Indian Institute of Technology Kharagpur

Computer Science and Engineering

CS 60064 Computational Geometry Spring 2022

Date: 25.02.2022 **Total points =** 120; Maximum points = 100

Online Test-01 Credit: 25% **Time:** 11:05 am -12:50 PM

Instructions

A. This is an OPEN-BOOK/OPEN-NOTES online test. Read instructions carefully. This question paper has **three pages**.

B. Submission of answers: Please create a pdf file including **your name, roll-number**, and your **answers**, and submit it to the CSE Moodle Page by **1:05 PM, Friday, 25 February 2022.**

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1. (15 points)

(a) Construct a 10-vertex simple polygon with all vertices in general positions (no three are collinear, etc.) such that it admits only a unique triangulation.

(b) An ortho-convex (simple) polygon is one with the following properties: its edges are parallel to the coordinate axes and its intersection with any horizontal or vertical straight line is either empty or a single straight line segment. Given a simple polygon *P*, suggest an efficient algorithm to check whether it is an ortho-convex polygon. Mention its time complexity. (7 + 8)

2. (20 points)

You are given a set of *n* points in the plane surrounded by a convex *k*-gon *P* as shown in Fig. 1 below. *P* is provided as an ordered cyclic sequence of vertices. Assume that all points including the vertices of the convex *k*-gon are in general positions. We want to triangulate the configuration such that all $(n + k)$ vertices participate in triangulation and only those.

(i) How many edges will be present in the triangulation (excluding the bounding edges of *P*)?

(ii) Sketch an algorithm for triangulating the configuration. Analyze its complexity. Just provide the conceptual scheme, no pseudo-code is necessary. (10 + 10)

P

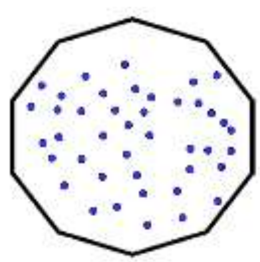
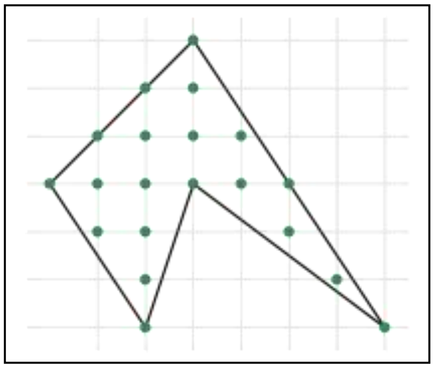


Fig. 1: Triangulation problem

Fig. 2: Lattice polygon P



3. (10 points) Let P be a simple polygon whose vertices have integer coordinates as shown in Fig. 2 (lattice polygon). Suggest an algorithm to find the number of integer points contained by P (interior plus boundary points). For example, the polygon in Fig. 2 contains 20 integer points.

4. (10 points)

A simple polygon P is said to be *strictly orthogonal* if all edges of P are axes-parallel (i.e., horizontal or vertical only), and no internal angle is π. Let such a polygon P comprise 12 vertices. We want to partition P into fewest convex pieces by inserting diagonals.
We claim that there exists such a polygon which cannot be so partitioned into fewer than 5 pieces. Justify the claim and show a supporting example, or argue that it is false.

5. (20 points) Two convex polygons P_1 (with m vertices) and P_2 (with n vertices) are given as CCW-ordered sequence of respective vertices.

(a) Sketch the outline of an $O(m + n)$-time algorithm that determines whether they are disjoint, intersecting, or one contains the other.

