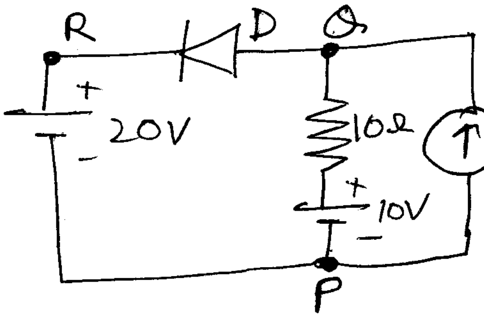
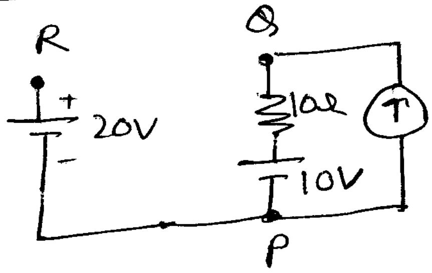
① **ckt A**

We will first solve ckt A using Thevenin's equivalance theorem. Note that ckt A has only one non-linear branch QR. The rest of the ckt is linear. Therefore we will treat the branch QR as external to the ckt. And find the ~~th~~ Thevenin equivalance ckt for the rest of the ckt.

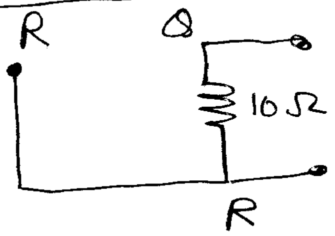


Thevenin Voltage (open ckt voltage) between Q, R



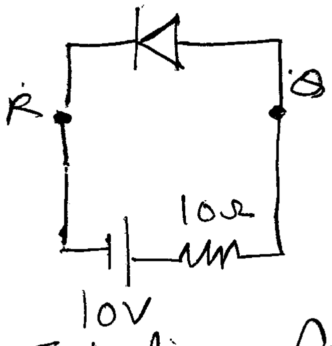
$$V_{QR} = 10\Omega \times 2A + 10V - 20V$$
$$= 20V + 10V - 20V$$
$$= 10V$$
$$\therefore V_{th} = 10V$$

Thevenin resistance between Q, R



Clearly $R_{th} = 10\Omega$

∴ Thevenin's equivalent ckt

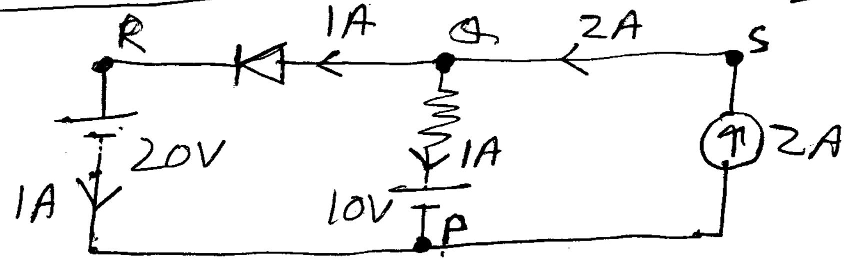


∴ Current through the diode

$$= \frac{10V}{10\Omega} = 1A \quad (\text{From Q to R})$$

> **Notation:**
> V_{XY} = Voltage drop from X to Y

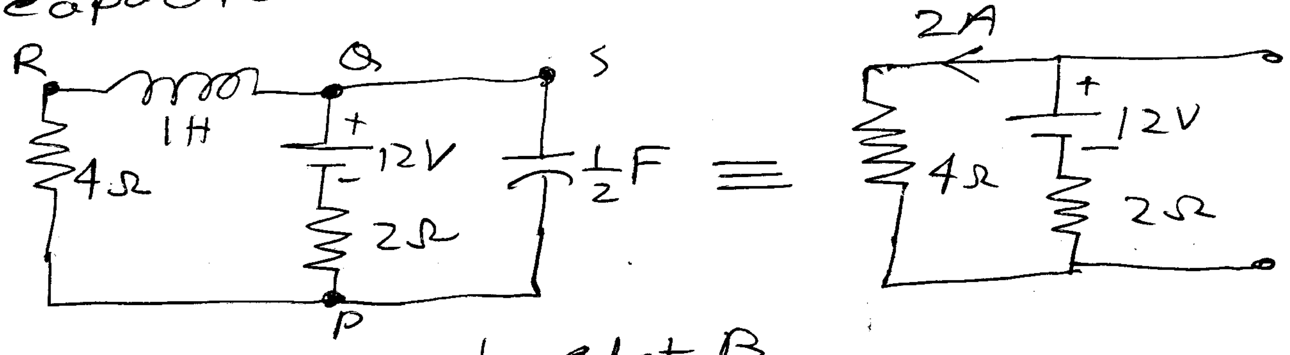
∴ Solution for ckt A



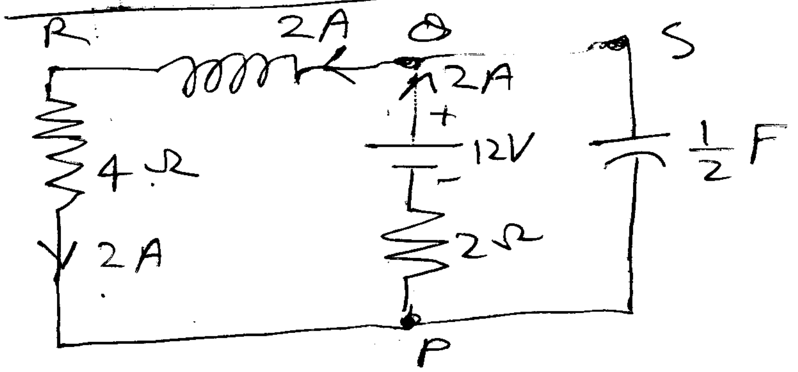
$V_{QR} = 0$
$V_{RP} = 20V$
$V_{QP} = 20V$
$V_{QSP} = 20V$

ckt B

Since the ckt has only DC sources ~~theref~~ therefore at steady state the inductor behaves like a short circuit & the capacitor as open circuit



∴ Solution of ckt B.



