Lab 1 Block 2 Report

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2023-11-27

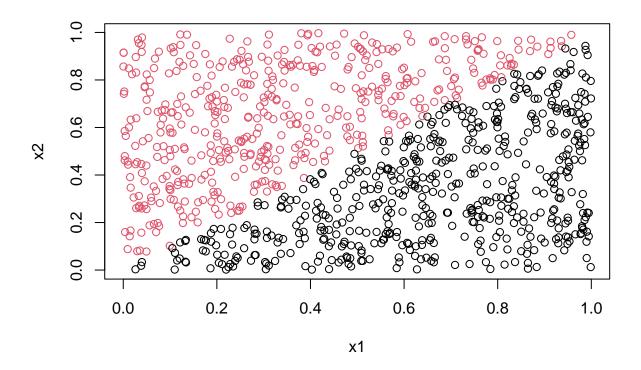
1. Ensemble Methods

Your task is to learn some random forests using the function randomForest from the R package randomForest. The training data is produced by running the following R code:

```
x1<-runif(100)
x2<-runif(100)
trdata<-cbind(x1,x2)
y<-as.numeric(x1<x2)
trlabels<-as.factor(y)
```

The task is therefore classifying Y from X1 and X2, where Y is binary and X1 and X2 continuous. You should learn a random forest with 1, 10 and 100 trees, which you can do by setting the argument ntree to the appropriate value. Use nodesize = 25 and keep.forest = TRUE. The latter saves the random forest learned. You need it because you should also compute the misclassification error in the following test dataset (use the function predict for this purpose):

```
set . seed (1234)
x1<-runif (1000)
x2<-runif (1000)
tedata<-cbind(x1,x2)
y<-as.numeric(x1<x2)
telabels<-as.factor(y)
plot(x1,x2,col=(y+1))</pre>
```



```
get_misc_err <- function(conf_mat){</pre>
  \#Given\ a\ confusion\ matrix , return the misclassification error
  (sum(conf_mat)-sum(diag(conf_mat)))/sum(conf_mat)
}
r1 <- randomForest(trdata, trlabels, ntree = 1, nodesize = 25, keep.forest =
   TRUE)
p1 <- predict(r1, tedata)
conf_mat1 <- table(p1, telabels)
get_misc_err(conf_mat1)
## [1] 0.206
cat("Mean_misclassification_error_for_1_tree:", get_misc_err(conf_mat1), "\n")
## Mean misclassification error for 1 tree: 0.206
r10 <- randomForest(trdata, trlabels, ntree = 10, nodesize = 25, keep.forest =
p10 <- predict(r10, tedata)
conf_mat10 <- table(p10, telabels)
get_misc_err(conf_mat10)
## [1] 0.106
cat("Mean_misclassification_error_for_10_trees:", get_misc_err(conf_mat10), "\
   n")
```

```
## Mean misclassification error for 10 trees: 0.106
r100 <- randomForest(trdata, trlabels, ntree = 100, nodesize = 25, keep.forest
     = TRUE)
p100 <- predict (r100, tedata)
conf_mat100 <- table(p100, telabels)
get_misc_err(conf_mat100)
## [1] 0.07
cat ("Mean_misclassification_error_for_100_trees:", get_misc_err(conf_mat100),
    "\n")
## Mean misclassification error for 100 trees: 0.07
Repeat the procedure above for 1000 training datasets of size 100 and report the mean and variance of the
misclassification errors. In other words, create 1000 training datasets of size 100, learn a random forest from
each dataset, and compute the misclassification error in the same test dataset of size 1000. Report results
for when the random forest has 1, 10 and 100 trees.
get_training <- function(){</pre>
  x1<-runif(100)
  x2<-runif(100)
  trdata < -cbind(x1, x2)
  y \le -as \cdot numeric(x1 \le x2)
  trlabels <-- as. factor (y)
  return(list(trdata, trlabels))
meR1 \leftarrow c()
meR10 \leftarrow c()
meR100 \leftarrow c()
for(i in 1:1000){
  \#cat("Iteration:", i, " \ r")
  #Creating the training data
  training <- get_training()
  trdata <- training [[1]]
  trlabels <- training [[2]]
  r1 <- randomForest(trdata, trlabels, ntree = 1, nodesize = 25, keep.forest =
       TRUE)
  p1 <- predict(r1, tedata)
  conf_mat1 <- table(p1, telabels)
  meR1 \leftarrow c(meR1, get_misc_err(conf_mat1))
  r10 <- randomForest(trdata, trlabels, ntree = 10, nodesize = 25, keep.forest
       = TRUE)
  p10 <- predict(r10, tedata)
```

