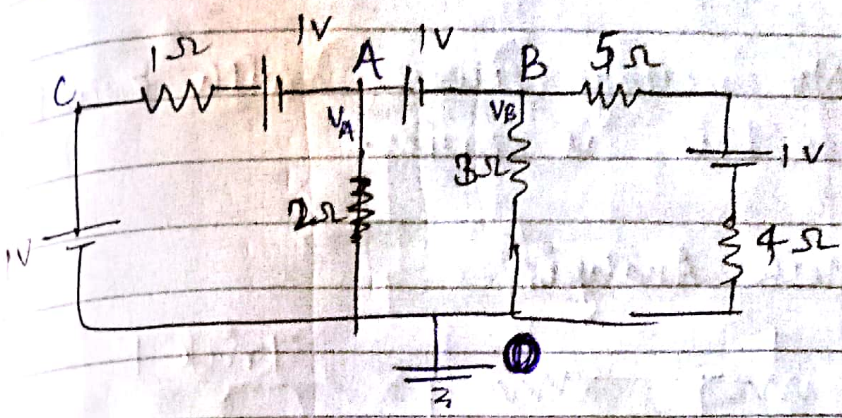
Network Theorems.



Nodal analysis,

KCL for node A,

$$\frac{V_A}{2} + \frac{V_A + 1 - V_c}{1} + \frac{V_A - 1}{0}$$

(Absurd....)

Trick-1

Consider A & B together,

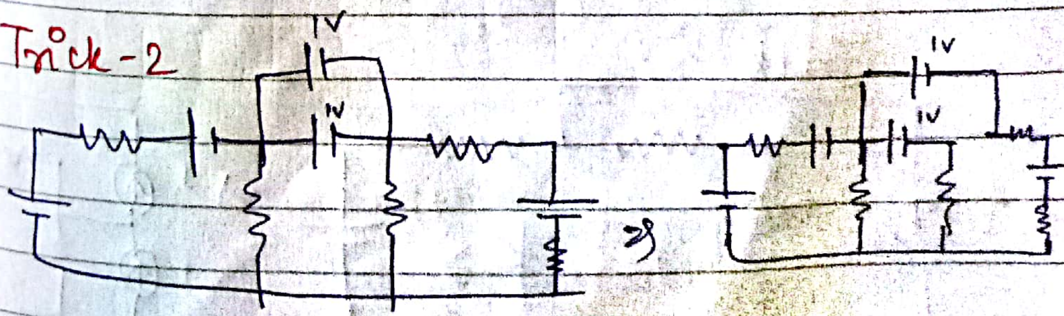
$$V_A = V_B + 1$$

Joint KCL for 'A' & 'B',

$$I_{CA} + I_{OA} + I_{OB} + I_{ODB} = 0$$

$$\frac{V_A}{1} + \frac{V_A}{2} + \frac{V_B}{3} + \frac{V_B - 1}{9} = 0$$

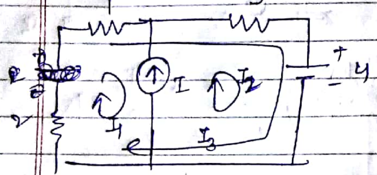
Trick-2



No voltage source with zero impedence so this is easier.

→ Mesh Analysis

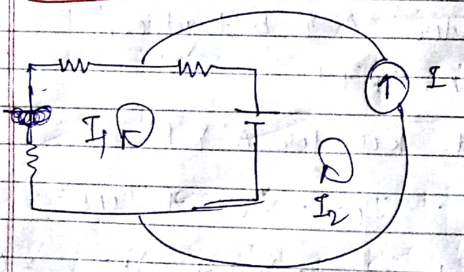
Trick 1



Outer loop ⇒ 1 eq.

$I_2 - I_1 = I$

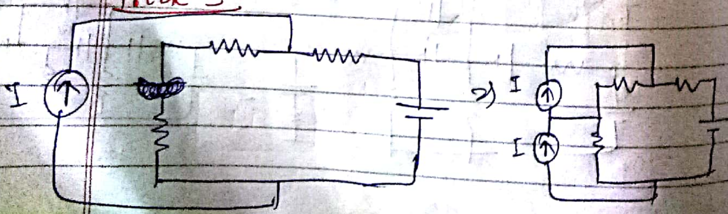
Trick 2



$I_2 = -I$

Write mesh for I_1.

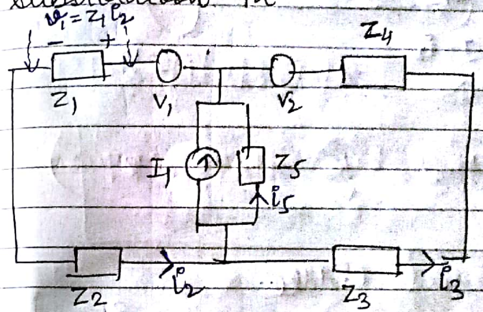
Trick-3



convert current ⇒ voltage source.

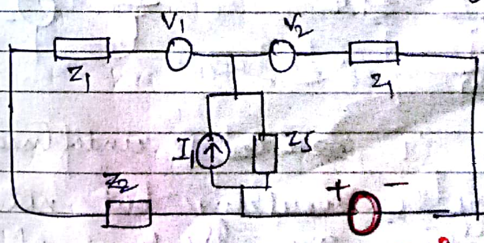
This is so simple that Trick 3 is not necessary but trick 3 is valid.

* Substitution Th:



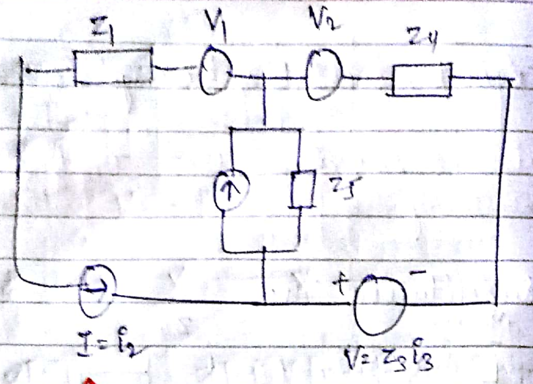
Suppose ckt is solved.

Subsitution with voltage source.



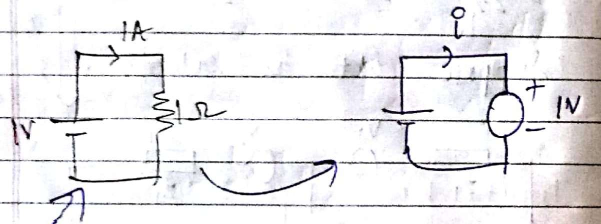
$V = Z_3 i_3$

To prove: ① KCL
② KVL
③ $V_k = Z_k i_k$



(Replaced by current source)

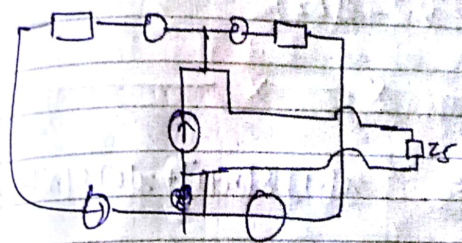
* Advanced Riddle:



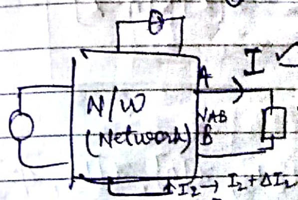
'i' ~~....~~ is undefined, it may be something but not of all solutions are solutions of

* compensation theo.

use Prev. ckt.



⇓



$I \to I+\Delta I$

$Z \to Z+\Delta Z$

V_{AB} when impedence $= Z$

Suppose N/W is solved once.

Modify $Z \to Z + \Delta Z$

$I \to I + \Delta I$

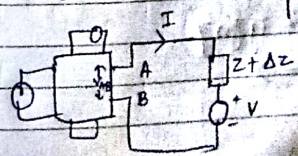
Task: To find

① $I + \Delta I = ?$

② $I_2 + \Delta I_2 = ?$

Answer 1 $\Rightarrow$ Solve again.

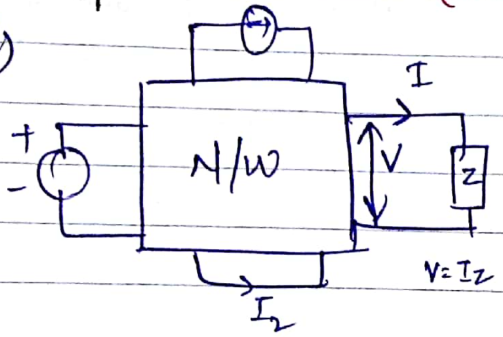
" 2 $\Rightarrow$ Use compensation theo.



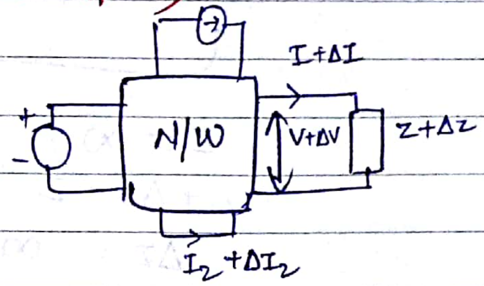
* Network theorems..

(1.) Compensation th. (Only for linear N/Ws)

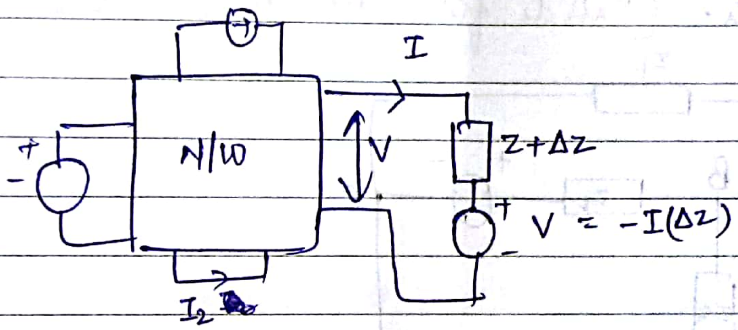
(i)



(ii).



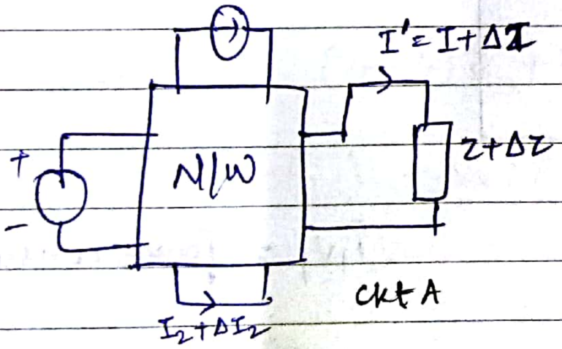
(iii)



$V = IZ$

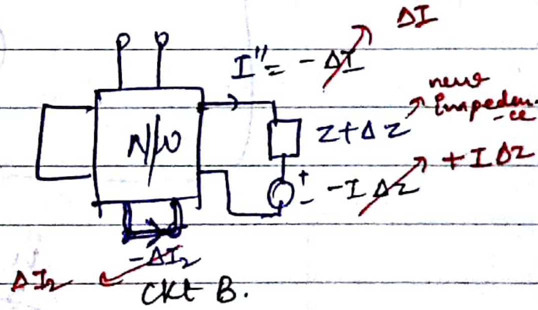
We used compensating V-source & change in impedence.

||



Ckt A

+



Ckt B.

$I' + I'' = I$

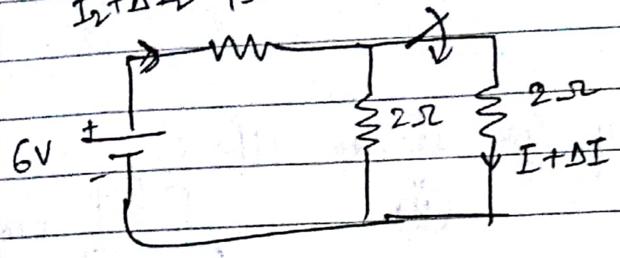
$\Rightarrow \boxed{I'' = -\Delta I}$

To get ΔI, Just solve ckt B

To get ΔI_2, Just solve ckt. B

H.W

Q.



find ΔI & ΔI_2 using compensation theo.

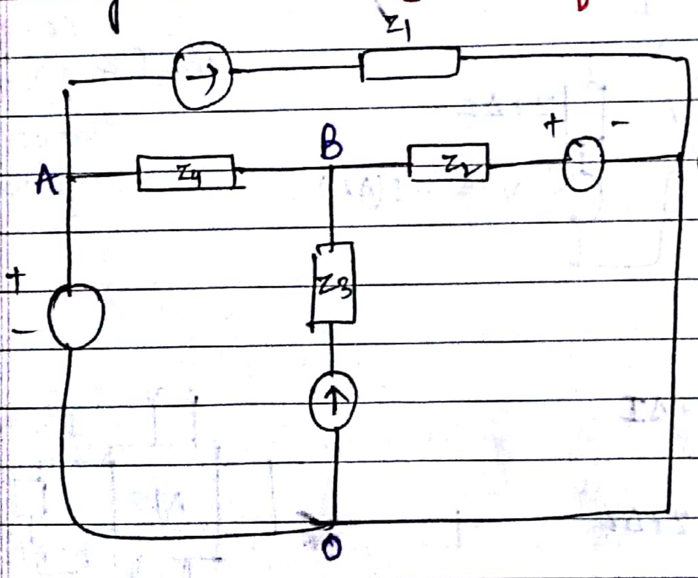
use compensating I-source & change in admittance.

