Assignment No 1

1) 1,1,1,1,1 $\quad \frac{5!}{5!} = 1 \times 5! = 120$

✓ 1,1,1,2,0 $\quad \cancel{\frac{5!}{3!\,2!} = \frac{20}{2} = 10} \quad \frac{5!}{2!\,3!} \times 5! = 1200$

1, 2,2,0,0 $\quad = \frac{5!}{2!\,2!\,2!\,2!} \times 5! = 900$

1,1,3,0,0 $\quad \frac{5!}{3!\,2!\,2!} \times 5! = 600$

1,4,0,0,0 $\quad \frac{5!}{4!\,3!} \times 5! = 100$

5,0,0,0,0 $\quad \frac{5!}{5!} \times \frac{5!}{4!} = 5$

(2,3,0,0,0) $= \frac{5!}{2!\,3!\,3!} \times 5! = 200$

$P(\text{exactly one zero}) = \frac{1200}{3125} = \frac{48}{125}$

2) $P(E \cap F) = P(E) \cdot P(F)$

$P(E \cap (F \cup G)) = P(E) \cdot P(F \cup G)$

$P(E \cap (F \cap G)) = P(E) \cdot P(F \cap G)$

$P(E \cap F \cup E \cap G) = P(E \cap F) + P(E \cap G)$

$P(E) \cdot (P(F) + P(G) - P(F \cap G)) \quad - P((E \cap F) \cap (E \cap G))$

$P(E) \cdot P(F) + P(E) \cdot P(G) - \cancel{P(E) \cdot P(F \cap G)} = P(E) P(F) + P(E \cap G) - \cancel{P(E) \cdot P(F \cap G)}$

$\therefore P(E \cap G) = P(E) \cdot P(G)$

even number appears

P = ((2,1),(2,2),(2,3)(2,4)(2,5),(2,6))
(4,1) (4,2) (4,3) (4,4) (4,5) (4,6)
(6,1), (6,2), (6,3) (6,4) (6,5) (6,6)
(1,2), (1,4), (1,6)
(3,2), (3,4), (3,6)
(5,2), (5,4), (5,6)

$P(E) = \frac{27}{36}$

$P(\text{sum of } 5) = (1,4),\ (4,1)\ (2,3),\ (3,2) = \frac{4}{36}$

4) $P(C) = 0.6 \quad P(H) = 0.3 \quad P(C \cap H) = 0.2$

$P(C \cap H') = 0.6 - 0.2$
$= 0.4$

$P(H \cap C') = 0.3 - 0.2 = 0.1$

$P(\text{car or house but not n\& both})$ $= 0.4 + 0.1$
$= 0.5$

5) $P(S) = 0.2$

$P\left(\frac{D}{S}\right) = 10\, P\left(\frac{D}{S'}\right)$

$0.006 = (0.2)\, P\left(\frac{D}{S}\right) + (0.8)\, P\left(\frac{D}{S'}\right)$

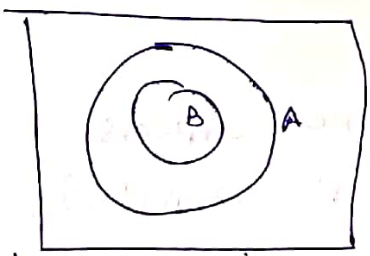
$0.006 = (0.2)\, P\left(\frac{D}{S}\right) + \frac{(0.8)}{10}\, P\left(\frac{D}{S}\right)$

$0.006 = \left(0.2 + \frac{0.8}{10}\right) P\left(\frac{D}{S}\right)$

$P\left(\frac{D}{S}\right) = \frac{3}{140}$

6) $\frac{P(A \cap B)}{P(B)} = 1$

$P(A \cap B) = P(B)$



from diagram $\longrightarrow \frac{P(A^c \cap B^c)}{P(A^c)} = \frac{P(A^c)}{P(A^c)} = 1$

7) $P(A^c) = 0.3 \quad P(B) = 0.4 \quad P(A \cap B^c) = 0.5$

$P(A^c \cup B) = 0.5$

$$P(B \mid A \cup B^c) = \frac{P(B \cap (A \cup B^c))}{P(A \cup B^c)}$$

$$P(B \cup (A \cup B^c)) = P(B) + P(A \cup B^c) - P(B \cap (A \cup B^c))$$

$$1 = 0.4 + 0.8 - P(B \cap (A \cup B^c))$$

$$P(B \cap (A \cup B^c)) = 0.2$$

$$P(A \cup B^c) = P(A) + P(B^c) - P(A \cap B^c)$$
$$= 0.7 + 0.6 - 0.5$$
$$= 1.3 - 0.5$$
$$= 0.8$$

$$P(B \mid A \cup B^c) = \frac{0.2}{0.8} = \frac{1}{4}$$

Q10) (i) ✓

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= 0.3 + 0.7 - 0.5$$
$$= 0.5$$

(ii)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$0.7 = 0.5 + 0.4 - P(A \cap B)$$
$$P(A \cap B) = 0.2$$
$$P(A) \cdot P(B) = 0.2$$

independent

$$P(A \cup B) = P(A) + P(B)$$
$$= 0.2 + 0.6$$
$$= 0.8$$

11)

$$n(S) = \frac{52!}{13!\,13!\,13!\,13!\,4!} \times 4! = \frac{\lfloor 52}{(\lfloor 13)^4}$$

$${}^{48}C_9 \times {}^{39}C_{13} \times {}^{26}C_{13} \times {}^{13}C_{13}$$

$$\frac{\lfloor 48}{\cancel{\lfloor 39}\lfloor 9} \times \frac{\cancel{\lfloor 39}}{\cancel{\lfloor 26}\lfloor 13} \times \frac{\cancel{\lfloor 26}}{\lfloor 13 \lfloor 13} = \frac{\lfloor 48}{\lfloor 13 \lfloor 13 \lfloor 13 \lfloor 9}$$

$$\frac{\lfloor 48}{(\lfloor 13)^3 \lfloor 9} \cdot \frac{(\lfloor 13)^4}{\lfloor 52}$$

$$\frac{13 \times 12 \times 11 \times 10}{52 \times 51 \times 50 \times 49}$$

$$= \frac{11}{4165}$$

14)

