1. Show that $f_0 f_1 + f_1 f_2 + \cdots + f_{2n-1} f_{2n} = f_{2n}^2$ ①

when n is a +ve integer and f_n is the n^{th} Fibonacci number

$f_0 = 0,\ f_1 = 1,\ f_2 = 1,\ f_3 = 2 \ldots$

Basis step

$n = 1$ $\quad f_0 f_1 + f_1 f_2 = 0.1 + 1.1 = 1 = 1^2 = f_2^2$

— true

Inductive step

Assume that the equation is true for ~~n =~~ k some integer

So, $f_0 f_1 + f_1 f_2 + \cdots + f_{2k-1} f_{2k} = f_{2k}^2$

Then $n = k+1$ $\quad f_0 f_1 + f_1 f_2 + \cdots + f_{2k+1} f_{2k+2}$

$= f_0 f_1 + f_1 f_2 + \cdots + f_{2k-1} f_{2k} + f_{2k} f_{2k+1} + f_{2k+1} f_{2k+2}$

$= f_{2k}^2 + f_{2k} f_{2k+1} + f_{2k+1} f_{2k+2}$

$= f_{2k}(f_{2k} + f_{2k+1}) + f_{2k+1} f_{2k+2}$

$= f_{2k} \cdot f_{2k+2} + f_{2k+1} f_{2k+2}$

$= f_{2k+2}(f_{2k} + f_{2k+1})$

$= f_{2k+2} \cdot f_{2k+2}$

$= f_{2k+2}^2 = f_{2(k+1)}^2$

So, for all +ve integer n the equation is true

Similar problem

$f_0 - f_1 + f_2 - \cdots - f_{2n-1} + f_{2n} = f_{2n-1} - 1$

n +ve integer, f_n – Fibonacci no.

2. Determine the no. of divisions used by the Euclidean algorithm to find the greatest common divisor of the Fibonacci numbers f_n and f_{n+1} when n is a nonneg integer. Verify your answer using mathematical induction.

Euclidean Algorithm (E.A)

procedure gcd (a, b : +ve integer)

$x = a$
$y = b$
while $y \neq 0$
begin $r = x \bmod y$
$x = y$
$y = r$
end { gcd (a, b) is x }

recursive procedure gcd (a, b) $a < b$
if $a = 0$ then $gcd(a, b) = b$
else $gcd(a, b) = gcd(b \bmod a, a)$

Ans.2 The no. of divisions used by E.A
to find $gcd(f_{n+1}, f_n)$ = 0 for $n = 0$
= 1 for $n = 1$
= $n - 1$ for $n \geq 2$

B.S for $n = 0$, $f_0 = 0$, $f_1 = 1$
$gcd(f_1, f_0) = gcd(1, 0) = 1$, no. of div = 0

$n = 1$ $gcd(f_2, f_1) = gcd(1,1) = 1$ ~~$= gcd(2, 1) = gcd(1, 0) = 1$~~
$gcd(f_3, f_2)$ $= gcd(2, 1) = gcd(1, 0) = 1$ no. of div = 1

I.S Assume $(k-1)$ divisions are required to find ~~gcd~~ $gcd(f_{k+1}, f_k)$; $\because k$.

To find $gcd(f_{k+2}, f_{k+1})$
divide f_{k+2} by f_{k+1} $f_{k+2} = 1 \cdot f_{k+1} + f_k$

After one division $\gcd(f_{k+2}, f_{k+1}) = \gcd(f_{k+1}, f_k)$ ②

By the inductive hypothesis exactly 1 more division is required

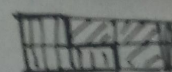
So $(k-1)+1 = k$ divisions are required to find $\gcd(f_{k+2}, f_{k+1})$

for true

3. (a) Show that any $(2i \times 3j)$ board, i, j are +ve integers, with no square missing can be tiled with trominoes.

B.S $i = 1, j = 1 \rightarrow (2 \times 3)$ board.

true



I.S ~~j, i ≥ 1~~ $i > 1, j > 1$

i) Let for $i = k$ & $j = k$ it is true

i.e $(2k \times 3k)$ the board is tiled.

for $(2(k+1) \times 3(k+1))$ board.

the size of board increased where a subboard to 2 row and 3 columns are added

ii) or if ~~j~~ for $(2(k+1) \times 3(m+1))$ board when $k \neq m$, the sub-board is of size

rows - multiple of 2 & columns multiple of 3.

by B.S it is already shown that (2×3) board can be tiled.

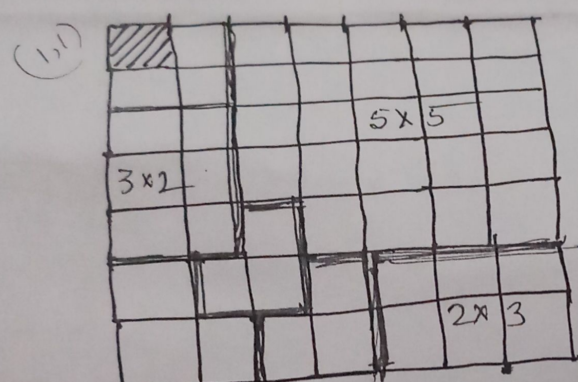
So it is proved (true)

Show that any 7×7 ~~(board)~~ deficient board can tiled with trominoes. Use the result of (a)

(i,j) denotes square in i^{th} row & j^{th} column.

Due to symmetry, we need only consider 7×7 boards of squares (i,j) removed where $i \le j \le 4$
empty

Solution when square $(1,1)$ is removed



Some (5×5) board can be tiled not all

(remember the result any $n\times n$ deficient board can be tiled with trominoes if (n^2-1) is divisible by 3 and $n \neq 5$)

n - need not be power of

& Any $2^n\times2^n$ deficient board can be tiled with trom

Work

Show that any (11×11) deficient board can be tiled with trominoes.