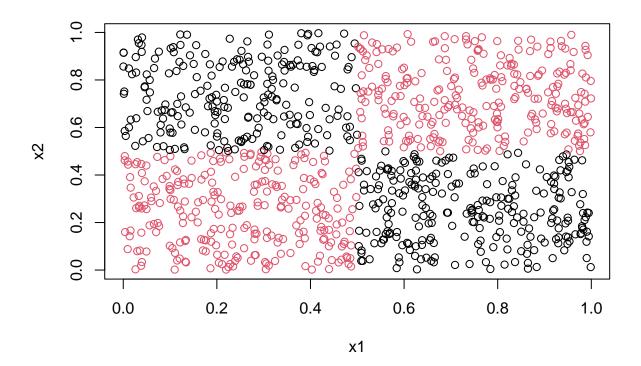
conf_mat10 <- table(p10, telabels)

 $meR10 \leftarrow c(meR10, get_misc_err(conf_mat10))$

```
r100 <- randomForest(trdata, trlabels, ntree = 100, nodesize = 25, keep.
      forest = TRUE)
  p100 <- predict (r100, tedata)
  conf_mat100 <- table(p100, telabels)
  meR100 <- c (meR100, get_misc_err (conf_mat100))
#Overall mean
\mathbf{cat}("Overall_{\square}mean_{\square}for_{\square}1_{\square}tree:", \mathbf{mean}(meR1), "\setminus n")
## Overall mean for 1 tree: 0.209284
cat("Overall_variance_for_1_tree:", var(meR1), "\n")
## Overall variance for 1 tree: 0.00309292
cat("Overall_mean_for_10_trees:", mean(meR10), "\n")
## Overall mean for 10 trees: 0.134774
cat("Overall_variance_for_10_trees:", var(meR10), "\n")
## Overall variance for 10 trees: 0.0008590079
cat("Overall_mean_for_100_trees:", mean(meR100), "\n")
## Overall mean for 100 trees: 0.111006
cat("Overall_variance_for_100_trees:", var(meR100), "\n")
## Overall variance for 100 trees: 0.0008635495
Repeat the exercise above but this time use the condition (x1<0.5) instead of (x1<x2) when producing the
training and test datasets.
set . seed (1234)
x1<-runif(1000)
x2<-runif(1000)
tedata < -cbind(x1, x2)
y < -as.numeric(x1 < 0.5)
telabels <- as. factor (y)
plot (x1, x2, col = (y+1))
```

```
get_training <- function(){</pre>
  x1<-runif(100)
  x2<-runif(100)
  trdata < -cbind(x1, x2)
  y<-as . \mathbf{numeric} ( \mathbf{x}1 < 0.5 )
  trlabels <- as. factor (y)
  return(list(trdata, trlabels))
}
meR1 \leftarrow c()
meR10 \leftarrow c()
meR100 \leftarrow c()
for(i in 1:1000){
  \#cat("Iteration:", i, " \mid r")
  #Creating the training data
  training <- get_training()</pre>
  trdata <- training [[1]]
  trlabels <- training [[2]]
  r1 <- randomForest(trdata, trlabels, ntree = 1, nodesize = 25, keep.forest =
       TRUE)
  p1 <- predict(r1, tedata)
```

```
conf_mat1 <- table(p1, telabels)
  meR1 <- c(meR1, get_misc_err(conf_mat1))
  r10 <- randomForest(trdata, trlabels, ntree = 10, nodesize = 25, keep.forest
       = TRUE)
  p10 <- predict(r10, tedata)
  conf_mat10 <- table(p10, telabels)
  meR10 \leftarrow c(meR10, get\_misc\_err(conf\_mat10))
  r100 <- randomForest(trdata, trlabels, ntree = 100, nodesize = 25, keep.
       forest = TRUE)
  p100 <- predict (r100, tedata)
  conf_mat100 <- table(p100, telabels)
  meR100 \leftarrow c(meR100, get_misc_err(conf_mat100))
}
#Overall mean
\mathbf{cat}(" \, \text{Overall}_{\square} \text{mean}_{\square} \text{for}_{\square} \mathbf{1}_{\square} \text{tree} : ", \, \mathbf{mean}(\text{meR1}), \, " \setminus n")
## Overall mean for 1 tree: 0.092951
cat("Overall_variance_for_1_tree:", var(meR1), "\n")
