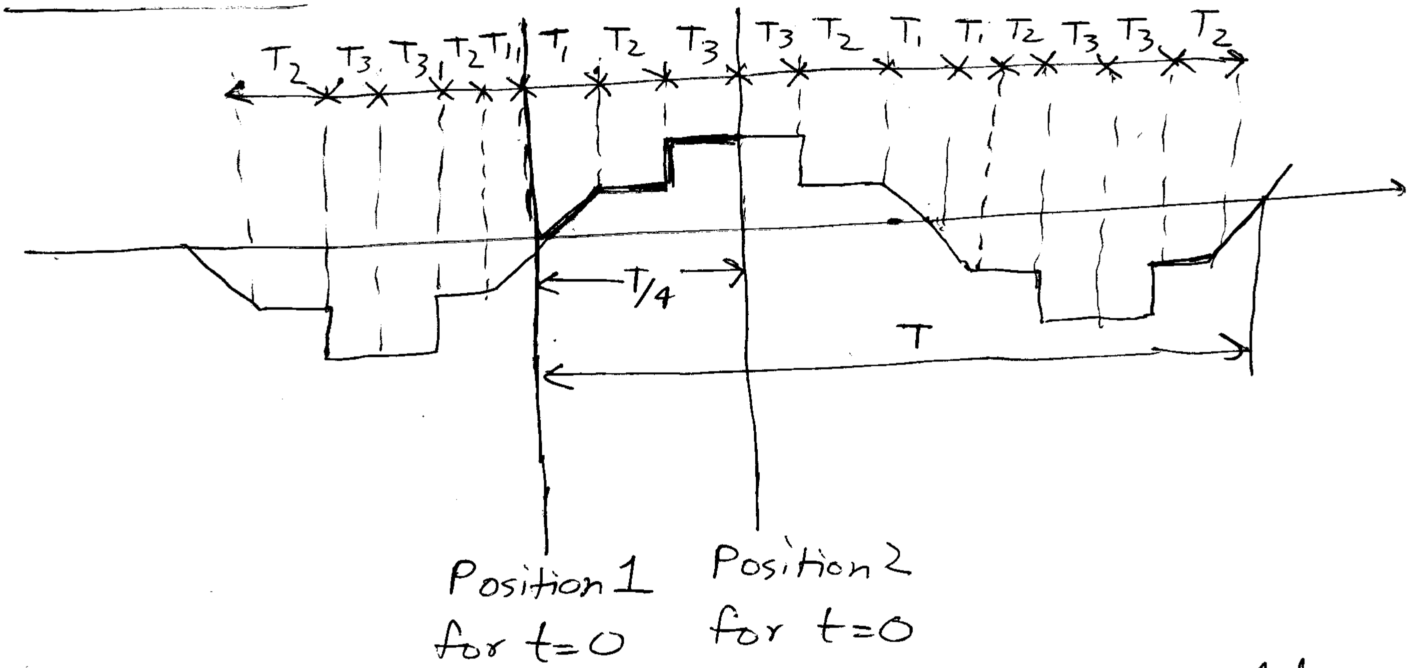
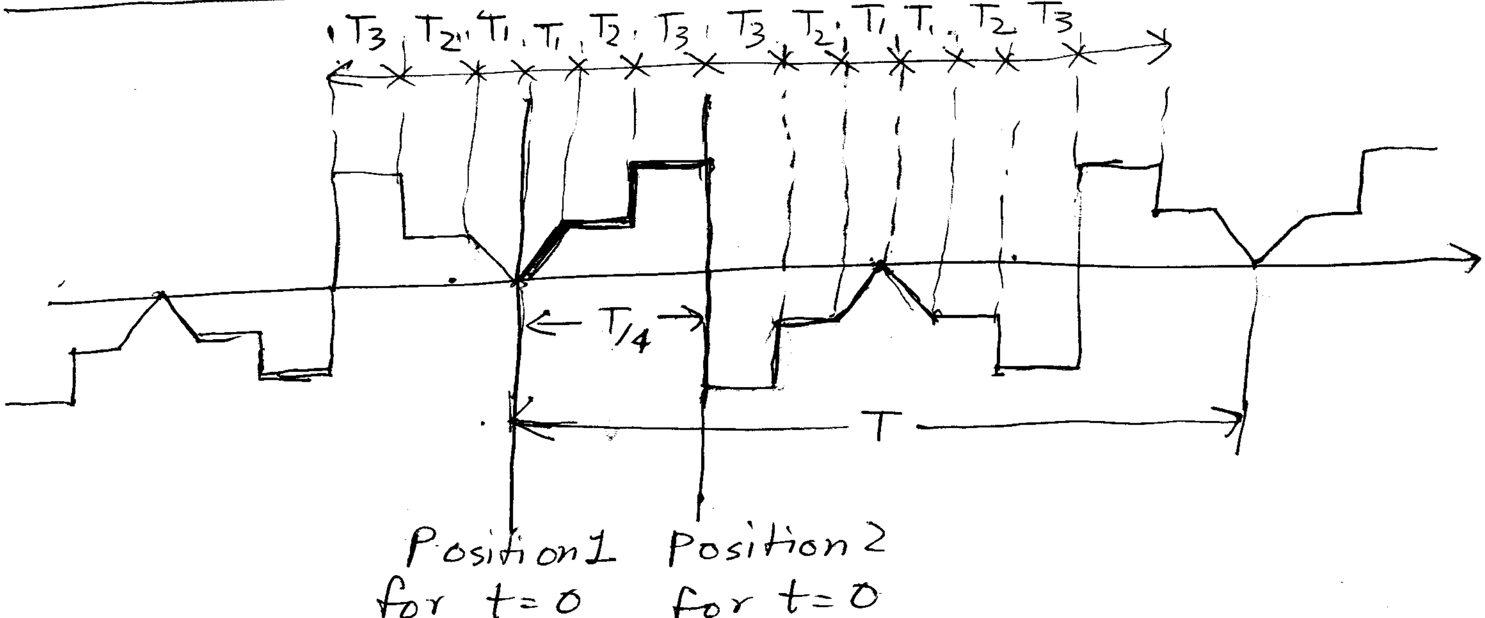
1) Two possible solutions:

