Discrete Structures

Tutorial 2

1st August, 2018

1. **Establish the validity of the following arguments.**

a)
$$\begin{array}{l} p \rightarrow (q \rightarrow r) \\ \neg q \rightarrow \neg p \\ p \\ \hline \therefore r \end{array}$$

b)
$$\begin{array}{l} p \wedge q \\ p \rightarrow (r \wedge q) \\ r \rightarrow (s \vee t) \\ \neg s \\ \hline \therefore t \end{array}$$

c)
$$\begin{array}{l} p \rightarrow (q \rightarrow r) \\ p \vee s \\ t \rightarrow q \\ \neg s \\ \hline \therefore \neg r \rightarrow \neg t \end{array}$$

d)
$$\begin{array}{l} p \vee q \\ \neg p \vee r \\ \neg r \\ \hline \therefore q \end{array}$$

2. Write each of the following arguments in symbolic form. Then establish the validity of the argument or give a counter-example to show that it is invalid.

a) If Dominic goes to the racetrack, then Helen will be mad. If Ralph plays cards all night, then Carmela will be mad. If either Helen or Carmela gets mad, then Veronica (their attorney) will be notified. Veronica has not heard from either of these two clients. Consequently, Dominic didn't make it to the racetrack and Ralph didn't play cards all night.

b) If there is a chance of rain or her red headband is missing, Lois will not mow her lawn. Whenever the temperature is over 80℉, there is no chance of rain. Today the temperature is 85℉ and Lois is wearing her red headband. Therefore, (sometime today) Lois will mow her lawn.

3. Write the following argument in symbolic form, then use resolution (along with the rules of inference and the laws of logic) to establish its validity.

Jonathan does not have his driver's license or his new car is out of gas.
Jonathan has his driver's license or he does not like to drive his new car.
Jonathan's new car is not out of gas or he does not like to drive his new car.
Therefore, Jonathan does not like to drive his new car.

4. Let NAND(p, q) = ¬(p ∧ q) , XOR(p, q) = (p ∧ ¬q) ∨ (¬p ∧ q).

a) Can we express XOR using only NAND?

b) Can we express NAND using only XOR?

5. For the universe of all integers, let p(x), q(x), r(x), s(x) and t(x) be the following open statements.

p(x): x > 0

q(x): x is even

r(x): x is a perfect square

s(x): x is exactly divisible by 4

t(x): x is exactly divisible by 5

a) Write the following statements in symbolic form:

1. At least one integer is even.
2. There exists a positive integer that is even.
3. If x is even, then x is not divisible by 5.
4. No even integer is divisible by 5.
5. There exists an even integer divisible by 5.
6. If x is even and x is a perfect square, then x is divisible by 4

b) Determine if each of the six statements in part (a) is true or false. For each false statement, provide a counter-example.

6. Let the universe for the variables in the following statements consist of all real numbers. In each case, negate and simplify the given statement.

a) $\forall x\, \forall y\, [(x > y) \rightarrow (x - y > 0)]$

b) $\forall x\, \forall y\, [(x < y) \rightarrow \exists z\, (x < z < y)]$

c) $\forall x\, \forall y\, [(|x| = |y|) \rightarrow (y = \pm x)]$