Problems on Maxwell's equations and Electromagnetic Waves

1. Determine charge density $\rho(r)$ in a spherically symmetric region for which electric field is $\vec{E}(\vec{r}) = ar^2\hat{r}$. $[\ \vec{\nabla}\cdot\vec{A} = \frac{1}{r^2}\frac{\partial^2}{\partial r^2}(r^2A_r)$ given for any vector $\vec{A}.]$

2. Determine current density $\vec{J}$ which produces magnetic filed $\vec{B} = a\sin(by)e^{bx}\hat{k}$, where a and b are constants.

3. Determine emf induced in a square loop with sides of length a lying in Y-Z plane in a region in which magnetic field changes over time as $\vec{B}(t) = B_0e^{-5t/t_0}\hat{i}$. Determine the induced electric field $\vec{E}$ in the region. Check the validity of Stoke's theorm of Curl.

4. Find displacement current density $\vec{J_d}$ in a R-C circuit, if the charge in the capacitor is given bt $Q(t) = Q_0e^{-t/(RC)}$.

5. Consider an electro-magnetic wave in free space with electric field $\vec{E} = E_0\cos(3\times 10^8t - \frac{x}{\sqrt{3}} - \frac{y}{\sqrt{3}} - \frac{z}{\sqrt{3}} + \pi/2)(\hat{i} - \hat{j})$. Determine wave-length and frequency of the wave. What is the direction of propgation of the wave? Find the corresponding magnetic field?

6. The magnetic field component of a plane wave in a non-magnetic dielecltric medium is given by $\vec{H} = 20\cos(10^8\pi t - 5x)\hat{k}$. Determine wave-length and wave-velocity of the wave. Find corresponding electric field $\vec{E}$ in the medium.