Solution 1



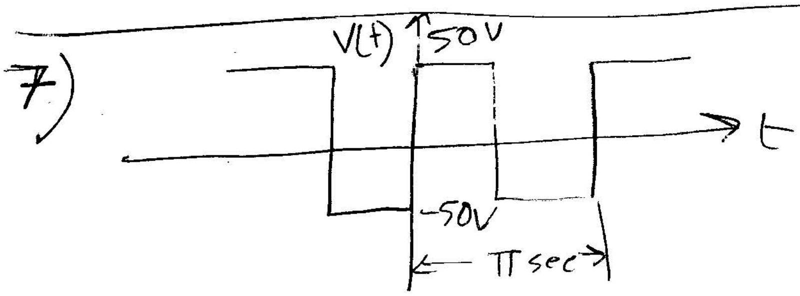
Due to quarter wave symmetry only odd harmonies will be present. ~~of~~ If $t=0$ is chosen to be at position 1 then it is an odd function so only sine terms will be present. If $t=0$ is chosen to be at position 2 then it becomes an even function so only cosine terms will be present. For any other position chosen for $t=0$ both sine & cosine terms can be present

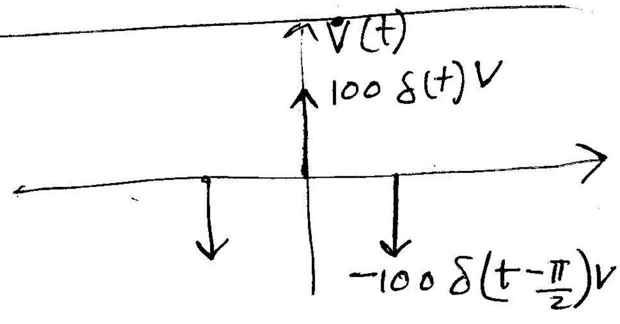
Solution 2



Due to quarter wave symmetry only odd harmonics will be there.
If position 1 is chosen for $t=0$ then it is an even function so only cosine terms will be there. If position 2 is chosen for $t=0$ then it becomes an odd function so only sine ~~ti~~ terms will be there.
For any other position of $t=0$ both sine & cosine terms will be there.

7)





F.S. coefficients of $\dot{V}(t)$ c_k'

$$= \frac{1}{T}\int_{0(t)}^{T(t)} \dot{V} e^{-jwkt}\, dt$$

$$= \frac{1}{T}\left(100 - 100 e^{-jwk\frac{\pi}{2}}\right)V$$

$$= \frac{100}{\pi}\left(1 - e^{-jk\frac{2\cancel{\pi}}{\cancel{\pi}}\times\frac{\pi}{\cancel{2}}}\right)V = \frac{100}{\pi}\left(1 - e^{-jk\pi}\right)$$

$\therefore$ F.S. coefficients of $V(t)$ $c_k = \frac{c_k'}{jwk}$

$$= \frac{100}{\pi jwk}\left(1 - e^{-jk\pi}\right) = \begin{cases} 0 & \text{for } k \text{ even} \\ \frac{200}{\pi jwk} & \text{for } k \text{ odd} \end{cases}$$

$$= \frac{200}{j\pi \frac{2\cancel{\pi}}{\cancel{\pi}} k} \text{ for } k \text{ odd} = \frac{100}{jk\pi} \text{ for } k \text{ odd.}$$

$\therefore c_k = \frac{200}{jk\pi}$ for k odd.

$\therefore a_k = c_k + c_k^* = \frac{100}{jk\pi} - \frac{100}{jk\pi} = 0$

& $b_k = \frac{c_k - c_k^*}{-j} = \left(\frac{100}{jk\pi} + \frac{100}{jk\pi}\right) \times \frac{1}{-j} = \frac{200}{k\pi}$

$\therefore v(t) = \sum_{k=1,3,5} \frac{200}{k\pi} \sin\left(k \frac{2\cancel{\pi}}{\cancel{\pi}} t\right)$

Now impedance $= 4\Omega + jk\omega 2\Omega$

$= \left(4 + jk \frac{2\cancel{\pi}}{\cancel{\pi}} 2\right)\Omega$

$= (4 + j4k)\Omega$

Harmonics	impedance	maximum Value of current	Phase angle of current
k= 1	$(4+4j)\Omega$ $= 4\sqrt{2}\angle 45^\circ \Omega$	$\frac{200}{k\pi} \times \frac{1}{4\sqrt{2}}$ A $= \frac{25\sqrt{2}}{\pi}$ A	-45°
k=3	$(4+12j)\Omega$ $= 4\sqrt{10}\angle 71\cdot6^\circ \Omega$	$\frac{\cancel{200}^{50}}{3\pi} \times \frac{1}{\cancel{4}\sqrt{10}}$ A $= \frac{5\sqrt{10}}{3\pi}$ A	$-71\cdot6^\circ$
k=5	$(4+20j)\Omega$ $= 4\sqrt{26}\angle 78\cdot7^\circ$	$\frac{200}{5\pi} \times \frac{1}{4\sqrt{26}}$ A $= \frac{10}{\pi\sqrt{26}}$ A	$-78\cdot7^\circ$

Since Ck=0 for even harmonics i.e. the applied voltage does not contain any even harmonics, the current is also zero for k=2,4,...

2) $x(t) = 6\sin 2t + 2\cos(4t + \pi/6) + 4\sin(6t - \pi/4)$

i) YES. It is a periodic signal.

The time periods of the three components are

$$T_1 = \frac{2\pi}{2}, \quad T_2 = \frac{2\pi}{4}, \quad T_3 = \frac{2\pi}{6}$$

Since $\frac{T_1}{T_2} = 2$, $\frac{T_2}{T_3} = \frac{3}{2}$, $\frac{T_3}{T_1} = \frac{1}{3}$

all these ratios are rational,

$x(t)$ is periodic.

