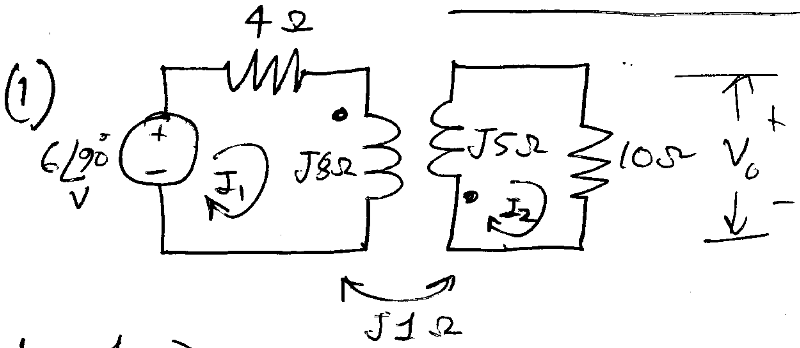
Tutorial 7 Solution

(1)



loop 1 ⇒

$$6J = (4 + J8)I_1 + J I_2$$

loop 2 ⇒ $(10 + J5) I_2 + J I_1 = 0$

$$\Rightarrow I_1 = \frac{-(10 + J5) I_2}{J}$$

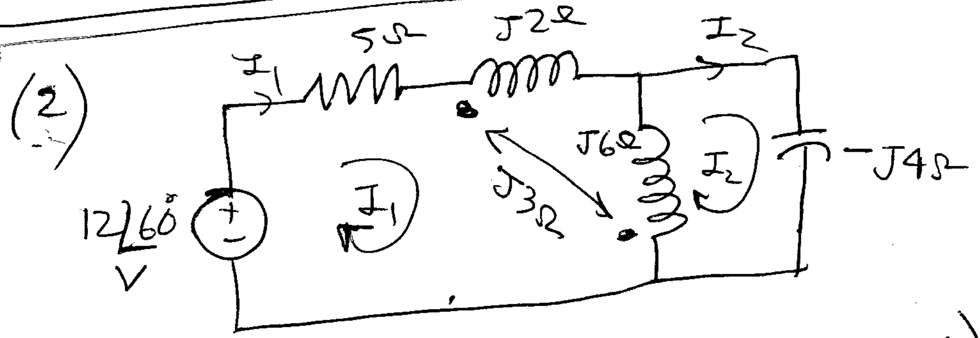
$$\therefore 6J = \left(\frac{-(4 + J8)(10 + J5)}{J} + J \right) I_2$$

$$\Rightarrow I_2 = 0.06 \angle -89.43^\circ$$

$$V_0 = 10 I_2 = 0.6 \angle -89.43^\circ$$

$=$

(2)



loop 1 ⇒ $12\angle 60^\circ = I_1 (5 + J2 + J6) - J6 I_2$

$+ (I_2 - I_1) J3 - I_1 (J3)$

$= I_1 (5 + J2) - I_2 (J3)$

loop 2 ⇒ $I_2 (-J4 + J6) - I_1 J6 + I_1 J3 = 0$

$\Rightarrow I_2 (J2) = I_1 J3$

$\Rightarrow I_2 = 1.5 I_1$

$$\therefore 12\angle 60^\circ = I_1(5+J2) - 4.5JI_1$$
$$= (5 - 2.5J)I_1 =$$
$$\Rightarrow 5.6\angle -26.6^\circ \, I_1 = 12\angle 60^\circ$$
$$\Rightarrow I_1 = 2.14\angle 86.6^\circ$$
$$\therefore I_2 = 3.2\angle 86.6^\circ$$

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(3)(a) $v_2 = 10 \times \frac{d}{dt}\left(-2e^{-5t}\right) = -20\left(-5e^{-5t}\right)$
$$= 100e^{-5t}$$

(b) $v_2 = -10\frac{d}{dt}\left(-2e^{-5t}\right) = -100e^{-5t}$

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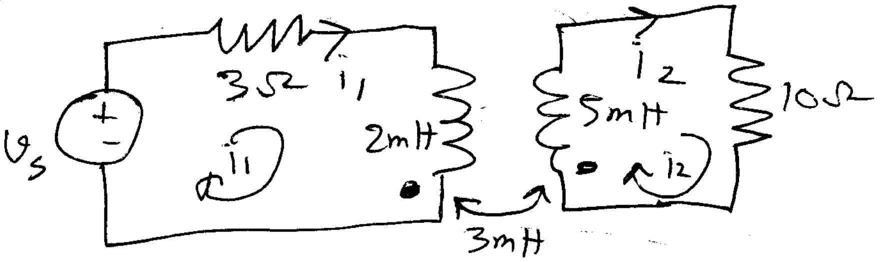
(4)



