

1.

Use polar co-ordinates and show the continuity of $(f(x, y))$.

Use definitions to show $f_x(0, 0)$ and $f_y(0, 0)$ does not exist. **2.**

a. $f_x = 2x, \quad f_y = 2y$

b. $f_x = 3 \cos(3x + 4y), \quad f_y = 4 \cos(3x + 4y)$

c. $f_x = -ye^{-x} + y, \quad f_y = e^{-x} + x$

3.

a. $f_x(0, 0) = 0, \quad f_y(0, 0) = 0, \quad f_x(0, y) = 1, \quad f_y(x, 0) = 1$

b. $f_x(0, 0) = 0, \quad f_y(0, 0) = 0, \quad f_x(0, y) = y, \quad f_y(x, 0) = x$

c. $f_x(0, 0) = 0, \quad f_y(0, 0) = 0, \quad f_x(0, y) = \text{does not exist}, \quad f_y(x, 0) = \text{does not exist}.$

4.

Use definition to show that the function $f(x, y)$ is differentiable at $(0, 0)$.

5.

Use definition to show that the function $f(x, y)$ is not differentiable at $(0, 0)$.

6.

Use definition to show that the function $f(x, y)$ is not differentiable at $(0, 0)$.

Use definitions to find $f_x(0, 0)$ and $f_y(0, 0)$ and their continuity.

7.

Use definition to show that the function $f(x, y)$ is not differentiable at $(0, 0)$.

8. $f_{xx}(0, 0) = 0, \quad f_{xy}(0, 0)$ does not exist, $f_{yx}(0, 0) = 0, \quad f_{yy}(0, 0)$ does not exist. [Hint: Use definition of partial derivatives]

The function $f(x, y)$ is not differentiable at $(0, 0)$.

9.

Use definition to find $f_{xy}(0, 0) = 1$ and $f_{yx}(0, 0) = 0$. Use definition to show that the function $f(x, y)$ is differentiable at $(0, 0)$.

10.

a. $f_{yxx}(x, y) = 36x^2 \cos 3y, \quad f_{xyx}(x, y) = 36x^2 \cos 3y.$

b. $f_{yxx}(x, y) = 60x^3 y^2, \quad f_{xyx}(x, y) = 60x^3 y^2.$

c. $f_{yxx}(x, y) = 2xe^{xy} \sec^2 x \tan x + 2xye^{xy} \sec^2 x + 2e^{xy} \sec^2 x + xy^2 e^{xy} \tan x + 2ye^{xy} \tan x + 12xy,$
 $f_{xyx}(x, y) = e^{xy} \sec^2 x + xye^{xy} \sec^2 x + 2xe^{xy} \sec^2 x \tan x + (xy + 1)e^{xy} (\sec^2 x + y) + ye^{xy} \tan x + 12xy.$

11.

a. $dw = (2x + y^2 + y^2 z^3)dx + 2xy(1 + z^3)dy + (3xy^2 z^2)dz.$

b. $dz = \frac{y}{x^2 + y^2}dx - \frac{x}{x^2 + y^2}dy.$

c. $du = 2e^{x^2+y^2+z^2}(xdx + ydy + zdz).$

d. $dw = 3 \cos(3x + 4y)dx + 4 \cos(3x + 4y)dy + 5e^z dz.$