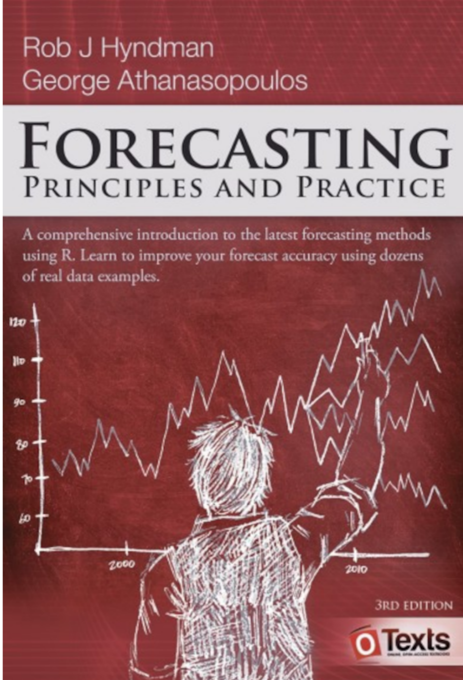
5. The forecaster's toolbox



5.4 Residual diagnostics

OTexts.org/fpp3/

Facebook closing stock price

```
fb_stock |> autoplot(Close)
```

Facebook closing stock price

```
fit <- fb_stock |> model(NAIVE(Close))
augment(fit)
```

```
## # A tsibble: 1,258 x 7 [1]
## # Key:       Symbol, .model [1]
##    Symbol .model       trading_day Close .fitted .resid .innov
##    <chr>  <chr>              <int> <dbl>   <dbl>  <dbl>  <dbl>
##  1 FB     NAIVE(Close)           1  54.7    NA   NA     NA
##  2 FB     NAIVE(Close)           2  54.6    54.7 -0.150 -0.150
##  3 FB     NAIVE(Close)           3  57.2    54.6  2.64   2.64
##  4 FB     NAIVE(Close)           4  57.9    57.2  0.720  0.720
##  5 FB     NAIVE(Close)           5  58.2    57.9  0.310  0.310
##  6 FB     NAIVE(Close)           6  57.2    58.2 -1.01  -1.01
##  7 FB     NAIVE(Close)           7  57.9    57.2  0.720  0.720
##  8 FB     NAIVE(Close)           8  55.9    57.9 -2.03  -2.03
##  9 FB     NAIVE(Close)           9  57.7    55.9  1.83   1.83
## 10 FB     NAIVE(Close)          10  57.6    57.7 -0.140 -0.140
## # ... with 1,248 more rows
```

Facebook closing stock price

```r
fit <- fb_stock |> model(NAIVE(Close))
augment(fit)
```

$\hat{y}_{t|t-1}$ e_t

```
## # A tsibble: 1,258 x 7 [1]
## # Key:       Symbol, .model [1]
##    Symbol .model       trading_day Close .fitted .resid .innov
##    <chr>  <chr>              <int> <dbl>   <dbl>  <dbl>  <dbl>
##  1 FB     NAIVE(Close)           1  54.7    NA   NA     NA
##  2 FB     NAIVE(Close)           2  54.6    54.7 -0.150 -0.150
##  3 FB     NAIVE(Close)           3  57.2    54.6  2.64   2.64
##  4 FB     NAIVE(Close)           4  57.9    57.2  0.720  0.720
##  5 FB     NAIVE(Close)           5  58.2    57.9  0.310  0.310
                                   6  57.2    58.2 -1.01  -1.01
                                   7  57.9    57.2  0.720  0.720
                                   8  55.9    57.9 -2.03  -2.03
                                   9  57.7    55.9  1.83   1.83
## 10 FB     NAIVE(Close)          10  57.6    57.7 -0.140 -0.140
## # ... with 1,248 more rows
```

Naïve forecasts:

$$\hat{y}_{t|t-1} = y_{t-1}$$

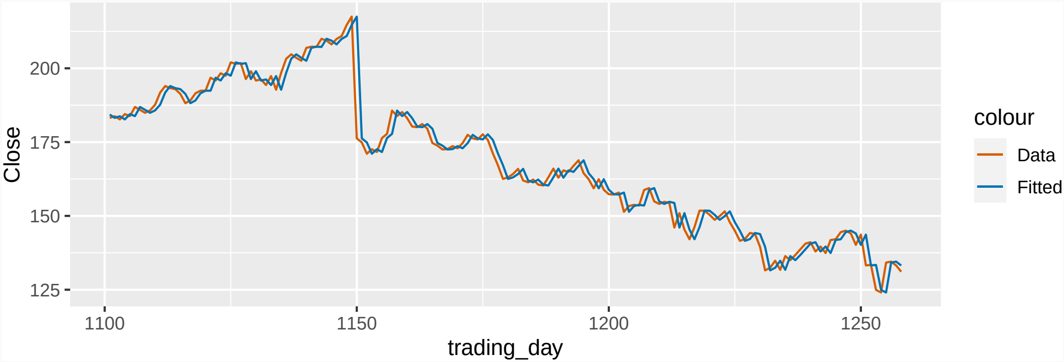
$$e_t = y_t - \hat{y}_{t|t-1} = y_t - y_{t-1}$$

