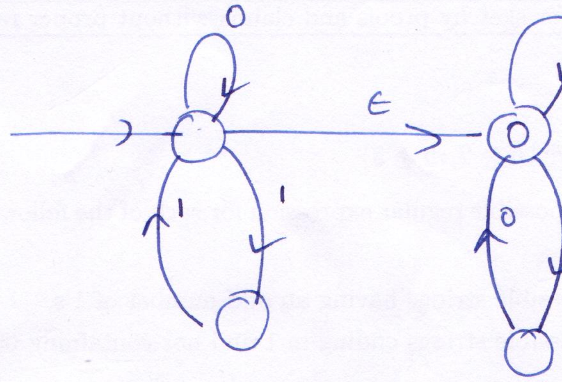
Regex ex

1.

$$(11+0)^*(00+1)^*$$

a



From Kozen

2.

$$(1+01+001)^*(\epsilon+0+00)$$

↓

simplify

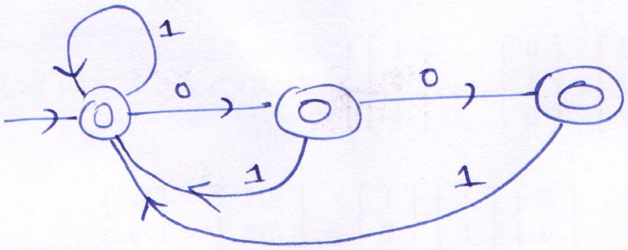
$$= ((\epsilon+0+00)1)^*(\epsilon+0+00)$$

[how ??]

$$= ((\epsilon+0)(\epsilon+0)1)^*(\epsilon+0)(\epsilon+0)$$

↑

set of all strings $\in \{0,1\}^*$ w/o string having more than 2 adjacent zeros.



Algebraic laws for Regex

'+'- associative, '+' commutative, idempotent w.r.t '+',

'$\cdot$' - associative,

$\epsilon = \{\epsilon\}$

$\Phi = \{\ \}$

$$\alpha + \Phi = \alpha$$

$$\epsilon\alpha = \alpha\epsilon = \alpha$$

$$\alpha(\beta+\gamma) = \alpha\beta + \alpha\gamma$$

$$(\alpha+\beta)\gamma = \alpha\gamma + \beta\gamma$$

$$\Phi\alpha = \alpha\Phi = \Phi$$

$$\epsilon + \alpha\alpha^* = \alpha\alpha^*$$

$$\epsilon + \alpha^*\alpha = \alpha^*\alpha$$

$$\beta + \alpha\gamma \leq \gamma \Longrightarrow \alpha^*\beta \leq \gamma$$

$$\beta + \alpha\gamma \leq \gamma \Rightarrow \beta\alpha^* \leq \gamma$$

'$\leq$' is subset ordering.

$$\alpha \leq \beta \Longleftrightarrow L(\alpha) \subseteq L(\beta)$$

$$\Longleftrightarrow L(\alpha+\beta) = L(\beta)$$

$$\Longleftrightarrow \alpha+\beta = \beta$$

The above laws do imply the following interesting equivalence of regular expressions.

$$(\alpha\beta)^*\alpha \equiv \alpha(\beta\alpha)^*$$

$$(\alpha^*\beta)^*\alpha^* \equiv (\alpha+\beta)^* \equiv \alpha^*(\beta\alpha^*)^*$$

$$(\epsilon+\alpha)^* \equiv \alpha^*$$

$$\alpha\alpha^* \equiv \alpha^*\alpha$$