

Answer sheet - 2

SPRING 2018

MATHEMATICS-II (MA10002)(Linear Algebra)

1. Ans: (a) and (c) form a basis.
2. (a) Ans: Basis : $\{(-5, 1, 3)\}$, $\dim U=1$.
(b) Ans: Basis : $\{(1, 0, 0, 0, -1), (0, 1, 1, 1, 0)\}$, $\dim U=2$.
(c) Ans: Basis : $\{x, -\frac{1}{3} + x^2, x^3, -\frac{1}{5} + x^4\}$, $\dim U=4$.
3. Ans: $\dim w_1 = 3$, $\dim w_2 = 2$, $\dim (w_1 + w_2) = 4$ and $\dim (w_1 \cap w_2) = 1$.
4. (a) Ans: No.
(b) Ans: Yes.
5. Ans: $\phi(z) = Re(z)$. Where $z \in \mathbb{C}$.
6. (a) $N(T) = \{(a_{ij})_{n \times n} \in \mathbb{M}_{n \times n} : a_{nn} = -(a_{11} + a_{22} + \dots + a_{n-1, n-1})\}$, $\dim N(T) = n^2 - 1$.
 $R(T) = \{\alpha = \sum_{i=1}^n a_{ij} : \alpha \in \mathbb{F}\}$, $\dim R(T) = 1$.
(b) $N(T) = \text{span}\{0\}$, $\dim N(T) = 0$.
 $R(T) = \text{span}\{3x, 2 + \frac{3}{2}x^2, 4x + x^3\}$, $\dim R(T) = 3$.
(c) $N(T) = \text{span}\{(0, 1, -1)\}$, $\dim N(T) = 1$.
 $R(T) = \text{span}\{(\frac{1}{2}, \frac{1}{2}), (\frac{1}{2}, 0)\}$, $\dim R(T) = 2$.
(d) $N(T) = \text{set of all } 2 \times 2 \text{ skew symmetric matrices}$. $\dim N(T) = 1$.
 $R(T) = \text{set of all } 2 \times 2 \text{ symmetric matrix}$. $\dim R(T) = 3$.
7. (a) Ans: $T(x, y) = (2x - y, x - y, 2x)$
(b) Ans: $T(x, y, z) = (x + 2y + z, -x + z, y + z)$
8. (a)
$$\begin{bmatrix} 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 6 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(b)
$$\begin{bmatrix} 0 & 1 & 0 \\ 2 & 2 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$
- 9.
10. (a) Ans: $(2c, -\frac{3}{2}c, c, 0)$
(b) Ans: $(0, -\frac{3}{2}, \frac{5}{2})$
11. Ans: Rank of A = 2.
12. (a) 2
(b) 3
13. Ans: $-\frac{1}{2}, 1, 1$

14. (a) Invertible and $A^{-1} = \begin{bmatrix} \frac{1}{8} & \frac{5}{4} & \frac{3}{4} \\ -\frac{1}{4} & \frac{3}{4} & -\frac{1}{2} \\ \frac{3}{8} & -\frac{3}{8} & \frac{1}{4} \end{bmatrix}$
 (b) Not invertible.

15. (a) $k=1$
 (b) $k=\frac{2}{3}, \frac{11}{3}, \frac{11}{3}$

16.

17. Ans: Not possible

18. Ans: (a) $a \neq 1$
 (b) $a=1, b \neq -1, 3$
 (c) $a=1, b=3$ or $a=1, b=-1$