## Overall variance for 1 tree: 0.01729452
cat ("Overall_mean_for_10_trees:", mean(meR10), "\n")
## Overall mean for 10 trees: 0.013772
cat("Overall_variance_for_10_trees:", var(meR10), "\n")
## Overall variance for 10 trees: 0.0004568148
cat("Overall_mean_for_100_trees:", mean(meR100), "\n")
## Overall mean for 100 trees: 0.006414
cat("Overall_variance_for_100_trees:", var(meR100), "\n")
## Overall variance for 100 trees: 6.676737e-05
Repeat the exercise above but this time use the condition ((x1<0.5 \& x2<0.5) | (x1>0.5 \& x2>0.5)) instead
of (x_1 < x_2) when producing the training and test datasets. Unlike above, use nodesize = 12 for this exercise.
\mathbf{set} . \mathbf{seed} (1234)
x1<-runif(1000)
x2<-runif(1000)
tedata < -cbind(x1, x2)
y \leftarrow as. numeric(((x1<0.5 \& x2<0.5) | (x1>0.5 \& x2>0.5)))
telabels<-as.factor(y)
\mathbf{plot}(x1, x2, \mathbf{col} = (y+1))
```



```
get_training <- function(){</pre>
  x1<-runif(100)
  x2<-runif(100)
  trdata < -cbind(x1, x2)
  y < -as.numeric((x1 < 0.5 \& x2 < 0.5) | (x1 > 0.5 \& x2 > 0.5))
  trlabels <- as. factor (y)
  return(list(trdata, trlabels))
}
meR1 \leftarrow c()
meR10 \leftarrow c()
meR100 \leftarrow c()
for(i in 1:1000){
  \#cat("Iteration:", i, " \mid r")
  #Creating the training data
  training <- get_training()</pre>
  trdata <- training [[1]]
  trlabels <- training [[2]]
  r1 <- randomForest(trdata, trlabels, ntree = 1, nodesize = 12, keep.forest =
       TRUE)
  p1 <- predict(r1, tedata)
```

```
conf_mat1 <- table(p1, telabels)
  meR1 <- c(meR1, get_misc_err(conf_mat1))
  r10 <- randomForest(trdata, trlabels, ntree = 10, nodesize = 12, keep.forest
       = TRUE)
  p10 <- predict (r10, tedata)
  conf mat10 <- table(p10, telabels)
  meR10 \leftarrow c(meR10, get\_misc\_err(conf\_mat10))
  r100 <-- randomForest(trdata, trlabels, ntree = 100, nodesize = 12, keep.
      forest = TRUE)
  p100 <- predict (r100, tedata)
  conf_mat100 <- table(p100, telabels)
  meR100 \leftarrow c(meR100, get_misc_err(conf_mat100))
}
#Overall mean
\mathbf{cat}(" \, \text{Overall}_{\square} \text{mean}_{\square} \text{for}_{\square} \mathbf{1}_{\square} \text{tree} : ", \, \mathbf{mean}(\text{meR1}), \, " \setminus n")
## Overall mean for 1 tree: 0.251183
cat("Overall_variance_for_1_tree:", var(meR1), "\n")
## Overall variance for 1 tree: 0.01325594
cat ( "Overall_mean_for_10_trees: ", mean(meR10), "\n")
## Overall mean for 10 trees: 0.122827
cat("Overall_variance_for_10_trees:", var(meR10), "\n")
## Overall variance for 10 trees: 0.00320077
cat("Overall_mean_for_100_trees:", mean(meR100), "\n")
## Overall mean for 100 trees: 0.076979
cat("Overall_variance_for_100_trees:", var(meR100), "\n")
## Overall variance for 100 trees: 0.001413212
```

