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Introduction

Buoyant force plays an important role in our universe and can observed everywhere in our daily lives. Buoyant force plays a very important role our society. They are what keeps the massive cargo ship on the ocean afloat, what allows fishes and submarines to control its vertical movement.

I am very interested in the battle ships that have searched in WW2, and I specifically find their large turret guns to be very cool. I have recently gotten the idea to create my own model of the Japanese ship Nagato. However, the several prototypes of the ship have either sunken because they were too heavy, or flipped because of lack of stability.

Just like how buoyant force keeps cargo ships afloat, it is also the force that bushes model ships up. The reason why my ship has sunken or is unstable is because it is either experiencing too less or too much buoyancy, thus causing problems to the ship.

In order to precisely design a ship that is stable enough for motorize operation while staying afloat, precise prediction of the buoyant force is required. Therefore, I have decided to investigate the cause of buoyant force, and how it can be controlled.

Theory

Research of buoyancy begun early in the human history. As time progressed, people modified and improved their models to explain and describe the nature of

buoyant force.

Archimedes' Principle

Greek physicist Archimedes have started his investigation in this topic back in 250 BCE. In his work *On Floating Bodies*, Archimedes explains what happens when a solid "heavier" that a liquid is placed in a liquid, and what happens when as solid "lighter" than the liquid is placed within it.[1] Archimedes describes that when the "heavier" solid is placed in the liquid, it will sink to the bottom, where as the "lighter" liquid will displace an amount of the fluid that adds up to its own weight.

The first part of this statement is obvious, as the fluid under the solid will experience less gravitational force, and the solid which experiences more gravitational force will use part of the energy generated by the force to move the liquid out of its way.

To demonstrate the second part of the principle (placing the "lighter" solid into the liquid). Suppose there exists solid A being heavier than a liquid, a liquid and solid B being lighter than the liquid, all with the same volume V. Also let $(G + H)$ represent the weight of solid A, G represent the weight of the liquid and $(G - H)$ to represent the weight of solid B. Under these assumptions, when

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[1] Back in the time of Archimedes, the concept of density has not been popularized. However, as Archimedes is investigating buoyant force, which is directly related to the density, he uses the words "lighter" and "heavier" to describe objects with less or more density.

mixing solid A and solid B together completely, they would have a combined mass of $G + H + G - H = 2G$, and a volume of $2V$. The density of the newly made solid will become $\frac{2G}{2V} = G/V$, which is the same as the density of the liquid. Since the newly made solid and the liquid have the same density, the new solid will remain stationary in the liquid.

Archimedes realized that the force pulling on solid A to sink down must be the same to the force which solid B experiences to push itself up in the liquid.

Imagine a glass full of water. When a solid denser than the liquid is placed in, some water will flood out as the solid sinks completely into the water. The amount of water that floods out will have the same volume as the solid after it sinks completely. This is because there is no creation or destruction of mass, and thus the mass of this closed system is conserved, and the sum of the masses of the solid and the water remains the same.

According to Newton's third law of motion states that for every force in nature, there is an equal and opposite reaction force. In this situation, there must be a force which pushes the water up against the force of gravity. With the environment being stationary, the only explanation for the force pushing the water up is that it is the reaction force of the force pulling the solid to sink down into the liquid.

Because of that a force and its reaction force has the same magnitude, the upward buoyant force pushing the solid up the fluid is the same as the force of gravity pulling the liquid down. The force pulling the liquid down is the weight of

the liquid of the same volume as the solid, meaning that the buoyant force which ab object experiences after being placed in a liquid is the same as the weight the displaced liquid, which has the same mass as the solid since the only liquid displaced is the liquid which was in the position where the solid is in now.

This important observation is known as the Archimedes' Principal, and it remains an important pillar to fluid mechanics to this day.

Water Pressure Interpretation

While the Archimedes' Principal uses intuition to help calculate the buoyant force which an object experiences, it does not explore the reason why the buoyant force exists in the first place.

Figure 1: Pressure experienced by any object in a liquid environment

On Earth, any environment filled with particles either with liquid or with gas will have pressure within it (Figure 1). Pressure is the force applied perpendicular to an objects surface measured in per unit area. The internal pressure of these substances on Earth is caused by the gravitational pull on the weight of each particle, pulling them together towards the ground.

