Practice Set I

1. Construct a truth table for each of these compound propositions.

a) $(p \vee q) \to (p \oplus q)$
b) $(p \oplus q) \to (p \wedge q)$
c) $(p \vee q) \oplus (p \wedge q)$
e) $(p \leftrightarrow q) \oplus (\neg p \leftrightarrow \neg r)$
d) $(p \leftrightarrow q) \oplus (\neg p \leftrightarrow q)$
f) $(p \oplus q) \to (p \oplus \neg q)$
d) $(p \leftrightarrow q) \oplus (\neg p \leftrightarrow q)$

2. Construct a truth table for each of these compound propositions

a) $p \to \neg q$
b) $\neg p \leftrightarrow q$
c) $(p \to q) \vee (\neg p \to q)$
d) $(p \to q) \wedge (\neg p \to q)$
e) $(p \leftrightarrow q) \vee (\neg p \leftrightarrow q)$
f) $(\neg p \leftrightarrow \neg q) \leftrightarrow (p \leftrightarrow q)$

3. Express these system specifications using the propositions p “The message is scanned for viruses“ and q “The message was sent from an unknown system“ together with logical connectives.
a) “The message is scanned for viruses whenever the message was sent from an unknown system.“
b) “The message was sent from an unknown system but it was not scanned for viruses.“
c) “It is necessary to scan the message for viruses whenever it was sent from an unknown system.“
d) “When a message is not sent from an unknown system it is not scanned for viruses.“

4. Show that each of these conditional statements is a tautology by using truth tables.
a) $[\neg p \wedge (p \vee q)] \to q$
b) $[(p \to q) \wedge (q \to r)] \to (p \to r)$
c) $[p \wedge (p \to q)] \to q$
d) $[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r$

5. Show that $(p \to q) \to (r \to s)$ and $(p \to r) \to (q \to s)$ are not logically equivalent.

6. The **dual** of a compound proposition that contains only the logical operators $\vee, \wedge$, and $\neg$ is the compound proposition obtained by replacing each $\vee$ by $\wedge$, each $\wedge$ by $\vee$, each T by F, and each F by T. The dual of s is denoted by s^*.

(i) Find the dual of each of these compound propositions.
$a)\ p \vee \neg q$
$b)\ p \wedge (q \vee (r \wedge T))$
$c)\ (p \wedge \neg q) \vee (q \wedge F)$

(ii) Find the dual of each of these compound propositions.
$a)\ p \wedge \neg q \wedge \neg r$
$b)\ (p \wedge q \wedge r) \vee s$
$c)\ (p \vee F) \wedge (q \vee T)$

7. Find a compound proposition involving the propositional variables p, q, and r that is true when exactly two of p, q, and r are true and is false otherwise. [Hint: Form a disjunction of conjunctions. Include a conjunction for each combination of values for which the propositional variable is true. Each conjunction should include each of the three propositional variables or their negations.]

8. Determine the truth value of each of these statements if the domain consists of all real numbers.
$a)\ \exists x(x^3 = -1)$
$b)\ \exists x(x^4 < x^2)$
$c)\ \forall x((-x)^2 = x^2)$
$d)\ \forall x(2x > x)$

9. Suppose that the domain of the propositional function $P(x)$ consists of the integers 0, 1, 2, 3, and 4. Write out each of these propositions using disjunctions, conjunctions, and negations.
$a)\ \exists x P(x)$
$b)\ \forall x P(x)]$
$c)\ \neg \exists x P(x)$
$d)\ \forall x \neg P(x)$
$f)\ \neg \forall x P(x)$

10. Translate in two ways each of these statements into logical expressions using predicates, quantifiers, and logical connectives. First, let the domain consist of the students in your class and second, let it consist of all people.
a) Everyone in your class has a cellular phone.
b) Somebody in your class has seen a foreign movie.
c) There is a person in your class who cannot swim.
d) All students in your class can solve quadratic equations.
e) Some student in your class does not want to be rich.

11. Translate each of these statements into logical expressions using predicates, quantifiers, and logical connectives.
a) Something is not in the correct place.
b) All tools are in the correct place and are in excellent condition.
e) Everything is in the correct place and in excellent condition.
d) Nothing is in the correct place and is in excellent condition.
e) One of your tools is not in the correct place, but it is in excellent condition.

12. Express each of these system specifications using predicates, quantifiers, and logical connectives.
a) When there is less than 30 megabytes free on the hard disk, a warning message is sent to all users.
b) No directories in the file system can be opened and no files can be closed when system errors have been detected.
c) The file system cannot be backed up ifthere is a user currently logged on.

13. Express each of these system specifications using predicates, quantifiers, and logical connectives.
a) Every user has access to an electronic mailbox.
b) The system mailbox can be accessed by everyone in the group if the file system is locked.
c) The firewall is in a diagnostic state only if the proxy server is in a diagnostic state.
d) At least one router is functioning normally if the throughput is between 100 kbps and 500 kbps and the proxy server is not in diagnostic mode.

14. Show that $\exists x(P(x) \vee Q(x))$ and $\exists xP(x) \vee \exists xQ(x)$ are logically equivalent.

15. Show that $\forall xP(x) \vee \forall xQ(x)$ and $\forall x(P(x) \vee Q(x))$ are not logically equivalent.

16. Show that $\forall xP(x) \wedge \forall xQ(x)$ and $\forall x(P(x) \wedge Q(x))$ are not logically equivalent.

17. Prove that if n is an integer and 3n + 2 is even, then n is even using
a) a proof by contraposition.
b) a proof by contradiction.

18. Use a proof by contradiction to show that there is no rational number r for which $r^3 + r + 1 = O$. [Hint: Assume that $r = a/b$ is a root, where a and b are integers and a/b is in lowest terms. Obtain an equation involving integers by multiplying by b^3. Then look at whether a and b are each odd or even.]

19. Prove that if n is a positive integer, then n is even if and only if $7n+4$ is even.

20. Show that these statements about the real number x are equivalent:
(i) x is rational, (ii) $x/2$ is rational, and (iii) 3x - 1 is rationaL

21. Prove that these four statements about the integer n are equivalent:
(i) n^2 is odd, (ii) $1 - n$ is even, (iii) n^3 is odd, (iv) $n^2 + 1$ is even.

22. Prove that $n^2 + 1 \geq 2^n$ when n is a positive integer with $1 \geq n \geq 4$.

23. Prove that $\sqrt[3]{2}$ is irrational.

24. a) Draw each of the five different tetrominoes, where a tetromino is a polyomino consisting of four squares.
b) For each of the five different tetrominoes, prove or disprove that you can tile a standard checkerboard using these tetrominoes.

25. Assuming the truth of the theorem that states that $\sqrt{n}$ is irrational whenever n is a poitive integer that is not a perfect square, prove that $\sqrt{2} + \sqrt{3}$ is irrational.