Computer Science & Engineering Department
I. I. T. Kharagpur

Principles of Programming Languages: CS40032

Elective

Assignment – 2: λ-Calculus *Marks: 20*
January 30, 2020

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1. Reduce the following λ-expressions. Show every step of α-, β-, η- and δ-reductions. **[2 * 6 = 12]**

   (a) $(\lambda z.\ z)\ (\lambda z.\ z\ z)\ (\lambda z.\ z\ y)$

   (b) $(\lambda x.\ \lambda y.\ x\ y\ y)\ (\lambda a.\ a)\ b$

   (c) $(\lambda x.\ \lambda y.\ x\ y\ y)\ (\lambda y.\ y)\ y$

   (d) $(\lambda x.\ x\ x)\ (\lambda y.\ y\ x)\ z$

   (e) $(\lambda x.\ (\lambda y.\ (x\ y))\ y)\ z$

   (f) $((\lambda x.\ (\lambda y.\ (x\ y))\ (\lambda y.\ y))\ w)$

2. Consider the recursive definition of $add(x, y), \forall x \geq 0, y \geq 0$:**[2 + 6 = 8]**

$$\begin{aligned} add(x,y) &= y && \text{, if x} = 0 \\ &= add(x-1, y+1) && \text{, if x} > 0 \end{aligned}$$

   (a) Using Y combinator, encode the above recursive definition of *add* as a λ-expression.

   (b) Reduce *add* 2 5. Show every step of β- and δ- reductions. You may skip the α-reductions with a mention of the step.