Now the time period of $x(t)$ is

$$T = L.C.M(T_1, T_2, T_3)$$
$$= L.C.M\left(\frac{2\pi}{2}, \frac{2\pi}{4}, \frac{2\pi}{6}\right)$$
$$= \frac{2\pi}{2}$$

$\therefore$ Fundamental period $= \frac{2\pi}{2}$

Fundamental Angular frequency $= \frac{2\pi}{(2\pi/2)} = 2$

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ii) $x(t) = 6\sin 2t + 2\cos\left(4t + \frac{\pi}{6}\right) + 4\sin\left(6t - \frac{\pi}{4}\right)$

$$= 6\,\frac{e^{J2t} - e^{-J2t}}{2J} + 2\,\frac{e^{J(4t+\pi/6)} + e^{-J(4t+\frac{\pi}{6})}}{2} + 4\,\frac{e^{J(6t-\pi/4)} - e^{-J(6t-\pi/4)}}{2J}$$

$$= -3Je^{J2t} + 3Je^{J(-2)t} + e^{J\pi/6}e^{J4t}$$
$$+ e^{-J\pi/6}e^{J(-4)t} - 2Je^{-J\pi/4}.e^{J6t}$$
$$+ 2Je^{J\pi/4}e^{J(-6)t}$$

$$= 3\angle{-90^\circ}\, e^{J2t} + 3\angle{90^\circ}\, e^{J(-2)t} + 1\angle{30^\circ}\, e^{J4t} + \cancel{1\angle 30^\circ e}$$
$$1\angle{-30^\circ}\, e^{J(-4)t} + 2\angle{-135^\circ}\, e^{J6t} + 2\angle{135^\circ}\, e^{J(-6)t}$$

$\boxed{\text{Fundamental freq} = 2}$

$\therefore\ C_{-3} = 2\angle 135^\circ$

$C_{-2} = 1\angle -30^\circ$

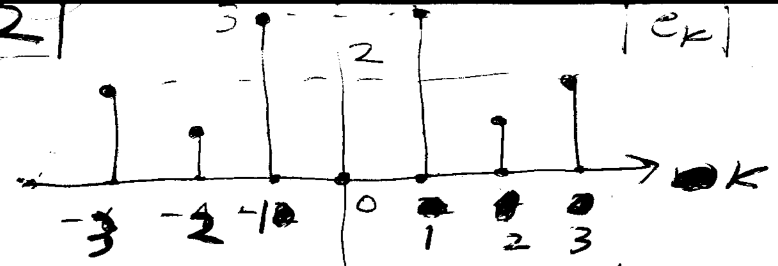
$C_{-1} = 3\angle 90^\circ$

$C_{1} = 3\angle -90^\circ$

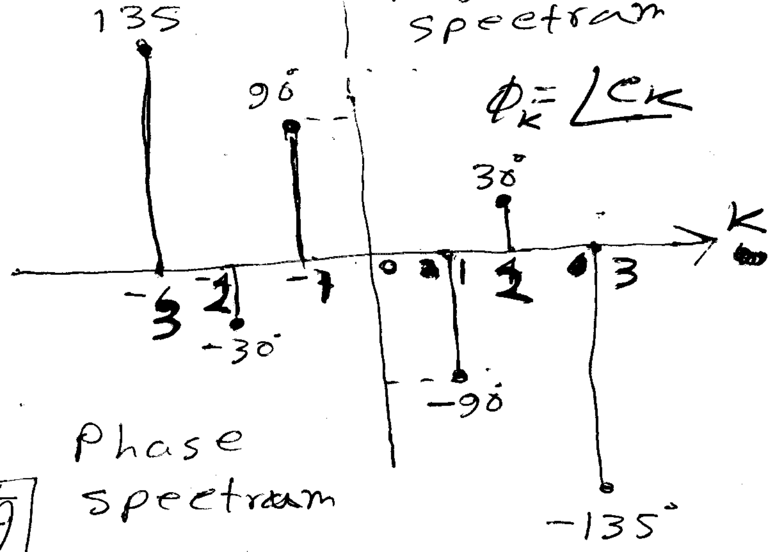
$C_{2} = 1\angle 30^\circ$

$C_{3} = 2\angle -135^\circ$

$C_k = 0$ for other values of K



Magnitude spectrum



Phase spectrum

$\cancel{x(t) = \sum_{\omega=\{2,-4,-6,2,4,6\}} C_\omega e^{j\omega t}}$

$$x(t) = \sum_{k=-\infty}^{\infty} C_k e^{j\omega t}$$ where C_k values are given above

3) Since the signal is a even function there will be know sine term. $\therefore b_k = 0\ \forall k$.

Again $x(t)$ is half wave symmetric so d.c value $= 0$ & only odd harmonics will be present.

Now $$a_k = \frac{2}{T}\int_{-T/2}^{T/2} x(t)\cos(k\omega t)\,dt$$

$$= \frac{2}{T}\left[\int_{-T/2}^{0} x(t)\cos(k\omega t)\,dt + \int_{0}^{T/2} x(t)\cos(k\omega t)\,dt\right]$$

$$= \frac{4}{T}\int_{0}^{T/2} x(t)\cos(k\omega t)\,dt \qquad \left[\because x(t)\cos(k\omega t) \text{ is an even function}\right]$$

$$= \frac{4}{T}\int_{0}^{T/2}\left(A - \frac{4A}{T}t\right)\cos(k\omega t)\,dt$$

$$= \frac{4A}{T}\int_{0}^{T/2}\left(\cos(k\omega t) - \frac{4t}{T}\cos(k\omega t)\right)dt$$

