Tutorial 1 Solution

1) i) $f(t) = \sin\left(2t + \frac{\pi}{2}\right)$

$= \cos(2t)$ (It is even by observation)

Even part $= \dfrac{f(t) + f(-t)}{2}$

$= \dfrac{\cos(2t) + \cos(2(-t))}{2}$

$= \cos(2t)$

Odd part $= \dfrac{f(t) - f(-t)}{2}$

$= \dfrac{\cos(2t) - \cos(2(-t))}{2}$

$= 0$

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ii) $f(t) = 1 - 2t + 3t^3$

Even part $= \dfrac{f(t) + f(-t)}{2}$

$= \dfrac{(1 - 2t + 3t^3) + (1 + 2t - 3t^3)}{2}$

$= 1$

odd part $= \dfrac{f(t) - f(-t)}{2}$

$= \dfrac{(1 - 2t + 3t^3) - (1 + 2t - 3t^3)}{2}$

$= -2t + 3t^3$

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iii) $f(t) = \sin(2t) + \sin(2t)\cos(2t) + \cos(2t)$

$= \sin(2t) + \frac{1}{2}\sin(4t) + \cos(2t)$

Even part $= \frac{f(t) + f(-t)}{2}$

$= \left[\left(\sin(2t) + \frac{1}{2}\sin(4t) + \cos(2t)\right) + \left(-\sin(2t) - \frac{1}{2}\sin(4t) + \cos(2t)\right)\right] \times \frac{1}{2}$

$= \cos(2t)$

Odd part $= \frac{f(t) - f(-t)}{2}$

$= \left(\left(\sin(2t) + \frac{1}{2}\sin(4t) + \cos(2t)\right) - \left(-\sin(2t) - \frac{1}{2}\sin(4t) + \cos(2t)\right)\right) \times \frac{1}{2}$

$= \sin(2t) + \frac{1}{2}\sin(4t)$

iv) $f(t) = e^{j2t}$

Even part $= \frac{f(t) + f(-t)}{2}$

$= \frac{e^{j2t} + e^{-j2t}}{2}$

$= \frac{\cos 2t + j\sin 2t + \cos(2t) + j\sin(-2t)}{2}$

$= \cos(2t)$

odd part $= \frac{f(t) - f(-t)}{2}$

$$= \frac{e^{J2t} - e^{-J2t}}{2}$$

$$= \frac{\cos 2t + J\sin 2t - \cos 2t + J\sin 2t}{2}$$

$$= J \sin 2t$$

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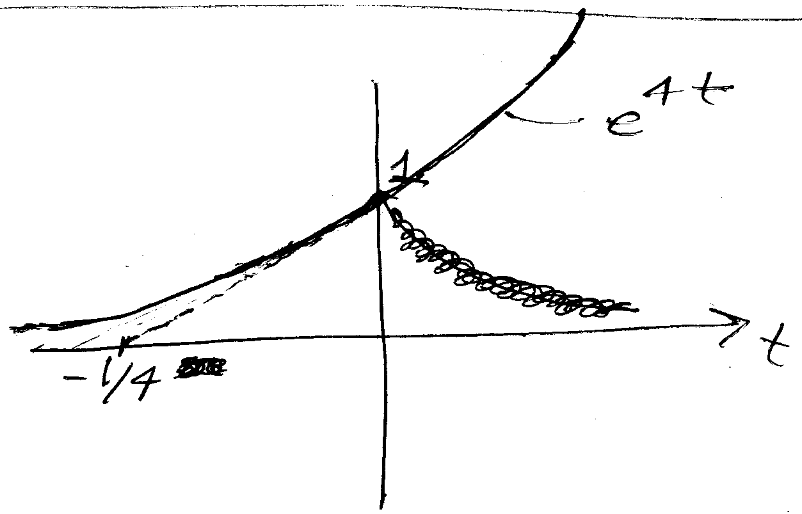
2) i) $f(t) = e^{4t}$

$f(-t) = e^{-4t}$

$\therefore f(-t) \neq f(t)$

and $f(-t) \neq -f(t)$

$\Rightarrow$ Neither even nor odd



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ii) $f(t) = u(t+2) - u(t-2)$

$f(-t) = u(-t+2) - u(-t-2)$

$= \left(u(-t+2) - 1\right) + \left(1 - u(-t-2)\right)$

$= -u(t-2) + u(t+2) = f(t)$

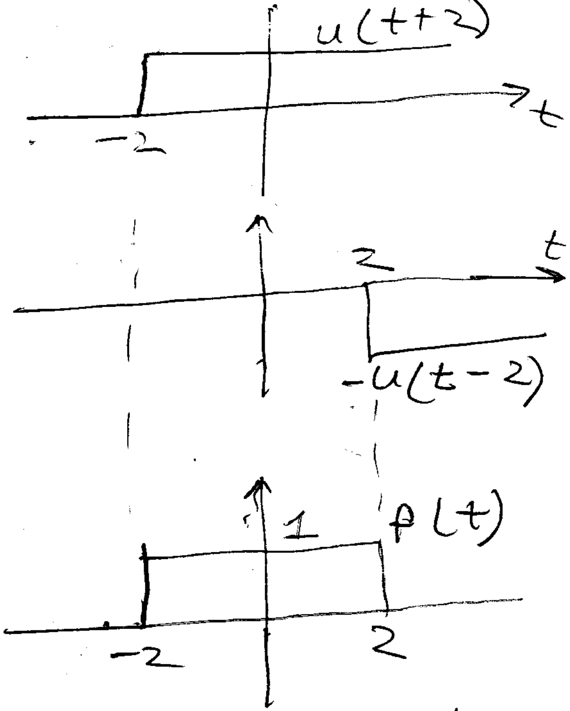
$\left[\because \boxed{1 - u(-t) = u(t)}\right.$

$\Rightarrow 1 - u(-(t-2)) = u(t-2)$

$\Rightarrow 1 - u(-t+2) = u(t-2)$

and $1 - u(-(t+2)) = u(t+2)$

$\left.\Rightarrow 1 - u(-t-2) = u(t+2)\right]$



from observation $f(t)$ is even.

Note: Only graphical proof is sufficient. Some students asked for an algebraic proof. So an algebraic proof is also given.

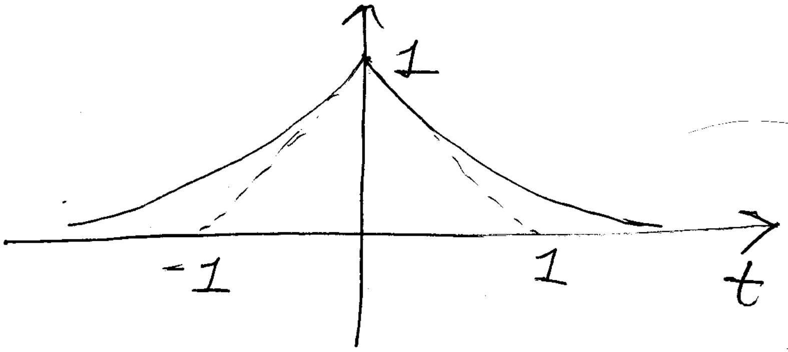
iii) $e^{-|t|} = f(t)$

$f(-t) = e^{-|-t|}$

$= e^{-|t|}$

$= f(t)$

$\therefore$ even function



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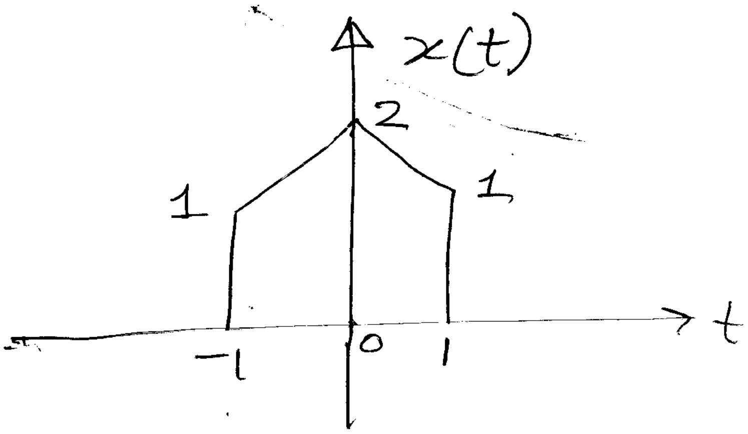
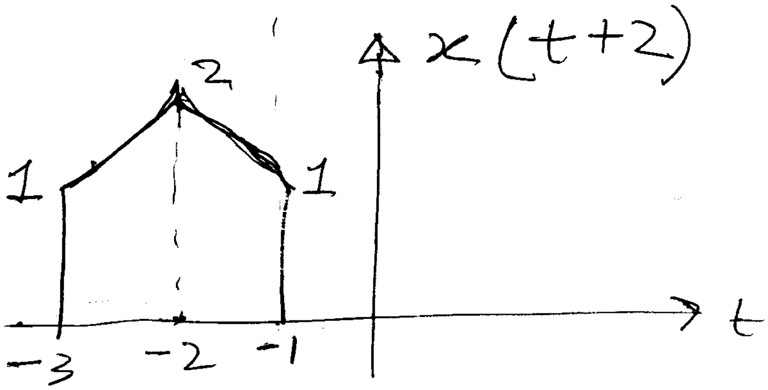
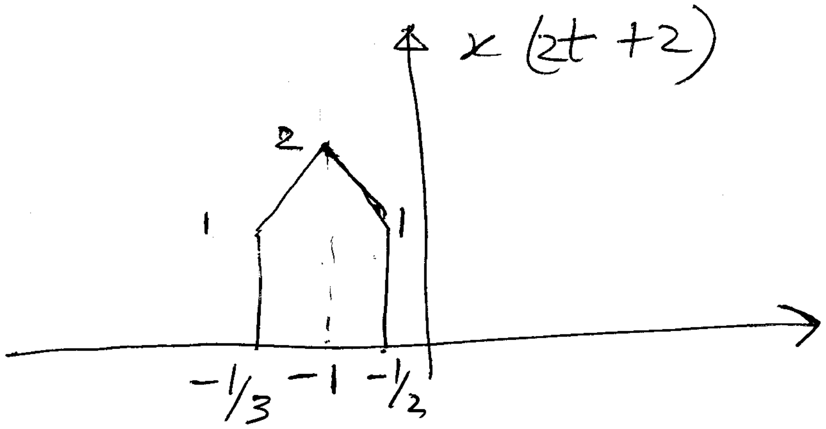
3)(i)



