1 Answers

1. (a) From the definition of the sequential Glauber dynamics:

$$T(s|F_i s) = \sigma(s_i h_i(F_i s)) = \sigma(s_i h_i(s))$$

The last step follows becasue $F_i s$ is the state s with the value of the ith neuron flipped ($s_i \to -s_i$). The local field h_i is given by

$$h_i = \sum_{j \neq i} w_{ij} s_j + \theta_i$$

and therefore does not depend on the value of s_i.
The reverse transition probability is

$$T(F_i s|s) = \sigma(-s_i h_i(s))$$

Therefore,

$$\begin{aligned} \frac{T(s|F_i s)}{T(F_i s|s)} &= \frac{\sigma(s_i h_i(s))}{\sigma(-s_i h_i(s))} \\ &= \frac{\exp(s_i h_i(s))}{\exp(h_i(s)) + \exp(-h_i(s))} \frac{\exp(h_i(s)) + \exp(-h_i(s))}{\exp(-s_i h_i(s))} \\ &= \exp(2 s_i h_i(s)) \end{aligned}$$

2. (a) The mean field equations are given by

$$m_1 = \tanh(w m_2 + \theta), \quad m_2 = \tanh(w m_1 + \theta)$$

When $w > 0$, the solution is of the form $m = m_1 = m_2$ and therefore we only have to solve

$$m_{mf} = \tanh(w m_{mf} + \theta)$$

for m_{mf}.

(b) See Matlab code.

The exact mean firing rates are normally intractable to compute for a large network, but for two neurons is easily computed. It is given by Eq. 29, which for the special case that $\theta = w$ becomes

$$m_{ex} = \frac{\exp(3w) - \exp(-w)}{3\exp(-w) + \exp(3w)}$$

For $\theta = w$, the solution for various values of w is plotted with the Matlab script.