$P(A)>0 \qquad P(A\cap B|A) \geq P(A\cap B|A\cup B)$

$$\frac{P(A\cap B\cap A)}{P(A)} \geq \frac{P((A\cap B)\cap(A\cup B))}{P(A\cup B)}$$

$$\frac{P(A\cap B)}{P(A)} \geq \frac{P(A\cap B)}{P(A\cup B)}$$

$$P(A\cup B) \geq P(A)$$

$$\frac{1}{P(A)} \geq \frac{1}{P(A\cup B)} \qquad \therefore \frac{P(A\cap B)}{P(A)} \geq \frac{P(A\cap B)}{P(A\cup B)}$$

$$\frac{P(A\cap B)}{P(A)} \geq \frac{P(A\cap B)}{P(A\cup B)}$$

12)

$b^2-4ac>0$

1: a b c — 1 2 1

3: a b c — 1 3 1; ~~a b c~~ 2 3 1; 1 3 2

8: a b c — 1 4 1; ~~a b c~~ 2 4 1; 1 4 2; 2 4 2; 3 4 1; 1 4 3; 4 4 1; 1 4 4

11, 12, 13, 14, 21, 22, 31, 41

12: a b c — 1 5 1; 1 5 2; 2 5 1; 1 5 3; 3 5 1; 4 5 1; 1 5 4; 2 5 2; ~~3 5 3~~; 5 5 1; 1 5 5; 2 5 3; 3 5 2; 1 5 6; 6 5 1

1 1, 2 1; 1 2, 2 2; 1 3, 2 3; 1 4, 3 1; 1 5, 3 2; 1 6, 4 1; 5 1; 6 1

17: a b c — 1 6 1; 1 6 2; 2 6 1; 2 6 2; 1 6 3; 3 6 1; 3 6 3; 1 6 4; 4 6 1; 5 6 1; 1 6 5; 6 6 1; 1 6 6

a b c — 2 6 3; 3 6 2; 2 6 4; 4 6 2

$P = \frac{43}{216}$

1 1, 1 2, 1 3, 1 4, 1 5, 1 6

2 1, 2 2, 2 3, 2 4

3×1, 3×2, 3×3

4×1, 4×2

5×1

6×1

9) $P(A_1) = \alpha * P_m + (1-\alpha) P_f$

$P\left(\frac{A_2}{A_1}\right) = \frac{P(A_2 \cap A_1)}{P(A_1)} = \frac{P_m^2 \alpha + (1-\alpha) P_f^2}{2 P_m + (1-\alpha) P_f}$

$= \frac{P_m^2 \alpha + (1-\alpha) P_f^2}{\alpha P_m + (1-\alpha) P_f}$

13) 5 courses a sem

1st semester $= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{32}$

$P(\text{N'al ex}) = \left(\frac{31}{32}\right)^{10} \quad 1 - \left(\frac{31}{32}\right)^{10}$

15) $S = \{1, 2, \dots n\}$

$P(X = i)$ number of elements in B

$P(X=i) = \frac{^nC_i}{2^n}$

$0 + {^nC_1}(2^1 - 1) + {^nC_2}(2^2 - 1) + {^nC_3}\, 2^3 - 1 \dots {^nC_n}\, 2^n - 1$

$P(A \subset B \mid X = i)$

$\frac{(2^{^nC_i} - 1)}{2^n} \times \frac{^nC_i}{2^n}$

${^nC_1}\, 2^1 + {^nC_2}\, 2^2 + {^nC_3}\, 2^3 \dots$

$(3^n - 1) - ({^nC_1} + {^nC_2} \dots {^nC_n})$

$(3^n - 1) - (2^n - 1)$

$\frac{3^n - 2^n}{2^n \times 2^n}$

$A \cap B \neq \phi \quad (3^n - 2^n)$

3^n

${^nC_0} \cdot 2^0 + {^nC_1}\, 2^1 + {^nC_2}\, 2^2 + {^nC_3}\, 2^3 \dots {^nC_n}\, 2^n$

$= 3^n$

$P(A \cap B = 0) \quad \frac{3^n}{2^n \cdot 2^n} = \frac{3^n}{4^n} = \left(\frac{3}{4}\right)^n$

17) $P(A|\bar{B}) = \dfrac{0.35}{0.85} = \dfrac{7}{17}$

$P(C|\bar{B}) = \dfrac{0.3}{0.85} = \dfrac{6}{17}$

$P(D|\bar{B}) = \dfrac{0.2}{0.85} = \dfrac{4}{17}$

16)

six T/F	two multiple	
$\frac{1}{\cancel{32}\ 64}$	$\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^2$	$= \frac{9}{16384} = \frac{9}{256} \times \frac{1}{64}$
five T/F	3 multip:	
$\frac{1}{64}$	$\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)$	$= \frac{3}{256} \times \frac{1}{64}$
4 T/F	4 multiple	
$\frac{1}{64}$	$\left(\frac{1}{4}\right)^4$	$\frac{1}{256} \times \frac{1}{64}$

8

9.

six T/F	3 multiple	
$\frac{1}{64}$	$\left(\frac{1}{4}\right)^3\left(\frac{3}{4}\right)$	$\left(\frac{3}{256}\right) \times \frac{1}{64}$
five T/F	4 multiple	
$\frac{1}{64}$	$\left(\frac{1}{4}\right)^4$	$\left(\cancel{\frac{1}{64}}\ \frac{1}{256}\right)\left(\frac{1}{64}\right)$

10

six	four	
$\left(\frac{1}{64}\right)$	$\left(\frac{1}{4}\right)^4$	$= \frac{1}{256} \times \frac{1}{64}$

$$\frac{13+3+1+1}{256 \times 64} = \frac{18}{256 \times 64} = \frac{\cancel{8}\,9}{8192}$$

18)

$n+1$ ~~guys~~ girls : $\left(\frac{1}{2}\right)^{2n}$

P(more girls than guys)

$= n\left(\frac{1}{2}\right)^{2n}$

19) A_i → ith person will get his right hat

$P(\cap A_i^c) = 1 - P(\bigcup_{i=1} A_i)$

$= 1 - \sum_{i=1} (-1)^{k} S_k$

$= 1 + \sum (-1)^{k-1} S_k$

$= 1 + (-1)^{k-1} \, {}^nC_k \dfrac{(n-k)!}{n!}$

$= 1 + \dfrac{(-1)^{k-1}}{k!}$

$\lim \left(1 + \dfrac{(-1)^{k-1}}{k!} \right) = e^{-1}$

$= \dfrac{D_n}{n_b}$

$= \dfrac{n! \left(1 - 1 + \frac{1}{2!} - \frac{1}{3!} \cdots (-1)^n \frac{1}{n!}\right)}{n!}$

20) k boxes.

n balls.