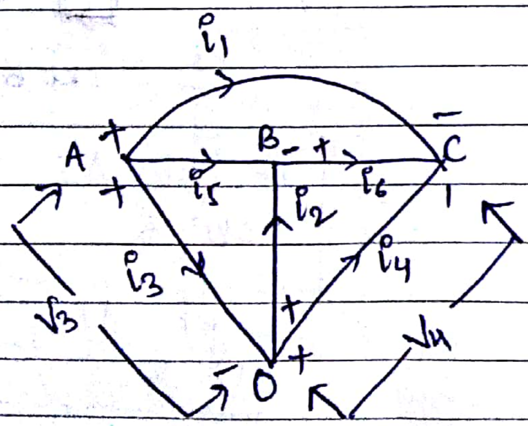
$Z = \infty$

$Z + \Delta Z = 2$

$\Delta Z = -\infty$

* Telegence th. (Valid for non linear ckts. also)



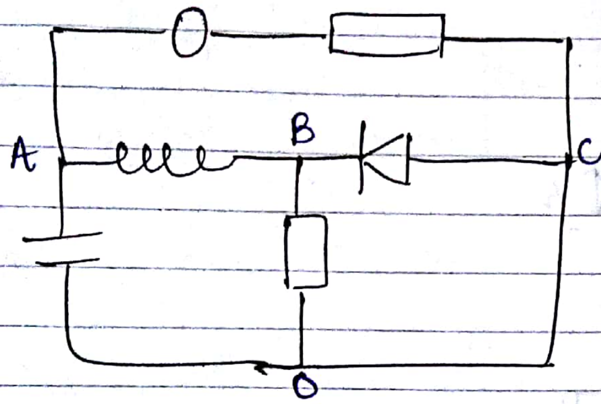


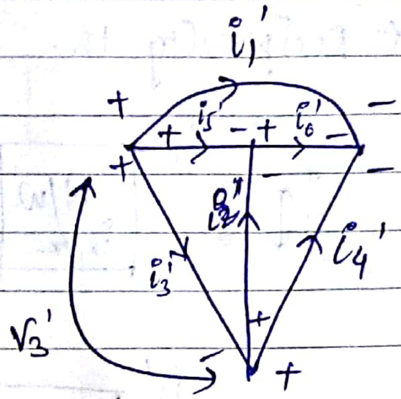
$i_1 V_1$ = Power consumed

$\sum_{k \in \{\text{all branches}\}} i_k V_k = 0$

Now, if the topology is same but elements are diff. for a ckt.

Scanned by CamScanner





Theorem says,

$$\sum_k i_k v_k' = 0$$

$$\sum_k i_k' v_k = 0$$

$$\sum v_k i_k' = 0$$

$\Rightarrow V_{OB}\, i_2' + V_{AC}\, i_1' + V_{AB}\, i_5' + V_{BC}\, i_6' + V_{AO}\, i_3' + V_{OC}\, i_4'$

$= (V_O - V_B)\, i_2' + (V_A - V_C)\, i_1' + (V_A - V_B)\, i_5' + (V_B - V_C)\, i_6' + (V_A - V_O)\, i_3' + (V_O - V_C)\, i_4'$

$= V_O\,(i_2' + i_4' - i_3') + V_A\,(i_1' + i_5' + i_3') + V_B(\ldots) + V_C(\ldots)$

$= 0$

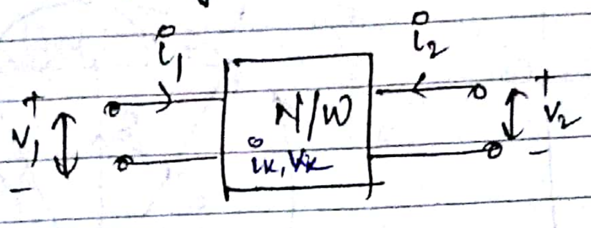
$$\sum v_k i_k' = 0$$

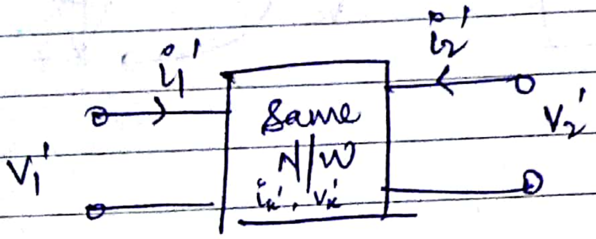
$$\sum v_k' i_k = 0$$

* Substitution:
A slight modification will make it cope with nonlinearity

Q. Is Telegence valid only for linear N/w ?
NO (as V=IR not used)

* Reciprocity th.





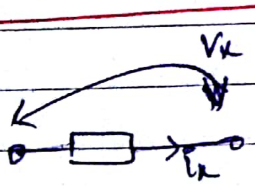
$$\sum_{k,1,2} i_k V_k' = 0$$

$$\Rightarrow i_1 V_1' + i_2 V_2' + \sum_k i_k V_k' = 0$$

$$\sum i_k' V_k = 0$$

$$\Rightarrow i_1' V_1 + i_2' V_2 + \sum_k i_k' V_k = 0$$

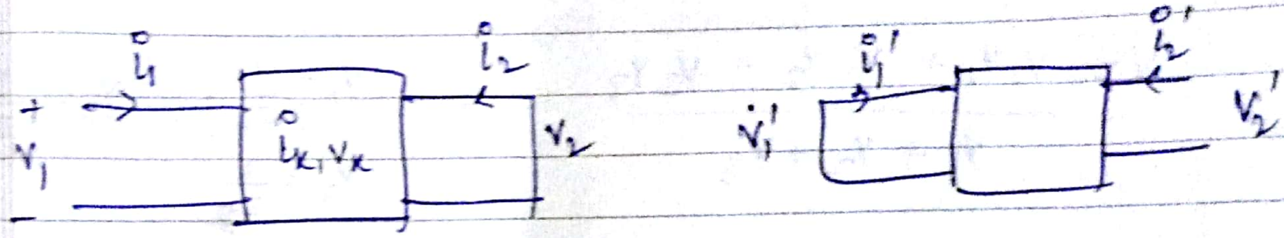
New conditions
1. N/W is linear $(V = IZ)$
2. N/W is passive. (No energy source)



$V_k = i_k Z_k$

$V_k' = i_k' Z_k$

$$\Rightarrow i_1 V_1' + i_2 V_2' = i_1' V_1 + i_2' V_2$$



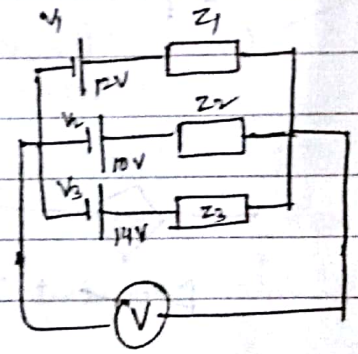
$\Rightarrow \quad i_2 V_2' = i_1' V_1$

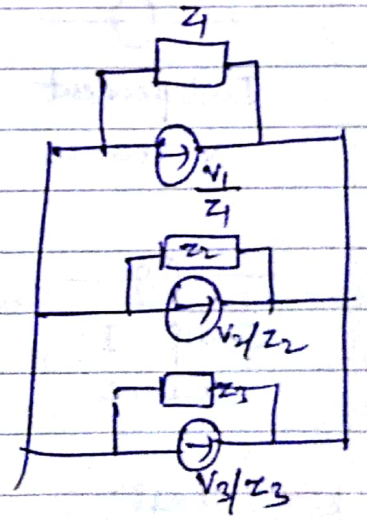
$\Rightarrow \quad \boxed{\frac{i_2}{V_1} = \frac{i_1'}{V_2'}}$

$\Rightarrow \quad \left.\frac{o/p}{i/p}\right)_{N/w_1} = \left.\frac{o/p}{i/p}\right)_{N/w_2}$

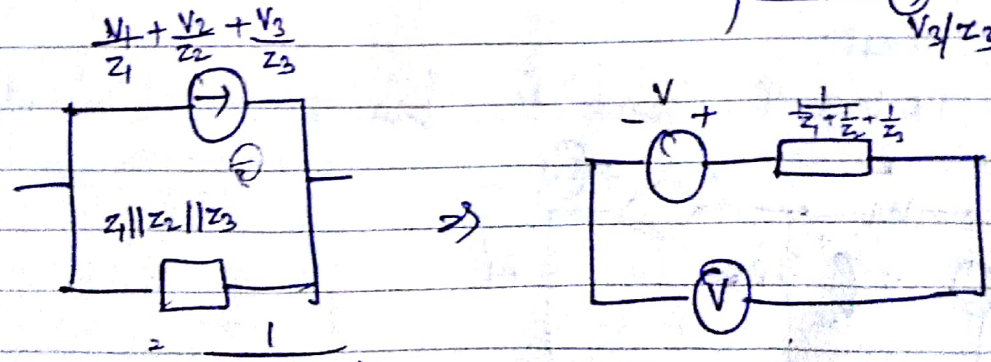
H.W.
Q. Consider,
I/p = current source
O/p = Voltage measured with voltmeters by open circuiting the O/p port.

* Millman's th:





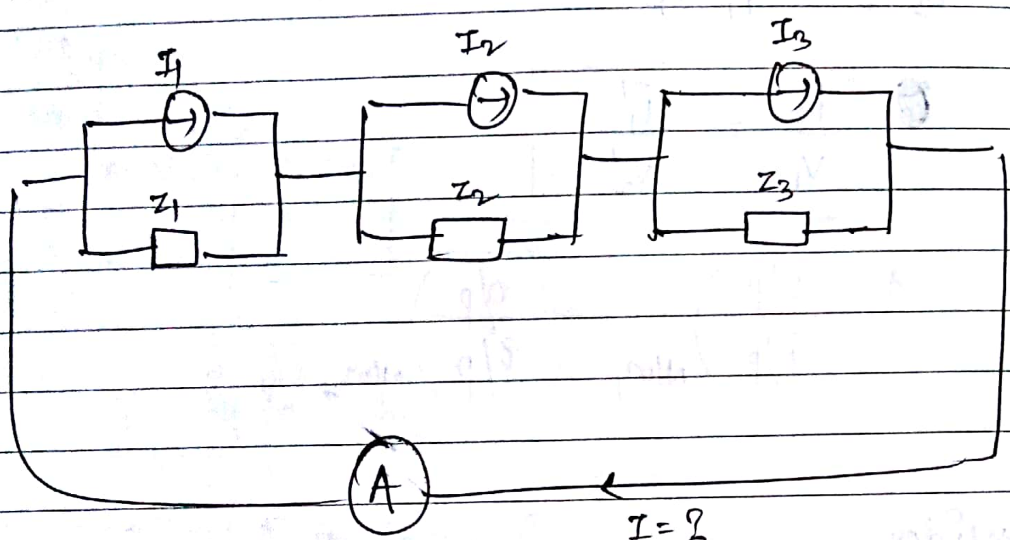
$= \frac{1}{\frac{1}{Z_1}+\frac{1}{Z_2}+\frac{1}{Z_3}}$



$$V = \frac{\frac{V_1}{Z_1}+\frac{V_2}{Z_2}+\frac{V_3}{Z_3}}{\frac{1}{Z_1}+\frac{1}{Z_2}+\frac{1}{Z_3}}$$

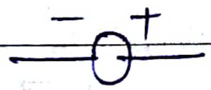
$$V = \frac{V_1 Y_1 + V_2 Y_2 + V_3 Y_3}{Y_1 + Y_2 + Y_3} \quad ; \quad Y_1 = \frac{1}{Z_1} \quad Y_2 = \frac{1}{Z_2} \quad Y_3 = \frac{1}{Z_3}$$

HW Q.

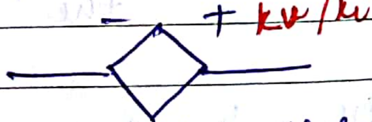


$$I = \frac{I_1 Z_1 + I_2 Z_2 + I_3 Z_3}{Z_1 + Z_2 + Z_3}$$

* Dependent sources.

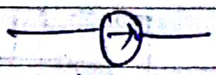


Independent V-source

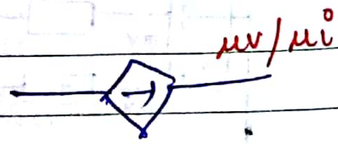


Dependent V-source.

- Only linear dependency is allowed.



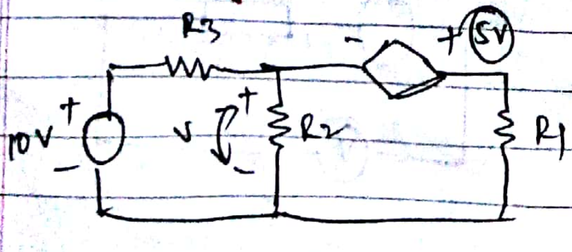
Indep. I-source

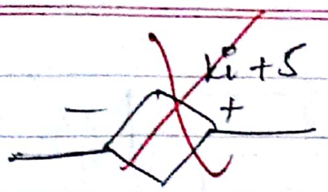


Dependent I-source.

k = const.

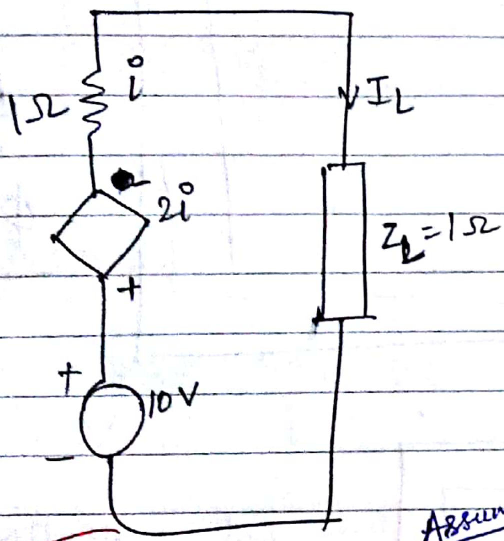
v = voltage in some other branch.

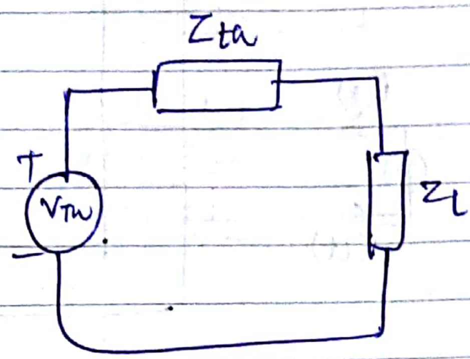




Not allowed.

Q.





