Problem Set - 8

AUTUMN 2017

MATHEMATICS-1(MA10001)

July 31,2017

1 Solve the following homogeneous differential equations:

a. $4\frac{d^2y}{dx^2} - 12\frac{dy}{dx} + 5y = 0$

b. $\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 16y = 0$

c. $\frac{d^2y}{dx^2} + 9y = 0$

d $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 0$

e. $\frac{d^4y}{dx^4} + 8\frac{d^2y}{dx^2} + 16y = 0$

f. $\frac{d^4y}{dx^4} + a^4y = 0$

g. $\frac{d^5y}{dx^5} - 3\frac{d^4y}{dx^4} + 3\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} = 0$

h. $\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 4y = 0$

i. $\frac{d^4y}{dx^4} + 4\frac{d^3y}{dx^3} + 8\frac{d^2y}{dx^2} + 8\frac{dy}{dx} + 4y = 0$

2 Solve the following initial value problems:

a. $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 12y = 0$ $\qquad y(0) = 3, y'(0) = 5$

b. $4\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 37y = 0$ $\qquad y(0) = 2, y'(0) = -4$

c. $9\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 5y = 0$ $\qquad y(0) = 6, y'(0) = 0$

d $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$ $\qquad y(0) = 1, y'(0) = 0$

e. $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = 0$ $\qquad y(0) = 1, y'(0) = 1$

f. $\dfrac{d^3y}{dx^3} - 5\dfrac{d^2y}{dx^2} - 22\dfrac{dy}{dx} + 56y = 0 \qquad y(0) = 1, y'(0) = -2, y''(0) = -4$

3 Solve the following differential equations:

a. $(D^2 - 4)y =$sin $2x$

b. $(D^2 - 1)y = x^2$cos x

c. $(D^2 - 2D - 3)y = 2e^{4x}$

d $(D^2 - 2D - 3)y = 2e^x - 10\text{sin}x$

e. $(D^2 - 7D - 18)y = x^2e^{-2x}$

f. $(D^2 - 3D + 2)y = 2x^2 + e^x + 2xe^x + 4e^{3x}$

g. $(D^2 - 2D + 2)y = e^x\text{sin}2x$

h. $(D^3 - 2D^2 - 5D + 6)y = (e^{2x} + 3)^2 + e^{3x}\text{cosh}x$

i. $(D^4 - 2D^3 + D^2)y = x^3$

4 Solve the following problems:

a. If $\dfrac{d^2x}{dt^2} + 2h\dfrac{dy}{dx} + (h^2 + p^2)x = ke^{-ht}\text{cos}pt$, then prove that $x = C_1e^{-ht}\text{cos}(pt + C_2) + \frac{k}{2p}te^{-ht}\text{sin}pt$

b. Using z=sin$x$, find y where $\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx}\text{tan}x + ycos^2x = 0$

c. $\dfrac{d^2y}{dt^2} + 2n\text{cos}\alpha\dfrac{dy}{dt} + n^2y = a\text{cos}nt$, given y=0, $\dfrac{dy}{dt} = 0$ at t=0

d Find the value of u which satisfies the equation $\dfrac{d^2u}{d\theta^2} + u = 2k\text{cos}\theta$ with the following condition

i. u has the same value when $\theta = \frac{\pi}{2} and - \frac{\pi}{2}$

ii. $\int_0^{\frac{\pi}{2}} ud\theta = 0$

e. $(D^4 - n^4)y = 0$ if $Dy = y = 0$, when x =0, $x = L$, prove that y=A(cosnx-coshnx)+B(sinnx-sinhnx) where A, B are arbitrary constants.