Computational Neuroscience - Problem Set 5 - Exercise 1 - Solutions

a) $p_1 = r\Delta t, p_0 = 1 - r\Delta t$

$$S_{bin} = -\sum_{i=1}^{2} p_i \log_2 p_i$$

$$= -r\Delta t \log_2 r\Delta t - (1 - r\Delta t) \log_2 (1 - r\Delta t)$$

Use:

(1) $\ln(1 - x) \approx -x$ around x = 0

(2) $\log_a b = \frac{\log_k b}{\log_k a}$

(3) $(r\Delta t)^2 \approx 0$

$$S_{bin} = \frac{-r\Delta t \ln r\Delta t + r\Delta t}{\ln 2} = \frac{r\Delta t}{\ln 2}(1 - \ln r\Delta t) = r\Delta t \left(\log_2 \frac{e}{r\Delta t}\right)$$

Entropy rate: $\dot{S} = r\left(\log_2 \frac{e}{r\Delta t}\right) = \frac{r}{\ln 2}[\ln 2 - \ln(r\Delta t)]$.

b) From DA, p. 145, the entropy rate is

$$\dot{S} = -r\int_0^\infty dt\, p[t]\log_2(p[t]\Delta t).$$

Substitute $p[t] = re^{-rt}$ and solve the integration, we have $\dot{S} = \frac{r}{\ln 2}[1 - \ln(r\Delta t)]$.