INDIAN INSTITUTE OF TECHNOLOGY

DATE 19/08/16 | EXPERIMENT No. 2 | SHEET NO 12

OBJECTIVE:-

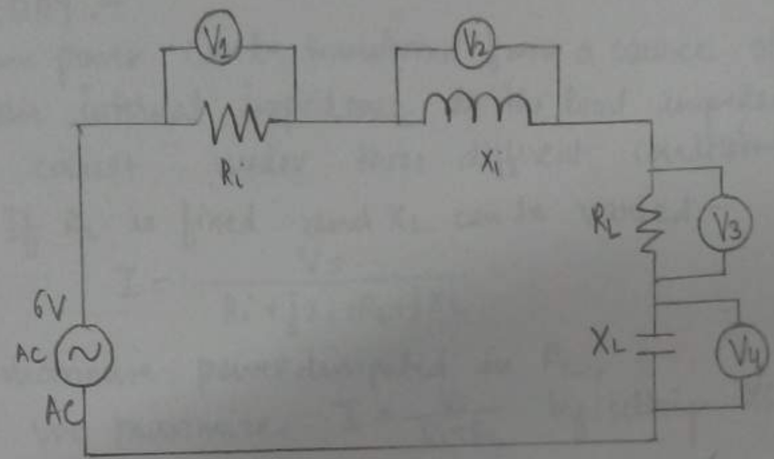
Verification of Maximum power theorem and Reciprocity theory.
(I) Verification of Maximum power theorem when the resistance and reactance of the load can be varied individually and seperated.

APPARATUS REQUIRED:-

S.No.	Apparatus Name	Quantity	Range/ Specifications
1.	A C Source	1	8V
2.	Resistors	2	100-200Ω
3.	Inductor	1	248mH.
4.	Variable Capacitor (Capacitance Box)	1	upto 20μF
5.	Capacitors	1	2μF
		1	8μF
		1	10μF
		1	21.6μF.
6.	RLC- Multimeter	1	
7.	Multimeter	1	
8.	Connecting Wires		
9.	Rheostats	2	0-85Ω
		3	0-185Ω
		2	0-1180Ω.

NEEDS

CIRCUIT DIAGRAM



THEORY:-

Maximum power can be transferred from a source of given voltage and an internal impedence to the load impedence Z_L in the circuit under three different conditions:-

(a) If R_L is fixed and X_L can be varied:-

$$I = \frac{V_s}{R_i + jX_i + R_L + jX_L}$$

To maximise power dissipated in R_L,
we maximize $I = \frac{V_s}{R_i + R_L}$ by setting $X_i = -X_L$

(b) If X_L is fixed and R_L can be varied:-

$$P = |I^2| R_L = \frac{V_s^2 R_L}{(R_i+R_L)^2 + (X_i+X_L)^2}$$

At max power

$$\frac{dP}{dR_L} = 0 \Rightarrow \frac{-1}{P^2}\frac{dP}{dR_L} = 0 \Rightarrow \frac{dP^{-1}}{dR_L} = 0$$

Hence, $\frac{d}{dR_L}\left\{\frac{(R_i+R_L)^2 + (X_i+X_L)^2}{V_s^2 * R_L}\right\} = 0$

$$\Rightarrow \frac{1}{V_s^2}\left\{\frac{-R_i^2}{R_L^2} + \cancel{2R_i} + 1\right\} + \frac{(X_i+X_L)^2}{V_s^2}\left(-\frac{1}{R_L^2}\right) = 0$$

$$\Rightarrow R_L = \sqrt{R_i^2 + (X_i+X_L)^2}$$

(c) If both R_L and X_L are adjustable:-
Then both the above solutions are simultaneously valid.

Hence, $X_i = -X_L$ and $R_L = R_i$

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OBSERVATIONS:-

- $L = 248\ mH$
- $R_{in} = 60.3\ \Omega$
- $R_L = 100.2\ \Omega$
- Expected value of $C = \frac{1}{4\pi^2 f^2 L} = 40.86\ \mu F$.

- $V_{in}(pp) / V_S = 8.81V$.

- $C_1 = 8.35\mu F$
- $C_2 = 10.2\mu F$
- $C_3 = 2.1\mu F$
- $C_4 = 20.2\mu F$.

TABLE-I

Sl. No	C (μF)	V_1 (V)	V_3 (V)	$V_1 V_3$ (Volt²)	Max. Value (V_1V_3) (Volt²)
1.	10.3	2.158	3.666	~~9.233~~ 7.911	
2.	11.4	2.271	3.851	8.746	
3.	12.4	2.360	4.000	9.44	
4.	13.5	2.420	4.100	9.922	
5.	14.6	2.448	4.153	~~35.74~~ 10.166	
6.	15.6	2.452	4.167	10.217	
7.	16.7	2.458	4.168	10.245	
8.	17.7 ~~2.440~~	2.440	4.127	10.069	10.24 Volt². at $C = 16.7\mu F$. $X_C = 190.60\ \Omega$
9.	18.7	2.449	4.102	10.025	
10.	19.8	2.421	4.091	9.904	
11.	20.8	2.408	4.067	9.793	
12.	21.8	2.38	4.06	9.663	
13.	28.7	2.255	3.821	8.616	
14.	29.7	2.226	3.790	8.436	

error calculation?

P.R.E.