$$= \frac{4A}{T}\left[\left[\frac{\sin k\omega t}{k\omega}\right]_{0}^{T/2} - \frac{4}{T}\left[\left[t\,\frac{\sin k\omega t}{k\omega}\right]_{0}^{T/2} - \frac{1}{k\omega}\int_{0}^{T/2}\sin k\omega t\,dt\right]\right]$$

$$= \frac{4A}{T}\left[\frac{\sin(k\omega T/2)}{k\omega} - \frac{4}{\cancel{T}}\frac{\cancel{T}}{2k\omega}\sin(k\omega T/2) + \frac{4}{T}\times\frac{1}{(k\omega)^2}\left[-\cos k\omega t\right]_{0}^{T/2}\right]$$

$$= \frac{4A}{T}\left[\sin(k\pi) - \frac{2}{k\omega}\sin(k\pi) + \frac{4}{T(k\omega)^2}(1-\cos k\pi)\right]$$

$$= \frac{4A}{T}\left[0 - 0 + \frac{4}{T(k\omega)^2}(1-\cos k\pi)\right]$$

$$= \frac{16A}{T^2(k\omega)^2}(1-\cos k\pi)$$

$$= \begin{cases} 0 & \text{if } k \text{ even} \\ \dfrac{A32}{T^2(k\omega)^2} & \text{if } k \text{ odd} \end{cases} \qquad = \frac{32A}{k^2\,\cancel{T^2}\,\dfrac{2^2\pi^2}{\cancel{T^2}}} \text{ for } k \text{ odd}$$

$$= \frac{8A}{k^2\pi^2} \text{ for } k \text{ odd}$$

ii) $$C_k = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, e^{-Jk\omega t}\, dt$$

$$= \frac{1}{T}\int_{-T/2}^{0} x(t)\, e^{-Jk\omega t}\, dt + \frac{1}{T}\int_{0}^{T/2} x(t)\, e^{-Jk\omega t}\, dt$$

$$= \frac{1}{T}\int_{-T/2}^{0} A\left(1+\frac{4t}{T}\right) e^{-Jk\omega t}\, dt + \frac{1}{T}\int_{0}^{T/2} A\left(1-\frac{4t}{T}\right) e^{-Jk\omega t}\, dt$$

$$= \frac{A}{T}\,\frac{\left[1-e^{Jk\omega T/2}\right]}{-Jk\omega} + \frac{4A}{T^2}\left(\left[\frac{t e^{-Jk\omega t}}{-Jk\omega}\right]_{-T/2}^{0} - \int_{-T/2}^{0}\frac{e^{-Jk\omega t}}{-Jk\omega}\,dt\right)$$

$$+ \frac{A}{T}\,\frac{\left[e^{-Jk\omega T/2}-1\right]}{-Jk\omega} - \frac{4A}{T^2}\left(\left[\frac{t e^{-Jk\omega t}}{-Jk\omega}\right]_{0}^{T/2} - \int_{0}^{T/2}\frac{e^{-Jk\omega t}}{-Jk\omega}\,dt\right)$$

$$= \frac{A}{T}\,\frac{e^{-Jk\omega T/2} - e^{Jk\omega T/2}}{-Jk\omega} + \frac{4A}{T^2}\left(\frac{T e^{Jk\omega T/2}}{2Jk\omega} - \frac{1}{(Jk\omega)^2}\left(1-e^{\frac{Jk\omega T}{2}}\right)\right)$$

$$- \frac{4A}{T^2}\left(\frac{T e^{-Jk\omega T/2}}{-2Jk\omega} - \frac{1}{(Jk\omega)^2}\left(e^{-Jk\omega T/2}-1\right)\right)$$

$$= \frac{2A}{Tk\omega}\sin k\omega T/2 + \frac{4A}{T^2}\left(\frac{-T}{k\omega}\sin(k\omega T/2) + \frac{1}{(k\omega)^2}\times \left(2-e^{Jk\omega T/2}-e^{-Jk\omega T/2}\right)\right)$$

$$= 0 + \frac{4A}{T^2}\times\frac{1}{(k\omega)^2}\left(2\left(1-\cos(k\omega T/2)\right)\right)$$

$$= \frac{8A}{T^2 k^2 \omega^2}(1-\cos(k\pi))$$

$$= \frac{2A}{k^2\pi^2}(1-\cos k\pi)$$

$$\therefore \boxed{c_k = \frac{2A}{k^2\pi^2}(1-\cos k\pi)}$$

iii) Now

$$\left.\begin{aligned} c_k &= \frac{a_k - Jb_k}{2} \\ c_k^* &= \frac{a_k + Jb_k}{2} \end{aligned}\right\} \Rightarrow \left\{\begin{aligned} c_k + c_k^* &= a_k \\ c_k - c_k^* &= -Jb_k \end{aligned}\right.$$

$$\therefore a_k = c_k + c_k^* = \frac{2A}{k^2\pi^2}(1-\cos k\pi) + \frac{2A}{k^2\pi^2}(1-\cos k\pi)$$

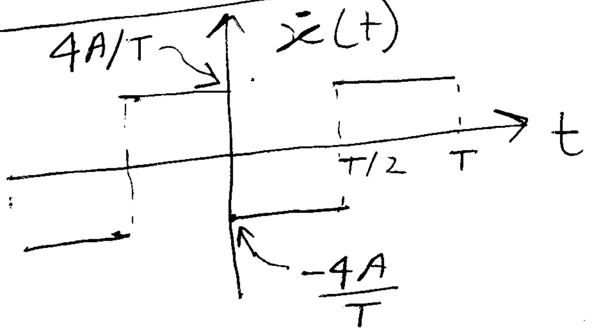
$$= \frac{4A}{k^2\pi^2}(1-\cos k\pi)$$

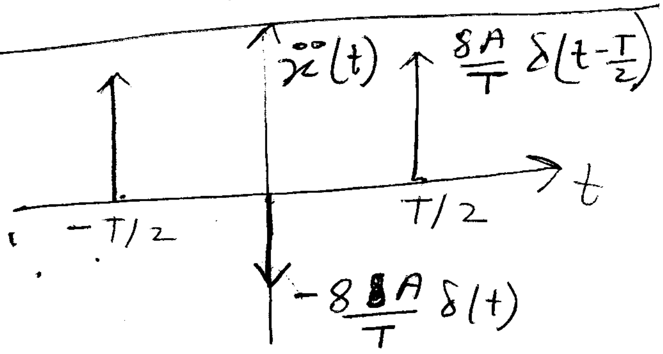
$$= \begin{cases} 0 & \text{if } k \text{ even} \\ \frac{8A}{k^2\pi^2} & \text{if } k \text{ odd.} \end{cases}$$

$$b_k = \frac{c_k - c_k^*}{-J} = 0 \quad [\because c_k \text{ is real}]$$

$\therefore$ This calculation matches with calculations in part (i)

iv)





