CS41001 : Theory of Computation
Autumn 2020

Test 2

6^{th} October, 2020

11:00 am - 11:45 am

1. Prove that given a CFG G, the following problems are undecidable.

   (a) Determine whether $L(G)$ contains a string of the form ww. (Hint: Consider $L(G_1)$ and $L(G_2)$, When will the language $L(G_1)L(G_2)$ have a string of the form ww?) **[5]**

   (b) Determine whether $L(G) = L(G)^{rev}$ **[5]**

2. (a) Given a polynomial $p()$, a TM M is said to run in $p()$-time if for any $n \geq 0$, and any n-length input string, M halts in at most $p(n)$ steps. Prove that $L(M)$ is a recursive set. **[1]**

   (b) Show that for any constant $c > 0$, there exists an n_c such that for all $n \geq n_c$, $cn \leq n^2$. **[1]**

   (c) A language L is said to be computable in linear time if $L = L(M)$ for a deterministic Turing Machine M where there is a constant $c > 0$ such that for any n, and an input string of length n, M halts in at most cn steps. Prove that there is a recursive set R that is not computable in linear time. **[3]**