Topic : Graph-I. (variable C, fixed R) Date :

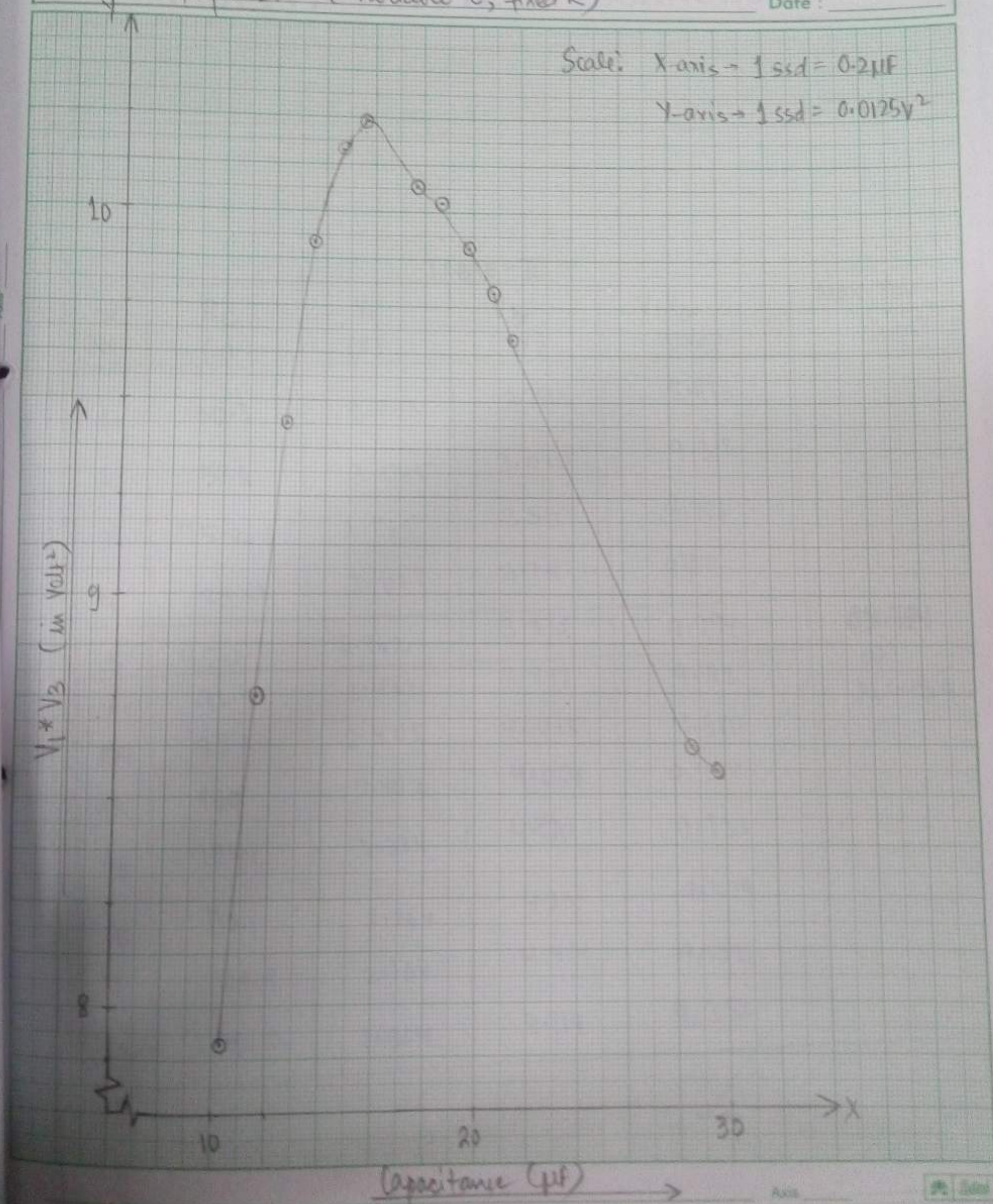

TABLE-II

Sl. No.	R_L (Ω)	V_1 (V)	V_3 (V)	$V_1 V_3$ ($Volt^2$)	Max. Value (V_1V_3) ($Volt^2$)
1.	50.3	3.263	2.750	8.973	
2.	59.8	3.045	3.060	9.318	
3.	75.2	2.763	3.513	9.706	
4.	85.0	2.607	3.741	9.750	
5.	95.0	2.462	3.978	9.794	09.794 at $R_L = 95\Omega$.
6.	96.5	2.442	3.977	9.712	
07	98.3	2.405	4.025	9.680	
08.	99.9	2.402	4.054	9.738	
09	105.1	2.334	4.170	9.733	
10.	110.1	2.264	4.231	9.579	
11.	119.9	2.164	4.414	9.552	
12.	140.0	1.984	4.714	9.352	

Topic : Graph-2 (variable R, fixed C) Date :

