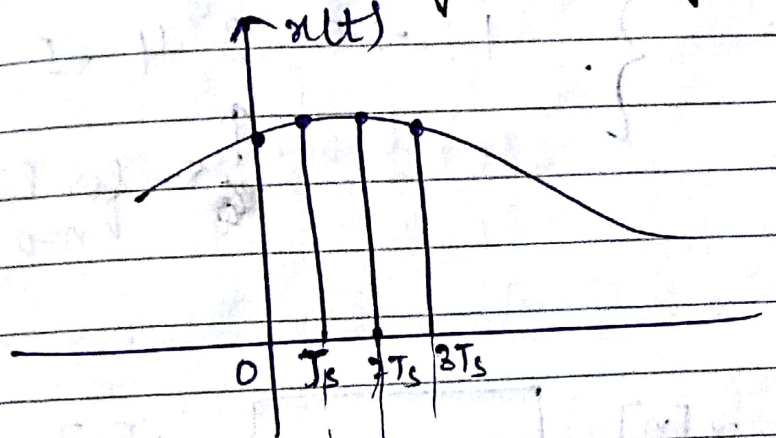
* L.T.

① Initial value theo. $\; x(t) = L^{-1}[X(s)]$, $X(s)$

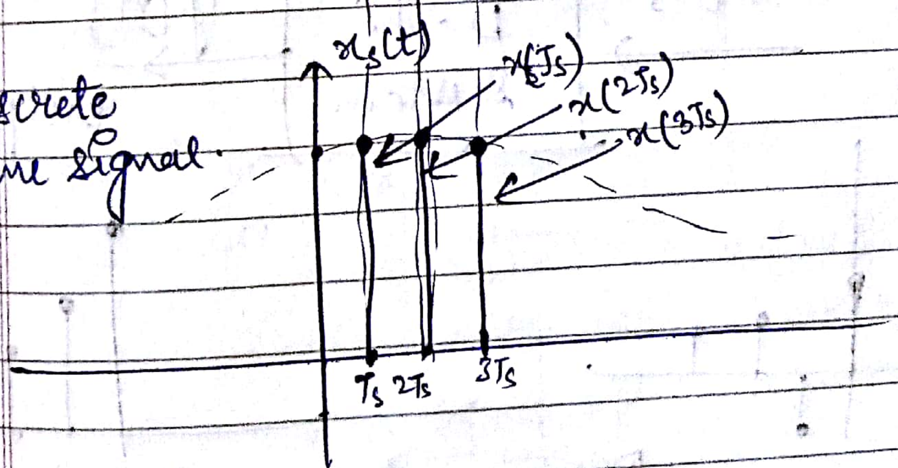
$$x(0) = \lim_{s \to \infty} sF(s)$$

② Final value theo. $\; x(\infty) = \lim_{s \to 0} sF(s).$

* Discrete Time signals & systems.



Discrete time signal.



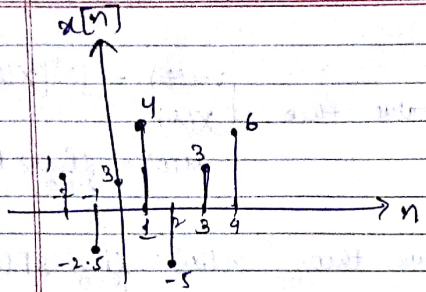
Conti. Time Signal

↓

Discrete time "

↓

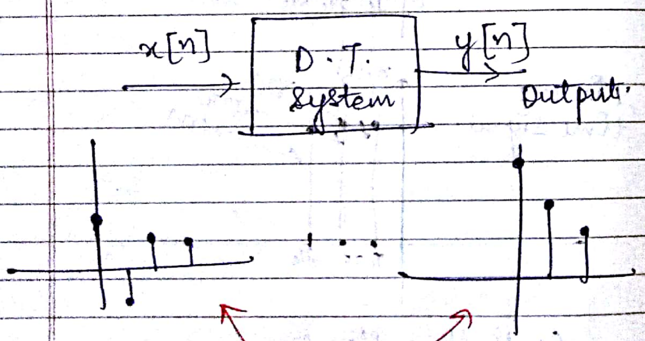
Digital Signal. (Even y-axis is quantized.)



