Assignment-7

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1. a) (hd(one cons(two cons nil)) cons nil)

= (hd(one cons(cases nil of
isunit() → inNat(two),
[].
)
) cons nil)

= (hd(one cons(cases inunit() of
isunit() → inNat(two),
[]
)
) cons nil)

= (hd (one cons inNat(two)) cons nil)

= (hd (cases inNat(two) of
isunit() → inNat(two)
[] isNat(inNat(two)) → inNatxNat
(one, inNat(two))
) cons nil)

= (hd (inNatxNat(one, inNat(two)))
cons nil)

= (one cons nil)

= cases inNil() of
isNil → inNat(one)
[]

= inNat(one)

b) $\lambda l . (null\ l) \to (zero\ cons\ nil)[\,](one\ cons\ nil)$
$(tl\ (one\ cons\ nil))$

$= \lambda l . (null\ l) \to (zero\ cons\ nil)[\,](one\ cons\ nil)(tl\ (in\ Nat\ (one)))$

$= \lambda l . (null\ l) \to (zero\ cons\ nil)[\,](one\ cons\ nil)($cases $inNat\ (one)$
isUnit () $\to$ inUnit;
[] isNat (isNat (one) $\to$ inUnit())
)

$= (\lambda l . (null\ l) \to (zero\ cons\ nil)[\,](one\ cons\ nil)\ isUnit())$

$= (null\ isUnit()) \to (zero\ cons\ nil)[\,](one\ cons\ nil)$

$=$ true $\to (zero\ cons\ nil)[\,](one\ cons\ nil)$

$= (zero\ cons\ nil)$

$= isNat(zero)$

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c) $((one\ cons\ (two\ cons\ nil))\ cons\ nil)$

$= ((one\ cons\ inNat\ two))\ cons\ nil)$

$\Rightarrow in\ Nat \times Nat\ (one, inNat\ two))$ does not belong to Nat

It belongs to Nat^*

But cons : $D \times D^* \to D^*$

cons (d, l) = case l of
is unit() $\to$ in D(d)
[]

Here, $l = nil = inUnit() \in Nat^*$
$d = in\ Nat \times Nat\ (one, inNat\ (two)) \in Nat^*$

$\therefore$ No rule in the algebra of lists

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d) $(\lambda l.\ hd\ l)\ (tl\ (zero\ cons\ (tl\ (two\ cons\ nil))))$

$= (\lambda l.\ hd\ l)\ (tl\ (zero\ cons\ (tl\ (in\ Nat\ two))))$

$= (\lambda l.\ hd\ l)\ (tl\ (zero\ cons\ nil))$

$= (\lambda l.\ hd\ l)\ (tl\ (in\ Nat\ (zero\ 0)))$

$= (\lambda l.\ hd\ l)\ nil$

$= hd\ nil \rightarrow$ Error

2 i) $0 \oplus 0 = 0$

$LHS = M[[0 \oplus 0]] = M[[0]] + M[[0]]$

$= 0 + 0 = 0$

$RHS = M[[0]] = 0$

$\therefore LHS = RHS$

ii) $0 \oplus 1 = 1$

$L\text{-}H\text{-}S = M[[0 \oplus 1]] = M[[0]] + M[[1]] = 0 + 1 = 1$

$R\text{-}H\text{-}S = M[[1]] = 1$

$\therefore L\text{-}H\text{-}S = R\text{-}H\text{-}S$

iii) $1 \oplus 1 = 10$

$L\text{-}H\text{-}S = M[[1 \oplus 1]] = M[[1]] + M[[1]] = 1 + 1 = 2$

$R\text{-}H\text{-}S = M[[10]] = 2 * M[[1]] = 2 * 1 = 2$

iv) $0X = X$

$R\text{-}H\text{-}S = M[[X]] = x_{n-1}2^{n-1} + x_{n-2}2^{n-2} \cdots\cdots 2x_1 + x_0$

$L\text{-}H\text{-}S = M[[0\ X]] = 0 \cdot 2^n + x_{n-1}2^{n-1} \cdots\cdots 2x_1 + x_0$