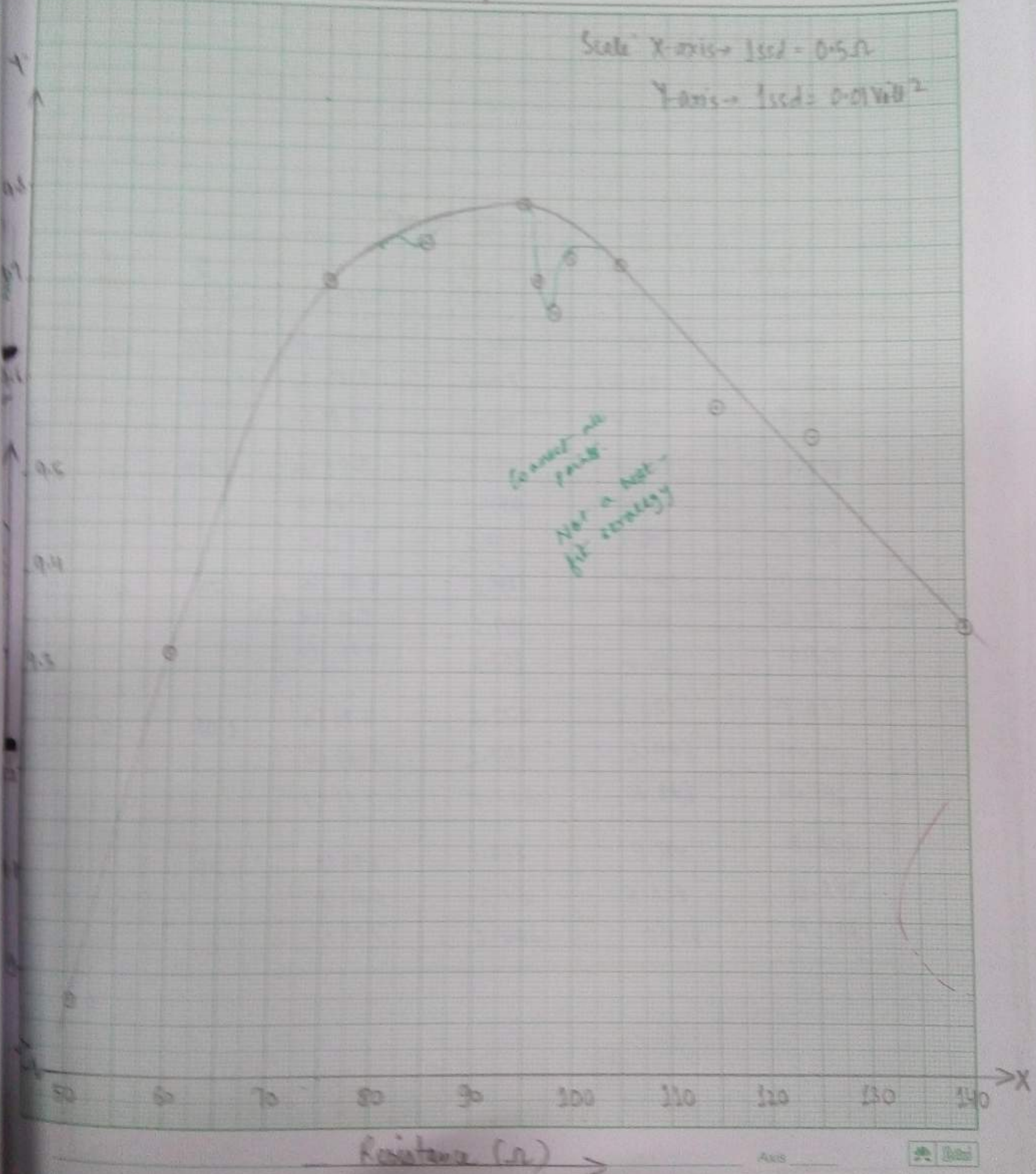

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TABLE III

S. No.	R (Ω)	C (μF)	V_1 (V)	V_3 (V)	$V_1 V_3$ (Volt^2)
1.	1022.2	a) 19.8	2.382	4.145	9.873
		b) 16.7	2.427	4.218	10.237
		c) 13.5	2.370	4.136	9.802
2.	977.9	a) 19.8	2.458	4.097	10.070
		b) 16.7	2.498	4.155	10.379
		c) 13.5	2.471	4.097	10.124
3.	935.1	a) 19.8	2.488	4.025	10.014
		b) 16.7	2.544	4.104	10.441 ← V_1V_3 max
		c) 13.5	2.503	4.022	10.067
4.	900.1	a) 19.8	2.555	3.883	9.921
		b) 16.7	2.588	3.957	10.241
		c) 13.5	2.536	3.863	9.796
5.	855.1	a) 19.8	2.633	3.784	9.963
		b) 16.7	2.672	3.854	10.298
		c) 13.5	2.606	3.749	9.769

P.R.E.

DISCUSSION :-

* Verification of Maximum power theorem:-

1. Sources of Error.

* Contact resistance at joints and multimeter probes.
* AC resistor of inductor (due to skin effect) was not added to the dc resistance we measured.
* ~~Loose~~ connections.
* Measurement of inductor resistance:- We passed a dc current and measured the resistance across the inductor. However, the method is not correct because ac circuit resistance differs from dc resistance because of proximity and skin effect, which are frequency dependent.

2. Precautions:

* The inductor should be kept at maximum distance from other circuit components as it may effect the power source or capacitance, resistance.

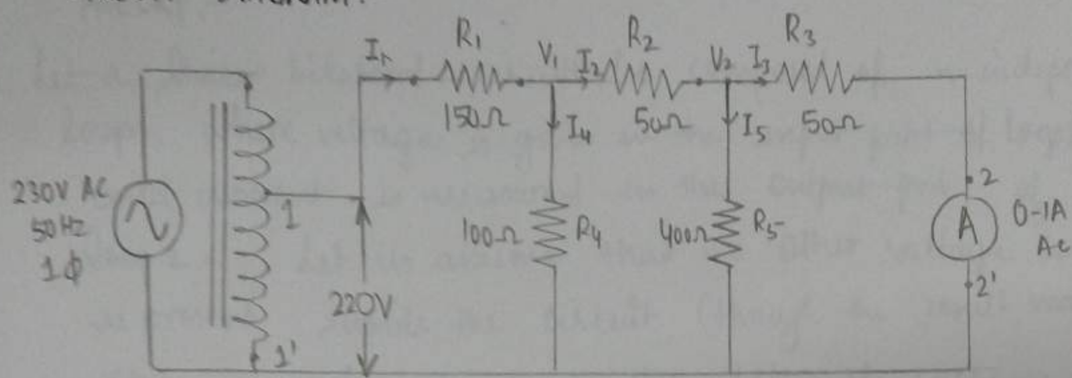
(II) AIM:-

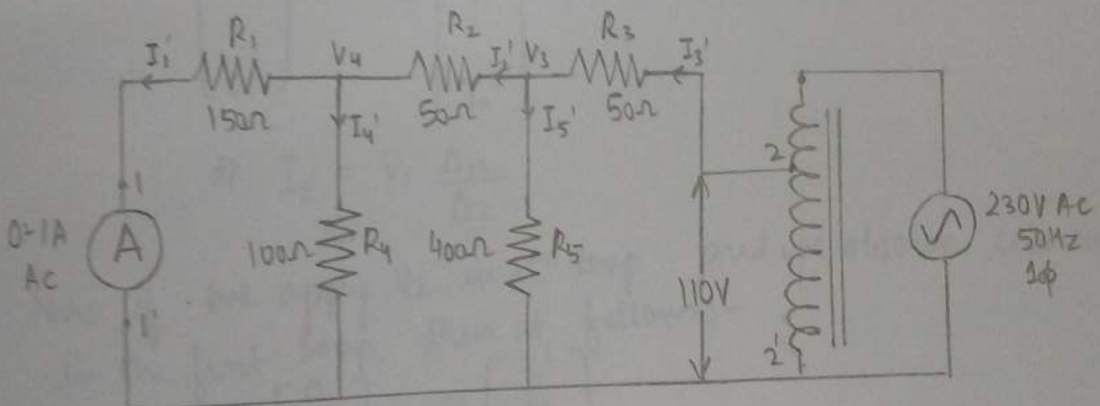
Verification of Reciprocity Theorem.

