Problem set 2, Computational Neuroscience, September 17, 2015
(Please hand in before September 24th, 5pm)

1. (0.5 pnt) Analyze the behavior of the following systems, with a special focus on the location and stability of fixed points. You should first try analytical methods and then follow up with numerical methods.

   a) Sketch the trajectories, indicate stability of fixed points.

$$\dot{x_1} = -x_2 + x_2^3$$
$$\dot{x_2} = -x_1 + x_1^3$$

   b) Find the two centers.

$$\dot{x_1} = 2x_1x_2$$
$$\dot{x_2} = \frac{1}{4} - x_1^2 + x_2^2$$

   c) Discuss the number of FPs, their location and their stability as you vary a.

$$\dot{x_1} = x_2 + x_1x_2 + ax_1x_2^2$$
$$\dot{x_2} = -x_1 - x_1^2 + x_2^2$$

2. (0.3 pnt) The Van der Pol oscillator is given by $\ddot{x} + C(x^2 - 1)\dot{x} + \omega^2 x = 0$and has a limit cycle.

   a) Show using a phase plane analysis (plot the null clines) that there is a limit cycle.

   b) Which parameter values (c or ω) influences the 1) amplitude of the oscillations; 2) the frequency; 3) the duration of the transient towards the stable limit cycle. You will need to use Matlab.

3. (0.2 pnt) Hopf bifurcations. Compare the dynamics of

$$\dot{r} = r(c - r^2) \qquad\qquad \text{with} \qquad\qquad \dot{r} = r(c + 2r^2 - r^4)$$
$$\dot{\theta} = 2\pi \qquad\qquad\qquad\qquad\qquad \dot{\theta} = 2\pi$$

by plotting the radius of the steady-state solution as a function of control parameter c, with the stability indicated. Discuss the difference between the two cases in relation to the switch in dynamics from c=-2 to 2 and which one is ‘dangerous’.