<u>loop 1</u> $\quad v_s = i_1(3) + \frac{di_1}{dt}\frac{2}{1000} - \frac{3}{1000}\frac{di_2}{dt}$

<u>loop 2</u> $\quad 10i_2 + \frac{5}{1000}\frac{di_2}{dt} - \frac{3}{1000}\frac{di_1}{dt}$

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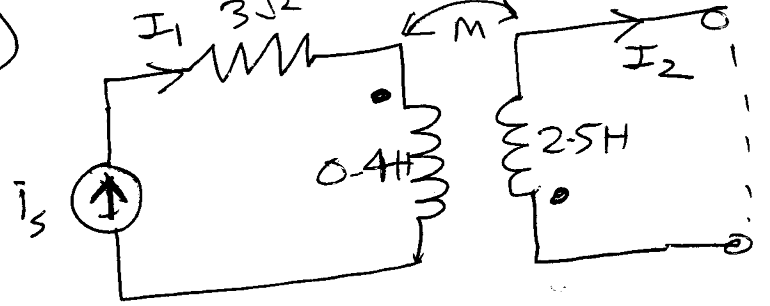
(5)



$k = 0.6$

$\therefore M = 0.6\sqrt{.4 \times 2.5}\, H$
$= 0.6\, H$

(a) If secondary is open then energy at $t = 0 = \frac{1}{2} \times .4 \times \left(2\cos(0)\right)^2 J$
$$= 0.8\, J$$

(b) If secondary is short then

~~primary loop =>~~

secondary loop =>

$$I_2(J10\times2.5) + I_1(J10\times0.6) = 0$$

$$\Rightarrow I_2 = -I_1 \times \frac{.6}{2.5} = -0.24\, I_1 = -0.24\times2\cos(10t)$$
$$= -0.48\cos(10t)$$

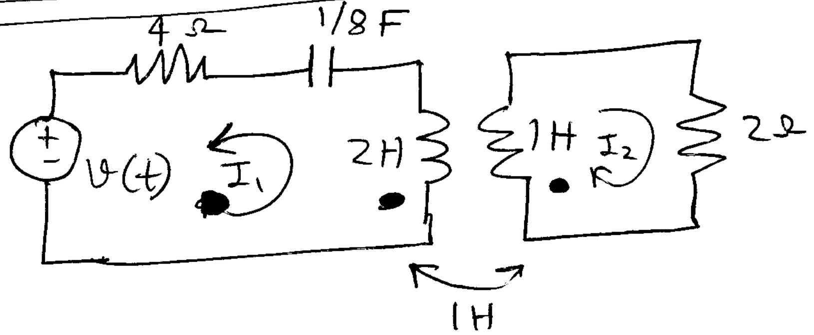
$\therefore$ Energy at $t=0$

$$= \left[\tfrac{1}{2}\times .4\times(2\cos(0))^2 + \tfrac{1}{2}\times 2.5\times(-.48\cos(0))^2 - 0.6\,(2\cos(0))(-.48\cos(0))\right] J$$

$$= [0.8 + 0.288 - 0.576]\, J$$

$$= 0.512\, J$$

(6)



$$K = \frac{M}{\sqrt{L_1 L_2}} = \frac{1}{\sqrt{2}} = 0.707$$

$v(t) = 20\cos(2t)$ V

$\omega = 2$

loop 2 $\Rightarrow$ $I_2(2+J2) + I_1(J2) = 0$

$$\Rightarrow I_1 = -\frac{I_2(J2+2)}{J2} = -I_2(1-J)$$

loop 1 $\Rightarrow$ $-v(t) = I_1\left(4 - J\frac{8}{2} + J2\times2\right) + I_2(J2)$

$$= I_1(4) + I_2(J2)$$
$$= (-4 + 4J + J2)\, I_2 = (-4+6J)\, I_2$$

$$\therefore I_2 = \frac{-v(t)}{-(4-6J)} = 0.139\times20\cos(2t+56^\circ)$$

~~[illegible]~~

$$I_1 = -I_2(1-J) = -1.414\, I_2 \angle -45^\circ$$

$$= -1.414 \times 0.139 \times 20 \cos(2t + 56^\circ - 45^\circ)$$

$$= -3.93 \cos(2t + 11^\circ)$$

$\cancel{\therefore I_1(0) = -3.859}$

$\cancel{\therefore \text{Energy at } 1.5s}$

$\cancel{= \frac{1}{2} \times 2 \times .7773 + \frac{1}{2}}$

$\cancel{= \left(\frac{1}{2} \times 2 \times (3.859)^2 + \frac{1}{2} \times 1 \times .7773^2\right.}$