$\cancel{{}^{n+k-1}C_{k-1}}$ = total possiblities

${}^{n \cdot k+r-1}C_{r-1} \cdot {}^{n-k+(k-r)-1}C_{(k-r)-1}$

$P = \dfrac{{}^{k+r-1}C_{r-1} \cdot {}^{n-k+k-r-1}C_{(k-r)-1}}{{}^{n+k-1}C_{k-1}}$

$\dfrac{({}^nC_k) \, r^k \times (k-r)^{n-k}}{k^n}$

21) $P = \dfrac{{}^{26}C_3 \, {}^{26}C_{10}}{{}^{52}C_{13}}$

22) $P \quad \dfrac{{}^{13}C_3 \cdot {}^{13}C_4 \cdot {}^{13}C_4 \cdot {}^{13}C_2}{{}^{52}C_{13}}$

23

a) $\Rightarrow \frac{4}{52} \times \frac{3}{52} \times \frac{2}{52} \times \frac{1}{52}$

$^{n+r-1}C_{r-1}$

b) $\frac{4}{52} \times \frac{4}{52} \times \frac{4}{52} \times \frac{4}{52}$

24) $^{n+n-1}C_{n-1} = {}^{2n-1}C_{n-1} = \frac{\lfloor 2n-1}{\lfloor n-1 \lfloor n}$

$x_1 \neq 0$ $\quad x_1 + x_2 - \cdots + x_n = n$

$x_2' + \cdots + x_n' = 1$

n

$(n-1)$ $\quad n$ $\quad {}^{1+(n-1)-1}C_{(n-1)-1}$

$$\frac{C_{n-1} \; (n-1)! \times (n-1)}{n^n}$$

Assignment No 2

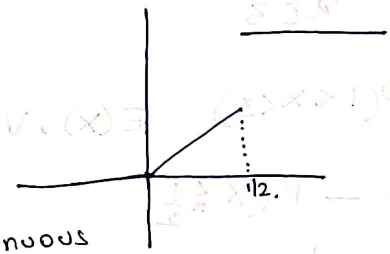
1) a) $F(x) = 0 \quad x < 0$

$x \quad 0 \le x < 1/2$

$1 \quad x \ge 1/2$

$F(-\infty) = 0$

$F(\infty) = 1$



No not right continuous at $x = 1/2$

$F(\frac{1}{2}) = 1/2 \quad F(\frac{1}{2}+\epsilon) = 1.$

x^2

$2x$

2

b) $\frac{1}{\pi} \tan^{-1}(x) \quad -\infty < x < \infty$

$x = \infty \quad \frac{1}{\pi} \tan^{-1}(\infty)$

$= \frac{\pi}{2 \times \pi} = 1/2$

$F(\infty)$ should be 1 but it is $1/2$ so not cdf.

c) $F(x) = 0$ if $x \le 1 \quad 1 - \frac{1}{x} \quad x > 1$

$\underset{x \to \infty}{Lt} F(x) = 1 - \frac{1}{x} = 1$

$F(-\infty) = 0$

$F'(x) = \frac{1}{x^2} > 0.$

always increasing, continuous

$F(x)$ is cdf

d) $F(x) = 1 - e^{-x}$ if $x \ge 0$

$0, x \le 0$

$F(-\infty) = 0 \quad F(\infty) = 1 - e^{-\infty} = 1.$

$F(0^+) = 1 - e^{-0} = 0 = F(0^-) = F(0)$

$F(x)$ is continuous and it is cdf

2)

$F(x) = 0 \quad x < 0$

$\frac{x}{4} \quad 0 \le x < 1$

$\frac{(x+1)}{4} \quad 1 \le x < 2$

$11/12 \quad 2 \le x < 3$

$1 \quad x \ge 3$

$P(\frac{1}{2} \le X \le \frac{5}{2})$, $P(1 < X < 3)$ $E(X), V(X)$ median of X.

$P(X \le 5/2) - P(X \le \frac{1}{2})$

$= \frac{11}{12} - \frac{1}{8}$

$= \frac{22-3}{24} = \frac{19}{24}.$

$P(1 < X < 3)$

$P(X<3) - P(X \le 1)$

$= \frac{11}{12} - \frac{2}{4}$

$= \frac{11-6}{12} = \cancel{\frac{8}{12} = \frac{2}{3}} \; \frac{5}{12}$

$P(1^-) = 1/4$

$P(1^+) = 2/4$

$P(X=1) = \frac{2}{4} - 1/4 = \frac{1}{4}$

$P(2^+) - P(2^-) = \frac{11}{12} - \frac{3}{4} = \frac{11-9}{12}$

$= 1 - \frac{11}{12}$

$E(X) = \int_0^1 x \frac{1}{4} dx + \int_1^2 \frac{x}{4} dx + 1 \times \frac{1}{4} + 2 \times \frac{1}{6} + 3 \times \frac{1}{12}$

$= \int_0^2 \frac{x}{4} dx = \left[\frac{x^2}{8}\right]_0^2 = \frac{4}{8} = 1/2 + \frac{1}{4} + \frac{1}{4} + \frac{1}{3}$

$1 + \frac{1}{3} = 4/3$

$E(X^2) = \int_0^2 \frac{x^2}{4} dx = \left[\frac{x^3}{12}\right]_0^2 = \frac{8}{12} = \frac{2}{3}$; $\frac{2}{3} + \frac{1}{4} + \frac{4}{6} + \frac{9}{12}$

$= \frac{4}{3} + 1 = \frac{7}{3}$

$V(X) = E(X^2) - (E(X))^2$

$= \cancel{\frac{2}{3} - \frac{1}{4} = \frac{8-3}{12}}$

$\cancel{V(X) = \frac{5}{12}}$

$V(X) = \frac{7}{3} - \frac{4}{3}$; $\frac{7}{3} - \frac{16}{9} = \frac{21-16}{9} = \frac{5}{9}$