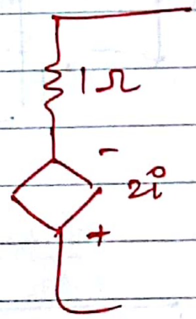
$V_{Th.} = 10\,V \quad (\because i = 0)$

Assume $Z_{th.} = 1\Omega$

$I_L = \frac{10}{1+1} = 5$ ✗

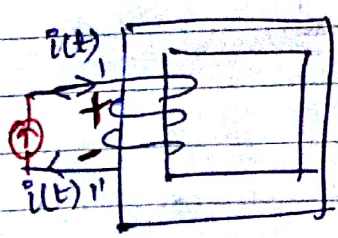
$10 - 2i - i - i = 0$

$i = 2.5$

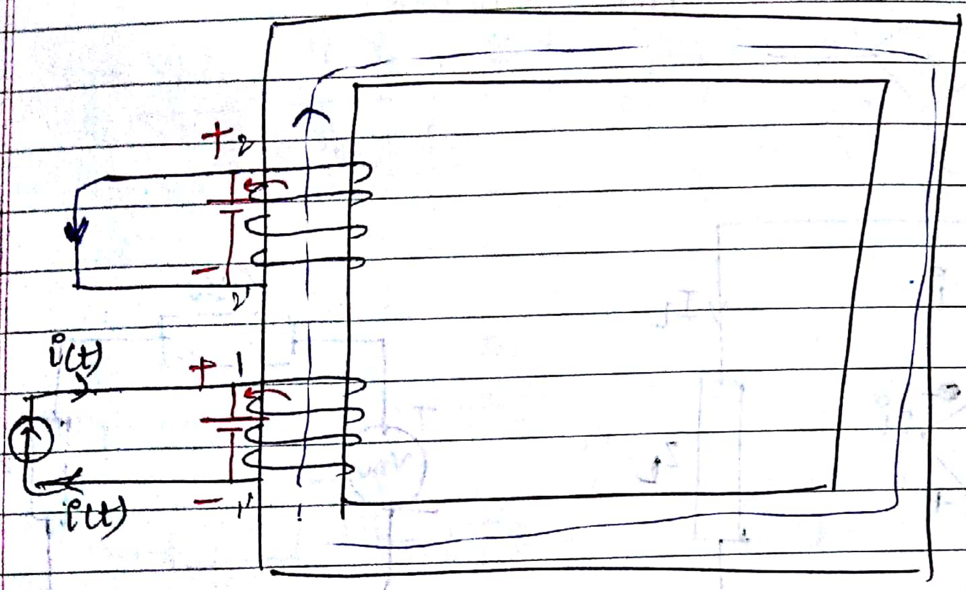


Note: Never remove dependent sources while applying thevenin's th., superposition th. etc.

* Mutual Inductance & transformer.

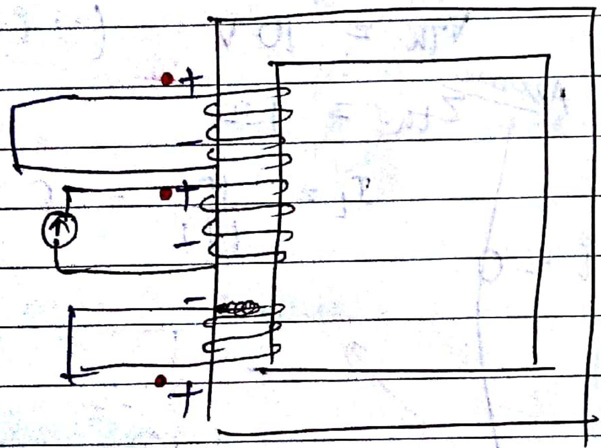


$\frac{di(t)}{dt} = +ve$



$\frac{di(t)}{dt} = +ve$

$\phi \uparrow$



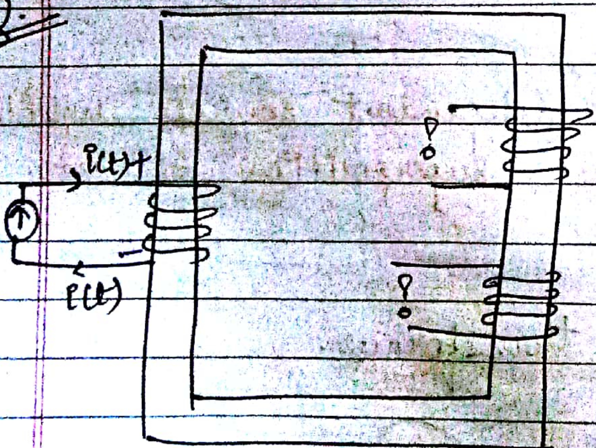
Dot convention

→ All +ve or All −ve together.

→ If current enters coils through dots the flux generated are all in the same direcⁿ.

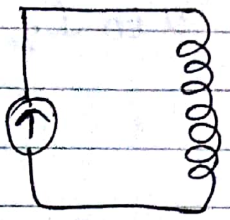
H.W

Q.



$\frac{di}{dt} = +ve$

* Energy stored in coils.



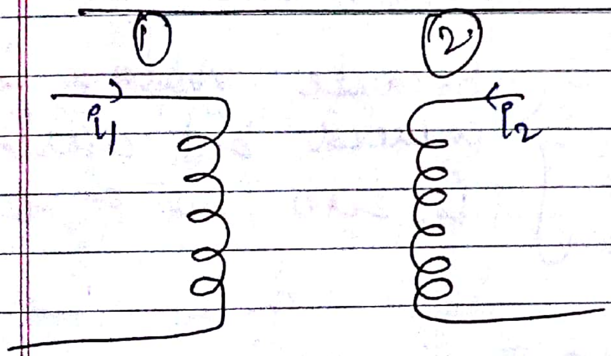
$$E = \frac{1}{2} L i^2$$

* Definitions.

$$V \neq \frac{d\phi}{dt}$$

(1). $L = \frac{\text{flux}}{i} = \frac{N\phi}{i}$ or $L = \frac{V}{(di/dt)}$ $\quad V = N\frac{d\phi}{dt} = \frac{d\psi}{dt}$

Flux linkage → $\psi = N\phi$



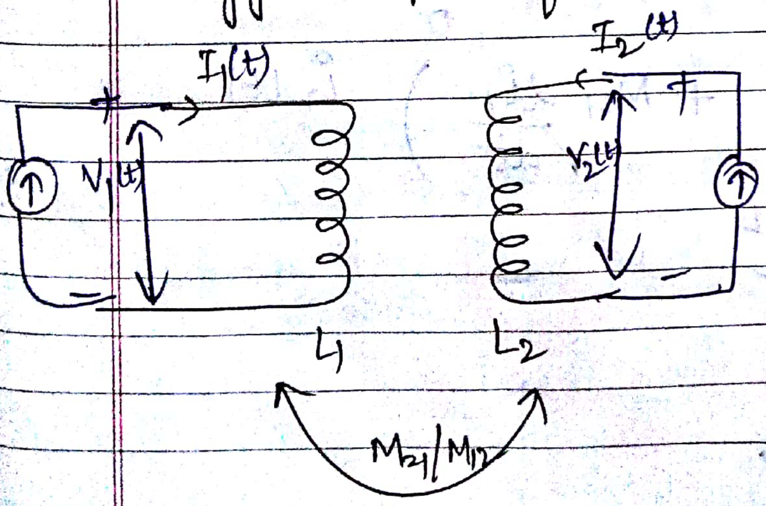
$$\phi_{21} = M_{21} i_1$$

Mutual inductance

$$\phi_{12} = M_{12} i_2$$

$$V_1 = M_{12}\frac{di_2}{dt} \; ; \; V_2 = M_{21}\frac{di_1}{dt}$$

* Energy in pair of coils.



At any instant, 't'

$$P(t) = \underbrace{V_1(t) I_1(t)}_{\text{func. of } I_1 \,\&\, I_2} + \underbrace{V_2(t) I_2(t)}_{\text{func. of } I_1, I_2}$$

For simplicity of calculation, I shall first increase I_1 from 0 to I_1 & then keeping I_1 const. increase I_2 from 0 to I_2

Phase 1

$I_2 = 0$

Power in coil 1

$$\int P_1(t)\,dt = \int v_1\, i(t)\, dt$$

$$= \int L_1 \frac{di_1}{dt} \cdot i_1(t)\, dt$$

$$E_1 = \frac{1}{2} L_1 i_1^2$$

$$E_2 = 0$$

Because voltage is induced but current is zero, so $E_2 = 0$.

Phase 2

$i_1(t) = I_1 = \text{const.}$

Power in coil 2

$$P_2(t) = v_2(t)\, i_2(t)$$

$$= \left(L_2 \frac{di_2}{dt} + M_{21} \cancel{\frac{di_1}{dt}}^{\,0} \right) i_2(t)$$

$$E_2(t) = \frac{1}{2} L_2 i_2^2$$

Power in coil 1,

$$P_1(t) = v_1(t)\, I_1$$

$$= M_{12}\frac{di_2}{dt}\cdot I_1$$

$$= M_{12} I_1 I_2$$

Total energy $= \frac{1}{2}L_1 I_1^2 + \frac{1}{2}L_2 I_2^2 + M_{12} I_1 I_2$

If we would have done this in oppo. order then we would have got,

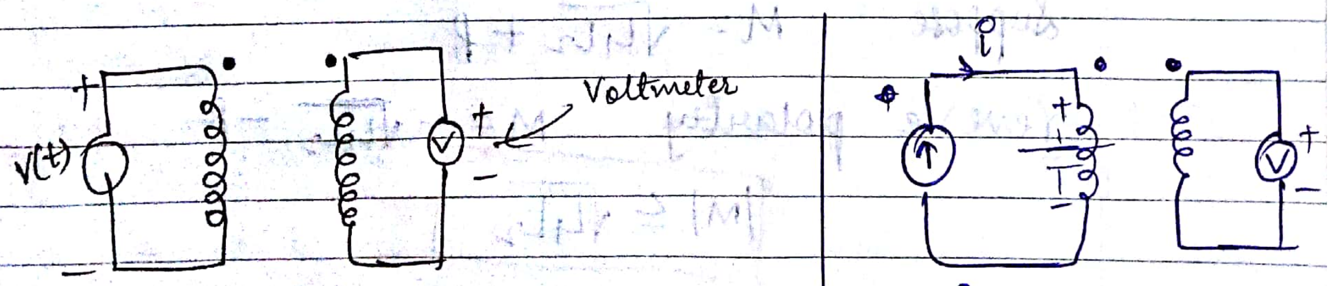
$$E_{Total} = \frac{1}{2}L_1 I_1^2 + \frac{1}{2}L_2 I_2^2 + M_{21} I_1 I_2$$

$$\therefore \boxed{M_{12} = M_{21}}$$

$$E = \underbrace{\frac{1}{2}}_{\geq 0}\Big[\underbrace{\left(\sqrt{L_1}\, I_1 - \sqrt{L_2}\, I_2\right)^2}_{\geq 0} + 2\underbrace{\left(\sqrt{L_1 L_2} + M\right)}_{>0} I_1 I_2\Big]$$

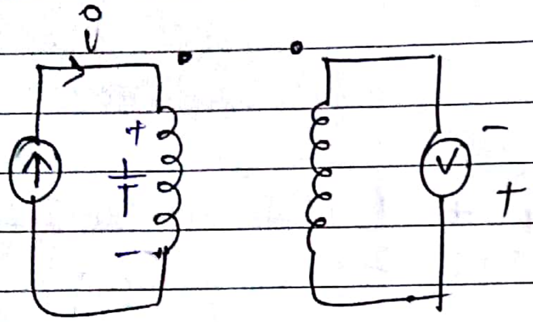
$$\boxed{M > -\sqrt{L_1 L_2}}$$

eg.



$t = t_0$, $v(t) > 0$ $\quad$ $V > 0$ $\quad$ $\frac{di}{dt} > 0 \not\Rightarrow V > 0$

$$M_{21} = \frac{V}{\frac{di}{dt}} > 0$$

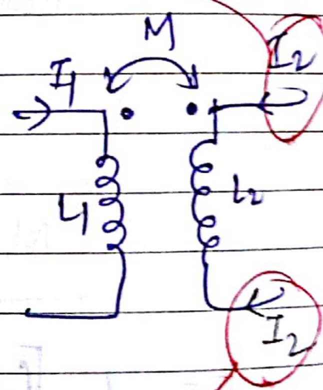


$\frac{di}{dt} > 0 \Rightarrow V < 0$

$$M_{21} = \frac{V}{\frac{di}{dt}} < 0$$

* Energy in a pair of coils.

$$\boxed{E = \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 \pm M I_1 I_2}$$



$$= \frac{1}{2}\left[\left(\sqrt{L_1} I_1 - \sqrt{L_2} I_2\right)^2 + 2\sqrt{L_1 L_2}\, I_1 I_2 + M I_1 I_2\right]$$

$$M + \sqrt{L_1 L_2} > 0$$

$$M > -\sqrt{L_1 L_2}$$

Suppose $M = \sqrt{L_1 L_2} + p$

reverse polarity $M = -\sqrt{L_1 L_2} - p$

$$\boxed{|M| \le \sqrt{L_1 L_2}}$$

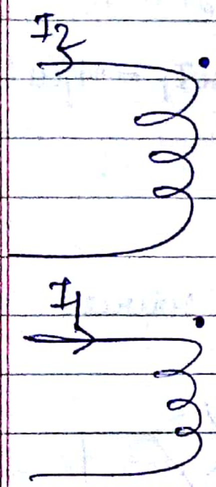
Not possible if $p > 0$ $\Rightarrow p = 0$ at ma

Def^n

$$\frac{M}{\sqrt{L_1 L_2}} := K \leq 1$$

$\uparrow$ coeff. of mutual Inductance.

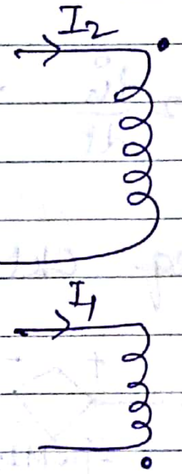
If $I_1 > 0$ & $I_2 > 0$



$E = E_1 + E_2$ + coupling energy

Very close

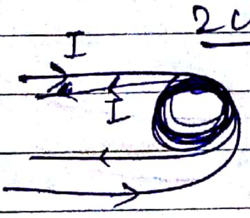
$K = 1 \Rightarrow |M| = \sqrt{L_1 L_2}$



$E = E_1 + E_2$ − coupling energy.