$\cancel{\left. - 1 \times 3.859 \times .7873\right) J}$

$\cancel{\therefore I_1(1.5s)}$ $\therefore$ at $t = 1.5s$

$$I_1 = -3.93 \cos\left(2 \times 1.5 \times \frac{180^\circ}{\pi} + 11^\circ\right)$$

$$= 3.885$$

and $I_2 = .139 \times 20 \cos\left(2 \times 1.5 \times \frac{180^\circ}{\pi} + 56^\circ\right)$

$$= -1.864$$

$$\therefore \text{Energy} = \left(\frac{1}{2} \times 2 \times 3.885^2 + \frac{1}{2} \times 1 \times 1.864^2 - 1 \times 3.885 \times 1.864\right) J$$

$$= 9.59\ J$$

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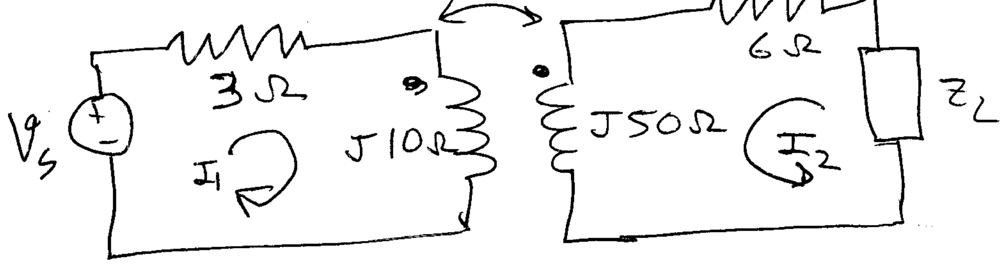
(7) $J\omega L_1 = J10\,\Omega$ $\quad J\omega L_2 = J50\,\Omega$

$J\omega M = J20\,\Omega$



$\underline{\text{loop 2}} \Rightarrow I_2(6+J50+Z_L) + I_1(J20) = 0$

$$\Rightarrow I_2 = \frac{-I_1(J20)}{6+J50+Z_L}$$

$\underline{\text{loop 1}} \Rightarrow V_s = I_1(3+J10) + I_2(J20)$

$$= I_1(3+J10) - \frac{I_1(J20)^2}{6+J50+Z_L}$$

$\therefore$ Input impedance $= \dfrac{V_s}{I_1}$

$$\text{or } Z_i = 3+J10 - \frac{(J20)^2}{6+J50+Z_L} = 3+J10+\frac{400}{6+J50+Z_L}$$

(a) $Z_L = 10\Omega$

$$\therefore Z_i = \left(3+J10+\frac{400}{16+J50}\right)\Omega$$

$$= (5.32+2.74J)\Omega$$

(b) $Z_L = J20\Omega$

$$\therefore Z_i = \left(3+J10+\frac{400}{6+J70}\right)\Omega$$

$$= (3.49+4.33J)\Omega$$

(c) $Z_L = 10+J20$

$$\therefore Z_i = \left(3+J10+\frac{400}{16+70J}\right)\Omega$$

$$= (4.24+J\,4.57)\Omega$$

(d) $Z_L = -J20\,\Omega$

$$\therefore Z_i = \left(3 + 10J + \frac{400}{6+30J}\right)\Omega$$

$$= (5.56 - J2.82)\,\Omega$$

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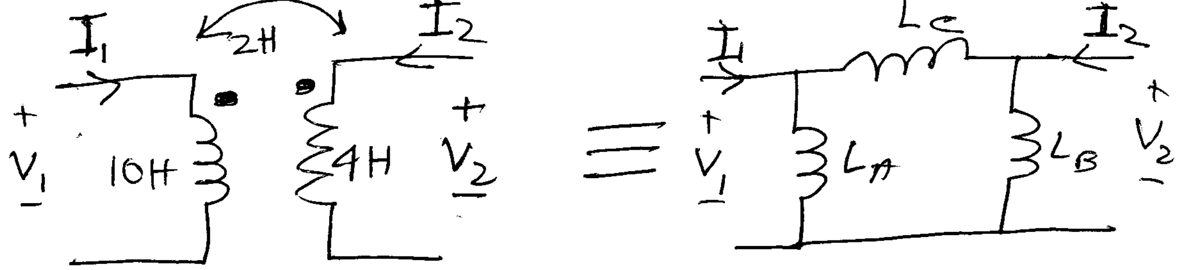
(8)