$P(X \le m) = 1/2$

$F(m) = \cancel{x+1}\left(\frac{m+1}{4}\right) = \frac{1}{2}$

$m = 1$

3) $P(X==1) = \frac{1}{4}$

$P(X==2) = \frac{3}{4} \times \frac{1}{3} = \frac{1}{4}$

$P(X==3) = \frac{3}{4} \times \frac{2}{3} \times \frac{1}{2} = \frac{1}{4}$

$\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

$\frac{3}{4} \times \frac{2}{3} \times \frac{1}{2} = \frac{1}{4}$

$p(X=x): f(x) = \frac{1}{4} \quad x=1$

$\frac{1}{4} \quad x=2$

$\frac{1}{2} \quad x=3$

~~$F(x) = \cancel{x<0}\ 0 \quad x<1$~~

~~$\frac{1}{4} \quad x \le x \quad 1 \le x\ 2$~~

~~$\frac{1}{2} \quad 1 < x \le 2$~~

~~$1 \quad x \le 3$~~

$P(X \le x)$ $F(x) = 0 \quad x < 1$

$\frac{1}{4} \quad 1 \le x < 2$

$\frac{1}{2} \quad 2 \le x < 3$

$1 \quad x \ge 3$

4) SS, SDS, SDD, DDDD, DDSS, DDSD, DDDS,

DSS DSD

$P(SS) = \frac{4}{9}$ $P(SDS) = \frac{2}{3} \times \frac{1}{3} \times \frac{2}{3} = \frac{4}{27}$ $P(SDD) = \frac{2}{3} \times \frac{1}{3} \times \frac{1}{3} = \frac{2}{27}$

$P(DDDD) = \frac{1}{3^4} = \frac{1}{81}$ $P(DDSS) = \frac{4}{81}$ $P(DDSD) = \frac{2}{81}$ $P(DDDS) = \frac{2}{81}$

$P(DSS) = \frac{4}{27}$ $P(DSD) = \frac{2}{27}$

X ~~set~~ number of survival

$P(X==0) = \frac{1}{81}$ $P(X==1) = \frac{2}{27} + \frac{2}{81} + \frac{2}{81} + \frac{2}{27} = \frac{16}{81}$

$P(X==2) = \frac{4}{9} + \frac{4}{27} + \frac{4}{81} + \frac{4}{27} = \cancel{\frac{149}{189}}\ \frac{64}{81}$

X' = number of deaths

$P(X' = 0) = \frac{4}{9}$

$P(X' = 1) = \frac{4}{27} + \frac{4}{27} = \frac{8}{27}$

$P(X' = 2) = \frac{2}{27} + \frac{4}{81} + \frac{2}{27} = \frac{16}{81}$

$P(X' = 3) = \frac{4}{81}$

$P(X' = 4) = \frac{1}{81}$

$M_{X}(s) = \frac{(1+e^s)^3}{8} = \left(\frac{1+e^s}{2}\right)^3$

$= \left(\frac{1}{2}e^s + 1 - \frac{1}{2}\right)^3$

$p = 1/2 \quad n = 3 \qquad X \sim bin(3, 1/2)$

$M_Y(t) = e^{(e^t - 1)} \qquad Y \sim P(\lambda)$

6)

$$f(x) = \begin{cases} x/2 & 0 \le x \le 1 \\ 1/2 & 1 < x \le 2 \\ \frac{3-x}{2} & 2 < x \le 3 \\ 0 & \text{else where} \end{cases}$$

$\frac{dF(x)}{dx} = f(x)$

$$F(x) = \begin{cases} c & x \le 0 \\ \frac{x^2}{4} + a & 0 \le x \le 1 \\ \frac{x}{2} + b & 1 < x \le 2 \\ \frac{3x}{2} - \frac{x^2}{4} + d & 2 < x \le 3 \\ e & x > 3 \end{cases}$$

$e = 1$

$\frac{3\times3}{2} - \frac{9}{4} + d = 1$

$\frac{9}{2} - \frac{9}{4} + d = 1$

$\frac{9}{4} + d = 1$

$d = 1 - \frac{9}{4}$

$d = -\frac{5}{4}$

$c = 0$

$a = 0$

$x = 1 \qquad \frac{1}{4} = \frac{1}{2} + b$

$b = -1/4$

$3 - 1\cancel{4} - \frac{5}{4}$

$2 - \frac{5}{4} = 3/4.$

$$F(x) = \begin{cases} 0 & x \le 0 \\ \frac{x^2}{4} & 0 \le x \le 1 \\ \frac{x}{2} - \frac{1}{4} & 1 < x \le 2 \\ \frac{3x}{2} - \frac{x^2}{4} - \frac{5}{4} & 2 < x \le 3 \\ 1 & x > 3 \end{cases}$$

mean = $E(x)$

$\int_0^1 x f(x)\,dx$

$\int^1 \frac{x^2}{2}\,dx = \frac{x^3}{6} = \frac{1}{6}$

$+ \cancel{\left[\frac{x}{2}\right]_1^2} \cdot 1$

$\left[\frac{x^2}{4}\right]_1^2 = \frac{3}{4}$

$\frac{x(3-x)}{2}$

$\frac{3-x}{2}$

$\int_2^3 \left(\frac{3x}{2} - \frac{x^2}{2}\right) dx$

$\frac{3x^2 - x^3}{2}$

$\left[\frac{3x^2}{4} - \frac{x^3}{6}\right]_2^3$

$\frac{3x^2}{62}$

$\frac{3}{4}\times5 - \frac{19}{6} = \cancel{25}\ \frac{15}{4} - \frac{19}{6} = \frac{45-38}{12} = \frac{7}{12}$

$\frac{1}{6} + \frac{7}{12} + \frac{3}{4} = \frac{2+7+9}{12} = \frac{18}{12} = \frac{3}{2}$

$P(x \le m) = 1/2 \qquad m = 3/2$ median

$F\left(\frac{3}{2}\right) = \frac{1}{2}$

$E(x^2) - (E(x))^2 = \int_0^1 \frac{x^3}{2}\,dx + \int_1^2 \frac{x^2}{2}\,dx + \int_2^3 \left(\frac{3x^2}{2} - \frac{x^3}{2}\right) dx$

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$\frac{5}{12} = E(x^2)$

$$= \left[\frac{x^4}{8}\right]_0^1 + \left[\frac{x^3}{3}\right]_1^2 + \left[\cancel{\frac{3x^2}{62} - \frac{x^3}{8}}\right]_2^3$$

$$= \frac{1}{8} + \frac{7}{3} + \cancel{\frac{19}{2}} \cancel{-} \cancel{\frac{65}{8}}$$

$\frac{3x^3}{6} - \frac{x^4}{8}$

$+\frac{19}{2} - \frac{65}{8}$

27

$$= \frac{23}{6}$$

$$\frac{23}{6} - \frac{9}{4} = \frac{46-27}{12} = \frac{19}{12}$$

7)

$$0 \to 0 \to 0$$

$$0 \to T \to 0$$
$$\searrow 0$$

$X \quad 1, 2, 3$

$Y \quad 1, 2, 3 \ldots\ldots 9$

$P(Y=1) = \frac{1}{3} \times \frac{1}{3} = \cancel{\frac{1}{81}} \cancel{27} = \frac{1}{9}$

