CS41001 : Theory of Computation
Autumn 2020

Assignment 3

Due Date: 8^{th} October, 2020, 23:59 hrs

Guidlines: Each question carries **5 marks.** *Do not search for solutions online.*

1. Assume that Turing Machines are encoded by strings over some alphabet Σ, and that $\# \notin \Sigma$. Consider the following language over the alphabet $\Sigma \cup \{\#\}$:

   $L' = \{M_1 \# M_2 \# M_3 | M_1, M_2, M_3 \text{ are Turing Machines with } L(M_1) \cap L(M_2) = L(M_3)\}$

   Prove that $\neg L'$ is not recursively enumerable.

2. A CFG is said to be unambiguous if each sentence has a unique left derivation (similarly unique right derivation). Show that it is undecidable whether a context-free grammar is ambiguous. (Hint: PCP is undecidable).

3. Prove the following extension of Rice's theorem (of which part I is a special case): Every non-trivial property of pairs of r.e. sets is undecidable.

   More formally, let $P : \{\text{r.e. sets}\} \times \{\text{r.e. sets}\} \rightarrow \{T, F\}$ be a non-trivial property on pairs of r.e. sets.
   Then show that $T_P = \{(M, N) | M \text{ and } N \text{ are TMs and } P(L(M), L(N)) = T\}$ is undecidable.