Let us compute the F.S for $\ddot{x}(t)$

$$c_k'' = \frac{1}{T}\int_{-\frac{T}{2}(+)}^{\frac{T}{2}(+)} \ddot{x}(t)\, e^{-Jk\omega t}\, dt$$

$$= \frac{8A}{T^2}\left(-1 + e^{-Jk\omega T/2}\right)$$

$$= \frac{8A}{T^2}\left(-1 + e^{-Jk\pi}\right)$$

$\therefore$ The F-S of $x(t)$

$$= \frac{c_k''}{(Jk\omega)^2} = -\frac{1}{(k\omega)^2} \times \frac{8A}{T^2}\left(-1 + e^{-Jk\pi}\right)$$

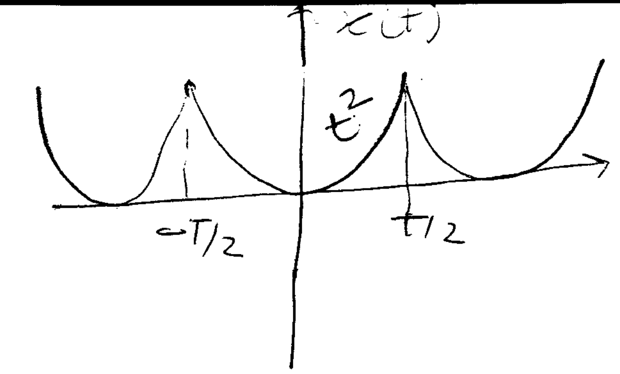
$$\cancel{= \frac{-8A}{4\pi^2}\left(e^{-\frac{Jk\pi}{2}} - e^{\frac{Jk\pi}{2}}\right) e^{-Jk\pi}}$$

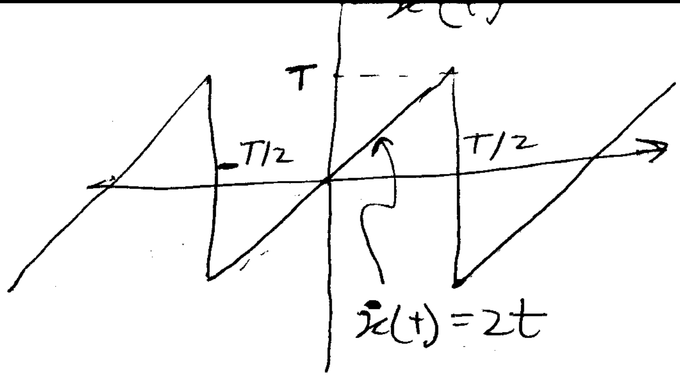
$$= \frac{-8A}{4\pi^2 k^2}\left(-1 + e^{-Jk\pi}\right) = \frac{2A}{\pi^2}\left(1 - e^{-Jk\pi}\right)$$

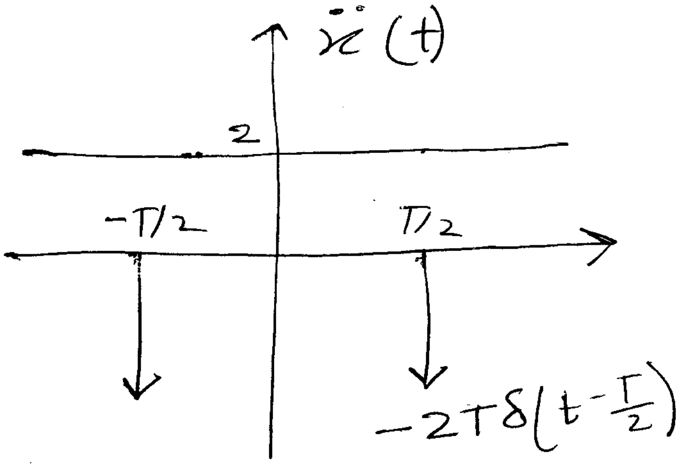
$$= \begin{cases} 0 & \text{for } k \text{ even} \\ \frac{2A}{k^2\pi^2}(1+1) & \text{for } k \text{ odd} \end{cases} = \begin{cases} 0 & \text{for } k \text{ even} \\ \frac{4A}{k^2\pi^2} & \text{for } k \text{ odd} \end{cases}$$

This calculation also matches calculation in part (ii)

5)







F.S coefficients of $\ddot{x}(t)$ c_k''

$$= \frac{1}{T}\int_{-\frac{T}{2}(+)}^{\frac{T}{2}(+)} \ddot{x}(t)\, e^{-jk\omega t}\, dt$$

$$= \frac{1}{T}\int_{-\frac{T}{2}(+)}^{\frac{T}{2}(+)} 2\, e^{-jk\omega t}\, dt - \frac{2T}{T} e^{-jk\omega T/2}$$

$$= \begin{cases} 2 - \cancel{\frac{2T}{T}} e^{-jk\pi} & \text{if } k=0 \\ -\cancel{\frac{2T}{T}} e^{-jk\pi} & \text{if } k\neq 0 \end{cases}$$

$$= \begin{cases} \left(2 - \frac{2T}{T}\right) = 0 & \text{if } k = 0 \\ -\cancel{\frac{2T}{T}} e^{-jk\pi} & \text{if } k \neq 0 \end{cases}$$