$\Rightarrow$



$\Rightarrow$



<u>Method</u>

Replace t with $t+2$ and shift the graph 2 units to the left

$\Downarrow$

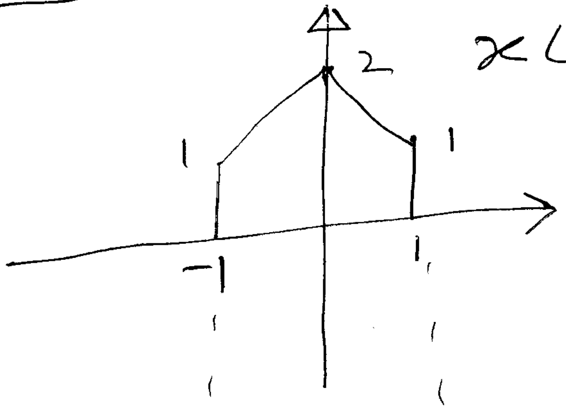
Replace t with $2t$ and squeeze the graph to half size horizont-ally. The squeezing should be arroud the y axis and NOT arround the centre of the graph

$\Downarrow$

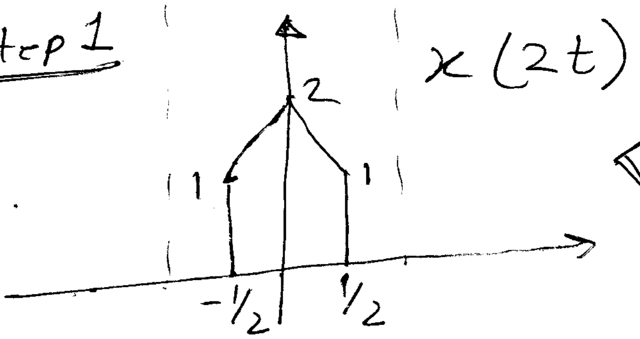
ANSWER

<u>Verify your answer</u>: The peak of $x(t)$ is at $t=0$. So the peak of $x(2t+2)$ should be at $(2t+2)=0 \Rightarrow t=-1$. Our answer is consistent with this requirement.

COMMON MISTAKE 1

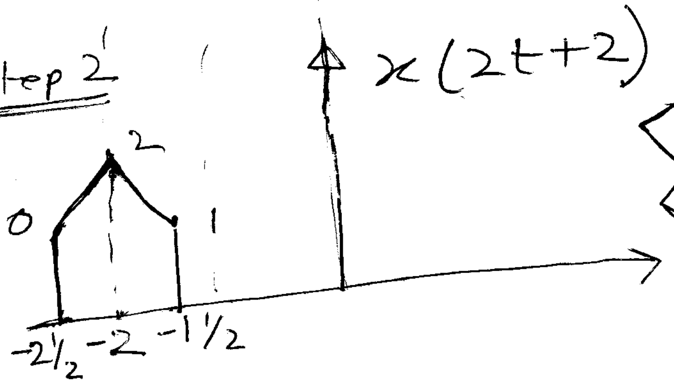


Step 1



Step 1 is correct. Here t is replaced with 2t and the graph is squeezed.

Step 2



Step 2 is incorrect because (2t) becomes (2t+2) $\Rightarrow$ (t) becomes (t+1)

$\boxed{2(t+1) = 2t+2}$

So the graph should shift 1 unit to the left & NOT to the left 2 units

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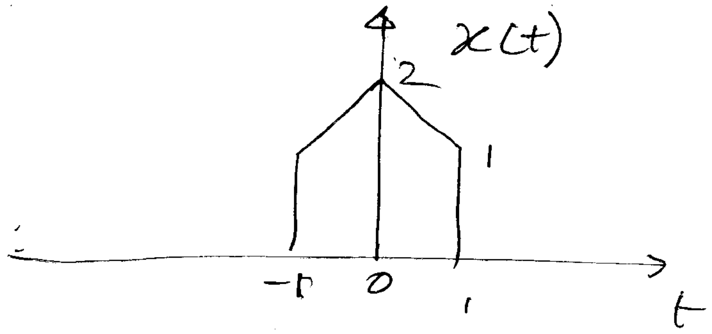
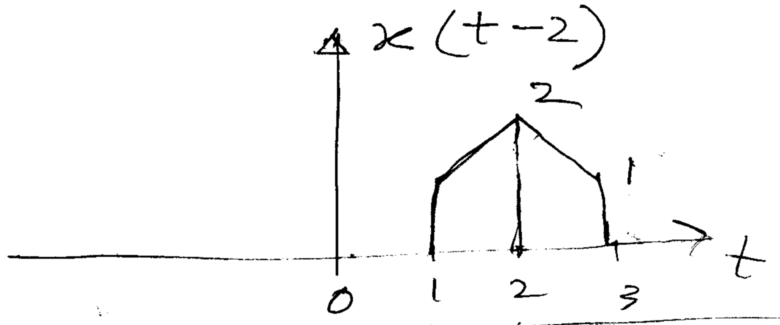
ALWAYS note what change is applied on t alone. Never look at what change is applied inside the brackets as a whole.

Again verify your answer; The peak of the graph should be at $2t+2=0$ $\Rightarrow t=-1$. But with the wrong method we got the peak at -2

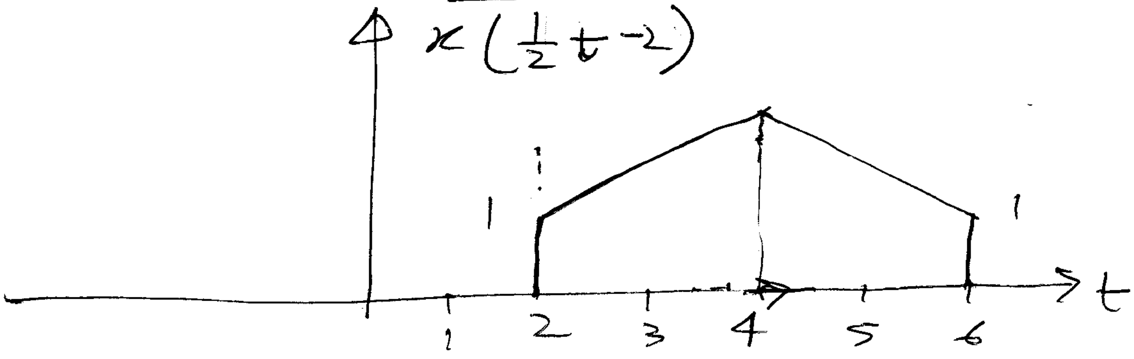
(ii)



Replace t with $(t-2)$ and shift the graph 2 units to the right.

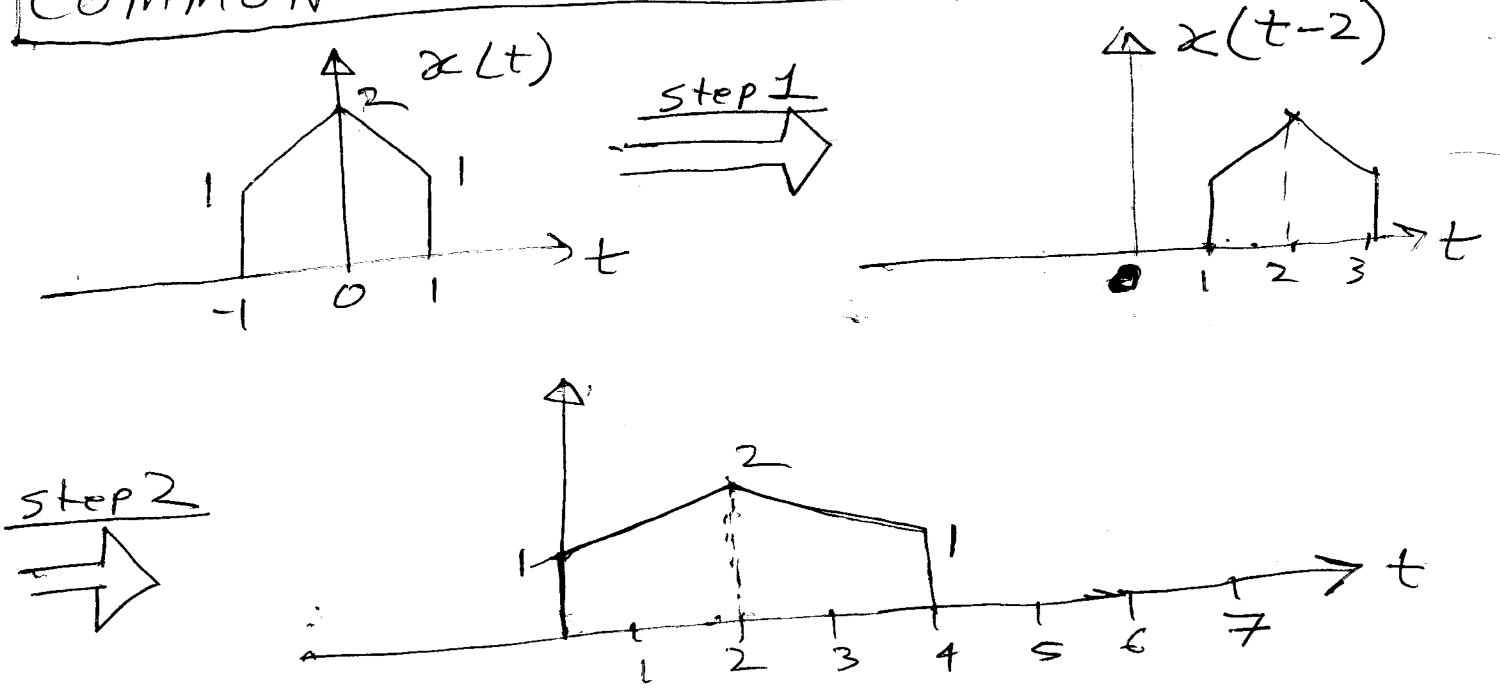


Replace t with $(1/2\, t)$ and expand the graph horizontaly by factor of.



