

# **Tiled Matrix Multiplication using Systolic Array**

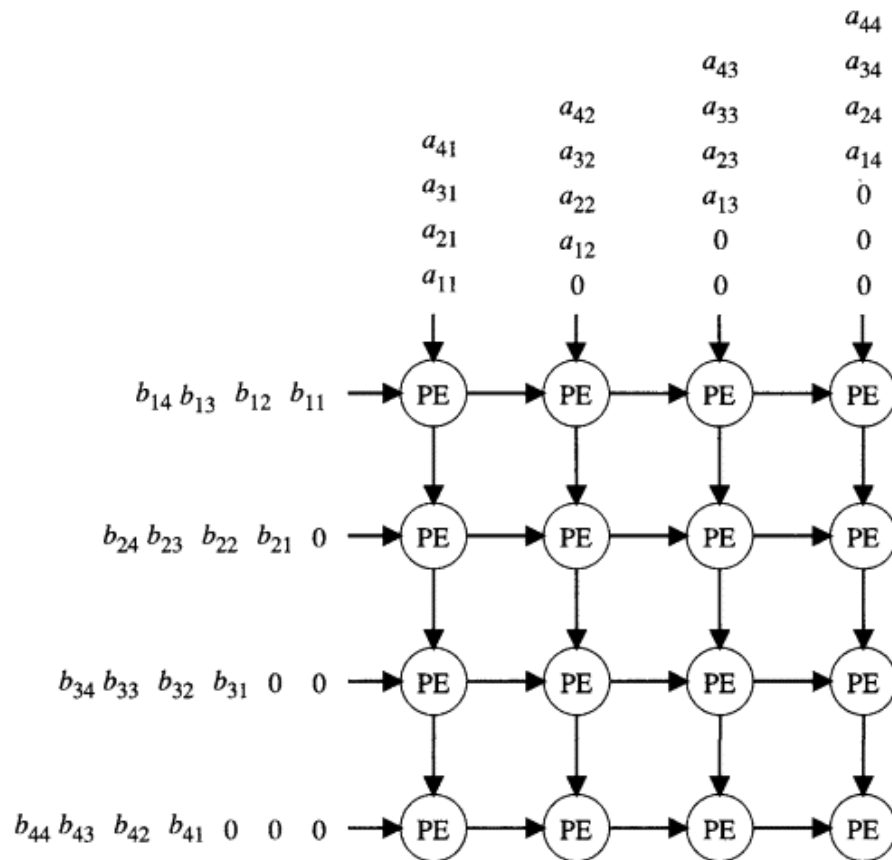
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# Problem Statement

- The sequential implementation of matrix multiplication is a very time-consuming task especially for large matrices.
- Systolic Array implementation of matrix multiplication is one approach to gain computational speed-up.
- Our project aims at accelerating matrix multiplication operation of large input matrices – 512x512 matrix multiplication.

# Systolic Array Design

## Design on paper



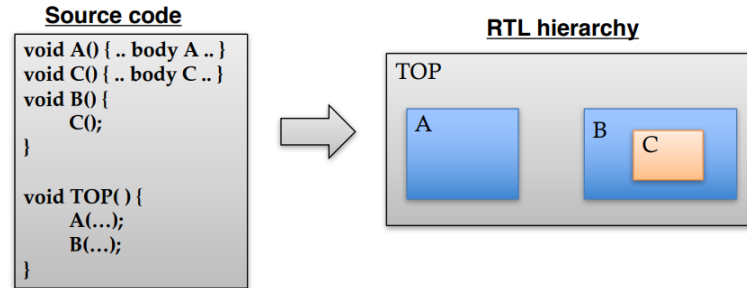
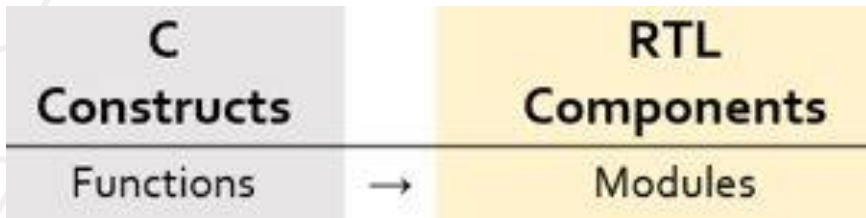
## HLS Implementation?

- How to implement the processing elements?
- How is data transferred between the processing elements?
- How to overlap the read, write and compute operations of the processing element?

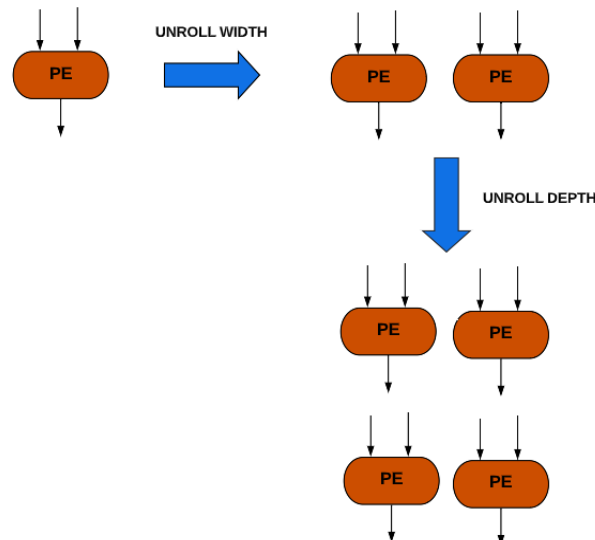
# HLS Implementation

## How to implement the processing elements?

- C functions translate to modules.



- Unrolling is scaling!



Has more meaning in FPGA than instruction-based architectures – Every "instruction" is a separate hardware.

