CNS 2013

Solutions to Paul Tiesinga Problem Set 5 - Exercise 2

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2.b Noise correlations for two neurons with linearly stimulus dependent firing rates

From the lecture notes:

$$\left(I_F(\vec{\theta})\right)_{ij} = \left[\frac{\partial \vec{\mu}}{\partial \theta_i}\right]^T \boldsymbol{C}^{-1} \left[\frac{\partial \vec{\mu}}{\partial \theta_j}\right] + \frac{1}{2}\mathrm{Tr}\left[\boldsymbol{C}^{-1}\left[\frac{\partial \boldsymbol{C}}{\partial \theta_i}\right]\boldsymbol{C}^{-1}\left[\frac{\partial \boldsymbol{C}}{\partial \theta_j}\right]\right] \tag{1}$$

In this case, $\boldsymbol{C}^{-1}$ does not depend on the stimulus value x, so the second term is zero. The inverse of the correlation matrix is:

$$\begin{aligned} \boldsymbol{C}^{-1} &= \frac{1}{\det \boldsymbol{C}}\ \mathrm{adj}\,\boldsymbol{C} \\ &= \frac{1}{\sigma^2(1-\rho^2)}\begin{pmatrix} 1 & -\rho \\ -\rho & 1 \end{pmatrix} \end{aligned} \tag{2}$$

So, for the Fisher Information:

$$\begin{aligned} I_F &= \left[\frac{\partial \vec{\mu}}{\partial x}\right]^T \boldsymbol{C}^{-1}\left[\frac{\partial \vec{\mu}}{\partial x}\right] \\ &= \begin{pmatrix} 1 \\ 1 \end{pmatrix}^T \frac{1}{\sigma^2(1-\rho^2)}\begin{pmatrix} 1 & -\rho \\ -\rho & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \\ &= \frac{2(1-\rho)}{\sigma^2(1-\rho^2)} \end{aligned} \tag{3}$$

This yields, for $\rho = 0$, $\rho \uparrow 1$ and $\rho \downarrow -1$ (singularities, but limits exist):

$$\begin{aligned} \rho = 0 \quad &\Rightarrow \quad I_F = \frac{2}{\sigma^2} \\ \lim_{\rho \uparrow 1} \quad &\Rightarrow \quad I_F = \frac{1}{\sigma^2} \\ \lim_{\rho \downarrow -1} \quad &\Rightarrow \quad I_F = \infty \end{aligned}$$

So, positive correlations reduce the Fisher information, while negative correlations increase the Fisher information and hence the discriminability.

2.c Two neurons with stimulus dependent noise correlations

As in b, but now the second term in equation **??** is not zero, while the first term is, because the mean firing rate of the neurons does not depend on the stimulus. Let's assume $\rho = \rho(x) = x$, with $x \in \langle -1, 1 \rangle$, such that:

$$\frac{\partial \boldsymbol{C}}{\partial x} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{4}$$

For the Fisher Information:

$$\begin{aligned} I_F &= \frac{1}{2}\mathrm{Tr}\left[\frac{1}{\sigma^2(1-\rho^2)}\begin{pmatrix} 1 & -\rho \\ -\rho & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\frac{1}{\sigma^2(1-\rho^2)}\begin{pmatrix} 1 & -\rho \\ -\rho & 1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\right] \\ &= \frac{1}{2}\frac{1}{(1-\rho^2)^2}\mathrm{Tr}\begin{pmatrix} 1+\rho^2 & -2\rho \\ -2\rho & 1+\rho^2 \end{pmatrix} \\ &= \frac{1+\rho^2}{(1-\rho^2)^2} \end{aligned} \tag{5}$$

This yields, for $\rho = 0$, $\rho \uparrow 1$ and $\rho \downarrow -1$:

$$\begin{aligned} \rho = 0 \quad &\Rightarrow \quad I_F = 1 \\ \lim_{\rho \uparrow 1} \quad &\Rightarrow \quad I_F = \infty \\ \lim_{\rho \downarrow -1} \quad &\Rightarrow \quad I_F = \infty \end{aligned}$$

So, in this case, both positive and negative correlations increase the Fisher information and the discriminability. Also, as was to be expected, the Fisher info does not depend on the noise variance σ^2, as the mean firing rates are the same for all stimuli.