Verify your answer: The peak should be at $\left(\frac{1}{2}t-2\right)=0 \Rightarrow t=4$. So our drawing is okay.

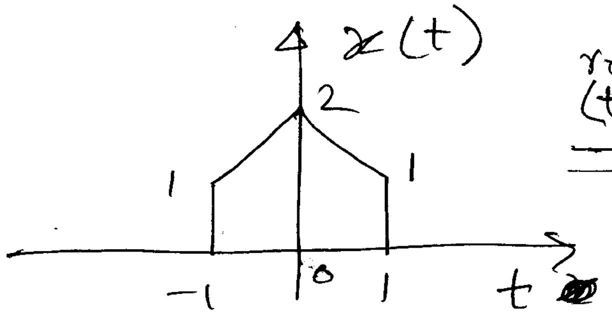
COMMON MISTAKE 2



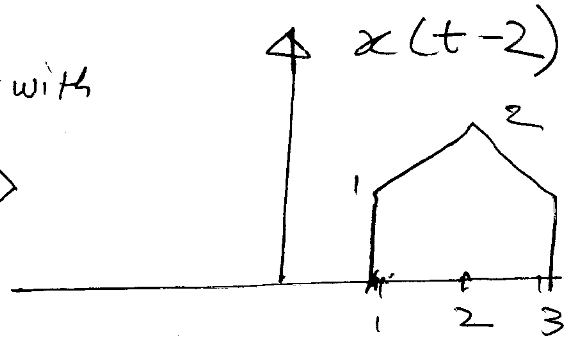
Step 1 is correct. But step 2 is wrong.
In step 2 the graph is expanded arround the center of the graph/ peak of the graph. But it should be expanded arround $t=0$ axis or y axis.

Verify the answer: The peak should be at $\frac{1}{2}t-2=0 \Rightarrow t=4$.
But with the wrong approach we got the peak at $t=2$.

(iii)

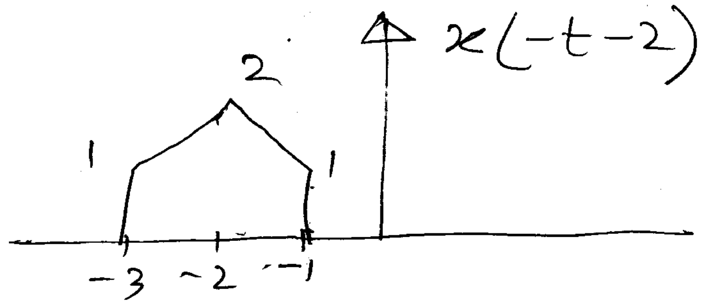


replace t with (t-2)



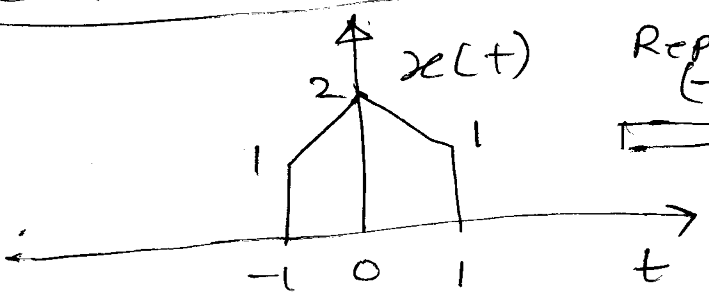
Replace t with (-t) and flip

arround y axis

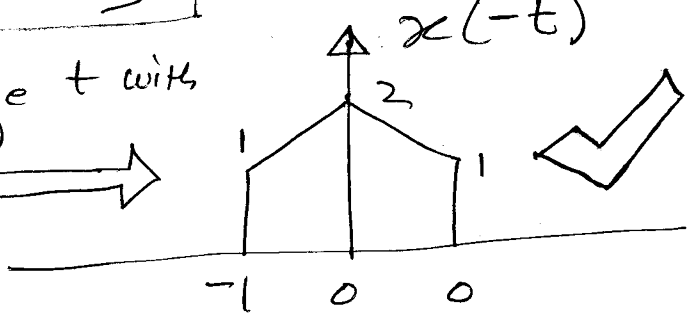


Verify peak should be at $-t-2=0 \Rightarrow t=-2$

COMMON MISTAKE 3

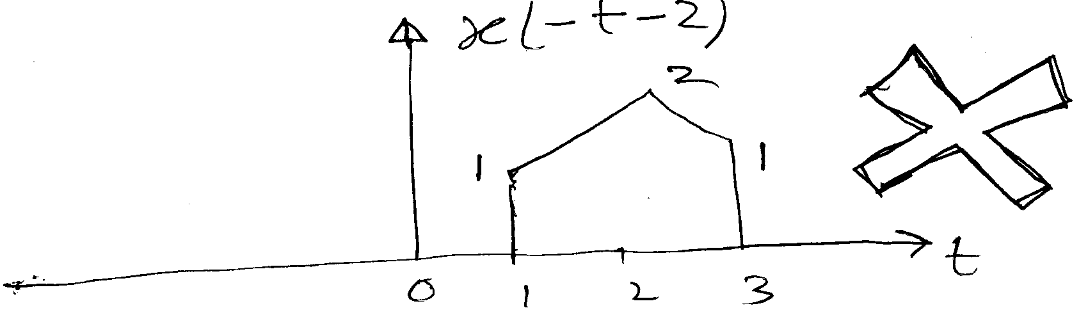


Replace t with (-t)



Step 2

wrong



Step 2 is wrong because we are changing (-t) to (-t-2) that means t is becoming (t+2).

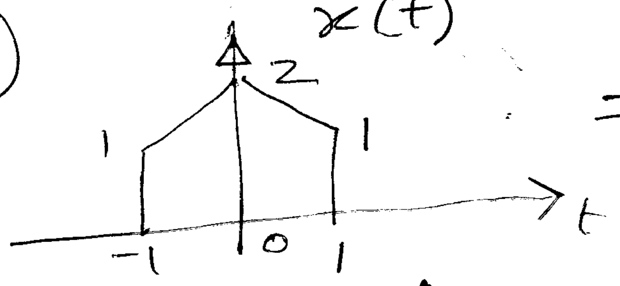
$$\boxed{-(t+2) = -t-2}$$

So it should shif towords left and

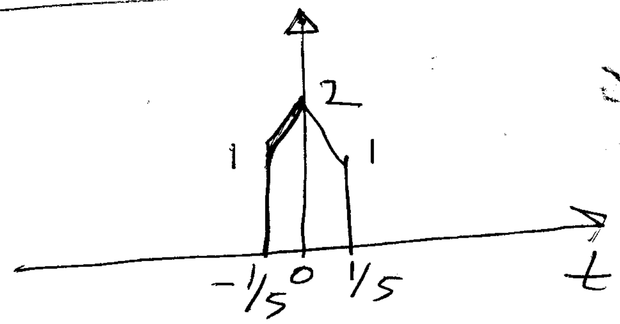
NOT towords right

<u>Verify</u> the peak should be at $(-t-2)=0 \Rightarrow t=-2$. But here we got the peak at $t=2$

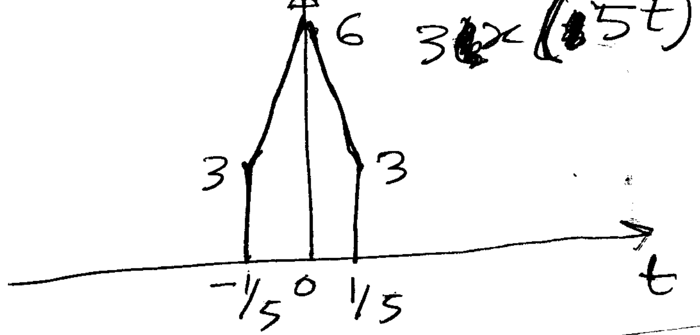
(iv)



$\Longrightarrow$

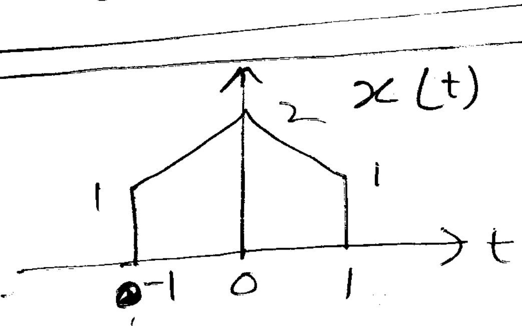


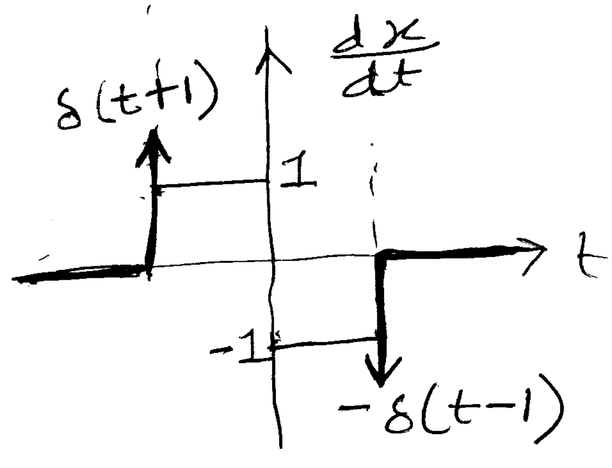
$\Longrightarrow$



My diagram is not upto the scale
But as long as I annotate the points on the time & y axis, it is acceptable

(v)





Note the two delta functions.
whenever there is a step Jump we have a delta fn.
If it is a step increase the positive delta & if it is a step decrease then a negative delta.