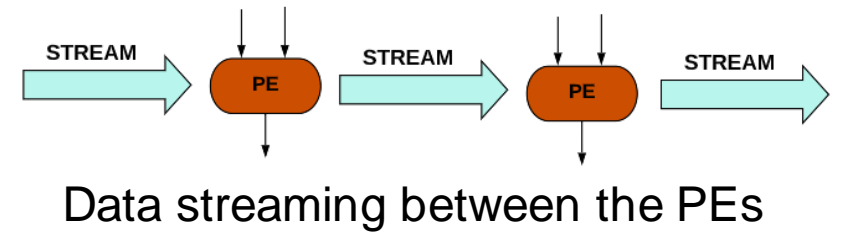
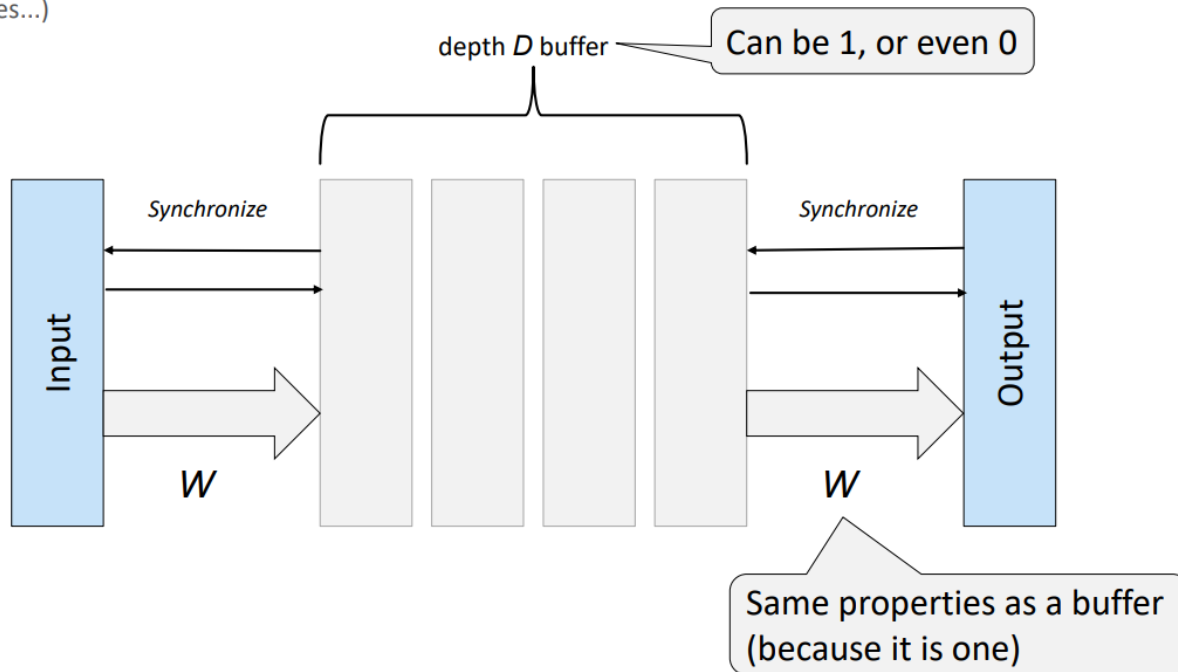
# HLS Implementation

## Data transfer between the processing elements

- Synchronous interface
- **Asynchronous interface**

### Streams

(FIFOs, queues...)

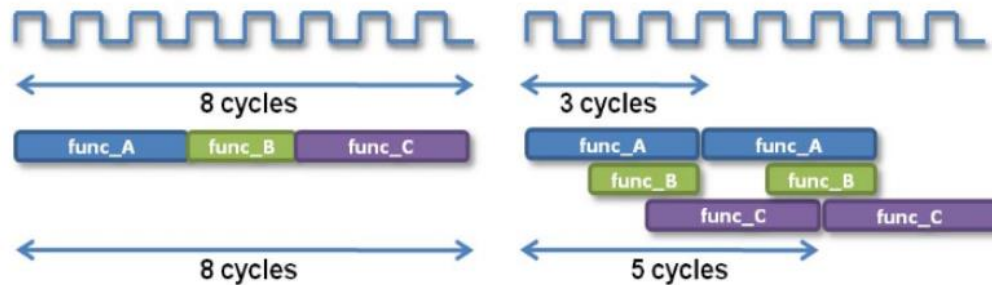


# HLS Implementation

## How to overlap execution in the processing element?

- Data flow to the rescue!

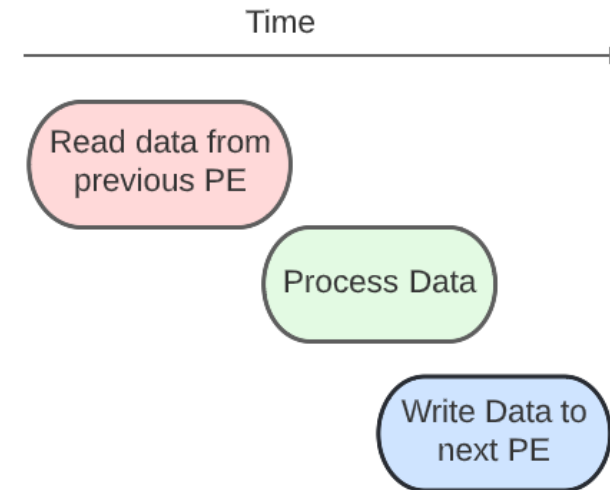
```
void top (a,b,c,d) {  
  ...  
  func_A(a,b,i1);  
  func_B(c,i1,i2);  
  func_C(i2,d)  
  return d;  
}
```



