DISCRETE STRUCTURES(CS21001)

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Let p, q, r denote primitive statements.

a) Use truth tables to verify the following logical equivalences.

i)
$$p \to (q \land r) \iff (p \to q) \land (p \to r)$$

ii)
$$[(p \lor q) \to r] \iff [(p \to r) \land (q \to r)]$$

iii)
$$[p \rightarrow (q \lor r)] \iff [\neg r \rightarrow (p \rightarrow q)]$$

b) Use the substitution rules to show that

$$[p \to (q \lor r)] \iff [(p \land \neg q) \to r].$$

For primitive statements p, q, r, is there any similar way to express the compound statement as follows:

$$\neg [\neg [(p \lor q) \land r] \lor \neg q]$$

Show that for primitive statements p, q,

$$p \veebar q \iff [(p \land \neg q) \lor (\neg p \land q)] \iff \neg (p \leftrightarrow q).$$

Verify for the primitive statements p, q, r $[(p \leftrightarrow q) \land (q \leftrightarrow r) \land (r \leftrightarrow p)] \iff [(p \rightarrow q) \land (q \rightarrow r) \land (r \rightarrow p)]$

Provide the steps and reasons to establish the following logical equivalences:

a)
$$[(p \lor q) \land (p \lor \neg q)] \lor q \iff p \lor q$$

b)
$$(p \to q) \land [\neg q \land (r \lor \neg q)] \iff \neg (q \lor p)$$

Provide the steps and reasons to establish the following logical equivalences:

a)
$$p \vee [p \wedge (p \vee q)] \iff p$$

b)
$$p \lor q \lor (\neg p \land \neg q \land r) \iff p \lor q \lor r$$

c)
$$[(\neg p \lor \neg q) \to (p \land q \land r)] \iff p \land q$$

For primitive statements p, q,

- a) verify that $p \to [q \to (p \land q)]$ is a tautology.
- **b**) verify that $(p \lor q) \to [q \to q]$ is a tautology by using the result from part (a) along with the substitution rules and the laws of logic.
- c) is $(p \lor q) \to [q \to (p \land q)]$ a tautology?

Express these system specifications using the propositions and logical connectives.

Propositions:

p: "The user enters a valid password"

q: "Access is granted"

r: "The user has paid the subscription fee"

- a) "The user had paid the subscription fee, but does not enter a valid password."
- b) "Access is granted whenever the user has paid the subscription fee and enters a valid password."
- c) "Access is denied if the user has not paid the subscription fee."
- d) "If the user has not entered a valid password but has paid the subscription fee, then access is granted."

Are these system specifications consistent? "Whenever the system software is being upgraded, users can not access the file system. If users can access the file system, then they can save new files. If users cannot save new files, then the system software is not being upgraded."