$P(Y=2) = \frac{1}{3} \times \frac{1}{3} + \frac{1}{3} \times \cancel{\left(\frac{1}{9}\right)^2} \left(\frac{1}{3}\right)^2 = \cancel{\frac{10}{243}} \; \frac{4}{27}$

$P(Y=3) = \cancel{\frac{1}{3}} \times \cancel{\frac{1}{9}} + \cancel{\frac{1}{3}} \times \cancel{\frac{1}{9}} \; \frac{1}{3} \times \frac{1}{3} + \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} \times 2 + \frac{1}{3} \times \left(\frac{1}{3}\right)^3$

$= \frac{16}{81}$

$P(Y=4) = \frac{1}{3} \times \left(\frac{1}{3}\right)^2 \times 3 + \frac{1}{3} \times \left(\frac{1}{3}\right)^3 \times 3 = \frac{4}{27}$

1,3
3,1
2,2

$P(Y=5) = \frac{1}{3} \times 2 \times \left(\frac{1}{3}\right)^2 + \frac{1}{3} \times 6 \times \left(\frac{1}{3}\right)^3 = \frac{4}{27}$

1, 1, 2 × 3

$P(Y=6) = \frac{1}{3} \times \left(\frac{1}{3}\right)^2 + \frac{1}{3} \times 7 \times \left(\frac{1}{3}\right)^3 = \frac{\cancel{8}\,10}{81}$

2,3
3,2 $\left(\frac{1}{3}\right)$

1,1,3 3
1,2,2 3

1,2,3 → 6
2,2,2 – 1

$P(Y=7) =$ 1, 3, 3. — 3 $1n_1 + 4n_2 + 9n_3$

2, 2, 3. — 3. $n_1 + 2n_2 + 3n_3$

$\frac{1}{3} \times 6 \times \left(\frac{1}{3}\right)^3 = \frac{2}{27}$

$P(Y=8) = 3 \times \frac{1}{81} = \frac{1}{27}$ 2, 3, 3. — 3

$P(Y=9) = \frac{1}{81}$

8) X = no of tests

blood tested together

$X = N\ 1$ ~~only first person tested~~

$P(\text{disease}) = 0.01$

$X = 11$ ~~all~~   blood tested together + individual 10

$P(X = 1) = (0.99)^{10}$

$P(X=11) = 1 - (0.99)^{10}$

$E(X) = 1 \times 0.99^{10} + 11(1-0.99)^{10}$

$= 1.956$

9)   m families.

$P(X=i) = \frac{n_i}{m}$.

Number of children $= \sum_{i=1}^{r} i n_i$

How to prove $E(Y) \geq E(X)$

$E(X) = \sum_{i=1}^{r} \frac{i \times n_i}{m}$

Y total no of children in the family

$E(Y) = \sum_{i=1}^{r} \frac{i^2 n_i}{\sum i n_i}$

$P(Y=i) = \frac{i n_i}{\sum_{i=1}^{r} i n_i}$

$E(Y) = \frac{1}{\sum_{i=1}^{r} i n_i} \sum_{i=1}^{r} i^2 n_i = \cancel{\sum_{i=1}^{r} i} \geq \sum_{i=1}^{r} \frac{i n_i}{m}$)

10)

$0 \leq \frac{1+3d}{4} \leq 1$ $0 \leq \frac{1-d}{4} \leq 1$ $d \leq 1$ $0 \leq \frac{1+2d}{4} \leq 1$

$0 \leq 1+3d \leq 4$ $1-d \leq 4$ $1+2d \leq 4$

$-\frac{1}{3} \leq d$ $3d \leq 3$ $-3 \leq d$ $2d \leq 3$

$d \leq 1$ $-\frac{1}{2} \leq d$ $d \leq 3/2$

$0 \le \frac{1-4d}{4} \le 1$

$1-4d \le 4 \qquad 4d \le 1$

$\frac{-3}{4} \le d \qquad d \le 1/4$

$\frac{-1}{3} \le d \le 1/4$

$E(x) = \frac{1+3d}{4} + \frac{1-d}{2} + \frac{3+6d}{4} + 1-4d$

$= \frac{5}{2} - \frac{9}{4}d$

$E(x^2) = \frac{1+3d}{4} + 1-d + \frac{9+18d}{4} + 4-16d$

$= \frac{15}{2} - \frac{47}{4}d$

$Var(x) = E(x^2) - E(x)^2$

$= \left(\frac{15}{2} - \frac{47}{4}d\right) - \left(\frac{5}{2} - \frac{9}{4}d\right)^2$

$= \left(\frac{15}{2} - \frac{47}{4}d\right) - \frac{25}{4} + \frac{45}{4}d - \frac{81}{16}d^2$

$Var(x) = \frac{5}{4} - \frac{2d}{4} - \frac{81}{16}d^2$

$Var(-1/3) = \frac{41}{48} = 0.8541$

$Var(1/4) = \frac{207}{256} = 0.8085$ Var(x) minimum at $d = 1/4$

11)

$k\left(\frac{1}{x+1} - \frac{1}{x+2}\right)$

$k\left(\frac{1}{1} - \frac{1}{2}\right) + k\left(\frac{1}{2} - \frac{1}{3}\right) \ldots + k\left(\frac{1}{n+1} - \frac{1}{n+2}\right) = 1$

$k\left(1 - \frac{1}{n+2}\right) = 1$

$k = 1$

$\frac{1}{x+1} - \frac{1}{x+2}$

$F(x \le a)$

$1/2$.

$F(x \le a)$

$F(x \le n)$

$1 - \frac{1}{n+2}$

$\frac{1}{1} = \frac{1}{2} + \frac{1}{2} - \frac{1}{3}$

$x < 0$ 0

$0 \le x < 1$ $1/2$

$1 \le x < 2$ $2/3$

$n-1 \le x < n$ $\frac{n}{n+1}$

$E(X) = \sum_{x=0}^{\infty} \frac{x}{(x+1)(x+2)} \not< \infty$ does not exist why?

median any m between 0 and 1

12) $E(X) = 100$ ~~$VAR(X) = 16$~~

$VAR(X) = 256$.

