Tutorial 4 Solution

(10) $\frac{d^2y}{dt^2} + 7\frac{dy}{dt} + 10y = x(t)$

$$\Rightarrow (j\omega)^2 Y(\omega) + 7(j\omega)Y(\omega) + 10\,Y(\omega) = X(\omega)$$

[taking Fourier transform on both the sides
$X(\omega) = \mathcal{F}\{x(t)\}$ and $Y(\omega) = \mathcal{F}\{y(t)\}$]

$$\Rightarrow \left((j\omega)^2 + 7(j\omega) + 10\right) Y(\omega) = X(\omega)$$

$$\Rightarrow Y(\omega) = \frac{X(\omega)}{(j\omega)^2 + 7(j\omega) + 10} = \frac{X(\omega)}{(j\omega+5)(j\omega+2)}$$

(a) $x(t) = \delta(t)$

$$\Rightarrow X(\omega) = 1$$

$$\therefore Y(\omega) = \frac{1}{(j\omega+5)(j\omega+2)} = \frac{1}{3}\left(\frac{1}{j\omega+2} - \frac{1}{j\omega+5}\right)$$

$$\therefore y(t) = \mathcal{F}^{-1}\left\{\frac{1}{3}\left(\frac{1}{j\omega+2} - \frac{1}{j\omega+5}\right)\right\}$$

$$= \frac{1}{3}\mathcal{F}^{-1}\left\{\frac{1}{j\omega+2}\right\} - \frac{1}{3}\mathcal{F}^{-1}\left\{\frac{1}{j\omega+5}\right\}$$

$$= \frac{1}{3}e^{-2t}u(t) - \frac{1}{3}e^{-5t}u(t)$$

$$= \frac{1}{3}\left(e^{-2t} - e^{-5t}\right)u(t).$$

(b) $x(t) = e^{-t}u(t) \Rightarrow X(\omega) = \frac{1}{j\omega+1}$

$$\therefore Y(\omega) = \frac{1}{(j\omega+1)(j\omega+5)(j\omega+2)}.$$

$$= \frac{A}{(j\omega+1)} + \frac{B}{(j\omega+2)} + \frac{C}{(j\omega+5)}$$

for some values of A, B and C

~~Such that~~

$$\cancel{\frac{1}{(J\omega+1)(J\omega+2)(J\omega+5)} = \frac{A(J\omega+2)(J\omega+5)+B(J\omega+1)(J\omega+5)+C(J\omega+1)(J\omega+2)}{}}$$

Such that

$$\frac{1}{(J\omega+1)(J\omega+2)(J\omega+5)} = \frac{A(J\omega+2)(J\omega+5)+B(J\omega+1)(J\omega+5)+C(J\omega+1)(J\omega+2)}{(J\omega+1)(J\omega+2)(J\omega+5)}$$

putting $J\omega=-1$ we get $A(-1+2)(-1+5)=1$

$$\Rightarrow A = 1/4$$

putting $J\omega=-2$ we get $B(-2+1)(-2+5)=1$

$$\Rightarrow B = -\frac{1}{3}$$

putting $J\omega=-5$ we get $C(-5+1)(-5+2)=1$

$$\Rightarrow C = \frac{1}{12}$$

$$\therefore Y(\omega) = \frac{1/4}{J\omega+1} - \frac{1/3}{J\omega+2} + \frac{1/12}{J\omega+5}$$

$$\therefore y(t) = \mathcal{F}^{-1}\{Y(\omega)\} = \left(\frac{1}{4}e^{-t} - \frac{1}{3}e^{-2t} + \frac{1}{12}e^{-5t}\right)u(t)$$

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(3) $x(t) = e^{-at}u(t)$

$$X(\omega) = \int_{-\alpha}^{\alpha} e^{-at}u(t)\,e^{-J\omega t}\,dt$$

$$= \int_{0}^{\alpha} e^{-(a+J\omega)t}\,dt$$

$$= \frac{-1}{a+J\omega}\left[e^{-(a+J\omega)t}\right]_0^{\alpha}$$

$$= \frac{e^{-(a+J\omega)t}\Big|_{t=0} - e^{-(a+J\omega)t}\Big|_{t=\alpha}}{a+J\omega}$$

$$= \frac{1-0}{a+J\omega}$$

$$\left[\because e^{-(a+J\omega)\alpha} = \left|e^{-(a+J\omega)\alpha}\right| \underline{\angle e^{-(a+J\omega)\alpha}}\right.$$