The internal pressure within liquids and gasses are also not constant. This is because the deeper the point of measurement, the more particles will be on top of the point of measurement. Since pressure is directly caused by the force of gravity, the particles on top will be pushing down, increasing the pressure as more particles are present above the point of measurement.
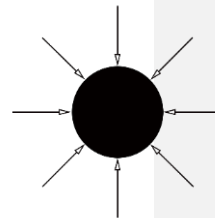
The internal pressure also acts towards all directions. This is because when all the particles have a positive temperature measured in K, they are all moving randomly. The pressure measured at any given point in the environment is caused by the moving particles colliding with the surface of measurement tool. Since the particles are all moving in random directions, the direction of force caused by the pressure will also be in all directions.

Since the pressure acts in all directions, the net force which it applies on a certain point cancels out, as the force it experiences from all directions are the same. However, when talking about something with actual dimensions, this assumption falls.
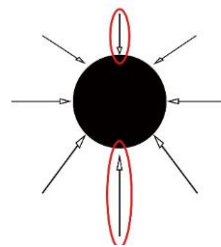
Figure 2: The top and bottom of an object experiences different magnitudes of pressure

While the forces acting perpendicular to the direction of gravity remains constant because the weight of the particles above them are equally distributed on each particle, the forces acting parallel to the direction of gravity is different (Figure 2). This is because when an object is placed completely within a uniform stationary liquid or gas environment, the top of the object will be positioned higher than the bottom of the object, thus causing the top and bottom to experience different magnitudes of forces due to pressure.

The bottom of the object would have more pressure since it has a further distance to the top, which means that the bottom will have more weight placed on it, thus increasing the pressure.

When taking a picture front the top and the bottom of any object, the outer

shape of the object will be inverted, but of the same size, meaning that the surface area of the top and bottom of any object are the same. Since there are more pressure at the bottom of any given object contained in a liquid or gas environment, the bottom of the object will experience more force, thus making it so that the net force which the object experiences is upwards.

This upwards force caused by the pressure difference is the buoyant force. It also makes intuitive sense when explaining the Archimedes' Principle. The upward force is caused by the pressure difference, which is exactly the amount of weight of the liquid taking in the place of where the object is. Thus, the buoyant force is equal to the weight of the displaced liquid.

Mathematical Proof of the Formula

In a resting fluid, the hydrostatic pressure is determined from the fallowing formula:

$$p = \rho g l + p_a$$

Where:

- P is the hydrostatic pressure (Pa)
- ρ is the density of the medium surrounding the point of measurement (kgm^{-3})
- g is the gravitational acceleration (ms^{-2})
- l is the distance from the point of measurement to the top of the medium parallel to the direction of gravity (m)

- p_a is the atmospheric pressure

Thus, the difference in pressure between the top and the bottom can be calculated by subtracting the distances:

$$\Delta p = p_1 - p_2$$

$$= (\rho g l_1 + p_a) - (\rho g l_2 + p_a)$$

$$= \rho g \times (l_1 - l_2)$$

$$= \rho g h$$

Where:

- l_1 is the distance from the top of the object to the top of the medium parallel to the direction of gravity (m)

- l_2 is the distance from the bottom of the object to the top of the medium parallel to the direction of gravity (m)

- h is the height of the object, which is also the distance between l_1 and l_2

Since buoyancy is the resultant force from the difference in pressure, it can be calculated using this formula:

$$F = pA$$

$$= \rho g h A$$

$$= \rho g V$$

$$= mg$$

Where:

- F is the buoyant force

- A is the area which exerts the force due to the pressure. (base area)

- V is the volume of the solid, simplified from hA (volume = base area × height)

- m is the mass of the medium displaced, simplified from ρv (mass = density × volume)

This makes sense as the medium displaced would want to return to its equilibrium position, which is where the object is. Thus, the gravitation force acting on the medium will be transferred to buoyant force, trying to push the object away in attempt to restore its equilibrium.

Therefore, the formula for calculating the buoyant force which an object experiences when placed in a liquid is $F = mg$

Exploration

Hypothesis

If the density of the medium is increased then the force due to gravity acting on the weight will decrease, because with a higher density but with the weight having a constant volume, more weight of the medium is displaced due to the weight, thus decreasing the force acting on the weight.