Answer the following questions: – What happens with the mean error rate when the number of trees in the random forest grows? Why?

The mean error rate decreases as the number of trees in the random forest grows. This is because as the number of tree increases, the "wisdom of crowd" effect intensifies

– The third dataset represents a slightly more complicated classification problem than the first one. Still, you should get better performance for it when using sufficient trees in the random forest. Explain why you get better performance.

We still get better performances because as the number of trees increases, we reduce the chances of overfitting by having many different trees that ignores the random variation in the training data and also reduces the variance of the ensemble model.

2. Mixture Models

Your task is to implement the EM algorithm for Bernoulli mixture model. Please use the R template below to solve the assignment. Then, use your implementation to show what happens when your mixture model has too few and too many clusters, i.e. set M=2, 3, 4 and compare results. Please provide a short explanation as well. A Bernoulli mixture model is

$$p(x) = \sum_{m=1}^{M} \pi_m Bern(x|\mu_m)$$

where $x = (x_1, ..., x_D)$ is a D-dimensional binary random vector, $\pi_m = p(y = m)$ and

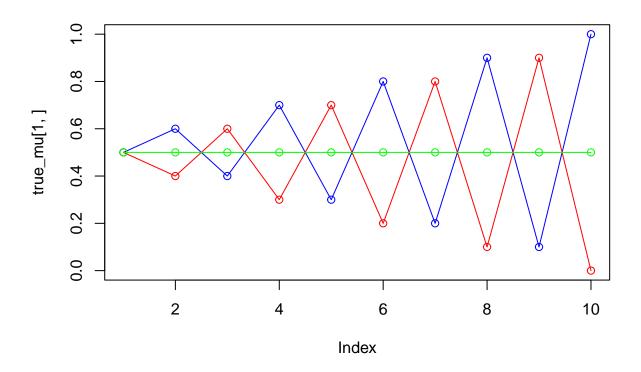
$$Bern(x|\mu_m) = \prod_{d=1}^{D} \mu_{m,d}^{x_d} (1 - \mu_{m,d})^{(1-x_d)}$$

where $\mu_m = (\mu_{m,1}, ..., \mu_{m,D})$ is a D-dimensional vector of probabilities. As usual, the log likelihood of the dataset $\{x_i\}_{i=1}^n$ is

$$\sum_{i=1}^{M} \log p(x_i)$$

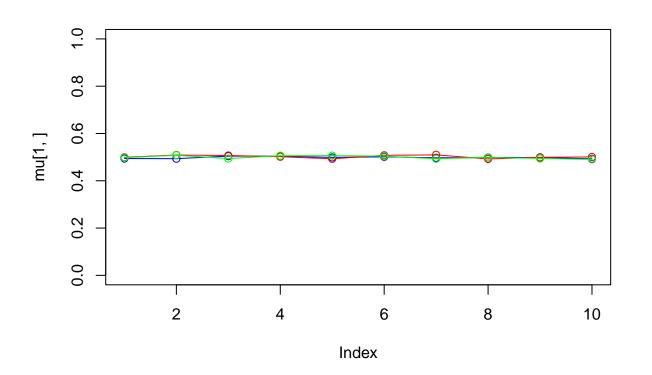
Finally, in the EM algorithm, the parameter updates for the Bernoulli mixture model are the same as for the Gaussian mixture model (see Equations 10.16a,b in the lecture slides).

```
set . seed (1234567890)
max_it <- 100 # max number of EM iterations
min_change <- 0.1 # min change in log lik between two consecutive iterations
n=1000 # number of training points
D=10 # number of dimensions
x <- matrix(nrow=n, ncol=D) # training data
true_pi <- vector(length = 3) # true mixing coefficients
true_mu <- matrix(nrow=3, ncol=D) # true conditional distributions
true_pi=c(1/3, 1/3, 1/3)
true_mu[1,]=c(0.5,0.6,0.4,0.7,0.3,0.8,0.2,0.9,0.1,1)
true_mu[2,]=c(0.5,0.4,0.6,0.3,0.7,0.2,0.8,0.1,0.9,0)
true_mu[3,]=c(0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5,0.5)
plot(true_mu[1,], type="o", col="blue", ylim=c(0,1))
points(true_mu[2,], type="o", col="red")
points(true_mu[3,], type="o", col="green")</pre>
```



```
# Producing the training data
for (i in 1:n) {
  m <- sample (1:3,1,prob=true_pi)
  for (d in 1:D) {
    x[i,d] \leftarrow \mathbf{rbinom}(1,1, true\_mu[m,d])
M=3 \# number of clusters
w \leftarrow matrix(nrow=n, ncol=M) \# weights
pi <- vector (length = M) # mixing coefficients
mu <- matrix (nrow=M, ncol=D) # conditional distributions
llik <- vector(length = max_it) # log likelihood of the EM iterations
\# Random initialization of the parameters
pi \leftarrow runif(M, 0.49, 0.51)
pi <- pi / sum(pi)
for (m in 1:M) {
  mu[m,] \leftarrow runif(D, 0.49, 0.51)
}
рi
\#\# [1] 0.3326090 0.3336558 0.3337352
mu
                          [,2]
                                     [,3]
                                                [,4]
                                                                      [,6]
##
               [,1]
                                                           [,5]
\#\# [1,] 0.4939877 0.4935375 0.5042511 0.5040286 0.4987810 0.5012754 0.4971036
\#\# [2,] 0.4993719 0.5088453 0.5068730 0.5016720 0.4929275 0.5077146 0.5095075
```

```
\#\# [3,] 0.4975302 0.5077926 0.4939841 0.5059821 0.5063490 0.5041462 0.4929400
##
                 [, 8]
                                         [,10]
                              [, 9]
## [1,] 0.4982144 0.4987654 0.4929075
## [2,] 0.4924574 0.4992470 0.5008651
## [3,] 0.4992362 0.4943482 0.4903974
for (it in 1:max_it) {
  \mathbf{plot} \left( \mathbf{mu} [1,], \mathbf{type} = \mathbf{0}, \mathbf{col} = \mathbf{blue}, \mathbf{ylim} = \mathbf{c} (0,1) \right)
  points(mu[2,], type="o", col="red")
points(mu[3,], type="o", col="green")
  \#points(mu[4,], type = "o", col = "yellow")
  Sys. sleep (0.1)
  \# E-step: Computation of the weights
  # Your code here
  \#Log\ likelihood\ computation.
  # Your code here
  \mathbf{cat}("iteration:_{\square}", it, "log_{\square}likelihood:_{\square}", llik[it], "\n")
  flush.console()
  # Stop if the lok likelihood has not changed significantly
  # Your code here
  #M-step: ML parameter estimation from the data and weights
  # Your code here
}
```