$V_{QR} = 0V$

$V_{RP} = 8V$

$V_{QP} = (12-4)V = 8V$

$V_{QSP} = 8V$

VERIFICATION OF TELLEGN'S THEOREM

(i) Using Voltages from ckt A & currents from ckt B

$$V^A_{QR} I^B_{QR} + V^A_{RP} I^B_{RP} + V^A_{QP} I^B_{QP} + V^A_{QSP} I^B_{QSP}$$

(superscripts denote which ckt the quantities are taken from)

$$= (0 \times 2 + 20 \times 2 + 20 \times (-2) + 20 \times 0)\ VA$$

$= 0$ (Tellegen's theorem obeyed)

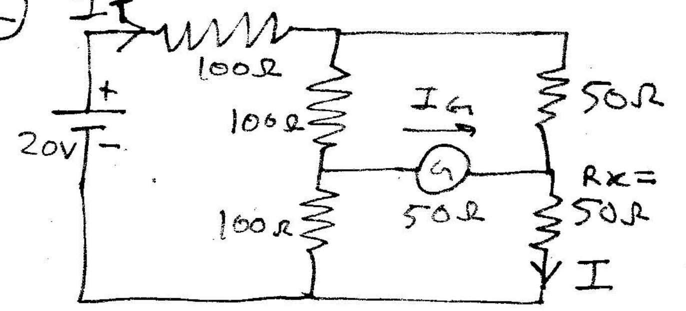
(ii) Using Voltages from ckt B & currents from ckt A

$$V^B_{QR} I^A_{QR} + V^B_{RP} I^A_{RP} + V^B_{QP} I^A_{QP} + V^B_{QSP} I^A_{QSP}$$

$$(0 \times 1 \quad + 8 \times 1 \quad + 8 \times 1 \quad 8 \times (-2))\ VA$$

$= 0$ (Tellegen's theorem obeyed.

② Nominal Circuit



Let the current through Rx be = I. Lets find I. One can find I using mesh, nodal analysis or thevenin theorem etc. But observe that I_G will be zero because G is ~~between~~ in a balanced Wheatstone bridge condition. So we can compute I easily

$$I = I_T \times \frac{200}{200+100}$$

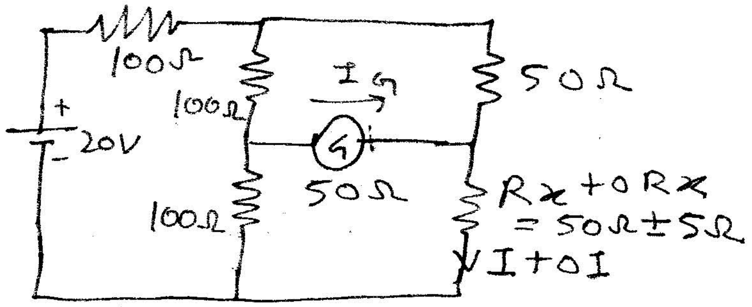
$$= \frac{20}{100 + (200 \| 100)} \times \frac{2}{3} A$$

$$= \frac{20}{100 + \frac{200}{3}} \times \frac{2}{3} A$$

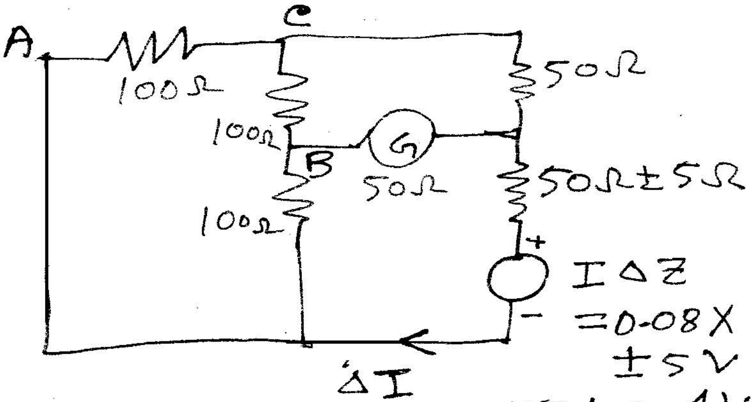
$$= \frac{40}{500} = \frac{4}{50} = 0.08 A$$

$$= 0.08 A$$

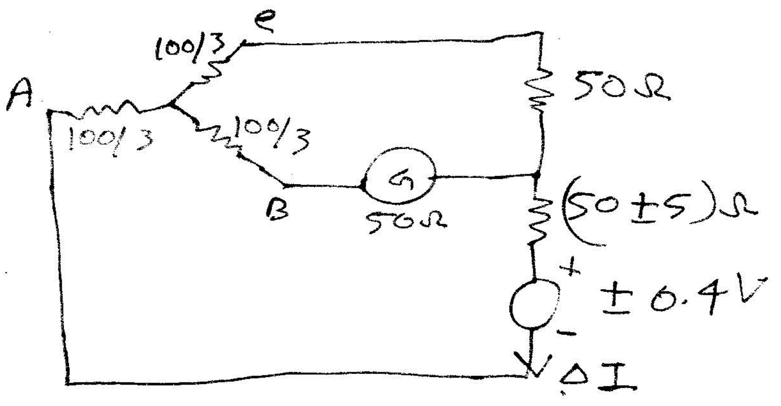
Circuit with ±10% = ±5Ω tolerance for Rx



Lets compute ΔI in this circuit using compensation theorem



Apply Δ-Y conversion for ΔABC



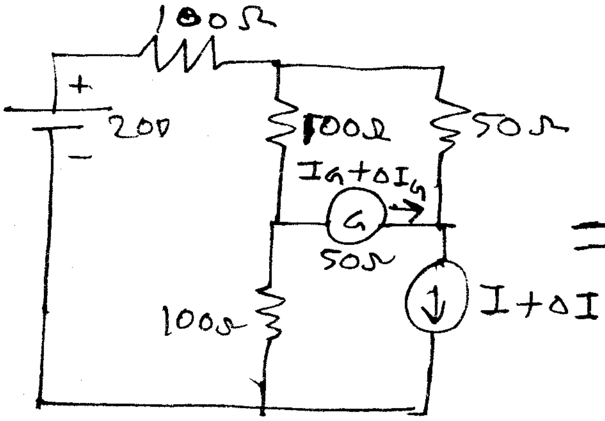
$$-\Delta I = \frac{\pm 0.4}{\frac{100}{3} + \frac{\frac{100}{3}+50}{2} + 50 \pm 5} A$$

$$= \frac{\pm 0.4}{\frac{100}{3} + \frac{250}{6} + 50 \pm 5} A$$

$$= \frac{\pm 0.4}{125 \pm 5} A = \frac{.4}{130} \text{ or } \frac{-.4}{120} A$$

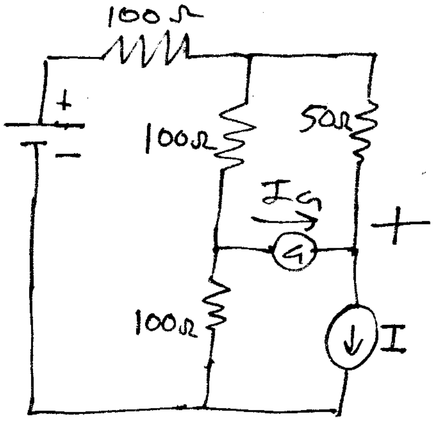
$$\therefore \Delta I = -.4/130 \text{ or } .4/120 A$$

