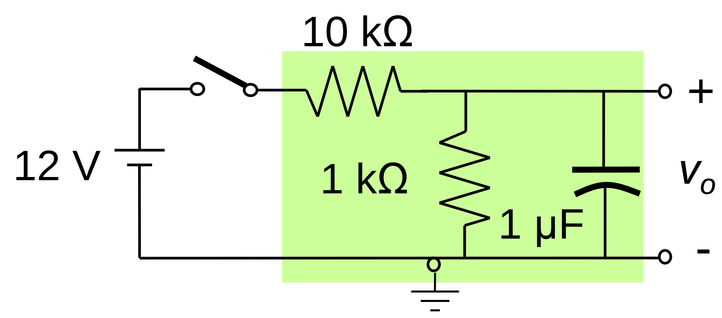
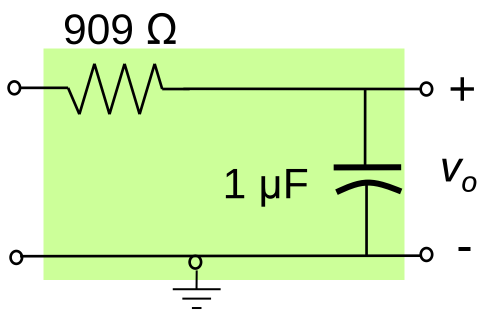
Tutorial - 1

Q1. (i) What type of filter is it? Calculate the cutoff frequency of the filter.
(ii) The switch is closed at t = 0, calculate the output voltage at t = 0 and at t = 10 mS.



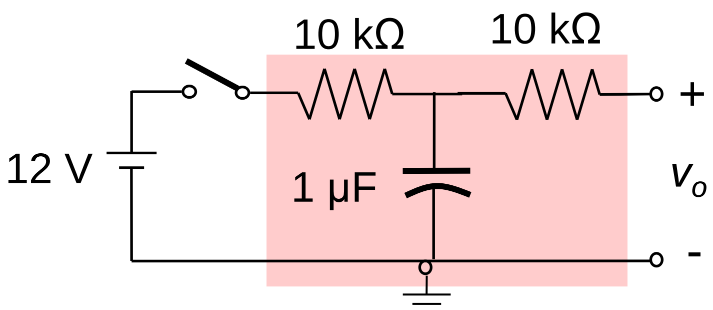
(i) Lowpass filter.
Cutoff frequency = 175.1 Hz.



(ii) At t = 0, the capacitor is shorted.
→V_o = 0 V.

Now, time constant = $R_{eq} \times C$ = 0.909 mS.
→$t \geq 5\tau$
→V_o = 1.09 V.

- Calculate the time constant of the following circuit.

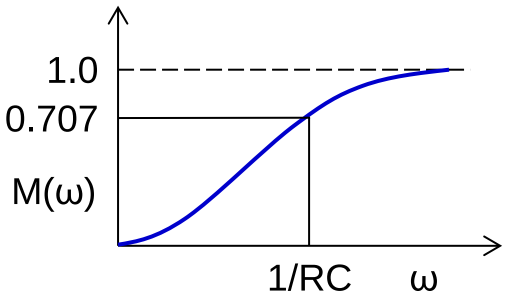


The output is open-circuited.
→time constant = $R \times C$ = 10 mS.

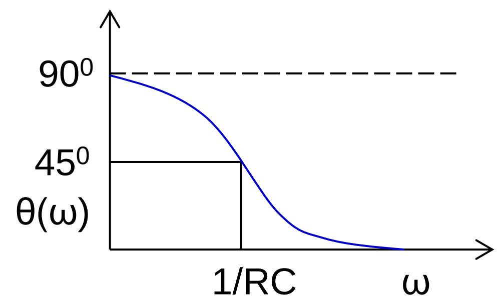
Questions

Q2. A Series RC circuit is excited by a voltage source of variable frequency. The output is taken across the *R*. Sketch the variation of the steady state transfer function with angular frequency ω.

Hints: Obtain H(jω) and represent in magnitude and phase form.

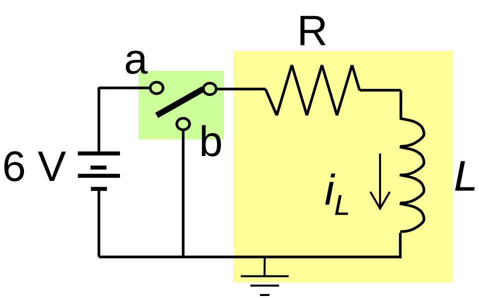


Magnitude response



Phase response

Q3. Initially the switch is connected to a and the circuit is in steady state. The switch is moved from *a* to *b* at $t = 0$. Find the current in the inductor. What is the power dissipated in *R* at $t = 0$ and $t = \infty$?



