

3. Time series decomposition

3.2 Time series components

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FORECASTING

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Time series patterns

Recall

Trend pattern exists when there is a long-term increase or decrease in the data.

Cyclic pattern exists when data exhibit rises and falls that are *not of fixed period* (duration usually of at least 2 years).

Seasonal pattern exists when a series is influenced by seasonal factors (e.g., the quarter of the year, the month, or day of the week).

Time series decomposition

$$y_t = f(S_t, T_t, R_t)$$

where y_t = data at period t

T_t = trend-cycle component at period t

S_t = seasonal component at period t

R_t = remainder component at period t

Time series decomposition

$$y_t = f(S_t, T_t, R_t)$$

where y_t = data at period t

T_t = trend-cycle component at period t

S_t = seasonal component at period t

R_t = remainder component at period t

Additive decomposition: $y_t = S_t + T_t + R_t$.

Multiplicative decomposition: $y_t = S_t \times T_t \times R_t$.

Time series decomposition

- Additive model appropriate if magnitude of seasonal fluctuations does not vary with level.
- If seasonal are proportional to level of series, then multiplicative model appropriate.
- Multiplicative decomposition more prevalent with economic series
- Alternative: use a Box-Cox transformation, and then use additive decomposition.
- Logs turn multiplicative relationship into an additive relationship:

$$y_t = S_t \times T_t \times R_t \quad \Rightarrow \quad \log y_t = \log S_t + \log T_t + \log R_t.$$

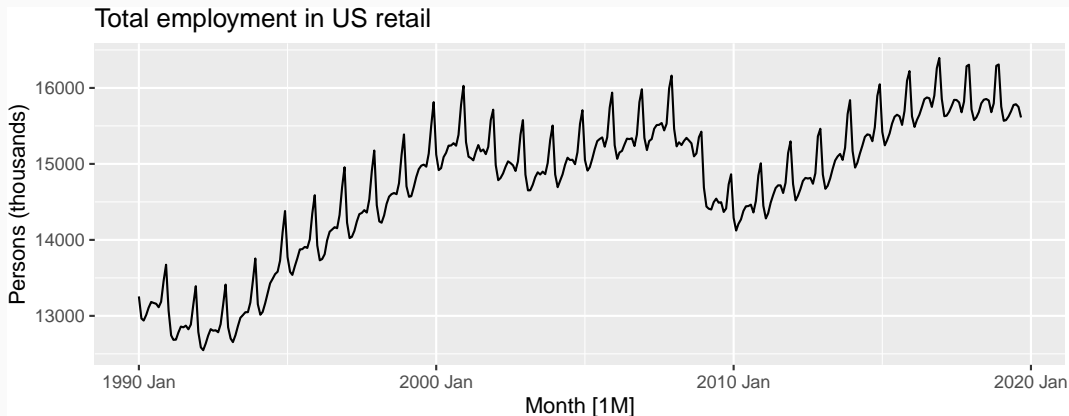
US Retail Employment

```
us_retail_employment <- us_employment |>
  filter(year(Month) >= 1990, Title == "Retail Trade") |>
  select(-Series_ID)
us_retail_employment
```

```
## # A tsibble: 357 x 3 [1M]
##       Month Title      Employed
##       <mth> <chr>      <dbl>
## 1 1990 Jan Retail Trade 13256.
## 2 1990 Feb Retail Trade 12966.
## 3 1990 Mar Retail Trade 12938.
## 4 1990 Apr Retail Trade 13012.
## 5 1990 May Retail Trade 13108.
## 6 1990 Jun Retail Trade 13183.
## 7 1990 Jul Retail Trade 13170.
## 8 1990 Aug Retail Trade 13160.
## 9 1990 Sep Retail Trade 13113.
```

US Retail Employment

```
us_retail_employment |>  
  autoplot(Employed) +  
  labs(y = "Persons (thousands)", title = "Total employment in US retail")
```