Now apply substitution theorem to the circuit with 10% tolerance to get $\boxed{\text{ckt A}}$



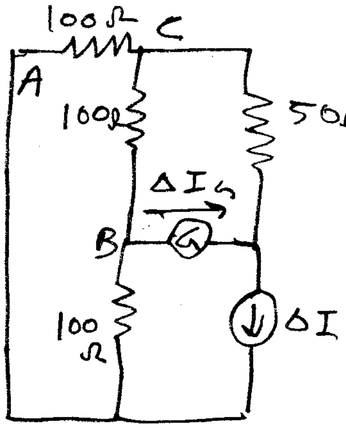
ckt A

=



ckt B

+



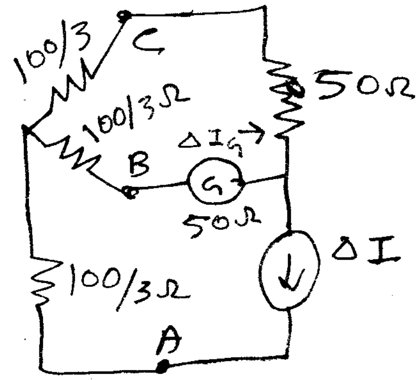
ckt C

$\boxed{\text{ckt A}}$ is broken into $\boxed{\text{ckt B}}$ + $\boxed{\text{ckt C}}$ using superposition theorem.

Now $\boxed{\text{ckt B}}$ is nothing but the nominal ckt with R_x substituted with a current source. Therefore, in $\boxed{\text{ckt B}}$ galvanometer current will be equal to the nominal current I_G. Therefore, the galvanometer current in $\boxed{\text{ckt C}}$ will be same as the error current ΔI_G.

So Let us solve $\boxed{\text{ckt C}}$ to get ΔI_G.

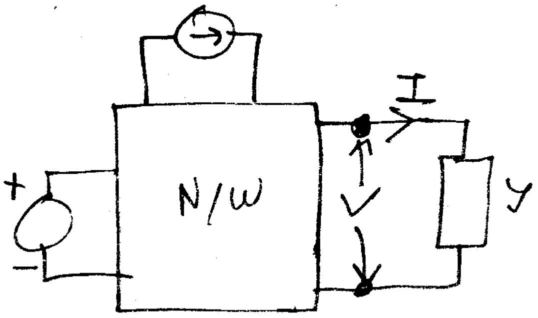
Apply Δ-Y conversion on ΔABC of $\boxed{\text{ckt C}}$

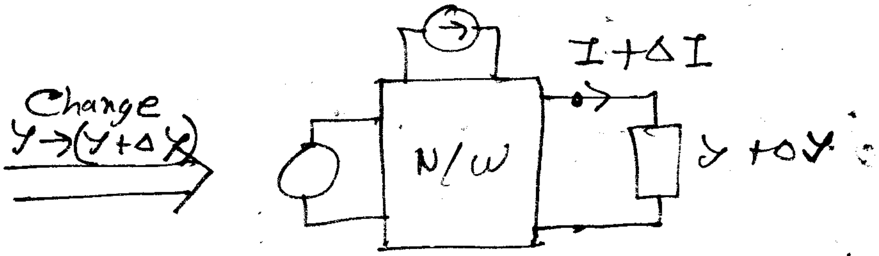


Clearly from current ~~divid~~ divider rule $\Delta I_G = \frac{1}{2} \Delta I$

$\therefore \Delta I_G = \frac{0.4}{130}$ or $+\frac{0.4}{120}$ A.

3) Relevant theory





Y is the admitance

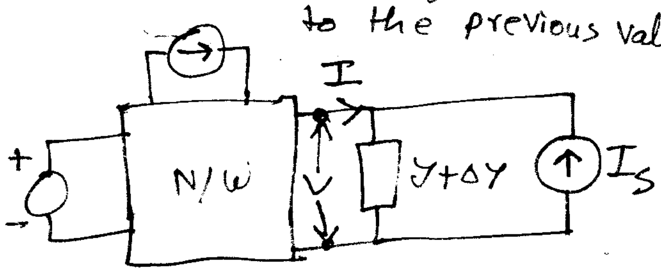
$Y = \frac{1}{Z}$

CKT 1

The current is changed from I to I+ΔI

CKT 2

Add a compensating current source I_s to bring the current to the previous value



CKT 3

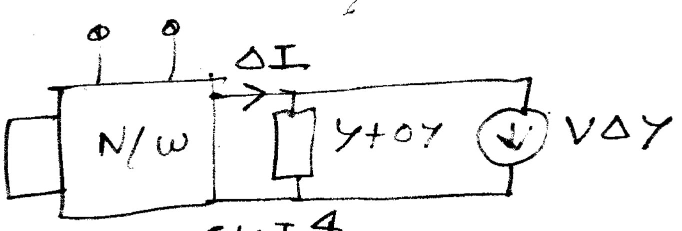
In CKT 3 we want to have same voltage V and current I as in CKT 1. But under a increased admitance ΔY, the Voltage V ~~should~~ would cause VΔY amount of more current to be drawn. I choose $I_s = V\Delta I$ so that the extra current drawn by the load comes exactly from I_s, which compensates the circuit.

$\therefore\ I_s = V\Delta Y$

Now,

I_s and N/W together supplies		current	I
(−) N/W alone	,,	,,	I+ΔI
	,,	,,	−ΔI
$\therefore I_s$,,			
$\Rightarrow (-)I_s$,,	,,	,,	+ΔI

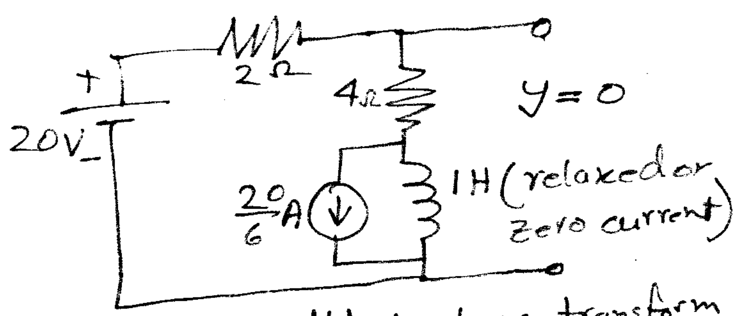
$\therefore$



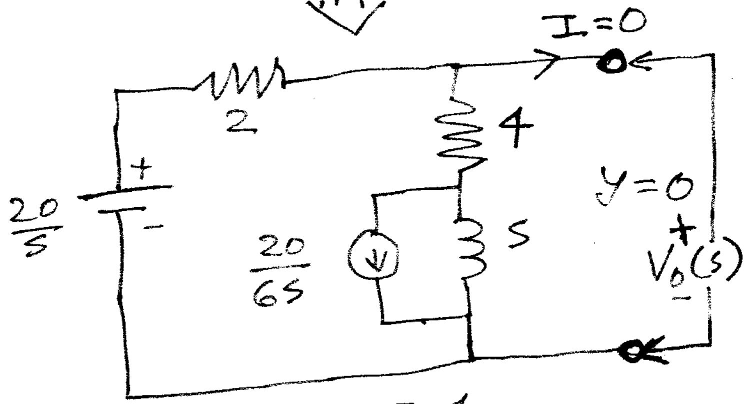
CKT 4

To find ΔI we can use ck4.. where the N/ω is inactive & a compensating ~~sot~~ current source VΔY is connected as shown.