Facebook closing stock price

```
augment(fit) |>
  ggplot(aes(x = trading_day)) +
  geom_line(aes(y = Close, colour = "Data")) +
  geom_line(aes(y = .fitted, colour = "Fitted"))
```

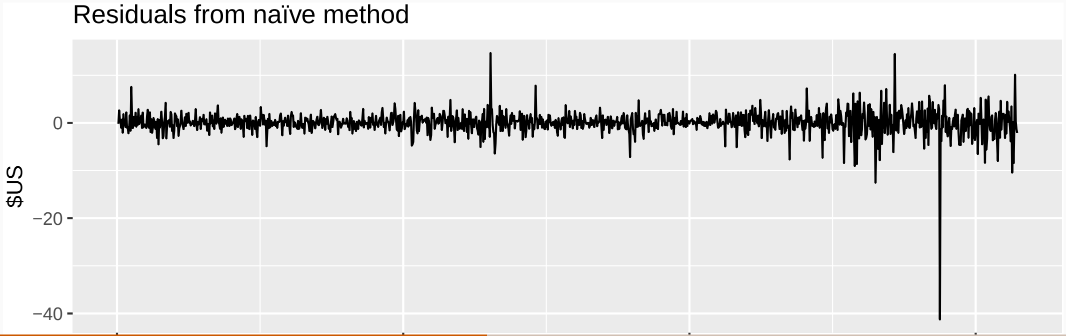
Facebook closing stock price

```
augment(fit) |>
  filter(trading_day > 1100) |>
  ggplot(aes(x = trading_day)) +
  geom_line(aes(y = Close, colour = "Data")) +
  geom_line(aes(y = .fitted, colour = "Fitted"))
```



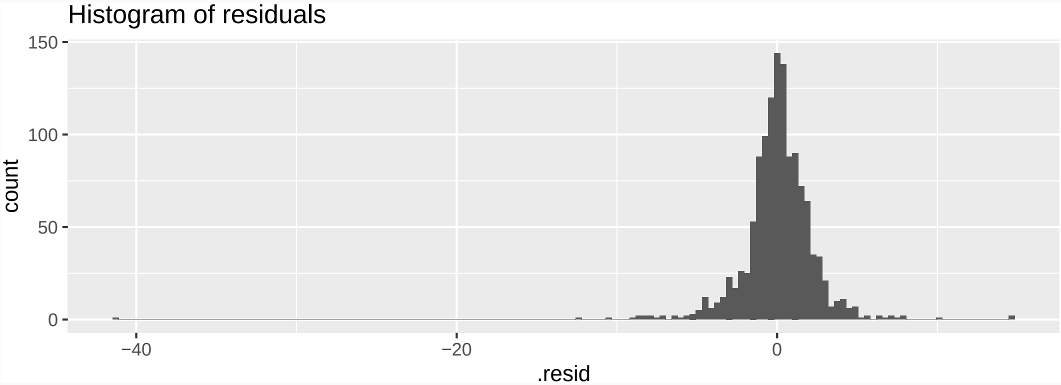
Facebook closing stock price

```
augment(fit) |>
  autoplot(.resid) +
  labs(
    y = "$US",
    title = "Residuals from naïve method"
  )
```