(A) Without Dataflow Pipelining

(B) With Dataflow Pipelining

Inside the processing element



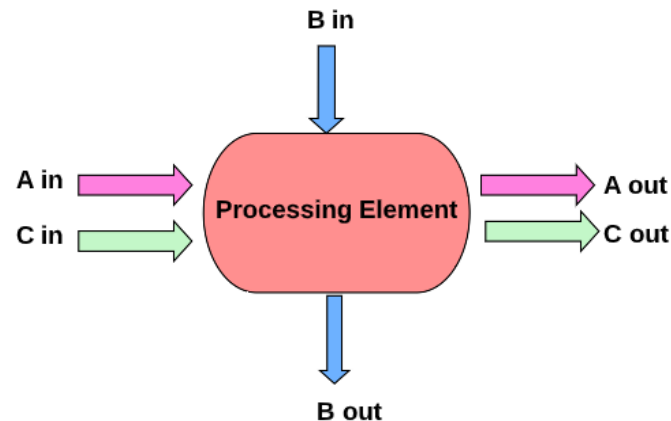
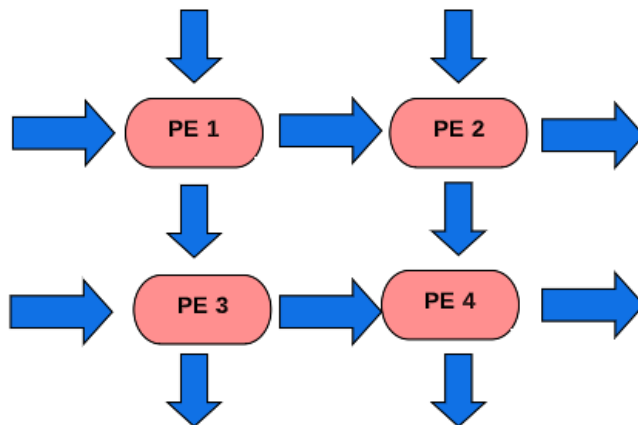
Pipelining in the function level

# HLS Implementation

```
ComputeLoop:
for (int i = 0; i < N; i++)
{
    #pragma HLS UNROLL

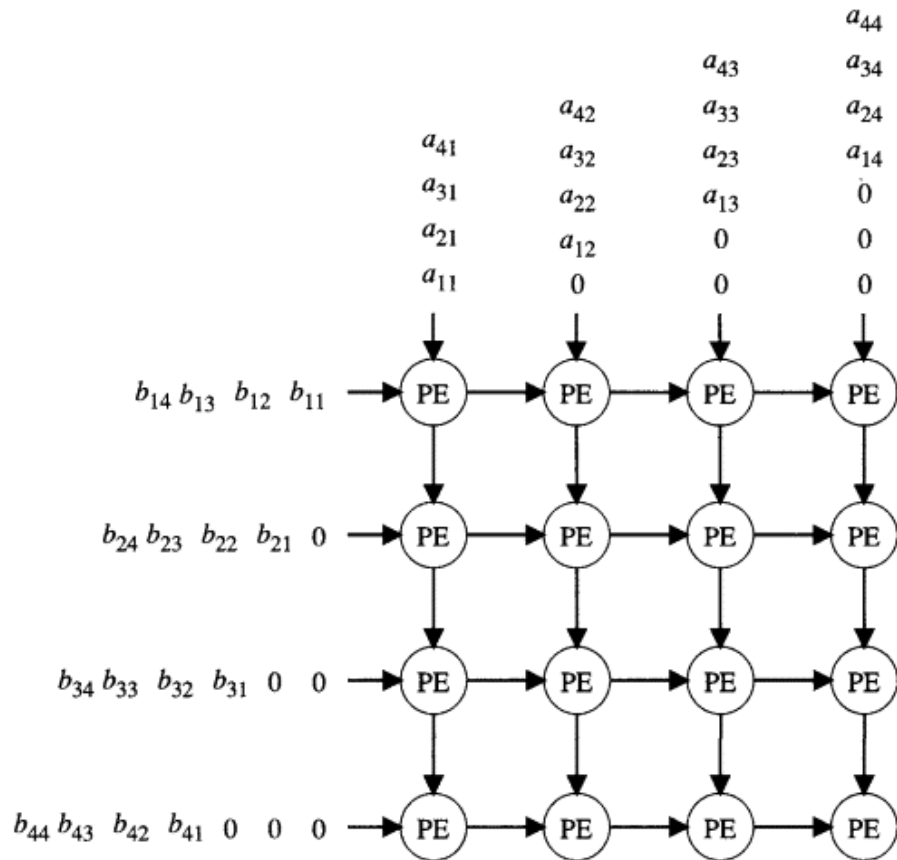
    for (int j = 0; j < N; j++)
    {
        #pragma HLS UNROLL
        //Creates nine instances of processing elements in hardware (Considering N=3)
        //Format is ProcessingElement(a_in, a_out, b_in, b_out, c_in, c_out)
        ProcessingElement(a_pipes[i][j], a_pipes[i][j + 1], b_pipes[i][j], b_pipes[i + 1][j], c_pipes[i][j], c_pipes[i][j+1], i, j);
    }
}
```

- Each PE is a function.
- 2D array of streams for data transfer.



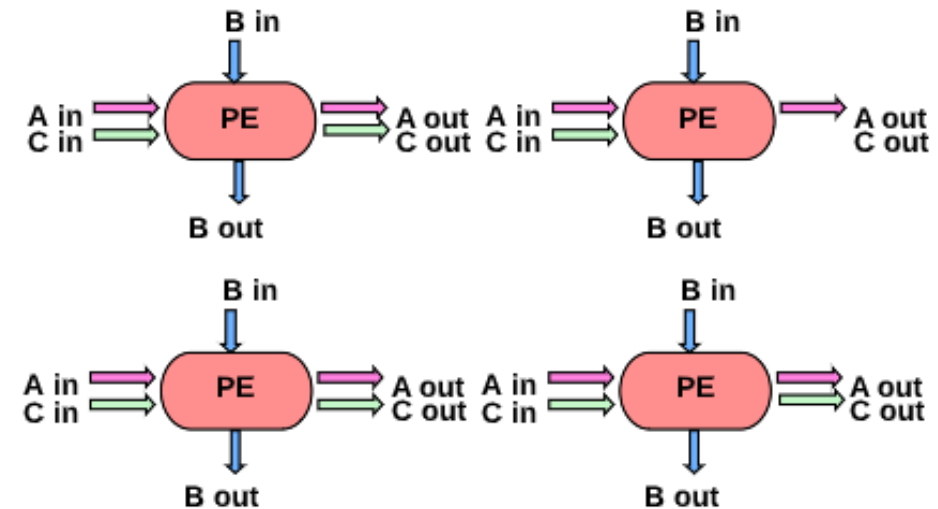
# Systolic Array Design – Putting it all together