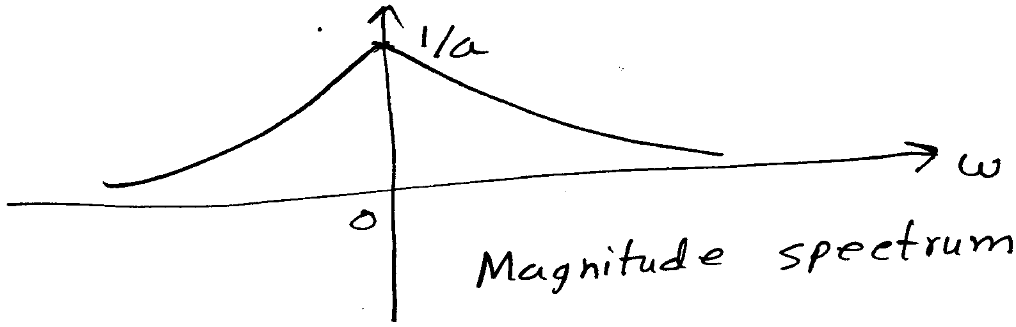
$$= \left|e^{-a\alpha}\right|\left|e^{-J\omega\alpha}\right| \underline{\angle e^{-(a+J\omega)\alpha}}$$

$$= 0 \times 1 \ \underline{\angle e^{-(a+J\omega)\alpha}}$$

$$\left. = 0 \right]$$

$$\therefore X(\omega) = \frac{1}{a+J\omega}$$

$$\therefore |X(\omega)| = \frac{1}{|a+J\omega|} = \frac{1}{\sqrt{a^2+\omega^2}}$$



Magnitude spectrum

9) a) $\mathcal{F}\left\{e^{-3t}\sin(4t)\,u(t)\right\}$

$$= \mathcal{F}\left\{\left(e^{-3t}u(t)\right)\frac{e^{J4t}-e^{-J4t}}{2J}\right\}$$

$$= \mathcal{F}\left\{\frac{e^{-3t}u(t)\,e^{J4t}}{2J}\right\} - \mathcal{F}\left\{e^{-3t}u(t)\,e^{-J4t}\right\}$$

$$= \frac{1}{2J}\,\frac{1}{J(\omega-4)+3} - \frac{1}{2J}\,\frac{1}{J(\omega+4)+3}$$

$\left[\because \text{since } \mathcal{F}\left\{e^{-3t}u(t)\right\} = \frac{1}{J\omega+3}\right.$

and modulation property implies

$\left.\mathcal{F}\left\{x(t)e^{J\omega_c t}\right\} = X(\omega-\omega_c)\right]$

$$= \frac{1}{2J}\left(\frac{\cancel{J\omega}+4J+\cancel{3}-\cancel{J\omega}+4J-\cancel{3}}{\left(J(\omega-4)+3\right)\left(J(\omega+4)+3\right)}\right) = \frac{4}{(J\omega+3)^2-(J4)^2}$$

$$= \frac{4}{(J\omega+3)^2+4^2}$$

Alternative solution

$$\mathcal{F}\{e^{-3t}\sin(4t)\,u(t)\} = \int_{-\alpha}^{\alpha} e^{-3t}\sin(4t)\,u(t)\,e^{-J\omega t}\,dt$$

$$= \int_{0}^{\alpha} e^{-3t}\sin(4t)\,e^{-J\omega t}\,dt$$

$$= \int_{0}^{\alpha} e^{-(3+J\omega)t}\,\frac{e^{J4t}-e^{-J4t}}{2J}\,dt$$

$$= \frac{1}{2J}\int_{0}^{\alpha} e^{-(3+J\omega-4J)t}\,dt - \frac{1}{2J}\int e^{-(3+J\omega+4J)t}\,dt$$

$$= \frac{1}{2J}\,\frac{\left[e^{-(3+J\omega-4J)t}\right]_{\alpha}^{0}}{3+J\omega-4J} - \frac{1}{2J}\,\frac{\left[e^{-(3+J\omega+4J)t}\right]_{\alpha}^{0}}{3+J\omega+4J}$$

$$= \frac{1}{2J}\,\frac{\left(1-e^{-(3+J\omega-4J)\alpha}\right)}{3+J\omega-4J} - \frac{1}{2J}\,\frac{\left(1-e^{-(3+J\omega+4J)\alpha}\right)}{3+J\omega+4J}$$

$$= \frac{1}{2J}\,\frac{(1-0)}{3+J\omega-4J} - \frac{1}{2J}\,\frac{(1-0)}{3+J\omega+4J}$$

