Problem Set - 9 (Hint & Solution)

MATHEMATICS-I (MA10001) *AUTUMN 2017*

1. Hint:-Check whether the given differential equations is a Cauchy-Euler form, if not, then transform to the Cauchy- Euler form, then put $x = e^z$ for questions $1(a)$ to $1(k)$ and the solutions are given below, where a, b and c are arbitrary constants:
   (a) $y(x) = \sqrt{x}[a\cos(\sqrt{3}/2\ln x) + b\sin(\sqrt{3}/2\ln x)] + x^2$,
   (b) $y(x) = ax^2 + bx^{-1} + 1/3\ln x(x^2 - x^{-1})$,
   (c) $y(x) = a + (b + c\ln x)x + x/2(\ln x)^2$,
   (d) $y(x) = a + bx^{-1} + 1/2(\ln x)^2 - \ln x$,
   (e) $y(x) = ax^{-1} + \sqrt{x}[b\cos(\sqrt{3}/2\ln x) + c\sin(\sqrt{3}/2\ln x)] + x/2 + \ln x$,
   (f) $y(x) = ax + bx^{5/3} + 1/34(\sin(\ln x) + 4\cos(\ln x))$,
   (g) $y(x) = ax + bx^2 - x\ln x + x^3/2 + x^2/2((\ln x)^2 - 2\ln x)$,
   (h) $y(x) = (a + b\ln x)\cos\ln x + (c + d\ln x)\sin\ln x + (\ln x)^2 + 2\ln x - 3$,
   (i) $y(x) = x[a\cos(\sqrt{3}\ln x) + b\sin(\sqrt{3}\ln x)] + 1/13[3\cos(\ln x) - 2\sin(\ln x)] + x/2\sin\ln x$,
   (j) $y(x) = x[a + b\cos(\ln x) + c\sin(\ln x)] + x^2/2(\ln x - 2) + 3x\ln x$,
   (k) $y(x) = a + b\ln x + 2(\ln x)^3$.

2. Hint:- put $ax + b = e^z$ for questions $2(a)$ to $2(c)$
   (a) $a = 1,\quad b = 1,\quad y(x) = a\cos\ln(1 + x) + b\sin\ln(1 + x) + 2\ln(1 + x)\sin\ln(1 + x)$,
   (b) $a = 1,\quad b = 3,\quad y(x) = a(x + 3)^2 + b(x + 3)^3 + (x + 2)/2$,
   (c) $a = 3,\quad b = 1,\quad y(x) = a(3x + 2)^2 + b(3x + 2)^{-2} + 1/108[(3x + 2)^2\ln(3x + 2) + 1]$.

3. Hint:-Find C.F. and P.I. of the differential equations. General solution = C.F.+ P.I., where a, b and c are arbitrary constants:
   (a) $y(x) = a\cos 2x + b\sin 2x - \cos 2x\ln(\sec 2x + \tan 2x)$,
   (b) $y(x) = ae^x + be^x - e^x - xe^{2x} + e^x\ln(e^{-x} + 1) + e^{2x}\ln(1 + e^x)$,
   (c) $y(x) = ax + bx^2 - \dfrac{x}{2}(\ln x)^2 - x(1 + \ln x)$,
   (d) $y(x) = \dfrac{a}{x} + \dfrac{b}{x}\ln x + \dfrac{\ln\{x(1 - x)\}}{x}$,
   (e) $y(x) = ae^x + be^{-2x} + \dfrac{x}{2} - \dfrac{3}{4} + \dfrac{3\sin x + \cos x}{10}$,
   (f) $y(x) = e^{-x}(a\cos x + b\sin x) + \dfrac{e^{-x}\cos 2x\ln(\cos 2x)}{4} + \dfrac{e^{-x}\sin 2x}{2}$,
   (g) $y(x) = a + b\cos x + c\sin x + \ln(\sec x + \tan x) - x\cos x + \sin x\ln(\cos x)$,
   (h) $y(x) = ae^x + be^{2x} + ce^{3x} - xe^{2x}$.

4. $y(x) = e^x(2 - x\sin x - 2\cos x)$.

5. Here a, b and c are arbitrary constants :

(a) $x = e^{3t}[a\cosh(t\sqrt{10}) + b\sinh(t\sqrt{10})], y = \dfrac{\sqrt{10}}{2}e^{3t}[a\cosh(t\sqrt{10}) + b\sinh(t\sqrt{10})]$,

(b) $y = (a+bt)e^{-4t} + \dfrac{1}{25}e^{t} + \dfrac{7}{36}e^{2t}, x = -(a+bt)e^{-4t} + be^{-4t}\dfrac{4}{25}e^{t} - \dfrac{1}{36}e^{2t}$,

(c) $x = ae^{-2t} + be^{-7t} + \dfrac{5}{14}t - \dfrac{31}{196} - \dfrac{1}{8}e^{t}, y = \dfrac{1}{3}[-2ae^{-2t} + 3be^{-7t} + \dfrac{5}{8}e^{t} + \dfrac{27}{98} - \dfrac{3}{7}t]$,

(d) $x = ae^{t} + be^{-5t} + \dfrac{3}{7}e^{2t} - \dfrac{1}{25}(10t+13), y = ae^{t} - be^{-5t} + \dfrac{4}{7}e^{2t} - \dfrac{3}{5}t - \dfrac{12}{25}$,

(e) $x = ae^{\sqrt{2}\ t} + be^{-\sqrt{2}\ t} + 3\cos t, y = (1+\sqrt{2})ae^{\sqrt{2}\ t} + (1-\sqrt{2})be^{\sqrt{2}\ t} + 2\sin t$.