$\therefore$ F.S. coefficients of $x(t)$ $c_k = \frac{c_k''}{(jk\omega)^2}$

$$= \frac{1}{(k\omega)^2}\, 2e^{-jk\pi} \text{ for } k\neq 0 = \frac{2e^{(-jk\pi)}T^2}{k^2 4\pi^2} \text{ for } k \neq 0$$

$$= \frac{T^2}{2k^2\pi^2} e^{-jk\pi} \quad \text{for } k \neq 0$$

We cannot compute c_k from c_k'' ~~because~~ when $k=0$, because then $c_k = \frac{c_k''}{(j\omega_k)^2}$ is undefined. So we compute c_0 directly

$$c_0 = \frac{1}{T} \int_{-T/2}^{T/2} t^2\, dt = \frac{1}{T} \times \frac{1}{3} \left[t^3 \right]_{-T/2}^{T/2}$$

$$= \frac{1}{3T} \times \frac{T^3}{4} = \frac{T^2}{12}$$

Now $a_k = c_k + c_k^* = \frac{T^2}{2k^2\pi^2} \left(e^{-jk\pi} + e^{jk\pi} \right)$

$$= \frac{T^2}{k^2\pi^2} \cos k\pi$$

and $b_k = \frac{c_k - c_k^*}{-j} = \frac{T^2}{2k^2\pi^2} \left(\frac{e^{-jk\pi} - e^{jk\pi}}{-j} \right)$

$$= \frac{T^2}{k^2\pi^2} \sin k\pi$$

Now $P_{av} = \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2\, dt = \frac{1}{T} \int_{-T/2}^{T/2} t^4\, dt$

$$= \frac{1}{5T} \left[T^5 \right]_{-T/2}^{T/2} = \frac{T^4}{80}$$

We can also compute P_{av} as

$$c_0^2 + \sum_{k \neq 0} |c_k|^2 = \frac{T^4}{12^2} + \sum_{k \neq 0} \frac{T^4}{4k^4\pi^4}$$

$$= \frac{T^4}{144} + 2 \left(\sum_{k=1}^{\infty} \frac{1}{k^4} \right) \times \frac{T^4}{4\pi^4}$$

$$= \frac{T^4}{144} + \cancel{2} \frac{\cancel{\pi^4}}{90} \times \frac{T^4}{\cancel{4\pi^4}_{2}} = T^4 \left(\frac{1}{144} + \frac{1}{180} \right)$$

$$= \frac{T^4}{80}$$

The value of $\sum_{k=1}^{\infty} \frac{1}{k^4} = \frac{\pi^4}{90}$ is found from internet.

The agreement ~~of~~ between the Pav computed with two different methods confirm the correctness of our calculation.

4) i) $c_0 = \frac{1}{T}\int_0^T x(t)\,dt = \frac{V}{T}\int_0^{T/2} \sin(\omega t)\,dt$

$= \frac{V}{T}\left[-\frac{\cos\omega t}{\omega}\right]_0^{T/2} = \frac{V}{\omega T}\left(1-\cos\pi\right)$

$= \frac{V}{2\pi} \times 2 = \frac{V}{\pi}$

$a_k = \frac{2}{T}\int_0^T x(t)\cos(k\omega t)\,dt$

$= \frac{2}{T} V \int_0^{T/2} \sin(\omega t)\cos(k\omega t)\,dt$

$= \frac{V}{T}\int_0^{T/2}\left(\sin((1+k)\omega t) + \sin((1-k)\omega t)\right) dt$

$= \frac{V}{T}\int_0^{T/2} \sin((1+k)\omega t)\,dt + \frac{V}{T}\int_0^{T/2} \sin((1-k)\omega t)\,dt$

$= \begin{cases} \frac{V}{T}\left[\frac{\cos((1+k)\omega t)}{(1+k)\omega}\right]_0^{T/2} + \frac{V}{T}\left[\frac{-\cos((1-k)\omega t)}{(1-k)\omega}\right]_0^{T/2} & \text{if } k \neq 1 \\ \frac{V}{T}\left[\frac{-\cos((1+k)\omega t)}{(1+k)\omega}\right]_0^{T/2} + 0 & \text{if } k = 1 \end{cases}$

$= \begin{cases} \frac{V}{T}\frac{\left(1-\cos((1+k)\pi)\right)}{(1+k)\omega} + \frac{V}{T}\frac{\left(1-\cos((1-k)\pi)\right)}{(1-k)\omega} & \text{if } k \neq 1 \\ 0 & \text{if } k = 1 \end{cases}$

$$= \begin{cases} \frac{V}{2\pi}\left(\frac{1+\cos k\pi}{1+k} + \frac{1+\cos k\pi}{1-k}\right) & \text{if } k \neq 1 \\ 0 \quad \text{if } k=1 \end{cases}$$

$$= \begin{cases} \frac{V}{2\pi}(1+\cos k\pi)\left(\frac{2}{1-k^2}\right) & \text{if } k \neq 1 \\ 0 \quad \text{if } k=1 \end{cases}$$

$$= \begin{cases} 0 \quad \text{if } k \text{ odd} \\ \frac{2V}{\pi(1-k^2)} & \text{if } k \text{ even.} \end{cases}$$

$$b_k = \frac{2}{T}\int_0^T x(t)\sin(k\omega t)\,dt$$

$$= \frac{2V}{T}\int_0^{T/2} \sin(\omega t)\sin(k\omega t)\,dt$$

$$= \frac{V}{T}\int_0^{T/2} \cos((1-k)\omega t)\,dt - \frac{V}{T}\int_0^{T/2} \cos((1+k)\omega t)\,dt$$