## Design on paper (4x4 systolic array)



Reference- [Evolutionary Mapping Techniques for Systolic Computing System](#)

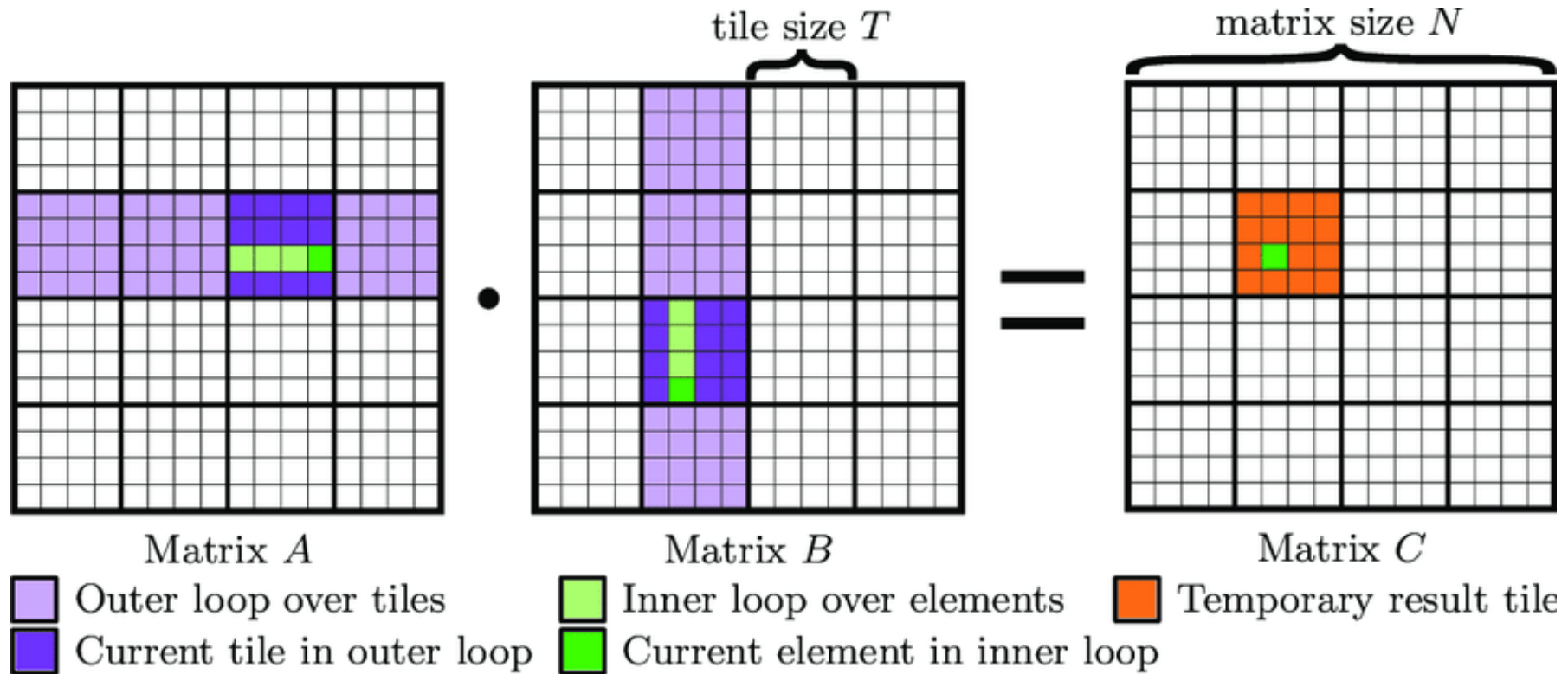
## HLS Implementation (2x2 systolic array)





# Tiling Matrix Multiplication

- Output stationary!



# BASELINE

```
455 void matrix_mul(int MatA_DRAM[M][N], int MatB_DRAM[N][K], int MatC_DRAM[M][K])
456 {
457     #pragma HLS interface m_axi depth = 100000 port = MatA_DRAM offset = slave bundle = memA
458     #pragma HLS interface m_axi depth = 100000 port = MatB_DRAM offset = slave bundle = memB
459     #pragma HLS interface m_axi depth = 100000 port = MatC_DRAM offset = slave bundle = memC
460     #pragma HLS interface s_axilite port = return
461     int MatA[TILE][TILE];
462     int MatB[TILE][TILE];
463     int MatC[TILE][TILE];
464     int MatC_partial[TILE][TILE];
465     InitC(MatC);
466     for (int mt = 0; mt < M; mt = mt + TILE)
467     {
468         for (int kt = 0; kt < K; kt = kt + TILE)
469         {
470             // InitC(MatC);
471             for (int nt = 0; nt < N; nt = nt + TILE)
472             {
473                 ReadInputs(MatA_DRAM, MatB_DRAM, MatA, MatB, mt, nt, kt);
474                 systolic_array(MatA, MatB, MatC_partial);
475                 addC(MatC, MatC_partial);
476             }
477             writeC(MatC, MatC_DRAM, mt, kt);
478         }
479     }
480 }
```

LATENCY -----> 1.8 seconds

# Optimization – II

## Pipeline

```
void ProcessingElement(hls::stream<int>& a_in, hls::stream<int>& a_out, hls::stream<int>& b_in, hls::stream<int>& b_out,
                      hls::stream<int>& c_in, hls::stream<int>& c_out, int pe_row, int pe_col)
{
    PE_Compute:
    int a_buffer, b_buffer;
    int c_buffer = 0;
    int count = 0;
    int c_prev;
    if (pe_col == 0)
    {
        c_prev = c_in.read();
    }
    // 3*N-2
    for (int k = 0; k < pe_row + pe_col + TILE; k++)
    {
#pragma HLS PIPELINE II = 1
        // Read a_buffer value from a_pipe_in
        if (pe_col == 0)
        {
            if (a_in.empty())
            {
                a_buffer = 0;
            }
            else if (count < pe_row)
            {
                a_buffer = 0;
                a_out.write(a_buffer);
            }
            else
            {
                a_buffer = a_in.read();
                a_out.write(a_buffer);
            }
        }
    }
}
```

```
        c_buffer += a_buffer * b_buffer;
        count++;
        if (!c_in.empty())
        {
            c_out.write(c_in.read());
        }
    }
    c_out.write(c_buffer);
} // End of processing element function
```

- Each PE element performs a matrix multiplication and accumulation operation.
- Within the PE these operations are pipelined across different cycles.