$|M| = \sqrt{L_1 L_2}$

2 coils

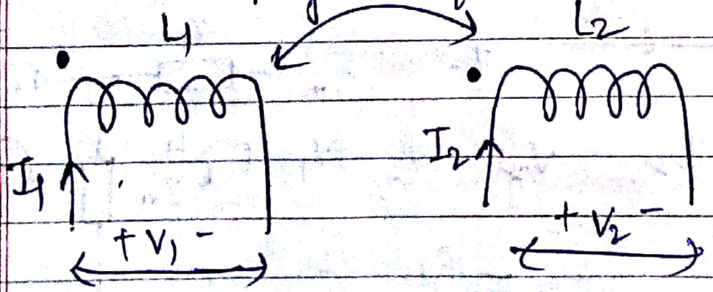


$I_1 = I, \; I_2 = -I, \; M = \sqrt{L_1 L_2}$

$$E = \frac{1}{2} L I^2 + \frac{1}{2} L I^2 - \sqrt{L_1 L_2}\, I^2$$

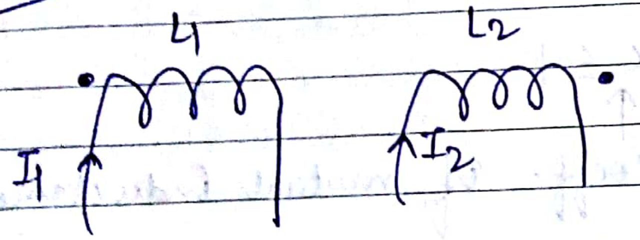
$$L I^2 - L I^2 = 0$$

Magnitude of mutual ind. $= M > 0$



$$V_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

Same

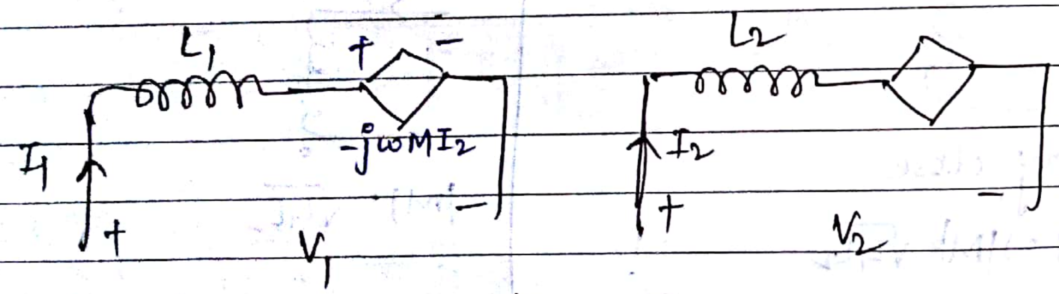


$$V_1 = L_1 \frac{di_1}{dt} - M\frac{di_2}{dt}$$

$$V_2 = L_2 \frac{di_2}{dt} - M\frac{di_1}{dt}$$

$I_1 = I_1 e^{j\omega t}$
$I_2 = I_2 e^{j\omega t}$
$\Rightarrow V_1 = L_1 j\omega I_1 - Mj\omega I_2$

* Draw an eq. ckt. with dependent sources



No coupling

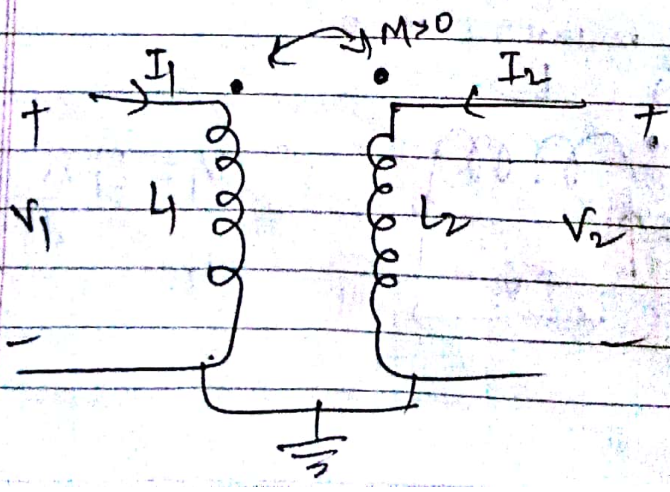
$$V_1 = j\omega L_1 I_1 - j\omega M I_2$$

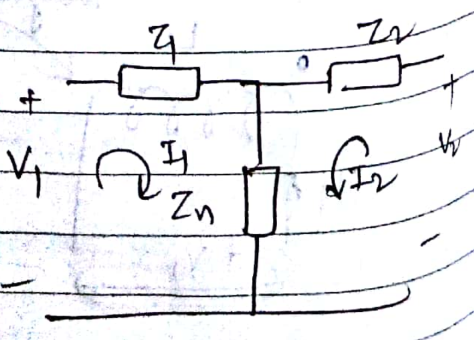
as V_1 should be
$L_1 j\omega I_1 - Mj\omega I_2$

Same

we get $V_{dependent\ source} = -j\omega M I_2$

Q.





$Ni = R\phi$; $Ni =$ MMF
$R =$ Reluctance.

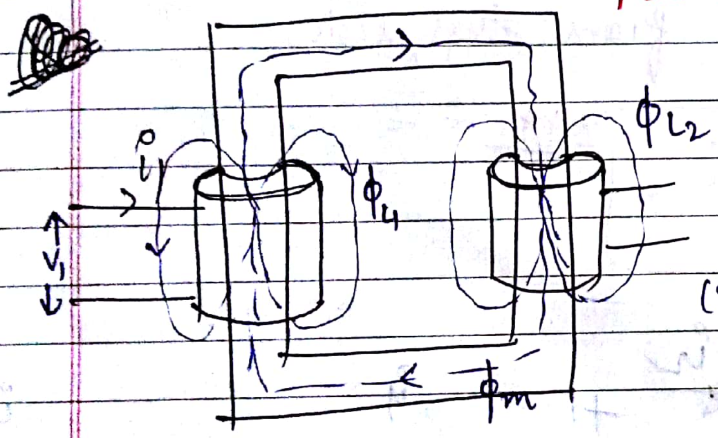
$$V_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$V_2 = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

Find Z_1, Z_2, Z_n for these circuits to be equivalent.

$$Z_1 = j\omega L_1 - j\omega M$$
$$Z_2 = j\omega L_2 - j\omega M$$
$$Z_n = j\omega M$$

For ideal transformer, $\phi_{L_1} = 0$, $\phi_{L_2} = 0$
(No leakage)



To find an expression for L_1, L_2. In terms of M.

$$V_1 \text{ (due to } i_1) = \frac{d\psi_1}{dt} = \frac{d(N_1\phi_T)}{dt}$$

$$= \frac{d}{dt} N_1(\phi_m + \phi_{L_1})$$

$$= N_1 \frac{d}{dt}\left(\frac{N_1 i_1}{R} + \frac{N_1 i_1}{R_a}\right)$$

$$V_2 \text{ (due to } i_1) = \frac{d}{dt} N_2(\phi_m) = \frac{d}{dt}\left(\frac{N_1 i_1}{R}\right) N_2$$

$$M_{21} = \frac{V_2}{\frac{di_1}{dt}} = \frac{N_1 N_2}{R} = M$$

$$L_1 = \frac{V_1}{\frac{di_1}{dt}} = \frac{N_1^2}{R} + \frac{N_1^2}{R_a}$$

$$L_1 = \frac{N_1}{N_2} \frac{N_1 N_2}{R} + \frac{N_1 N_1}{R_a} \rightarrow \text{Reluctance of air.}$$

$$L_1 = \left(\frac{N_1}{N_2}\right) M + L_{l1} = aM + L_{l1}$$

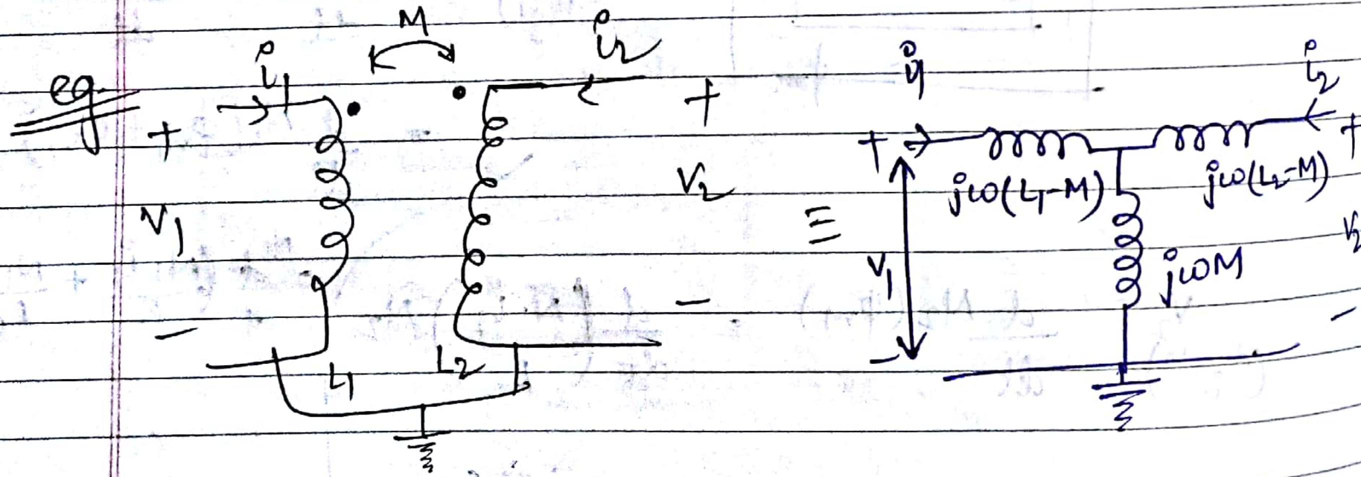
↑ Turns. Ratio

$$\therefore \boxed{L_1 = aM + L_{l1}} \quad ***$$

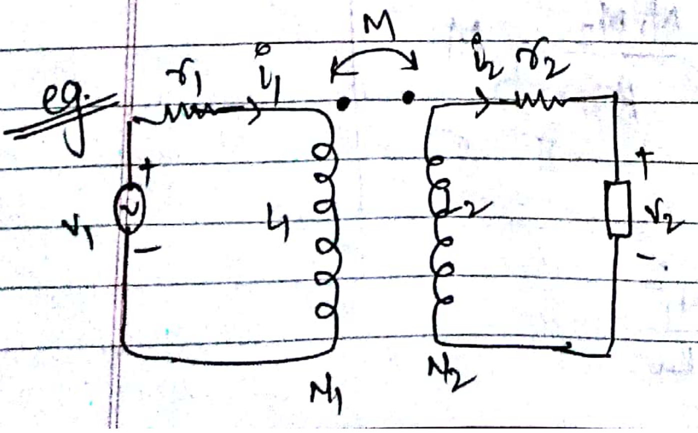
||ly, by exciting from other side,

$$\boxed{L_2 = \frac{M}{a} + L_{l2}} \quad ***$$

eg.



eg.



$a = \frac{N_1}{N_2}$

Transformer.

Find eq. ckt.

$$V_1 = i_1 r_1 + L_1 \frac{di_1}{dt} - M\frac{di_2}{dt}$$

$$V_2 = -i_2 r_2 - L_2 \frac{di_2}{dt} + M\frac{di_1}{dt}$$

→ why "plus" as current in coil 1 enters dot ⇒ $\left(+M\frac{di_1}{dt}\right)$

coil 2 ~~dot~~ dot = +ve voltage

$$V_2 = \frac{V_1}{a}$$

$$i_2 = i_1 a$$

$V_2 a := V_2'$ (defined as) → Just to bring the voltages on both sides to a similar range.

$$\frac{i_2}{a} = i_2' \rightarrow$$

$$\therefore aV_2 = a^2\left(-\frac{i_2}{a}\right)r_2 + a^2 L_2 \frac{d}{dt}\left(-\frac{i_2}{a}\right) + aM\frac{di_1}{dt}$$

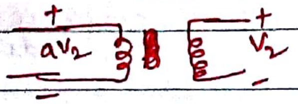
Put ~~ø~~ the value of L_1 & L_2 $\left(\begin{matrix} L_1 = aM + L_{l_1} \\ L_2 = \frac{M}{a} + L_{l_2} \end{matrix}\right)$

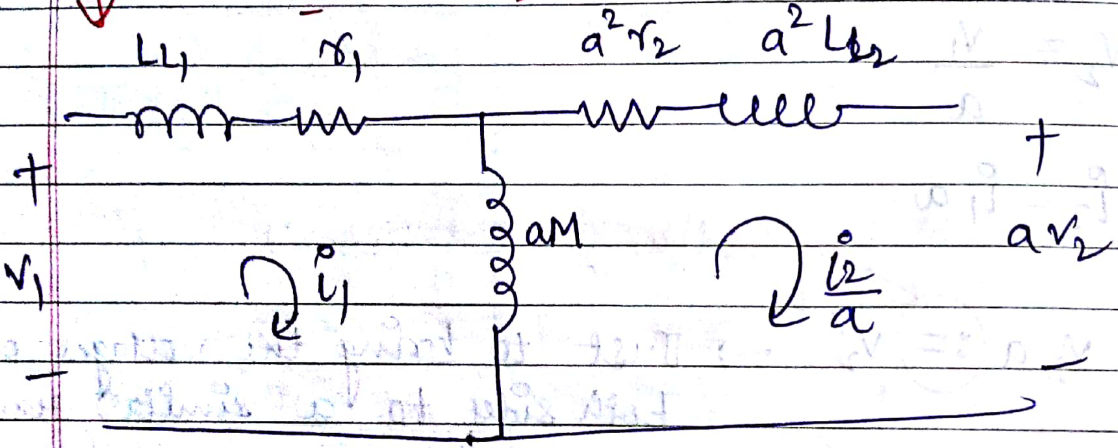
(1) $$V_1 = i_1 r_1 + aM\frac{di_1}{dt} + L_{l_1}\frac{di_1}{dt} + aM\frac{d}{dt}\left(-\frac{i_2}{a}\right)$$

(2) $$aV_2 = a^2\left(\frac{-i_2}{a}\right)r_2 + a^2 \frac{M}{a}\frac{d}{dt}\left(-\frac{i_2}{a}\right) + a^2 L_{l_2}\frac{d}{dt}\left(-\frac{i_2}{a}\right)$$
$$+ aM\frac{di_1}{dt}$$

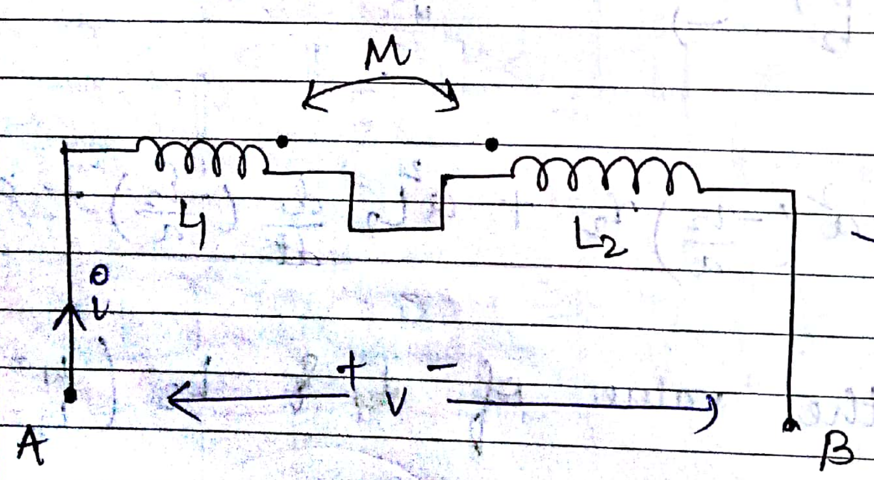
$$\Rightarrow \quad aV_2 = \left(\frac{-i_2}{a}\right) r_2 a^2 + a^2 L_{l2}\frac{d}{dt}\left(\frac{-i_2}{a}\right) + aM\frac{d}{dt}\left(\frac{-i_2}{a}\right)$$
$$+ aM\frac{d}{dt} i_1$$

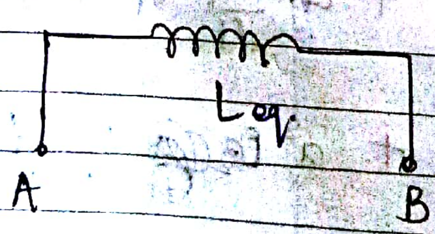
Eq. ckt. + Ideal transformer = complete eq. ckt.