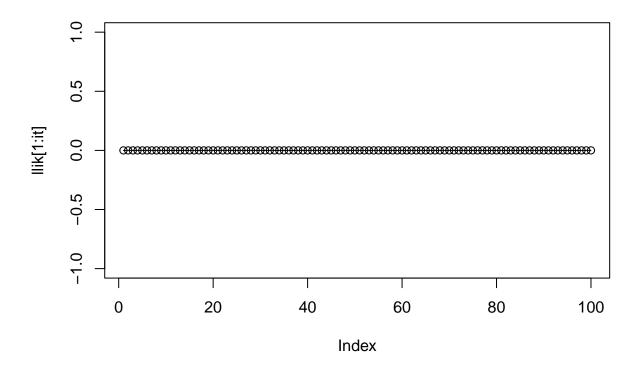
iteration: 1 log likelihood: FALSE
iteration: 2 log likelihood: FALSE

```
3 log likelihood:
## iteration:
                                   FALSE
## iteration:
                4 log likelihood:
                                   FALSE
## iteration:
                5 log likelihood:
                                   FALSE
## iteration:
                6 log likelihood:
                                   FALSE
## iteration:
                7 log likelihood:
                                   FALSE
## iteration:
                8 log likelihood:
                                   FALSE
## iteration:
                9 log likelihood:
                                   FALSE
## iteration:
                10 log likelihood:
                                    FALSE
                11 log likelihood:
                                    FALSE
## iteration:
## iteration:
                12 log likelihood:
                                    FALSE
## iteration:
                13 log likelihood:
                                    FALSE
                14 log likelihood:
## iteration:
                                    FALSE
## iteration:
                15 log likelihood:
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## iteration:
                16 log likelihood:
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## iteration:
                17 log likelihood:
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                18 log likelihood:
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                19 log likelihood:
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                20 log likelihood:
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                21 log likelihood:
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                22 log likelihood:
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                23 log likelihood:
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                25 log likelihood:
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                26 log likelihood:
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                30 log likelihood:
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                34 log likelihood:
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## iteration:
                35 log likelihood:
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                36 log likelihood:
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                37 log likelihood:
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                38 log likelihood:
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                41 log likelihood:
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                44 log likelihood:
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                48 log likelihood:
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                49 log likelihood:
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                50 log likelihood:
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## iteration:
                51 log likelihood:
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                53 log likelihood:
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                54 log likelihood:
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                56 log likelihood:
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                57 log likelihood:
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                                    FALSE
## iteration:
                58 log likelihood:
```

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                59 log likelihood:
                                    FALSE
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                60 log likelihood:
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                61 log likelihood:
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                62 log likelihood:
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                63 log likelihood:
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## iteration:
                64 log likelihood:
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## iteration:
                65 log likelihood:
                                    FALSE
## iteration:
                66 log likelihood:
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## iteration:
                67 log likelihood:
                                    FALSE
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                68 log likelihood:
                                    FALSE
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                69 log likelihood:
                                    FALSE
                70 log likelihood:
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## iteration:
                71 log likelihood:
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                73 log likelihood:
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                74 log likelihood:
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                75 log likelihood:
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                76 log likelihood:
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                77 log likelihood:
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                78 log likelihood:
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                79 log likelihood:
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## iteration:
                80 log likelihood:
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                81 log likelihood:
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## iteration:
                82 log likelihood:
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                85 log likelihood:
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## iteration:
                86 log likelihood:
                                    FALSE
```

