24/1/20

POPL

Assignment - I

Kousshik Raj
17CS30022

1 a) $\lambda x. x\ z\ \lambda y. x\ y$

$= (\lambda x. (x\ z\ \lambda y. (x\ y)))$

$= (\lambda x. ((x\ z)\ (\lambda y. (x\ y))))$

b) $(\lambda x. x\ z)\ \lambda y. w\ \lambda w. w\ y\ z\ x$

$= (\lambda x. (x\ z))\ (\lambda y. w\ \lambda w. w\ y\ z\ x)$

$= (\lambda x. (x\ z))\ (\lambda y. (w\ (\lambda w. (w\ y\ z\ x))))$

$= ((\lambda x. (x\ z))\ (\lambda y. (w\ (\lambda w. (((w\ y)\ z)\ x)))))$

c) $\lambda x. x\ y\ \lambda x. y\ x$

$= \lambda x. (x\ y\ \lambda x. y\ x)$

$= (\lambda x. (x\ y\ (\lambda x. (y\ x))))$

$= (\lambda x. ((x\ y)\ (\lambda x. (y\ x))))$

$= (\lambda x. ((x\ y)\ (\lambda x. (y\ x))))$

B - bound
F - free

2. a) $\lambda x. (x\ z\ \lambda y. x\ y$

$= (\lambda x. (\underset{B}{x}\ \underset{F}{z}\ \lambda y. (\underset{B}{x}\ \underset{B}{y})))$

Kousshik Raj
17CS30022

b) $(\lambda x.\ x\ z)\ \lambda y.\ w\ \lambda w.\ w\ y\ z\ x$

$= ((\lambda x.\ \underset{B}{x}\ \underset{F}{z})(\lambda y.\ (\underset{F}{w}\ \lambda w.\ (\underset{B}{w}\ \underset{B}{y}\ \underset{F}{z}\ x))))$

c) $\lambda x.\ x y\ \lambda x.\ y x$

$= (\lambda x.\ (x\ \underset{F}{y} (\lambda x.\ (\underset{F}{y}\ x))))$

3 a) NOT (NOT TRUE) = TRUE

NOT = $\lambda x.\ ((x$ FALSE) TRUE)

TRUE = $\lambda x.\ \lambda y.\ x$

FALSE = $\lambda x.\ \lambda y.\ y$

• (NOT TRUE) = $\lambda x.\ ((x$ FALSE) TRUE)
TRUE

~~= λx.((TRUE FALSE)~~

= (TRUE FALSE) TRUE
{β-red}

= $(\lambda x.\ \lambda y.\ x$ FALSE) TRUE
{Def. of True}

= $(\lambda y.$ FALSE) TRUE
{β-red}

Kousthik Ray
17CS30022

$= \emptyset\ FALSE$

$\{\beta\text{-red}\}$

$\Rightarrow$ NOT TRUE = FALSE —— (1)

NOT FALSE $= \lambda x.((x\ FALSE)\ TRUE)\ FALSE$

$= (FALSE\ FALSE)\ TRUE$

$\{\beta\text{-red}\}$

$= (\lambda x.\lambda y.y\ FALSE)\ TRUE$

$\{FALSE\ \text{expan.}\}$

$= \lambda y.y\ TRUE$

$\{\beta\text{-red}\}$

$\Rightarrow$ NOT FALSE = TRUE —— (2)

From (1) & (2),

NOT(NOT TRUE) = NOT FALSE

= TRUE

Hence proved.

b) OR FALSE TRUE = TRUE

$OR = \lambda x.\lambda y.((x\ TRUE)\ y)$

$TRUE = \lambda x.\lambda y.x$

$FALSE = \lambda x.\lambda y.y$

OR FALSE TRUE $= \lambda x.\lambda y.((x\ TRUE)\ y)\ FALSE\ TRUE$

$= FALSE\ TRUE\ TRUE$

$\{\beta\text{-red}\}$

$= \lambda x.\lambda y.y\ TRUE\ TRUE$

$\{FALSE\ \text{expan}\}$

$= TRUE$ $\{\beta\text{-red}\}$

Hence Proved

Kousshik Raj
17CS30026

c) $SUCC\ 2 = 3$

$$2 = \lambda f.\ \lambda y.\ f\ (f\ y)$$
$$3 = \lambda f.\ \lambda y.\ f\ (f\ (f\ y))$$
$$SUCC = \lambda z.\ \lambda f.\ \lambda y.\ f\ (z\ f\ y)$$