$\Big[\because\ e^{-(3+J\omega-4J)\alpha} = e^{-3\alpha}\times e^{(J\omega-4J)\alpha}$

and $|e^{-3\alpha}| = 0$ but $|e^{(J\omega-4J)\alpha}| = 1$

so $e^{-(3+J\omega-4J)\alpha} = 0$

similarly $e^{-(3+J\omega+4J)} = 0\Big]$

$$= \frac{1}{2J}\left(\frac{\cancel{3}+\cancel{J\omega}+4J-\cancel{3}-\cancel{J\omega}+4J}{(3+J\omega)^2-(4J)^2}\right)$$

$$= \frac{4}{(3+J\omega)^2+4^2}$$

(b) $\mathcal{F}\{\delta(t+4)+\delta(t+2)+\delta(t-2)+\delta(t-4)\}$

$$= \mathcal{F}\{\delta(t+4)\}+\mathcal{F}\{\delta(t+2)\}+\mathcal{F}\{\delta(t-2)\}+\mathcal{F}\{\delta(t-4)\}$$

$$= 1\times e^{J\omega 4} + 1\times e^{J\omega 2} + 1\times e^{-J\omega 2} + 1\times e^{-J\omega 4}$$

$[\because \mathcal{F}\{\delta(t)\} = 1$ and using time shifting property $]$

$$= \left(e^{J\omega 4}+e^{-J\omega 4}\right) + \left(e^{J\omega 2}+e^{-J\omega 2}\right)$$

$$= 2\cos(4\omega) + 2\cos(2\omega)$$

Alternative solution

$\mathcal{F}\{\delta(t+4)+\delta(t+2)+\delta(t-2)+\delta(t-4)\}$

$$= \int_{-\infty}^{\infty}\left(\delta(t+4)+\delta(t+2)+\delta(t-2)+\delta(t-4)\right)e^{-J\omega t}\,dt$$

$$= \int_{-\infty}^{\infty}\delta(t+4)e^{-J\omega t}\,dt + \int_{-\infty}^{\infty}\delta(t+2)e^{-J\omega t}\,dt$$

$$+ \int_{-\infty}^{\infty}\delta(t-2)e^{-J\omega t}\,dt + \int_{-\infty}^{\infty}\delta(t-4)e^{-J\omega t}\,dt$$

$$= e^{J\omega 4}+e^{J\omega 2}+e^{-J\omega 2}+e^{-J\omega 4}$$

$= 2\cos(4\omega) + 2\cos(2\omega)$

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7) a) $x_1(t) = 2e^{-2t}u(t)$

$\therefore X_1(\omega) = 2\frac{1}{J\omega+2}$

$x_2(t) = e^{-4t}u(t)$

$\therefore X_2(\omega) = \frac{1}{J\omega+4}$

$\therefore \mathcal{F}\{x_1(t) * x_2(t)\} = X_1(\omega)X_2(\omega)$

$= \frac{2}{(J\omega+2)(J\omega+4)}$

$= \frac{1}{J\omega+2} - \frac{1}{J\omega+4}$

$\therefore x_1(t) * x_2(t) = \mathcal{F}^{-1}\left\{\frac{1}{J\omega+2} - \frac{1}{J\omega+4}\right\}$

$= e^{-2t}u(t) - e^{-4t}u(t)$

$= (e^{-2t} - e^{-4t})u(t)$

(b) $x_1(t) = te^{-t}u(t)$

$\therefore X_1(\omega) = J\frac{d}{d\omega}\left(\frac{1}{J\omega+1}\right) = J\frac{-1}{(J\omega+1)^2} \times J = \frac{1}{(J\omega+1)^2}$

$x_2(t) = e^{-2t}u(t)$

$\therefore X_2(\omega) = \frac{1}{J\omega+2}$

$$\therefore \mathcal{F}\{x_1(t) * x_2(t)\} = X_1(\omega)\,X_2(\omega)$$

$$= \frac{1}{(j\omega+1)^2(j\omega+2)}$$

$$= \frac{(j\omega+2)-(j\omega+1)}{(j\omega+1)^2(j\omega+2)}$$

$$= \frac{1}{(j\omega+1)^2} - \frac{1}{(j\omega+1)(j\omega+2)}$$

$$= \frac{1}{(j\omega+1)^2} - \left(\frac{1}{j\omega+1} - \frac{1}{j\omega+2}\right)$$

$$= \frac{1}{(j\omega+1)^2} - \frac{1}{j\omega+1} + \frac{1}{j\omega+2}$$

$$\therefore x_1(t) * x_2(t) = \mathcal{F}^{-1}\left\{\frac{1}{(j\omega+1)^2} - \frac{1}{j\omega+1} + \frac{1}{j\omega+2}\right\}$$