$x[n] \rightarrow$ Sequence

$$= \{ \ldots \; 1 \;\; -2.5 \;\; \underset{\uparrow}{3} \;\; 4 \;\; -5 \;\; 3 \;\; 6 \ldots \}$$

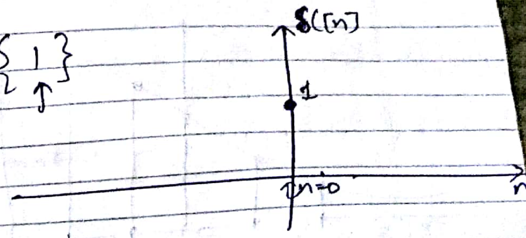
← for indicating n=0.

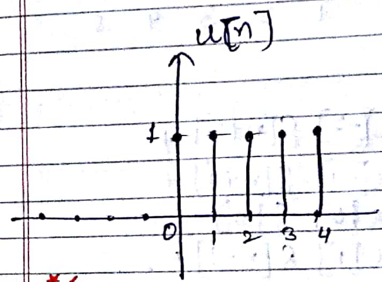


connected by difference eqⁿ.

$$\delta[n] = \{ \underset{\uparrow}{1} \}$$

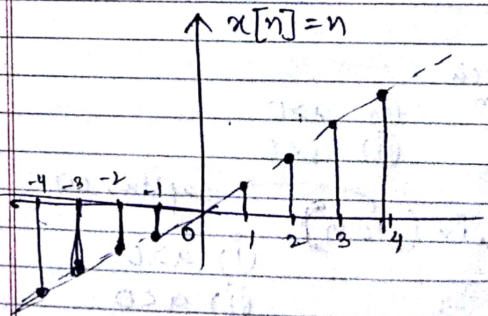
Unit Impulse seq.



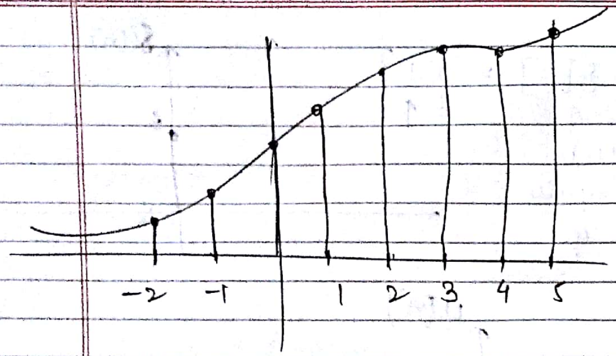


$$u[n] = \delta[n] + \delta[n-1] + \delta[n-2] + \ldots$$

$$\boxed{u[n] - u[n-1] = \delta[n]}$$



$$\boxed{n\,u[n] = r[n]}$$



$$x[n] = x[-2]\,\delta[n+2] + x[-1]\,\delta[n+1] + x[0]\,\delta[n] + x[1]\,\delta[n-1] + \ldots$$

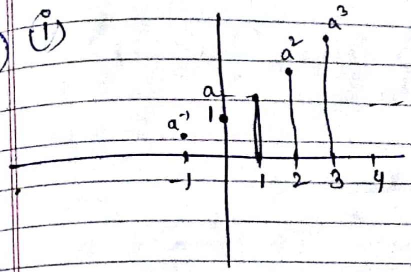
$$\therefore \boxed{x[n] = \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k]}$$

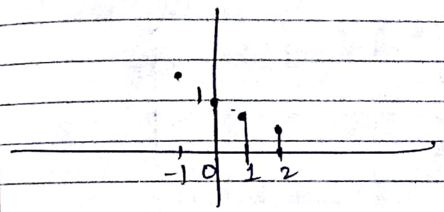
Q. Sketch

1. a^n (i) $a>0$ (ii) $a<0$

2. $a^n \sin(\Omega n)$ — capital ω
   (i) $a>0$
   (ii) $a<0$

1. (i)





2.

$V_{AB} = I(z+\Delta z) + V$
$\| $
Iz

$V = -I(\Delta z)$

$x[n] = \ldots x[-1]\,\delta[n+1] + x[0]\,\delta[n] + x[1]\,\delta[n-1] + \ldots$

$\therefore \boxed{x[n] = \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k]}$ ***

$x[n] = \{\ldots x[-1], \underset{\uparrow}{x[0]}, x[1], x[2] \ldots\}$

$x[n] = \{\ldots 1, 5, \underset{\uparrow n=0}{-2}, 3, 4 \ldots\}$

$x[n-1] = \{\ldots 1, \underset{\uparrow}{5}, -2, 3, 4 \ldots\}$

$x[-n] = \{4, 3, \underset{\uparrow}{-2}, 5, 1\}$

$x[n] \rightarrow \boxed{DTS} \rightarrow y[n]$

$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$

LTI

$\delta[n] \rightarrow \boxed{DTS} \rightarrow h[n]$ ← Impulse Resp.

