NULL && data >= head-
>link[level]->data || level == 0) {
        updateInsert(head->link[level], data, update,
level);
    } else {
        updateInsert(head, data, update, level-1);
    }
}
```



INSERTION

```
int SkipList::randomLevel()
{
    float r = (float)rand() / RAND_MAX;
    int lvl = 0;
    while (r < P && lvl < MAXLVL)
    {
        lvl++;
        r = (float)rand() / RAND_MAX;
    }
    return lvl;
};
```



STREAMLINED

```
int coinFlipLevel()
{
    int newLevel, coin;
    for(newLevel = 0, coin = rand() % 2; coin &&
newLevel < MAXLEVEL; newLevel++, coin = rand() % 2)
    {}
    return newLevel;
}
```



INTERNET

DELETION

```
void SkipList::deleteElement(int key)
{
    Node *current = header;
    Node *update[MAXLVL+1];
    memset(update, 0, sizeof(Node*)*(MAXLVL+1));
    for(int i = level; i >= 0; i--) {
        while(current->forward[i] != NULL &&
              current->forward[i]->key < key)
            current = current->forward[i];
        update[i] = current;
    }
    current = current->forward[0];
    if(current != NULL and current->key == key)
    {
        for(int i=0;i<=level;i++)
        {
            if(update[i]->forward[i] != current)
                break;
            update[i]->forward[i] = current-
>forward[i];
        }

        // Remove levels having no elements
        while(level>0 &&
              header->forward[level] == 0)
            level--;
    }
};
```



STREAMLINED

```
void deleteData(SkipList *list, int data) {
    Node *update = (struct
node**)calloc(sizeof(struct node*),MAXLEVEL+1);

    Node node = updateDelete(list->head, data,
update, list->level);

    if(node != NULL && node->data == data) {
        int i;
        for(i = 0; i <= list->level && update[i]-
>link[i] == node; i++) {
            update[i]->link[i] = node->link[i];
        }
        for(; list->level > 0 && list->head->link[list-
>level] == NULL; list->level--) {}
        free(node);
    }

    free(update);
}
```



INTERNET

DELETION

```
void SkipList::deleteElement(int key)
{
    Node *current = header;
    Node *update[MAXLVL+1];
    memset(update, 0, sizeof(Node*)*(MAXLVL+1));
    for(int i = level; i >= 0; i--) {
        while(current->forward[i] != NULL &&
              current->forward[i]->key < key)
            current = current->forward[i];
        update[i] = current;
    }
    current = current->forward[0];
    if(current != NULL and current->key == key)
    {
        for(int i=0;i<=level;i++)
        {
            if(update[i]->forward[i] != current)
                break;
            update[i]->forward[i] = current-
>forward[i];
        }

        // Remove levels having no elements
        while(level>0 &&
              header->forward[level] == 0)
            level--;
    }
};
```



STREAMLINED

```
Node updateDelete(Node head, int data, Node
*update, int level) {
    if(head == NULL || head->data >= data){
        return head;
    }

    update[level] = head;

    if (head->link[level] != NULL && data > head-
>link[level]->data || level == 0) {
        updateDelete(head->link[level], data, update,
level);
    } else {
        updateDelete(head, data, update, level-1);
    }
}
```



INTERNET SEARCH

```
void SkipList::searchElement(int key)
{
    Node *current = header;

    for(int i = level; i >= 0; i--)
    {
        while(current->forward[i] &&
              current->forward[i]->key < key)
            current = current->forward[i];
    }
    current = current->forward[0];

    if(current and current->key == key)
        cout<<"Found key: "<<key<<"\n";
};
```



STREAMLINED

```
Node searchData(Node head, int level, int data) {
    if(head == NULL || head->data >= data ){
        return head;
    }

    if (head->link[level] != NULL && data >= head-
        >link[level]->data || level == 0) {
        searchData(head->link[level], level, data);
    } else {
        searchData(head, level-1, data);
    }
}
```





THANK YOU

END SLIDE

