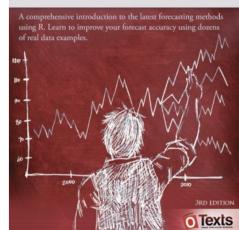
5. The forecaster's toolbox

5.5 Distributional forecasts
OTexts.org/fpp3/

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FORECASTING PRINCIPLES AND PRACTICE



```
brick_fit <- aus_production |>
  filter(!is.na(Bricks)) |>
  model(
    Seasonal_naive = SNAIVE(Bricks),
    Naive = NAIVE(Bricks),
    Drift = RW(Bricks ~ drift()),
    Mean = MEAN(Bricks)
)
```

Forecast distributions

- A forecast $\hat{y}_{T+h|T}$ is (usually) the mean of the conditional distribution $y_{T+h} \mid y_1, \dots, y_T$.
- Most time series models produce normally distributed forecasts.
- The forecast distribution describes the probability of observing any future value.

Forecast distributions

Assuming residuals are normal, uncorrelated, sd = $\hat{\sigma}$:

Mean:
$$y_{T+h|T} \sim N(\bar{y}, (1+1/T)\hat{\sigma}^2)$$

Naïve:
$$y_{T+h|T} \sim N(y_T, h\hat{\sigma}^2)$$

Seasonal naïve:
$$y_{T+h|T} \sim N(y_{T+h-m(k+1)}, (k+1)\hat{\sigma}^2)$$

Drift:
$$y_{T+h|T} \sim N(y_T + \frac{h}{T-1}(y_T - y_1), h^{\frac{T+h}{T}}\hat{\sigma}^2)$$

where k is the integer part of (h-1)/m.

Note that when h = 1 and T is large, these all give the same approximate forecast variance: $\hat{\sigma}^2$.

Prediction intervals

- A prediction interval gives a region within which we expect y_{T+h} to lie with a specified probability.
- Assuming forecast errors are normally distributed, then a 95% PI is

$$\hat{\mathbf{y}}_{\mathsf{T}+\mathsf{h}|\mathsf{T}} \pm 1.96\hat{\sigma}_{\mathsf{h}}$$

where $\hat{\sigma}_h$ is the st dev of the *h*-step distribution.

■ When h = 1, $\hat{\sigma}_h$ can be estimated from the residuals.

Prediction intervals

```
brick_fit <- aus_production |>
  filter(!is.na(Bricks)) |>
  model(Seasonal_naive = SNAIVE(Bricks)) |>
  forecast(h = "5 years") |>
  hilo(level = 95)
```

Prediction intervals

- Point forecasts often useless without a measure of uncertainty (such as prediction intervals).
- Prediction intervals require a stochastic model (with random errors, etc).
- For most models, prediction intervals get wider as the forecast horizon increases.
- Use level argument to control coverage.
- Check residual assumptions before believing them.
- Usually too narrow due to unaccounted uncertainty.