APPARATUS REQUIRED :-

S.No.	Apparatus Name	Quantity	Range / Specifications
1.	Voltage Supply (AC)	1	
2.	Auto Transformer	1	270V-3A.
3.	Resistor (from Rheostat)	2	50Ω
		1	100Ω
		1	150Ω
		1	400Ω
4.	Ammeter	1	AC(MI)
5.	Multimeter/Voltmeter	1.	

CIRCUIT DIAGRAM :-





THEORY:-

Let a linear bilateral circuit be composed of n independent loops, where voltage is given in the input port of loop1 and current is measured in the output port of loop2. Let us assume that no other voltage source is present inside the circuit (though the result would remain unchanged, and the additional voltages can be taken care of by superposition.) Hence, it follows:-

$$\begin{bmatrix} V_1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = Z \begin{bmatrix} I_1 \\ I_2' \\ \vdots \\ I_n' \end{bmatrix}$$

$$\Rightarrow I_2' = V_1 \frac{\Delta_{12}}{\Delta Z}$$

Now if we apply V_2 in 2^{nd} loop and we observe current I_1 in the first loop, then it follows:-

$$\begin{bmatrix} 0 \\ V_2 \\ \vdots \\ 0 \end{bmatrix} = Z \begin{bmatrix} I_1' \\ I_2 \\ \vdots \\ I_n \end{bmatrix}$$

$$\Rightarrow I_1' = V_2 \frac{\Delta_{21}}{\Delta Z}$$

Comparing, $\frac{I_2'}{V_1} = \frac{I_1'}{V_2}$. Thus the ratio of the response to the excitation is invariant to an interchange in position of excitation and response.

OBSERVATIONS:

V_s (V)	I_3 (A)	V_s/I_3 (V/A)	V_s' (V)	I_1' (A)	V_s'/I_1' (V/A)
201.3	0.485	415.05	100.2	0.25	400.8

$$\frac{V_s}{I_3} \simeq \frac{V_s'}{I_1'}$$

Percentage error

$$= \frac{415.05 - 400.8}{415.05} \times 100 = 3.433\%$$

DISCUSSION:-

* Verification of Reciprosity Theorem.

→ Sources of Error:-

* The multimeters' internal resistance, which needs to be added to the circuit across 1-1' at one time & across 2-2' in the other time. However, for reciprocity theorem to be valid, the circuit must be unchanged. Only the position of voltage source and current distribution should change. Thus the result is errorneous.
* Resistance becomes large at any point of loose contact - eg. the probes of multimeter or any loose connection. Thus, unless the circuit is perfect, the distribution of resistance do not become exact.

→ Branch Analysis:-

(i) First case.

$V = 201.3V$

$$R_{eq} = 150 + (100 \| (50 + (400 \| 50)))$$

$$= 150 + (100 \| (50 + \tfrac{8}{9} \times 50))$$

$$= 150 + (100 \| (\tfrac{17}{9} \times 50))$$

$$= 150 + 340/7$$

$$= 198.57\Omega.$$

$$I_1 = \frac{V}{R_{eq}} = 1.01374A.$$

$\therefore I_2 = I_1 \times \frac{100}{100 + \frac{17}{9} \times 50}$

$= 0.521 A.$

$\therefore I_3 = I_2 \times \frac{400}{450}$

$= 0.463 A. \rightarrow$ Theoritical

Practically Measured $I_3 = 0.485 A.$

$\therefore I_3$ (theoritical) $\approx I_3$ (practical).

(ii) Second case

$V = 101.2V$

$R_{eq} = (((150 \| 100) + 50) \| 400) + 50$

$= (110 \| 400) + 50$

$= 86.274 + 50$

$= 136.274.$

$\therefore I_3' = \frac{V}{R_{eq}} = 0.74262 A.$

$\therefore I_2' = I_3' \times \frac{400}{510} = 0.58244 A.$

$\therefore I_1' = I_2' \times \frac{100}{250} = 0.233 A$ (Theoretical)

I_1' (practical) $= 0.250 A.$

$\therefore I_1'$(practical) $\approx I_1'$(theoretical).

