CS41001 : Theory of Computation
Autumn 2020

Class Test 3

Date: 5th November, 2020

Time Limit: 1 hour

Guidlines: Each question carries **5 marks.** *Do not search for solutions online.*

1. For a given constant $c > 0$, what is the relationship between P and $DSPACE(n^c)$?

2. Let CL_1 and CL_2 be two time complexity classes or two space complexity classes. Show that if $CL_1(f(n)) \subseteq CL_2(g(n))$, then $CL_1(f(n^c)) \subseteq CL_2(g(n^c))$.
   Hint: Try the trick of padding.

3. In the generalized version of the game Tic-Tac-Toe, 2 players place marks X(crosses) and O(noughts) on an $m \times n$ grid. A player wins if she is the first to place k marks in a row, column or diagonal. The game ends in a draw if no such sequence is present when all the mn cells of the grid are filled. Assuming that X always starts, show that the language

   $GTICTACTOE = \{< m, n, k, c > \mid c$ is an intermediate configuration on the $m \times n$ board with next move by X and $\exists$ a winning strategy for $X.\}$

   is in $PSPACE$.