Before switching ON

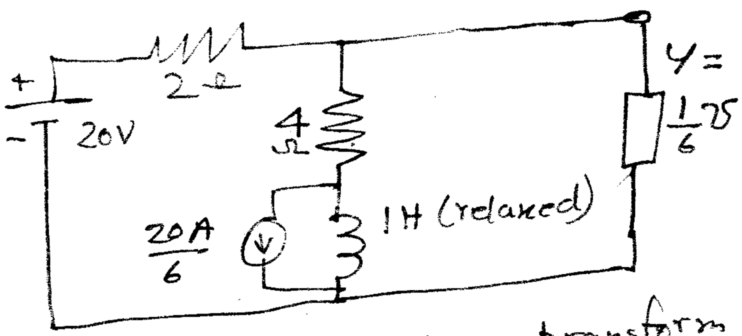


⇓ Laplace transform

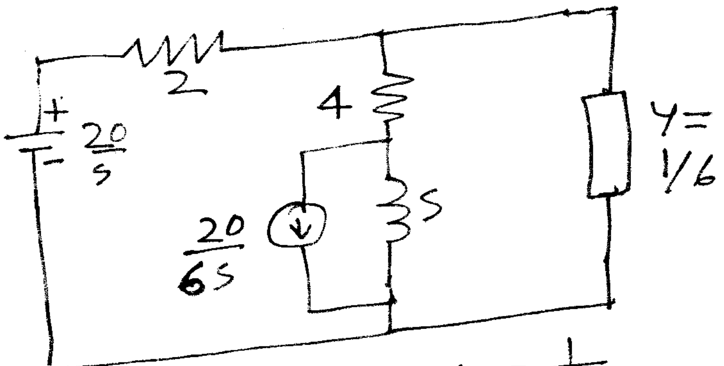


CKT 1

After switching ON



⇓ Laplace transform



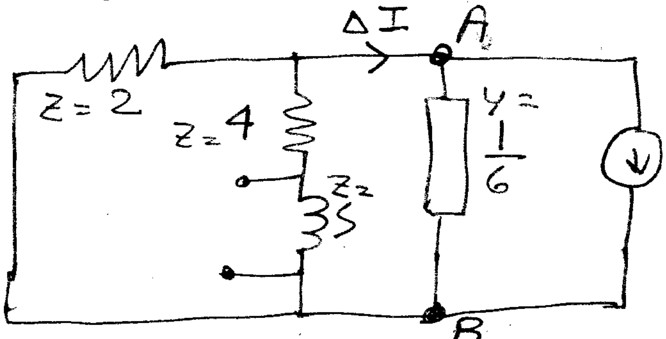
$\Delta Y = \frac{1}{6} - 0 = \frac{1}{6}$

~~Using compensation theorem we get.~~

Lets compute $V_o(s)$ first (see CKT 1)

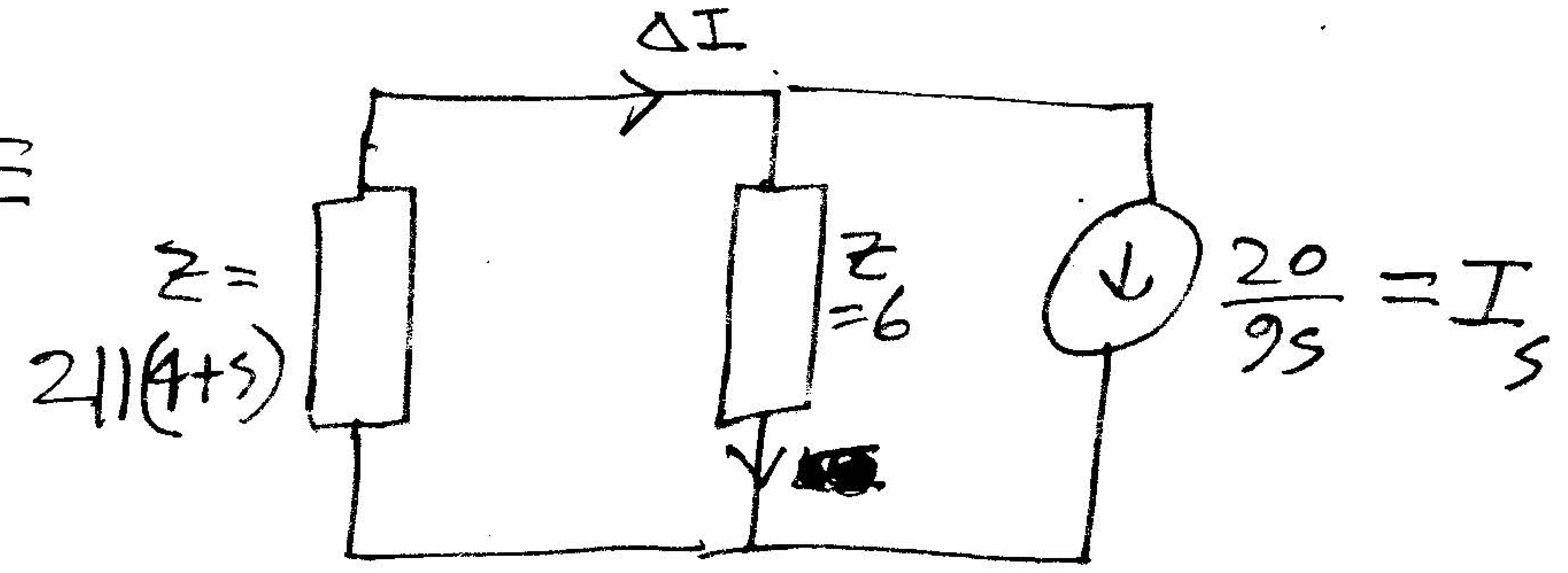
clearly $V_o(s) = \frac{20}{6s} \times 4$.