$$= te^{-t}u(t) - e^{-t}u(t) + e^{-2t}u(t)$$

$$= \left((t-1)e^{-t} + e^{-2t}\right)u(t)$$

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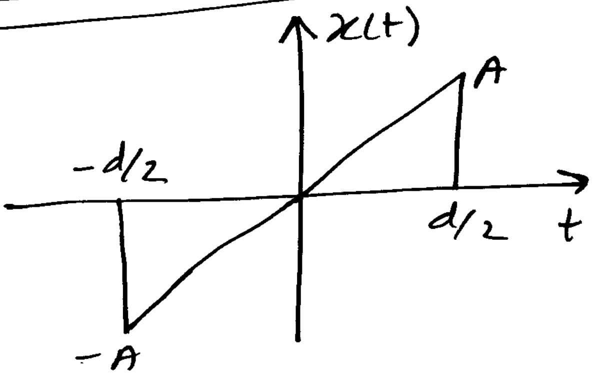
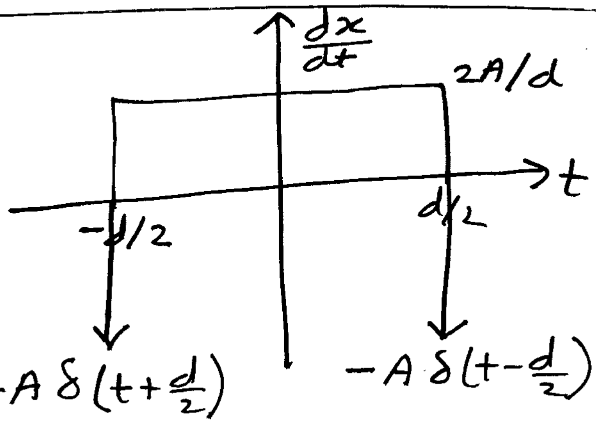
6)



$\Rightarrow$



$$\mathcal{F}\left\{\frac{dx}{dt}\right\} = \int_{-\infty}^{\infty} \frac{dx}{dt}\, e^{-j\omega t}\, dt$$

$$= \int_{-\frac{d}{2}}^{\frac{d}{2}} \frac{2A}{d} e^{-j\omega t} dt + \int_{-\frac{d}{2}^-}^{-\frac{d}{2}^+} \left(-A\,\delta\left(t+\frac{d}{2}\right)\right) e^{-j\omega t} dt + \int_{\frac{d}{2}^-}^{\frac{d}{2}^+} \left(-A\,\delta\left(t-\frac{d}{2}\right)\right) e^{-j\omega t} dt$$

$$= \frac{2A}{d} \frac{\left[e^{-j\omega t}\right]_{-d/2}^{d/2}}{-j\omega} - A e^{-j\omega\left(-\frac{d}{2}\right)} - A e^{-j\omega\frac{d}{2}}$$

$$= \frac{2A}{d} \frac{e^{j\omega\frac{d}{2}} - e^{-j\omega\frac{d}{2}}}{j\omega} - A\left(e^{j\omega\frac{d}{2}} + e^{-j\omega\frac{d}{2}}\right)$$

$$= \frac{4A}{d\omega} \sin\left(\omega\frac{d}{2}\right) - 2A\cos\left(\omega\frac{d}{2}\right)$$

$$= 2A\left[\frac{\sin\left(\frac{\omega d}{2}\right)}{\left(\frac{\omega d}{2}\right)} - \cos\left(\omega\frac{d}{2}\right)\right]$$

$$\therefore \mathcal{F}\left\{\frac{dx}{dt}\right\} = 2A\left[\frac{\sin\left(\frac{\omega d}{2}\right)}{\left(\frac{\omega d}{2}\right)} - \cos\left(\frac{\omega d}{2}\right)\right] = X'(\omega) \text{ (say)}$$

Now $X'(0) = 0$

$$\therefore \mathcal{F}\{x(t)\} = \frac{X'(\omega)}{j\omega} + \pi\,\delta(\omega)\,X'(0)$$

$$= \frac{X'(\omega)}{j\omega} + 0$$

$$= \frac{2A}{j\omega}\left[\frac{\sin\left(\frac{\omega d}{2}\right)}{\left(\frac{\omega d}{2}\right)} - \cos\left(\frac{\omega d}{2}\right)\right]$$