Experiment
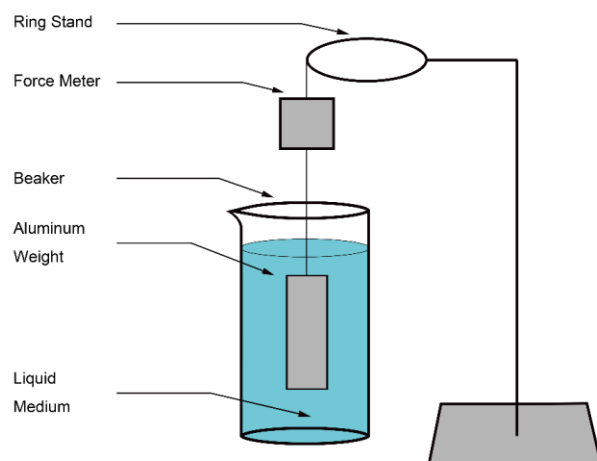
Note: The liquid medium will be changed to different solutions of different densities. For instance, sugar water solution and alcohol.

Material list

Ring Stand - 1

Ring Clamp - 1

Force Meter - 1

String (25cm) - 1

Aluminum weight (0.06kg) - 1

Beaker (250ml) - 3

Graduated cylinder(100ml) - 1

Electronic scale - 1

Glycerine – 300ml

Tap water – 300ml

Methyl – 300ml

Procedure

1, Mix 120ml of glycerine with 80ml of tap water from graduated cylinder to a beaker

2, Mix 70ml of methyl with 130 ml of tap water from graduated cylinder to a beaker

3, Connect the aluminum weight, force meter and the ring clamp with a string

4, Pour 200 ml of glycerine into a beaker

5, Submerge the aluminum weight

6, Record the weight from the force meter

7, Pour 100ml of glycerine into the graduated cylinder

8, Measure its weight

9, Clean the beaker and the graduated cylinder with water

10, Repeat step 4 to 9 with tap water, glycerine water solution, methyl, methyl water solution

Points of Caution

Measuring device

I used an old force meter to measure the buoyant force, which lacked in accuracy and precision. I tried to calibrate the force meter using its calibration function. However, even after calibrating the device, the reading I have received fluctuating reading on the last decimal place on the meter. I have tried to balance this out by increasing the uncertainty of the force measured by $\pm 0.005N$, since fluctuation on the last digit means that there is 0.01N uncertainty in total.

Mixing of different liquid mediums

When pouring and changing of liquid mediums, there might be some mixing of the mediums happening. While mixing in the container itself will not have a big impact, since the amount mixed is so small that it would be negligible. There will also be some liquid that sticks on to the sides of the weight and is absorbed by the rope. This is a problem because this weight is added directly on the weight. It will increase the buoyant force of the new medium is denser than the previous medium or decrease the buoyancy if the new medium is less dense than the previous medium, thus directly changing the results of the experiment.

To prevent the mixing of the liquid in effecting the results, I wash the containers and the weight involved after every trial with water, and wiped it clean with paper towel. I have also used to paper towel to suck out all of the liquid in the rope to prevent the liquid in the rope from adding any additional weight.

Deepness submerged

The buoyant force will be the same as the weight of the liquid of the same volume to the weight. Since the pressure in a liquid will increase as moving deeper, the deeper molecules will be packed together more closely. Therefore, the deeper liquid will have a greater density than the surface, meaning that the weight of liquid with the same volume as the aluminum weight will be heavier. Thus, the weight will experience more buoyant force when at a deeper depth.

To make sure the weight is at the same depth for every trial, I filled the beaker containing the liquid with the same volume of liquid every time, and used the same rope to hang the aluminum weight to the force meter. I have also kept the ring clamp in the same place.

However, to maintain an extra level of caution, the height was adjusted so that the liquid just barely covers the top. This is because with the different densities of the liquid, the same amount of deepness can still result in different amount of increase of buoyant force. By keeping the depth as shallow as possible, the change in force is kept as small as possible. While this should not make a difference in the results, I still did this as an extra level of caution.

Analysis

After measuring the downward force acting on the aluminum weight, some calculation is required to see the relationship between the density of the liquid and the buoyant force experienced by the weight.