$\therefore$ Using compensation theorem we can draw the following ckt



$V_o(s)\Delta Y = \frac{20 \times 4}{6s \times 6} = \frac{20}{9s}$

$\equiv$



$$\Delta I = I_s \times \frac{6}{6+\left(2 \| (4+s)\right)} = \frac{20}{9s} \times \frac{6}{6+\frac{8+2s}{6+s}}$$

$$= \frac{20}{9s} \times \frac{6(6+s)}{44+8s} = \frac{20}{3s} \times \frac{6+s}{4s+22} = \frac{10}{3s} \times \frac{6+s}{2s+11}$$

$$= \frac{10}{3}\left(\frac{6}{s} + \frac{-1}{2s+11}\right)\frac{1}{11}$$

$$\therefore i(t) = \mathcal{L}^{-1}\{\Delta I\} = \frac{10}{33}\left[6u(t) - \frac{1}{2}e^{-11/2\,t}u(t)\right]$$

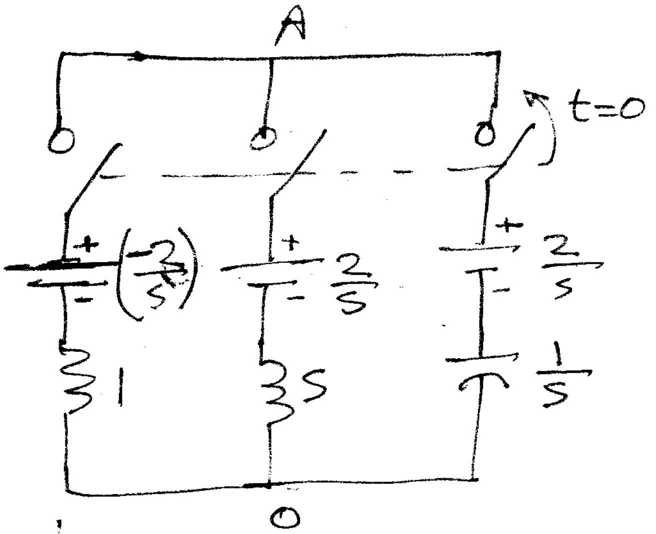
$$= \left(\frac{20}{11} - \frac{5}{33}e^{-\frac{11t}{2}}\right)u(t)$$

<u>Verification</u>: ① $\lim_{t\to\infty} i(t) = \frac{20}{11}$. From D.E. circuit analysis (by shorting the inductor) we also get $i(t) = \frac{20}{11}$ at steady state.

② Time constant: $\frac{L}{R} = \frac{1}{4+2\|6} = \frac{1}{4+\frac{12}{8}} = \frac{8}{44} = \frac{2}{11}$

This also matches with our answer.

(4) Zero initial condition.



Using Millman's theorem

$V_{AO}(s)$

$$= \frac{\left(\frac{-2}{s}\right)\times 1 + \left(\frac{2}{s}\right)\times\frac{1}{s} + \left(\frac{2}{s}\right)\times s}{1+\frac{1}{s}+s}$$

$$= \frac{\frac{2}{s}\left(s+\frac{1}{s}-1\right)}{\left(s+\frac{1}{s}+1\right)} = \frac{\frac{2}{s}(s^2-s+1)}{(s^2+s+1)}$$

$$\therefore V_{AO}(s) = \frac{2(s^2-s+1)}{s(s^2+s+1)} = \frac{2(s^2+s+1-2s)}{s(s^2+s+1)}$$

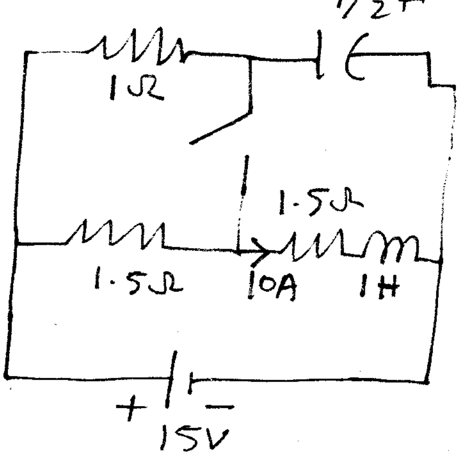
$$= 2\left[\frac{1}{s} - \frac{2}{s^2+s+1}\right]$$

$$= 2\left[\frac{1}{s} - \frac{2}{\left(s+\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}\right]$$

$$\therefore V_{AO}(t) = 2\left[u(t) - 2\times\frac{2}{\sqrt{3}}\sin\left(\frac{\sqrt{3}}{2}t\right)e^{-\frac{1}{2}t}u(t)\right]$$

$$= 2\left(1-\frac{4}{\sqrt{3}}\sin\left(\frac{\sqrt{3}}{2}t\right)e^{-t/2}\right)u(t)$$

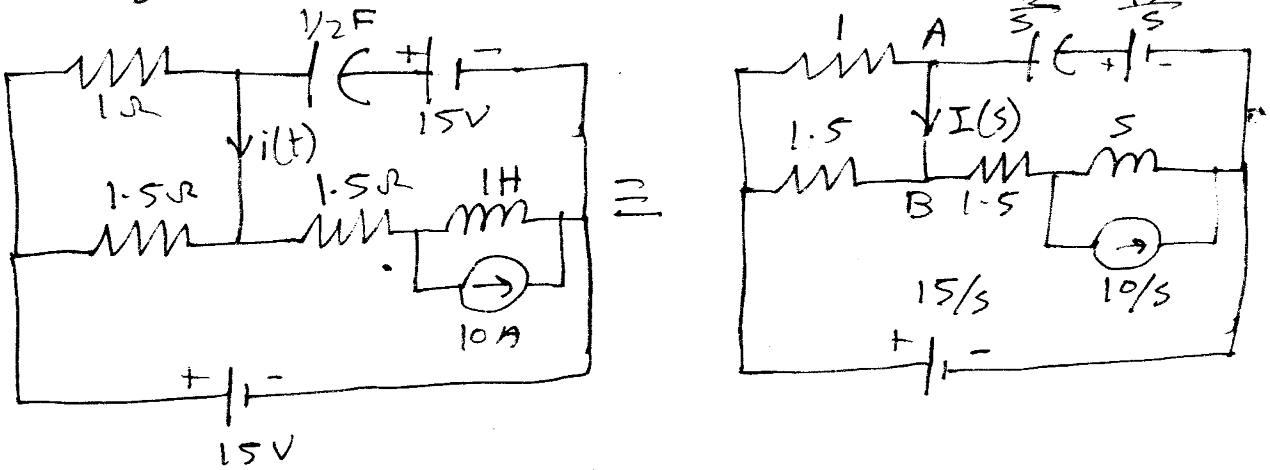
(5)



