Final Exam Computational Neuroscience NM047B, Jan 24th 2013 – 90 minutes

Please write your name on each sheet.
Please write clearly, if I cannot read it, you will not receive credit for your answer.
You can use a calculator.
Each problem is worth 10 points.

Problem 1. (Nonlinear dynamics of neurons)

The Izhikevich model is given in its non-dimensional form by

$$\frac{dv}{dt} = I + v^2 - u \qquad \text{with when } v \geq 1 \text{ then} \quad \begin{array}{l} v \leftarrow c \\ u \leftarrow u + d \end{array}$$
$$\frac{du}{dt} = a(bv - u)$$

with variables u and v and parameters a,b,c,d and I.
Consider the parameter setting a=0.05, b=0.2,c=-0.1,d=0, I=0.

(a) Explain (briefly) the meaning of each variable and the parameters I and d.
(b) Draw the phase plane with the null clines and fixed points, including their stability (note that I=0).
(c) Explain how and why, by varying I, you can make the neuron spike periodically.

Problem 2. (Fisher information and information theory)

A variable x is drawn from the distribution $p(x \mid \theta)$, here x represents the response of a neuron, such as firing rate, and θ a stimulus parameter, such as orientation, that we want to estimate.

(a) Give an expression for the Fisher information and explain how this can be used to estimate how good the possible estimates of θ based on a measurement x can be.

(b) Assume that p is Gaussian distribution with standard deviation σ_x and that the mean of x is given by $\mu = \exp(-(\theta - \theta_p)^2/(2\sigma_\theta^2))$. Let the preferred 'orientation' be θ_p=0. What is the Fisher information as a function of θ?

In the lecture notes we had the expression

$$\left(I_F(\vec{\theta})\right)_{ij} = \left[\frac{\partial \vec{\mu}}{\partial \theta_i}\right]^T C^{-1}\left[\frac{\partial \vec{\mu}}{\partial \theta_j}\right] + \tfrac{1}{2} Tr\left[C^{-1}\left[\frac{\partial C}{\partial \theta_i}\right] C^{-1}\left[\frac{\partial C}{\partial \theta_j}\right]\right]$$

for a Gaussian multivariate distribution with a mean $\vec{\mu}(\vec{\theta})$ and covariance matrix $C(\vec{\theta})$.

(c) Use this formula to consider the case of two neurons x_1 and x_2, and preferred 'orientations' be $\theta_{p1}=-1$, $\theta_{p2}=1$, and a one-dimensional stimulus parameter θ. The variance of each x is still given by σ_x. In addition, the two responses x_1 and x_2 could be (anti)correlated, which is denoted by the parameter ρ (please define this parameter*). Do correlations increase Fisher information?* For simplicity consider the case $\theta=0$ (Please document your answer as the correct answer without documentation will not receive credit).

(d) Why is there a bias in the mutual information and what is the sign of this bias?

Problem 3. (Firing rate models and topographic maps)

(a) Explain what firing rate models are and why they might be useful.

Consider the following set of equations for the activity of population E_1 and E_2.

$$\tau \frac{dE_1}{dt} = -E_1 + S(K_1 - 3E_2)$$
$$\tau \frac{dE_2}{dt} = -E_2 + S(K_2 - 3E_1)$$

With response function $S(x) = \dfrac{100x^2}{120^2 + x^2}$ for x>0 and zero otherwise and τ=20ms. Set $K_1=K_2=120$.

(b) There are three fixed points. Contrary to the case discussed during the lectures, only one of them is for $E_1=E_2$, for the other two one of the variables is zero, because of the x<0 in the response function. *Show that (20,20), (50,0) and (0,50) are fixed points and determine the stability of the fixed points*. (Hint: Two fixed points are stable and one is a saddle point.)

(c) Draw the phase plane and describe what this circuit does.