Problem Set - 11

AUTUMN 2017

MATHEMATICS-I (MA10001)

1. (a) Ans : $\frac{248}{15}$ (Hint:Write Γ and the integrand in terms of x and y and then integrate)
   (b) (i) Ans:$\frac{10}{21}+i\frac{14}{15}$ (Hint:Put $z(t)=t+it^3$,$0\le t\le 1$;in the integral).
   (ii) Ans:$\frac{1}{3}(a^2+b^2)(a+ib)$ (Hint:Put $z(t)=(a+ib)t$, $0\le t\le 1$;$a,b\in\mathbb{R}$).
   (c) (i) Ans: $\frac{511}{3}-i\frac{49}{5}$ (Hint:$z=x+iy$ and put $y=2x^2$)
   (ii) Ans:$\frac{518}{3}-57i$ (Hint:First take x=constant and integrate w.r.t y then y=constant and integrate w.r.t x)
   (iii) Ans:$\frac{518}{3}-8i$ (Hint:Write down the straight line equation joining $(1,1)$ and $(2,8)$.Then change the integral in terms of a single variable)

2. (a) Ans :$\frac{\pi a}{(a^4-1)^2}$ (Hint: Use ML inequality formulae)
   (b) Ans :2 (Hint: Use ML inequality formulae)
   (c) Ans: $-i\pi$ (Hint:Put $z=e^{i\theta}$, $0\le\theta\le\pi$).

3. (a) Ans: 0 (Hint:Apply cauchy integral theorem.)
   (b) Ans: 0 (Hint:Apply cauchy integral theorem.)

4. (a) Ans: $\frac{\pi}{5}$ (Hint:Apply Cauchy integral formulae)
   (b) Ans: $2\pi i(e+1)$ (Hint:Apply Cauchy integral formulae).
   (c) (i) πi (Hint:Use Cauchy integral formulae)
   (ii) Ans: 0 (Hint:Apply cauchy integral theorem.)
   (d) Ans:$\frac{\pi i}{4}$ (Hint:Apply Cauchy integral formulae.)
   (e) Ans: $\pi\cos 6$ (Hint:Apply cauchy integral formulae.)
   (f) Ans: $4\pi i$ (Hint:Apply cauchy integral formulae.)
   (g) (Hint:Apply cauchy integral formulae.)

5. (a) Ans: $-\frac{2\pi i(a-3)}{a^3}$ (Hint:Applying Cauchy integral formulae for derivative)
   (b) Ans: $-i\frac{8}{3}\pi e^{-2}$ (Hint:Use Cauchy's Integral formulae for n'th derivative)

6. (a) Ans: e^2 (Hint:Apply cauchy integral formulae)
   (b) Ans: 0 (Hint:Apply cauchy integral theorem)

7. (a) Ans: $\frac{z}{9} - \frac{z^5}{9^2} + \frac{z^9}{9^3} - \frac{z^{13}}{9^4}$. (Hint:write $\frac{z}{z^4+9} = \frac{z}{9}(1+\frac{z^4}{9})^{-1}$ .Then expand)

(b) Ans: $z - \frac{z^2}{2} + \frac{z^3}{3} - z^4 4 + ...$ (Hint:Use Taylor's theorem about the point $z = 0$)

(c) Ans: $\frac{z-1}{2} - \frac{(z-1)^2}{2^2} + \frac{(z-1)^3}{2^3} - .....$ (Hint:Let $z - 1 = u$)

(d) Ans: $\frac{\sqrt{2}}{2}[1 + (z - \frac{\pi}{4}) - \frac{(z-\frac{\pi}{4})^2}{2!} - .....]$ (Hint:Use Taylor's series formulae)

8. (i) Ans:$\frac{1}{3}[1 - \frac{z}{3} - \frac{z^2}{18} + ...]$ (Hint:Use expansion of e^z)

(ii) Ans:$1 + z + \frac{z^2}{2} + ...$ (Hint:Use expansion of e^x)