Alternative Solution

$$\mathcal{F}\{x(t)\} = \int_{-\alpha}^{\alpha} x(t)\, e^{-J\omega t}\, dt$$

$$= \int_{-d/2}^{d/2} \frac{2A}{d}\, t\, e^{-J\omega t}\, dt$$

$$= \frac{2A}{d}\left[\left[t\, \frac{e^{-J\omega t}}{-J\omega}\right]_{-d/2}^{d/2} - \int_{-d/2}^{d/2} \frac{e^{-J\omega t}}{-J\omega}\, dt\right]$$

$$= \frac{2A}{d}\left[\frac{\left(\frac{d}{2}\, e^{-J\omega\frac{d}{2}} + \frac{d}{2}\, e^{J\omega\frac{d}{2}}\right)}{-J\omega} + \frac{1}{J\omega} \times \frac{1}{(-J\omega)}\left[e^{-J\omega t}\right]_{-d/2}^{d/2}\right]$$

$$= \frac{2A}{d}\left[-\frac{d}{J\omega}\, \overset{\cos}{\cancel{\sin}}\left(\frac{\omega d}{2}\right) + \frac{1}{\omega^2} \times (-2J)\sin\left(\frac{\omega d}{2}\right)\right]$$

$$= 2A\left[-\frac{1}{J\omega}\, \overset{\cos}{\cancel{\sin}}\left(\frac{\omega d}{2}\right) + \frac{2}{\omega J} \times \frac{1}{\omega d}\sin\left(\frac{\omega d}{2}\right)\right]$$

$$= 2A\left[-\frac{\overset{\cos}{\cancel{\sin}}\left(\frac{\omega d}{2}\right)}{J\omega} + \frac{1}{\omega J}\, \frac{\sin\left(\frac{\omega d}{2}\right)}{\left(\frac{\omega d}{2}\right)}\right]$$

$$= \frac{2A}{J\omega}\left[\frac{\sin\left(\frac{\omega d}{2}\right)}{\frac{\omega d}{2}} - \cos\left(\frac{\omega d}{2}\right)\right]$$

8) a) $x(t) = \mathcal{F}^{-1}\{X(\omega)\} = \mathcal{F}^{-1}\left\{\frac{J\omega}{(J\omega+3)^2}\right\}$

$= \mathcal{F}^{-1}\left\{\frac{J\omega+3-3}{(J\omega+3)^2}\right\}$

$= \mathcal{F}^{-1}\left\{\frac{1}{J\omega+3} - \frac{3}{(J\omega+3)^2}\right\}$

$= \mathcal{F}^{-1}\left\{\frac{1}{J\omega+3}\right\} - 3\,\mathcal{F}^{-1}\left\{\frac{1}{(J\omega+3)^2}\right\}$

$= e^{-3t}u(t) - 3\,t e^{-3t}u(t)$

$= (1-3t)e^{-3t}ut$

$\cancel{b)\ x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\, e^{J\omega t} d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-4\omega} u(\omega)}$

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b) $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega) e^{J\omega t} d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-4\omega} u(\omega) e^{J\omega t} d\omega$

$= \frac{1}{2\pi}\int_{0}^{\infty} e^{-4\omega} e^{J\omega t} d\omega = \frac{1}{2\pi}\int_{0}^{\infty} e^{(Jt-4)\omega} d\omega$

$= \frac{\left[e^{(Jt-4)\omega}\right]_0^{\infty}}{2\pi(Jt-4)} = \frac{0-1}{2\pi(Jt-4)} = \frac{1}{(4-Jt)2\pi}$

Alternative solution.

$\mathcal{F}\{x(t)\} = e^{-4\omega}u(\omega)$

$\therefore \mathcal{F}\{e^{-4t}u(\omega)\} = 2\pi\, x(-\omega)$ [Duality property]

$\Rightarrow \frac{1}{J\omega+4} = 2\pi\, x(-\omega)$

$\Rightarrow x(\omega) = \frac{1}{2\pi}\,\frac{1}{4-J\omega} \Rightarrow x(t) = \frac{1}{2\pi(4-Jt)}$

4) a) $\mathcal{F}\{x(t)\} = \mathcal{F}\{e^{-2t}(u(t-1)-u(t-2))\}$

$$= \int_{-\infty}^{\infty} e^{-2t}(u(t-1)-u(t-2))\, e^{-J\omega t}\, dt$$

$$= \int_{1}^{2} e^{-2t} e^{-J\omega t}\, dt = \int_{1}^{2} e^{-(2+J\omega)t}\, dt$$

$$= \frac{\left[e^{-(2+J\omega)t}\right]_1^2}{-(2+J\omega)} = \frac{e^{-(2+J\omega)} - e^{-2(2+J\omega)}}{2+J\omega}$$