~~256~~

$P(|X - 100| > 48) \le \frac{Var(X)}{48^2}$

$\le \frac{256}{(48)^2} \le 1/9$.

$\frac{20}{32}$ $\frac{1}{2}$ $\frac{6}{8}$ $\frac{3}{4}$ $\frac{12}{16}$ $\frac{3}{4}$ $\frac{26}{32}$ $\frac{5}{8}$

Tutorial 3

1)

$$\cancel{{}^7C_3 \left(\frac{4}{7}\right)^2\left(\frac{3}{7}\right) + {}^7C_3\left(\frac{4}{7}\right)^3} \qquad \frac{{}^4C_2 \times {}^3C_1 + {}^4C_3}{{}^7C_3} = \frac{22}{35}$$

$$= \cancel{\frac{16}{49}}$$

2)

$P(B) = \frac{1}{2}$

$$\cancel{E(X) = 2\times\left(\frac{1}{2}\right)^2 + 3\times {}^2C_1 \times\left(\frac{1}{2}\right)^3 + 4\times {}^3C_2 \times \left(\frac{1}{2}\right)^4 + 5\times {}^4C_3\left(\frac{1}{2}\right)^5 \ldots}$$

$$\sum \cancel{\frac{n(n-1)}{2^n}} \qquad \cancel{\left(\frac{1}{2}\right)^2 + 2\left(\frac{1}{2}\right)^3 + 3\left(\frac{1}{2}\right)^4 + 4\left(\frac{1}{2}\right)^5 +}$$

$p = \frac{1}{2}$ $q = \frac{1}{2}$

X bombs are direct hits

$$P(X >= 2) = 0.99$$

$$1 - P(X < 2) \geq 0.99$$

$$P(X<2) \leq 0.01$$

$${}^nC_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^n + {}^nC_1\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^{n-1}$$

$$\left(\frac{1}{2}\right)^n + n\left(\frac{1}{2}\right)^n \leq 0.01$$

$$\left(\frac{1}{2}\right)^n(n+1) \leq 0.01$$

3)

$n = 11$

$$P(X_1 \geq 3) = {}^5C_3(p)^3(1-p)^2 + {}^5C_4(p)^4(1-p) + {}^5C_5(p^5)(1-p)^0$$

$$P(X_2 \geq 2) = {}^3C_2(p)^2(1-p) + {}^3C_3 p^3$$

$$P(X_1 \geq 3) > P(X_2 \geq 2)$$

$$10(p)(1-p)^2 + 5p^2(1-p) + p^3 > 3(1-p) + p$$

$$10p(1-2p+p^2) + 5p^2 - 5p^3 + p^3 > 3 - 3p + p$$

$$10p - 20p^2 + 10p^3 + 5p^2 - 5p^3 + p^3 > 3 - 2p$$

$$6p^3 - 15p^2 + 12p - 3 > 0$$

$$2p^3 - 5p^2 + 4p - 1 > 0$$

$$p-1 \overline{\big) 2p^3 - 5p^2 + 4p - 1}$$ quotient $2p^2 - 3p + 1$

$2p^3 - 2p^2$

$(-) \quad (+)$

$-3p^2 + 4p$

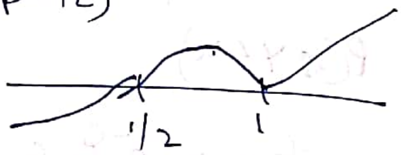
$-3p^2 + 3p$

$(+) \quad (-)$

$p - 1$

$= 2p^2 - 3p + 1$

$(p-1)^2(p - 1/2)$



$\frac{1}{2} < p < 1$

4) Probability that ~~tese that~~ pack will not be returned

$^{10}C_0 \times (0.01)^0 (0.99)^{10} + {}^{10}C_1 (0.01)(0.99)^9$

$= 0.995734$

X: No of packs returned

$P(X=0) + P(X=1)$

$(^3C_0)(0.995734)^3 + {}^3C_1 \times (0.995734)^2 (1 - 0.995734)$

Probability that atmost one pack will be returned

5) $= 0.99994555$

$e^{\lambda x} = \sum \frac{\lambda^k x^k}{k!}$ pmf $f(x) = \frac{e^{-\lambda} \lambda^x}{x!}$

6) $P_i(0) = 0.0497$ initial

$P_f(0) = 0.13533$ final

A = ~~reduced parameter~~ drug ~~successful~~ beneficial

$P(A) = 0.75$

$P(A \mid \text{no cold})$

$$P(\text{beneficial}) \frac{0.75 \times (0.13533)}{0.75 \times 0.13533 + 0.25 \times 0.0497}$$

$P(\text{no cold}/A)$

$= 0.8909$ $P(A^c)$ $P(\text{no cold}/A^c)$

7) 300 d.

in 2% area average number of errors = 6

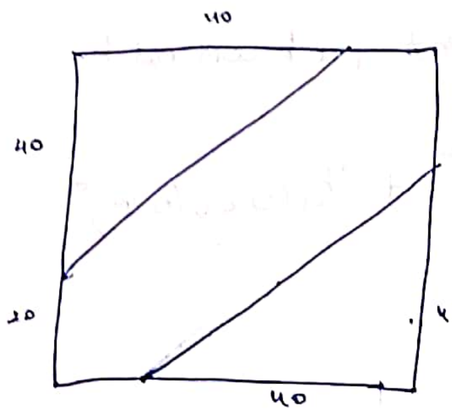
$Y \sim P(6)$

$P(Y \le 4)$

$= e^{-6} + \frac{e^{-6} \times 6}{1!} + \frac{e^{-6} \times (6)^2}{2!} + \frac{e^{-6}(6)^3}{3!} + \frac{e^{-6} 6^4}{4!}$

$= 0.285$

8)



$|x-y| < 20$

$40 \times 40 = 1600$

$60 \times 60 = 3600$

$\frac{3600-1600}{3600} = \frac{2000}{3600} = \frac{5}{9}$

9) $U \sim \left(\frac{3}{4}c, 2c\right)$

$\cancel{\frac{x - \frac{3}{4}c}{2c - \frac{3}{4}c} = F(x)}$

$\cancel{\frac{x - \frac{3}{4}c}{\frac{5}{4}c} = F(x)}$

$\cancel{\text{Profit} = \left(\frac{x-\frac{3}{4}c}{\frac{5}{4}c}\right)(x-c) = 0}$

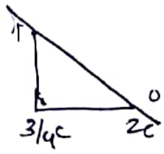
$\cancel{x^2 - \frac{7}{4}xc + \frac{3}{4}c^2}$