~~with secondary open circuited~~

~~For left circuit $V_1 = 10 I_1$~~

~~for right circuit~~

Let the frequency (angular) = ω rad/sec

~~with secondary open circuited~~

~~For left~~

For left circuit

$$V_1 = J\omega 10 I_1 + J\omega 2 I_2$$

$$V_2 = J\omega 4 I_2 + J\omega 2 I_1$$

$$\Rightarrow \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} J\omega 10 & J\omega 2 \\ J\omega 2 & J\omega 4 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

For the right circuit

$$I_1 = \frac{V_1}{J\omega L_A} + \frac{V_1 - V_2}{J\omega L_e} = \frac{V_1}{J\omega}\left(\frac{L_A + L_e}{L_A L_e}\right) - \frac{V_2}{J\omega L_e}$$

and $I_2 = \dfrac{V_2}{j\omega L_B} + \dfrac{V_2 - V_1}{j\omega L_C}$

$= \dfrac{V_2}{j\omega}\left(\dfrac{L_B + L_C}{L_B L_C}\right) - \dfrac{V_1}{j\omega L_C}$

$$\Rightarrow \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \frac{1}{j\omega}\begin{bmatrix} \dfrac{L_A + L_C}{L_A L_C} & -\dfrac{1}{L_C} \\ -\dfrac{1}{L_C} & \dfrac{L_B + L_C}{L_B L_C} \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = j\omega L_C \begin{bmatrix} \dfrac{L_A + L_C}{L_A} & -1 \\ -1 & \dfrac{L_B + L_C}{L_B} \end{bmatrix}^{-1}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$= \frac{j\omega L_C \begin{bmatrix} \dfrac{L_B + L_C}{L_B} & 1 \\ 1 & \dfrac{L_A + L_C}{L_C} \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}}{\left(\dfrac{(L_A + L_C)(L_B + L_C)}{L_A L_B} - 1\right)}$$

$$= \begin{bmatrix} j\omega 10 & j\omega 2 \\ j\omega 2 & j\omega 4 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \quad \text{---- (i)}$$

[$\because$ right side circuit is equivalent to the left side circuit]

Comparing the two matrices we can write

$$\frac{\dfrac{L_B + L_C}{L_B}}{1} = \frac{j\omega 10}{j\omega 2} = 5 \Rightarrow L_B + L_C = 5L_B$$

$$\Rightarrow L_C = 4L_B$$

And $\dfrac{L_A + L_C}{\frac{L_C}{1}} = \dfrac{4J\omega}{2J\omega} = 2$

$\Rightarrow L_A + L_C = 2L_C \Rightarrow L_A = L_C = 4L_B$

So equation ① can be written interms of L_B as

$$\frac{4J\omega L_B \begin{bmatrix} \frac{L_B + 4L_B}{L_B} & 1 \\ 1 & \frac{4L_B + 4L_B}{4L_B} \end{bmatrix}}{\left(\frac{(4L_B + 4L_B)(L_B + 4L_B)}{4L_B^2} - 1\right)} = J\omega \begin{bmatrix} 10 & 2 \\ 2 & 4 \end{bmatrix}$$