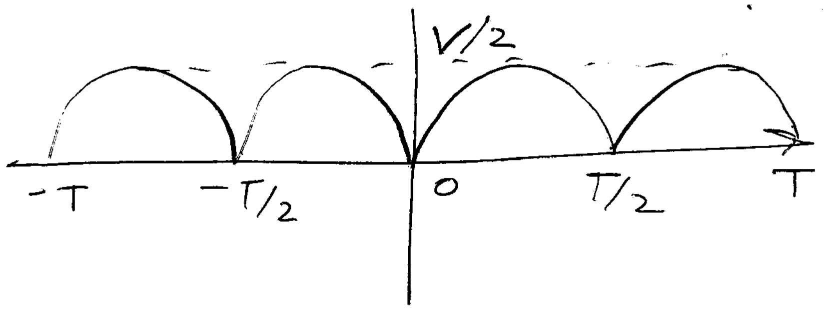
$$= \begin{cases} \frac{V}{T}\left[\frac{\sin((1-k)\omega t)}{(1-k)\omega t}\right]_0^{T/2} - \frac{V}{T}\left[\frac{\sin((1+k)\omega t)}{(1+k)\omega t}\right]_0^{T/2} & \text{if } k \neq 1 \\ \frac{V}{T}\frac{T}{2} - \frac{V}{T}\left[\frac{\sin((1+k)\omega t)}{(1+k)\omega t}\right]_0^{T/2} & \text{if } k = 1 \end{cases}$$

$$= \begin{cases} \frac{V}{T}\times 0 - \frac{V}{T}\times 0 & \text{if } k \neq 1 \\ \frac{V}{2} - 0 & \text{if } k = 1 \end{cases}$$

$$= \frac{V}{2} \text{ if } k=1, \quad \text{else } 0$$

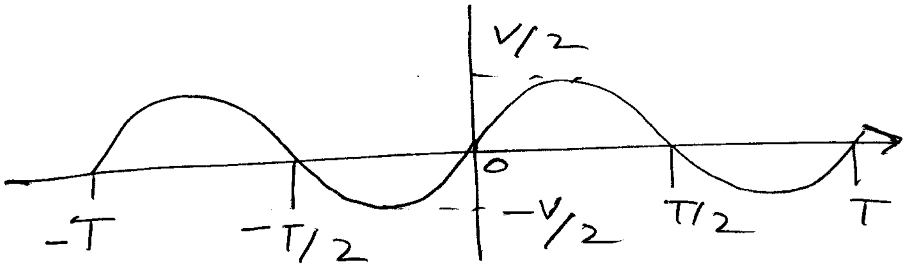
ii) Even part of $x(t)$

$$= \frac{x(t) + x(-t)}{2} = \left|\frac{V}{2}\sin\left(\frac{2\pi}{T}t\right)\right|$$



odd part of $x(t)$

$$= \frac{x(t) - x(-t)}{2} = \frac{V}{2}\sin\left(\frac{2\pi}{T}t\right)$$



Now Even part $= \frac{x(t)}{2} + \frac{x(t-T/2)}{2}$

From the previously computed F.S coefficients of $x(t)$ we can write

$$x(t) = \frac{V}{\pi} + \sum_{k=2,4,6\ldots} \frac{2V}{\pi(1-k^2)}\cos(k\omega t) + \frac{V}{2}\sin(\omega t)$$

$$\therefore x\left(t-\frac{T}{2}\right) = \frac{V}{\pi} + \sum_{k=2,4,6\ldots} \frac{2V}{\pi(1-k^2)}\cos\left(k\omega t - k\omega\frac{T}{2}\right) + \frac{V}{2}\sin\left(\omega t - \omega\frac{T}{2}\right)$$

$$= \frac{V}{\pi} + \sum_{k=2,4,\ldots} \frac{2V}{\pi(1-k^2)}\cos(k\omega t - k\pi) + \frac{V}{2}\sin(\omega t - \pi)$$

$$= \frac{V}{\pi} + \sum_{k=2,4\ldots} \frac{2V}{\pi(1-k^2)} \cos(k\omega t) + \frac{V}{2}\sin(\omega t)$$

~~Clearly the F.S of even part is given by~~ $\cancel{c_0 = \frac{V}{\pi}}$

$\cancel{a_k = \frac{2V}{\pi(1-k^2)}}$

$\therefore$ Even part $= \dfrac{x(t)}{2} + \dfrac{x(t-\frac{T}{2})}{2}$

$$= \frac{\cancel{2}V}{\cancel{2}\pi} + \sum_{k=2,4\ldots} \frac{2V}{\pi(1-k^2)} \cos(k\omega t) + 0$$

$\therefore$ F.S. coefficients of even part are

$$c_{0_{even}} = \frac{V}{\pi}, \quad a_{k_{even}} = \begin{cases} \frac{2V}{\pi(1-k^2)} & \text{for even } k \\ 0 & \text{for odd } k \end{cases}$$

and $b_{k_{even}} = 0$

Now since odd part is $\frac{V}{2}\sin\frac{2\pi}{T}t$

$\therefore$ the F.S. coefficients of odd part are

$$c_{0_{odd}} = 0, \quad a_{k_{odd}} = 0$$

$$b_{k_{odd}} = \begin{cases} \frac{V}{2} & \text{for } k=1 \\ 0 & \text{else} \end{cases}$$

Now adding the F.S. coefficients of odd & even part we get

$$c_0 = c_{0_{even}} + c_{0_{odd}} = \frac{V}{\pi} + 0 = \frac{V}{\pi}$$

$$a_k = a_{k_{even}} + a_{k_{odd}} = \begin{cases} \frac{2V}{\pi(1-k^2)} & \text{for even } k \\ 0 & \text{for odd } k \end{cases}$$

$$b_k = b_{k_{even}} + b_{k_{odd}} = \frac{V}{2} \text{ for } k=1, \quad 0 \text{ otherwise.}$$

This coefficients matches with those calculated in part (i)