Also the value (coefficient / strength) of the delta f^n is equal to the amount of step jump

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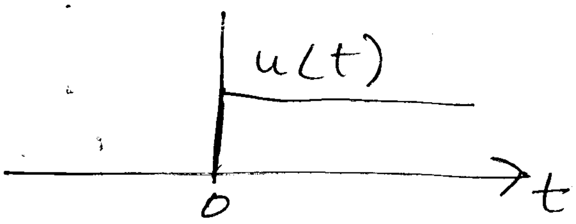
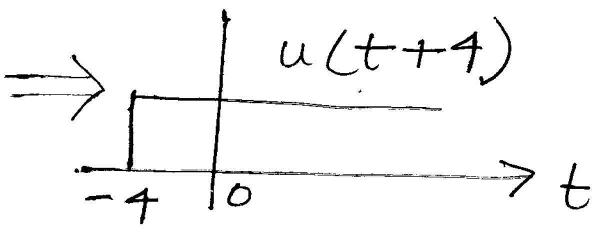
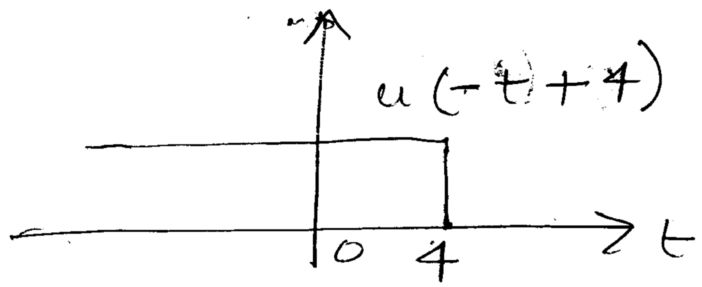
5) i)







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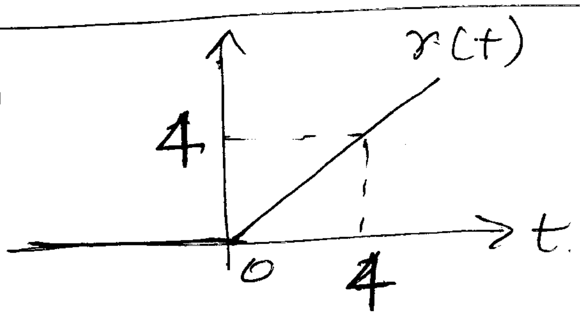
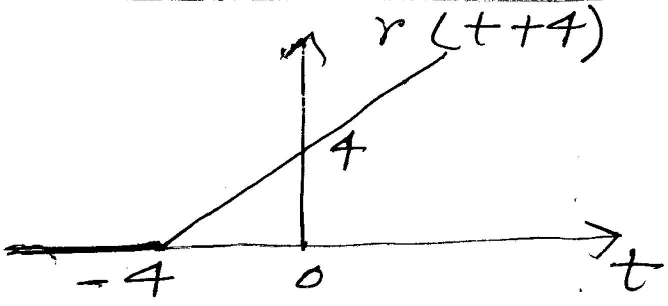
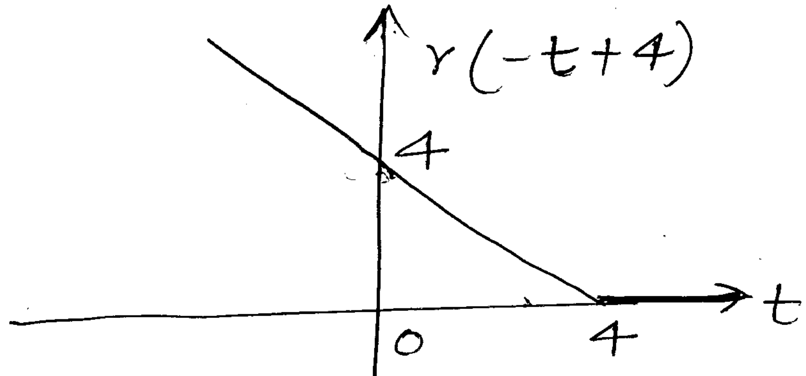
ii)







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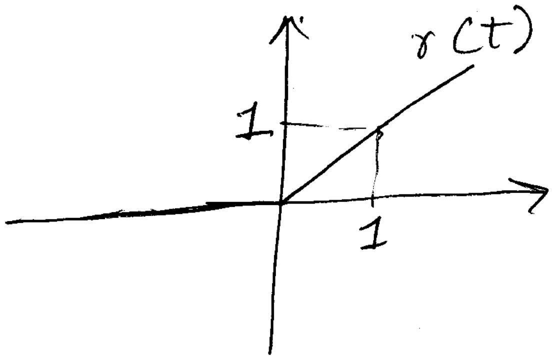
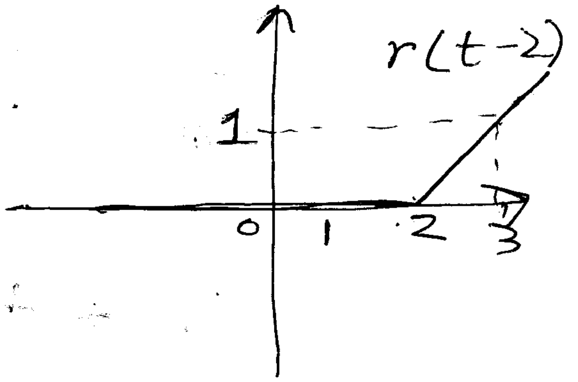
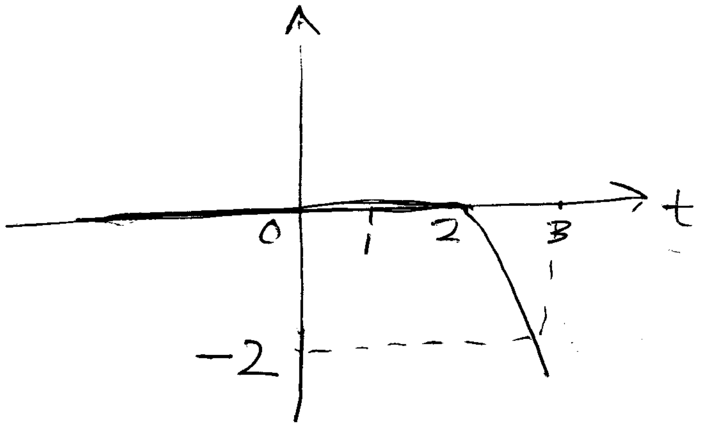
iii)



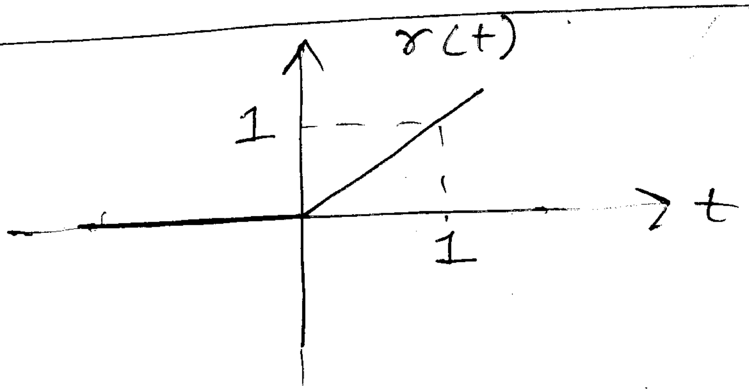
$\Rightarrow$

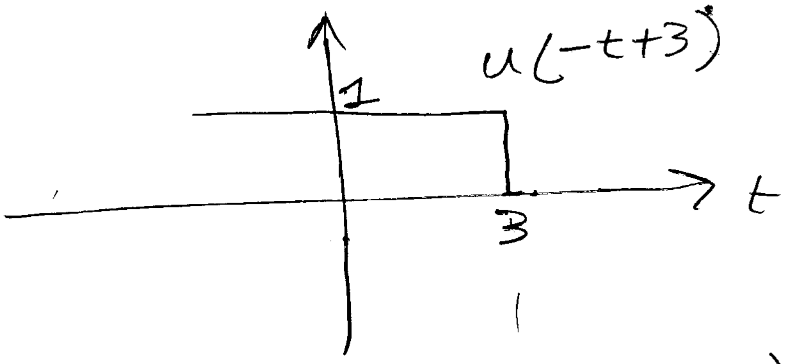


$\Rightarrow$

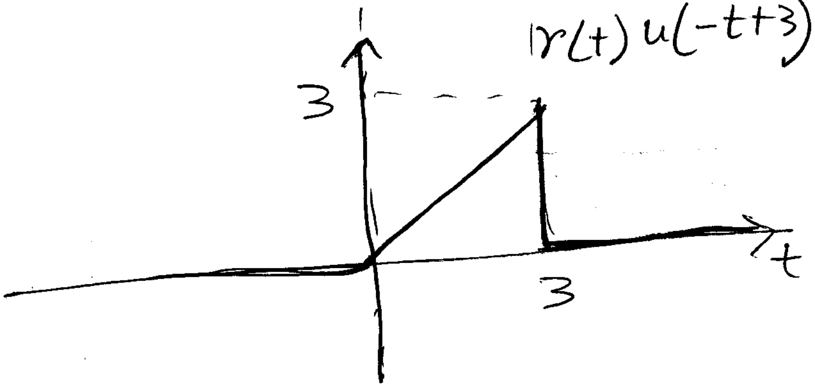


iv)

