Answer and hints of tutorial Sheet - 1 **AUTUMN 2017**

MATHEMATICS-I (MA10001) *August 1, 2017*

1. $c = \frac{6 \pm 4\sqrt{3}}{6}$.

2. Use the concept of Rolle's theorem that is between any two roots of $f(x) = 0$ there is a root of $f'(x) = 0$.

3. An odd degree polynomial has at least one real root(show). For the given polynomial apply Rolle's theorem to argue that it has exactly one real root(can be proofed by method of contradiction).

4. Apply Rolle's theorem on [a,b] to prove the result.

5. Apply Rolle's theorem and argue by contradiction.

6. i. Choose $g(x) = e^{-x} \int_0^x f(t)\, dt$ and apply Rolle's theorem on [0,1].

   ii. Consider $\phi(x) = f(x) - \frac{(x-b)(x-c)}{(a-b)(a-c)} f(a) - \frac{(x-c)(x-a)}{(b-a)(b-c)} f(b) - \frac{(x-a)(x-b)}{(c-a)(c-b)} f(c)$ and apply Rolle's theorem on ϕ and ϕ'.

7. Take $k(x) = \gamma(f_2(b)-f_2(a))(f_3(b)-f_3(a))(f_1(x)-f_1(a))+\alpha(f_1(b)-f_1(a))(f_3(b)-f_3(a))(f_2(x)-f_2(a)) + \beta(f_1(b) - f_1(a))(f_2(b) - f_2(a))(f_3(x) - f_3(a))$ and apply Rolle's theorem.

8. Choose $f(x) = e^{\alpha x} p(x)$ and apply Rolle's theorem on $[a, b]$, where $p(a) = p(b) = 0$.

9. $f'(x) = 4(x^3 - 1) = 4(x - 1)(x^2 + x + 1)$ has only one real root. By Rolle's theorems it has two different real roots.

10. i. 7.

    ii. 3.037(Choose $f(x) = \sqrt[3]{x}$ and apply Lagrange's MVT on [27,28]).

11. Choose $h(x) = f(x) - g(x)$ and Lagrange's MVT on $[a, x]$.

12. i. Choose $f(x) = \tan^{-1} x + \cot^{-1} x$ on $[0, x]$.

    ii. Choose $h(x) = 3\cos^{-1} x - \cos^{-1}(3x - 4x^3)$ on $[0, x]$.

13. Consider $g(x) = (f(x))^2$ and apply Lagrange's MVT on [0,2].

14. Choose
$$g(x) = \begin{vmatrix} f(x) & f(b) \\ \phi(x) & \phi(b) \end{vmatrix}$$
and apply Lagrange's MVT on $[a, b]$.

15. Consider $f(t) = (1 + t)^n$ and apply Lagrange's MVT on $[0, x]$.

16. i. Consider $g(x) = e^{-x} f(x)$ and apply Lagrange's MVT on $[0, x]$.

    ii. Consider $g(x) = f(x)(a - x)(b - x)$ and apply Lagrange's MVT on $[a, b]$.

17. i. Consider $f(x) = \sin\ x$ and apply Lagrange's MVT on $[0, x]$ for right inequality. Consider $f(x) = \frac{\sin\ x}{x}$ and apply Lagrange's MVT on $[x, \pi/2]$ for left inequality.

    ii. Consider $g(x) = \tan\ x - x$ and apply Lagrange's MVT on $[0, x]$, $x < \frac{\pi}{4}$, for left inequality. Consider $g(x) = \frac{\tan\ x}{x}$ and apply Lagrange's MVT on $[x, \frac{\pi}{4}]$ for right inequality.

    iii. Choose $f(x) = \sin^{-1} x$ and apply Lagrange's MVT on $[0, x]$ for left inequality. Choose $g(x) = \sin^{-1} x - \frac{x}{\sqrt{1-x^2}}$ apply Lagrange's MVT on $[0, x]$ for right inequality.

    iv. Consider $f(x) = \log(1 + x)$ apply Lagrange's MVT on $[0, x]$ for right inequality. Consider $g(x) = \log(1 + x) - \frac{x}{1+x}$ apply Lagrange's MVT on $[0, x]$ for left inequality. Properly choose necessary values of x in the above inequalities and add the inequalities to get the next result.

18. $f'(x) = nx^{n-1} + p$ has only one real root when n is even and not more than two real roots when n is odd.

19. i. Consider $h(x) = \frac{f(x)}{x}$, $g(x) = \frac{1}{x}$ and apply Cauchy's MVT on $[a, b]$.

    ii. Consider $h(x) = f(x)$ and $g(x) = x^2$ and apply Cauchy's MVT on $[0, 1]$.

    iii. Consider $h(x) = f(x)$ and $k(x) = x, x^2, x^3$ successively and apply Cauchy's MVT on $[a, b]$ to compare the values of $\frac{f(b)-f(a)}{b-a}$.

20. Consider $f(x) = 1 - \cos\ x$ and $g(x) = \frac{x^2}{2}$ and apply Cauchy's MVT on $[0, x]$.