Before the switch is closed at steady state the capacitor behaves like a open circuit and the inductor as short ckt. Therefore, s.s current through the inductor $= \frac{15}{2 \times 1.5} A = 5 A = 5A$ (left to right)

The capacitor voltage = 15V

∴ Equivalent ckt after $t \geq 0$



a) Thevenin Theorem

Thevenin impedance between A and B

$$= \left(1.5 \,||\, (1.5+s)\right) + \left(1 \,||\, \left(\frac{2}{s}\right)\right)$$

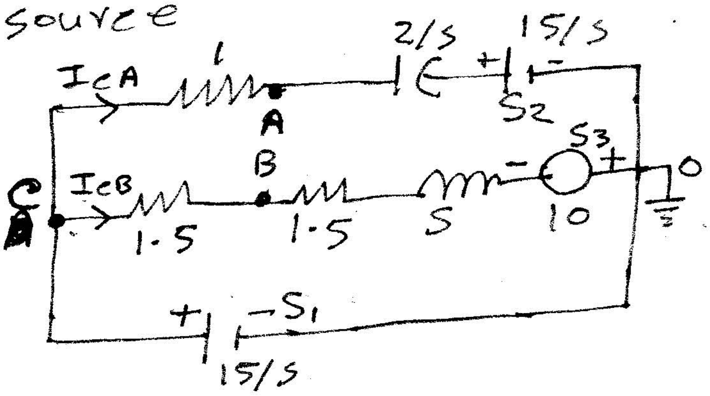
$$= \frac{2.25+1.5s}{3+s} + \frac{\frac{2}{s}}{1+\frac{2}{s}} = \frac{2.25+1.5s}{3+s} + \frac{2}{2+s}$$

$$= \frac{4.5+3s+2.25s+1.5s^2+6+2s}{(3+s)(2+s)}$$

$$= \frac{1.5s^2+7.25s+10.5}{(3+s)(2+s)} = Z_{th} \text{ (say)}$$

Thevenin voltage between A and B

Lets convert the current source into a voltage source



Clearly $V_c = \frac{15}{s}$

$\therefore V_{AB} = V_A - V_B = -I_{cA} \times 1 + I_{cB} \times 1{\cdot}5$

$= -\frac{\left(\frac{15}{s} - \frac{15}{s}\right)}{1+\frac{2}{s}} \times 1 + \frac{\frac{15}{s}+10}{3+s} \times 1{\cdot}5$

$= 0 + \frac{(3+2s)\,7{\cdot}5}{s(s+3)} = 7{\cdot}5\left(\frac{1}{s+3} + \frac{1}{s}\right)$

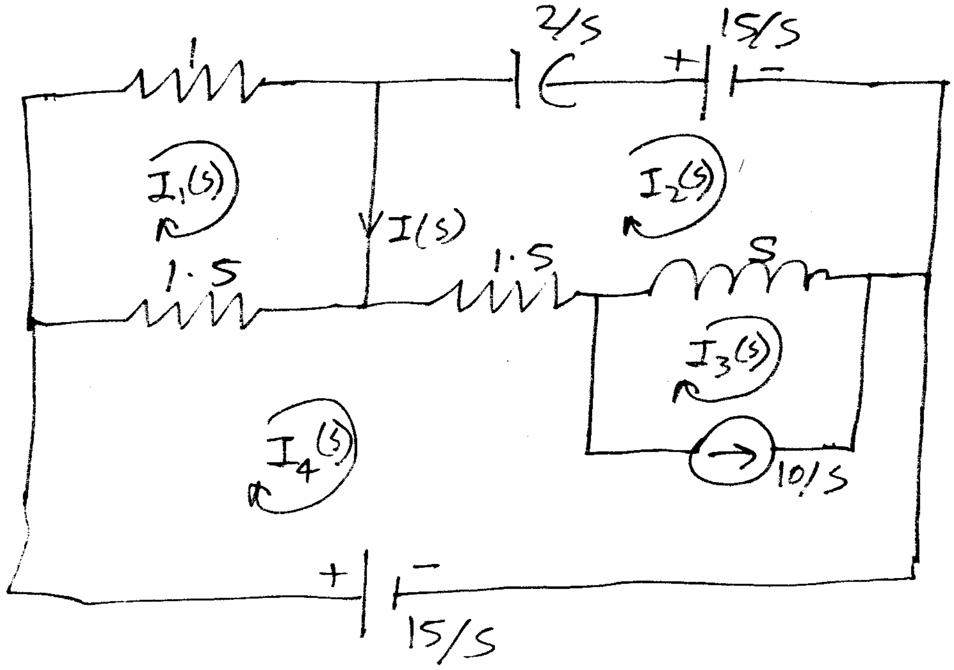
$\therefore V_{th}(s) = 7{\cdot}5\left(\frac{1}{s+3} + \frac{1}{s}\right) = \frac{7{\cdot}5\,(2s+3)}{s(s+3)}$

$\therefore I(s) = \frac{V_{th}(s)}{Z_{th}+0} = \frac{7{\cdot}5(2s+3)}{s\cancel{(s+3)}} \times \frac{\cancel{(3+s)}(2+s)}{1{\cdot}5s^2 + 7{\cdot}25s + 10{\cdot}5}$

$= \frac{750\,(2s+3)(2+s)}{s(150s^2 + 725s + 1050)}$

$= \frac{30\,(2s+3)(s+2)}{s(6s^2 + 29s + 42)}$

(b) Mesh analysis



KVL on loop 1: $2.5\, I_1(s) = 1.5\, I_4(s)$

$$\Rightarrow I_4(s) = \frac{5}{3} I_1(s) \quad \text{---- (i)}$$

Current source between loop 3 an4 :

$$I_4(s) - I_3(s) = \frac{10}{s}$$

$$\Rightarrow \frac{5}{3} I_1(s) - I_3(s) = \frac{10}{s} \quad \text{(using (i))}$$

$$\Rightarrow I_3(s) = \frac{5}{3} I_1(s) - \frac{10}{s} \quad \text{---- (ii)}$$

KVL on loop 2 :

$$I_2(s)\left(\frac{2}{s} + s + 1.5\right) - I_3(s)\, s - I_4(s)\, 1.5 = -\frac{15}{s}$$

$$\Rightarrow I_2(s)\left(\frac{s^2+1.5s+2}{s}\right) - \frac{5s}{3} I_1(s) + 10 - \frac{5}{2} I_1(s) = -\frac{15}{s}$$

$$\Rightarrow I_2(s)\left(\frac{s^2+1.5s+2}{s}\right) - I_1(s)\left(\frac{10s+15}{6}\right) = -\frac{15}{s} - 10 = -\frac{15+10s}{s}$$

$$\Rightarrow I_1(s) = \left(I_2(s)\left(\frac{s^2+1.5s+2}{s}\right) + \frac{15+10s}{s}\right)\frac{\times 6}{(10s+15)} \quad \text{---- (iii)}$$

