Problem Set - 7

AUTUMN 2017

MATHEMATICS - I(MA10001)

August , 2017

1. **Find the order and degree of the following differential equation:**

   (i) $\sqrt{y + \left(\frac{dy}{dx}\right)^2} = 1 + x$

   (ii) $\left(\frac{d^3y}{dx^3}\right)^3 + \left(\frac{d^2y}{dx^2}\right)^2 + \left(\frac{dy}{dx}\right)^4 = 0$

   (iii) $y^2\left(\frac{d^2y}{dx^2}\right)^2 + 2y\left(\frac{dy}{dx}\right) + x = 0$

   (iv) $\left(\frac{d^2y}{dx^2}\right) = 5\left(1 + \left(\frac{dy}{dx}\right)^2\right)^{3/2}$

   (v) $\sqrt{1 - x^2}\ dx + dy = 0$

2. **Form the ODE by eliminating the arbitrary constants:**

   (i) $\big(y - B\big)^2 = 4a(x - A)$

   (ii) $y = \big(A + Bx\big)e^{2x}$

   (iii) $y = A\cos\ x + B\cosh\ x$

   (iv) $e^{2y} + 2kxe^y + k^2 = 0$

   (v) Find the differential euation of all circle touching the y axis at the origin.

   (vi) Find the differential equation of all parabolas whose axis are parallel to y axis.

3. **Solve the following Initial Value Problems:**

   (i) $\frac{dy}{dx} + 2y(\tan\ x) = \sin\ x \quad ; \quad y(\frac{\pi}{3}) = 0$

   (ii) $(x^2 + y^2 + 2x)dx + 2ydy = 0 \quad ; \quad y(1) = 0$

4. **Check if the differential equations are homogeneous (reduced it to homogeneous if not), then solve it:**

   (i) $x\ \cos\ \frac{y}{x}(y\ dx + x\ dy) = y\ \sin\ \frac{y}{x}(x\ dy - y\ dx)$

   (ii) $y^3\frac{dy}{dx} + x + y^2 = 0$

   (iii) $y(2xy + 1)dx + x(1 + 2xy + x^2y^2)dy = 0$

5. **Check if the differential equations are exact (if not, reduced it to exact using proper Integrating Factor), then solve it:**

   (i) $(a^2 - 2xy - y^2)dx - (x + y)^2dy = 0$

   (ii) $(4x^3y)dx + (x^4 + y^4)dy = 0$

   (iii) $(x^2y - 2xy^2)dx + (3x^2y - x^3)dy = 0$

(iv) $(xy + 2x^2y^2)ydx + (xy - x^2y^2)xdy = 0$

(v) $(x^2 + y^2 + 2x)dx + 2ydy = 0$

(vi) $(3x^2y^4 + 2xy)dx + (2x^3y^3 - x^2)dy = 0$

6. **Solve the following ODEs by reducing them to linear differential equations:**

(i) $\cos^2 x \ \frac{dy}{dx} + y = \ \tan\ x$

(ii) $\frac{dy}{dx} + \frac{2}{x}y = \ 3x^2y^{4/3}$

(iii) $\frac{dy}{dx} + y\frac{df}{dx} = f(x)\ \frac{df}{dx}$

(iv) $\frac{dy}{dx} + \frac{1}{x}\ \tan\ y = \frac{1}{x^2}\ \tan\ y\ \sin\ y$

(v) $xy - \frac{dy}{dx} = y^3e^{-x^2}$

(vi) $\sec^2 y\ \frac{dy}{dx} + 2x\ \tan\ y = x^3$

(vii) $\frac{dy}{dx} + \frac{y}{x}\ \ln\ y = \frac{y}{x^2}(\ln\ y)^2$