$$SUCC\ 2 = (\lambda z.\ \lambda f.\ \lambda y.\ f\ (z\ f\ y))\ 2$$
$$= \lambda f.\ \lambda y.\ f\ (2\ f\ y) \quad \{\beta\text{-red}\}$$

$$= \lambda f.\ \lambda y.\ f\ ((\lambda f.\ \lambda y.\ f\ (f\ y))\ f\ y)$$
$\{$Expand 2$\}$

$$= \cancel{\lambda f.\ \lambda y.\ f\ (\lambda f.\ \lambda y.}$$

$$= \lambda f.\ \lambda y.\ f\ ((\lambda y.\ f\ (f\ y))\ y)$$
$\{\beta\text{-red}\}$

$$= \lambda f.\ \lambda y.\ f\ (f\ (f\ y))$$

$$SUCC\ 2 = 3$$

Hence proved

d) $(Y\ FACT)\ 2 = 2$

$$Y = \lambda f.(\lambda x.\ f\ (x\ x))(\lambda x.\ f\ (x\ x))$$

$FACT = \lambda f.\ \lambda n$ IF $n = 0$ THEN 1 ELSE $n * (f\ (n-1))$

$\cancel{(Y\ FACT)\ 2 =}$

Kausshik Raj
17CS30022

$(Y\ FACT)\ 2 = ((\lambda f \cdot (\lambda x \cdot f(x\ x))(\lambda x \cdot f(x\ x)))\ FACT)\ 2$

{ Expansion }

$= ((\lambda x \cdot FACT(x\ x))(\lambda x \cdot FACT(x\ x)))\ 2$

{ β-red } — ①

$= FACT((\lambda x \cdot FACT(x\ x))(\lambda x \cdot FACT(x\ x)))\ 2$

{ β-red }

$= (\lambda f \cdot \lambda n\ \text{IF } n=0 \text{ then } 1 \text{ else } n * (f(n-1)))\ ((\lambda x \cdot FACT(x\ x))\ (\lambda x \cdot FACT(x\ x)))\ 2$

Sub $f \rightarrow (\lambda x \cdot FACT(x x)(\lambda x \cdot FACT(x\ x))$

$n \rightarrow 2$

From ①

$f \rightarrow Y\ FACT$

$= 2 * ((Y\ FACT)\ 1)$

III ly

$= 2 * (1 * ((Y\ FACT)\ 0))$

$= 2 * (1 * 1)$

$= 2 * 1$

$= 2$

Hence Proved

Scanned by CamScanner

e) $exp\ \bar{0}\ \bar{n} = \lambda x . x$

Koushik Ra[illegible]
17CS3002[illegible]

$exp = \lambda m . \lambda n . (m\ n)$

$exp\ \bar{0}\ \bar{n} = (\lambda m . \lambda n . (m\ n))\ \bar{0}\ \bar{n}$

$= \bar{0}\ \bar{n}$ {β-reduction}

$= (\lambda f . \lambda x . x)\ \bar{n}$

{Expanding $\bar{0}$}

$= \lambda x . x$ {β-reduction}

f) $add\ \bar{6}\ \bar{2}$

$add = \lambda n . \lambda m . \lambda f . \lambda x . n f (m f x)$

$add\ \bar{6}\ \bar{2} = (\lambda n . \lambda m . \lambda f . \lambda x . n f (m f x))\ \bar{6}\ \bar{2}$

{Expansion}

$= \lambda f . \lambda x\ \bar{6}\ f\ (\bar{2}\ f\ x)$

{β-red}

$= \lambda f . \lambda x . (\lambda f . \lambda x\ f^6 x)\ f((\lambda f . \lambda x\ f^2 x)\ f\ x)$