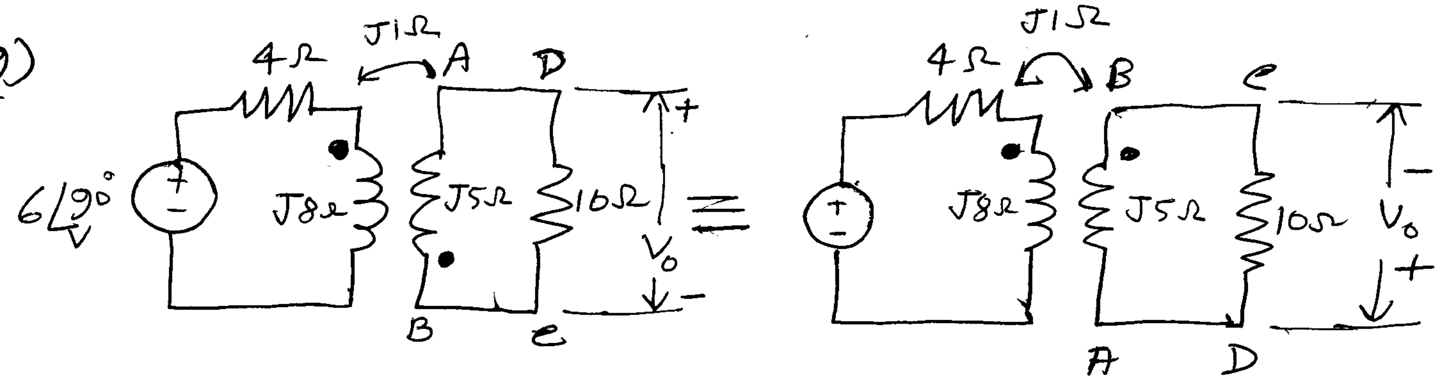
$$\Rightarrow \frac{4L_B \begin{bmatrix} 5 & 1 \\ 1 & 2 \end{bmatrix}}{\frac{8 \times 5}{4} - 1} = \begin{bmatrix} 10 & 2 \\ 2 & 4 \end{bmatrix}$$

$\Rightarrow \dfrac{4L_B}{9} = 2 \quad \Rightarrow L_B = \dfrac{18}{4} = 4.5$

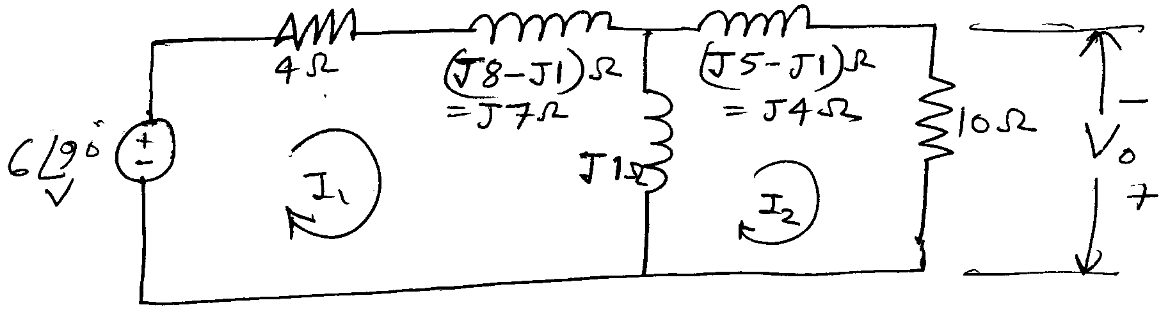
$\therefore L_A = L_C = 4L_B = 18$

$$\left.\begin{aligned} L_A &= 18\ H \\ L_B &= 4.5\ H \\ L_C &= 18\ H \end{aligned}\right\} \underline{\text{Ans}}$$

(9)



$\therefore$ T-equivalent ckt :



Loop 1: $6\angle 90^\circ = (4+8J)I_1 - J I_2$

loop 2 : $(10+5J)I_2 - J I_1 = 0 \Rightarrow I_1 = \dfrac{10+5J}{J} I_2$

$\therefore 6\angle 90^\circ = (4+8J)\left(\dfrac{10+5J}{J}\right) I_2 - J I_2$

$\Rightarrow 6J = \left((4+8J)\left(\dfrac{10+5J}{J}\right) - J\right) I_2$

$\Rightarrow I_2 = -0.06\angle{-89.43^\circ}$ A

$\therefore V_o = -10 I_2 = 0.6\angle{-89.43^\circ}$ A

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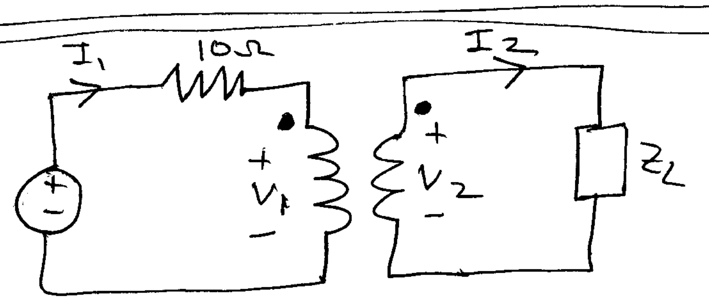
(10)