2/9

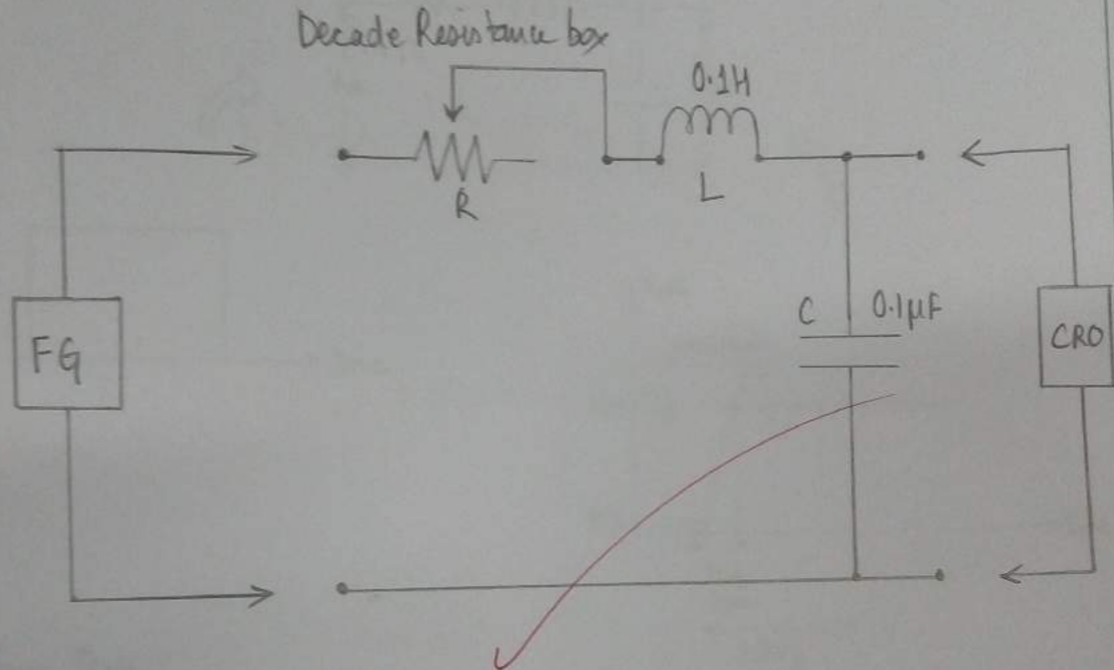
OBJECTIVE:-

To study Transient and frequence response of R-L-C series circuit.

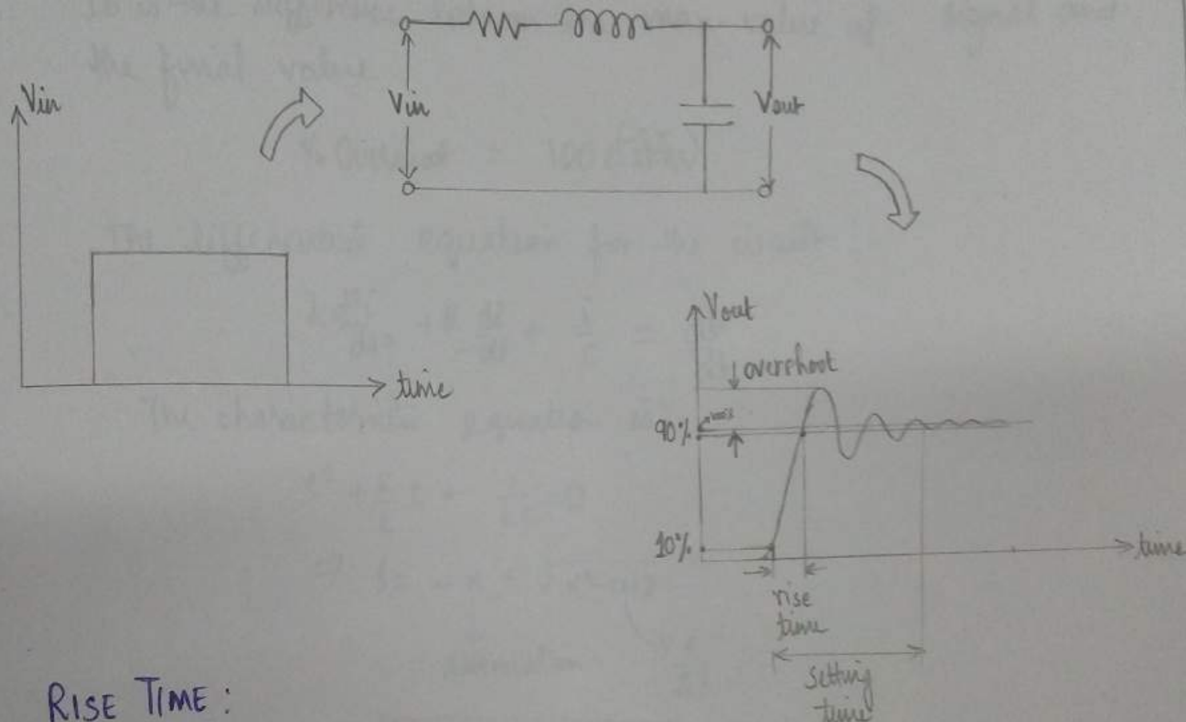
APPARATUS REQUIRED:-

S.No.	APPARATUS	QUANTITY	SPECIFICATIONS.
1.	FUNCTION GENERATOR	1	-
2.	Decade Resistance Box	1	Range: 2k Ω
3.	Inductor	1	~~[illegible]~~
4.	Capacitor	1	
5.	Cathode Ray Oscilloscope	1	

CIRCUIT DIAGRAM:-



THEORY :-



RISE TIME :

It is the time taken by the signal to reach from 10% to 90% of of its final value

$$\text{Rise time} \quad t_r = \frac{1}{\omega_0\sqrt{1-\xi^2}}\left(\pi - \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}\right)$$

SETTING TIME :

It is the time taken by the signal to reach from 10% of its final value till it settles within 2% error band of it's final value.

$$t_s = -\frac{\ln(\%\text{ of error band})}{\xi\omega_0}$$

OVERSHOOT :

It is the difference between the max. value of signal and the final value.

$$\% \text{ Overshoot} = 100\, e^{\left(\frac{-\xi\pi}{\sqrt{1-\xi^2}}\right)}$$

The differential equation for the circuit :-

$$L\frac{d^2 i}{dt^2} + R\frac{di}{dt} + \frac{i}{C} = \frac{dv}{dt}$$

The characteristic equation is :-

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

$$\Rightarrow s = \underbrace{-\alpha}_{\text{attenuation}} \pm \sqrt{\alpha^2 - \omega_0^2}$$

$$\omega_0 \to \frac{1}{\sqrt{LC}}$$

$$\Rightarrow \boxed{\xi = \frac{\alpha}{\omega_0} = \frac{R}{2}\sqrt{\frac{C}{L}}}$$