US Retail Employment

```
us_retail_employment |>  
  model(stl = STL(Employed))
```

```
## # A mable: 1 x 1  
##      stl  
##    <model>  
## 1    <STL>
```

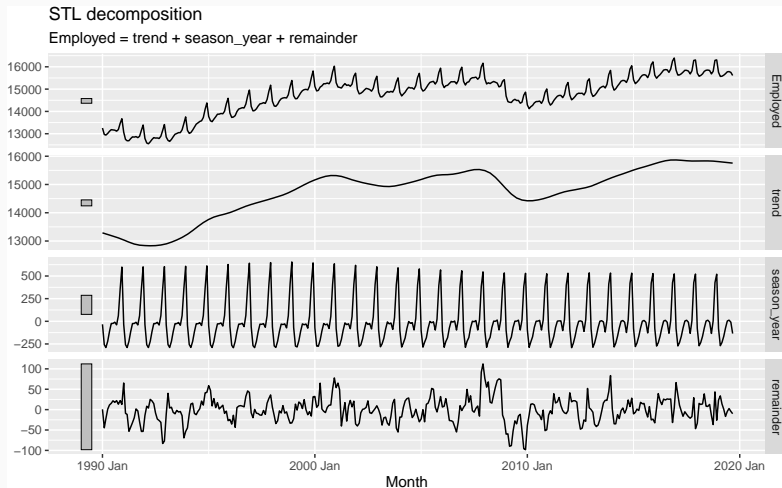

US Retail Employment

```
dcmp <- us_retail_employment |>
  model(stl = STL(Employed))
components(dcmp)
```

```
## # A dable: 357 x 7 [1M]
## # Key:      .model [1]
## # :      Employed = trend + season_year + remainder
##   .model      Month Employed  trend season_year remainder season_adjust
##   <chr>      <mth>    <dbl>  <dbl>      <dbl>      <dbl>      <dbl>
## 1 stl       1990 Jan    13256. 13288.      -33.0        0.836    13289.
## 2 stl       1990 Feb    12966. 13269.     -258.       -44.6    13224.
## 3 stl       1990 Mar    12938. 13250.     -290.       -22.1    13228.
## 4 stl       1990 Apr    13012. 13231.     -220.        1.05    13232.
## 5 stl       1990 May    13108. 13211.     -114.       11.3    13223.
## 6 stl       1990 Jun    13183. 13192.     -24.3       15.5    13207.
## 7 stl       1990 Jul    13170. 13172.     -23.2       21.6    13193. 8
## 8 stl       1990 Aug    13168. 13151.     -17.0       17.0    13168.
```

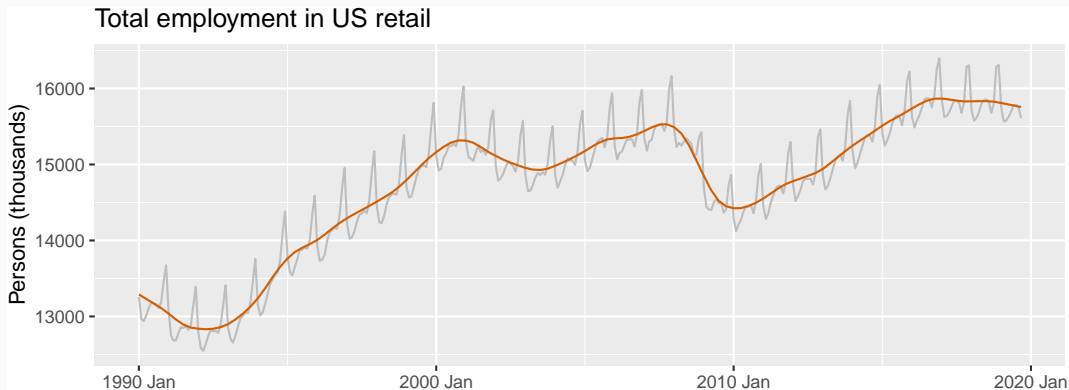
US Retail Employment

```
components(dcmp) |> autoplot()
```



