Problem Set - 9

AUTUMN 2017

MATHEMATICS-I (MA10001)

July 26, 2017

1. Solve the following differential equations:

   (a) $x^2y'' + y = 3x^2$,
   (b) $x^2y'' - 2y = x^2 + x^{-1}$,
   (c) $xy''' + y'' = x^{-1}$,
   (d) $x^2y'' + 2xy' = \ln x$,
   (e) $x^3y''' + 3x^2y'' + xy' + y = \ln x + x$,
   (f) $3x^2y'' - 5xy' + 5y = \sin(\ln x)$,
   (g) $x^2y'' - 2xy' + 2y = x + x^2 \ln x + x^3$,
   (h) $x^4y^{iv} + 6x^3y''' + 9x^2y'' + 3xy' + y = (1 + \ln x)^2$,
   (i) $x^2y'' - xy' + 4y = \cos(\ln x) + x\sin(\ln x)$,
   (j) $x^2y''' + 2y' - 2x^{-1}y = x\ln x + 3$,
   (k) $y'' + \dfrac{1}{x}y' = \dfrac{12\ \ln x}{x^2}$.

2. Solve the following differential equations:

   (a) $(1 + x)^2y'' + (1 + x)y' + y = 4\cos(\ln(1 + x))$,
   (b) $(x + 3)^2y'' - 4(x + 3)y' + 6y = x$,
   (c) $(3x + 2)^2y'' + 3(3x + 2)y' - 36y = 3x^2 + 4x + 1$.

3. Apply the method of variation of parameters to solve the following differential equations:

   (a) $y'' + 4y = 4\ \tan 2x$,
   (b) $y'' - 3y' + 2y = \dfrac{e^x}{(1 + e^x)}$,
   (c) $x^2y'' - 2xy' + 2y = x\ \ln x, \quad x > 0$,
   (d) $x^2y'' + 3xy' + y = \dfrac{1}{(1 - x)^2}$,
   (e) $y'' + 3y' + 2y = x + \cos x$,
   (f) $y'' + 2y' + 5y = e^{-x}\sec 2x$,
   (g) $y''' + y' = \sec x$,
   (h) $y''' - 6y'' + 11y' - 6y = e^{2x}$.

4. Using the method of variation of parameters, solve

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x \sin x$$

   with $y(0) = 0$ and $(\frac{dy}{dx})_{x=0} = 0$.

5. Solve the following system of differential equations :

(a) $\dfrac{dx}{dt} = 3x + 2y, \qquad \dfrac{dy}{dt} = 5x + 3y,$

(b) $\dfrac{dx}{dt} + 5x + y = e^t, \qquad \dfrac{dy}{dt} = x - 3y + e^{2t},$

(c) $\dfrac{dx}{dt} + 4x + 3y = t, \qquad \dfrac{dy}{dt} + 2x + 5y = e^t,$

(d) $\dfrac{dx}{dt} + 2x - 3y = t, \qquad \dfrac{dy}{dt} - 3x + 2y = e^{2t},$

(e) $\dfrac{dx}{dt} + \dfrac{dy}{dt} - 2y = 2\cos t - 7\sin t, \qquad \dfrac{dx}{dt} - \dfrac{dy}{dt} + 2x = 4\cos t - 3\sin t.$