{Expansion}

$= \lambda f . \lambda x . (\lambda f . \lambda x\ f^6 x)\ f\ (f^2 x)$

{β-reduction}

~~$= \lambda f . \lambda x . (\lambda f . \lambda x\ f^6 x)\ f^2$~~

$= \lambda f . \lambda x . (f^6 f^2 x)$

{β-reduction}

$= \lambda f . \lambda x . f^8 x$

$\boxed{add\ \bar{6}\ \bar{2} = \bar{8}}$

Kousshik Ray
17CS30022

g) IF FALSE THEN x ELSE y = y

IF a THEN b ELSE c = a b c

TRUE = $\lambda x . \lambda y . x$

FALSE = $\lambda x . \lambda y . y$

IF FALSE THEN x ELSE y = FALSE x y {Using def.}

$= (\lambda x . \lambda y . y)\, x\, y$ {Expand}

$= (\lambda y . y)\, y$ {β-red}

$= y$

Hence proved.

h) add & mul are associative

mul = $\lambda n . \lambda m . \lambda x\ (n\ (m\ x))$

add = $\lambda n . \lambda m . \lambda f . \lambda x .\ n\ f\ (m\ f\ x)$

i) add

T.P

add a (add b c) = add (add a, b) c

L.H.S $= (\lambda n . \lambda m . \lambda f . \lambda z\ \ n\ f\ (m\ f\ z))\ a\ (\text{add}\ b\ c)$ {Expand}

$= \lambda f . \lambda z\ \ a\ f\ ((\text{add}\ b\ c)\ f\ z)$ {β-red}

$= \lambda f . \lambda z\ \ a\ f ((\lambda g . \lambda x\ \ b\ g\ (c\ g\ x)) f\ z$ {α-conv, β-redc}

$= \lambda f . \lambda z\ \ a\ f\ (b\ f\ (c\ f\ z))$

$= \lambda f . \lambda z\ \ f^{a+b+c}\ z$

Kousshik Raj
17CS30028

IIIly R.H.S

$= (\text{add } (\text{add } a\ b)\ c)$

$= \text{add } ((\lambda n.\lambda m.\lambda f.\lambda x\ n\ f\ (m\ f\ x))\ a\ b)\ c$

$= \text{add} (\lambda f.\lambda x\ a\ f\ (b\ f\ x))\ c$

$= (\lambda n.\lambda m.\lambda g.\lambda z\ n\ g\ (m\ g\ z))$
$(\lambda f.\lambda x\ a\ f\ (b\ f\ x))\ c$

$= \lambda g.\lambda z\ (\lambda f.\lambda x\ a\ f(b\ f\ x))\ g\ (c\ g\ z)$

$= \lambda g.\lambda z\ a\ g\ (b\ g\ (c\ g\ z))$

$= \lambda g.\lambda z\ g^{a+b+c}\ z$

$= \lambda f.\lambda x\ f^{a+b+c}\ x$

$\Rightarrow$ L.H.S = R.H.S

$\therefore$ Proved

i) mul

Kaushik Roy
12CS30022

~~R.H.S~~

T.P $\text{mul } a \ (\text{mul } b \ c) = \text{mul } (\text{mul } a \ b) \ c$

R.H.S

$= \text{mul } ((\lambda n.\lambda m.\lambda x \ n \ (m \ x)) \ a \ b) \ c$

$= \text{mul } (\lambda x. \ a \ (b \ x)) \ c$ {B-red}

$= (\lambda n.\lambda m.\lambda z \ n \ (m \ z)) \ (\lambda z. \ a \ (b \ x)) \ c$ {Expansion}

$= \lambda z \ (\lambda x. \ a \ (b \ x)) \ (c \ z)$ {B-red}

$= \lambda z. \ a \ (b \ (c \ z))$ {B-red}

L.H.S

$= \text{mul } a \ ((\lambda n.\lambda m.\lambda x \ n \ (m \ x)) \ b \ c)$

$= \text{mul } a \ (\lambda x. \ b \ (c \ x))$ {B-red}

$= (\lambda n.\lambda m.\lambda z \ n \ (m \ z)) \ a \ (\lambda x. \ b \ (c \ x))$

$= \lambda z. \ a \ ((\lambda x. \ b \ (c \ x)) \ z)$ {B-red}

$= \lambda z. \ a \ (b \ (c \ z))$ {B-red}

$\Rightarrow$ L.H.S = R.H.S

$\therefore$ Proved.