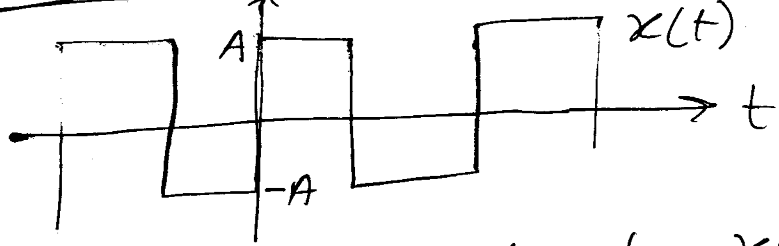
6) ii) ~~The F.S. of the $x(t)$ in Q3~~ is $c_k = \frac{2A}{k^2\pi^2}(1-\cos k\pi)$

$= \begin{cases} 0 & \text{for even } k \\ \frac{4A}{k^2\pi^2} & \text{for odd } k \end{cases}$

$\therefore$ The average power P_{av}

$= \sum_{k \text{ odd}} |c_k|^2 = \sum_{k \text{ odd}} \frac{4A}{k}$

6) i)



The F.S. coefficients of $x(t)$ (as computed in Q7) are

$c_k = \frac{2A}{jk\pi}$ for k odd, 0 for k even

$\therefore$ Average power $P_{av} = \sum_k |c_k|^2$

$= \sum_{k=\{1,3,5\ldots, -1,-3,-5\ldots\}} \frac{4A^2}{k^2\pi^2} = 2\sum_{k=\{1,3,5\ldots\}} \frac{4A^2}{k^2\pi^2}$

$= \frac{8A^2}{\pi^2}\left(\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \text{--- upto } \infty\right)$

Again $P_{av} = \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2 dt$

$$= \frac{1}{T}\int_{-T/2}^{0} (-A)^2 dt + \frac{1}{T}\int_{0}^{T/2} A^2 dt$$

$$= A^2$$

$$\therefore \frac{8A^2}{\pi^2}\left(\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots \text{upto } \alpha\right) = A^2$$

$$\Rightarrow \left(\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots \text{upto } \alpha\right) = \frac{\pi^2}{8}$$

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ii) The F.S. of the triangular $x(t)$ in Q3

Alternative

is $c_k = \frac{2A}{k^2\pi^2}(1 - \cos k\pi)$

$$= \begin{cases} 0 & \text{for even } k \\ \frac{4A}{k^2\pi^2} & \text{for odd } k \end{cases}$$

$$\therefore x(t) = \sum_{k=\{\pm1,\pm3,\pm5,\ldots\}} \frac{4A}{k^2\pi^2} e^{jk\omega t}$$

$$\therefore x(0) = \sum_{k=\{\pm1,\pm3\ldots\}} \frac{4A}{k^2\pi^2} = A$$

$$\Rightarrow 2\sum_{k=\{1,3,5\ldots\}} \frac{4A}{k^2\pi^2} = A$$

$$\Rightarrow \frac{8A}{\pi^2}\left(\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots \text{upto } \alpha\right) = A$$

$$\Rightarrow \left(\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots \text{upto } \alpha\right) = \frac{\pi^2}{8}$$

6) ii) The F.S. coefficient of $x(t)$ in Q5 are given as

$$c_k = \begin{cases} \frac{T^2}{12} & \text{for } k=0 \\ \frac{T^2}{2k^2\pi^2} e^{-jk\pi} & \text{for } k \neq 0 \end{cases}$$

$$= \begin{cases} \frac{T^2}{12} & \text{for } k=0 \\ (-1)^k \frac{T^2}{2k^2\pi^2} & \text{for } k \neq 0 \end{cases}$$

$$\therefore x(t) = \frac{T^2}{12} + \sum_{\substack{k=\{2,4,\ldots\} \\ \{-2,-4\ldots\}}} \frac{T^2 e^{j\omega kt}}{2k^2\pi^2} - \sum_{\substack{k=\{1,3\ldots\} \\ \{-1,-3\ldots\}}} \frac{T^2}{2k^2\pi^2} e^{j\omega kt}$$

$$\therefore x(0) = \frac{T^2}{12} + \frac{T^2 \times \cancel{2}}{\cancel{2}\pi^2}\left(\frac{1}{2^2} + \frac{1}{4^2} + \cdots \text{upto } \alpha\right) - \frac{T^2 \times \cancel{2}}{\cancel{2}\pi^2}\left(\frac{1}{1^2} + \frac{1}{3^2} + \cdots \text{upto } \alpha\right)$$

$$= 0$$

$$\Rightarrow \frac{1}{12} + \frac{1}{\cancel{2}\pi^2}\left(\frac{1}{2^2} + \frac{1}{4^2} + \cdots \text{upto } \alpha\right) = \frac{1}{\cancel{2}\pi^2}\left(\frac{1}{1^2} + \frac{1}{3^2} + \cdots \text{upto } \alpha\right)$$

$$\Rightarrow \frac{1}{12} + \frac{1}{\cancel{2}\pi^2}\left(\frac{1}{2^2} + \frac{1}{4^2} + \cdots \text{upto } \alpha\right) = \frac{1}{\cancel{2}\cancel{\pi^2}} \times \frac{\cancel{\pi^2}}{8} = \frac{1}{8} = \frac{1}{8}$$

$$\Rightarrow \frac{1}{\cancel{2}\pi^2}\left(\frac{1}{2^2} + \frac{1}{4^2} + \cdots \text{upto } \alpha\right) = \frac{1}{8} - \frac{1}{12} = \frac{1}{24}$$

$$\Rightarrow \left(\frac{1}{2^2} + \frac{1}{4^2} + \cdots \text{upto } \alpha\right) = \frac{\pi^2}{24}$$

iii) ~~B~~ using the results of 6(i) & 6(ii)

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4} + \cdots \text{ upto } \alpha$$

$$= \frac{\pi^2}{8} + \frac{\pi^2}{24} = \frac{\pi^2}{6}.$$