```
87 log likelihood:
## iteration:
               88 log likelihood:
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               89 log likelihood:
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               90 log likelihood:
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## iteration:
               91 log likelihood:
                                    FALSE
## iteration:
               92 log likelihood:
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               93 log likelihood:
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## iteration:
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               94 log likelihood:
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               95 log likelihood:
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               96 log likelihood:
## iteration:
                                    FALSE
## iteration:
               97 log likelihood:
                                    FALSE
               98 log likelihood: FALSE
## iteration:
               99 log likelihood: FALSE
## iteration:
## iteration:
               100 log likelihood: FALSE
рi
## [1] 0.3326090 0.3336558 0.3337352
mu
##
                        [,2]
                                  [,3]
                                                                 [, 6]
              [,1]
                                            [,4]
                                                       [,5]
\#\# [1,] 0.4939877 0.4935375 0.5042511 0.5040286 0.4987810 0.5012754 0.4971036
\#\# [2,] 0.4993719 0.5088453 0.5068730 0.5016720 0.4929275 0.5077146 0.5095075
\#\# [3,] 0.4975302 0.5077926 0.4939841 0.5059821 0.5063490 0.5041462 0.4929400
              [, 8]
                        [, 9]
                                 [,10]
##
## [1,] 0.4982144 0.4987654 0.4929075
## [2,] 0.4924574 0.4992470 0.5008651
## [3,] 0.4992362 0.4943482 0.4903974
plot (llik [1: it], type="o")
```



Appendix

```
knitr::opts\_chunk\$set (echo = TRUE)
library(randomForest)
x1<-runif(100)
x2<-runif(100)
trdata < -cbind(x1, x2)
y < -as. numeric(x1 < x2)
trlabels<-as.factor(y)
set . seed (1234)
x1<-runif(1000)
x2<-runif(1000)
tedata \leftarrow cbind(x1, x2)
y < -as. numeric(x1 < x2)
telabels <- as. factor (y)
\mathbf{plot}(x1, x2, \mathbf{col} = (y+1))
get_misc_err <- function(conf_mat){</pre>
  #Given a confusion matrix, return the misclassification error
  (sum(conf_mat)-sum(diag(conf_mat)))/sum(conf_mat)
}
r1 <- randomForest(trdata, trlabels, ntree = 1, nodesize = 25, keep.forest =
   TRUE)
```

```
p1 <- predict(r1, tedata)
conf_mat1 <- table(p1, telabels)
get misc err (conf mat1)
cat("Mean_misclassification_error_for_1_tree:", get_misc_err(conf_mat1), "\n")
r10 <- randomForest(trdata, trlabels, ntree = 10, nodesize = 25, keep.forest =
     TRUE)
p10 <- predict(r10, tedata)
conf_mat10 <- table(p10, telabels)
get_misc_err(conf_mat10)
\mathbf{cat}("\mathrm{Mean}_{\square}\,\mathrm{misclassification}_{\square}\,\mathrm{error}_{\square}\,\mathrm{for}_{\square}10_{\square}\,\mathrm{trees}:",\ \mathbf{get}_{\underline{\hspace{-0.5em}}\,\mathrm{misc}}\,\mathrm{err}\,(\,\mathrm{conf}_{\underline{\hspace{-0.5em}}\,\mathrm{mat}}10)\,,\ "\setminus
    n")
r100 <-- randomForest(trdata, trlabels, ntree = 100, nodesize = 25, keep.forest
     = TRUE
p100 <- predict(r100, tedata)
conf_mat100 <- table(p100, telabels)
get misc err (conf mat100)
cat ("Mean_misclassification_error_for_100_trees:", get_misc_err(conf_mat100),
    "\n")
get_training <- function(){</pre>
  x1<-runif(100)
  x2<-runif(100)
  trdata < -cbind(x1, x2)
  y < -as.numeric(x1 < x2)
  trlabels <-- as. factor (y)
  return(list(trdata, trlabels))
meR1 \leftarrow c()
meR10 \leftarrow c()
meR100 \leftarrow c
for (i in 1:1000) {
  \#cat("Iteration:", i, " \mid r")
  #Creating the training data
  training <- get_training()
  trdata <- training [[1]]
  trlabels <- training [[2]]
  r1 <- randomForest(trdata, trlabels, ntree = 1, nodesize = 25, keep.forest =
       TRUE)
  p1 <- predict(r1, tedata)
  conf_mat1 <- table(p1, telabels)
  meR1 <- c(meR1, get_misc_err(conf_mat1))
  r10 <- randomForest(trdata, trlabels, ntree = 10, nodesize = 25, keep.forest
       = TRUE)
```

