

Ch3. Time series decomposition

3.2 Time series components

OTexts.org/fpp3/

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FORECASTING

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Time series patterns

Recall

Trend pattern exists when there is a long-term increase or decrease in the data.

Cyclic pattern exists when data exhibit rises and falls that are *not of fixed period* (duration usually of at least 2 years).

Seasonal pattern exists when a series is influenced by seasonal factors (e.g., the quarter of the year, the month, or day of the week).

Time series decomposition

$$y_t = f(S_t, T_t, R_t)$$

where y_t = data at period t

T_t = trend-cycle component at period t

S_t = seasonal component at period t

R_t = remainder component at period t

Time series decomposition

$$y_t = f(S_t, T_t, R_t)$$

where y_t = data at period t

T_t = trend-cycle component at period t

S_t = seasonal component at period t

R_t = remainder component at period t

Additive decomposition: $y_t = S_t + T_t + R_t$.

Multiplicative decomposition: $y_t = S_t \times T_t \times R_t$.

Time series decomposition

- Additive model appropriate if magnitude of seasonal fluctuations does not vary with level.
- If seasonal are proportional to level of series, then multiplicative model appropriate.
- Multiplicative decomposition more prevalent with economic series
- Alternative: use a Box-Cox transformation, and then use additive decomposition.
- Logs turn multiplicative relationship into an additive relationship:

$$y_t = S_t \times T_t \times R_t \quad \Rightarrow \quad \log y_t = \log S_t + \log T_t + \log R_t.$$

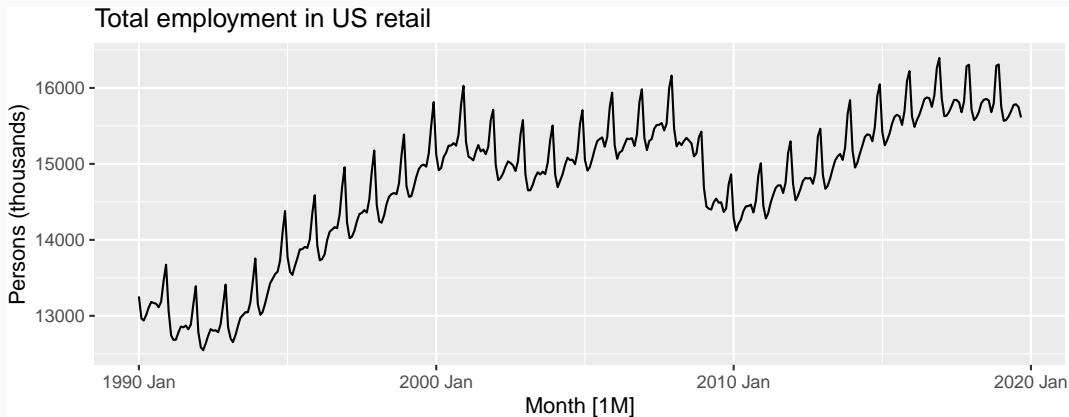
US Retail Employment

```
us_retail_employment <- us_employment %>%  
  filter(year(Month) >= 1990, Title == "Retail Trade") %>%  
  select(-Series_ID)  
us_retail_employment
```

```
## # A tsibble: 357 x 3 [1M]  
##       Month Title      Employed  
##       <mth> <chr>      <dbl>  
## 1 1990 Jan Retail Trade 13256.  
## 2 1990 Feb Retail Trade 12966.  
## 3 1990 Mar Retail Trade 12938.  
## 4 1990 Apr Retail Trade 13012.  
## 5 1990 May Retail Trade 13108.  
## 6 1990 Jun Retail Trade 13183.  
## 7 1990 Jul Retail Trade 13170.  
## 8 1990 Aug Retail Trade 13160.  
## 9 1990 Sep Retail Trade 13112.
```

US Retail Employment

```
us_retail_employment %>%  
  autoplot(Employed) +  
  labs(y="Persons (thousands)", title="Total employment in US retail")
```