Data Processing

Table 1: Weight of Aluminum weight (N) when placed into mediums of different densities (g/100ml)

Medium	Density	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Average
C3H8O3	122.62	0.301	0.298	0.307	0.294	0.303	0.301
C3H8O3 + H2O	112.31	0.319	0.330	0.315	0.326	0.322	0.322
H2O	97.95	0.377	0.369	0.365	0.376	0.378	0.373
CH3OH + H2O	90.71	0.383	0.387	0.395	0.392	0.385	0.388
CH3OH	77.60	0.421	0.425	0.418	0.413	0.427	0.421

Note: The Average is calculated by adding the values of trial 1 ~ 5 and dividing the sum by 5

$$w_{average} = (w_1 + w_2 + w_3 + w_4 + w_5) \div 5$$

Table 2: Buoyant force experienced by the aluminum weight when placed into mediums of different densities and uncertainties of measurements

Density (N/100ml)	Weight (N)	Buoyant force (N)	Density uncertainty	Weight uncertainty	Force Uncertainty
1.203	0.301	0.312	±0.022	±0.012	±0.012
1.102	0.322	0.291	±0.021	±0.012	±0.012
0.9609	0.373	0.24	±0.020	±0.011	±0.011
0.8899	0.388	0.225	±0.019	±0.012	±0.012
0.7613	0.421	0.192	±0.018	±0.013	±0.013

Note: The uncertainties use the same unit as their parent unit

The force density (N/100ml) is calculated by multiplying the mass density by firstly dividing it by 100 to turn the units into kg/100ml, and then multiplying it by g to turn it into force density.

$$\rho_F = \rho_M \div 1000 \times g$$

The buoyant force is calculated by distracting the weight measured from the original weight of the aluminum weight (0.613N)

$$F_B = W_O - W_M = 0.613\ N - W_M$$

The Density uncertainty is determined by the smallest unit of measurement used to measure the weight and volume, and then adding them together as percentage uncertainty

$$u_D = u_{\%w} + u_{\%v}$$

The uncertainty of the weight and Force is determined by finding the difference between the minimum, average, maximum, and taking the largest value to incorporate all of the value, and then adding it the device uncertainty. However, the device showed fluctuation on the values when taking the readings, so I have added an extra 0.005 to the uncertainty to counter the fluctuation of the value on the force meter.

$$u_{W/F} = \max(u_{max} - u_{avg}, u_{avg} - u_{min}) + 0.005$$

**Table 3: Buoyant force predicted using the formula with calculations**

Density (kg/ml)	Mass displaced (kg)	Calculated force (N)	Measured force (N)
0.0012262	0.031145	0.305537	0.312
0.0011231	0.028527	0.279847	0.291
0.0009795	0.024879	0.244066	0.24
0.0009071	0.02304	0.226026	0.225
0.000776	0.01971	0.193359	0.192

Note: Density is changed to kg/ml from g/100ml by dividing each value by 100,000. Mass displaced is calculated by multiplying the density of the liquid to the volume of the aluminum weight (25.4ml). The force is calculated by multiplying the mass to the gravitational acceleration.

$$\text{F} = \rho V g$$

$$= mg$$

Graph 1: Buoyant Force Experienced by the Aluminum weight as force density of medium increases
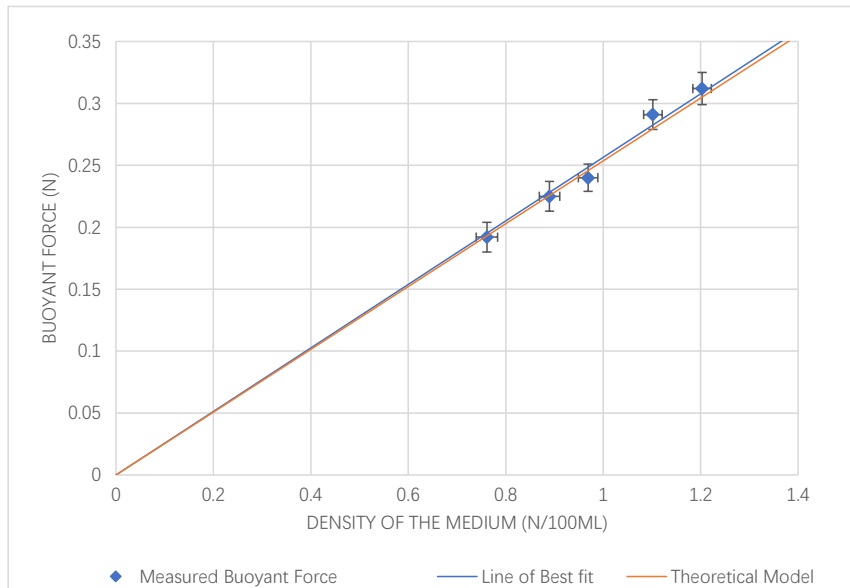
This graph is produced based on the measured buoyant force and calculated theoretical force for each density of the liquids from table 3.

