# Answer Set - 7

AUTUMN 2017

MATHEMATICS - I(MA10001)

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1. **Find the order and degree of the following differential equation:**

   (i) Order: 1 ; Degree: 2

   (ii) Order: 3 ; Degree: 3

   (iii) Order: 2 ; Degree: 2

   (iv) Order: 2 ; Degree: 2

   (v) Order: 1 ; Degree: 1

2. **Form the ODE by eliminating the arbitrary constants:**

   (i) $\left(\frac{dy}{dx}\right)^3 + 2a\,\frac{d^2y}{dx^2} = 0$

   (ii) $\frac{d^2y}{dx^2} - 4\,\frac{dy}{dx} + 4y = 0$

   (iii) $(\tanh x \;+\; \tan x)\,\frac{d^2y}{dx^2} - 2\,\frac{dy}{dx} + (\tanh x \;-\; \tan x)\,y = 0$

   (iv) $(1-x^2)\left(\frac{dy}{dx}\right)^2 + 1 = 0$

   (v) $2xy\,\frac{dy}{dx} = y^2 - x^2 \quad \left[hint : (x-a)^2 + y^2 = a^2\right]$

   (vi) $\frac{d^3y}{dx^3} = 0 \qquad \left[hint : y = ax^2 + bx + c\right]$

3. **Solve the following Initial Value Problems:**

   (i) $y = \cos x - 2\cos^2 x$

   (ii) $(x-1) + \ln(x^2+y^2) = 0$

4. **Check if the differential equations are homogeneous (reduced it to homogeneous if not), then solve it:**

   (i) Homogeneous ; $\sec\frac{y}{x} = Cxy$

   (ii) Not Homogeneous ; $\ln\,(2x^2 + 2xy^2 + y^4) = 2\,\tan^{-1}\frac{x+y^2}{x} + C$
   [ hint: use the substitution $y^2 = v - x$]

   (iii) Not Homogeneous ; $2x^2y^2\ln y - 4xy - 1 = Cx^2y^2$
   [ hint: use the substitution $xy = v$]

5. **Check if the differential equations are exact (if not, reduced it to exact using proper Integrating Factor), then solve it:**

   (i) Exact ; $a^2x - x^2y - xy^2 - \frac{1}{3}y^3 = C$

   (ii) Exact ; $5x^4y + y^5 = C$

   (iii) Not Exact ; $\frac{x}{y} - 2\ln x + 3\ln y = C \quad$ [ hint: IF $= \frac{1}{x^2y^2}$]

(iv) Not Exact ; $x^2 = Cye^{1/xy}$ $\left[\text{ hint: IF} = \frac{1}{x^3y^3}\right]$

(v) Not Exact ; $(x^2 + y^2)e^x = C$ $\left[\text{ hint: IF} = e^x\right]$

(vi) Not Exact ; $x^3y^2 + \frac{x^2}{y} = C$ $\left[\text{ hint: IF} = \frac{1}{y^2}\right]$

6. **Solve the following ODEs by reducing them to linear differential equations:**

(i) $y = (\tan x - 1) + Ce^{-\tan x}$ $\left[\text{ hint: divide by } \cos^2 x\right]$

(ii) $y^{-1/3}x^{-2/3} = -\frac{3}{7}\, x^{7/3} + C$ $\left[\text{ hint: put } y^{-1/3} = v\right]$

(iii) $y = f(x) - 1 + Ce^{-f(x)}$

(iv) $\frac{1}{x\ siny} = \frac{1}{2x^2} + C$ $\left[\text{ hint: put cosec } y = v\right]$

(v) $e^{-x^2} = y^2(2x + C)$ $\left[\text{ hint: put } -y^{-2} = v\right]$

(vi) $2 \tan y = (x^2 - 1) + Ce^{-x^2}$ $\left[\text{ hint: put } \tan y = v\right]$

(vii) $\frac{1}{x \ln y} = \frac{1}{2x^2} + C$ $\left[\text{ hint: put } \frac{1}{\ln y} = v\right]$