# Optimization – II

## Pipeline + Loop Fusion

```
void ReadInputs(int MatA_DRAM[][N], int MatB_DRAM[][K], int MatA[][TILE], int MatB[][TILE], int m0, int n0, int k0)
{
    #pragma HLS pipeline off
    for (int i = 0; i < TILE; i++)
    {
        #pragma HLS LOOP_TRIPCOUNT min = 16 max = 16
        for (int j = 0; j < TILE; j++)
        {
            #pragma HLS PIPELINE
            #pragma HLS LOOP_TRIPCOUNT min = 16 max = 16
            MatA[i][j] = MatA_DRAM[i + m0][j + n0];
            MatB[i][j] = MatB_DRAM[i + n0][j + k0];
        }
    }
}
```

- Pipelining is also performed during memory access operations
- Since we use 512 x 512 matrices both the arrays can be accessed in the same loop

**LATENCY -----> 0.348 sec**

# Optimization – III

## Ping Pong Buffers

```
449 void matrix_mul(int MatA_DRAM[M][N], int MatB_DRAM[N][K], int MatC_DRAM[M][K])
450 {
451     #pragma HLS interface m_axi depth = 100000 port = MatA_DRAM offset = slave bundle = memA
452     #pragma HLS interface m_axi depth = 100000 port = MatB_DRAM offset = slave bundle = memB
453     #pragma HLS interface m_axi depth = 100000 port = MatC_DRAM offset = slave bundle = memC
454     #pragma HLS interface s_axilite port = return
455     int MatAO_pong[TILE][TILE];
456     int MatBO_pong[TILE][TILE];
457     int MatCO_pong[TILE][TILE];
458     int MatAO_ping[TILE][TILE];
459     int MatBO_ping[TILE][TILE];
460     int MatCO_ping[TILE][TILE];
461     int MatC[TILE][TILE];
462     #pragma HLS array_partition variable=MatC dim=1 complete
463     #pragma HLS array_partition variable=MatC dim=2 complete
464     #pragma HLS array_partition variable=MatCO_ping type= complete
465     #pragma HLS array_partition variable=MatCO_pong type= complete
466     InitC(MatC);
```

```
ReadInputs(MatA_DRAM, MatB_DRAM, MatAO_ping, MatBO_ping, mt, 0, kt);
for (int nt = 0; nt < N; nt = nt + TILE)
{
    if (nt % 2 == 0)
    {
        //Execute from ping buffer
        systolic_array(MatAO_ping, MatBO_ping, MatCO_ping);

        addC(MatC, MatCO_ping);
        //Read pong buffer
        if (nt + 3*TILE < N)
        {
            ReadInputs(MatA_DRAM, MatB_DRAM, MatAO_pong, MatBO_pong, mt, nt + 2*TILE, kt);
        }
    }
    else
    {
        //Execute from pong buffer
        systolic_array(MatAO_pong, MatBO_pong, MatCO_pong);

        addC(MatC, MatCO_pong);
        //Read ping buffer
        if (nt + 3*TILE < N)
        {
            ReadInputs(MatA_DRAM, MatB_DRAM, MatAO_ping, MatBO_ping, mt, nt + 2 * TILE, kt);
        }
    }
}
writeC(MatC, MatC_DRAM, mt, kt);
```

- Ping Pong Buffers are used to overlap read operation(for the next tile) and systolic array (current tile) operation.

LATENCY -----> 0.269 sec

# Optimization – IV

## 2 Parallel Systolic Arrays

```
451 #pragma HLS interface m_axi depth = 100000 port = MatA_DRAM offset = slave bundle = memA
452 #pragma HLS interface m_axi depth = 100000 port = MatB_DRAM offset = slave bundle = memB
453 #pragma HLS interface m_axi depth = 100000 port = MatC_DRAM offset = slave bundle = memC
454 #pragma HLS interface s_axilite port = return
455 int MatAO_ping[TILE][TILE];
456 int MatBO_ping[TILE][TILE];
457 int MatCO_ping[TILE][TILE];
458 int MatAO_pong[TILE][TILE];
459 int MatBO_pong[TILE][TILE];
460 int MatCO_pong[TILE][TILE];
461 int MatA1_ping[TILE][TILE];
462 int MatB1_ping[TILE][TILE];
463 int MatC1_ping[TILE][TILE];
464 int MatA1_pong[TILE][TILE];
465 int MatB1_pong[TILE][TILE];
466 int MatC1_pong[TILE][TILE];
467 int MatC[TILE][TILE];
468 #pragma HLS array_partition variable=MatC dim=1 complete
469 #pragma HLS array_partition variable=MatC dim=2 complete
470 #pragma HLS array_partition variable=MatCO_ping type= complete
471 #pragma HLS array_partition variable=MatCO_pong type= complete
472 #pragma HLS array_partition variable=MatC1_ping type= complete
473 #pragma HLS array_partition variable=MatC1_pong type= complete
```

```
ReadInputs(MatA_DRAM, MatB_DRAM, MatAO_ping, MatBO_ping, mt, 0, kt);
ReadInputs(MatA_DRAM, MatB_DRAM, MatA1_ping, MatB1_ping, mt, TILE, kt);
for (int nt = 0; nt < N; nt = nt + 2*TILE)
{
    if (nt % 2 == 0)
    {
        //Execute from ping buffer
        systolic_array(MatAO_ping, MatBO_ping, MatCO_ping);
        systolic_array(MatA1_ping, MatB1_ping, MatC1_ping);
        addC(MatC, MatCO_ping, MatC1_ping);
        //Read pong buffer
        if (nt + 3*TILE < N)
        {
            ReadInputs(MatA_DRAM, MatB_DRAM, MatAO_pong, MatBO_pong, mt, nt + 2*TILE, kt);
            ReadInputs(MatA_DRAM, MatB_DRAM, MatA1_pong, MatB1_pong, mt, nt + 3*TILE, kt);
        }
    }
    else
    {
        //Execute from pong buffer
        systolic_array(MatAO_pong, MatBO_pong, MatCO_pong);
        systolic_array(MatA1_pong, MatB1_pong, MatC1_pong);
        addC(MatC, MatCO_pong, MatC1_pong);
        //Read ping buffer
        if (nt + 3*TILE < N)
        {
            ReadInputs(MatA_DRAM, MatB_DRAM, MatAO_ping, MatBO_ping, mt, nt + 2 * TILE, kt);
            ReadInputs(MatA_DRAM, MatB_DRAM, MatA1_ping, MatB1_ping, mt, nt + 3 * TILE, kt);
        }
    }
}
writeC(MatC, MatC_DRAM, mt, kt);
```