Answer:

$$i = -6/R\, e^{-Rt/L} \text{ A}, \quad t \geq 0 \qquad P_R = 36/R \text{ W},$$

$$= 6/R \text{ A}, \quad t < 0 \qquad = 0 \text{ W}.$$

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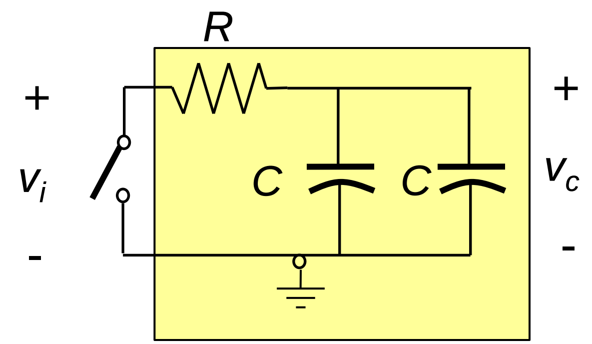
Q4. In the circuit, the capacitors are fully charged at $t = 0_-$ so that $v_i = 12$ V ($C = 47$ µF, and $R = 1$ kΩ).

(i) The switch is closed at $t = 0$, calculate the time when $v_c = 6$ V.

(ii) Calculate the minimum power rating of the resistor.



(i) $C_{eq} = 94$ µF.

$$v_C = V_o e^{-t/RC_{eq}}$$

$$\Rightarrow \ln\frac{v_C}{V_o} = -t/RC_{eq}$$

$$\Rightarrow t = 65.2\ mS.$$

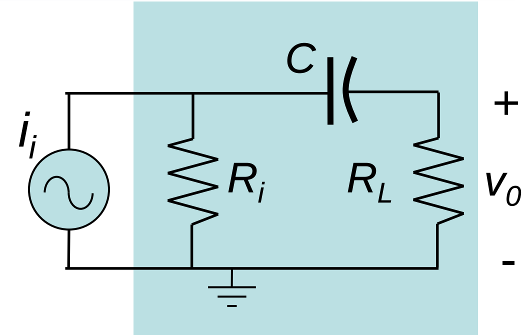
(ii). $$\text{Power rating} = \left(i_{peak}\right)^2 R$$

$$= \left(\frac{12}{1k}\right)^2 1k$$

$$= 144\ mW.$$

Tutorial - 1

Q5. In the following circuit, a current source i_i = sin(2π*ft*) mA with internal resistance R_i = 10 kΩ is connected to a *RC* circuit. Calculate the output voltages (magnitudes) at *f* = 10 kHz and 100 kHz. Given that *C* = 2.2 nF, and R_L = 10 kΩ.



Transform the current source into a voltage source v_i.

$$\therefore v_i = i_i R_i = 10\sin(2\pi ft) \text{ V.}$$

$$\text{Now, } \left|\frac{v_{R_L}}{v_i}\right| = \frac{\omega C R_L}{\sqrt{1+\omega^2 C^2 (R_i + R_L)^2}}$$

$$\therefore at\ 10\ kHz, \quad \omega C R_L = 1.382, \qquad \omega^2 C^2 (R_i + R_L)^2 = 7.643 \qquad \Rightarrow \left|\frac{v_{R_L}}{v_i}\right| = 0.47$$

$$\therefore v_{R_L}\Big|_{10\ kHz} = 4.7\sin\left(20\times 10^3 \pi t\right) \text{ V.}$$

$$\therefore at\ 100\ kHz, \quad \left|\frac{v_{R_L}}{v_i}\right| = \frac{13.82}{\sqrt{1+764.3}} = 0.4996 \quad \Rightarrow v_{R_L}\Big|_{100\ kHz} = 5\sin\left(20\times 10^4 \pi t\right) \text{ V.}$$

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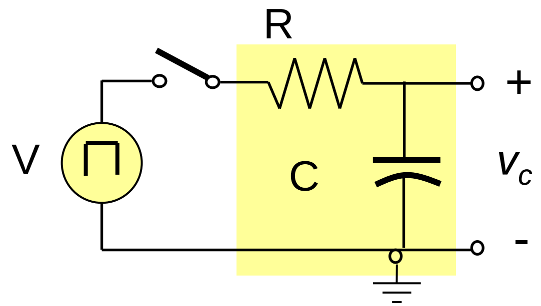
Q6. In the following circuit, a pulse of height *V* and width *a* is applied at *t* = 0. Find an expression for the current.



$$\therefore v_{in}(t) = V\left[U(t) - U(t-a)\right].$$

Now, applying KVL, $v_c(0_-) + \dfrac{1}{C}\displaystyle\int_{0_-}^{t} i\, dt + Ri = V\left[U(t) - U(t-a)\right]$