$x[n] \rightarrow \boxed{DTS} \rightarrow y[n] = ?$

$\delta[n] \rightarrow h[n]$

Only for LTI sys. $\Rightarrow \delta[n-k] \rightarrow h[n-k]$

Linearity $\Rightarrow x[k]\,\delta[n-k] \rightarrow x[k]\,h[n-k]$

superposition

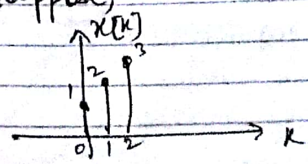
$\sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k] \rightarrow \sum_{k=-\infty}^{\infty} x[k]\,h[n-k]$

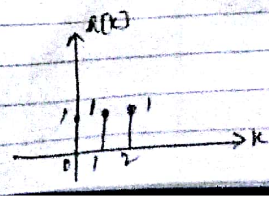
$\therefore \boxed{y[n] = \sum_{k=-\infty}^{\infty} x[k]\,h[n-k]}$ convolution sum

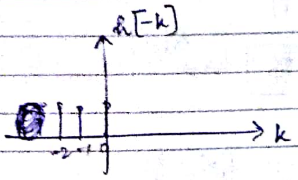
$= \sum_{k=-\infty}^{\infty} h[k]\,x[n-k]$

$\Rightarrow y[n] = x[n] * h[n] = h[n] * x[n]$

Suppose,







Go ahead but it is tedious to get $y[n]$.

Method-2

Solve using this.

$x[n] = \{x_{-3} \ x_{-2} \ x_{-1} \ \overset{\downarrow}{x_0} \ x_1 \ x_2\}$ ← l_1 No. of elements (length)

$h[n] = \{h_{-1} \ h_0 \ h_1\}$ ← l_2

$y[n] = \boxed{x[n] * h[n]}$ ← $l_1 + l_2 - 1$ (No. of elements in 'y'[n])

		x_{-3}	x_{-2}	x_{-1}	x_0	x_1	x_2
				h_{-1}	h_0	h_1	×
		h_1x_{-3}	h_1x_{-2}	h_1x_{-1}	h_1x_0	h_1x_1	h_1x_2
	h_0x_{-3}	h_0x_{-2}	h_0x_{-1}	h_0x_0	h_0x_1	h_0x_2	×
$h_{-1}x_{-3}$	$h_{-1}x_{-2}$	$h_{-1}x_{-1}$	$h_{-1}x_0$	$h_{-1}x_1$	$h_{-1}x_2$	×	×
$y[-4]$	$y[-3]$	$y[-2]$	$y[-1]$	$y[0]$	$y[1]$	$y[2]$	$y[3]$

← Add + vertically without + any carry forward.

Obtained by adding subscripts of above numbers.

---

n→ -3 -2 -1 0 1 2 3 4

$x[n] = \{1 \ 4 \ 3 \ \overset{\downarrow}{-2} \ 5 \ 6 \ 8 \ 10\}$

$x[2n] = \{4 \ \overset{\downarrow}{-2} \ 6 \ 10\}$

$x[n/2] = \{1, 0, 4, 0, 3, 0, \overset{\downarrow}{-2}, 0, 5, 0, 6, 0, 8, 0, 10\}$

Q. Find out $x[-2n+2]$ if

$x[n] = \{1 \ 4 \ 3 \ \overset{\downarrow}{-2} \ 5 \ 6 \ 8 \ 10\}$

$x[n+2] = \{1 \ 4 \ 3 \ -2 \ 5 \ \overset{\downarrow}{6} \ 8 \ 10\}$

$x[2n+2] = \{4 \ -2 \ \overset{\downarrow}{6} \ 10\}$

$x[-2n+2] = \{10 \ \overset{\downarrow}{6} \ -2 \ 4\}$

Q. $x[n] = \{\overset{\downarrow}{1} \ 2 \ 3 \ 4\}$

$h[n] = \{\underset{\uparrow}{-1} \ -2 \ 4\}$

$y[n] = ?$

~~y[n] = {1 1 -1 -2 4 16}~~

$x[n] \rightarrow$ / $h[n] \downarrow$	1	2	3	4
-1	-1	-2	-3	-4
-2	-2	-4	-6	-8
4	4	8	12	16

$y[n] = \{-1 \quad -4 \quad -3 \quad -2 \quad 4 \quad 16\}$

* convolution sum :- graphically.