Note that this is an ideal transformer

$N_1 = 1000$

$N_2 = 5000$

$Z_L = (500 - J400)\,\Omega$

(a) $I_2 = 1.4\angle 20^\circ A$ rms

Load power $= (1.4)^2 \times 500$ W

$\simeq 980W$

(b) $V_2 = 900\angle 40^\circ$ V rms

$I_2 = \frac{V_2}{Z_L} = \frac{900\angle 40^\circ}{500 - j400}$ A

$\therefore |I_2| = \frac{9}{\sqrt{25+16}}$ A

$\therefore$ Load power $= \left(\frac{9}{\sqrt{25+16}}\right)^2 \times 500$ W

$= 987.8$ W

(c) $V_1 = 80\angle 100^\circ$ V rms

$\therefore V_2 = 80 \times \frac{5000}{1000} \angle 100^\circ V = 400\angle 100^\circ V$

$I_2 = \frac{400\angle 100^\circ}{500 - j400}$ A $\Rightarrow |I_2| = \frac{4}{\sqrt{41}}$ A

$\therefore$ Load power $= \left(\frac{4}{\sqrt{41}}\right)^2 \times 500$ W

$= 195.1$ W

(d) $I_1 = 6\angle 45^\circ$ A rms

$\therefore I_2 = 6\angle 45^\circ \times \frac{1000}{5000}$ A $= 1.2\angle 45^\circ$ A

$\therefore$ Load power $= 1.2^2 \times 500$ W

$= 720$

(e) $V_s = 200\angle 0^\circ$ V rms

Loop 1: $V_s - 10 I_1 = V_1$

loop2: $V_2 = I_2 Z_L = \frac{I_1}{5} Z_L$

$$\Rightarrow 5V_1 = \frac{I_1}{5} Z_L \Rightarrow V_1 = \frac{I_1 Z_L}{25}$$

$$\therefore V_s - 10 I_1 = V_1 = \frac{I_1 Z_L}{25}$$

$$\Rightarrow V_s = \left(10 + \frac{Z_L}{25}\right) I_1 = \left(10 + \frac{500 - J400}{25}\right) I_1$$

$$\Rightarrow 200\angle 0^\circ = V_s = (30 - 16J)\, I_1$$

$$\Rightarrow I_1 = \frac{200\angle 0^\circ}{30 - 16J}\, A$$

$$\therefore I_2 = \frac{I_1}{5} = \frac{40\angle 0^\circ}{30-16J} A \Rightarrow |I_2| = \frac{40}{\sqrt{30^2 + 16^2}} = \frac{40}{34} A$$

$$\therefore \text{Load power} = \left(\frac{40}{34}\right)^2 \times 500\,W = 692\,W.$$

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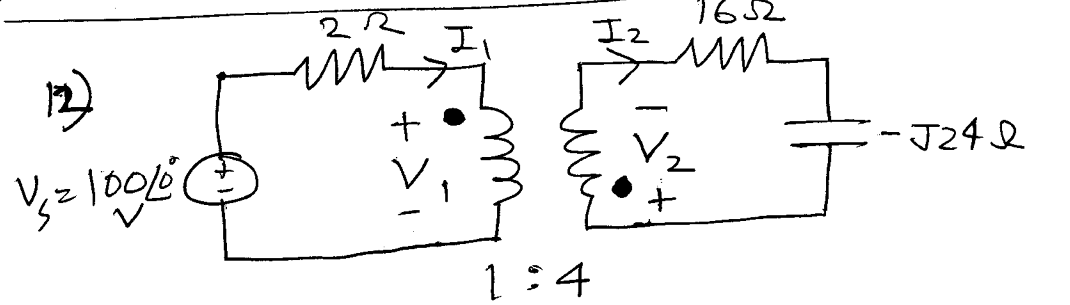
12)



The transformer is ideal.

From loop 1 : $100\angle 0^\circ = 2I_1 + V_1$

from loop 2 : $V_2 = (-J24 + 16)(-I_2)$

$$\Rightarrow 4V_1 = (-J24 + 16)\frac{I_1}{4}$$

$$\Rightarrow V_1 = \frac{16 - 24J}{16} I_1$$

$$\therefore 100\angle 0^\circ = 2I_1 + \frac{16 - 24J}{16} I_1$$

$$= (2 + 1 - 1.5J) I_1$$

$$= (3 - 1.5J) I_1$$

$$\therefore I_1 = \frac{100}{3 - 1.5J} A$$

∴ Complex power supplied by the source

$$= V_s I_1^* = 100 \times \frac{100}{3+1.5J} \text{ VA}$$

$$= 2981.4 \angle -26.6^\circ \text{ VA}$$

(ii)