$\therefore$ Taking Laplace transform,

$$\frac{v_c(0_-)}{s} + \frac{I(s)}{Cs} + R\,I(s) = \frac{V}{s}\left[1 - e^{-as}\right]$$

Assuming $v_c(0_-) = 0$, $I(s) = VC\dfrac{1-e^{-as}}{1+CRs} = \dfrac{V}{R}\left[\dfrac{1}{s+1/CR} - \dfrac{e^{-as}}{s+1/CR}\right]$

$\therefore$ Taking inverse Laplace transform,

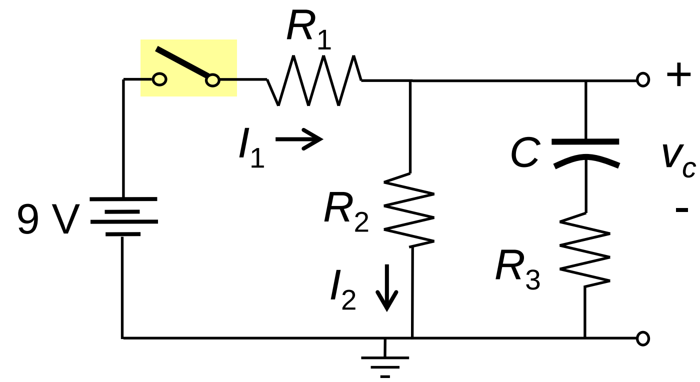
$$i(t) = \frac{V}{R}\left[e^{-t/CR}U(t) - e^{-(t-a)/CR}U(t-a)\right].$$

Tutorial - 1

Q7. In the following circuit, charge on the capacitor is zero for t <0. R_1 = 10 kΩ, R_2 = 10 kΩ R_3 = 1 kΩ C = 10 μF.

(i) At t = 0 Sec, the switch is closed. Find I_1 and I_2 at t = 0 and at t = 1 Sec.

(ii) The switch is reopened at t = 2 Sec. Find I_1 and I_2 at t = 2 Sec.



Answer:

(i) At $t = 0$, the capacitor is shorted.

$$\therefore I_1(0) = 9/\left(R_1 + R_2 \parallel R_3\right) = 0.825 \text{ mA. and}$$

$$I_2(0) = R_3 \times 0.825/\left(R_2 + R_3\right) = 0.075 \text{ mA.}$$

$$\tau = R_{eq}C = 60\, mS << 1 \text{ Sec} \quad \Rightarrow \text{the capacitor is fully charged.}$$

$$\therefore I_1(1\text{Sec}) = I_2(1\text{Sec}) = 9/\left(R_1 + R_2\right) = 0.45 \text{ mA.}$$

(ii) At $t = 2$ Sec, left-hand part is open.

$$\therefore I_1(2 \text{ Sec}) = 0 \text{ and } \; I_2(2 \text{ Sec}) = 4.5/\left(R_2 + R_3\right) = 0.409 \text{ mA.}$$

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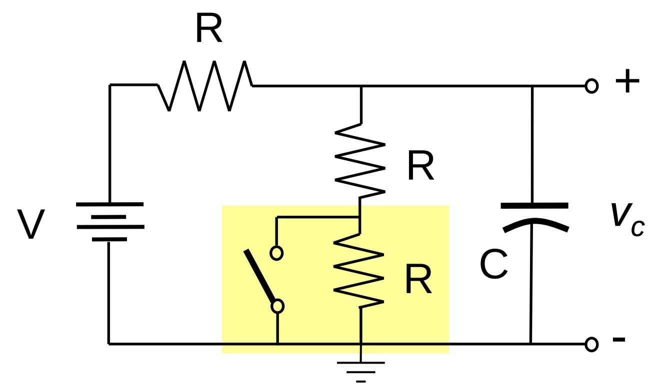
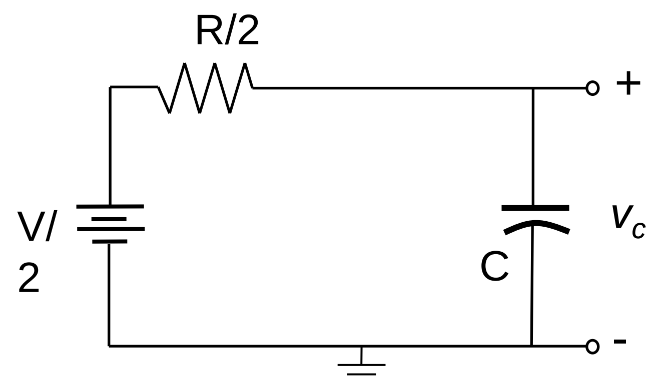
Q8. The circuit is in steady state. The switch is closed at $t = 0$. Find an expression for the v_c.



$$\therefore v_c(0_-) = \frac{2}{3}V.$$

But, for $t > 0$, looking from the capacitor terminal,

the Thevenin's voltage = $\dfrac{V}{2}$.



