

5. The forecaster's toolbox

5.6 Forecasting using transformations

OTexts.org/fpp3/

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FORECASTING

PRINCIPLES AND PRACTICE

A comprehensive introduction to the latest forecasting methods using R. Learn to improve your forecast accuracy using dozens of real data examples.

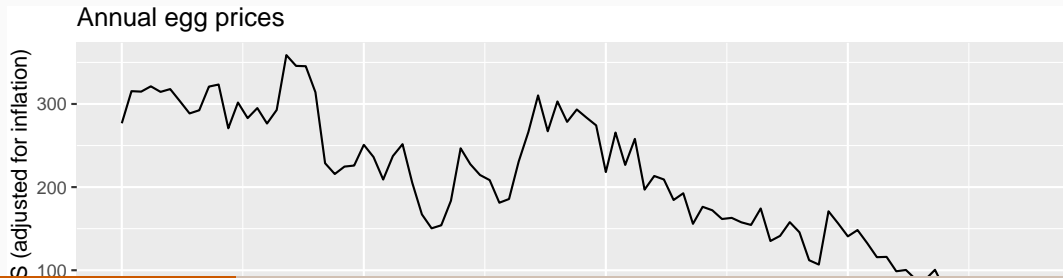


3RD EDITION

oTexts
OPEN TEXTS

Modelling with transformations

```
eggs <- prices |>
  filter(!is.na(eggs)) |>
  select(eggs)
eggs |> autoplot() +
  labs(
    title = "Annual egg prices",
    y = "$US (adjusted for inflation)"
  )
```



Modelling with transformations

Transformations used in the left of the formula will be automatically back-transformed. To model log-transformed egg prices, you could use:

```
fit <- eggs |>  
  model(RW(log(eggs) ~ drift()))  
fit
```

```
## # A mable: 1 x 1  
##   `RW(log(eggs) ~ drift())`  
##                               <model>  
## 1                               <RW w/ drift>
```

Forecasting with transformations

```
fc <- fit |>  
  forecast(h = 50)  
fc
```

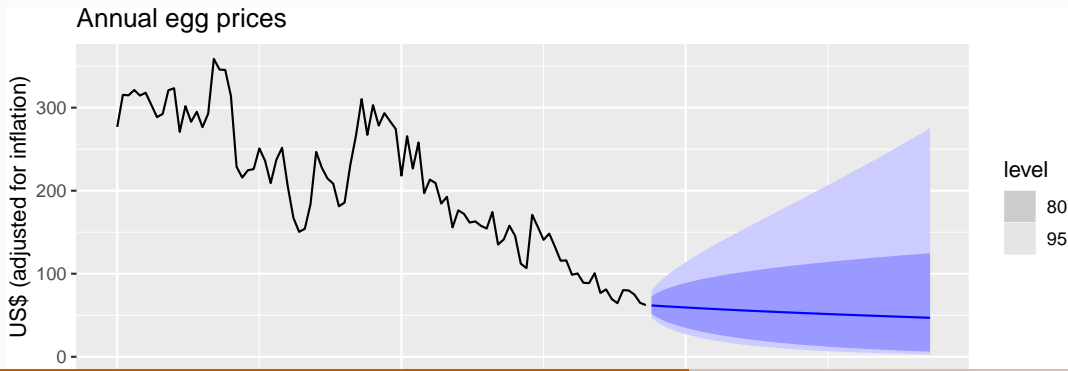
```
## # A tibble: 50 x 4 [1Y]
```

```
## # Key:           .model [1]
```

##	.model	year	eggs	.mean
##	<chr>	<dbl>	<dist>	<dbl>
##	1 RW(log(eggs) ~ drift())	1994	t(N(4.1, 0.018))	61.8
##	2 RW(log(eggs) ~ drift())	1995	t(N(4.1, 0.036))	61.4
##	3 RW(log(eggs) ~ drift())	1996	t(N(4.1, 0.055))	61.0
##	4 RW(log(eggs) ~ drift())	1997	t(N(4.1, 0.074))	60.6
##	5 RW(log(eggs) ~ drift())	1998	t(N(4.1, 0.093))	60.2
##	6 RW(log(eggs) ~ drift())	1999	t(N(4, 0.11))	59.8
##	7 RW(log(eggs) ~ drift())	2000	t(N(4, 0.13))	59.4
##	8 RW(log(eggs) ~ drift())	2001	t(N(4, 0.15))	59.0

Forecasting with transformations

```
fc |> autoplot(eggs) +  
  labs(  
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  )
```



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- Back-transformed point forecasts are medians.
- Back-transformed PI have the correct coverage.

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Back-transformed means

Let X be have mean μ and variance σ^2 .

Let $f(x)$ be back-transformation function, and $Y = f(X)$.

Taylor series expansion about μ :

$$f(X) = f(\mu) + (X - \mu)f'(\mu) + \frac{1}{2}(X - \mu)^2f''(\mu).$$

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$$E[Y] = E[f(X)] = f(\mu) + \frac{1}{2}\sigma^2f''(\mu)$$

Bias adjustment

Box-Cox back-transformation:

$$y_t = \begin{cases} \exp(w_t) & \lambda = 0; \\ (\lambda W_t + 1)^{1/\lambda} & \lambda \neq 0. \end{cases}$$

$$f(x) = \begin{cases} e^x & \lambda = 0; \\ (\lambda x + 1)^{1/\lambda} & \lambda \neq 0. \end{cases}$$

$$f''(x) = \begin{cases} e^x & \lambda = 0; \\ (1 - \lambda)(\lambda x + 1)^{1/\lambda - 2} & \lambda \neq 0. \end{cases}$$

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$$E[Y] = \begin{cases} e^{\mu} \left[1 + \frac{\sigma^2}{2} \right] & \lambda = 0; \\ (\lambda \mu + 1)^{1/\lambda} \left[1 + \frac{\sigma^2(1-\lambda)}{2(\lambda \mu + 1)^2} \right] & \lambda \neq 0. \end{cases}$$

Bias adjustment

```
fc |>  
  autoplot(eggs, level = 80, point_forecast = lst(mean, median)) +  
  labs(  
    title = "Annual egg prices",  
    y = "US$ (adjusted for inflation)"  
  )
```