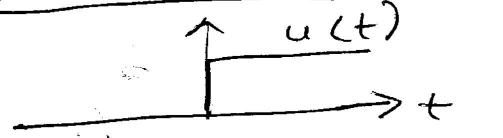




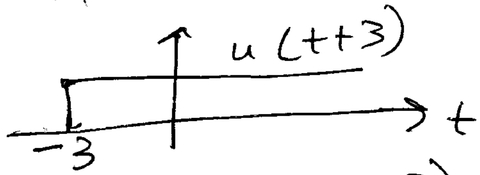
$\Rightarrow$



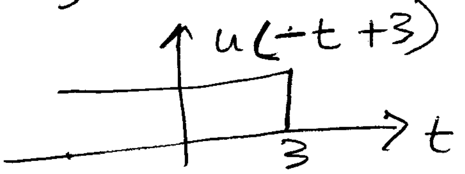
<u>side note</u>



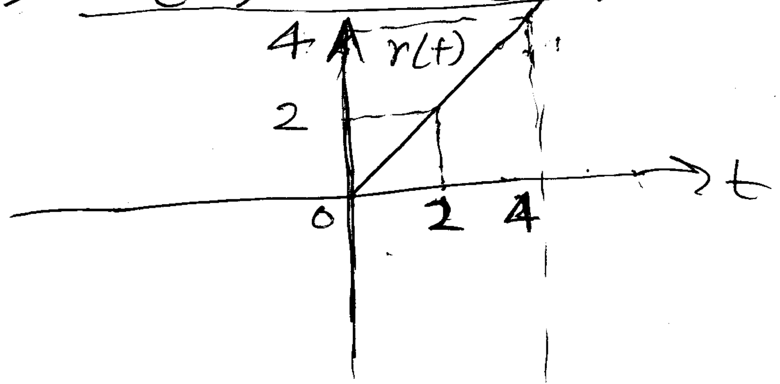
$\Rightarrow$

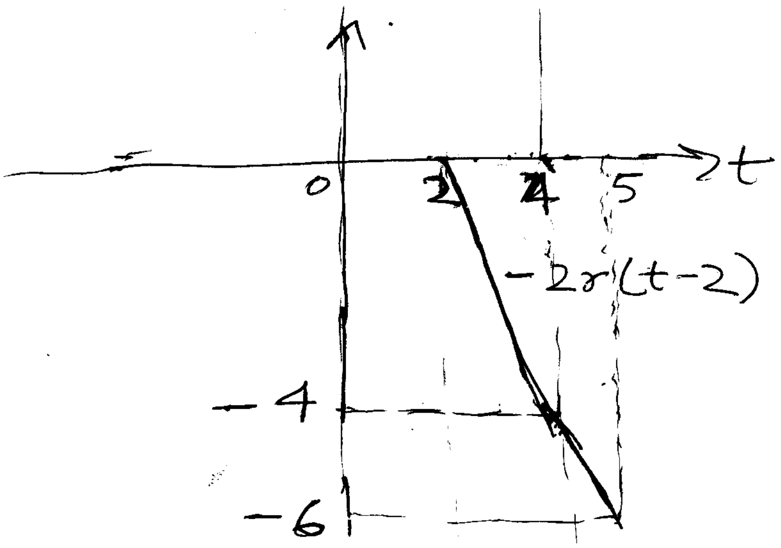


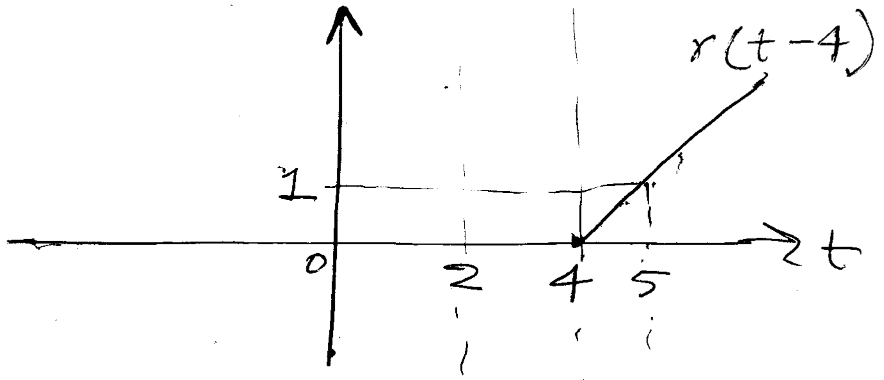
$\Rightarrow$

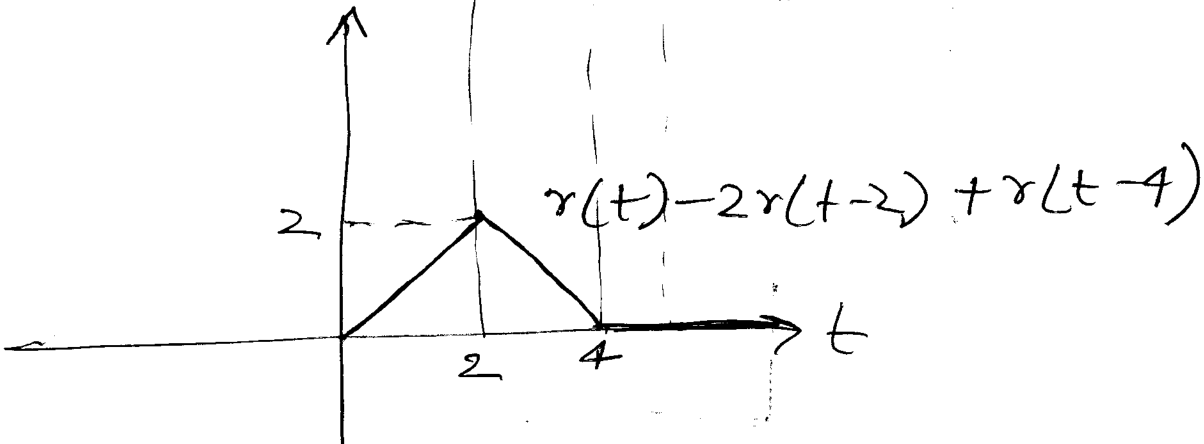


v) $r(t) - 2r(t-2) + r(t-4)$









6) i) $$\int_{-\alpha}^{\alpha} e^{-t^2}\delta(t-3)\,dt$$

$$= \int_{3^-}^{3^+} e^{-t^2}\delta(t-3)\,dt$$ [$\because$ for all time other than between 3^- and t^+ $\delta(t-3)=0$]

$$= \int_{3^-}^{3^+} e^{-3^2}\delta(t-3)\,dt$$

$$= e^{-9}\int_{3^-}^{3^+}\delta(t-3)\,dt$$

$$= e^{-9}$$

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ii) $$\int_{-\alpha}^{\alpha}\delta(t+3)e^{-2t}dt = \int_{-3^-}^{-3^+}\delta(t-(-3))\,e^{-2t}\,dt$$

$$= e^{-2(-3)}\int_{-3^-}^{-3^+}\delta(t-(-3))\,dt = e^{6}.1 = e^{6}$$

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iii) $$\int_{-3}^{3}\delta(t)\sin(5\pi t)\,dt = \int_{0^-}^{0^+}\delta(t)\sin(0)\,dt = 0$$

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iv) $$\int_{-\alpha}^{\alpha}[\delta(t)\cos 2t + \delta(t-2)\sin 2t]\,dt$$

$$= \int_{-\alpha}^{\alpha}\delta(t)\cos 2t\,dt + \int_{-\alpha}^{\alpha}\delta(t-2)\sin 2t\,dt = \cos 0 + \sin 4t$$

$$= 1 + \sin(4t)$$

v) $\int_{-\infty}^{\infty} \delta(4t)e^{-t}\,dt$ [One may apply the relevant formula directly or follow the steps as done here]

Put $4t = \tau$

$\Rightarrow dt = \frac{d\tau}{4}$

$$\therefore \int_{-\infty}^{\infty} \delta(4t)\,e^{-t}\,dt = \int_{-\infty}^{\infty} \delta(\tau)\,e^{-\frac{\tau}{4}}\,\frac{d\tau}{4}$$

$$= \frac{1}{4}\int_{-\infty}^{\infty} \delta(\tau)\,e^{-\tau/4}\,d\tau = \frac{1}{4}$$

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vi) $\int_{-\infty}^{\infty} \delta(2t+3)\,t^2\,dt$

Put $2t+3 = \tau$

$\Rightarrow dt = \frac{d\tau}{2}$

$$= \int_{-\infty}^{\infty} \delta(\tau)\left(\frac{\tau-3}{2}\right)^2 \frac{d\tau}{2} = \frac{1}{2} \times \left(\frac{-3}{2}\right)^2$$

$$= \frac{9}{8}$$

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vii) $\int_{-\infty}^{\infty} \delta(t^2+t-6)\cos t\,dt = I$ (say)

$$= \int_{-\infty}^{\infty} \delta\big((t+3)(t-2)\big)\cos t\,dt$$

$$= \int_{-3^-}^{-3^+} \delta\big((t+3)(t-2)\big)\cos t\,dt + \int_{2^-}^{2^+} \delta\big((t+3)(t-2)\big)\cos t\,dt$$

$$= \cos(-3)\int_{-3^-}^{-3^+} \delta\big((t+3)(t-2)\big)dt + \cos(2)\int_{2^-}^{2^+} \delta\big((t+3)(t-2)\big)dt$$

It is easier to apply the direct formula here which gives

$$I = \frac{\cos(-3)}{\left|\frac{d}{dt}(t+3)(t-2)\right|_{t=-3}} + \frac{\cos(2)}{\left|\frac{d}{dt}\left((t+3)(t-2)\right)\right|_{t=2}}$$

$$= \frac{\cos(3)}{|2t+1|_{t=-3}} + \frac{\cos 2}{|2t+1|_{t=2}}$$

$$= \frac{\cos(3)}{|-5|} + \frac{\cos 2}{|5|} = \frac{1}{5}(\cos 2 + \cos 3)$$

OR you may follow the steps mentioned below (This method is actually developed by some of you students)

$$I = \int_{-3^-}^{-3^+} \delta(t^2+t-6)\cos t\, dt + \int_{2^-}^{2^+} \delta(t^2+t-6)\cos t\, dt$$

$$= \cos(-3)\int_{-3^-}^{-3^+} \delta(t^2+t-6)\,dt + \cos(2)\int_{2^-}^{2^+} \delta(t^2+t-6)\,dt$$

$$= \cos 3\int_{-3^-}^{-3^+} \delta(t^2+t-6)\,dt + \cos 2\int_{2^-}^{2^+} \delta(t^2+t-6)\,dt$$

