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TOC
Assignment 3

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1. We will show this through a reduction of $\overline{HP}$ to $\overline{L_1}$. Since, $\overline{HP}$ is non recursive-enumerable, then $\overline{L_1}$ is also non recursive enumerable, if there is a reduction.

We also know that,

$$\overline{HP} \leq_m \overline{L_1} \iff HP \leq_m L_1.$$

So it is enough if we show

$$M\#x \in HP \iff \sigma(M\#x) \in L_1$$

where $\sigma(M\#x) = M_1 \# M_2 \# M_3$

Reduction :-

So, given $M\#x$, construct $M_1 \# M_2 \# M_3$,

- M_1, that accepts & halts for all input.
- M_2, that accepts & halts for all input
- M_3, on input y, accepts if M halts on x, else rejects it.

Reduction validity :-

$\Rightarrow L(M_1) = L(M_2) = \Sigma^*$

* M halts on $x \Rightarrow L(M_3) = \Sigma^* = L(M_1) \cap L(M_2)$

$\Rightarrow \boxed{M_1 \# M_2 \# M_3 \in L_1}$

* M does not halt on $x \Rightarrow L(M_3) = \phi \Rightarrow \boxed{M_1 \# M_2 \# M_3 \notin L_1}$

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2. $L = \{\langle G\rangle \mid G \text{ is CFG } \& \text{ ambiguous}\}$

We will show a reduction from PCP to L. Since, we know PCP is undecidable, so is L.

- $G = (V, \Sigma, R, S)$

$V = \{S, S_1, S_2\}$

$R = \{ S \to S_1 \mid S_2,$
$S_1 \to \alpha_1 S_1 \sigma_1 \mid \dots \mid \alpha_m S_1 \sigma_m,$
$S_1 \to \alpha_1 \sigma_1 \mid \dots \mid \alpha_m \sigma_m,$
$S_2 \to \beta_1 S_2 \sigma_1 \mid \dots \mid \beta_1 S_2 \sigma_m,$
$S_2 \to \beta_1 \sigma_1 \mid \dots \mid \beta_m \sigma_m \}$

$\Sigma = \{\sigma_1, \sigma_2, \dots, \sigma_m\}$

& $\alpha_1 \dots \alpha_m, \beta_1 \dots \beta_m$ are strings where Σ is not present in them.

Assume G is ambiguous

$\Rightarrow \exists x \in L(G)$ such that x can be derived in 2 different ways.

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=> It is obvios that in the derivations, one start with $S \to S_1$, & the other with $S \to S_2$. Because, if both start with same $S \to S_1 (S_2)$, we uniquely choose a reduction because, the terminal alphabets at the end of x are uniquely determined.

$$\therefore \quad S \to S_1 \xrightarrow{*} x \quad -① $$
$$S \to S_2 \xrightarrow{*} x \quad -② $$

=> From ① & ②

$$x = \alpha_{i_1} \alpha_{i_2} \cdots \alpha_{i_k} \sigma_{i_1} \sigma_{i_2} \cdots \sigma_{i_k}$$
$$= \beta_{i_1} \beta_{i_2} \cdots \beta_{i_k} \sigma_{i_1} \sigma_{i_2} \cdots \sigma_{i_k}$$

Since σ_i is not present in α_j & β_k $\forall i, j, k$, we see that

$$\alpha_{i_1} \cdots \alpha_{i_k} = \beta_{i_1} \cdots \beta_{i_k}$$

Thus we found a solution for the PCP

~~Thus, we found a solution for PCP. If L is decidable, then PCP is decidable. PCP is decidable which is wrong.~~

=> Similarly, if there is no ambiguity, then PCP cannot be solved.

$\therefore$ L decidable => PCP decidable.

Hence, L is undecidable.

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3. $T_P = \{ (M,N) \mid M \text{ \& } N \text{ are TMs}$
$P(L(M), L(N)) = T \}$

P is a non trivial property

$\Rightarrow P(\phi, \phi) = F$ {Assume}
$P(L_1, L_2) = T$ {Non-triviality}

Let A, B be TMs such
$L(A) = L_1,$
$L(B) = L_2.$

we now show a reduction from HP to T_P (i.e) $HP \leq_m T_P$.

Reduction:-

- Given $\langle M, x\rangle$, construct TM Y such that,
  - i) On input y, accepts if M halts on x & A accepts y.
- construct TM Z such that
  - i) on input y, accepts if M halts on x & B accepts y.

Reduction validity :-

- $\langle M, x\rangle \in HP \Rightarrow L(Y) = L_1 \text{ \& } L(Z) = L_2$
$\Rightarrow P(L(Y), L(Z)) = T$
$\Rightarrow \langle Y, Z\rangle \in T_P$

- $\langle M, x\rangle \notin HP \Rightarrow L(Y) = L(Z) = \phi$
$\Rightarrow P(L(Y), L(Z)) = F$
$\Rightarrow \langle Y, Z\rangle \notin T_P$

Thus T_P is undecidable