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b) $\mathcal{F}\{x(t)\} = \mathcal{F}\{e^{-2t}(u(t-1)-u(t-2))\}$

$$= \mathcal{F}\{e^{-2t}u(t-1)\} - \mathcal{F}\{e^{-2t}u(t-2)\}$$

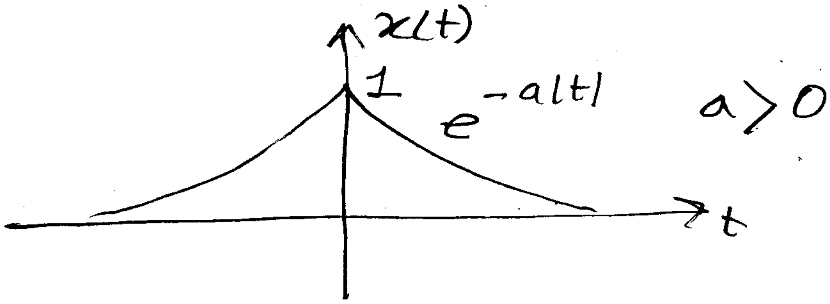
$$= \mathcal{F}\{e^{-2(t-1)}u(t-1)e^{-2}\} - \mathcal{F}\{e^{-2(t-2)}u(t-2)e^{-4}\}$$

$$= e^{-2}\mathcal{F}\{e^{-2(t-1)}u(t-1)\} - e^{-4}\mathcal{F}\{e^{-2(t-2)}u(t-2)\}$$

$$= e^{-2}\frac{1}{J\omega+2} \times e^{-J\omega} - e^{-4}\frac{1}{J\omega+2} e^{-J2\omega}$$ [Time shifting]

$$= \frac{e^{-(2+J\omega)} - e^{-(4+2J\omega)}}{J\omega+2}$$

5)



(i) $X(w) = \int_{-\infty}^{\infty} x(t) e^{-Jwt} dt$

$= \int_{-\infty}^{0} e^{-a(-t)} e^{-Jwt} dt + \int_{0}^{\infty} e^{-at} e^{-Jwt} dt$

$= \int_{-\infty}^{0} e^{(a-Jw)t} dt + \int_{0}^{\infty} e^{-(a+Jw)} dt$

$= \frac{\left[e^{(a-Jw)t}\right]_{-\infty}^{0}}{a-Jw} + \frac{\left[e^{-(a+Jw)t}\right]_{0}^{\infty}}{-(a+Jw)}$

$= \frac{1-0}{a-Jw} + \frac{0-1}{-(a+Jw)}$

$= \frac{a+\cancel{Jw}+a-\cancel{Jw}}{a^2+w^2} = \frac{2a}{a^2+w^2}$

(ii) $X(w) = \mathcal{F}\{x(t)\} = \mathcal{F}\{e^{-at}u(t) + e^{+at}u(-t)\}$

$= \frac{1}{Jw+a} + \frac{1}{J(-w)+a}$ [using time reversal property]

$= \frac{2a}{a^2+w^2}$

Calculation in time domain

(iii) Energy $= \int_{-\infty}^{\infty} |x(t)|^2 dt$

$$= 2\int_{0}^{\infty} |x(t)|^2 dt \quad [\because x(t) \text{ is an even function}]$$

$$= 2\int_{0}^{\infty} (e^{-at})^2 dt = 2\int_{0}^{\infty} e^{-2at} dt$$

$$= 2\frac{\left[e^{-2at}\right]_0^{\infty}}{-2a} = -\frac{1}{a}(0-1) = \frac{1}{a}$$

Calculation in frequency domain

$$\text{Energy} = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(\omega)|^2 d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \left(\frac{2a}{a^2+\omega^2}\right)^2 d\omega$$

$$= \frac{4a^2}{2\pi} \times 2\int_{0}^{\infty} \frac{1}{(a^2+\omega^2)^2} d\omega \quad [\because X(\omega) \text{ is even function}]$$

$$= \frac{4a^2}{\pi}\int_{0}^{\infty} \frac{1}{(a^2+\omega^2)^2} d\omega$$

putting $\omega = a\tan\theta$

$\Rightarrow d\omega = a\sec^2\theta \, d\theta$

ω	0	∞
θ	0	$\pi/2$

$$\therefore \text{Energy} = \frac{4a^2}{\pi}\int_0^{\pi/2} \frac{1}{(a^2+a^2\tan^2\theta)^2} \times a\sec^2\theta \, d\theta$$

$$= \frac{4a^2}{\pi}\int_0^{\pi/2} \frac{a\sec^2\theta \, d\theta}{a^4\sec^4\theta}$$