- Start two systolic arrays at the same time.
- Integrate Ping Pong buffer operation.

**LATENCY -----> 0.222 sec (2 Systolic Arrays)**



# Optimization – V

## 4 Parallel Systolic Arrays

```
int MatC[TILE][TILE];
InitC(MatC);
for (int mt = 0; mt < M; mt = mt + TILE)
{
    for (int kt = 0; kt < K; kt = kt + TILE)
    {
        // InitC(MatC);
        for (int nt = 0; nt < N; nt = nt + 4*TILE)
        {
            ReadInputs(MatA_DRAM, MatB_DRAM, MatA0, MatB0, mt, nt, kt);
            ReadInputs(MatA_DRAM, MatB_DRAM, MatA1, MatB1, mt, nt + TILE, kt);
            ReadInputs(MatA_DRAM, MatB_DRAM, MatA2, MatB2, mt, nt + 2*TILE, kt);
            ReadInputs(MatA_DRAM, MatB_DRAM, MatA3, MatB3, mt, nt + 3*TILE, kt);

            systolic_array(MatA0, MatB0, MatC0);
            systolic_array(MatA1, MatB1, MatC1);
            systolic_array(MatA2, MatB2, MatC2);
            systolic_array(MatA3, MatB3, MatC3);

            addC(MatC, MatC0, MatC1, MatC2, MatC3);
        }
        writeC(MatC, MatC_DRAM, mt, kt);
    }
}
```

- Start 4 systolic\_arrays at the same time.
- Removed the Ping Pong Buffer implementation as the resource utilization is above 100%

**LATENCY -----> 0.189 sec (4 Systolic Arrays)**

# Results

## Latency of the design

Modules & Loops	Issue Type	Violation Type	Distance	Slack	Latency(cycles)	Latency(ns)
▼ matrix_mul				-	22177093	2.220E8
▶ matrix_mul_Pipeline_VITIS_LOOP_414_1_VITIS_LOOP_416_2				-	258	2.580E3
▶ ReadInputs				-	465	4.650E3
▶ systolic_array				-	322	3.220E3
▶ systolic_array				-	322	3.220E3
▶ writeC				-	449	4.490E3
addC				-	8	80.000
▶ VITIS_LOOP_475_1				-	22176832	2.220E8

## Resource Utilization of the design

Name	BRAM_18K	DSP	FF	LUT	URAM
DSP	-	-	-	-	-
Expression	-	-	0	80	-
FIFO	-	-	-	-	-
Instance	12	1536	346892	360786	0
Memory	8	-	2048	256	0
Multiplexer	-	-	-	6497	-
Register	-	-	24865	-	-
Total	20	1536	373805	367619	0
Available	4032	9216	2592000	1296000	960
Available SLR	1344	3072	864000	432000	320
Utilization (%)	~0	16	14	28	0
Utilization SLR (%)	1	50	43	85	0

➤ Memory access is the bottleneck!



# Results

## *PEs in the initial columns*

▼ systolic_array	-	322	3.220E3
▶ ● read_inputAB11	-	258	2.580E3
● read_inputC12	-	0	0.0
▶ ● ProcessingElement13	-	22	220.000
▶ ● ProcessingElement14	-	22	220.000
▶ ● ProcessingElement29	-	23	230.000
▶ ● ProcessingElement15	-	23	230.000

## *PEs in the final rows and columns*

▶ ● ProcessingElement25	-	50	500.000	-	50
▶ ● ProcessingElement26	-	50	500.000	-	50
▶ ● ProcessingElement26	-	51	510.000	-	51
▶ ● write_resultC269	-	258	2.580E3	-	258

