1. Prove vector identity $\nabla \times \nabla\phi = 0$.

2. Prove vector identity $\nabla \times \nabla \times \mathbf{A} = \nabla(\nabla.\mathbf{A}) - \nabla^2\mathbf{A}$.

3. For the vector field $\mathbf{A} = -\frac{y}{x^2+y^2}\hat{i} + \frac{x}{x^2+y^2}\hat{j}$ find $\nabla.\mathbf{A}$ and $\nabla \times \mathbf{A}$. Identify the nature of vector field. [Ans0,0 except at the origin]

4. Find constants a, b, c so that $\mathbf{V} = (x + 2y + az)\mathbf{i} + (bx - 3y - z)\mathbf{j} + (4x + cy + 2z)\mathbf{k}$ is irrotational.[$Ans.a = 4, b = 2, c = -1$]

5. For the problem stated above show that $\mathbf{V}$ can be expressed as the gradient of a scalar function i.e. $\mathbf{V} = \nabla\phi(x, y, z)$. Identify $\phi(x, y, z)$ if $\phi(0, 0, 0) = 0$. [$Ans.\phi = \frac{x^2}{2} - 3\frac{y^2}{2} + z^2 + 2xy + 4xz - yz$]

6. If $\mathbf{A} = 2x^2\mathbf{i} - 3yz\mathbf{j} + xz^2\mathbf{k}$ and $\phi = 2z - x^3y$, find $\mathbf{A}.\nabla\phi$ and $\mathbf{A} \times \nabla\phi$ at the point(1,-1,1). [$Ans.$5,$7\mathbf{i} - \mathbf{j} - 11\mathbf{k}$]

7. Find $\phi(r)$ such that $\nabla\phi = \frac{\mathbf{r}}{r^5}$ and $\phi(1) = 0$. [$Ans.\frac{1}{3}(1 - \frac{1}{r^3})$]

8. If $\mathbf{F} = (2x + y)\mathbf{i} + (3y - x)\mathbf{j}$, evaluate $\int_{\mathcal{C}} \mathbf{F}.d\mathbf{r}$ where $\mathcal{C}$ is the curve in the xy plane consisting of the straight lines from (0,0) to (2,0) and then to (3,2).[$Ans.$ 11]

9. If $\mathbf{F} = (5xy - 6x^2)\mathbf{i} + (2y - 4x)\mathbf{j}$, evaluate $\int_{\mathcal{C}} \mathbf{F}.d\mathbf{r}$ where $\mathcal{C}$ is the curve in the xy plane, $y = x^3$ from the point $(1, 1)$ to $(2, 8)$ [$Ans.$35]

10. If $\mathbf{A} = (y - 2x)\mathbf{i} + (3x + 2y)\mathbf{j}$, compute the circulation of $\mathbf{A}$ about a circle $\mathcal{C}$ in the xy plane with center at the origin and radius 2, if $\mathcal{C}$ is traversed in the positive direction.[$Ans.8\pi$]

11. (a) If $\mathbf{A} = (4xy - 3x^2z^2)\mathbf{i} + 2x^2\mathbf{j} - 2x^3z\mathbf{k}$, prove that $\int_{\mathcal{C}} \mathbf{A}.d\mathbf{r}$ is independent of the curve $\mathcal{C}$ joining two given points. (b) Show that there is a differentiable function ϕ such that $\mathbf{A} = \nabla\phi$ and find it. [$Ans.$(b)$\phi = 2x^2y - x^3z^2 + \text{constant}$]

12. Evaluate $\int\int_S \mathbf{r}.\mathbf{n}dS$ over: (a) the surface S of the unit cube bounded by the cordinate planes $x = 1, y = 1, z = 1$; (b) the surface of a sphere of radius a with center at $(0, 0, 0)$. [$Ans.$(a)3 (b)$4\pi a^3$]

13. If $\mathbf{F} = (2x^2 - 3z)\mathbf{i} - 2xy\mathbf{j} - 4x\mathbf{k}$, evaluate $(a) \int\int_V\int \nabla.\mathbf{F}dV$ and (b) $\int\int_V\int \nabla \times \mathbf{F}dV$, where V is the closed region bounded by the planes $x = 0, y = 0, z = 0$ and $2x + 2y + z = 4$.[$Ans.$(a)$\frac{8}{3}$ (b)$\frac{8}{3}(\mathbf{j} - \mathbf{k})$]

14. For the vector field $\mathbf{A} = -\frac{1}{2}By\mathbf{i} + \frac{1}{2}Bx\mathbf{j}$ verify Stoke's theorem $\int_S\int(\nabla \times \mathbf{A}).\hat{n}dS = \oint \mathbf{A}.d\mathbf{l}$ for a circular disk of radius R in xy-plane. [$Ans.$Both sides πR^2B]

15. For the vector field $\mathbf{F} = 4xz\mathbf{i} - y^2\mathbf{j} + yz\mathbf{k}$ verify divergence theorem $\int\int_V\int \nabla.\mathbf{F}dV = \int_S\int \mathbf{F}.\mathbf{n}dS$, where S is the surface of the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$.[$Ans.$Both sides $\frac{3}{2}$]