KVL on combined loop 3 & 4:

$$\frac{15}{s} = -1.5I_1(s) - I_2(s)(s+1.5) + I_3(s)s + I_4(s)(1.5+1.5)$$

$$= -1.5I_1(s) - I_2(s)(s+1.5) + \left(\frac{5}{3}I_1(s) - \frac{10}{s}\right)s + \frac{5}{\cancel{3}}I_1(s) \times \cancel{3}$$

$$= 3.5I_1(s) + \frac{5s}{3}I_1(s) - I_2(s)(s+1.5) - 10$$

$$\Rightarrow I_1(s)\left(\frac{10.5+5s}{3}\right) - I_2(s)(s+1.5) = \frac{15+10s}{s}$$

using the value of $I_1(s)$ from (iii)

$$\Rightarrow \frac{(10.5+5s)}{\cancel{3}} \times \left(I_2(s)\frac{(s^2+1.5s+2)}{s} + \frac{15+10s}{s}\right) \times \frac{\cancel{6}\,2}{(10s+15)}$$
$$- I_2(s)(s+1.5) = \frac{10s+15}{s}$$

$$\Rightarrow \frac{2(10.5+5s)}{(15+10s)}\left(I_2(s)\frac{(s^2+1.5s+2)}{s}\right) - I_2(s)(s+1.5)$$
$$+ \frac{2(10.5+5s)}{s} = \frac{10s+15}{s}$$

$$\Rightarrow I_2(s)\left(\frac{21+10s}{15+10s} \times \frac{s^2+1.5s+2}{s} - \frac{s^2+1.5s}{s}\right) = \frac{-6}{s}$$

$$\Rightarrow I_2(s)\left(\frac{(21+10s)(s^2+1.5s)+2(21+10s)-(s^2+1.5s)(15+10s)}{15+10s}\right) = -6$$

$$\Rightarrow I_2(s)\left((s^2+1.5s)6+42+20s\right) = -90-60s$$

$$\Rightarrow I_2(s) = \frac{-30(3+2s)}{6s^2+29s+42} \quad \text{------ (iv)}$$

Now from (iii)

$$I_1(s) = I_2(s)\frac{(s^2+1.5s+2)}{s}\frac{6}{(15+10s)} + \frac{6}{s}$$

Now $I(s) = I_1(s) - I_2(s)$

$$= I_2(s)\left(\frac{6(s^2+1.5s+2)}{(15+10s)s} - 1\right) + \frac{6}{s}$$

$$= \frac{-30\cancel{(3+2s)}}{6s^2+29s+42}\left(\frac{6s^2+9s+12-10s^2-15s}{s\,\cancel{(15+10s)}\,5}\right) + \frac{6}{s}$$

[using equation (iv)]

$$= \frac{-30}{6s^2+29s+42}\frac{(-4s^2-6s+12)}{5s} + \frac{30}{5s}$$

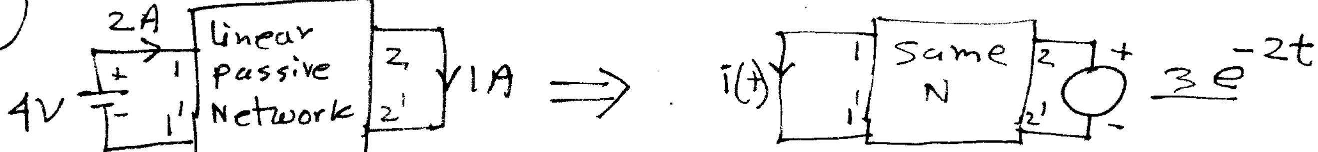
$$= \frac{30}{6s^2+29s+42}\left[\frac{4s^2+6s-12}{5s} + \frac{6s^2+29s+42}{5s}\right]$$

$$= \frac{30}{6s^2+29s+42}\left[\frac{10s^2+35s+30}{5s}\right]$$

$$= \frac{30(2s+3)(s+2)}{(6s^2+29s+42)s}$$

Now $i(t)$ can be computed by taking a Laplace inverse of $I(s)$ using partial fraction method which is left for the students.

(6)



ckT 1 ckT 2

- ~~D~~ using compensation theorem, $i(t)$ in ckT2 is given by $\frac{3}{4}e^{-2t}$.

- From ckT 1, the impedance as viewed from 1-1' port (i.e input impedance of 1-1') is

$$Z_{th} = \frac{4V}{2A} = 2\Omega$$

- The short circuit current in 1-1' of ckT2

$$I_{sc} = \frac{3}{4}e^{-2t}\ A$$

- The impedance of 1-1' of ckT2 is same as the impedance of 1-1' of ckT 1 $= Z_{th} = 2\Omega$

- Now if we replace the short ckt ~~at~~ at 1-1' in ckT 2 with a 4Ω resistance then the current in 1-1' of ckT can be found using Norton's theorem

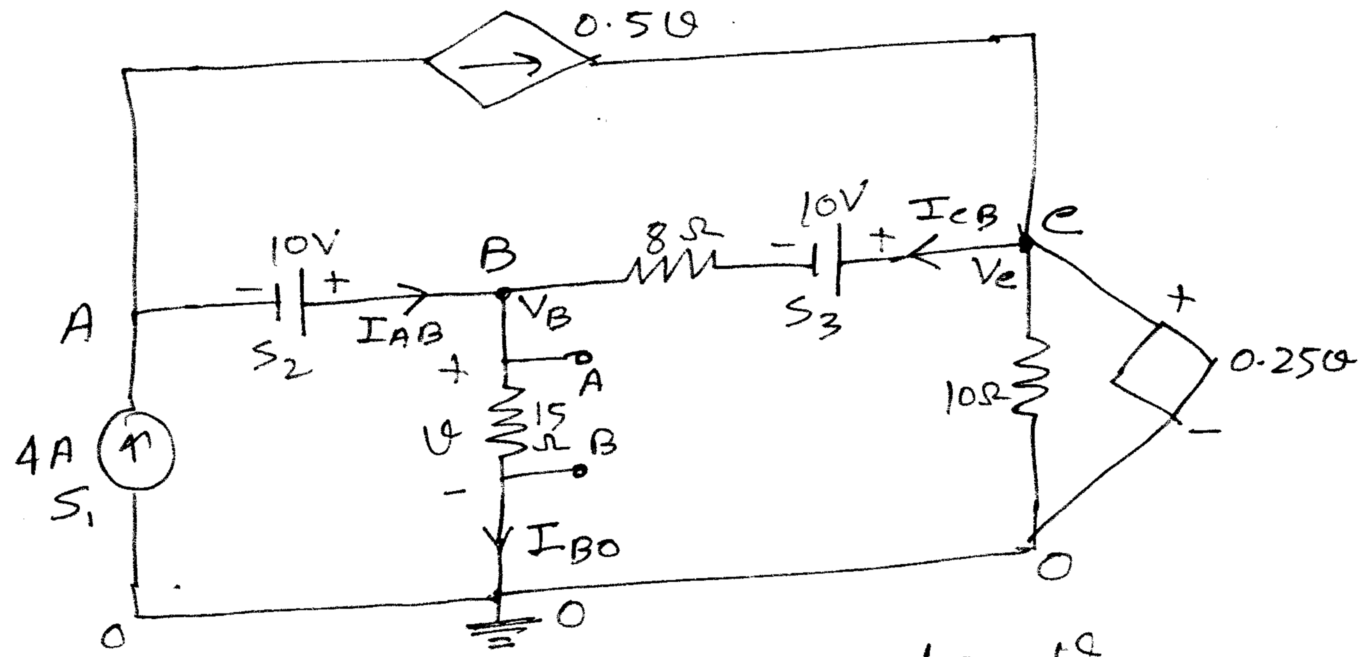
$$i = I_{sc} \times \frac{Z_{th}}{Z_{th} + 4\Omega} = \frac{3}{4}e^{-2t} \times \frac{2}{2+4}\ A$$