$$= \frac{4}{\pi a}\int_0^{\pi/2} \cos^2\theta \, d\theta$$

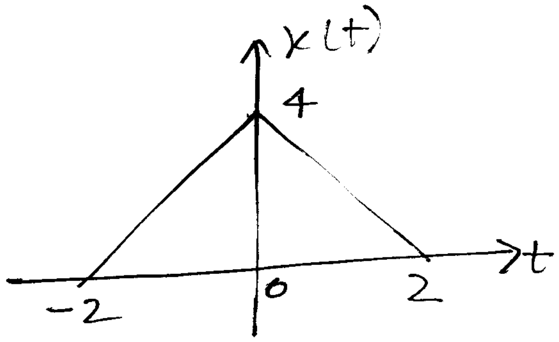
$$= \frac{4}{\pi a}\int_0^{\pi/2} \frac{\cos 2\theta + 1}{2} \, d\theta$$

$$= \frac{4}{2\pi a}\left[\frac{\sin 2\theta}{2} + \theta\right]_0^{\pi/2}$$

$$= \frac{4}{2\pi a}\left(\frac{\pi}{2}\right)$$

$$= \frac{1}{a}$$

(2)



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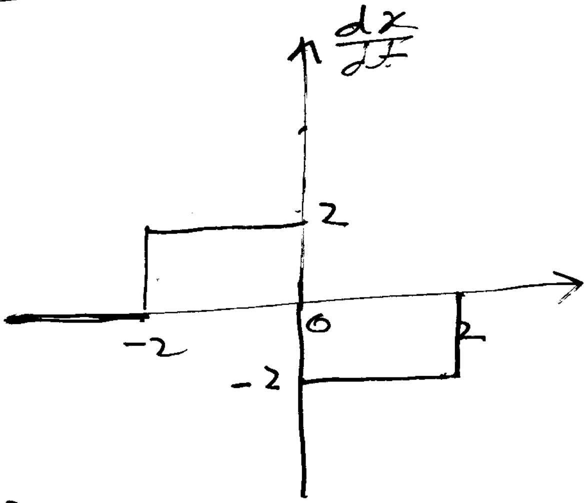

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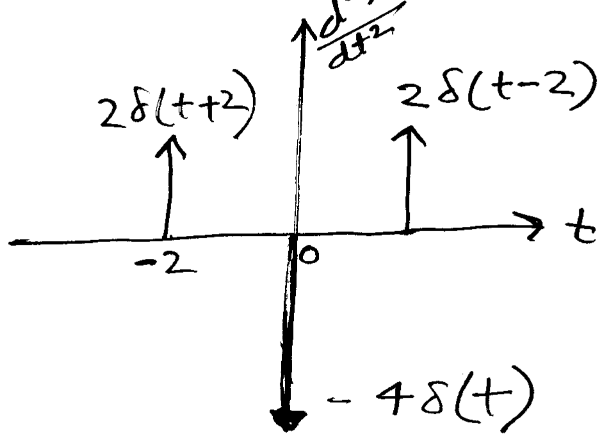


$$\therefore \mathcal{F}\left\{\frac{d^2x}{dt^2}\right\} = \int_{-\infty}^{\infty} \left(2\delta(t+2)+2\delta(t-2)-4\delta(t)\right)e^{-j\omega t}\,dt$$

$$= 2e^{-j\omega(-2)} + 2e^{-j\omega 2} - 4e^{j\omega 0}$$

$$= 2(2\cos(2\omega)) - 4$$

$$= 4(\cos 2\omega - 1)$$

$$= 4(1-2\sin^2\omega - 1)$$

$$= -8\sin^2\omega$$

$$\therefore \mathcal{F}\left\{\frac{dx}{dt}\right\} = -8\sin^2\omega \times \left(\frac{1}{j\omega} + \pi\delta(\omega)\right)$$

$$= \frac{-8\sin^2\omega}{j\omega} + \pi\delta(\omega)\times(-8\sin^2(0))$$

$$= \frac{-8\sin^2\omega}{j\omega} + 0$$

$$\therefore \mathcal{F}\{x(t)\} = \frac{-8\sin^2\omega}{j\omega} \times \left(\frac{1}{j\omega} + \pi\delta(\omega)\right)$$

