7/2/20

Assignment - 2

Kausshik Raj
17CS30022

1. a) $(\lambda z.\ z)(\lambda z.\ z\ z)(\lambda z.\ z\ y)$

$= ((\lambda z.\ z)(\lambda z.\ z\ z))(\lambda z.\ z\ y)$

$\Rightarrow_\beta (\lambda z.\ z\ z)(\lambda z.\ z\ y)$

$\Rightarrow_\beta (\lambda z.\ z\ y)(\lambda z.\ z\ y)$

$\Rightarrow_\beta (\lambda z.\ z\ y)\ y$

$\Rightarrow_\beta (y\ y)$

b) $(\lambda x.\ \lambda y.\ x\ y\ y)(\lambda a.\ a)\ b$

$\Rightarrow_\beta (\lambda y.((\lambda a.\ a)\ y\ y))\ b$

$\Rightarrow_\beta (\lambda y.\ y\ y)\ b$

$\Rightarrow_\beta (b\ b)$

c) $(\lambda x.\ \lambda y.\ x\ y\ y)(\lambda y.\ y)\ y$

$\Rightarrow_\alpha (\lambda x.\ \lambda z.\ x\ z\ z)(\lambda a.\ a)\ y$

$\Rightarrow_\beta (\lambda z.((\lambda a.\ a)\ z\ z))\ y$

$\Rightarrow_\beta (\lambda z.\ z\ z)\ y$

$\Rightarrow_\beta (y\ y)$

Kaushik Roy
17CS30025

d) $(\lambda x \cdot x\ x)(\lambda y \cdot y\ z)\ z$

$\Rightarrow_\beta ((\lambda y \cdot y\ x)(\lambda y \cdot y\ x))\ z$

$\Rightarrow_\beta ((\lambda y \cdot y\ x)\ x)\ z$

$\Rightarrow_\beta ((z\ x)\ z)$

e) $(\lambda x \cdot (\lambda y \cdot (x\ y))\ y)\ z$

$= (\lambda x \cdot ((\lambda y \cdot (x\ y))\ y))\ z$

$\Rightarrow_\beta (\lambda x \cdot (x\ y))\ z$

$\Rightarrow_\beta (z\ y)$

f) $((\lambda x \cdot \lambda y \cdot x\ y)(\lambda y \cdot y))\ w)$

$\Rightarrow_\beta ((\lambda x \cdot \lambda y \cdot x\ y)(\lambda a \cdot a))\ w)$

$\Rightarrow_\beta ((\lambda y \cdot ((\lambda a \cdot a)\ y))\ w)$

$\Rightarrow_\beta ((\lambda y \cdot y)\ w)$

$\Rightarrow_\beta w$

Koustuk Ray
17CS30022

2 a)

$Y = \lambda t.(\lambda x.\, t\,(x\,x))(\lambda x.\, t\,(x\,x))$

{Y-combinator}

Let $T = \lambda f.\lambda x.\lambda y.\ if\ (=x\ 0)\ y\ (f\ (-x\ 1)\ (+y\ 1))$

add = Y T

b) add 25 = (Y T) 25

= (T (Y T)) 2 5 {property of Y-combinator}

$\Rightarrow_\beta (\lambda x.\lambda y.\ if\ (=x\ 0)\ y\ ((YT)\ (-x\ 1)\ (+y\ 1)))\ 2\ 5$

{Expand T & β-red}

$\Rightarrow_\beta if\ (=2\ 0)\ 5\ ((YT)\ (-2\ 1)\ (+5\ 1))$

{Sub x & y}

$\Rightarrow (YT)\ (-2\ 1)\ (+5\ 1)$ {if statement}

$\Rightarrow_\delta (YT)\ 1\ 6$ {δ-reduction}

$\Rightarrow (T\ (YT))\ 1\ 6$ {property of Y-combinator}

$\Rightarrow_\beta (\lambda x.\lambda y.\ if\ (=x\ 0)\ y\ ((YT)\ (-x\ 1)\ (+y\ 1)))\ 1\ 6$

{Expand T & β-red}

$\Rightarrow_\beta if\ (=1\ 0)\ 6\ ((YT)\ (-1\ 1)\ (+6\ 1))$

{Sub x & y}

$\Rightarrow YT\ (-1\ 1)\ (+6\ 1)$ {Property of if}

$\Rightarrow_\delta YT\ 0\ 7$ {δ-reduction}

$\Rightarrow (T\ (Y\ T))\ 0\ 7$ {Property of Y-combinator}

$\Rightarrow_\beta ((\lambda x.\ \lambda y.\ \text{if}\ (=x\ 0)\ y\ (Y\ T)\ (-x\ 1)\ (+y\ 1)))\ 0\ 7$ {Expand T & β-red}

$\Rightarrow_\beta \text{if}\ (=0\ 0)\ 7\ ((Y\ T)\ (-x\ 1)\ (+y\ 1))$ {β-red of x & y}

$\Rightarrow 7$ {Property of if}

add 2 5 = 7

Kausshik Raj
17CS30022