Now put $z = t^2+t-6$

$$\therefore dz = \frac{d}{dt}(t^2+t-6)\,dt = (2t+1)\,dt$$

t	-3^-	-3^+	2^-	2^+
z	0^+	0^-	0^-	0^+

t	-3	2
dz	$-5dt$	$5dt$

$$\therefore I = \cos(3)\int_{0^+}^{0^-} \delta(z)\,\frac{dz}{-5} + \cos(2)\int_{0^-}^{0^+} \delta(z)\,\frac{dz}{5}$$

$$= \frac{\cos(3)}{5}\int_{0^-}^{0^+} \delta(z)\,dz + \frac{\cos(2)}{5}\int_{0^-}^{0^+} \delta(z)\,dz$$

$$= \frac{1}{5}\left(\cos(2) + \cos(3)\right)$$

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viii) $\displaystyle\int_{-\infty}^{\infty} e^{-t}\,\frac{d\delta}{dt}\,dt$

[Again one can apply direct formula ~~these~~ taught in the class. Here we are using integration by parts]

$$= \left[e^{-t}\,\delta(t)\right]_{-\infty}^{\infty} - \int_{-\infty}^{\infty} -e^{-t}\,\delta(t)\,dt$$

$$= 0 + \int_{-\infty}^{\infty} e^{-t}\,\delta(t)\,dt = e^{0}\int_{0^-}^{0^+} \delta(t)\,dt$$

$$= e^{0} = 1$$

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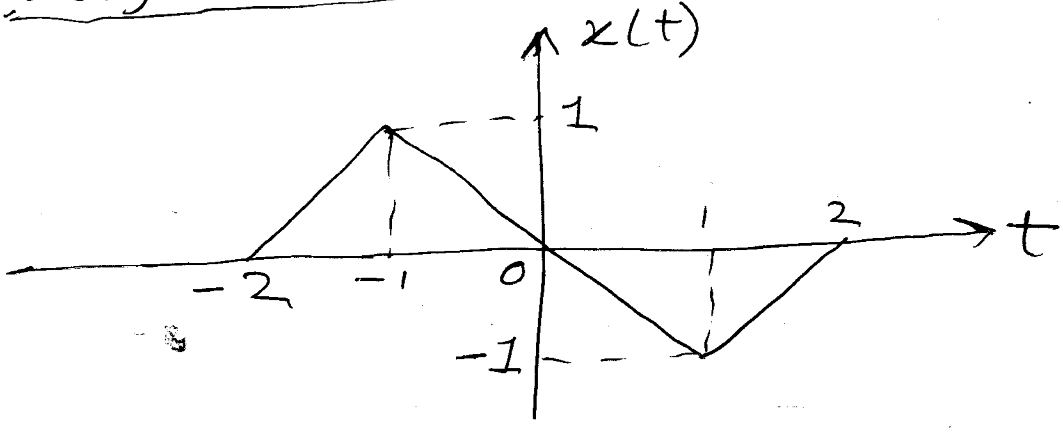
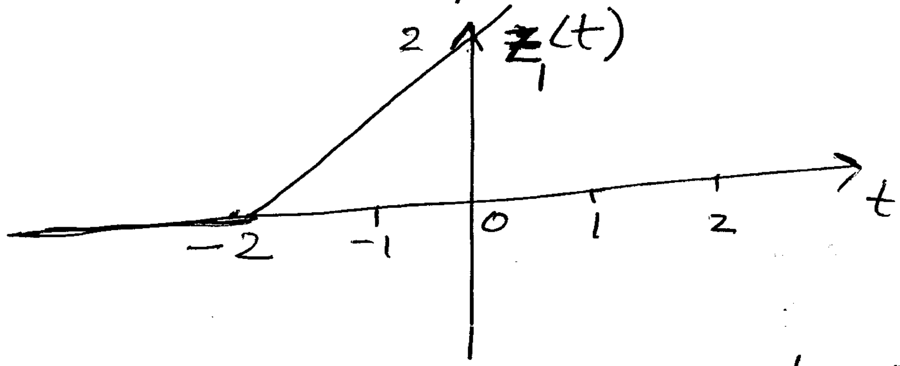
4) i) ~~Target~~ function to be constructed $x(t)$

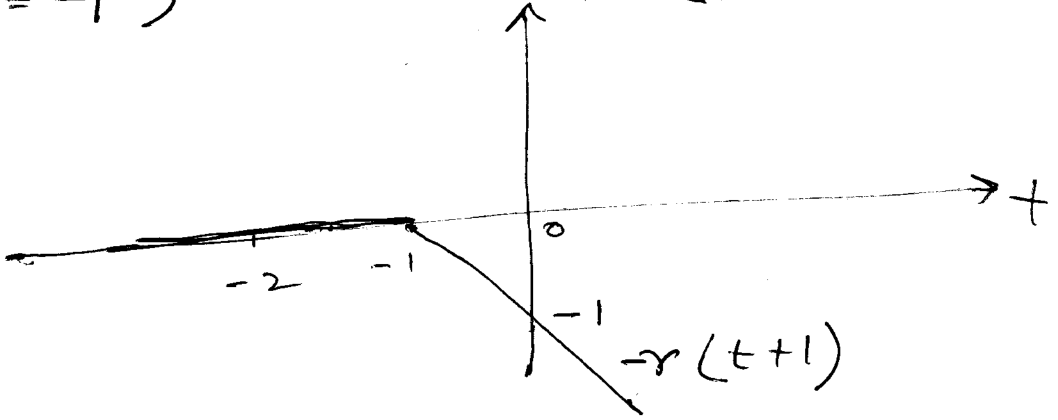


We will construct this function from left side to right side

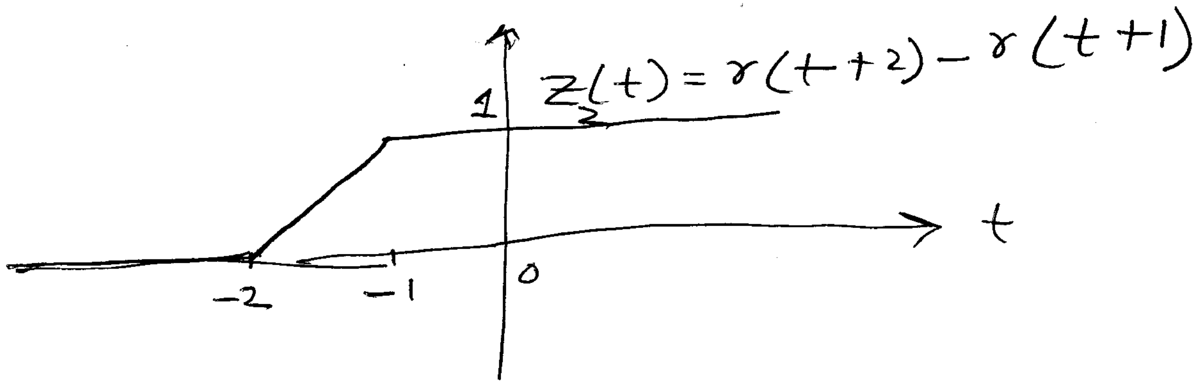
First take $z_1(t) = r(t+2)$



Now to stop the increase of this function at $t=-1$, Lets add $(-r(t+1))$

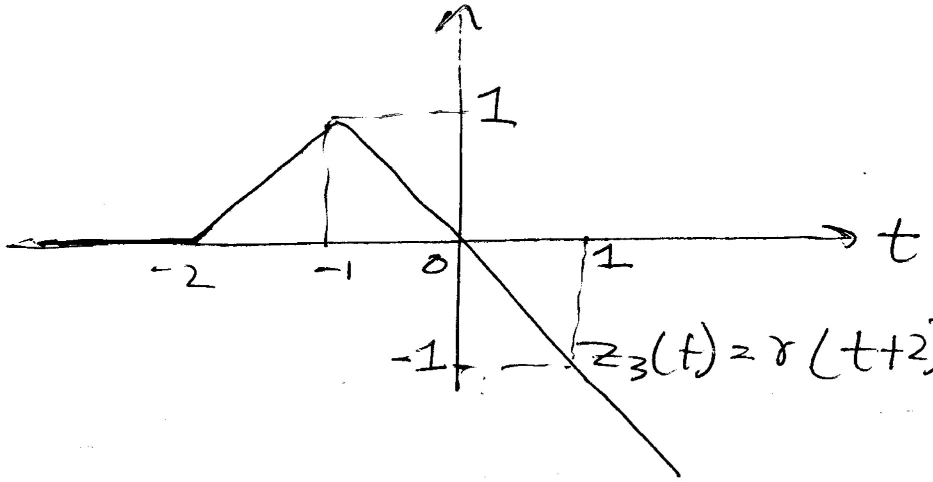


Let
$z_2(t) = r(t+2) - r(t+1)$



Now to bend down this function at $t=-1$, add one more $-r(t+1)$

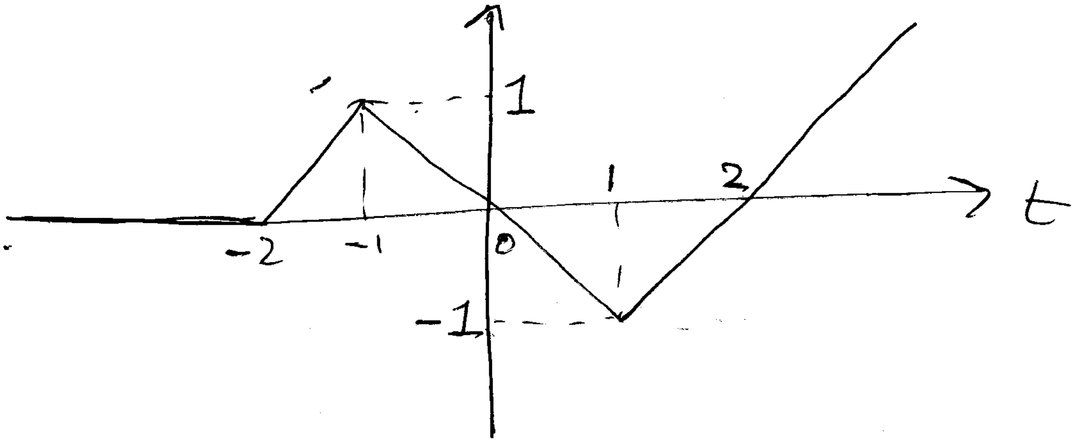
Let $z_3(t) = r(t+2) - r(t+1) - r(t+1)$
$= r(t+2) - 2r(t+1)$



