

2.

Let  $Q_A$ .

a)

Let the given strings be  $s_1, s_2, \dots, s_k$ .

Define a matrix  $A_{ij} = S_{ij}$ .

- Now each column ~~has~~ can be partitioned based on the letters occurring in it. ~~Rows~~ Rows of the column in which same letter occur are grouped into the same partition. This partitioning is done on every column. So, we can define  $C_p$  has the number of columns having the same partition.

- If  $C$  is a class of the partitioning  $P$ , then let  $Z_{P,C}$  be the number of columns where solution agrees with class  $C$ . That is, ~~the corresponding~~ ~~to the~~ ~~partition~~ Here, class refers to the partitioning of the column.

By the definition of Closest-String problem

$$\sum_{C \in P} Z_{P,C} \leq C_P \quad \forall P \quad \left\{ \begin{array}{l} \text{The number of} \\ \text{columns of a} \\ \text{particular partition} \end{array} \right\}$$

$$\sum_{i \in P, C \in P} Z_{P,C} \leq d \quad \forall 1 \leq i \leq K \quad \left\{ \begin{array}{l} \text{The number} \\ \text{of} \\ \text{where the} \\ \text{class doesn't} \\ \text{agree with the} \\ \text{solution should} \\ \text{be less} \\ \text{than } d \end{array} \right\}$$

$$Z_{P,C} \geq 0 \quad \forall P, C \quad \left\{ \begin{array}{l} \text{Trivial} \\ \text{constraint} \end{array} \right\}$$

(2)

Thus, if closest string has a solution, the ILP also has a solution.

Kaushik Ray  
176530022

b) Similarly, if we have the ILP and we can find its solution,

we can choose  $\mathbb{Z}_p, c$  columns that match in the solution, ~~and the~~ having the corresponding partition of the rest of the columns with that partition we can choose a solution string that disagrees.

Thus, if the ILP has a solution, then the closest string also has a solution.

c) We know that ILP can be solved in  $O(p^{2.5p+o(p)} \cdot L)$  where  $p$  is the number of variables.

Here the number of variables is  $|P|k$ , where  $|P|$  is the number of partitions of the instance.

So, time is  $O(|P|k)^{O(|P|k)}$  and thus closest string is FPT.

(3)

Koushik Ray

17CS30022

1. We will first try to reduce the given instance.

- Each vertex can have only at most  $r$  edges of the same colour. This is because if we choose any at most  $r$  vertex neighboring to this in the matching set's vertices.  

$$\Rightarrow \max_{v \in V(G)} \deg(v) = r^2$$

- Now we just branch on the choice of edges. Assuming there is an edge in the solution from this vertex and the neighboring vertex, we choose that edge & remove all edges with that color and edges incident on those 2 vertices and decrease  $r$  by 1.

- If not then we branch on the choice of the edges in the solution from neighbor to neighbors' edges incident on those vertices and the edges of corresponding colour.

- Tree depth  $= r$   
 Number of branches  $= r^2 + r^4 + \dots + r^{4r}$   
 $\Rightarrow \text{Total time} = O(r^{4r} \cdot n)$   
 $\Rightarrow$  This is an FPT