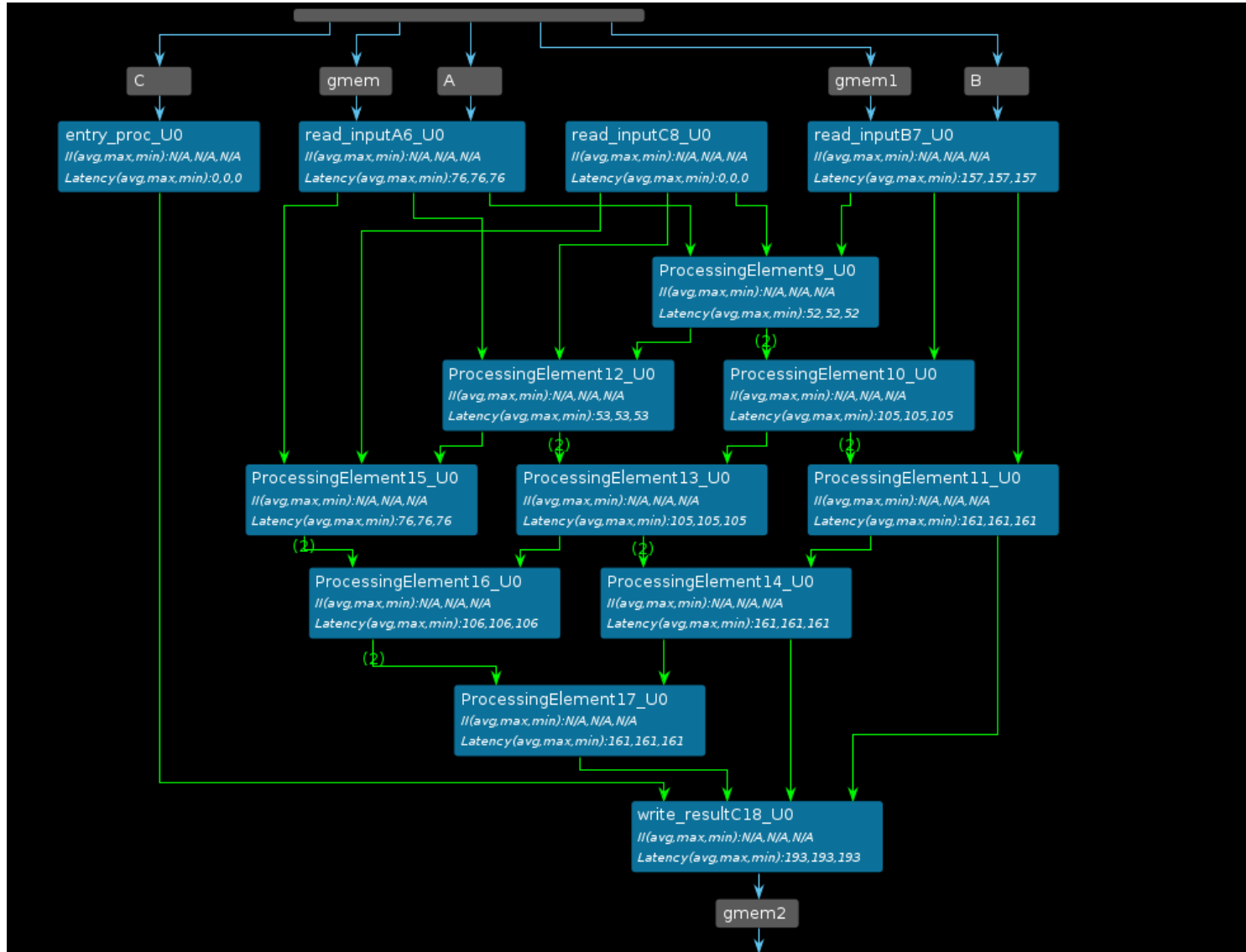
# Results

```
matrix_mul_ProcessingElement131 ProcessingElement131_U0(  
    .ap_clk(ap_clk),  
    .ap_rst(ap_rst),  
    .ap_start(ProcessingElement131_U0_ap_start),  
    .ap_done(ProcessingElement131_U0_ap_done),  
    .ap_continue(ProcessingElement131_U0_ap_continue),  
    .ap_idle(ProcessingElement131_U0_ap_idle),  
    .ap_ready(ProcessingElement131_U0_ap_ready),  
    .a_in_07125_dout(a_pipes_V_7_6_dout),  
    .a_in_07125_empty_n(a_pipes_V_7_6_empty_n),  
    .a_in_07125_read(ProcessingElement131_U0_a_in_07125_read),  
    .a_out_022390_din(ProcessingElement131_U0_a_out_022390_din),  
    .a_out_022390_full_n(a_pipes_V_7_7_full_n),  
    .a_out_022390_write(ProcessingElement131_U0_a_out_022390_write),  
    .b_in_037638_dout(b_pipes_V_7_6_dout),  
    .b_in_037638_empty_n(b_pipes_V_7_6_empty_n),  
    .b_in_037638_read(ProcessingElement131_U0_b_in_037638_read),  
    .b_out_054908_din(ProcessingElement131_U0_b_out_054908_din),  
    .b_out_054908_full_n(b_pipes_V_8_6_full_n),  
    .b_out_054908_write(ProcessingElement131_U0_b_out_054908_write),  
    .c_in_0691155_dout(c_pipes_V_7_6_dout),  
    .c_in_0691155_empty_n(c_pipes_V_7_6_empty_n),  
    .c_in_0691155_read(ProcessingElement131_U0_c_in_0691155_read),  
    .c_out_0841412_din(ProcessingElement131_U0_c_out_0841412_din),  
    .c_out_0841412_full_n(c_pipes_V_7_7_full_n),  
    .c_out_0841412_write(ProcessingElement131_U0_c_out_0841412_write)  
);
```

