TOC - TEST 2

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1 a) ~~To show whether~~

Let G, G_1, G_2 be grammars such that

$L(G) = L(G_1) L(G_2)$

- To show whether string of form $ww \in L(G)$ is decidable, it is enough if we show that $L(G_1) \cap L(G_2) = \phi$ is decidable or not

- We show that $L(G_1) \cap L(G_2) = \phi$ is undecidable by a reduction from PCP (which we know is undecidable).

Reduction:-

Let $\alpha_1, \alpha_2 \ldots \alpha_m$ & $\beta_1, \beta_2 \ldots \beta_m$ be an instance of PCP over Σ. Let $\Sigma' = \Sigma \cup (\sigma_1, \sigma_2 .. \sigma_m)$

Define 2 CFGs G_1, G_2, such that

i) $G_1 = (\{S_1\}, \Sigma', P_1, S_1)$

$P_1 = \{ S_1 \to \alpha_i S_1 \sigma_i \mid \alpha_i \sigma_i \ \forall 1 \le i \le m \}$

ii) $G_2 = (\{S_2\}, \Sigma', P_2, S_2)$

$P_2 = \{ S_2 \to \beta_i S_2 \sigma_i \mid \beta_i \sigma_i \ \forall 1 \le i \le m \}$

Reduction Validity:-

- If the PCP has a solution sequence $i_1, i_2 \ldots i_k$, let $x = \alpha_{i_1} \alpha_{i_2} \ldots \alpha_{i_k} = \beta_{i_1} \beta_{i_2} \ldots \beta_{i_k}$. Then, from the reduction rules we can see that $x \sigma_{i_k} \ldots \sigma_{i_1} \in L(G_1)$ & $x \sigma_{i_k} \ldots \sigma_{i_1} \in L(G_2)$

Thus $L(G_1) \cap L(G_2) \neq \phi$.

- Now if $L(G_1) \cap L(G_2) \neq \phi$. By a similar backward trace of the previous argument, we arrive at a solution sequence for the PCP from the common string x belonging to $L(G_1) \cap L(G_2)$.

Thus $L(G_1) \cap L(G_2) \stackrel{?}{=} \phi$ is undecidable, and by extension whether $ww \in L(G)$ is undecidable.

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b) ~~Given ... instan~~

We will show this as a reduction from $\overline{HP}$.

<u>Reduction:-</u>

Given instance $\langle M, x \rangle$ of $\overline{HP}$, define G such that $L(G) = \overline{VALCOMPS_{M,x}}$.

- Let Σ be the alphabet of $VALCOMPS_{M,x}$.

<u>Reduction Validity :-</u>

- If $\langle M, x \rangle \in \overline{HP}$ $\Rightarrow VALCOMPS_{M,x} = \phi$
$\Rightarrow \overline{VALCOMPS_{M,x}} = \Sigma^*$
$\Rightarrow L(G) = L(G)^R = \Sigma^*$

- If $\langle M, x \rangle \notin \overline{HP}$ $\Rightarrow \overline{VALCOMPS_{M,x}} \neq \Sigma^*$
$\exists \alpha \in VALCOMPS_{M,x}$
but $\alpha^R = \alpha \notin L(G)$ since starting & accept states are distinct.
$\Rightarrow L(G) \neq L(G)^R$.

Thus, $L(G) \stackrel{?}{=} L(G)^R$ is undecidable.

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2. a) ~~Let~~ Since n is finite, $p(n)$ is also finite

Since TM M halts in atmost $p(n)$ steps, we can say it takes finite time for completion.

$\Rightarrow$ Accept or reject is determined in finite time

$\Rightarrow$ $L(M)$ is recursive

b) $n^2 \geq nc$

$\Rightarrow n(n-c) \geq 0$

$\Rightarrow$ For, $\forall n_c \geq c$, this condition is satisfied

Thus $n^2 \geq nc$, holds true $\forall n \geq c$.

$\Rightarrow \exists n_c$ s.t $n \geq n_c$ & $n^2 \geq nc$.

c) ~~Ackerman set is~~

Ackerman function is total recursive but not computable in linear time.