$z_3(t) = r(t+2) - 2r(t+1)$

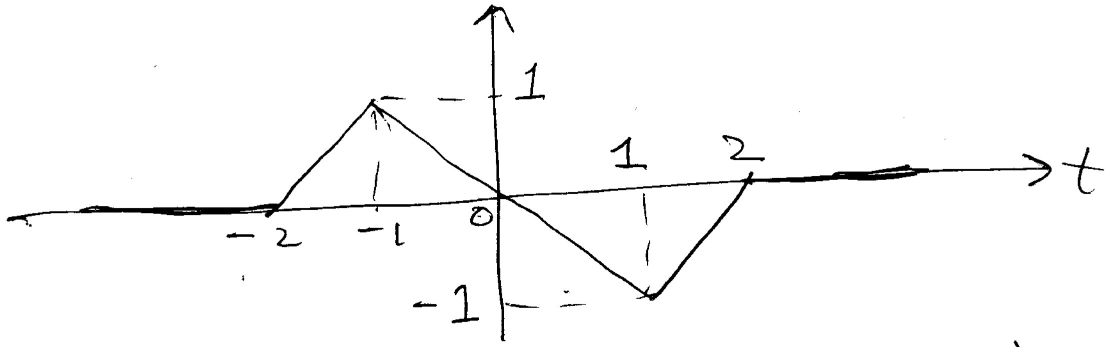
Now similarly, add $2r(t-1)$

$$z_4(t) = z_3(t) + 2r(t-1)$$
$$= r(t+2) - 2r(t+1) + 2r(t-1)$$



Finally add $(-r(t-2))$

$$z_5(t) = z_4(t) - r(t-2)$$
$$= r(t+2) - 2r(t+1) + 2r(t-1) - r(t-2)$$



This is our desired function $x(t)$

$$\therefore x(t) = z_5(t)$$
$$= r(t+2) - 2r(t+1) + 2r(t-1) - r(t-2)$$

We will give you the final answers for (4) ii) – (4) v). Please verify the answers yourselves following the steps as in (4) i)

ii) $x(t) = 2u(t) - r(t) + r(t-2)$ .... for 1st triangle
$+2u(t-4) - r(t-4) + r(t-6)$ ---- for 2nd "

iii) $x(t) = r(t+2) - r(t+1) - 2u(t-1) + \frac{1}{2}u(t-2)$
$+\frac{1}{2}r(t-2) - \frac{1}{2}r(t-3)$

iv) $x(t) = u(t-2) - r(t-2) + 2r(t-3) - r(t-4)$
$-u(t-4)$

v) $x(t) = 2r(t) - 2r(t-1) - u(t-1) + u(t-2)$
$-2r(t-2) + 2r(t-3)$

Additional) $2\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + y(t) = x(t)$

The characteristic equation is given by
$2D^2 + 3D + 1 = 0$

The roots of char. eqn. are
$$\frac{-3 \pm \sqrt{9-8}}{4} = \frac{-3 \pm 1}{4} = -1 \text{ and } -\frac{1}{2}$$

B.C. $y(0^-) = 0$ and $\frac{dy}{dt}(0^-) = 1$

o) $x(t) = 5$

$\dot{x}(t) = 0$

$\therefore$ Forced response

$$y_f(t) = k_1 x(t) + k_2 \dot{x}(t) + \cdots$$

$$= k_1 5$$

Now $y_f(t)$ should satisfy the diff. eqn

$$\therefore \quad 2\frac{d^2 y_f}{dt^2} + 3\frac{dy_f}{dt} + y_f(t) = 5$$

$$\Rightarrow 2\frac{d^2}{dt^2}(k_1 5) + 3\frac{d}{dt}(k_1 5) + k_1 5 = 5$$

$$\Rightarrow 0 + 0 + k_1 5 = 5 \Rightarrow k_1 = 1 \Rightarrow y_f(t) = 5$$

Now natural response

$$y_n(t) = c_1 e^{-1t} + c_2 e^{-\frac{1}{2}t}$$

$\therefore$ Total response

$$y(t) = y_f(t) + y_n(t)$$

$$= 5 + c_1 e^{-t} + c_2 e^{-\frac{1}{2}t}$$

$$\therefore y(0^-) = 5 + c_1 + c_2 = 0 \quad \text{(from B.C.)} \quad \text{---(i)}$$

~~and $\dot{y}(0)$~~

$$\text{Now } \dot{y}(t) = -c_1 e^{-t} - \frac{1}{2}c_2 e^{-\frac{1}{2}t}$$

$$\Rightarrow \dot{y}(0^-) = -c_1 - \frac{1}{2}c_2 = 1 \quad \text{(from B.C.)} \quad \text{---(ii)}$$

from (i) and (ii) we have

$$c_1 + c_2 = -5$$

$$\text{and } -c_1 - \frac{1}{2}c_2 = 1$$

$$\frac{1}{2}c_2 = -4$$

$$\Rightarrow c_2 = -8$$

$$\therefore c_1 = 3$$

$\therefore\ y(t) = 3e^{-t} - 8e^{-t/2} + 5$

Alternative ~~way~~ solution

$$x(t) = 5 = 5e^{0t}$$

The forced response $= \dfrac{5e^{0t}}{\left[2D^2+3D+1\right]_{D=0}}$

$$\Rightarrow y_f(t) = \frac{5}{1} = 5$$

The natural response and total response can be computed as ~~&~~ in the previous approach.

i) $x(t) = t^2$

$\dot{x}(t) = 2t$

$\ddot{x}(t) = 2$

$\therefore$ forced response $y_f(t) = k_1 t^2 + k_2 t + k_3$

$y_f(t)$ must satisfy

$$2\frac{d^2}{dt^2}y_f + 3\frac{d}{dt}y_f + y_f(t) = x(t)$$

$$\Rightarrow 2(2k_1) + 3(2k_1 t + k_2) + (k_1 t^2 + k_2 t + k_3) = t^2$$

$\therefore\ k_1 = 1$

$k_2 + 6k_1 = 0 \Rightarrow k_2 = -6$

$k_3 + 3k_2 + 4k_1 = 0 \Rightarrow k_3 - 18 + 4 = 0$

$\Rightarrow k_3 = 14$

$\therefore\ y_f(t) = t^2 - 6t + 14$

Now Natural response $y_n(t) = c_1 e^{-t} + c_2 e^{-t/2}$

$\therefore$ Total response $y(t) = y_n(t) + y_e(t)$

$\Rightarrow y(t) = c_1 e^{-t} + c_2 e^{-t/2} + t^2 - 6t + 14$

$\Rightarrow \dot{y}(t) = -c_1 e^{-t} - \frac{1}{2} c_2 e^{-t/2} + 2t - 6$

From B.C.

$y(0^-) = c_1 + c_2 + 14 = 0$ ---- (i)

$\dot{y}(0^-) = -c_1 - \frac{1}{2} c_2 - 6 = 1$ ---- (ii)

from (i) and (ii) $\quad c_2 = -14$

$\quad c_1 = 0$

$\therefore y(t) = -14 e^{-t/2} + t^2 - 6t + 14$

---

iii) $x(t) = \cos(2t + \frac{\pi}{6})$

$$\cos\left(2t + \frac{\pi}{6}\right) = \frac{e^{j(2t+\frac{\pi}{6})} + e^{-j(2t+\frac{\pi}{6})}}{2}$$

$$= \frac{1}{2} e^{j\pi/6} e^{j2t} + \frac{1}{2} e^{-j\pi/6} e^{-j2t}$$

Now the ~~natural~~ forced response due to $\frac{1}{2} e^{j\pi/6} e^{j2t}$ is given by

$$y_{f1}(t) = \frac{\frac{1}{2} e^{j\pi/6} e^{j2t}}{\left[2D^2 + 3D + 1\right]_{D=2j}}$$

$$= \frac{\frac{1}{2} e^{j\pi/6} e^{j2t}}{-7 + 6j}$$

And the forced response due to $\frac{1}{2} e^{-j\pi/6} e^{-j2t}$ is given by

$$y_{f2}(t) = \frac{\frac{1}{2} e^{-J\pi/6} e^{-2Jt}}{\left[2D^2+3D+1\right]_{D=-2J}}$$

$$= \frac{\frac{1}{2} e^{-J\pi/6} e^{-J2t}}{-7-6J}$$

∴ forced response due to

$$\cos(2t+\pi/6) \left[= \frac{1}{2} e^{J\pi/6} e^{J2t} + \frac{1}{2} e^{-J\pi/6} e^{-J2t}\right]$$

is given by

$$y_f(t) = y_{f1}(t) + y_{f2}(t)$$

$$= \frac{\frac{1}{2} e^{J\pi/6} e^{J2t}}{-7+6J} + \frac{\frac{1}{2} e^{-J\pi/6} e^{-J2t}}{-7-6J}$$

$$= \frac{\frac{1}{2} e^{\frac{J\pi}{6}} e^{J2t}(-7-6J)}{49+36} + \frac{\frac{1}{2} e^{-\frac{J\pi}{6}} e^{-J2t}(-7+6J)}{49+36}$$

$$= \frac{1}{170}\left(-7\left(e^{J(2t+\pi/6)} + e^{-J(2t+\pi/6)}\right) - 6J\left(e^{J(2t+\pi/6)} - e^{-J(2t+\pi/6)}\right)\right)$$

$$= \frac{1}{170}\left(-7\times 2\cos(2t+\pi/6) - 6J\times 2J\sin(2t+\pi/6)\right)$$

$$= \frac{1}{170}\times 2\left(-7\cos(2t+\pi/6) + 6\sin(2t+\pi/6)\right)$$

$$= \frac{1}{85}\left(6\sin\left(2t+\pi/6\right) - 7\cos\left(2t+\pi/6\right)\right)$$