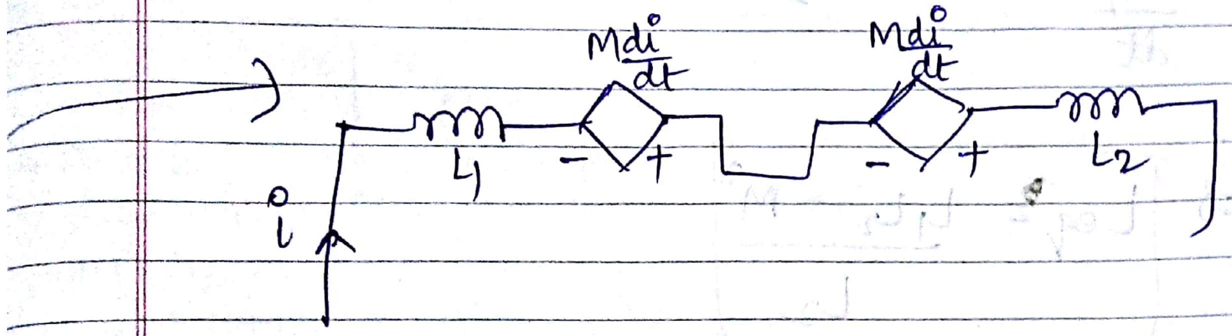


Q.





$V = L\frac{di}{dt}$



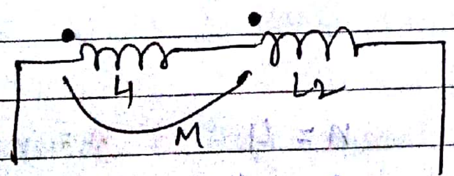
$$V = L_1\frac{di}{dt} - M\frac{di}{dt} - M\frac{di}{dt} + L_2\frac{di}{dt}$$

$$= (L_1+L_2-2M)\frac{di}{dt}$$

$$= L_{eq}\,\frac{di}{dt}$$

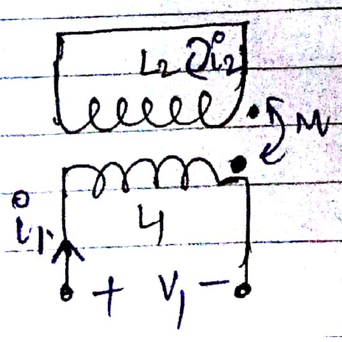
$$\Rightarrow \boxed{L_{eq} = L_1+L_2-2M}$$

Q.



$$\boxed{L_{eq} = L_1+L_2+2M}$$

Q.



Find eq. Inductance.

$$V_1 = L_1\frac{di_1}{dt} - M\frac{di_2}{dt}$$

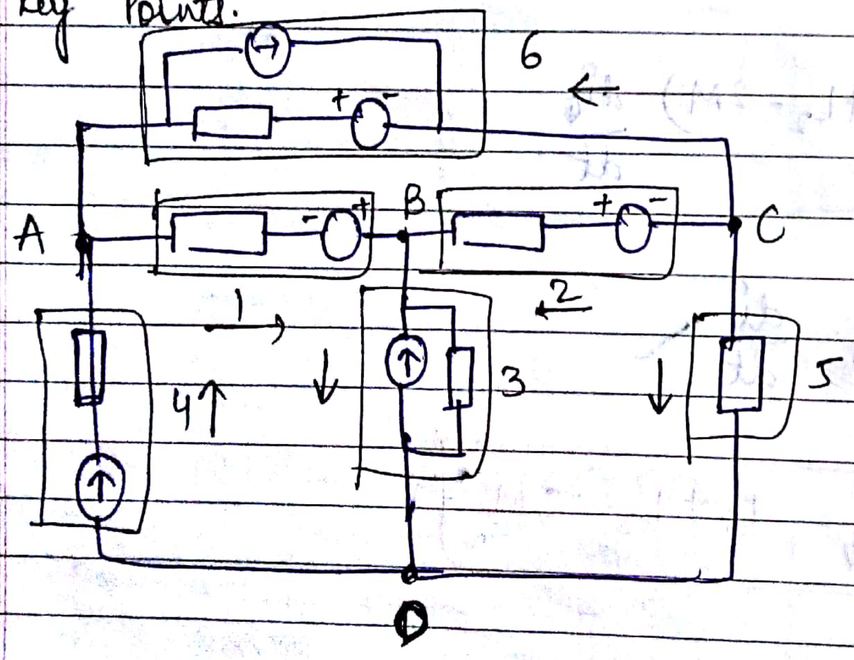
$$0 = L_2\frac{di_2}{dt} - M\frac{di_1}{dt} \Rightarrow \frac{M}{L_2}\frac{di_1}{dt} = \frac{di_2}{dt}$$

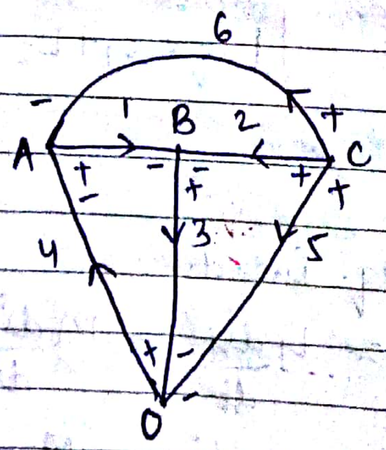
$$\frac{V_1}{\frac{di_1}{dt}} = L_1 - \frac{M^2}{L_2}$$

$$\Rightarrow \boxed{L_{eq} = \frac{L_1 L_2 - M^2}{L_2}}$$

* Graph Theory.

* Key Points.

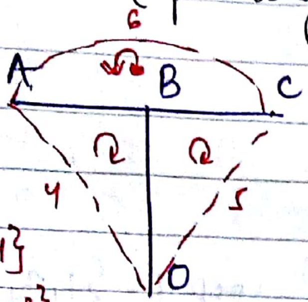




$n = 4$ $\quad$ n = nodes

$e = 6$ $\quad$ e = edges.

* TREE (Spanning tree)



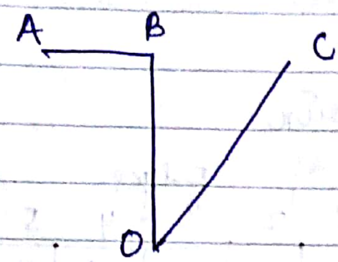
Edges.

loop4 = {1, 3, 4}

loop5 = {5, -3, -2}

loop6 = {1, -2, 6}

or



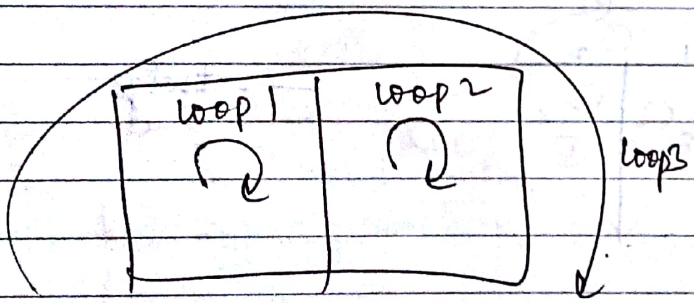
or - - -

→ Is TREE Unique? NO

→ How many branches should a tree a contain? n-1

→ # loops formed ~~if~~ if we join nodes → e-(n-1)
(Fundamental)
↳ (Independent)

* Ex



KVL loop1 +
" loop2 =

" loop3

∴ loop 1 & loop 2 are Fundamental (Independent)

OR loop 1 & loop 3 are Fund— (Indepen—)

* Incidence Matrix

Edges

Nodes	1	2	3	4	5	6
A	1	0	0	-1	0	-1
B	-1	-1	1	0	0	0
C	0	1	0	0	1	1
D	~~0~~	~~0~~	~~-1~~	~~1~~	~~-1~~	~~0~~

= [A]

All Rows are not linearly independent.

1 → current coming out of that node

-1 → " going in
" " "

0 → NO Relation.

Remove any 1 row
⇒ "Reduced Incidence Matrix".

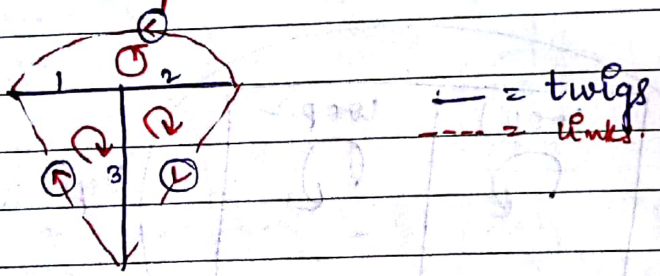
* Loop Matrix

$$\begin{array}{c c} & \text{Edges} \\ & \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \text{Independent loops} \begin{array}{c} 4 \\ 5 \\ 6 \end{array} & \begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & -1 & -1 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 & 0 & 1 \end{bmatrix} = [B] \\ & \underbrace{\quad\quad\quad}_{\text{twigs}} \; \underbrace{\quad\quad\quad}_{\text{links}} \end{array}$$

loop 4 = {1, 3, 4}
loop 5 = {5, -3, -2}
loop 6 = {1, -2, 6}

1 ⇒ current aligned to assumed direcⁿ
-1 ⇒ " not " " "
0 ⇒ No Relation.

* This direcⁿ of loop is direcⁿ of edges added.



V_{ek} = voltage across edge k
V_{nk} = voltage across at node n_k
i_{ek} = current in edge k

* Relation betⁿ node & edge voltages.

$$A^T \begin{bmatrix} V_{nA} \\ V_{nB} \\ V_{nC} \\ V_{nD} \end{bmatrix} = ?$$

2)
$$\begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & 1 & -1 \\ -1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} V_{nA} \\ V_{nB} \\ V_{nC} \\ V_{nD} \end{bmatrix} = \begin{bmatrix} V_{e_1} \\ V_{e_2} \\ V_{e_3} \\ V_{e_4} \\ V_{e_5} \\ V_{e_6} \end{bmatrix}$$

(1) ***
$$\therefore \boxed{\begin{bmatrix} V_{e_1} \\ V_{e_2} \\ \vdots \\ V_{e_e} \end{bmatrix} = A^T \begin{bmatrix} V_{n_1} \\ V_{n_2} \\ \vdots \\ V_{n_n} \end{bmatrix}}$$

$$A \begin{bmatrix} i_{e_1} \\ i_{e_2} \\ \vdots \\ i_{e_6} \end{bmatrix} = \begin{bmatrix} ? \end{bmatrix}$$

(2) ***
$$\boxed{A \begin{bmatrix} i_{e_1} \\ i_{e_2} \\ \vdots \\ i_{e_e} \end{bmatrix} = 0}$$ $\boxed{\text{KCL}}$

check if $B \begin{bmatrix} V_{e_1} \\ V_{e_2} \\ \vdots \\ V_{e_6} \end{bmatrix} = 0$

(3) ***
$$\boxed{B \begin{bmatrix} V_{e_1} \\ V_{e_2} \\ \vdots \\ V_{e_e} \end{bmatrix} = 0}$$ KVL

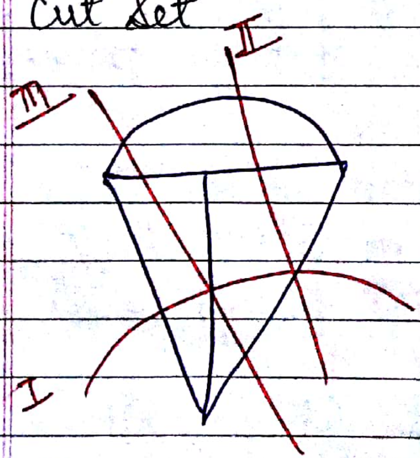
* Relation betⁿ branch & loop currents.

$$B^T \begin{bmatrix} i_{L4} \\ i_{L5} \\ i_{L6} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & -1 \\ 1 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_{L4} \\ i_{L5} \\ i_{L6} \end{bmatrix} = \begin{bmatrix} i_{e1} \\ i_{e2} \\ i_{e3} \\ i_{e4} \\ i_{e5} \\ i_{e6} \end{bmatrix}$$

(4.) $B^T \begin{bmatrix} \text{loop} \\ \text{currents} \end{bmatrix} = \begin{bmatrix} \text{edge} \\ \text{current} \end{bmatrix}$

* Cut Set

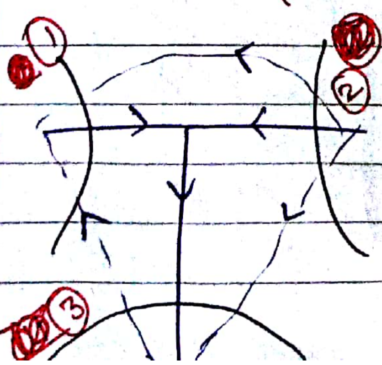


Edges

$I = \{4, 3, 5\}$

$II = \{6, 2, 5\}$

$III = \{6, 1, 3, 5\}$



Fundamental cut → cuts which passes through only one twig.