$$= \frac{-8\sin^2\omega}{j^2\omega^2} + \pi\delta(\omega)\times 0$$

$$= \frac{8\sin^2\omega}{\omega^2}$$

$$= 8\,\mathrm{sinc}^2\omega$$

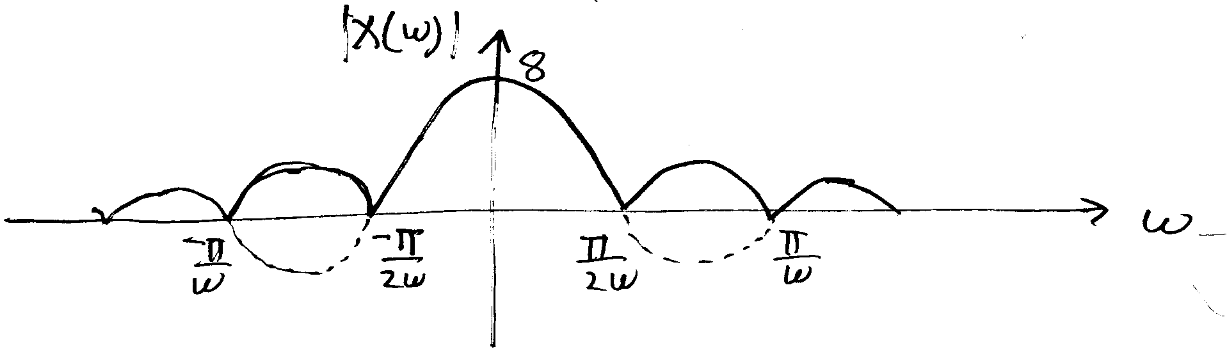
(1) a) $\mathcal{F}\{x(t)\} = \int_{-\infty}^{\infty} x(t) e^{-j\omega t}\, dt$

$$= \int_{-2}^{2} 2e^{-j\omega t}\, dt \quad [\because x(t) = 0 \text{ if } t < -2 \text{ or } t > 2]$$

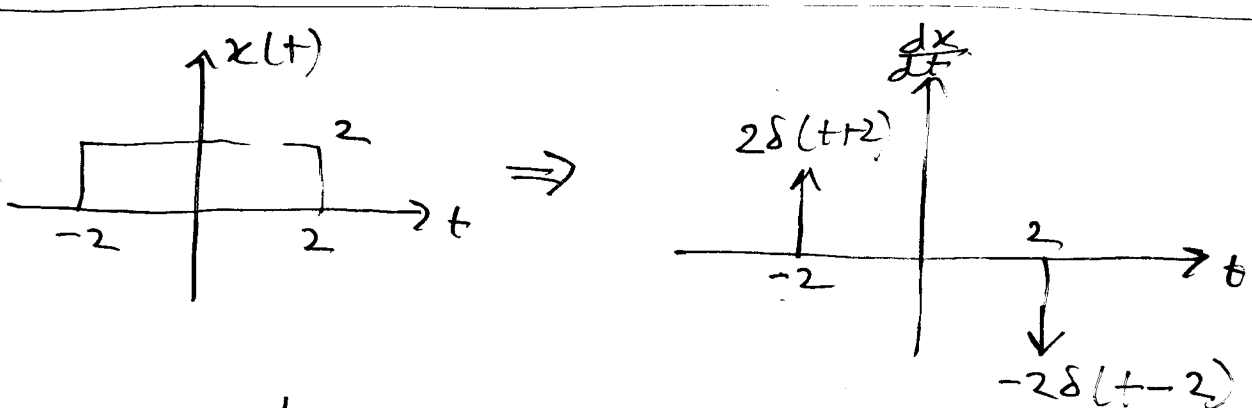
$$= \left[2\frac{e^{-j\omega t}}{-j\omega}\right]_{-2}^{2}$$

$$= 2\,\frac{e^{j\omega 2} - e^{-j\omega 2}}{j\omega}$$

$$= \frac{4}{\omega}\sin 2\omega = 8\,\frac{\sin 2\omega}{2\omega} = 8\ \mathrm{sinc}\,2\omega$$



(b)



$$\mathcal{F}\left\{\frac{dx}{dt}\right\} = \int_{-\infty}^{\infty} \frac{dx}{dt} e^{-j\omega t}\, dt$$

$$= \int_{-2^-}^{-2^+} 2\delta(t+2) e^{-j\omega t}\, dt + \int_{2^-}^{2^+} -2\delta(t-2) e^{-j\omega t}\, dt$$

$$= 2e^{-j\omega(-2)} - 2e^{-j\omega 2} = 4j\sin(2\omega)$$

(e) $x(t) = \frac{dx}{dt} * u(t)$

$$\therefore \mathcal{F}\{x(t)\} = \mathcal{F}\{\tfrac{dx}{dt}\}\,\mathcal{F}\{u(t)\}$$

$$= 4j\sin(2w) \times \left(\frac{1}{jw} + \pi\delta(w)\right)$$

$$= \frac{4\sin(2w)}{w} + \pi\delta(w) \times 4j\sin(0)$$

$$= \frac{8\sin(2w)}{2w} + 0$$

$$= 8\,\mathrm{sinc}(2w)$$