*A simple instantiation of a PE module in the matrix\_mul.v module (Verilog)*

# FPGA Implementation

- Implemented 3x3 systolic array



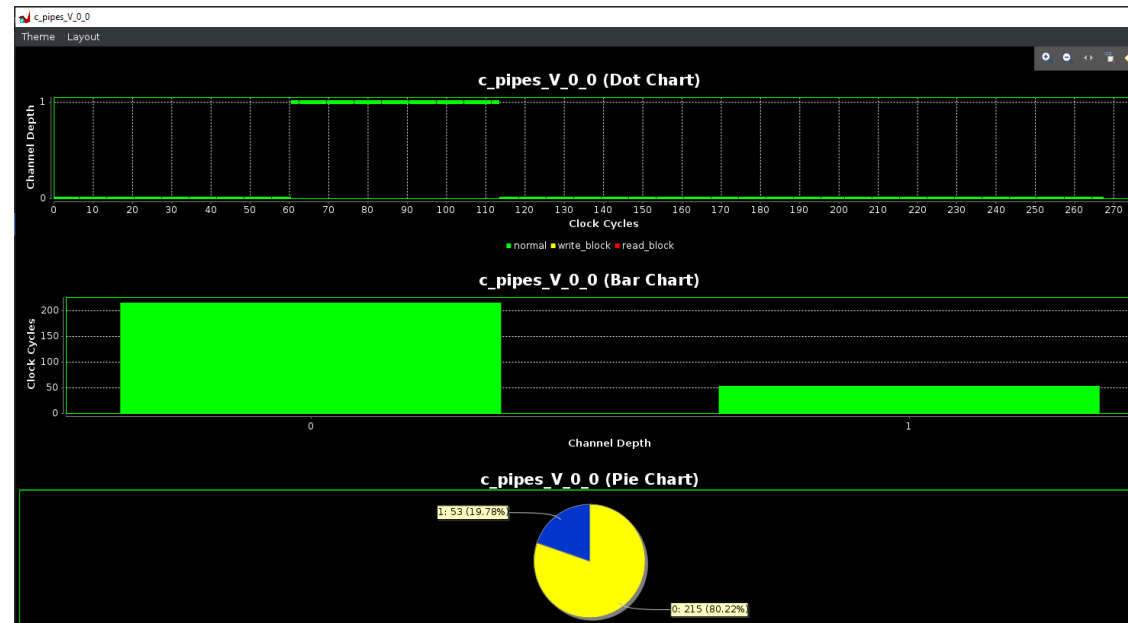
Data flow  
viewer

# FPGA Implementation

## Data flow properties

Process	Channel											
Name	Cosim Category	Cosim Read Block Time	Cosim Write Block Time	Cosim Max Depth	Depth	Type	Sub-Type	BitWidth	Producer	Consumer	Cosim Distribution Graph	
c_c_channel	none	0.00%	0.00%	1	7	FIFO	TaskLevel	64	entry_proc_U0	write_resultC18_U0	<a href="#">Link</a>	
a_pipes_V_0_0	read_block	5.97%	0.00%	2	10	FIFO	Stream	32	read_inputA6_U0	ProcessingElement9_U0	<a href="#">Link</a>	
a_pipes_V_1_0	read_block	15.30%	0.00%	1	10	FIFO	Stream	32	read_inputA6_U0	ProcessingElement12_U0	<a href="#">Link</a>	
a_pipes_V_2_0	read_block	24.25%	0.00%	1	10	FIFO	Stream	32	read_inputA6_U0	ProcessingElement15_U0	<a href="#">Link</a>	
b_pipes_V_0_0	read_block	16.79%	0.00%	1	10	FIFO	Stream	32	read_inputB7_U0	ProcessingElement9_U0	<a href="#">Link</a>	
b_pipes_V_0_1	read_block	35.45%	0.00%	1	10	FIFO	Stream	32	read_inputB7_U0	ProcessingElement10_U0	<a href="#">Link</a>	
b_pipes_V_0_2	read_block	54.48%	0.00%	1	10	FIFO	Stream	32	read_inputB7_U0	ProcessingElement11_U0	<a href="#">Link</a>	
c_pipes_V_0_0	none	0.00%	0.00%	1	15	FIFO	Stream	32	read_inputC8_U0	ProcessingElement9_U0	<a href="#">Link</a>	
c_pipes_V_1_0	none	0.00%	0.00%	1	15	FIFO	Stream	32	read_inputC8_U0	ProcessingElement12_U0	<a href="#">Link</a>	
c_pipes_V_2_0	none	0.00%	0.00%	1	15	FIFO	Stream	32	read_inputC8_U0	ProcessingElement15_U0	<a href="#">Link</a>	
a_pipes_V_0_1	none	0.00%	0.00%	3	10	FIFO	Stream	32	ProcessingElement9_U0	ProcessingElement10_U0	<a href="#">Link</a>	

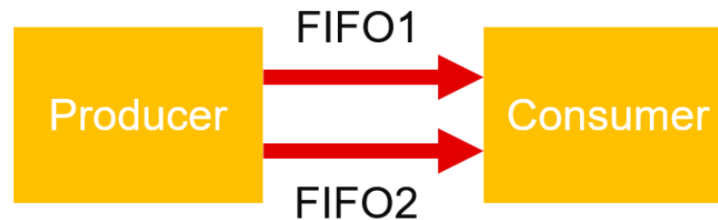
Process	Channel											
Name	Cosim Category	Cosim Stalling Time	Cosim Read Block Time									
entry_proc_U0	none	0.00%	0.00%									
read_inputA6_U0	none	0.00%	0.00%									
read_inputB7_U0	none	0.00%	0.00%									
read_inputC8_U0	none	0.00%	0.00%									
ProcessingElement9_U0	read_block	16.79%	16.79%									
ProcessingElement10_U0	read_block	35.45%	35.45%									
ProcessingElement11_U0	read_block	54.48%	54.48%									
ProcessingElement12_U0	read_block	16.42%	16.42%									
ProcessingElement13_U0	read_block	34.70%	34.70%									
ProcessingElement14_U0	read_block	51.49%	51.49%									
ProcessingElement15_U0	read_block	24.25%	24.25%									
ProcessingElement16_U0	read_block	34.33%	34.33%									
ProcessingElement17_U0	read_block	50.37%	50.37%									
write_resultC18_U0	read_block	59.33%	59.33%									





# FPGA Implementation - Points to note

- Each stream should have one consumer and one producer.
- Insufficiently sized FIFOs can cause deadlocks



- Blocking statement calls – Stall the flow (Can check the Cosim stall time)

```
if(a_in.empty())  
{  
    a_buffer=0;  
}
```



# Scope for future work

- Address the memory accessing bottleneck using techniques like **data packing, ping pong buffers, etc.**
- Try implementing a larger systolic array to increase the resource utilization as it is available.
- Scale our design to multiply **asymmetric matrices.**
- We would like to extend our systolic array implementation to convolution as a follow-on work.