$\therefore L\text{-}H\text{-}S = R\text{-}H\text{-}S$

v) $x \oplus y = y \oplus x$

L·H·S = $M[[x \oplus y]] = M[[x]] + M[[y]]$

R·H·S = $M[[y \oplus x]] = M[[y]] + M[[x]]$

$\therefore$ L·H·S = R·H·S

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vi) $x \oplus (y \oplus z) = (x \oplus y) \oplus z$

L·H·S = $M[[x \oplus (y \oplus z)]]$

$= M[[x]] + M[[y]] + M[[z]]$

= R·H·S

vii) $x0 \oplus y0 = (x \oplus y)0$

L·H·S = $M[[x0 \oplus y0]] = M[[x0]] + M[[y0]]$

$= 2 \times (M[[x]] + M[[y]])$

$= 2 \times (x \oplus y)$

$= (x \oplus y)0$

= R·H·S

ix) $x1 \oplus y1 = (x \oplus y \oplus 1)0$

L·H·S = $M[[x1 \oplus y1]] = M[[x1]] + M[[y1]]$

$= 2 * (M[[x]] + M[[y]]) + 2$

$= 2 \times (M[[x]] + M[[y]] + 1)$

$= 2 \times (M[[x \oplus y]] + M[[1]])$

$= 2 \times (M[[x \oplus y \oplus 1]])$

$= (x \oplus y \oplus 1)0$

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2. Axioms for Queue

deq(Q) ∉ Q, if empty(Q) = false
deq(Q) ∉ = error, else
deq(new()) = error
deq(enq(new(), E)) = E
front(new()) = error
front(enq(Q, E)) = E
front(Q) = error, if empty(Q) = true
back(new()) = error
back(enq(new(), E)) = E
back(enq(Q, E)) ≠ E if empty(Q) = false
back(enq(Q, E)) = E if empty(Q) = true
size(new()) = 0
empty(new()) = true
empty(enq(Q, E)) = false

3. digit[[0]] = 0
digit[[1]] = 1
digit[[2]] = 2
⋮
digit[[9]] = 9

value[[Digit]] = digit[[Digit]]
value[[numeral Digit]] = 10 × value[[numeral]] + digit[[Digit]]

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a) value [[98]]
= 10* value [[9]] + digit [[8]]
= 10* digit [[9]] + 8 = 98

b) value [[0021]] = 10 * value [[002]] + digit [[1]]
= 10 × (10 × value [[00]] + digit [[2]])
+ 1
= 10 × (10 × (10 × value [[0]] + digit [[0]])
+ ~~digit~~ + 2) + 1
= 10 × (10 × (10 × 0 + 0) + 2) + 1
= 10 × 2 + 1 = 21

4 i) P[[postfix 3 3 sub swap pop]] [56, 90]
= (λi* if (length i*) = N [[3]]
then ....
else errorAnswer) [56, 90]

= if (length [56, 90] = 3)
then ....
else errorAnswer
⇒ errorAnswer

ii) P[[postfix 2 5 swap sub pop]] [7, 8]
= (λi*. if length (i*) = N [[2]]
then rest Ans top Q [[Q]] value* ⇒ stack (map (Int ⇒ value) i*)
else errorAnswer) [7, 8]

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= if length [[7,8]] =>
then restoAns top Q[[5 swap sub pop]] value* → stack
(map (Int → value)
[7,8])

else error Answer

= restoAns top Q[[5 swap sub pop]] ∘ C[[5]] (7,8)$_5$

= restoAns top Q[[swap sub pop]] push value →
Result (Int → Value [[5]])
(7,8)$_5$

= restoAns top (λS · Q[[sub pop]] (C [[swap]]$_S$)) (7,8,5)$_5$

= restoAns top Q[[sub pop]] push (top pop (7,8),5)$_5$)

= restoAns top Q[[sub pop]] (push 8 (7,5)$_5$)

= restoAns top Q[[pop]] ∘ ([[sub]] (7,5,8)$_5$)

= restoAns top Q[[pop]] ([[sub]] (7,5,8)$_5$

= restoAns top Q[[pop]] (7,3)$_5$

= restoAns top Q[[]] pop (7,3)$_5$

= restoAns top (λs · s) (7)$_5$

= restoAns top (7)$_5$

= restoAns 7

=7