

## 5. The forecaster's toolbox

### 5.5 Distributional forecasts

[OTexts.org/fpp3/](http://OTexts.org/fpp3/)

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# FORECASTING

## PRINCIPLES AND PRACTICE

A comprehensive introduction to the latest forecasting methods using R. Learn to improve your forecast accuracy using dozens of real data examples.



3RD EDITION

 **OTexts**  
OPEN ACCESS TEXTBOOKS

```
brick_fit <- aus_production |>
  filter(!is.na(Bricks)) |>
  model(
    Seasonal_naive = SNAIVE(Bricks),
    Naive = NAIVE(Bricks),
    Drift = RW(Bricks ~ drift()),
    Mean = MEAN(Bricks)
  )
```

# Forecast distributions

- A forecast  $\hat{y}_{T+h|T}$  is (usually) the mean of the conditional distribution  $y_{T+h} \mid y_1, \dots, y_T$ .
- Most time series models produce normally distributed forecasts.
- The forecast distribution describes the probability of observing any future value.

# Forecast distributions

Assuming residuals are normal, uncorrelated,  $\text{sd} = \hat{\sigma}$ :

**Mean:**  $y_{T+h|T} \sim N(\bar{y}, (1 + 1/T)\hat{\sigma}^2)$

**Naïve:**  $y_{T+h|T} \sim N(y_T, h\hat{\sigma}^2)$

**Seasonal naïve:**  $y_{T+h|T} \sim N(y_{T+h-m(k+1)}, (k+1)\hat{\sigma}^2)$

**Drift:**  $y_{T+h|T} \sim N(y_T + \frac{h}{T-1}(y_T - y_1), h\frac{T+h}{T}\hat{\sigma}^2)$

where  $k$  is the integer part of  $(h - 1)/m$ .

Note that when  $h = 1$  and  $T$  is large, these all give the same approximate forecast variance:  $\hat{\sigma}^2$ .

# Prediction intervals

- A prediction interval gives a region within which we expect  $y_{T+h}$  to lie with a specified probability.
- Assuming forecast errors are normally distributed, then a 95% PI is

$$\hat{y}_{T+h|T} \pm 1.96\hat{\sigma}_h$$

where  $\hat{\sigma}_h$  is the st dev of the  $h$ -step distribution.

- When  $h = 1$ ,  $\hat{\sigma}_h$  can be estimated from the residuals.

# Prediction intervals

```
brick_fit <- aus_production |>  
  filter(!is.na(Bricks)) |>  
  model(Seasonal_naive = SNAIVE(Bricks)) |>  
  forecast(h = "5 years") |>  
  hilo(level = 95)
```

# Prediction intervals

- Point forecasts often useless without a measure of uncertainty (such as prediction intervals).
- Prediction intervals require a stochastic model (with random errors, etc).
- For most models, prediction intervals get wider as the forecast horizon increases.
- Use `level` argument to control coverage.
- Check residual assumptions before believing them.
- Usually too narrow due to unaccounted uncertainty.