Now the natural response is

$$y_n(t) = c_1 e^{-t} + c_2 e^{-t/2}$$

$\therefore$ Total response

$$y(t) = c_1 e^{-t} + c_2 e^{-t/2} + \frac{1}{85}\left(6\sin\left(2t+\frac{\pi}{6}\right) - 7\cos\left(2t+\pi/6\right)\right)$$

$$\therefore y(0^-) = c_1 + c_2 + \frac{6}{85}\sin\frac{\pi}{6} - \frac{7}{85}\cos\frac{\pi}{6}$$

$$= c_1 + c_2 + \frac{6}{85}\times\frac{1}{2} - \frac{7}{85}\frac{\sqrt{3}}{2} = 0 \ldots (i)$$

(from B.C)

$$\dot{y}(t) = -c_1 e^{-t} - \frac{1}{2}c_2 e^{-t/2} + \frac{12}{85}\cos\left(2t+\frac{\pi}{6}\right) + \frac{14}{85}\sin\left(2t+\frac{\pi}{6}\right)$$

$$\therefore \dot{y}(0^-) = -c_1 - \frac{c_2}{2} + \frac{12}{85}\cos\frac{\pi}{6} + \frac{14}{85}\sin\frac{\pi}{6}$$

$$= -c_1 - \frac{c_2}{2} + \frac{\cancel{12}^{\,6}\sqrt{3}}{85\times\cancel{2}} + \frac{7}{85} = 1 \ldots (ii)$$

(From B.C.)

from (i) + (ii)

$$\frac{1}{2}c_2 + \frac{1}{85}\left(3 - \frac{7\sqrt{3}}{2} + 6\sqrt{3} + 7\right) = 1$$

$$\Rightarrow \frac{1}{2}c_2 + \frac{1}{85}\left(10 + \frac{5\sqrt{3}}{2}\right) = 1$$

$$\Rightarrow c_2 = 2\left(1 - \frac{10}{85} - \frac{5\sqrt{3}}{2\times 85}\right)$$

$$= \frac{30-\sqrt{3}}{17}$$

$\therefore$ from (i)

$$c_1 = \frac{7}{85}\frac{\sqrt{3}}{2} - \frac{3}{85} - c_2$$

$$= \frac{7}{85}\frac{\sqrt{3}}{2} - \frac{3}{85} - \frac{30}{17} + \frac{\sqrt{3}}{17}$$

$$= \frac{1}{17}\left(\frac{7}{10}\sqrt{3} - \frac{3}{5} - 30 + \sqrt{3}\right)$$

$$= \frac{1}{17}\left(\frac{17\sqrt{3}}{10} - \frac{153}{5}\right) = \frac{1}{170}\left(17\sqrt{3} - 306\right)$$

$$= \frac{\sqrt{3} - 18}{10}$$

$$\therefore y(t) = \frac{\sqrt{3}-18}{10}e^{-t} + \frac{30-\sqrt{3}}{17}e^{-t/2} + \frac{6}{85}\sin\left(2t+\frac{\pi}{6}\right) - \frac{7}{85}\cos\left(2t+\frac{\pi}{6}\right)$$

<u>Alternative approach</u>

$$x(t) = \cos\left(2t+\frac{\pi}{6}\right)$$

$$\dot{x}(t) = -2\sin\left(2t+\frac{\pi}{6}\right)$$

$$\ddot{x}(t) = -4\cos\left(2t+\frac{\pi}{6}\right)$$

$\therefore$ forced response will have the form

$$y_f(t) = k_1\cos\left(2t+\frac{\pi}{6}\right) + k_2\sin\left(2t+\pi/6\right)$$

Now $y_f(t)$ must satisfy

$$2\frac{d^2}{dt^2}y_f(t) + 3\frac{d}{dt}y_f(t) + y_f(t) \cancel{\cos} = \cos\left(2t+\frac{\pi}{6}\right)$$

$$\Rightarrow 2\left(-4k_1\cos(2t+\pi/6) - 4k_2\sin(2t+\pi/6)\right) + 3\left(-2k_1\sin(2t+\pi/6) + 2k_2\cos(2t+\pi/6)\right) + k_1\cos(2t+\pi/6) + k_2\sin(2t+\pi/6) = \cos(2t+\pi/6)$$

$$\Rightarrow -8k_1 + 6k_2 + k_1 = 1$$

and $-8k_2 - 6k_1 + k_2 = 0$

$$\Rightarrow \left.\begin{array}{l} -7k_1 + 6k_2 = 1 \\ -6k_1 - 7k_2 = 0 \end{array}\right\} \Rightarrow \begin{array}{l} -49k_1 + 42k_2 = 7 \\ -36k_1 - 42k_2 = 0 \end{array}$$

$$\Rightarrow k_1 = \frac{-7}{85} \quad \text{and} \quad k_2 = \frac{1}{6}\left(1 - \frac{49}{85}\right) = \frac{6}{85}$$

$$\therefore y_f(t) = \frac{-7}{85}\cos\left(2t+\frac{\pi}{6}\right) + \frac{6}{85}\sin(2t+\pi/6)$$

Then the natural and the total response can be computed as in the previous approach.

v) $x(t) = \delta(t)$

we shall solve ~~a~~ the diff. eqn in three steps :

Step 1 : for time interval 0^- to 0^+
Step 2 : for time interval 0^+ to α
Step 3 : for time interval $-\alpha$ to 0^-

Step 1

$$2\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + y(t) = x(t) = \delta(t)$$

$$\Rightarrow 2\int_{0^-}^{0^+} \frac{d^2y}{dt^2}\,dt + 3\int_{0^-}^{0^+} \frac{dy}{dt}\,dt + \int_{0^-}^{0^+} y(t)\,dt = \int_{0^-}^{0^+} \delta(t)$$

[by mulitipleying both sides with dt & integrating ove 0^- to 0^+]

$$\Rightarrow 2\int_{0^-}^{0^+} \frac{d}{dt}\left(\frac{dy}{dt}\right)dt + 3\left(y(0^+) - y(0^-)\right) + 0 = 1$$

[because the interval 0^- to 0^+ is so small that for any value of $y(t)$ in this interval $\int_{0^-}^{0^+} y(t)\,dt = 0$]

$$\Rightarrow 2\left(\frac{dy}{dt}(0^+) - \frac{dy}{dt}(0^-)\right) + 3\left(y(0^+) - y(0^-)\right) = 1$$

$$\Rightarrow 2\left(\frac{dy}{dt}(0^+) - 1\right) + 3\times 0 = 1$$

[because $y(0^+)$ must be equal to $y(0^-)$, Otherwise there will be a step jump from $y(0^-)$ to $y(0^+)$. Therefore the derivative of $y(t)$ will produce a delta function & the second derivative of $y(t)$ will produce a $\dot{\delta}(t)$ function.

However, the RHS of the diff eqn does not have any $\dot{\delta}(t)$ term]

$$\therefore 2\frac{dy}{dt}(0^+) - 2 = 1$$

$$\Rightarrow \frac{dy}{dt}(0^+) = \frac{3}{2} \quad \text{------ (i)}$$

Also as argued before

$$y(0^+) = y(0^-) = 0 \quad \text{--- (ii)}$$

Step 2

Now we can solve the diff eqn for interval 0^+ to α using (i) and (ii) as B.C. In this interval $x(t) = 0$

$$\therefore 2\frac{d^2}{dt^2}y + 3\frac{dy}{dt} + y(t) = 0$$

$\therefore$ forced response will be 0 (zero)

$$\therefore y(t) = y_n(t) = c_1 e^{-t} + c_2 e^{-t/2}$$

and $\dot{y}(t) = -c_1 e^{-t} - \frac{c_2}{2} e^{-t/2}$

$\therefore\ y(0^+) = c_1 + c_2 = 0$ - - - - - (iii)

(from B-C)

and $\dot{y}(0^+) = -c_1 - \frac{c_2}{2} = \frac{3}{2}$ - - - - (iv)

(from B-C-)

from (iii) and (iv) $c_2 = 3$ and $c_1 = -3$

$\therefore\ y(t) = -3e^{-t} + 3e^{-t/2}$ for $0^+ < t < \alpha$

Step 3

Now for interval $-\alpha$ to 0^- the

$x(t) = 0$

$\Rightarrow 2\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + y(t) = 0$

$\Rightarrow$ forced response will be zero.

$\therefore\ y(t) = c_4 e^{-t} + c_5 e^{-t/2}$

and $\dot{y}(t) = -c_4 e^{-t} - \frac{c_5}{2} e^{-t/2}$

$\therefore\ y(0^-) = c_4 + c_5 = 0$ - - - - - (v)

(from B-C-)

and $\dot{y}(0^-) = -c_4 - \frac{c_5}{2} = 1$ - - - (vi)

(from B-C)

from (v) & (vi) $c_5 = 2,\ c_4 = -2$

$\therefore\ y(t) = -2e^{-t} + 2e^{-t/2}$

for $-\alpha < t < 0^-$

$$\therefore\ y(t) = \begin{cases} -2e^{-t} + 2e^{-t/2} & \text{for } -\alpha < t < 0^- \\ 0 & \text{for } 0^- < t < 0^+ \\ -3e^{-t} + 3e^{-t/2} & \text{for } 0^+ < t < \alpha \end{cases}$$