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Para-Algo
Test-3

2.

~~Lets define $I[X]$ = maximum number of independent sets~~

Lets define for $X \subseteq V(G)$,

$I[X]$ = minimum k such that, $G[X]$ can be partitioned into k independent sets

$C[X]$ = minimum k such that, $G[X]$ can be partitioned into k cliques

Recursions:-

$$I[X] = \begin{cases} 0 & X = \phi \\ \min\limits_{\substack{Y \subseteq X \\ Y \text{ is independent} \\ Y \neq \phi}} (1 + I[X-Y]) & \text{else} \end{cases}$$

$$C[X] = \begin{cases} 0 & X = \phi \\ \min\limits_{\substack{Y \subseteq X \\ Y \text{ is clique} \\ Y \neq \phi}} (1 + C[X-Y]) \end{cases}$$

As, this is an exhaustive recursion, we can see the transitions are optimal

$\boxed{\text{If } \exists_{X \subseteq V(G)} (I[X] + C[V-X] = k), \text{ we say that } G \text{ is a } k\text{-IC-graph.}}$

Total number of states $= O(2^n)$
Transition $= O(n^{O(1)})$
checking feasibility $= O(2^n)$

Total time :- $O(2^n \cdot n^{O(1)})$

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②

1.

~~partition($C_1, C_2, \ldots, C_K$) = $\forall_{v \in V(G)}$~~

$$\text{partition}(C_1, C_2, \ldots, C_K) = \forall_{v \in V(G)} \Big[(v \in C_1 \wedge v \notin C_2 \wedge \ldots v \notin C_K)$$
$$\vee (v \notin C_1 \wedge v \in C_2 \wedge \ldots v \notin C_K)$$
$$\vee (v \notin C_1 \wedge v \notin C_2 \wedge v \in C_3 \ldots \wedge v \notin C_K)$$
$$\vdots$$
$$\vee (v \notin C_1 \wedge v \notin \ldots \wedge v \in C_K) \Big]$$

$$\text{dist}1(u, v) = (\exists_{e \in E(G)} \text{inc}(u,e) \wedge \text{inc}(v,e))$$

$$\text{dist}2(u, v) = \exists_{w \in V(G)} \text{dist}1(u, w) \wedge \text{dist}1(w, v)$$

$$Q(C_1, C_2, \ldots, C_K) = \text{partition}(C_1, C_2, \ldots, C_K) \wedge$$
$$\Big[(\forall_{u,v \in C_1} \neg\text{dist}1(u,v) \wedge \neg\text{dist}2(u,v)) \wedge$$
$$(\forall_{u,v \in C_2} \neg\text{dist}1(u,v) \wedge \neg\text{dist}2(u,v)) \wedge$$
$$\vdots$$
$$(\forall_{u,v \in C_K} \neg\text{dist}1(u,v) \wedge \neg\text{dist}2(u,v)) \wedge$$
$$(\forall_{u \in C_1} \forall_{v \in C_2} \neg\text{dist}1(u,v)) \wedge$$
$$(\forall_{u \in C_2} \forall_{v \in C_3} \neg\text{dist}1(u,v)) \wedge$$
$$\vdots$$
$$(\forall_{u \in C_{K-1}} \forall_{v \in C_K} \neg\text{dist}1(u,v) \Big).$$

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(3)

Q accepts k free parameters $C_1, C_2 \ldots C_K$ which is a partition of $V(G)$ and correspond to the vertices with color i.

Q is a MSO_2 accepting $L(2,1)$-k-coloring.

$$\|Q\| = k^2$$

$\therefore$ By Courcelle's Theorem, there exists an FPT that verifies Q in time $f(\|Q\|, t)$ for some computable function f. Here, t is treewidth.

$$\boxed{\text{Thus, } p^*\text{-tw-}L(2,1)\text{-coloring is FPT}}$$