PA - Test 4

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1. Let the given instance be (G, s, k) where k is the parameter. If we can build a boolean circuit with wgt-1, then by the definition of W[1]-class, we can say Partial Vertex Cover belongs to W[1] complexity class.

Let $V = V(G)$,

$E = E(G)$,

$F = \{X \mid X \subseteq 2^{E(G)}, |X| = k\}$ ↳ all subsets of

create boolean circuit such that:-

i) Create a input node v, $\forall v \in V$

ii) Create an or-node e_{uv}, $\forall (u,v) \in E$.
Make an edge from input node u to e_{uv} and an edge from input node v to e_{uv}.

iii) Create an and-node X, $\forall X \in F$.
Make an edge from node e_{ij} to node X, if (i,j) edge $\in X$.

iv) Create an and-node O, which is also the output node. Make an edge from all nodes of the form

X, where X ∈ F.

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Here, we can see
nodes of i) has 0 - indegree,
ii) has 2 - indegree
iii) has ℓ - indegree
iv) has 2^{ℓ} - indegree.

It can be seen that every path has exactly 1 node from i), ii), iii), iv). And only nodes from iv) are large nodes. Hence, the above circuit is a weft-1 circuit. (iii) is not a large node as ℓ is a constant)

$\Rightarrow$ Partial Vertex Cover ∈ W[I].

2. ~~This problem~~ we will use the result that UNIT-DISK-INDEP-SET has no $f(k)\,n^{o(\sqrt{k})}$ algorithm unless ETH fails, (Lyaan et. al Pg. 500)

we can show that there is a reduction from Unit-Disk-Independent-set to Exact cover on Squares where parameter transform is linear.

∴ If Exact cover on Squares can be solved in $f(k)\,n^{o(\sqrt{k})}$ then UNIT-DISK-INDEP-SET can be solved in $f(k)\,n^{o(\sqrt{k})}$. This is false,

unless ETH fails.

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$\Rightarrow$ For no function $f(k)n^{O(k)}$ can be a algorithm

~~So if we use unit~~

The reduction actually reduces this to UNIT-SQUARE-INDEP-SET which is the same as the UNIT-CIRCLE-INDEP-SET.