GAIN :

$$\frac{V_c(j\omega)}{V_s(j\omega)} = \frac{1/j\omega C}{R + j\omega L + 1/j\omega C} = \frac{\omega_0^2}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4\xi^2\omega_0^2\omega^2}} \angle\phi_c$$

$$\text{where, } \tan\phi_c = \frac{-2\xi\omega_0\omega}{\omega_0^2 - \omega^2}$$

OVERSHOOT :

It is the difference between the max. value of signal and the final value.

$$\% \text{ Overshoot} = 100 e^{\left(\frac{-\xi\pi}{\sqrt{1-\xi^2}}\right)}$$

The differential equation for the circuit :-

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{i}{C} = \frac{dv}{dt}$$

The characteristic equation is :-

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

$$\Rightarrow s = \underbrace{-\alpha}_{\text{attenuation}} \pm \sqrt{\alpha^2 - \underbrace{\omega_0^2}_{\frac{1}{\sqrt{LC}}}}$$

$$\Rightarrow \boxed{\xi = \frac{\alpha}{\omega_0} = \frac{R}{2}\sqrt{\frac{C}{L}}}$$

GAIN :

$$\frac{V_c(j\omega)}{V_s(j\omega)} = \frac{1/j\omega C}{R + j\omega L + 1/j\omega C} = \frac{\omega_0^2}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4\xi^2\omega_0^2\omega^2}} \angle \phi_c$$

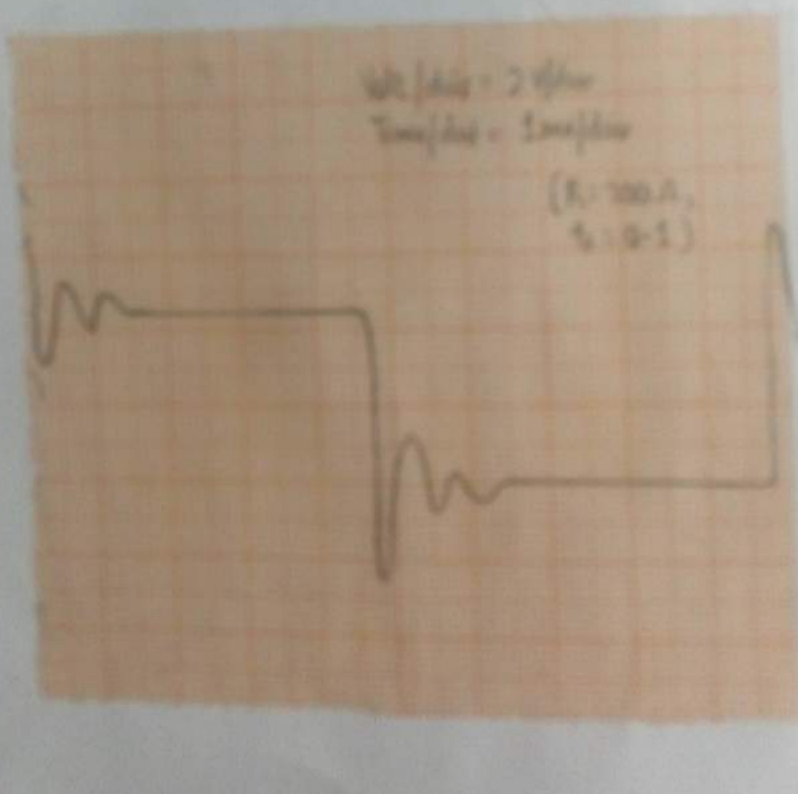
$$\text{where, } \tan \phi_c = \frac{-2\xi\omega_0\omega}{\omega_0^2 - \omega^2}$$

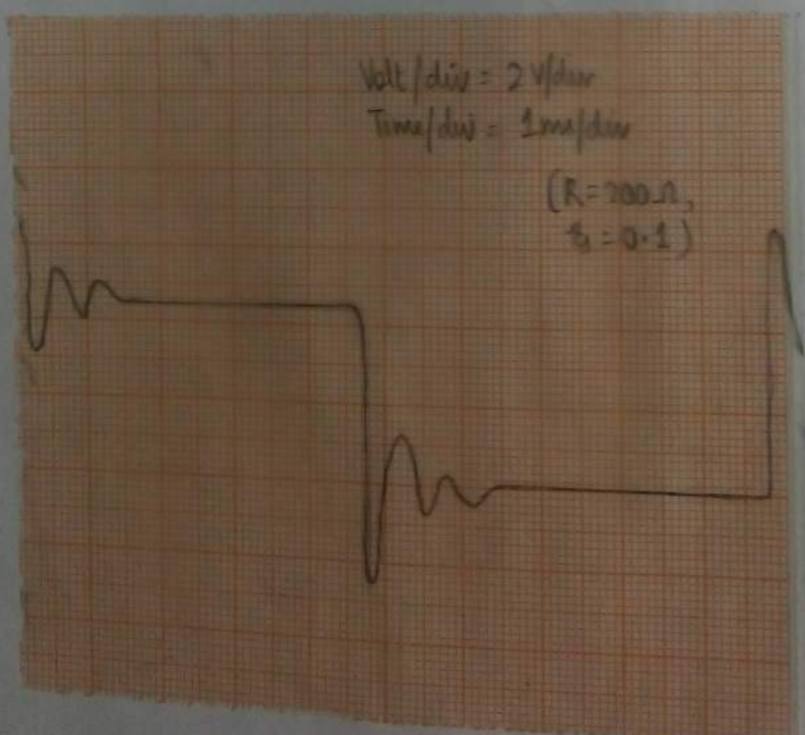
$$\frac{V_R(j\omega)}{V_S(j\omega)} = \frac{2\xi\omega_0\omega}{\sqrt{(\omega_0^2-\omega^2)^2+4\xi\omega_0^2\omega^2}} \angle\phi_R.$$

where $\phi_R = \frac{\pi}{2} - \tan^{-1}\left(\frac{2\xi\omega\omega_0}{\omega_0^2-\omega^2}\right)$.

$$\frac{V_L(j\omega)}{V_S(j\omega)} = \frac{\omega^2}{(\omega_0^2-\omega^2)^2+4\xi^2\omega^2\omega_0^2} \angle\phi_L.$$

Where $\phi_L = \pi - \tan^{-1}\left(\frac{2\xi\omega\omega_0}{\omega_0^2-\omega^2}\right)$


Volt/div = 2 V/div
Time/div = 1ms/div
(R = 100 Ω,
ζ = 0.1)



Volt/div = 2 V/div
Time/div = 1ms/div
(R = 200 Ω,
ξ = 0.1)

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OBSERVATIONS:-

C = 0.1μF , L = 0.1H

V_{input} = 2.06 V.