```
p10 <- predict(r10, tedata)
   conf_mat10 <- table(p10, telabels)
  meR10 <- c(meR10, get_misc_err(conf_mat10))
  r100 <- randomForest(trdata, trlabels, ntree = 100, nodesize = 25, keep.
       forest = TRUE)
  p100 <- predict (r100, tedata)
  conf_mat100 <- table(p100, telabels)
  meR100 <- c (meR100, get_misc_err (conf_mat100))
}
#Overall mean
cat("Overall_mean_for_1_tree:", mean(meR1), "\n")
\mathbf{cat}(" \, Overall_{\sqcup} \, variance_{\sqcup} \, for_{\sqcup} 1_{\sqcup} \, tree : ", \, \mathbf{var}(meR1), \, " \setminus n")
\mathbf{cat} \, (\, "\, O\, verall_{\,\sqcup} mean_{\,\sqcup} \, for_{\,\sqcup} \, 10_{\,\sqcup} \, tre\, es: \, "\, , \  \, \mathbf{mean} \, (\, meR10\,) \, , \  \, "\, \backslash n \, "\,)
{f cat} ("Overall_variance_for_10_trees:", {f var} (meR10),
\mathbf{cat} \, (\, "\, O\, verall \, \bot \, mean \, \bot \, for \, \bot \, 100 \, \bot \, trees : \, "\, , \  \, \mathbf{mean} (\, meR100) \, , \  \, "\, \backslash n \, "\, )
cat("Overall_variance_for_100_trees:", var(meR100), "\n")
set . seed (1234)
x1 < -runif(1000)
x2 < -runif(1000)
tedata < -cbind(x1, x2)
y < -as. numeric (x1 < 0.5)
telabels <- as. factor (y)
plot (x1, x2, col = (y+1))
get_training <- function(){</pre>
  x1<-runif(100)
  x2<-runif(100)
  trdata < -cbind(x1, x2)
  y < -as.numeric(x1 < 0.5)
   trlabels <-- as. factor (y)
  return(list(trdata, trlabels))
meR1 \leftarrow c()
meR10 \leftarrow c()
meR100 \leftarrow c()
for(i in 1:1000){
  \#cat("Iteration:", i, " \ r")
  #Creating the training data
   training <- get_training()
   trdata <- training [[1]]
   trlabels <- training [[2]]
  r1 <- randomForest(trdata, trlabels, ntree = 1, nodesize = 25, keep.forest =
        TRUE)
  p1 <- predict(r1, tedata)
   conf_mat1 <- table(p1, telabels)
```

```
meR1 \leftarrow c(meR1, get\_misc\_err(conf\_mat1))
  r10 <- randomForest(trdata, trlabels, ntree = 10, nodesize = 25, keep.forest
       = TRUE)
  p10 <- predict(r10, tedata)
  conf mat10 <- table(p10, telabels)
  meR10 \leftarrow c(meR10, get misc err(conf mat10))
  r100 <- randomForest(trdata, trlabels, ntree = 100, nodesize = 25, keep.
      forest = TRUE)
  p100 <- predict (r100, tedata)
  conf_mat100 <- table(p100, telabels)
  meR100 <- c(meR100, get_misc_err(conf_mat100))
}
#Overall mean
cat("Overall_mean_for_1_tree:", mean(meR1), "\n")
cat("Overall_variance_for_ltree:", var(meR1), "\n")
\mathbf{cat}(" \, Overall_{\sqcup} mean_{\sqcup} \, for_{\sqcup} 10_{\sqcup} trees : ", mean(meR10), " \ " \ "
cat("Overall_variance_for_10_trees:", var(meR10), "
cat("Overall_mean_for_100_trees:", mean(meR100), "\n")
cat("Overall_variance_for_100_trees:", var(meR100), "\n")
\mathbf{set} . \mathbf{seed} (1234)
x1<-runif(1000)
x2<-runif(1000)
tedata < -cbind(x1, x2)
y < -as.numeric(((x1<0.5 \& x2<0.5) | (x1>0.5 \& x2>0.5)))
telabels <- as. factor (y)
plot (x1, x2, col = (y+1))
get_training <- function(){
  x1 < -runif(100)
  x2<-runif(100)
  trdata < -cbind(x1, x2)
  y < -as.numeric((x1 < 0.5 \& x2 < 0.5) | (x1 > 0.5 \& x2 > 0.5))
  trlabels <-- as. factor (y)
  return(list(trdata, trlabels))
meR1 \leftarrow c()
meR10 \leftarrow c
meR100 \leftarrow c
for (i in 1:1000) {
  \#cat("Iteration:", i, " \ r")
  \#Creating the training data
  training <- get_training()
  trdata <- training [[1]]
  trlabels <- training [[2]]
  r1 <-- randomForest(trdata, trlabels, ntree = 1, nodesize = 12, keep.forest =
```