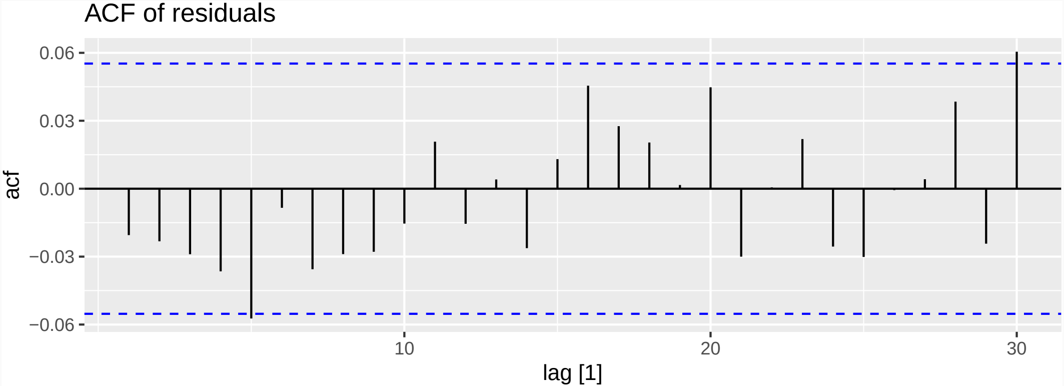
Facebook closing stock price

```r
augment(fit) |>
  ggplot(aes(x = .resid)) +
  geom_histogram(bins = 150) +
  labs(title = "Histogram of residuals")
```



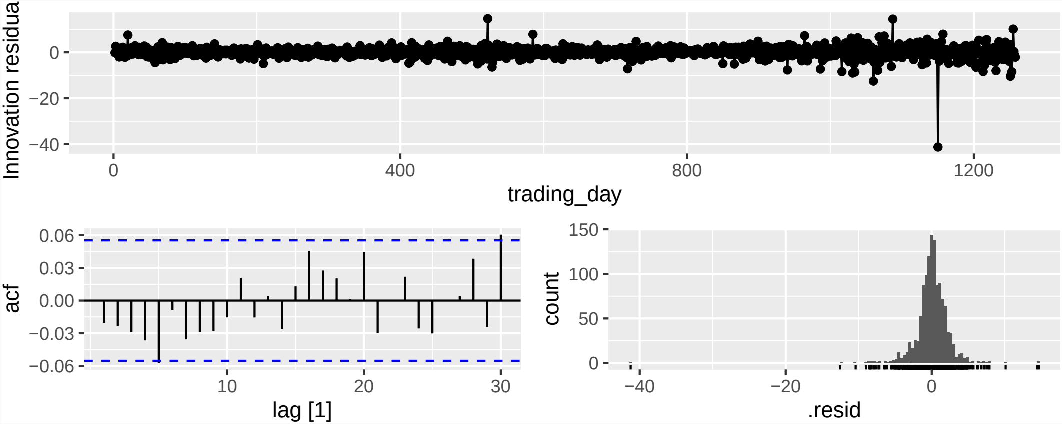
Facebook closing stock price

```
augment(fit) |>
  ACF(.resid) |>
  autoplot() + labs(title = "ACF of residuals")
```



gg_tsresiduals() function

```
gg_tsresiduals(fit)
```



ACF of residuals

- We assume that the residuals are white noise (uncorrelated, mean zero, constant variance). If they aren't, then there is information left in the residuals that should be used in computing forecasts.
- So a standard residual diagnostic is to check the ACF of the residuals of a forecasting method.
- We *expect* these to look like white noise.

Portmanteau tests

r_k = autocorrelation of residual at lag k

Consider a *whole set* of r_k values, and develop a test to see whether the set is significantly different from a zero set.

Portmanteau tests

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Consider a *whole set* of r_k values, and develop a test to see whether the set is significantly different from a zero set.

Box-Pierce test

$$Q = T\sum_{k=1}^{\ell} r_k^2$$

where ℓ is max lag being considered and T is number of observations.

- If each r_k close to zero, Q will be **small**.
- If some r_k values large (positive or negative), Q will be **large**.

Portmanteau tests

r_k = autocorrelation of residual at lag k

Consider a *whole set* of r_k values, and develop a test to see whether the set is significantly different from a zero set.

Ljung-Box test

$$Q^* = T(T+2)\sum_{k=1}^{\ell}(T-k)^{-1}r_k^2$$

where ℓ is max lag being considered and T is number of observations.

- My preferences: $\ell = 10$ for non-seasonal data, $h = 2m$ for seasonal data (where m is seasonal period).
- Better performance, especially in small samples.

Portmanteau tests

- If data are WN, Q^* has χ^2 distribution with $(\ell - K)$ degrees of freedom where K = no. parameters in model.
- When applied to raw data, set $K = 0$.
- `lag` = ℓ, `dof` = K

```
augment(fit) |>
  features(.resid, ljung_box, lag = 10, dof = 0)
```

```
## # A tibble: 1 x 4
##   Symbol .model        lb_stat lb_pvalue
##   <chr>  <chr>           <dbl>     <dbl>
## 1 FB     NAIVE(Close)     12.1     0.276
```