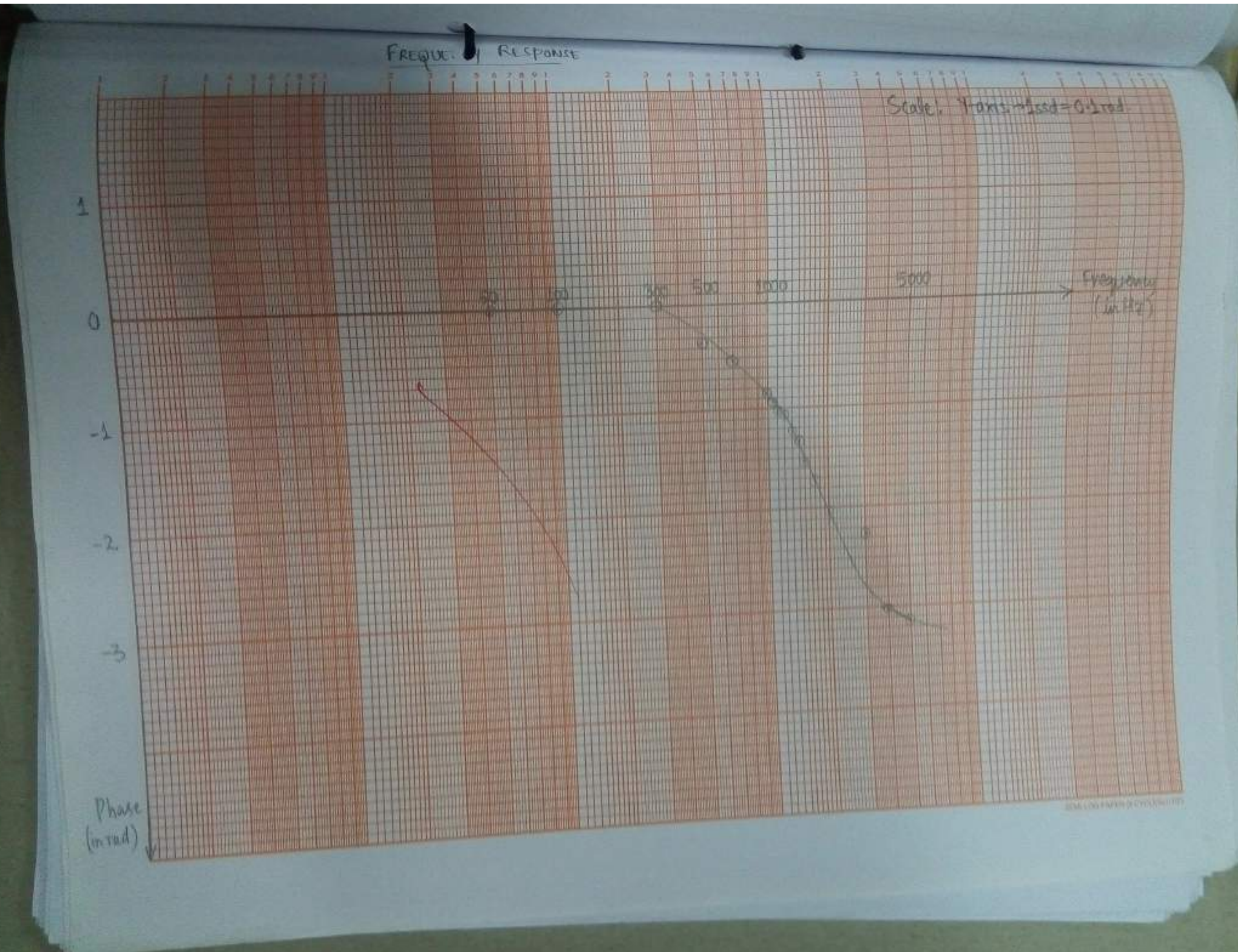
S.No.	ξ	R (in Ω)	V_{output} (in V)	Overshoot (in V)
1.	0.1	200	4.4	2.60
2.	0.2	400	4.4	1.80
3	0.3	600	4.4	1.20
4.	0.5	1000	4.4	0.45
5.	0.7	1400	4.4	0.08
6	0.8	1600	4.4	0.00.

