# 5. The forecaster's toolbox

5.5 Distributional forecasts

OTexts.org/fpp3/

```
brick_fit <- aus_production |>
  filter(!is.na(Bricks)) |>
  model(
    Seasonal_naive = SNAIVE(Bricks),
    Naive = NAIVE(Bricks),
    Drift = RW(Bricks ~ drift()),
    Mean = MEAN(Bricks)
  )
```

# Forecast distributions

- A forecast $\hat{y}_{T+h|T}$ is (usually) the mean of the conditional distribution $y_{T+h} \mid y_1, \ldots, y_T$.
- Most time series models produce normally distributed forecasts.
- The forecast distribution describes the probability of observing any future value.

## Forecast distributions

Assuming residuals are normal, uncorrelated, sd = $\hat{\sigma}$:

**Mean:** $\quad y_{T+h|T} \sim N(\bar{y}, (1 + 1/T)\hat{\sigma}^2)$

**Naïve:** $\quad y_{T+h|T} \sim N(y_T, h\hat{\sigma}^2)$

**Seasonal naïve:** $\quad y_{T+h|T} \sim N(y_{T+h-m(k+1)}, (k + 1)\hat{\sigma}^2)$

**Drift:** $\quad y_{T+h|T} \sim N(y_T + \frac{h}{T-1}(y_T - y_1), h\frac{T+h}{T}\hat{\sigma}^2)$

where $k$ is the integer part of $(h - 1)/m$.

Note that when $h = 1$ and $T$ is large, these all give the same approximate forecast variance: $\hat{\sigma}^2$.

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# Prediction intervals

- A prediction interval gives a region within which we expect $y_{T+h}$ to lie with a specified probability.

- Assuming forecast errors are normally distributed, then a 95% PI is

$$\hat{y}_{T+h|T} \pm 1.96\hat{\sigma}_h$$

  where $\hat{\sigma}_h$ is the st dev of the $h$-step distribution.

- When $h$ = 1, $\hat{\sigma}_h$ can be estimated from the residuals.

# Prediction intervals

```
brick_fit <- aus_production |>
  filter(!is.na(Bricks)) |>
  model(Seasonal_naive = SNAIVE(Bricks)) |>
  forecast(h = "5 years") |>
  hilo(level = 95)
```

# Prediction intervals

- Point forecasts often useless without a measure of uncertainty (such as prediction intervals).
- Prediction intervals require a stochastic model (with random errors, etc).
- For most models, prediction intervals get wider as the forecast horizon increases.
- Use `level` argument to control coverage.
- Check residual assumptions before believing them.
- Usually too narrow due to unaccounted uncertainty.