→ $(n-1)$ (No. of twigs)

$1 = \{1, -6, -4\}$ '−' for 'i' direcⁿ oppo. to branch/twig
'k' for k^{th} cut.

$2 = \{6, 2, 5\}$

$3 = \{5, 3, -4\}$

Cut	1	2	3	4	5	6
1	1	0	0	−1	0	−1
2	0	1	0	0	1	1
3	0	0	1	−1	1	0

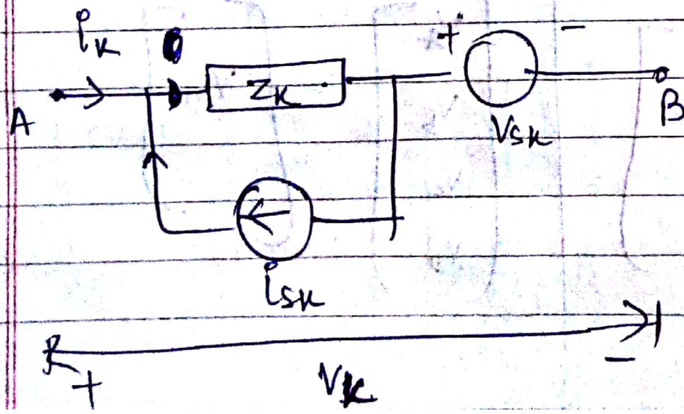
(Edges across the top) $= [Q]$

Q. $Q \begin{bmatrix} i_{e_1} \\ i_{e_2} \\ \vdots \\ i_{e_6} \end{bmatrix} = 0$ $i_{e_1} - i_{e_4} - i_{e_6} = 0$

(3) $\boxed{Q \begin{bmatrix} i_{e_1} \\ \vdots \end{bmatrix} = 0}$ ***

General representation of any branch.

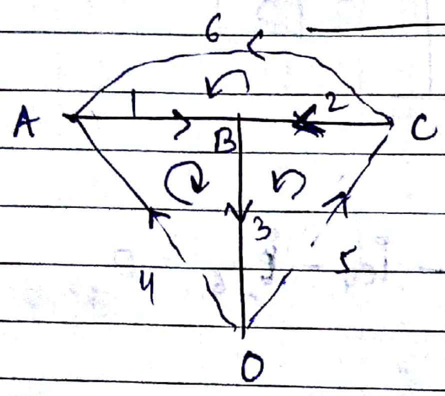
$Y_k = \dfrac{1}{Z_k}$



$Y_k(V_k - V_{sk}) - i_{sk} = i_k$

$V_k = V_{sk} + Z_k(i_k + i_{sk})$

$$\underbrace{\begin{bmatrix} V_1 \\ V_2 \\ \vdots \end{bmatrix}}_{\#\text{edges}} = \overset{\#\text{edges}}{\begin{bmatrix} Z_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & Z_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & Z_3 & 0 & 0 & 0 \\ & & \vdots & & & \end{bmatrix}} \left(\begin{bmatrix} i_{e_1} \\ i_{e_2} \\ \vdots \end{bmatrix} + \underbrace{\begin{bmatrix} i_{s_1} \\ i_{s_2} \\ \vdots \end{bmatrix}}_{\#\text{edges}} \right) + \underbrace{\begin{pmatrix} V_{s_1} \\ V_{s_2} \\ \vdots \end{pmatrix}}_{\#\text{edges}}$$



Tree : $n-1$

$n = \#$nodes

$e = \#$ edges

$l = e - \#$twigs $= e-(n-1)$

Nodal mtd $\Rightarrow$ $n-1$ unknowns (ϕ as one $= 0$)

$n-1$ eq^{ns} (KCL)

Mesh $\Rightarrow$ l unknowns

l KVL eq^{ns}

$$\begin{bmatrix} i_{e_1} \\ i_{e_2} \\ | \\ i_{e_6} \end{bmatrix} = \begin{bmatrix} i_e \end{bmatrix} \qquad \Bigg| \qquad \begin{bmatrix} V_A \\ V_B \\ V_C \\ V_D \end{bmatrix} = \begin{bmatrix} V_n \end{bmatrix}$$

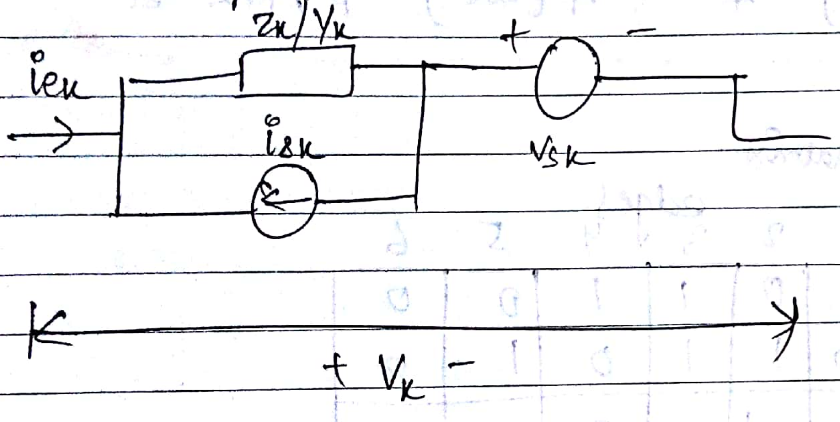
Rules of eq^ns

① KCL   N

② KVL   M

③ Branch eq^ns   N, M.

Branch eq^ns



KCL

$i_{ek} = (V_k - V_{sk}) Y_k - i_{sk}$

$$Y_k = \begin{bmatrix} Y_{e1} & & & 0 \\ & Y_{e2} & & \\ & & Y_{e3} & \\ 0 & & & Y_{ee} \end{bmatrix}$$

$$\begin{bmatrix} i_{e1} \\ i_{e2} \\ \vdots \\ i_{ee} \end{bmatrix} = \begin{bmatrix} Y_{e1} & & \\ & Y_{e2} & \\ & & \ddots & \\ & & & Y_{ee} \end{bmatrix} \left(\begin{bmatrix} V_{e1} \\ V_{e2} \\ \vdots \\ V_{ee} \end{bmatrix} - \begin{bmatrix} V_{se1} \\ V_{se2} \\ \vdots \\ V_{see} \end{bmatrix} \right) - \begin{bmatrix} i_{se1} \\ i_{se2} \\ \vdots \\ i_{see} \end{bmatrix}$$

KCL

$$[i_e] = [Y_k]\left([V_e] - [V_s]\right) - [i_{se}]$$

KVL

$$[A][i_e] = A\left([Y_k]\left((V_e) - (V_s)\right) - (i_{se})\right) = 0$$

As $[V_e] = A^T[V_N]$

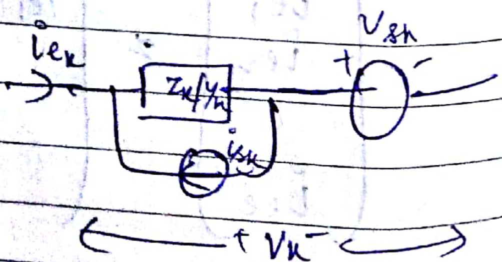
$$A Y_k \left[A^T[V_N] - [V_{se}]\right) - A[i_{se}] = 0$$

$$\boxed{[A Y_k A^T] V_N = A[i_{se}] + A Y_k V_{se}}$$ ★★★

$B =$ loop Matrix

loops \ edges	1	2	3	4	5	6
4	1	0	1	1	0	0
5	0	1	1	0	1	0
6	1	-1	0	0	0	1

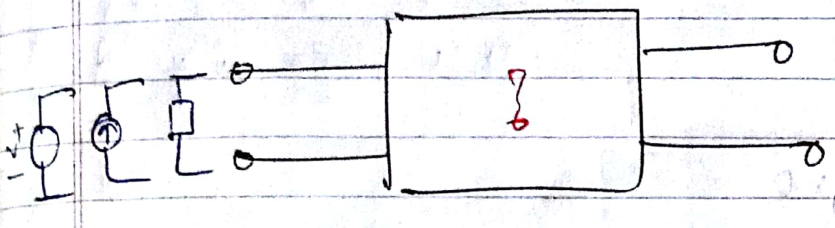
$$\begin{bmatrix} i_{e_1} \\ i_{e_2} \\ \vdots \\ i_{ee} \end{bmatrix} = \underbrace{\begin{array}{c|ccc} & 4 & 5 & 6 \\ \hline 1 & 1 & 0 & 1 \\ & & & \\ & & & \end{array}}_{B^T} \begin{bmatrix} i_{L4} \\ i_{L5} \\ i_{L6} \end{bmatrix}$$



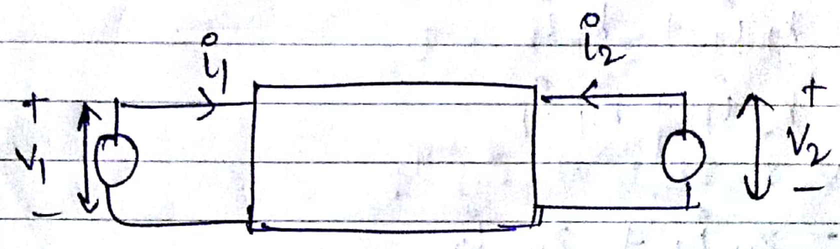
$$V_k = Z_k(i_k + i_{sk}) + V_{sk}$$

$$\begin{bmatrix} V_{e_1} \\ V_{e_2} \\ \vdots \\ V_{ee} \end{bmatrix} = \begin{bmatrix} Z_{e_1} & & \\ & \ddots & \\ & & Z_{ee} \end{bmatrix} \left(\begin{bmatrix} i_{e_1} \\ i_{e_2} \\ \vdots \\ i_{ee} \end{bmatrix} + \begin{bmatrix} i_{s_1} \\ i_{s_2} \\ \vdots \\ i_{se} \end{bmatrix} \right) + \begin{bmatrix} V_{s_1} \\ \vdots \\ V_{s6} \end{bmatrix}$$

* Two Port Networks.



Inside n/w No Independent sources.



$i_1 = K_1 V_1 + K_2 V_2$ (By superposition theo.)

$i_2 = K_3 V_1 + K_4 V_2$

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} K_1 & K_2 \\ K_3 & K_4 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$i_1 = Y_{11} V_1 + Y_{12} V_2$

$i_2 = Y_{21} V_1 + Y_{22} V_2$

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Short ckt admittance parameters.

$$Y_{11} = \left.\frac{i_1}{V_1}\right)_{V_2=0}$$

$$Y_{12} = \left.\frac{i_1}{V_2}\right)_{V_1=0}$$

Transfer admittance.

Now,

$$V_1 = k_1 i_1 + k_2 i_2$$
$$V_2 = k_3 i_1 + k_4 i_1$$

$$V_1 = Z_{11} i_1 + Z_{12} i_2$$
$$V_2 = Z_{21} i_1 + Z_{22} i_2$$

$$Z_{11} = \left.\frac{V_1}{i_1}\right)_{i_2=0}$$

| | |

* Also,

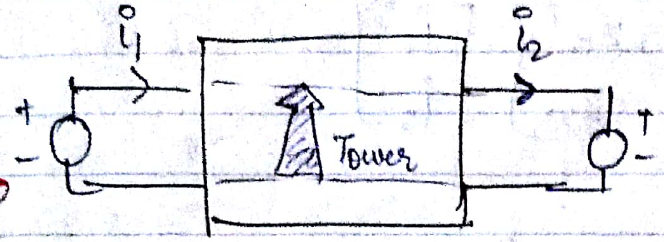
$$\begin{bmatrix} V_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -i_2 \end{bmatrix}$$

Primary

Secondary.

Transmission parameters /

T — 4



(If i_1 goes in, i_2 has to come out.)

* $$\begin{bmatrix} V_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ V_2 \end{bmatrix}.$$

Hybrid parameters.

* Relation betn the parameters.

(1) Z & Y

$$[V] = [Z][I] \qquad Z = Y^{-1}$$

$$[I] = [Y][V]$$

(2) $$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$ ✓

$$\begin{bmatrix} V_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -i_2 \end{bmatrix}$$

H.W.s

* Properties:

Reciprocity

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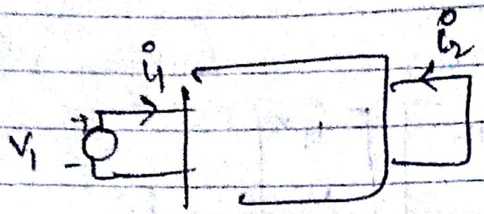
$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

eg:



When $V_2 = 0$; To find $\frac{V_1}{I_2}$

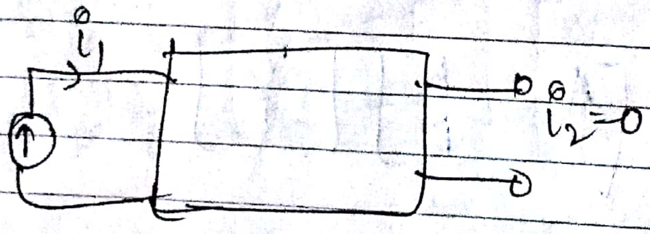
$$Z_{21} i_1 + Z_{22} i_2 = 0$$

$$V_1 = Z_{11} i_1 + Z_{12} i_2$$

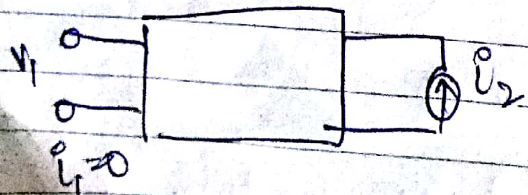
$$V_1 = -Z_{11} \frac{Z_{22}}{Z_{21}} i_2 + Z_{12} i_2$$

$$\frac{V_1}{i_2} = Z_{12} - \frac{Z_{11} Z_{22}}{Z_{21}}$$

eg:



$\frac{V_{21}}{i_1} = Z_{21}$ ✓



$\frac{V_1}{i_2} = Z_{12}$

For Reciprocity

$Z_{21} = Z_{12}$

* Y & Reciprocity.

$$Y_{12} = Y_{21}$$

* ABCD & Reciprocity.

$$\begin{bmatrix} V_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} A & -B \\ C & -D \end{bmatrix} \begin{bmatrix} V_2 \\ i_2 \end{bmatrix}$$

Apply $i_2 = 0$ & current source on primary.