```
TRUE)
  p1 <- predict(r1, tedata)
   conf mat1 <- table(p1, telabels)
  meR1 <- c(meR1, get_misc_err(conf_mat1))
  r10 <- randomForest(trdata, trlabels, ntree = 10, nodesize = 12, keep.forest
        = TRUE)
  p10 <- predict(r10, tedata)
   conf_mat10 <- table(p10, telabels)
  meR10 \leftarrow c(meR10, get\_misc\_err(conf\_mat10))
  r100 <- randomForest(trdata, trlabels, ntree = 100, nodesize = 12, keep.
       forest = TRUE
  p100 <- predict (r100, tedata)
   conf_mat100 <- table(p100, telabels)
  meR100 <- c(meR100, get_misc_err(conf_mat100))
}
#Overall mean
\mathbf{cat}(" \, Overall_{\sqcup} mean_{\sqcup} \, for_{\sqcup} 1_{\sqcup} \, tree : ", \; \mathbf{mean}(meR1), \; " \setminus n")
cat("Overall_{\square} variance_{\square} for_{\square} 1_{\square} tree:", var(meR1), "\n")
\textbf{cat} \, (\, \texttt{"} \, O\, verall_{\,\sqcup} mean_{\,\sqcup} \, for_{\,\sqcup} \, 10_{\,\sqcup} \, tre \, es: \, \texttt{"} \, , \,\, \, \textbf{mean} (\, meR10) \, , \,\, \, \, \texttt{"} \, \backslash n \, \texttt{"} \, )
cat("Overall_variance_for_10_trees:", var(meR10), "\n")
\textbf{cat} \left( \, "\, \, O\, verall \, \sqcup \, mean \, \sqcup \, for \, \sqcup \, 100 \, \sqcup \, tr\, ees: \, "\, \, , \, \, \, \textbf{mean} \left( \, meR100 \, \right) \, , \, \, \, "\, \, \backslash n \, " \, \right)
cat("Overall_variance_for_100_trees:", var(meR100), "\n")
set . seed (1234567890)
max_it <- 100 # max number of EM iterations
min_change <- 0.1 # min change in log lik between two consecutive iterations
n=1000 # number of training points
\mathbf{D}\!\!=\!\!10\ \#\ number\ of\ dimensions
x <- matrix (nrow=n, ncol=D) # training data
true_pi <- vector(length = 3) # true mixing coefficients
true_mu <- matrix(nrow=3, ncol=D) # true conditional distributions
true_pi=c(1/3, 1/3, 1/3)
true_{mu}[1,] = c(0.5, 0.6, 0.4, 0.7, 0.3, 0.8, 0.2, 0.9, 0.1, 1)
true_{mu}[2,] = c(0.5, 0.4, 0.6, 0.3, 0.7, 0.2, 0.8, 0.1, 0.9, 0)
\mathbf{plot}\left(\mathbf{true\_mu}[1,],\ \mathbf{type="o"},\ \mathbf{col}="\mathbf{blue"},\ \mathbf{ylim=c}(0,1)\right)
points(true_mu[2,], type="o", col="red")
points(true_mu[3,], type="o", col="green")
# Producing the training data
for(i in 1:n){
  m \leftarrow sample(1:3,1,prob=true\_pi)
  for (d in 1:D) {
     x[i,d] \leftarrow \mathbf{rbinom}(1,1,\mathrm{true\_mu[m,d]})
M=3 \# number of clusters
w \leftarrow matrix(nrow=n, ncol=M) \# weights
pi <- vector(length = M) # mixing coefficients
mu <- matrix (nrow=M, ncol=D) # conditional distributions
# Random initialization of the parameters
pi \leftarrow runif(M, 0.49, 0.51)
```

```
pi <- pi / sum(pi)
for (m in 1:M) {
  mu[m,] \leftarrow runif(D, 0.49, 0.51)
рi
mu
for (it in 1:max_it) {
  \mathbf{plot}\left(\mathbf{mu}[1\,,]\,,\ \mathbf{type} = \mathbf{"o"}\,,\ \mathbf{col} = \mathbf{"blue"}\,,\ \mathbf{ylim} = \mathbf{c}\left(0\,,1\right)\right)
  points (mu[2,], type="o", col="red")
  points(mu[3,], type="o", col="green")
  \#points(mu[4,], type = "o", col = "yellow")
  Sys. sleep (0.1)
  \# E	ext{-step}: Computation of the weights}
  # Your code here
  \#Log\ likelihood\ computation.
  # Your code here
  cat("iteration: ", it, "log likelihood: ", llik[it], "\n")
  flush.console()
  # Stop if the lok likelihood has not changed significantly
  # Your code here
  \#M\!\!-\!step: ML parameter estimation from the data and weights
  # Your code here
рi
mu
plot (llik [1:it], type="o")
```