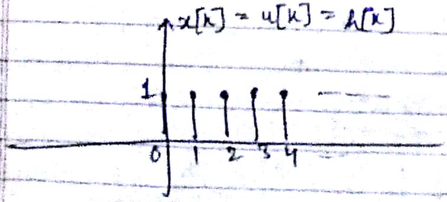
$x[n] = u[n]$

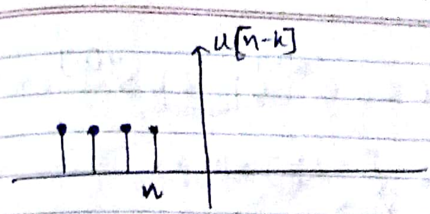
$h[n] = u[n]$

$y[n] = x[n] * h[n] = u[n] * u[n]$

$$y[n] = \sum_{k=-\infty}^{\infty} u[k]\, u[n-k]$$

$$y[1] = \sum_{k=-\infty}^{\infty} u[k]\, u[1-k]$$

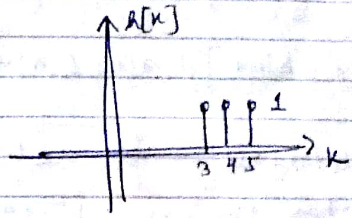


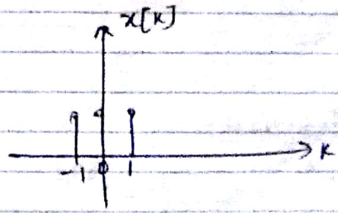


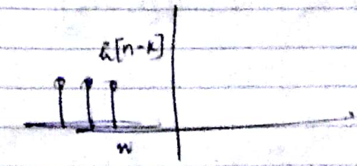
y(0) $\quad n<0$ no over lap

$n \geq 0$

eg.







(1) Linear

$x[n] \rightarrow \boxed{\quad} \rightarrow y[n]$

$x_1[n] \rightarrow y_1[n]$

$x_2[n] \rightarrow y_2[n]$

$\alpha x_1[n] + \beta x_2[n] \rightarrow \alpha y_1[n] + \beta y_2[n]$

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② causal.

$x[n] \to \boxed{\quad} \to y[n]$

$$y[n] = \sum_{k=-\infty} h[k]\, x[n-k]$$

$$= \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

$$\ldots + x_{-1} h[n+1] + x[0]\, h[n] + x[1]\, h[n-1] + \ldots$$

$$\boxed{\begin{array}{l} h[n] = 0 \quad \text{for } n < 0 \\ \text{then the sys. is causal} \end{array}}$$

③ Time Invariant.

$x[n] \to \boxed{\quad} \to y[n]$

$$\boxed{x[n-n_0] \to y[n-n_0]}$$

④ Stability.

$$y[k] = \sum_{k=-\infty}^{\infty} h[k]\, \underbrace{x[n-k]}_{\text{bounded}}$$

$$|y[k]| \le \sum_{k=-\infty}^{\infty} |h[k]|\, |x[n-k]|$$

$$\boxed{\therefore \sum_{k=-\infty}^{\infty} |h[k]| < \infty}$$

For memory less sys.,
$h[n] = k\,\delta[n]$.

⑤ Invertible.

$x[n] \to \boxed{h_1} \to \boxed{h_2} \to x[n]$

$$x[n] = (x[n] * h_1) * h_2$$
$$= x * (h_1 * h_2)$$

$$\boxed{\begin{array}{l} \text{For Invertibility,} \\ (h_1 * h_2) = \delta[n] \end{array}}$$

* Exponential f^n & Difference eq^n

$$y[n] - \alpha\, y[n-1] = x[n]$$

$x[n] \to \boxed{\quad} \to y[n]$

$$y[n] - \alpha\, y[n-1] = 0$$

let $y[n] = k a^n$

$$y[n-1] = \frac{k}{a} a^n$$

$y[0] = \alpha\, y[-1]$
$y[1] = \alpha\, y[0]$

Shift operator.