$\cancel{2x - \frac{7}{4}c}$

$x = \frac{7}{8}c$

$5/4c \quad P = 1$

$2c \quad P = 0$



$y - 1 = \frac{1}{5/4c}(x - 3/4c)$

$\left(\frac{3}{4}c, 1\right) \quad (2c, 0)$

$y = \frac{\left(x - \frac{3}{4}c\right)}{\frac{5}{4}c} + 1$

$y = -\frac{1}{\frac{5}{4}c}(x - 2c)$

$y = \left(\frac{2c - x}{\frac{5}{4}c}\right)($

Profit $= \left(\frac{2c-x}{\frac{5}{4}c}\right)(x - c)$

$= 1 - P(A_1^c) \cdot P(A_2^c) \cdot P(A_3^c) = 1 - p$

$\dfrac{2cx - 2c^2 - x^2 + xc}{\frac{5}{4}c} = \dfrac{3cx - x^2 - 2c^2}{\frac{5}{4}c}$

$= -2x + 3c = 0$

$x = \dfrac{3c}{2}$

maximize his profit $= \dfrac{3c}{2}$

10) 10 bulbs.

~~$\lambda = 50$ hrs.~~

$E(z) = \dfrac{1}{\lambda}$

$\lambda = \dfrac{1}{50}$

$P(z > 100) = e^{-\lambda z} = e^{-\frac{1}{50} \times 100} = e^{-2} = 0.13533$

X = bulbs are working

$P(X = 0) + P(X = 1)$

${}^{10}C_0 (0.86466)^{10} + {}^{10}C_1 (0.864664)^9 (1 - 0.864664)$

P(atmost one bulb is working) $= 0.599$

P(atleast 2 bulbs are working) $= 1 - 0.599 = 0.4$

11)

A: $\lambda = \dfrac{1}{5}$, $P(A) = \dfrac{1}{4}$

B: $\lambda = \dfrac{1}{2}$, $P(B) = 3/4$

P(atleast 5 months) $= \left(\dfrac{1}{4}\right) \times e^{-\frac{1}{5} \times 5} + \dfrac{3}{4} e^{-\frac{1}{2} \times 5}$

$= 0.1535$

12) $P(X_j < t, X_i > t \ \ i \neq j)$

$P(X_j < t) \cdot P(X_i > t)$

$\Rightarrow (1 - e^{-\lambda_j t}) \times \left(\prod_{\substack{i \\ i \neq j}}^{n} e^{-\lambda_i t}\right) \Big/ \left(1 - \prod_{i=1}^{n} e^{-\lambda_i t}\right)$

A_i - ith ~~event~~ component fails before time t

$\dfrac{P\left(A_j \cap \left(\bigcap_{\substack{i=1 \\ i \neq j}}^{n} A_i^c\right) \middle| A\right)}{P(A)}$

$= 1 - P\left(\bigcap_{i=1}^{n} A_i^c\right)$

$= 1 - \prod_{i=1}^{n} e^{-\lambda_i t}$

13)

$E(X) = \dfrac{\alpha}{\lambda} = 20$

$5\alpha = 20$

$\alpha = 4.$

$\sigma = 10$

$\sigma^2 = 100 = \dfrac{\alpha}{\lambda^2}$

$100 = \dfrac{20}{\lambda}$

$\lambda = 1/5$

$$P(X \le 15) = \int_0^{15} \frac{\left(\frac{1}{5}\right)^4 e^{-\frac{y}{5}} y^3}{\Gamma 4}\, dy$$

$$= 1 - \int_{15}^{\infty} \left(\tfrac{1}{5}\right)^4 e^{-\frac{y}{5}} y^3\, dy$$

$\frac{y}{5} = t$

$\frac{dy}{5} = dt$

$$= 1 - \int_{\cancel{15}\,3}^{\infty} \frac{\left(\frac{1}{5}\right)^4 e^{-t} \times (5)^3 t^3\, 5\, dt}{6}$$

$$= 1 - \frac{1}{6}\int_3^{\infty} e^{-t}\, t^3\, dt$$

ILATE

$$\int_3^{\infty} e^{-t} t^3\, dt$$

$$\left[\frac{t^3 e^{-t}}{-1}\right]_3^{\infty} + \int \left(\frac{e^{-t}}{+1}\right)(3t^2)\, dt$$

$$\left[\frac{e^{-t} t^3}{-1}\right]_3^{\infty} + \left[\left(\frac{e^{-t}}{-1}\right)(3t^2)\right]_3^{\infty} + \int \left(\frac{e^{-t}}{+1}\right)(6t)\, dt$$

$$27e^{-3} + 27e^{-3} + \left[\frac{e^{-t}}{-1} 6t\right]_3^{\infty} + \int_3^{\infty} \frac{e^{-t}}{+1} \times 6\, dt$$

$$54e^{-3} + 18e^{-3} + 6e^{-3} \qquad \left[\frac{6e^{-t}}{-1}\right]_3^{\infty}$$

$$54e^{-3} + 24e^{-3}$$

$$P = 1 - \frac{78e^{-3}}{6}$$

$$= 0.3527$$

14) $f_X(x) = \alpha\beta\, x^{\beta-1} e^{-\alpha x^{\beta}} \qquad \alpha, \beta, x > 0$

