TOC - Test I

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3. $L = \{a^n b^{2n} c^{3n} : n \geq 1\}$

$S \to aBSccc$

$S \to aBccc$

$Ba \to aB$

$Bccc \to bbccc$

$Bbb \to bbbb$

1. Let $g(x,y) = \begin{cases} y & \text{if } y \text{ is prime} \\ f_{x+1}(y) + f_{x+2}(y) & \text{otherwise} \end{cases}$

There exists ~~a~~ such a partial recursive function g.

Let M be a multitape Turing machine that does the following on input x, y:-

i) check if y is prime (using sieve of Eratosthenes).
~~[illegible]~~ If it is prime then write y on the "output" tape & stop.

ii) Otherwise simulate the TM with index $x+1$ on 1 tape, and TM with index $x+2$ on another tape. It takes the output from both tapes and ~~[illegible]~~ ~~[illegible]~~ writes the sum of both on the "output" tape.

We can see that M computes $g(x,y)$.

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Smn theorem :-

By Smn theorem, there exists a total recursive function $\sigma : N \to N$ s.t $\forall y$, $f_{\sigma(x)}(y) = g(x, y)$.

As σ is total recursive, σ has a fixed point x_0 ~~for~~, according to Recursion theorem.

Thus $\forall y$, $\boxed{f_{x_0}(y) = f_{\sigma(x_0)}(y) = g(x, y)}$

2. a) ~~f(a,b)~~ $f(a, b, c) = add(a, b, c) = a + b + c$

i) $g(x) = \pi_1(x) = x$ is primitive recursive (projection rule)

ii) $\pi_4(x, y, z, a) = a$ is primitive recursive (projection rule)
$\pi_3(x, y, z) = z$

iii) $S(x) = x + 1$ is primitive recursive (successor rule)

iv) $h(x, y, z, a) = S(\pi_4(x, y, z, a)) = a + 1$ (composition rule)
$h_1(x, y, z) = S(\pi_3(x, y, z))$

v) $f(x, 0) = \pi_1(x) = x$
$f(x, n+1) = h_1(x, n, f(x, n)) = S(\pi_3(x, n, f(x, n)))$
$= f(x, n) + 1$

$f(x, y, 0) = \pi_1(f(x, y)) = f(x, y)$
$f(x, y, n+1) = h(x, y, n, f(x, y, n))$
$= S(\pi_4(x, y, n, f(x, y, n)))$
$= f(x, y, n) + 1$

Thus $f(a, b, c) = add(a, b, c) = a + b + c$ is a primitive recursion function

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b) $succ = \lambda n . \lambda f . \lambda x (f ((n f) x))$

$f = \lambda m . \lambda n . \lambda f . \lambda x . ((((m\ succ) n) f) x)$

$add = (\lambda a . \lambda b . \lambda c . (f_0\ a\ (f_0\ b\ c)))$

thus, $add\ (a\ b\ c) = a + b + c$

$\Rightarrow$ f corresponds to succ function applied $\bar{n}$ times on $\bar{m}$