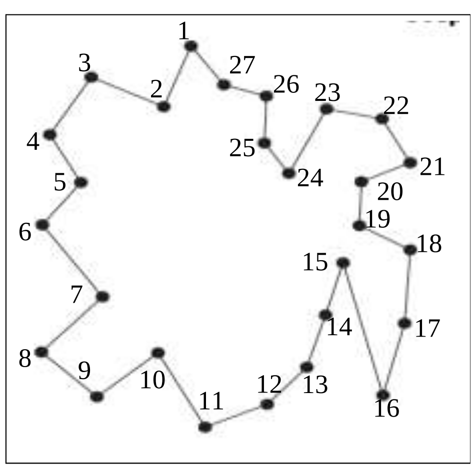
(b) Discuss an $O(m + n)$-time algorithm to construct the convex hull of $P_1 \cup P_2$.

In both cases, write down the steps only and justify the correctness of algorithm (no pseudo-code is required). (10 + 10)

6. (15 points)

(a) Which diagonals do you need to add to partition P (as in Fig. 3) into minimum number of y-monotone polygons (just write down the pair of vertices that define the diagonals, no need to show algorithmic steps)? Justify why your solution is indeed *minimum*.

Fig. 3: Monotone partitioning

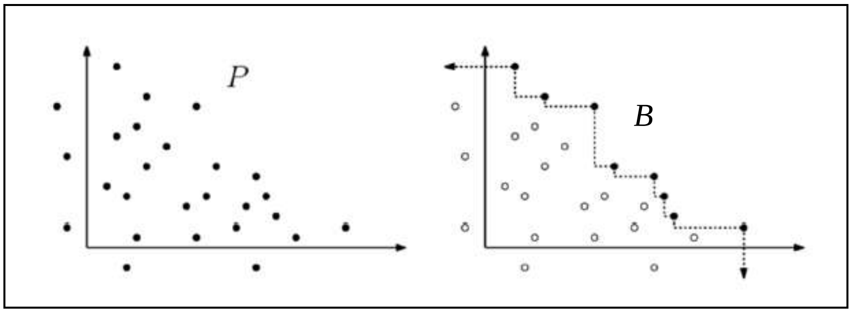


(b) Let P be a simple polygon with n vertices and assume that P have $r \geq 4$ reflex vertices. Show that r vertex guards are sometimes necessary and always sufficient to see the interior of P.

(7 + (4+4))

7. (10 points)

Fig. 4:

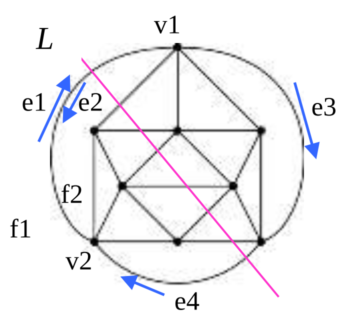


You are given a set P of n points, $n \geq 3$, being randomly placed in the plane as in Fig. 4. We want to construct a staircase-boundary B as shown above such that it tightly surrounds all the points lying in the first quadrant where all edges of B are either horizontal and vertical, and the area under B and the two axes is minimized. Note that a corner of B may not always coincide with a point in P. Design an $O(n \log n)$ algorithm for constructing B.

8. (10 points)

A planar subdivision of complexity $O(n)$ as in Fig. 5 is described in DCEL.
(a) Write the record for half-edge e1 in DCEL format.
(b) Given a straight-line L on the plane, and a half edge (say, e1) hit by it, as in Fig. 5, discuss a scheme with time complexity, to report all the faces intercepted by L. (5 + 5)

Fig. 5: DCEL



9. (10 points) How many times will turn-checking be needed while computing the *upper chain* of the convex hull using Sort-hull algorithm (a modification of Graham's Scan) for the following point-set shown in Fig. 6? Also, find out how many of them will be Left-Turn. (6 + 4)

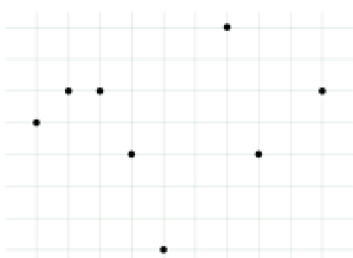


Fig. 6: Sort-Hull