$0 \qquad 0 \quad x \le 0$

$$P(X \le x) = \int_{-\infty}^{x} 2\alpha x\, e^{-\alpha x^2}\, dx$$

$\alpha x^2 = y$

$2\alpha x\,dx = dy$

$$\int_{0}^{\alpha x^2} e^{-y}\,dy = \left(\frac{e^{-y}}{-1}\right)$$

$= e^{-\alpha x^2} - (1)$

$P(X \le 100) - P(X \le 90)$

$1 - e^{-\alpha(100)^2} - (1 - e^{-\alpha(90)^2})$

$e^{-\alpha(90^2)} - e^{-\alpha(100)^2}$

$P(X \le 90) = 1 - e^{-\alpha x^2}$

$= 1 - e^{-\alpha(90)^2}$

$P(X \le 100 \mid X \ge 90) = 0.15$

$$\frac{e^{-\alpha(90^2)} - e^{-\alpha(100)^2}}{e^{-90\alpha 2}} = 0.15$$

$P(X > 90) = e^{-\alpha(90)^2}$

$P(X \le 100) = 1 - e^{-\alpha(100)^2}$

$0.15\left(e^{-\alpha(90)^2}\right) = 1 - e^{-\alpha(100)^2}$

$0.15\, e^{-8100\alpha} = 1 - e^{-10000\alpha}$

$0.15\, e^{-8100\alpha} + e^{-10000\alpha} = 1$

$\alpha = \frac{1}{10^5}$

$P(X > 80) = e^{-\frac{(80)^2}{10^5}}$

$= 0.938$

$0.85\, e^{-\alpha(90)^2} = e^{-\alpha(100)^2}$

$0.85 = \frac{e^{-\alpha(10000)}}{e^{-\alpha(8100)}}$

$0.85 = e^{-\alpha(1900)}$

$\log 0.85 = -\alpha(1900)$

$\alpha = 8.55 \times 10^{-5}$

15) $P(X > 80)$

$= e^{-(8.55)(10^{-5})(80^2)}$

$= 0.578.$

45

45 - 60

60 - 75

75+

16) $\sigma = 0.005\text{cm}$

$Var = \sigma^2 = (0.005)^2 =$

$X \sim N(3, (0.005)^2)$

P(ball bearing to be scrapped) $: 1 - P(2.99 < X < 3.01)$

$X = 3 + (0.05)z$

$3.01 - 3 = 0.05\, z$

$z = 2$

$P(z > 2) = (P \ldots\; 0.4772 = 0.0228$

On both sides $= 0.0228 \times 2 = 0.0456 = P(\text{scraped})$

17)

$P(\text{defective}) = 1 - P(0.895 < x < 0.905)$

$P(z<1.66) - P(z<-1.66)$

$\phi(1.66) - 1 + \phi(1.66)$

$2(\phi(1.66)) - 1$

$X = \mu + \sigma Z$

$0.905 = 0.9 + 0.003z$

$z = 1.667$

$0.5 - 0.4525$

$= 0.0475$ on one side

$P(\text{defective}) = 0.095$

$\% = 9.5\%$

$P = 0.01$

$0.5 - 0.05$

$= 0.45$ $\quad Z = \cancel{1.65}\ 2.57$

$P(0.895 < X < 0.905) \geq 0.99$

$2\phi\left(\frac{0.05}{\sigma}\right) - 1 \geq 0.99$

$\phi\left(\frac{0.05}{\sigma}\right) \geq 0.995$

$0.9905 = 0.9 + (1.65)(\sigma)$

$0.005 = (2.57)\sigma$

$\sigma \leq 0.0194$

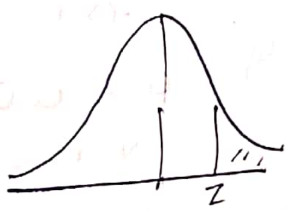
$\frac{0.05}{\sigma} \geq 2.58$

$\sigma \leq 0.194.$

0.005

18)

$P(X > x) = 0.95$



$0.5 - 0.05$
$= 0.45$

$P(X < x) = 0.05$

$X = 200 + Z(10)$

$X = 200 + -1.65 \times 10$

$x = 183.5$ cm

$P(X > x) = 0.1$

$0.5 - 0.1 = 0.4$

$P(x \leq x) = 0.9$

$z = 1.29 \quad x = 200 + (1.29)(10) = 212.9$

19) $N(74, 62.41)$

$P(X<x) = 0.1$

$P(X<x) = 0.9$ $\quad Z = 1.29$

$\quad Z = -1.29$

$X = 74 + (\overset{7.9}{\cancel{62.41}}) Z$

$= 63.809 = 64$

b) $P(X<x) = 0.95$

$X = 74 + (1.65)(7.9)$

$= 87$

c) $P(X<x) = 0.65$

$X = 74 + (0.39)(7.9)$

$X = 77$

15) $P(\cancel{X \leq \text{distinction}}) = 0.95$

$X \leq 75$

$75 = \mu + 1.65\sigma$

$45 = \mu - 1.28\sigma$

$\mu = 58.1$ $\quad \sigma = 10.238$

$60 = 58.105 + Z(10.238)$

$Z = 0.185$

$0.57535 - 0.10$

$P(\text{II nd class}) = 0.47$

$P(\text{1st class}) = 75 = 58.105 + (10.238)Z$

$Z = 1.65$

$: 0.950$

$= 0.38$

20)

Profit $= C_0 \, P(6<X<8) - C_1 P(X<6) - C_2 P(X>8)$

$= C_0 (\phi(8-\mu) - \phi(6-\mu)) - C_1 \phi(6-\mu) - C_2 \phi(\mu-8)$

$0 = C_0 e^{-\frac{(8-\mu)^2}{2}}(-1) + C_0 e^{-\frac{(6-\mu)^2}{2}} + C_1 e^{-\frac{(6-\mu)^2}{2}} - C_2 e^{-\frac{(\mu-8)^2}{2}}$

$$(C_0+C_2)e^{-\frac{(8-\mu)^2}{2}} = (C_0+C_1)\, e^{-\frac{(6-\mu)^2}{2}}$$

$$e^{\frac{(6-\mu)^2}{2}-\frac{(8-\mu)^2}{2}} = \frac{C_1+C_0}{C_0+C_2}$$

$$= 2\ln\left(\frac{C_1+C_0}{C_0+C_2}\right)$$

$$\begin{array}{r} 64 \\ 36 \\ \hline 28 \end{array}$$

$$(36-12\mu+\mu^2) - (64-16\mu+\mu^2)$$

$$4\mu - 28 = 2\ln\left(\frac{C_1+C_0}{C_0+C_2}\right)$$

$$\mu = 7+\frac{1}{2}\ln\left(\frac{C_1+C_0}{C_0+C_2}\right)$$

$$P(Y>2.7) = P(\ln Y > 0.9933) = P(Z>1.93)$$

$$\sigma Z + \mu = \log Y$$

$$Z = 1.93 \qquad 0.5-0.4732$$

$$P(Z>1.93) = 0.5 - 0.4732$$

$$= 0.0268$$

$$P(0.8-c<\ln Y < 0.8+c) = 0.95$$

$0.8+c$

$$P(-10c < Z < 10c) = 0.95$$

$$P(Z<10c) - P(Z<-10c) = 0.95$$

$$P(Z<10c) - (1-P(Z<10c)) = 0.95$$

$$2P(Z<10c) - 1 = 0.95$$

$$P(Z<10c) = 0.975$$

$$10c = \cancel{2.58}\ 1.96$$

$$c = \cancel{0.258}\ 0.196$$

$$1.829 < Y < 2.707$$

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