Apply i_1 & measure V_2

$$i_1 = C V_2 \Rightarrow \frac{V_2}{i_1} = \frac{1}{C}$$

Now,

Apply i_2 & measure V_1

$$i_1 = 0$$

$$0 = C V_2 - D i_2$$

$$V_1 = A V_2 - B i_2$$

$$V_1 = A \frac{D i_2}{C} - B i_2$$

$$\frac{V_1}{i_2} = \left(\frac{AD}{C} - B\right)$$

By Reciprocity $\frac{\text{Out/p}}{\text{inp/p}} = \frac{\text{Out/p}}{\text{Inp/p}} \Rightarrow \frac{V_2}{i_1} = \frac{V_1}{i_2}$

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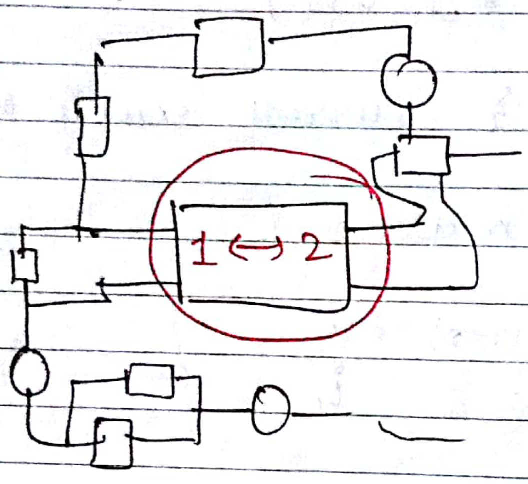
$$\frac{1}{C} = \frac{AD}{C} - B$$

$$AD - BC = 1$$

Determinant.

* Symmetry



If we change 1 & 2 no change occurs, this is symmetry.

① Reciprocal

② I/p impedence of port 1 & 2 should be equal

eg $\begin{bmatrix} V_1 \\ i_1 \end{bmatrix} = \begin{bmatrix} A & -B \\ C & -D \end{bmatrix} \begin{bmatrix} V_2 \\ i_2 \end{bmatrix}$ (given)

$\begin{bmatrix} V_2 \\ i_2 \end{bmatrix} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix} \begin{bmatrix} V_1 \\ i_1 \end{bmatrix}$ (to find)

X matrix

Since, the network is symmetric

X matrix is (ABCD) inverse

X matrix $= \dfrac{\begin{pmatrix} -D & B \\ -C & A \end{pmatrix}}{\text{Determinant} = -1}$ divided by

$$X = \frac{1}{-1}\begin{pmatrix} -D & B \\ -C & A \end{pmatrix}$$

As N/w is symm.

$$\begin{pmatrix} A & -B \\ C & -D \end{pmatrix} = \begin{pmatrix} +D & -B \\ +C & -A \end{pmatrix}$$

$\Rightarrow$ $A = D$, ...

* Eigen vector of a matrix

Output vector whose direcⁿ is same as input

$$\begin{bmatrix} \\ \\ \\ \end{bmatrix}_{3\times1} = \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & 10 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix} = 10 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

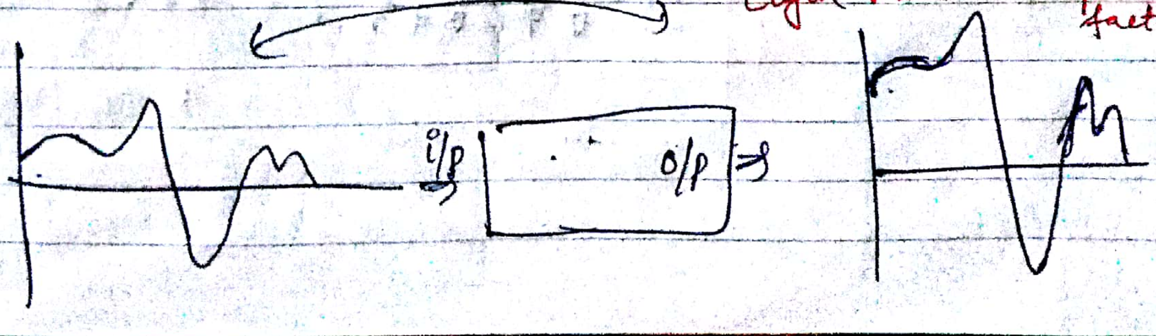
Eigen value

Doubt → Eigen vector. ??

Both of them can be considered as eigen vectors. as only direcⁿ matters.

* Eigen func. of a sys. Same pattern, magnitude may differ.

Eigen value = Multiplication factor.



eg. $1 \cdot \frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 3y(t) = x(t)$ —— (1)

Can we find the eigen f^n?

Check whether the following are eigen f^n.

a) $x(t) = \delta(t)$ ✗

b) $x(t) = u(t)$ ✗

c) $= \sin\omega t$ ✗

d) $= \cos\omega t$ ✗

e) $= e^{\sigma t}$ ✓

f) $= e^{-\sigma t}$ ✓

g) $= e^{-j\omega t}$ ✓

(b) Let $y = k\,u(t)$ if $x(t)$ is eigen f^n

Put in (1)

$$k\dot{\delta} + 2k\,\delta(t) + k\,u(t) = u(t) \quad ✗ \text{ Not possible}$$

(e) $x(t) = e^{\sigma t}$; $y(t) = k\,e^{\sigma t}$

$$k\sigma^2 e^{\sigma t} + 2k\sigma e^{\sigma t} + 3k\,e^{\sigma t} = e^{\sigma t}$$

$$k\sigma^2 + 2k\sigma + 3k - 1 = 0$$

$$\boxed{k = \frac{1}{\sigma^2 + 2\sigma + 3}}$$ Eigen value

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Page

(d) $y = K\cos\omega t$; $x(t) = \cos\omega t$

--- X

*** For any LTI sys., exponential f^n are eigen f^{ns}

Proof :- For any LTI sys.

$\delta(t) \longrightarrow h(t)$

$\boxed{y(t) = x(t) * h(t)}$ valid only for LTI sys.

$$= \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$

Choose $x(t) = e^{st}$; s = real/complex/anything.

$$= \int_{-\infty}^{\infty} h(\tau)\, e^{st} e^{-s\tau}\, d\tau$$

$$= e^{st} \int_{-\infty}^{\infty} h(\tau)\, e^{-s\tau}\, d\tau$$

Being a definite integral, it is a const

$y(t) = k\, e^{st}$.

$\therefore e^{st}$ is eigen f^{ns}.

$$\boxed{\text{Eigen value} = \int_{-\infty}^{\infty} h(t)\, e^{-st}\, dt}$$ ***

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$$e^{at} \longrightarrow \boxed{LTI} \longrightarrow ke^{at} \qquad \left(\text{k can be complex}\right)$$

$$k = \mathcal{L}\{h(t)\}\Big|_{s=\alpha}$$

(1)

I/p	O/p
$e^{j\omega t}$	$k_1 e^{j\omega t}$
$e^{-j\omega t}$	$k_2 e^{-j\omega t}$

$$\left.\begin{aligned} k_1 &= \int_{-\infty}^{\infty} h(t)\, e^{-j\omega t}\, dt \\ k_2 &= \int_{-\infty}^{\infty} h(t)\, e^{j\omega t}\, dt \end{aligned}\right\} \Rightarrow k_1 = k_2^*$$

for Real systems as $h(t)$ has to be Real.

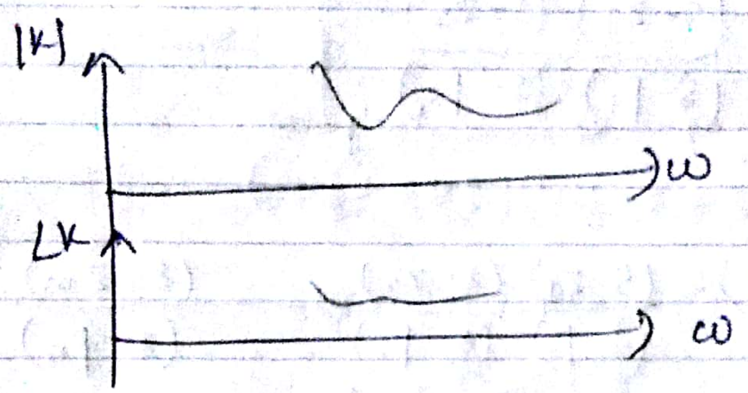
(2)

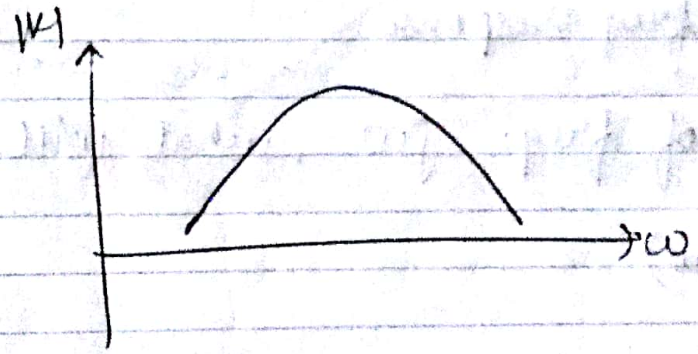
I/p	O/p
$\cos\omega t$	
$\dfrac{e^{j\omega t} + e^{-j\omega t}}{2}$	$\dfrac{k_1 e^{j\omega t}}{2} + \dfrac{k_2 e^{-j\omega t}}{2} = Re^{j\theta}\dfrac{e^{j\omega t}}{2} + Re^{-j\theta}\dfrac{e^{-j\omega t}}{2}$
	$= R\cos(\omega t + \theta)$

$H(s) = \mathcal{L}\{h(t)\}$ = called Transfer func.

To know any LTI sys. ≡ knowing all eig. values of exponential f^n

Take all ω's f^n of sines & cosines as input & $k = Re^{j\theta}$





Q. $a\frac{d^2y}{dt^2} + b\frac{dy}{dt} + c\,y(t) = x(t)$ $k = \mathcal{L}\{h(t)\}$

To find $\mathcal{L}\{h(t)\}$

$a\ddot{h} + b\dot{h} + c\,h(t) = \delta(t)$

$a s^2 H(s) + b s H(s) + c H(s) = 1$ $(h(0) = h'(0) = 0$ as for LTI initial condition = 0)

$$H(s) = \frac{1}{as^2 + bs + c}$$

$$a y'' + b y' + c y = p x(t) + q(\dot{x}) + r \ddot{x}$$

$$H(s) = \frac{r s^2 + q s + p}{a s^2 + b s + c}$$ where $x(t) = \delta(t)$

$$= \frac{(s - z_1)(s - z_2)}{(s - p_1)(s - p_2)}$$

zeroes

poles.

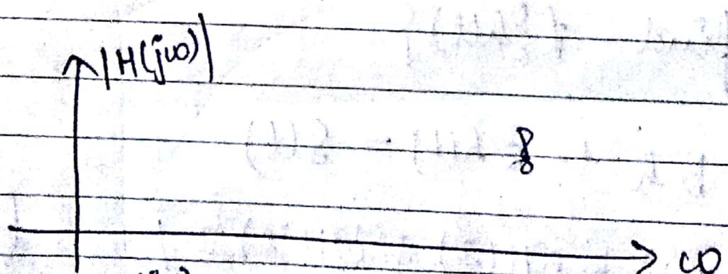
Given $H(s) = \dfrac{(s - z_1)(s - z_2) \;---\; (s - z_m)}{(s - p_1)(s - p_2) \;---\; (s - p_n)}$

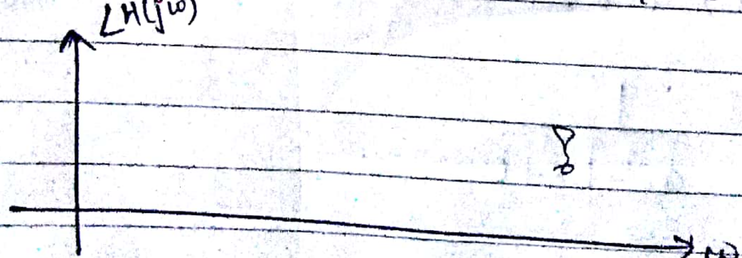
To find freq. response

If i/p is of freq. $j\omega$, what will be k (eigen value)

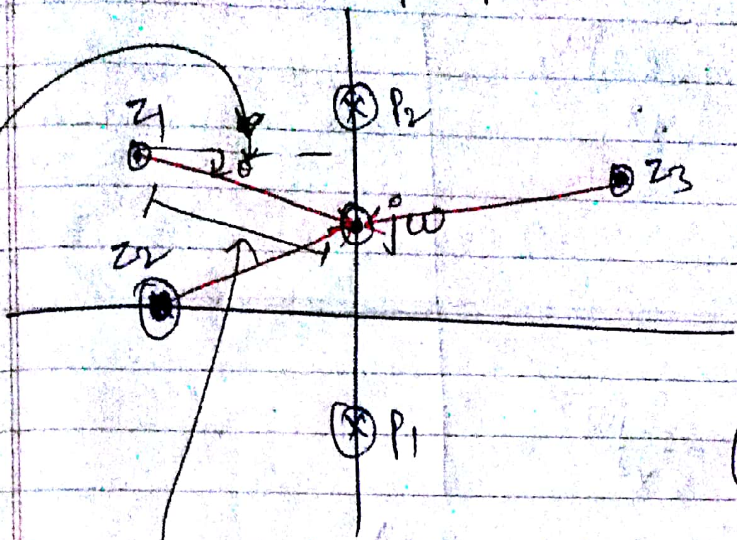
$$k = H(j\omega)$$

$$= \frac{(j\omega - z_1)(j\omega - z_2) \;---}{(j\omega - p_1)(j\omega - p_2) \;---}$$





Complex Plane



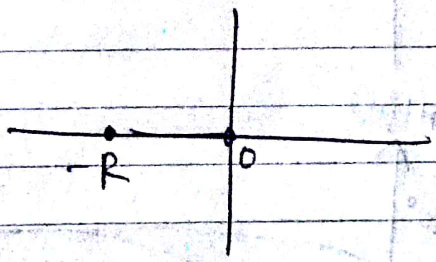
$\longrightarrow = j\omega - z_1$

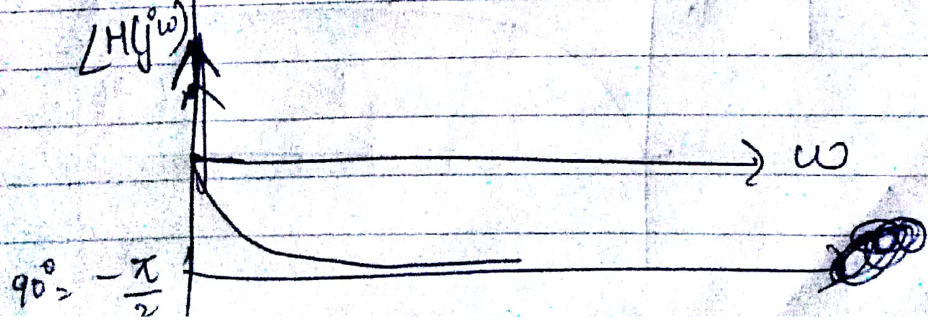
Pole - Zero diagram.
(Draw zeroes as dots & poles as cross)

$$|H(j\omega)| = \frac{|j\omega - z_1|\,|j\omega - z_2| \cdots}{|j\omega - p_1|\,|j\omega - p_2| \cdots}$$