$$\boxed{\begin{array}{l} E\{y[n]\} = y[n+1] \\ y[n] = E^{-1}\{y[n+1]\} \end{array}}$$

$n \to n+1$

$$y[n+1] - \alpha\, y[n] = 0$$
$$E\,y[n] - \alpha\, y[n] = 0$$
$$\underbrace{(E-\alpha)}_{\text{ch. eq.}^n}\, y[n] = 0$$

$m - \alpha = 0$
$m = \alpha$
$y[n] = k\alpha^n$

eg. $y[n] - \alpha\, y[n-1] - \beta\, y[n-2] = 0$

$y[n] - \alpha E^{-1} y[n] - \beta E^{-2} y[n] = 0$

$(1 - \alpha E^{-1} - \beta E^{-2})\, y[n] = 0$

$\underbrace{(E^2 - \alpha E - \beta)}_{\text{ch. eq}^n}\, y[n] = 0$

$m^2 - \alpha m - \beta = 0 < \begin{matrix} m_1 \\ m_2 \end{matrix}$

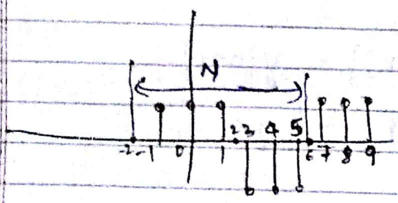
$\boxed{y[n] = c_1 m_1^n + c_2 m_2^n}$

* Periodic f^n in DTS.

$x[n] = x[n \pm N]$

N = funda. period.

$\Omega = \dfrac{2\pi}{N}$



Trigonometric f^n

$\cos(\Omega n)$

If it is to be periodic

$\cos(\Omega n) = \cos(\Omega(n+N))$

$N\Omega = k\, 2\pi$ ← Integer

$\boxed{\Omega = 2\pi\left(\dfrac{k}{N}\right)}$ ← Rational

eg: $\cos 2t$

$\Omega = 2 = 2\pi\left(\dfrac{1}{\pi}\right)$ ← Irrational → Non periodic.

eg. $\sin\left[\dfrac{1}{5}\pi n + 3\right]$

$\Omega = \dfrac{\pi}{5} = 2\pi\left(\dfrac{1}{10}\right)$ → Rational → Periodic

(radian) not rad/sec

fundamental Period = 10

$T = \dfrac{2\pi}{(\pi/5)} = 10$

Q. $x[n] = \cos(\pi n + 3) + \sin(0.1\pi n)$

$\Omega_1 = \pi$ $\Omega_2 = 0.1\pi$

$N_1 = \dfrac{2\pi}{\pi} = 2$ $N_2 = \dfrac{2\pi}{0.1\pi} = 20$

Period of the multitone signal

$LCM(2, 20) = 20$

Period $(x[n]) = 20$

Q. Find out the convolution of.

① $x_1[n] = u(n)$

$x_2(n) = \left(\frac{1}{2}\right)^n u(n)$

② $y[n] = \frac{1}{3}\left(x[n] + x[n-1] + x[n-2]\right)$

If $x[n] = u[n]$ sketch $y[n]$

Q. Find out if the following systems are causal & stable?

(i) $h[n] = 2^n u[-n]$

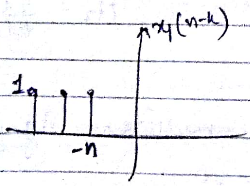
(ii) $h[n] = e^{2n} u[n-1]$

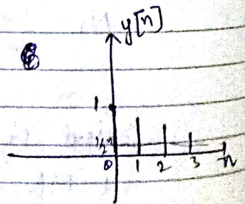
(iii) $5^n u[3-n]$

(iv) $e^{-6|n|}$

① $x_1(n-k) = u(n-k)$

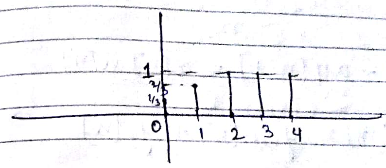
$x_2(k) = \frac{1}{2^k} u(k)$





$n > 0$	$n < 0$
$y[n] = 0$	$y[n] = \sum_{k=0}^{n} \frac{1}{2^k} = 2\left(1-\frac{1}{2}\right)$ (G.P. sum)

② $y[n] = \frac{1}{3}\left(u[n] + u[n-1] + u[n-2]\right)$



③ (i) Non causal ; stable.