Trends and Reliability

After plotting the results of the experiment on the graph, they show a direct linear relationship between the density of the medium and the buoyant force experienced since it intercepts the origin. By graphing the theoretical model with the line of best fit of the results, it can be seen that the line of best fit is very similar to the theoretical formula.

The results of the experiment have high accuracy. It has a very high r^2 value

of 0.9817. The slope of the theoretical model is 0.2542, the slope of the line of best fit of the experiment is 0.2564, and they only have 0.0012 difference. The similarity of the slopes also shows that the data have a high precision. Since the theoretical model fits within all of the uncertainties of the experiment, and it has high accuracy and precision, the results of the experiment are very reliable.

The results of the experiment show that as the density of the liquid increases, the buoyant force which the weight experiences also increases. Experiments from other researchers also support the trend. When placing an egg in normal water, it sinks to the bottom of the container, but as salt is added to the water, the egg starts to float. (Tommy's egg project)

批注 [LC2]: Change to proper citation
http://www.ggteks.net/tommy/Eggproject.html

Sources of Error

Prior to preforming the experiment, I have thought of some sources which may cause errors in the experiment, and have taken action in trying to minimalize the effects of these errors on the result of the experiment (see "Points of Caution"). However, I also noticed some additional sources of error that I have not expected.

Variety of Data

The densities of the different liquid which I have used are very similar. This small difference is caused by the availability of materials. Having a small

difference in samples means that the experiment might not create an accurate representation of trend, since it only makes up a small portion of the entire trend. This will cause the conclusions of the experiment to be inaccurate due to the lack of variety in data. This is a systematic error, because it comes from the procedure and effects all of the values.

This can be improved by using lower liquids or even gas, such as hydrocarbon. Higher density liquids such as mercury might not be appropriate, because they would create too much buoyant force, causing the aluminum weight to float above the surface. However, this can be fixed my holding the from inside the liquid and directly measuring the pulling force, and adding the force of gravity to that.

Dissolution of different liquids

In my experiment, I needed to mix different chemicals in order to create liquids with different densities. The different liquid I used to mix also have different viscosities. When I was mixing some of them, I have noticed that parts of the solution resisted my motion more than some other parts of the solution. This means that the solution is not mixed evenly, and thus the average density of the solution is different from the density of the solution at different points.

This might be caused by the solubility of the different liquids not being able to hold as much mass of each other as I have placed, or it might be because the liquid required more stirring to completely mix. After doing some research on the

solubilities of the different liquids, I have realized that the methyl is only 25% present soluble with water at room temperature.

A possible improvement is to only use chemicals that can completely mix with each other, or just to use more chemicals with a smaller difference in density. This way, the density of the liquid will be even no matter the point of measure, and thus make the results of the experiment more accurate.

Conclusion

The buoyant force has long played an important roll in maintaining the world as it is as well as providing many conveniences to our daily life. The goal of this experiment is to find out more specifically the causes of the force, and how to accurately predict the force

Graph 1 graphs the experimental results along with the expected results from the hypothesized formula. The trend generated by the formula fits with-in the uncertainty range of the experimental results, and resembles the results of the experiment closely. It only has 0.0012 difference in slope from the line of best fit generated from the experimental results.

After analyzing the trend and reliability of the results, it can be seen that my hypothesis is clearly supported by the results of the experiment. The buoyant force is caused by the difference in water pressure at different heights, and the magnitude of the force is the same as the weight of the displaced liquid.

This means that when I create my ship model, the weight of the water with

the same volume of the parts of the ship underwater will be the buoyant force experienced by the ship. Therefore, the parts of the ship that will be under water can be estimated by adding water to a hallow model of the boat. When the weight of the boat doubles, the top of the water will roughly resemble the place where water will reach.

http://www.ggteks.net/tommy/Eggproject.html