$$= \frac{1}{4}e^{-2t}\ A.$$

<u>Note</u>

In this problem we have implicitely assumed that the network is instantaneou ~~oo~~ i.e. has no transient behaviour ~~ororob~~ i.e. resistive in nature.

(7)



(a) Superposition theorem to compute υ

Step 1: S_1 Active, S_2 & S_3 short circuited

$I_{AB} = 4 - 0.5\upsilon$

$V_e = 0.25\upsilon$ and $V_B = \upsilon$

$\therefore I_{cB} = \frac{V_e - V_B}{8} = \frac{0.25\upsilon - \upsilon}{8} = -\frac{3}{32}\upsilon$

$\therefore I_{BO} = I_{AB} + I_{cB} = 4 - 0.5\upsilon - \frac{3}{32}\upsilon$

$\therefore V_B = \upsilon = 15 \times I_{BO} = 15\left(4 - \frac{1}{2}\upsilon - \frac{3}{32}\upsilon\right)$

$= 60 - \left(\frac{15\times16}{32} + \frac{45}{32}\right)\upsilon$

$\Rightarrow \upsilon\left(1 + \frac{15\times16}{32} + \frac{45}{32}\right) = 60$

$\Rightarrow \upsilon = \frac{60\times32}{317}$

(b) Step 2: S_1 opened, S_2 active, S_3 shorted

$I_{AB} = -\frac{1}{2}\upsilon$

$I_{cB} = -\frac{3}{32}\upsilon$ (as before)

$\therefore \upsilon = 15 \times I_{BO} = 15(I_{AB} + I_{cB}) = 15\left(-\frac{1}{2} - \frac{3}{32}\right)\upsilon$

$\Rightarrow v = 0$

step 3 : S_1 open, S_2 shorted, S_3 active

$\therefore I_{AB} = -\frac{1}{2} v$

from branch BC

$v + 8 I_{CB} + 10 = \frac{v}{4}$

$\Rightarrow 8 I_{CB} = -10 - \frac{3}{4} v$

$\Rightarrow I_{CB} = -\frac{1}{8}\left(10 + \frac{3}{4} v\right)$

$\therefore V_{BO} = v = 15 \times I_{BO} = 15(I_{AB} + I_{CB})$

$= 15\left(-\frac{1}{2} v - \frac{10}{8} - \frac{3}{32} v\right)$

$= 15\left(-\frac{10}{8} - \frac{19}{32} v\right)$

$\Rightarrow v\left(1 + \frac{15 \times 19}{32}\right) = \frac{-150}{8}$

$\Rightarrow v\left(\frac{317}{32}\right) = \frac{-150}{8} \Rightarrow v = \frac{-150}{\cancel{8}} \times \frac{\cancel{32}\,4}{317} = \frac{-600}{317}$

$\therefore$ By adding the results for individual sources (superposition theorem)

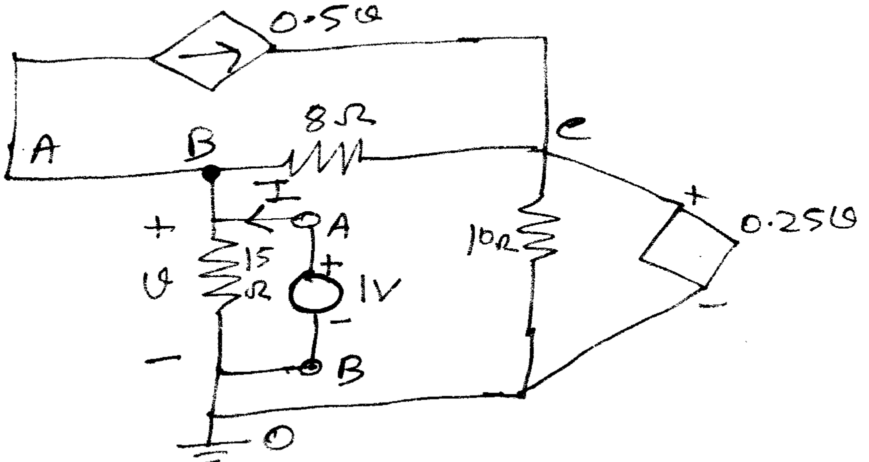
$v = \frac{60 \times 32}{317} + 0 - \frac{600}{317} = \frac{60 \times 22}{317} = \frac{1320}{317}$

$\therefore V_{AB} = v = \frac{1320}{317}$

(b) Thevenin voltage as computed in part (a)

$= \frac{1320}{317}$ V

To get thevenin impedance we connect a 1V source between A and B & deactivate all the internal independent sources



Clearly now

$v = 1$

we need to find the current I

Applying KCL at node B

$$I - 0.5v - (v - 0.25v)/8 - \frac{v}{15} = 0$$

$$\Rightarrow I = \frac{v}{15} + \frac{v}{2} + \frac{3}{32}v = \frac{1}{15} + \frac{1}{2} + \frac{3}{32} \quad [\because v = 1]$$

$$= \frac{32 + 16\times15 + 3\times15}{32\times15} = \frac{32 + 240 + 45}{480}$$

$$= \frac{317}{480}$$

$$\therefore Z_{th} = \frac{1}{I} = \frac{480}{317}$$

$\therefore$ Thevenin equv. ckt between AB is