

TOC CS41001 Test 4

Time Limit : 1hr

12 November 2020

1. 5 marks

Is the union of 2 NL-complete languages NL-complete ?

2. 5 marks

Show that $BIPARTITE = \{G \mid G \text{ is bipartite}\}$ is in NL. Use the fact that a graph is bipartite if and only if it does not have an odd cycle.

3. 5 marks

A map $h : \Sigma^* \rightarrow \tau^*$ is a homomorphism if $h(xy) = h(x)h(y)$ for all strings $x, y \in \Sigma^*$ (here xy denotes concatenation of x and y). It follows that $h(\epsilon) = \epsilon$. A homomorphism is non-erasing if $h(a) \neq \epsilon$ for all $a \in \Sigma$. For a language $A \subset \Sigma^*$, define $h(A) = \{h(x) \mid x \in A\} \subseteq \tau^*$. Prove that Σ_i^p is closed under non-erasing homomorphisms i.e., $A \in \Sigma_i^p \implies h(A) \in \Sigma_i^p$.

Hint: What happens to the length of a string after mapping by h ?