US Retail Employment

```
us_retail_employment %>%  
  model(stl = STL(Employed))
```

```
## # A mable: 1 x 1  
##      stl  
##    <model>  
## 1    <STL>
```

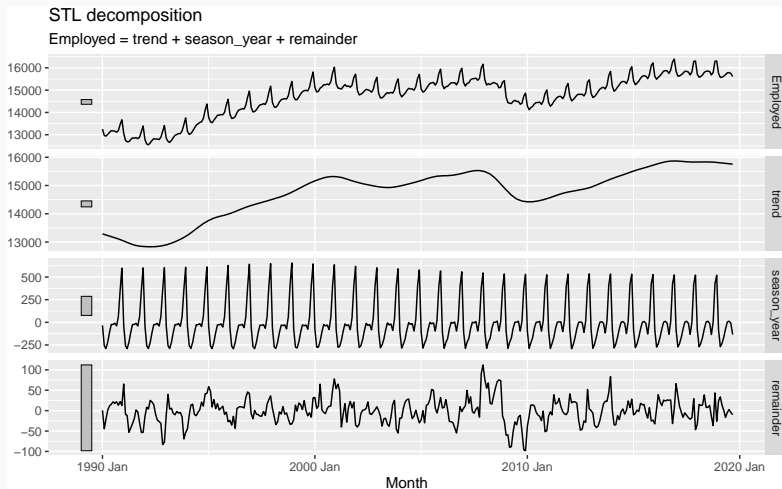

US Retail Employment

```
dcmp <- us_retail_employment %>%  
  model(stl = STL(Employed))  
components(dcmp)
```

```
## # A dable: 357 x 7 [1M]  
## # Key:      .model [1]  
## # :      Employed = trend + season_year + remainder  
##   .model      Month Employed  trend season_year remainder season_adjust  
##   <chr>      <mth>    <dbl>  <dbl>      <dbl>      <dbl>      <dbl>  
## 1 stl      1990 Jan    13256. 13288.    -33.0      0.836     13289.  
## 2 stl      1990 Feb    12966. 13269.   -258.     -44.6     13224.  
## 3 stl      1990 Mar    12938. 13250.   -290.     -22.1     13228.  
## 4 stl      1990 Apr    13012. 13231.   -220.      1.05     13232.  
## 5 stl      1990 May    13108. 13211.   -114.     11.3     13223.  
## 6 stl      1990 Jun    13183. 13192.   -24.3     15.5     13207.  
## 7 stl      1990 Jul    13170. 13172.   -23.2     21.6     13193. 8  
## 8 stl      1990 Aug    13160. 13151.    -9.5     17.0     13160.
```

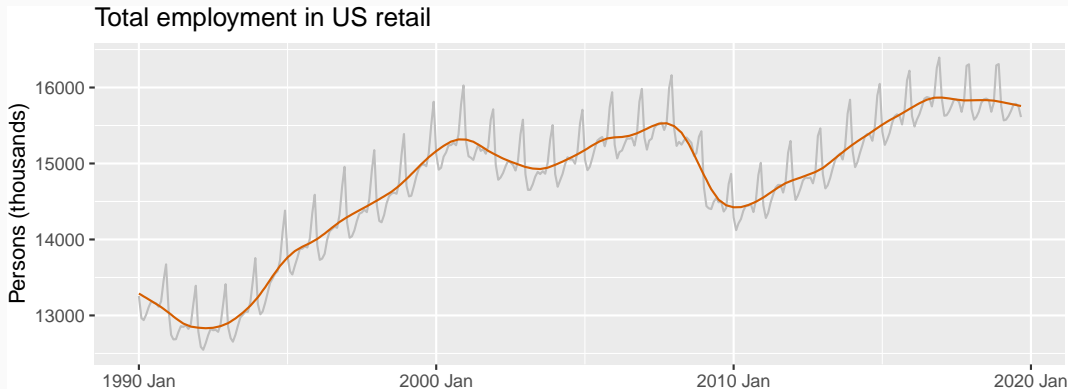
US Retail Employment

```
components(dcmp) %>% autoplot()
```



