Answer Set - 12

AUTUMN 2017

MATHEMATICS - I(MA10001)

August , 2017

1. **Find Laurent series expansion of the function:**

   (a) $1 + \frac{3}{2}(1 - \frac{z}{2} + \frac{z^2}{2^2} - ...) - \frac{8}{3}(1 - \frac{z}{3} + \frac{z^2}{3^2} - ...)$

   (b) $1 + \frac{3}{z}(1 - \frac{2}{z} + \frac{2^2}{z^2} - ...) - \frac{8}{3}(1 - \frac{z}{3} + \frac{z^2}{3^2} - ...)$

   (c) $1 + \frac{3}{z}(1 - \frac{2}{z} + \frac{2^2}{z^2} - ...) - \frac{8}{z}(1 - \frac{3}{z} + \frac{3^2}{z^2} - ...)$

2. **Find Laurent series expansion in power of z of the function:**

   – Answer: $\frac{1}{z^3} - \frac{1}{z^5} + \frac{1}{z^7} - ...$

3. **Find Laurent series expansion in the given region:**

   – Answer: $\frac{-1}{6}(1 + \frac{z-2}{2} + \frac{(z-2)^2}{2^2} + ...) - \frac{1}{3(z-2)}(1 - \frac{1}{z-2} + \frac{1}{(z-2)^2} - \frac{1}{(z-2)^3} + ...)$

4. **Find the principal part of the following Laurent series:**

   (a) $\frac{1}{5}[\frac{1}{z}(1 - \frac{2}{z} + \frac{2^2}{z^2} - \frac{2^3}{z^3} + ...) - \frac{1}{z}(1 - \frac{1}{z^2} + \frac{1}{z^4} - ...)(1 - \frac{2}{z})$

   (b) $\frac{-1}{z}$ in the region $0 < |z| < 1$ and $\frac{1}{z^3} - \frac{1}{z^5} + \frac{1}{z^7} - ...$ in the region $|z| > 1$

   (c) $\frac{1}{z^2} - \frac{1}{z^4} + \frac{1}{z^6} - \frac{1}{z^8} + ...$ in the region $1 < |z| < \sqrt{2}$ and $\frac{1}{z^4} - \frac{3}{z^6} + \frac{7}{z^8} - ....$ in the region $|z| > \sqrt{2}$

   (d) $\frac{1}{z^3} - \frac{1}{3!z}$

   (e) $\frac{-3}{z} - \frac{1}{3!z^2} + \frac{3}{3!z^3} + ...$

5. **Find the principal part of the Laurent expansion of the following functions at the given point:**

   (a) Principal part $\frac{e^2}{(z-2)^2} + \frac{e^2}{z-2}$

   (b) Principal part $\frac{1}{z^2} - \frac{2}{z} + ...$

6. **Find the singularity of the following functions and classify them:**

   (i) $z = \infty$ [non isolated essential singularity].

   (ii) $z = 1$ [isolated essential singularity].

   (iii) $z = \infty$ [isolated essential singularity].

   (iv) $z = 0$ [non isolated essential singularity].

   (v) $z = 0$ [non isolated essential singularity].

   (vi) $z = \infty$ [isolated essential singularity].

   (vii) $z = 1$ [isolated essential singularity] and $z = 0$ is a pole of order 2.

   (viii) $z = \infty$ [non isolated essential singularity].

(ix) $z = 0$ [non isolated essential singularity].

(x) $z = -2$ [isolated essential singularity].

(xi) $z = 2i, -2i$ are simple poles.

(xii) $z = \infty$ [non isolated essential singularity].

(xiii) $z = \frac{\pi}{4}$ simple pole.

(xiv) $z = \infty$ [non isolated essential singularity].

7. **Find poles of the following functions and determine their order:**

(a) $z = 0$ is a pole of order 2 $z = 1$ is a pole of order 3.

(b) $z = i$ is a pole of order 2 $z = -i$ is a pole of order 2.

8. **Find each pole and its order and calculate residue at each of the pole:**

(a) $z = 1$ [pole of order 2],$z = -2$ [simple pole] [res at $z = 1$ is $\frac{5}{9}$ and res at $z = -2$ is $\frac{4}{9}$].

9. **Find residue of the functions:**

(a) res at $z = i$ is $\frac{6}{32i}$ and res at $z = -i$ is $\frac{-6}{32i}$

(b) res at $z = ia$ is $\frac{ai}{2}$

(c) res at $z = a$ is $\frac{-\pi}{(sin\pi a)^2}$

(d) res at $z = 0$ is -$\frac{1}{2}$

10. **Express the function in a series of positive and negative powers of** $(z - 1)$

$$\frac{-1}{2(z-1)} - \frac{3}{4}(1 + \frac{z-1}{2} + \frac{(z-1)^2}{2^2} + ...)$$

11. **Expand the function in Laurent series about** $z = 2$

$$1 + \frac{z}{z-2} + \frac{z^2}{2!(z-2)^2} + \frac{z^3}{3!(z-2)^3} + ...$$

11. **Using cauchy residue theorem evaluate the following integrals:**

(a) $-16\pi i$

(b) $-4\pi i$

(c) $\frac{-2\pi i}{9}$

(d) $2\pi i(1 + (\frac{-e^{\pi}}{4} - \frac{e^{-}\pi}{4}) + (-\frac{e^{\pi}}{4} - \frac{e^{-}\pi}{4}))$