

**Algorithms -1**  
**Tutorial 3**  
**August 17, 2018**

1. Show all steps in sorting the array  $A = (38\ 81\ 22\ 48\ 13\ 69\ 93\ 14\ 45\ 58\ 79\ 72)$  using Quicksort always choosing the pivot element to be the element in position  $(\text{left} + \text{right})/2$ .
2. Suppose you have to sort the integers 1342, 233, 145, 8757, 23, 1888, 2545, 3245, 191 using radix sort. Show the sorted sequences after each digit is sorted.
3. How can you use the partitioning idea of quicksort to give an algorithm that finds the median element of an array of  $n$  integers? What is the time complexity of your algorithm?
4. An array  $A$  of  $n$  integers consists of only the numbers 0, 1, and 2. Design an  $O(n)$  time algorithm to sort the array using (i)  $O(n)$  extra space, (ii)  $O(1)$  extra space.
5. Given an Array  $A$  with  $n$  elements, two elements  $A[i]$  and  $A[j]$  form an inversion if  $A[i] > A[j]$  and  $i < j$ . Count the number of Inversions in the array in  $O(n \log n)$  time.
6. Given  $n$  integers in the range 0 to  $k$ , design an algorithm that pre-processes its input in  $O(n + k)$  time, then answers a query about how many of the  $n$  integers fall into a range  $[a, b]$  in  $O(1)$  time. You can use  $O(k)$  additional space.
7. You are given  $n$  intervals in the form of  $[l, r]$  where  $l$  and  $r$  are integers. An interval is said to be active at  $i$  if  $l \leq i \leq r$ . Design an  $O(n \log n)$  time algorithm to find the maximum number of intervals that are active at any integer.