Equivalent circuit for $t > 0$.

Now, applying KVL, $\dfrac{R}{2} i + v_c(0_-) + \dfrac{1}{C}\displaystyle\int_{0_-}^{t} i \; dt = \dfrac{V}{2}$

Taking Laplace transform,

$$\frac{R}{2} I(s) + \frac{v_c(0_-)}{s} + \frac{I(s)}{Cs} = \frac{V}{2s},$$

$$\Rightarrow \frac{R}{2} I(s) + \frac{2V}{3s} + \frac{I(s)}{Cs} = \frac{V}{2s},$$

$$\Rightarrow I(s) = -\frac{V/(3R)}{s + 2/(CR)}.$$

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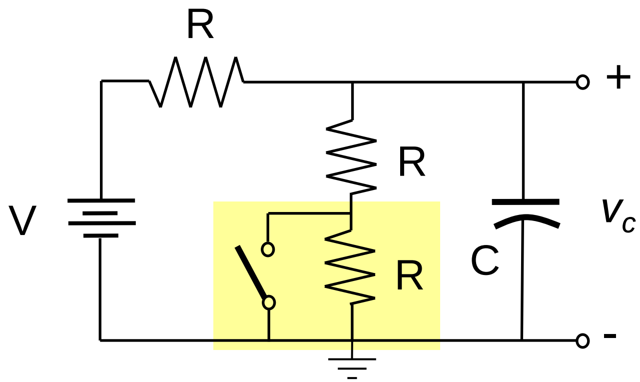

$\therefore$ Capacitor voltage for $t>0$,

$$V_c(s) = \frac{v_c(0\_)}{s} + \frac{I(s)}{Cs} = \frac{2V}{3s} - \frac{V/(3RC)}{s[s+2/(CR)]},$$

$$= \frac{(2/3)Vs + V/(RC)}{s[s+2/(CR)]} = \frac{A}{s} + \frac{B}{s+2/(CR)}.$$

Now, expanding into partial fractions,

$$A = \left.\frac{(2/3)Vs + V/(RC)}{[s+2/(CR)]}\right|_{s=0} = \frac{V}{2}$$

$$B = \left.\frac{(2/3)Vs + V/(RC)}{s}\right|_{s=-2/CR} = \frac{V}{6}.$$

$$\therefore V_c(s) = \frac{V}{2s} + \frac{V}{6[s+2/(CR)]}.$$

$\therefore$ Taking inverse Laplace transform,

$$v_c(t) = \frac{V}{2} + \frac{V}{6}e^{-2t/CR}, \quad (\text{for } t>0).$$

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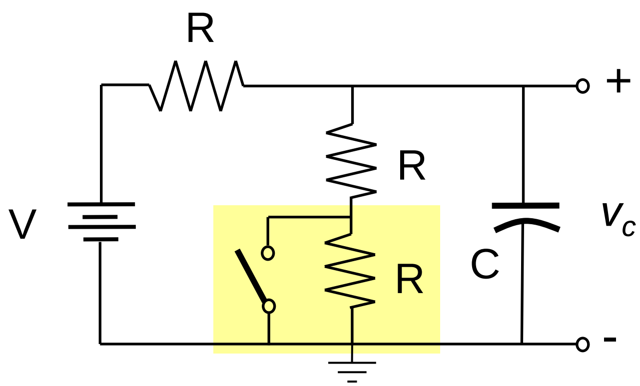
$\therefore$ Capacitor voltage for $t > 0$,

$$V_c(s) = \frac{v_c(0_-)}{s} + \frac{I(s)}{Cs} = \frac{2V}{3s} - \frac{V/(3RC)}{s\left[s+2/(CR)\right]},$$

$$= \frac{(2/3)Vs + V/(RC)}{s\left[s+2/(CR)\right]} = \frac{A}{s} + \frac{B}{s+2/(CR)}.$$



Now, expanding into partial fractions,

$$A = \left.\frac{(2/3)Vs + V/(RC)}{\left[s+2/(CR)\right]}\right|_{s=0} = \frac{V}{2}$$

$$B = \left.\frac{(2/3)Vs + V/(RC)}{s}\right|_{s=-2/CR} = \frac{V}{6}.$$

$$\therefore V_c(s) = \frac{V}{2s} + \frac{V}{6\left[s+2/(CR)\right]}.$$

$\therefore$ Taking inverse Laplace transform,

$$v_c(t) = \frac{V}{2} + \frac{V}{6}e^{-2t/CR}, \quad (\text{for } t > 0).$$