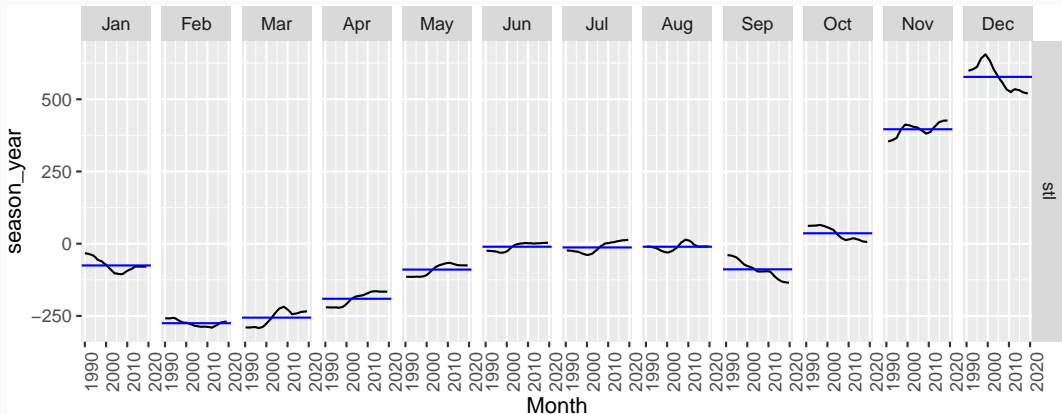
US Retail Employment

```
us_retail_employment %>%  
  autoplot(Employed, color='gray') +  
  autolayer(components(dcmp), trend, color='#D55E00') +  
  labs(y="Persons (thousands)", title="Total employment in US retail")
```



US Retail Employment

```
components(dcmp) %>% gg_subseries(season_year)
```



Seasonal adjustment

- Useful by-product of decomposition: an easy way to calculate seasonally adjusted data.
- Additive decomposition: seasonally adjusted data given by

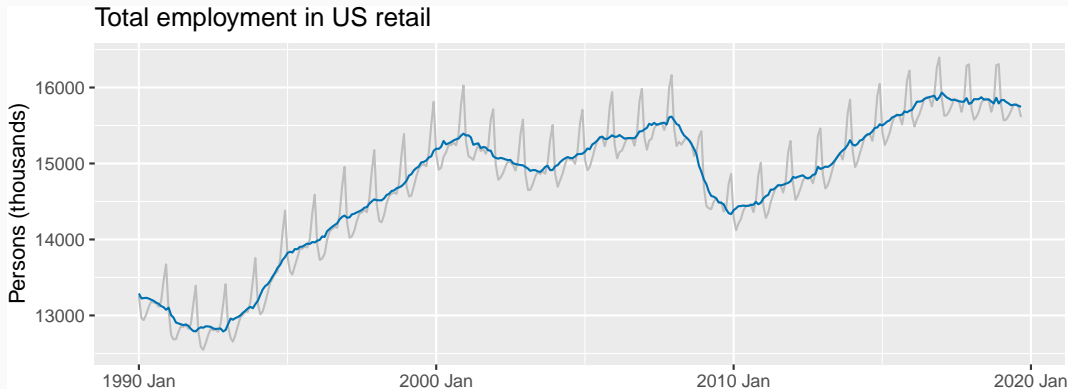
$$y_t - S_t = T_t + R_t$$

- Multiplicative decomposition: seasonally adjusted data given by

$$y_t/S_t = T_t \times R_t$$

US Retail Employment

```
us_retail_employment %>%  
  autoplot(Employed, color='gray') +  
  autolayer(components(dcmp), season_adjust, color='#0072B2') +  
  labs(y="Persons (thousands)", title="Total employment in US retail")
```



Seasonal adjustment

- We use estimates of S based on past values to seasonally adjust a current value.
- Seasonally adjusted series reflect **remainders** as well as **trend**. Therefore they are not “smooth” and “downturns” or “upturns” can be misleading.
- It is better to use the trend-cycle component to look for turning points.