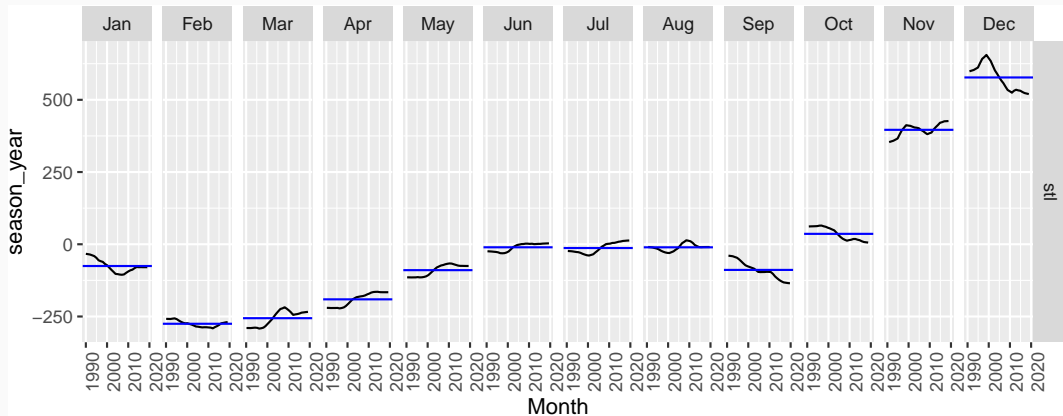
US Retail Employment

```
us_retail_employment |>  
  autoplot(Employed, color = "gray") +  
  autolayer(components(dcmp), trend, color = "#D55E00") +  
  labs(y = "Persons (thousands)", title = "Total employment in US retail")
```



US Retail Employment

```
components(dcmp) |> gg_subseries(season_year)
```



Seasonal adjustment

- Useful by-product of decomposition: an easy way to calculate seasonally adjusted data.
- Additive decomposition: seasonally adjusted data given by

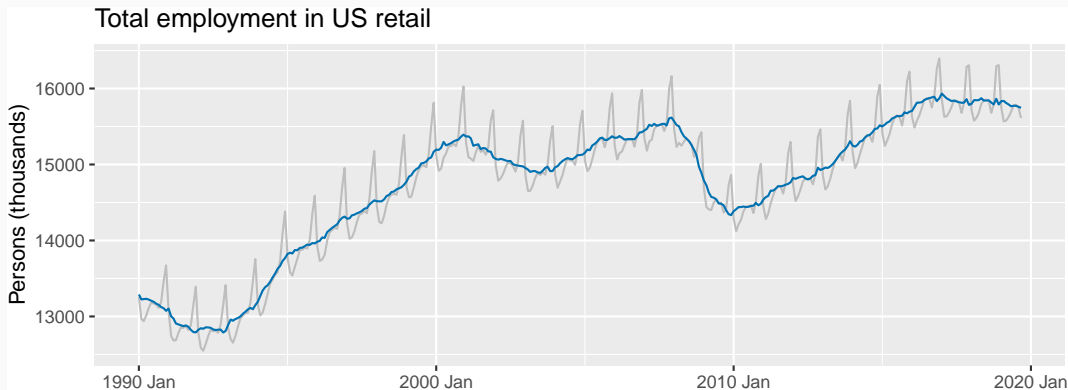
$$y_t - S_t = T_t + R_t$$

- Multiplicative decomposition: seasonally adjusted data given by

$$y_t/S_t = T_t \times R_t$$

US Retail Employment

```
us_retail_employment |>  
  autoplot(Employed, color = "gray") +  
  autolayer(components(dcmp), season_adjust, color = "#0072B2") +  
  labs(y = "Persons (thousands)", title = "Total employment in US retail")
```



Seasonal adjustment

- We use estimates of S based on past values to seasonally adjust a current value.
- Seasonally adjusted series reflect **remainders** as well as **trend**. Therefore they are not “smooth” and “downturns” or “upturns” can be misleading.
- It is better to use the trend-cycle component to look for turning points.