DISCRETE STRUCTURES (CS21001)
Tutorial-6

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Q1. A vending machine dispensing books of stamps accepts only dollar coins, \$1 bills, and \$5 bills.

a) Find a recurrence relation for the number of ways to deposit *n* dollars in the vending machine, where the order in which the coins and bills are deposited matters.

b) What are the initial conditions?

c) How many ways are there to deposit \$10 for a book of stamps?

Q2. a) Find a recurrence relation for the number of ternary strings that do not contain two consecutive 0s or two consecutive 1s.

b) What are the initial conditions?

c) How many ternary strings of length six do not contain two consecutive 0s or two consecutive 1s?

<u>Note</u>: A string that contains only 0s, 1s, and 2s is called a ternary string.

Q3. a) Find a recurrence relation for the number of strictly increasing sequences of positive integers that have 1 as their first term and n as their last term, where n is a positive integer. That is, sequences $a_1, a_2, \ldots, a_k$, where $a_1 = 1$, $a_k = n$, and $a_j < a_{j+1}$ for $j = 1, 2, \ldots, k - 1$.

b) What are the initial conditions?

c) How many sequences of the type described in (a) are there when n is a positive integer with $n \geq 2$?

Q4. In how many ways can a $2 \times n$ rectangular checkerboard be tiled using 1 x 2 and 2 x 2 pieces?

Q5. Find all solutions of the recurrence relation $a_n = 6a_{n-1} - 9a_{n-2} + 2^n + 3n$ with $a_0=1$ and $a_1=2$.

HOME-WORK

Q6. a) Find a recurrence relation for the number of bit strings of length *n* that contain the string 01.

b) What are the initial conditions?

c) How many bit strings of length seven contain the string 01?

Q7. a) Find the recurrence relation satisfied by R_n , where R_n is the number of regions that a plane is divided into by n lines, if no two of the lines are parallel and no three of the lines go through the same point.

b) Find R_n using iteration.

Q8. a) Find a recurrence relation for the number of ways to completely cover a $2 \times n$ checkerboard with 1×2 dominoes.

b) What are the initial conditions for the recurrence relation in part (a)?

c) How many ways are there to completely cover a 2×17 checkerboard with 1×2 dominoes?

Q9. Messages are transmitted over a communications channel using two signals. The transmittal of one signal requires 1 microsecond, and the transmittal of the other signal requires 2 microseconds.

a) Find a recurrence relation for the number of different messages consisting of sequences of these two signals, where each signal in the message is immediately followed by the next signal, that can be sent in n microseconds.

b) What are the initial conditions?

c) How many different messages can be sent in 10 microseconds using these two signals?

Q10. Solve the recurrence relation

$$T(n) = n\,T^2(n/2)$$

with initial condition $T(1) = 6$.

[Hint: Let $n = 2^k$ and then make the substitution $a_k = \log T(2^k)$ to obtain a linear nonhomogeneous recurrence relation.]

Thank You