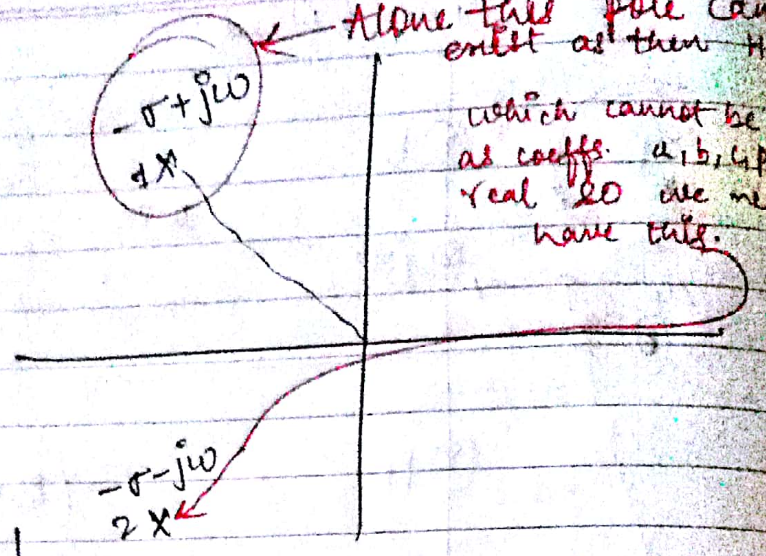
$$\angle H(j\omega) = \angle j\omega - z_1 + \angle j\omega - z_2 + \cdots - (\angle j\omega - p_1 + \angle j\omega - p_2 + \cdots)$$

Q. $\dfrac{1}{s+R}$



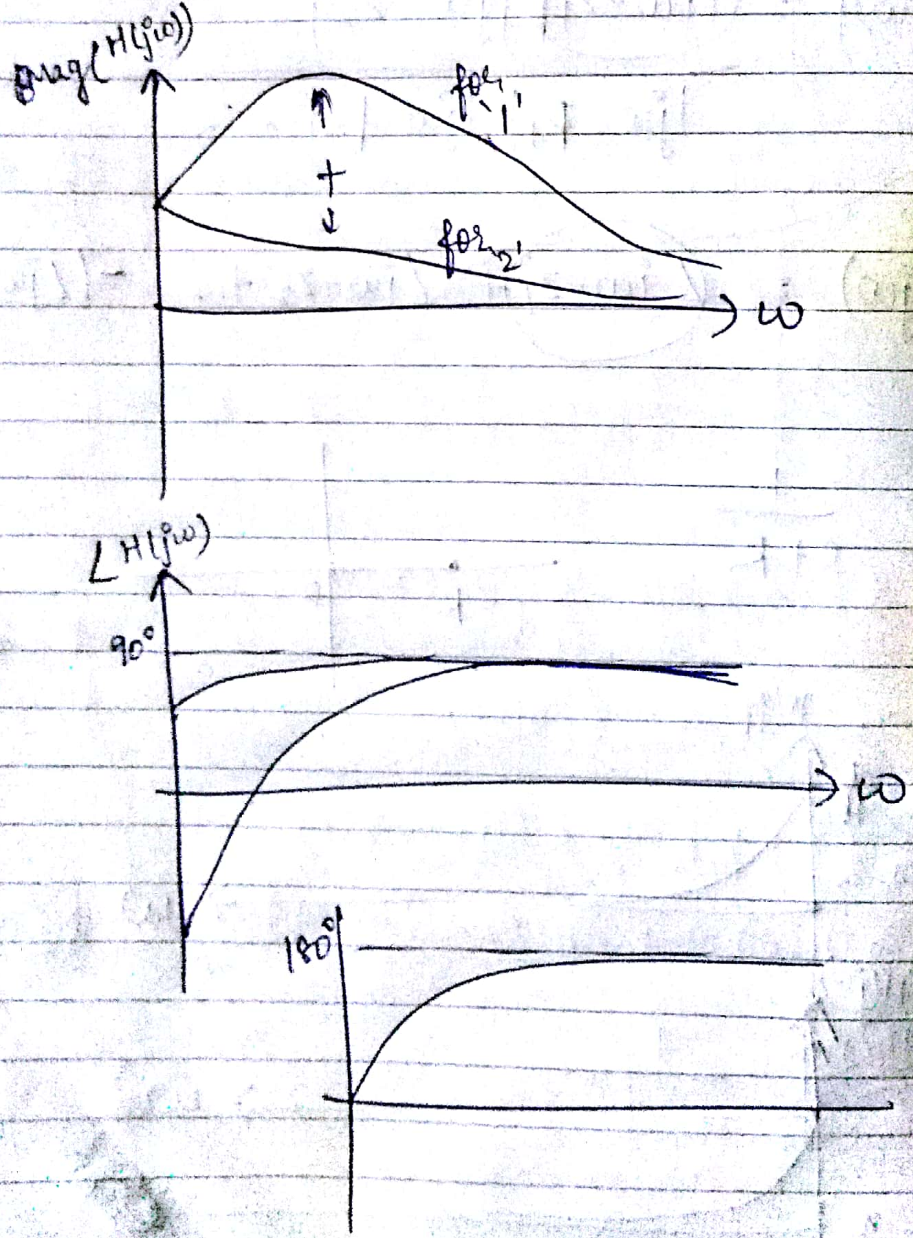


Q. H(s) =

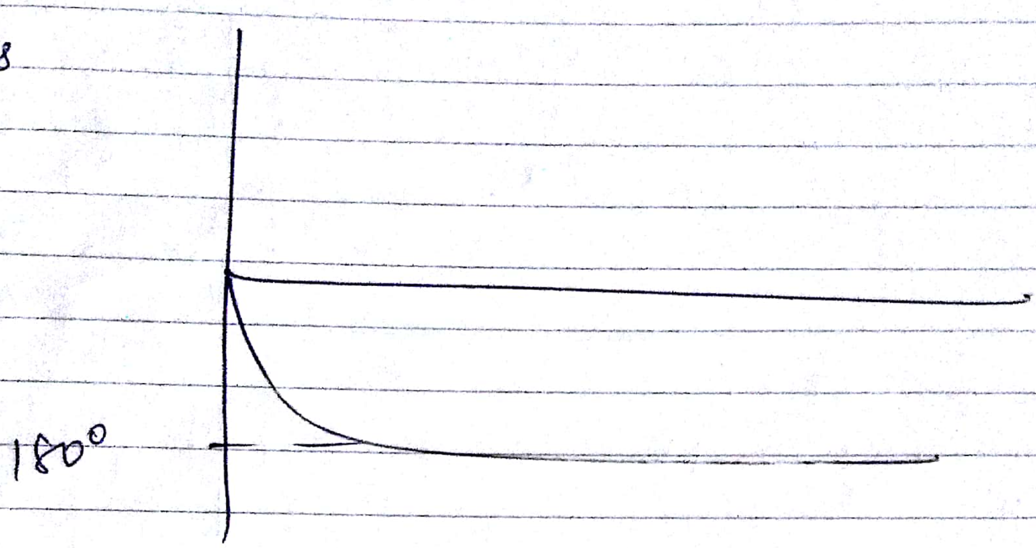


Alone this pole cannot exist as then H(s) ... which cannot be complex as coeffs. a, b, c, p, q, r are real so we need to have this.

$$H(s) = \frac{1}{(s+\sigma-j\omega)(s+\sigma+j\omega)}$$



Ans



as we have the poles in denominator.

* Bode Plots.

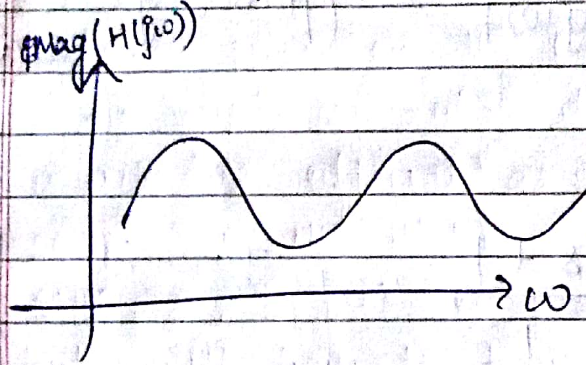
$i/p = e^{\alpha t}$

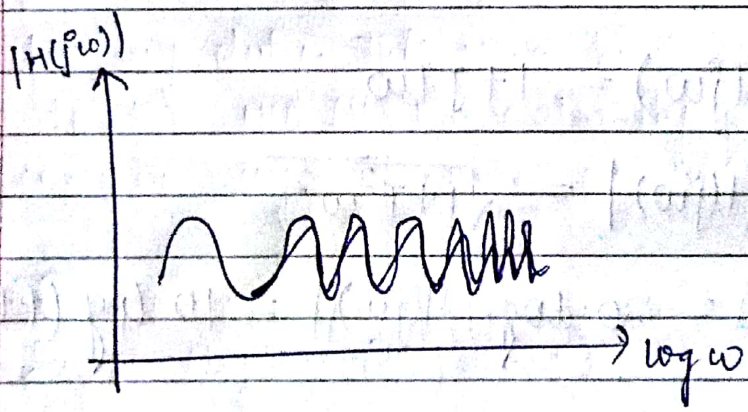
$o/p = k e^{\alpha t}$

$k = H(s)\big)_{s=\alpha}$ where $H(s) = \mathcal{L}[h(t)]$

α = real / imag. / complex.

= complex freq.





Magnitude gain $|H(j\omega)|$

$= \dfrac{\text{Amplitude of } o/p}{\text{" " } i/p}$

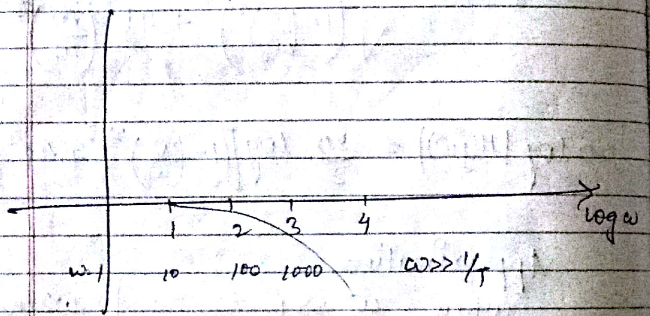
Power / Energy = amplitude^2

(2) $H(s) = \frac{1}{sT+1}$

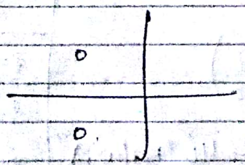
$20 \log |H(j\omega)| = -20 \log (|1 + j\omega T|)$

$\omega = 1/T \longrightarrow 20 \log |H(j\omega)| = -3 dB.$

$\log \omega = -\log T$



(3) $H(s) = (s-(\alpha+j\beta)) \times (s-(\alpha-j\beta))$



$H(s) = (s-\alpha)^2 + \beta^2$

$= s^2 + \underbrace{\alpha^2+\beta^2}_{\omega_0^2} - 2\underbrace{\alpha}_{\xi\omega_0} s$

$\frac{o/p \ Power}{i/p \ Power} = |H(j\omega)|^2$

$\log(\ '' \ ) = 2 \log |H(j\omega)|$

$10 \log |H(j\omega)|$ (decibel)

$20 \log |H(j\omega)|$ (power gain in decibel)

* Examples.

(1) $H(s) = \frac{Ts+1}{1}$; T = const.

$H(j\omega) = 1 + jT\omega$

$|H(j\omega)| = \sqrt{1+T^2\omega^2}$

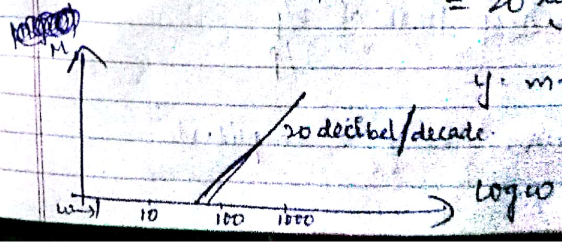
$M = 20 \log |H(j\omega)| = 10 \log (1+T^2\omega^2)$

Approximation

when $\omega >> \frac{1}{T} \Rightarrow M \approx 20 \log(\omega T)$

$= 20 \underbrace{\log \omega}_{x} + 20 \log T$

$y = mx + c$



$$= s^2 - 2\zeta\omega_0 s + \omega_0^2$$

$$= \omega_0^2\left[1 - 2\zeta\left(\frac{s}{\omega_0}\right) + \left(\frac{s}{\omega_0}\right)^2\right]$$

$$H(j\omega) = \omega_0^2\left[\underbrace{1 - 2\zeta\left(\frac{j\omega}{\omega_0}\right) - \frac{\omega^2}{\omega_0^2}}\right]$$

$$|H(j\omega)| = \sqrt{\left(1-\left(\frac{\omega}{\omega_0}\right)^2\right)^2 + 4\zeta^2\left(\frac{\omega}{\omega_0}\right)^2}$$

$$20\log|H(j\omega)| = \frac{20}{2}\log\left[\left(1-\left(\frac{\omega}{\omega_0}\right)^2\right)^2 + 4\zeta^2\left(\frac{\omega}{\omega_0}\right)^2\right]$$

Approximation

when $\frac{\omega}{\omega_0} >> 1$

$$4\times 10\log\left(\frac{\omega}{\omega_0}\right)$$

$$= 40\log\left(\frac{\omega}{\omega_0}\right)$$

$$= 40\log\omega - 40\log(\omega_0)$$

when $\frac{\omega}{\omega_0} << 1$

$M = 0$

$\zeta = 1/2 \quad M = 0$

$\zeta = 1;\ M = \log 9\times 10$

$0 < \zeta < 1/2$