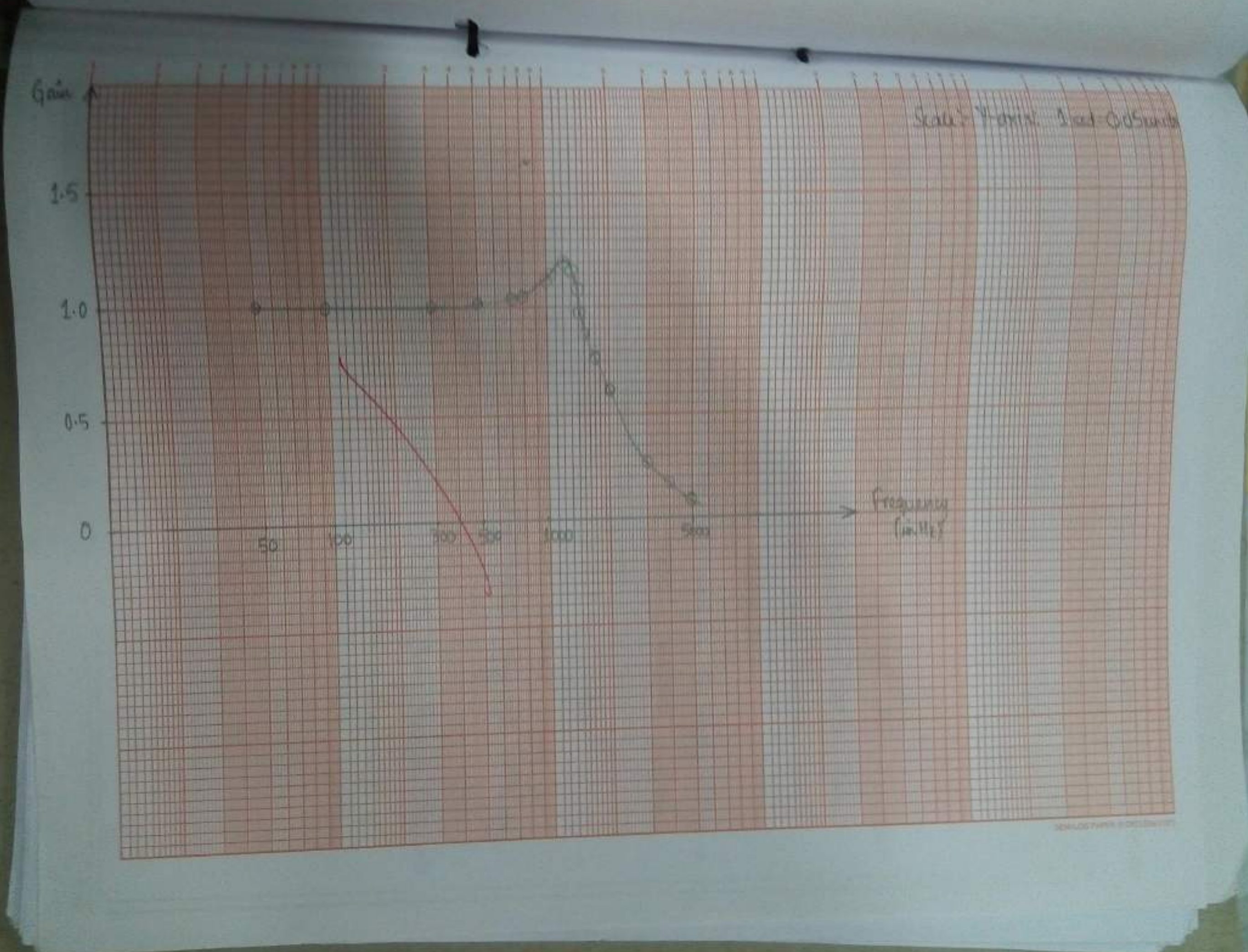
S.No	ξ	Theoritical % Overshoot	Actual % Overshoot
1.	0.1	72.92	59.09
2.	0.2	52.66	40.91
3.	0.3	37.23	27.27
4.	0.5	16.30	10.23
5.	0.7	4.60	1.82
6.	0.8	1.18	0

FREQUENCY RESPONSE

$\xi = 0.5$, $V_{in} = 4.6V$.

S.No.	Frequency (in Hz)	Vc (in V) (Vout)	Gain	Phase (in rad).
1.	50	4.6	1	0
2.	100	4.6	1	0
3.	300	4.6	1	0
4.	500	4.7	1.02	-0.377
5.	700	4.8	1.04	-0.528
6.	1000	5.2	1.13	-0.880
7.	1050	5.3	1.15	-0.923
8.	1100	5.4	1.17	-1.037
9.	1150	5.5	1.19	-1.084
10.	1200	5.4	1.17	-1.056
11.	1300	5.0	1.09	-1.225
12.	1400	4.5	0.98	-1.319
13.	1500	4.0	0.87	-1.508
14.	1700	3.5	0.76	-1.709
15.	2000	2.8	0.61	-2.011
16.	3000	1.2	0.26	-2.262
17.	4000	0.8	0.17	-3.016
18.	5000	0.4	0.09	-3.142


FREQUE RESPONSE
Scale: Y axis: 1 sqd = 0.1 rad
Frequency (in Hz)
Phase (in rad)
1
0
-1
-2
-3
50
100
300
500
1000
5000



Gain
Scale: Y-axis: 1cm = 0.05units
1.5
1.0
0.5
0
50
100
300
500
1000
5000
Frequency (in Hz)

DISCUSSION :

* In the percentage overshoot part of the experiment, the theoritical and practical values may not have matched due to loss of precision in the instruments which might have been caused due to loose connections or user mishandlings.
* The voltage across capacitor starts increasing from the supply voltage as we start from low frequency. It reaches a maxima and then asymptotically decreases to zero.

  The theoritical value of maxima is 115.47 % gain.

  The theoritical value of freq. at which maxima occurs is $\omega_n\sqrt{1-2\xi^2}$ = 1125.34 Hz and it tallies very well with the experimental values.

  The phase angle decreases from 0 to $-\pi$ going through $-\pi/2$ at ω_n (natural freq.). Experimentally we obtain a phase angle of -1.5 radians & -1.7 radians at 1300 Hz & 1400 Hz respectively.

  This confirms the predictions.

DISCUSSION :

* In the percentage overshoot part of the experiment, the theoritical and practical values may not have matched due to loss of precision in the instruments which might have been caused due to loose connections or user mishandlings
* The voltage across capacitor starts increasing from the supply voltage as we start from low frequency. it reaches a maxima and then asymptotically decreases to zero.

The theoritical value of maxima is 115.47 % gain.

The theoritical value of freq. at which maxima occurs is $\omega_0\sqrt{1-2\zeta^2}$ = 1125.39 Hz and it tallies very well with the experimental values.

The phase angle decreases from 0 to $-\pi$ going through $-\pi/2$ at ω_0 (Natural freq.). Experimentally we obtain a phase angle of -1.5 radians & -1.7 radians at 1500 Hz & 1100 Hz respectively.

This confirms the predictions.

28/9/16