Problem set 5

Spring 2018

MATHEMATICS-II (MA10002)(Numerical Analysis)

1. Find $f(0.05)$ using the Newton's forward difference formula from the given table:

x	0	0.1	0.2	0.3	0.4
$f(x)$	1	1.2214	1.4918	1.8221	2.2255

2. Using Newton's forward difference formula find $f(1.5)$ from the given table

x	0	2	4	6	8
$f(x)$	-1	13	43	89	151

3. Given:

x	2.0	2.2	2.4	2.6	2.8	3.0
$f(x) = \log_{10} x$	0.30103	0.34242	0.38021	0.41497	0.44716	0.47721

   Find the value of $\log_{10} 2.91$ using Newton's backward difference formula.

4. Find the value of $f(1.45)$ using Newton's backward difference formula.

x	1.0	1.1	1.2	1.3	1.4	1.5
$f(x)$	0.24197	0.21785	0.19419	0.17137	0.14973	0.12952

5. In an examination the number of candidates who secured marks between certain limit were as follows:

Marks	0-19	20-39	40-59	60-89	80-99
No. of candidates	41	62	65	50	17

   Estimate the number of candidates getting marks less than 70.

6. A certain function f, defined on the interval $(0, 1)$ is such that $f(0) = 0, f(1/2) = -1, f(1) = 0$. Find the quadratic polynomial $p(x)$ which agrees with $f(x)$ for $x = 0, 1/2, 1$.
   If $|\frac{d^3f}{dx^3}| \leq 1$ for $0 \leq x \leq 1$. Show that $|f(x) - p(x)| \leq \frac{1}{12}$ for $0 \leq x \leq 1$.

7. Show that the sum of Lagrangian functions or coefficients is unity, i.e., $\sum_{r=0}^{n} w_r(x) = 1$.

8. Use Lagrange's formula to find the value of y when $x = 102$, from the given data:

x	93	96.2	100	104.2	108.7
$y = f(x)$	11.38	12.80	14.70	17.07	19.91

9. Find by Lagrange's formula the interpolation polynomial which corresponds to the following data:

x	-1	0	2	5
$f(x)$	9	5	3	15

10. Evaluate $\int_0^1 (4x - 3x^2)dx$, taking ten equal intervals, by (i)trapezoidal rule, (ii)Simpson's one-third rule. Compute the exact value and find the errors in your result.

11. Evaluate $\int_0^1 \frac{1}{1+x^2}dx$, by (i) trapezoidal rule and (ii) Simpson's one-third rule taking six equal intervals, correct up to three decimal places and find the errors in both the methods.

12. Find the value of $\int_0^{\pi/2} e^{\sin x}dx$, by (i) trapezoidal rule and (ii) Simpson's one-third rule taking $h = \frac{\pi}{12}$, correct up to five decimal places.

13. Find the value of $\int_0^1 \cos x dx$, taking five equal intervals. Explain the reason behind your choice of the integration formula used.

14. Find the value of $\int_0^1 e^{-x^2}dx$, taking 10 equal intervals by Simpson's one-third method and estimate the error.

15. Find the Lagrange's interpolating polynomial of degree 2, approximating the function $f(x) = \ln x$ defined by the following table of values.

x	2	2.5	3.0
$f(x)$	0.69315	0.91629	1.09861

Hence determine the value of $y = \ln(2.7)$. Also estimate the error in the value of y.

16. Let $f(x) = \ln(1 + x)$, $x_0 = 1$ and $x_1 = 1.1$. Use linear Lagrange interpolation to calculate an approximate value of $f(1.04)$ and obtain an upper bound on the error.

17. Determine the appropriate step size to use, in the construction of a table of $f(x) = (1 + x)^6$ on [0, 1]. The error for linear interpolation is to be bounded by 5×10^{-5}.

18. a) Show that the error of quadratic interpolation in an equidistance table is bounded by: $(\frac{h^3}{9\sqrt{3}}) \max|f'''(\xi)|$.

    b) If we want to set up an equidistance table of $f(x) = x^2 \ln(x)$ in the interval [5, 10], evaluate the step size h which is to be used to yield a total error less than 10^{-5} on quadratic Lagrange interpolation in this table.