(ii) Causal ; unstable

(iii) Non causal ; Stable

(iv) Non-causal ; Stable

* $y[n] + \alpha y[n-1] = x[n] = k b^n u[n]$

Let $y_f[n] = Y b^n$

$\left(1 + E^{-1}\alpha\right) y[n] = k b^n u[n]$

$Y b^n + \alpha Y b^{n-1} = k b^n u[n] = x[n]$

$Y b^n = x[n] / \left(1 + \frac{\alpha}{b}\right)$

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Also $y[n] = \frac{x[n]}{(1+\alpha E^{-1})}\Big|_{E=b}$

* $y[n] - \alpha y[n-1] = x[n]\,u[n]$

$y[n] = y_{nat}[n] + y_f[n]$

$y[n+1] - \alpha y[n] = 0$

$(E-\alpha)\,y[n] = 0$

$\boxed{m=\alpha}$

$\boxed{y_{nat}[n] = k_1 \alpha^n}$

If $x[n] = cb^n u[n]$

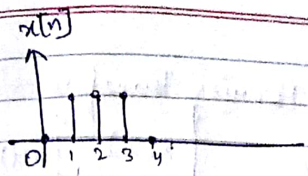
$$\boxed{y_f[n] = \frac{x[n]}{(E-\alpha)}\Big|_{E=b}}$$

* Discrete time Fourier series.

If $x[n]$ is periodic with fund. period 'N'

$c_k = \frac{1}{T}\int_{\langle T\rangle} x(t)\, e^{-jk\omega t}\, dt$

$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega t}$



$N=4$

$\Omega = \frac{2\pi}{4} = \frac{\pi}{2}$

$$\boxed{x_k = \frac{1}{N}\sum_{\langle N\rangle} x[n]\, e^{-jk\Omega n} \qquad x[n] = \sum_{k} x_k\, e^{jk\Omega n}}$$

$x_0 = \frac{1}{4}\sum_{\langle N\rangle} x[n]\, e^{0} = \frac{1}{4}\left[0\times1 + 1 + 1 + 1\right]$

$x_1 = \frac{1}{4}\sum_{\langle N\rangle} x[n]\, e^{-j\Omega n} = \frac{1}{4}\left[0 + e^{-j\Omega} + e^{-2j\Omega} + e^{-3j\Omega}\right]$

$e^{-jk\Omega n} = e^{-jk\Omega(n+N)}$ (as $N\Omega = 2\pi$)

eg. $x[n] = \cos \pi n + \cos 2\pi n$

$T_1 = \frac{2\pi}{\pi} = 2$; $T_2 = \frac{2\pi}{2\pi} = 1$

$\Omega = \frac{2\pi}{2} = \pi$

$$x[n] = \underbrace{\frac{1}{2} e^{j\pi n}}_{x_1} + \underbrace{\frac{1}{2} e^{-j\pi n}}_{x_{-1}} + \underbrace{\frac{1}{2} e^{j2\pi n}}_{x_2} + \underbrace{\frac{1}{2} e^{-j2\pi n}}_{x_{-2}}$$

* Discrete time Fourier Transform

$y[n] = x[n] * h[n]$
$= h[n] * x[n]$

$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$

$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega$

$y[n] = \sum_{k=-\infty}^{\infty} h[k] x[n-k]$

$x[n] \rightarrow \boxed{} \rightarrow y[n]$ Let $x[n] = a^n$

$y[n] = \sum_{k=-\infty}^{\infty} h[k] a^{n-k} = \left(\sum_{k=-\infty}^{\infty} h[k] a^{-k}\right) a^n$

Let $x[n] = e^{j\Omega n} = (e^{j\Omega})^n$

$a = e^{j\Omega}$

$\therefore y[n] = \left(\sum_{k=-\infty}^{\infty} h[k] e^{-j\Omega k}\right) \cdot e^{j\Omega n}$

***

$$\boxed{\begin{gathered} y[n] = (H(\Omega)) e^{j\Omega n} \\ H(\Omega) = \sum_{k=-\infty}^{\infty} h[k] e^{-j\Omega k} = \text{DTFT of } h[k] \end{gathered}}$$

$\therefore X(\Omega) = \sum_{k=-\infty}^{\infty} x[k] e^{-j\Omega k}$