Problem set 4, Computational Neuroscience, October 1st, 2015
(Please hand in before October 8th, 2015 5pm)

Problem 1: *Fisher information for an array of orientation tuned cells.*
a) Calculate the Fisher information for estimating the stimulus orientation θ using an estimator $\hat{\theta}$ based on the tuning function $f_i(\theta_s) = \exp\left(-\frac{(\theta_m-\theta_i)^2}{2\sigma^2}\right)$ assuming a Poisson distribution for the spike count and a measurement interval of length T. What orientation can you estimate best?
b) Assume that you have an array of N cells, with tuning functions f_i, i=1,..,N for which the preferred orientationθ_s is uniformly distributed and for which the resulting spike trains are independent. Determine the Fisher information as a function of the standard deviation σ. (You can calculate this numerically – for a number of stimulus orientations, or you can analytically approximate the sum by an integral, in which case you will have to assume that the preferred orientations are across a range much larger than the sigma, so that you can take the integral from – to +infinity, this is also done in Dayan & Abbott Eq. 3.47).
c) Explain using (a) why you would expect there to be an optimal width σ when N is kept constant. In (b) you will find analytically that Fisher information goes as $1/\sigma$. Explain.
d) Derive the maximum likelihood estimator of the stimulus orientation for this population of neurons.
e) Implement a program to generate the response – spike counts across the N different cells -- on one trial to a particular stimulus with stimulus orientation θ_s. *Note that there is a matlab function that generates Poisson deviates in the statistics toolbox*.
f) Estimate the bias and variance of the ML estimator by using the program in e). Compare this to the Fisher information (as written down analytically, but evaluated numerically). Does it match?

Problem 2: *The role of correlations in coding information.*
a) Review the portion of the Gielen lecture notes where the Fisher information is calculated for the case with correlations in the form of a covariance matrix 'Q(x)'.

We will study the case of 2 neurons. For this case the Gaussian probability distribution can be written in terms of the variables σ (standard deviation) & ρ (crosscorrelation), in terms of which the covariance matrix is $Q = \sigma^2 \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$ and the mean is $(\mu_1(x), \mu_2(x))^T$ for stimulus x.

b) Assume that the means are linear in x, $\mu_1(x) = x, \mu_2(x) = x$ and determine the Fisher Information as a function of σ, ρ and x. Compare $\rho = -1,0,1$.Note that there could be a singularity when you take $\rho \to -1$. Can you explain why the correlations might improve the Fisher information? (Hint: find the eigenvectors of the covariance matrix and draw the probability functions in the 2D plane. The relevant quantity is $(x_2 - x_1) = \sigma$ which should be of order one, where x_1 and x_2 are the two stimuli you want to distinguish. For the best result they need to have the least overlap).

c) Study the case where the means are constant as a function of x but the correlations ρ are linear in the stimulus x (note that can only range from -1 to 1). What is the Fisher information now? What is the maximum likelihood estimator for the stimulus x?

Problem 3: *The role of heterogeneity in coding information.*

(a) Download and read the Chelaru & Dragoi paper (http://www.pnas.org/content/105/42/16344). Summarize their main findings on the Fisher information.
(b) Interpret their results in terms of Eq. (31) in the Gielen lecture notes. Does this formula predict that heterogeneity in the tuning curves improves coding?