Algorithms -1
Tutorial 1
August 2, 2018

1. Prove mathematically that $3n^2 + 4n + 1 = O(n^2)$

2. Arrange the following in increasing order of growth (show your calculations):
   $(\lg n)^{\lg n}$, $n^{(1/(\lg n))}$, $\lg(\lg n)$, $(\log_{10} n)^2$, n, $(\text{sqrt}(2))^{\lg n}$

3. Given a sorted sequence (ascending order) of n distinct integers a[1], a[2],...,a[n], design an O(log n) time algorithm to find if there exists an index i such that a[i] = i.

4. A cyclically sorted array is the array obtained by sorting an array and then doing a cyclic rotation. For example, $[1,2,3], [3,1,2], [2,3,1]$ are cyclically sorted arrays. Design a O(log n) time algorithm to search for an element in a cyclically sorted array of n integers.

5. Is the delete operation in a BST commutative (i.e., given a BST and two nodes x and y in it, does deleting first x then y and deleting first y then x result in the same final BST)?

6. Professor X designs a BST with one additional field per node which stores the number of nodes in the subtree rooted at that node (including the node itself). He claims that if a BST satisfies the property that the number of nodes in the left subtree is the same as the number of nodes in the right subtree for every node then the BST is a balanced BST. Is he right? Justify your answer.

7. What is the largest possible number of internal nodes in a red-black tree with black-height k? What is the smallest possible number? Justify your answer. Note that black height of a red-black tree is the black height of its root.

8. Can you design an algorithm to find the median element in a Balanced BST with n nodes in $O(\log n)$ time? If n is even, choose the larger of the two median elements.

9. Design an ADT that stores 2-tuples of the form (x, y), where x and y are integers. It is given that for any two tuples (x1, y1) and (x2, y2) in the ADT, x1, x2, y1, y2 are all distinct. The ADT must support the following operations:
   a. Search(a, b) – returns all tuples with x = a OR y = b OR both
   b. Delete(a, b) – deletes all tuples with x = a OR y = b OR both
   c. Insert(a, b) – inserts the tuple (a,b) in the ADT

   All the operations must run in O(log n) time. Write psuedocodes for the operations.

10. Design an ADT that stores (element, value) pairs (x, v) where x and v are integers. Assume that all x's (elements) are distinct, though two elements may have the same value. The ADT must support the following operations:
    a. FindValue(x, v) – finds if a pair (x,v) is in the ADT or not
    b. Insert(x, v) – inserts the pair (x,v) in the ADT
    c. AddValueToAll(v) – adds the value v to the existing values of **all** elements in the tree

    All of the operations must run in O(log n) time.