Problem Set - 11

AUTUMN 2017

MATHEMATICS-I (MA10001)

1. (a) Evaluate $\int_\Gamma (\bar{z}^2 dz + z^2 d\bar{z})$, where $\Gamma :\ z^2 + 2z\bar{z} + \bar{z}^2 = (2-2i)z + (2+2i)\bar{z}$ joins the points $z = 1$ and $z = 2 + 2i$.

   (b) Compute $I = \int_\Gamma |z|^2 dz$, where

      (i) $\Gamma = t + it^3$, $0 \le t \le 1$.

      (ii) Γ is the straight line segment from 0 to $a + ib$.(a,b $\in \mathbb{R}$).

   (c) Evaluate $\int_C (x^2 - iy^2) dz$ along

      (i) the parabola $y = 2x^2$ from $(1,1)$ to $(2,8)$.

      (ii) the straight lines from $(1,1)$ to $(1,8)$ and then from $(1,8)$ to $(2,8)$.

      (iii) the straight line from $(1,1)$ to $(2,8)$.

2. (a) Find an upper bound of $\int_C \frac{1}{(z^4+1)^2} dz$, where C is the upper half circle $|z| = a$, $a > 1$,traversed once in the counter clock-wise direction.

   (b) Show that $|\int_C \frac{1}{z^2} dz| \le 2$, where C is the straight line joining the points i and $2 + i$.

   (c) Evaluate $\int_\Gamma \bar{z} dz$, where Γ is the upper half of the circle $|z| = 1$ from $z = -1$ to $z = 1$.

3. (a) Evaluate $\int_C \frac{z+4}{z^2+2z+5} dz$, where C is the circle $|z+1|$=1.

   (b) Let C be a circle centered at $4 + i$ with radius 1. Without any calculation explain why $\int_c \frac{1}{z^2+2z+5} dz = 0$.

4. (a) Evaluate $\int_C \frac{z}{(9-z^2)(z+i)} dz$, where C is the circle $|z| = 2$.

   (b) Evaluate $\int_C \frac{e^z + z^3}{z-1} dz$, where C is the circle $|z| = 2$.

   (c) Evaluate $\int_C \frac{z}{z^2+1} dz$, where C is the path

      (i) $C :\ |z - i| = \frac{1}{2}$

      (ii) $C :\ |z| = \frac{1}{2}$.

   (d) Evaluate $\int_C \frac{\cos z}{z(z^2+8)} dz$ over the square with vertices at (1,1),(-1,1),(-1,-1),(1,-1).

   (e) Evaluate $\int_C \frac{e^{3z}}{z^2+4} dz$ over the contour C, where C:$|z| = 4$.

(f) Evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$, where C is the circle $|z-i| = 3$.

(g) Show that $\frac{1}{2\pi i} \int_C \frac{e^{zt}}{z^2+1} dz = \sin t$ if t>0, where C:$|z| = 3$.

5. (a) Evaluate $\int_{|z|=1} \frac{z+3}{z^4 + az^3} dz$, $(|a| > 1)$.

   (b) Evaluate $\int_C \frac{e^{2z}}{(z+1)^4} dz$, where C is the circle $|z| = 3$.

6. Evaluate $\frac{1}{2\pi i} \int_C \frac{e^z}{z-2} dz$, where C is

   (a) the circle $|z| = 3$.

   (b) the circle $|z| = 1$.

7. Find the Taylor series expansions of the following functions

   (a) $f(z) = \frac{z}{z^4 + 9}$ about $z = 0$

   (b) $f(z) = \log(1+z)$ about $z = 0$

   (c) $f(z) = \frac{z-1}{z+1}$ about $z = 0$ and $z = 1$

   (d) $f(z) = \sin z$ about $z = \frac{\pi}{4}$.

8. Obtain the first three non zero terms in Taylor's series expansion of the following function about $z = 0$

   (i) $\frac{1}{2+e^z}$

   (ii) $e^{\frac{z}{\cos z}}$