DATE

EXPERIMENT – 5

SHEET NO.

PART-1

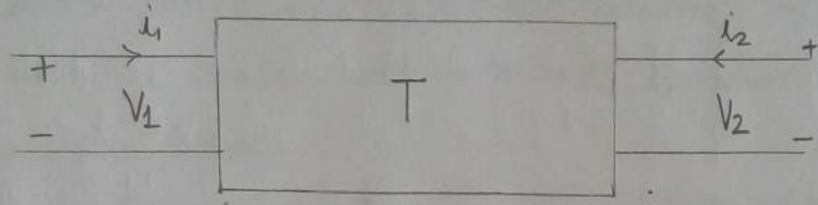
OBJECTIVE:-

To determine Y, Z and ABCD parameters of single and cascaded two Port networks experimentally and verify their interrelation ships.

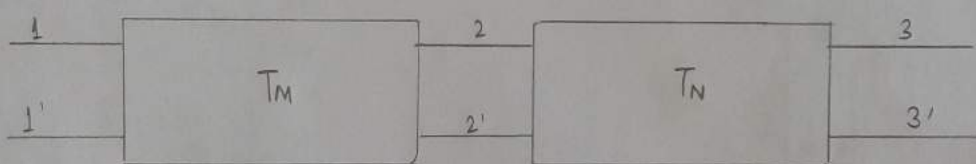
APPARATUS REQUIRED:-

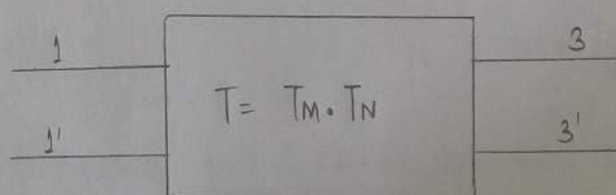
S.No.	APPARATUS NAME	QUANTITY
1.	Two Port Networks	2 (M, N)
2.	Multimeter	1
3.	12V DC Supply	1
4.	Connecting wires	-

CIRCUIT DIAGRAMS:-



$T = T_M$ or $T = T_N$.





THEORY:-

Consider a passive 2-port (4 Terminal) network. The voltages V_1, V_2 and currents I_1, I_2 can be related in terms of Z parameters & Y parameters as shown below:-

$$\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} \qquad \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$$

where $Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$ $\qquad Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$

$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$ $\qquad Y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$

$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$ $\qquad Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$

$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$ $\qquad Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$

Voltages & currents of port 1 can be represented in terms of those of port 2 as follows:-

$$\begin{pmatrix} V_1 \\ I_1 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} V_2 \\ -I_2 \end{pmatrix} = T \begin{pmatrix} V_2 \\ -I_2 \end{pmatrix}$$

$A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$ $\qquad C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$

$-B = \left.\frac{V_1}{I_2}\right|_{V_2=0}$ $\qquad -D = \left.\frac{I_1}{I_2}\right|_{V_2=0}$

OBSERVATION TABLE

1-1' = Input   2-2' = Output.

Port	Conditions	V_1 (in V)	I_1 (in mA)	V_2 (in V)	I_2 (in mA)
M	O/P open	11.97	0.28	7.39	0
	O/P short	11.98	0.46	0	29
	I/P Open	7.41	0	11.98	0.28
	I/P ~~ess~~ short	0	28	11.95	0.46
N	O/P open	11.97	0.33	4.013	0
	O/P short	11.96	0.38	0	12
	I/P open	3.97	0	11.93	0.33
	I/P short	0	13	11.97	0.37
MN	O/P open	11.97	0.35	1.43	0
	O/P short	11.98	0.36	0	4
	I/P Open	1.41	0	11.98	0.35
	I/P Short	0	4	11.98	0.35

CALCULATIONS :-

	M	N	MN
$Z_{11} = \frac{V_1}{I_1}\Big\vert_{I_2=0}$	42.75kΩ	36.273kΩ	34.2kΩ
$Z_{12} = \frac{V_1}{I_2}\Big\vert_{I_1=0}$	26.464kΩ	12.03kΩ	4.028kΩ
$Z_{21} = \frac{V_2}{I_1}\Big\vert_{I_2=0}$	26.393kΩ	12.16kΩ	4.086kΩ
$Z_{22} = \frac{V_2}{I_2}\Big\vert_{I_1=0}$	42.786kΩ	36.152kΩ	34.228kΩ
$Y_{11} = \frac{I_1}{V_1}\Big\vert_{V_2=0}$	3.839×10^{-5} S	3.178×10^{-5} S	3.008×10^{-5} S
$Y_{12} = \frac{I_1}{V_2}\Big\vert_{V_1=0}$	2.339 mS	1.086 mS	0.334 mS
$Y_{21} = \frac{I_2}{V_1}\Big\vert_{V_2=0}$	2.421 mS	1.002 mS	0.334 mS
$Y_{22} = \frac{I_2}{V_2}\Big\vert_{V_1=0}$	3.849×10^{-5} S	3.091×10^{-5} S	2.921×10^{-5} S
$A = \frac{V_1}{V_2}\Big\vert_{I_2=0}$	1.6197	2.983	8.371
$B = \frac{-V_1}{I_2}\Big\vert_{V_2=0}$	- 413.103	-996.67	- 2995
$C = \frac{I_1}{V_2}\Big\vert_{I_2=0}$	3.789×10^{-5}	8.223×10^{-5}	2.448×10^{-4}
$D = -\frac{I_1}{I_2}\Big\vert_{V_2=0}$	- 0.0159	- 0.03167	- 0.09

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REPORT :-

1. Theoritical relationships between Y, Z and ABCD parameters :-

$[V]=[Z][I]$ and $[I]=[Y][V]$

$\to [Z]^{-1}[V]=[I]$ $\quad$ $\to [Y]^{-1}[I]=[V]$

$\Downarrow$ $\quad$ $\Downarrow$

$[Y]=[Z]^{-1}$ and $[Z]=[Y]^{-1}$

$[Z]=\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$

$[Y]=\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$

$\Rightarrow \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \frac{1}{|Z|}\begin{bmatrix} Z_{22} & -Z_{12} \\ -Z_{21} & Z_{11} \end{bmatrix}$ $\quad$ $\Rightarrow \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \frac{1}{|Y|}\begin{bmatrix} Y_{22} & -Y_{12} \\ -Y_{21} & Y_{11} \end{bmatrix}$

$$* \ \underline{A} = \left.\frac{V_1}{V_2}\right|_{I_2=0} = \frac{\left.\frac{V_1}{I_1}\right|_{I_2=0}}{\left.\frac{V_2}{I_1}\right|_{I_2=0}} = \frac{Z_{11}}{Z_{21}} = \frac{-Y_{22}}{Y_{21}}$$

Similarly, theoritically we get :-

$$B = \frac{-1}{Y_{21}} = \frac{|Z|}{Z_{21}} \qquad C = \frac{1}{Z_{21}} = \frac{-|Y|}{Y_{21}} \qquad D = \frac{-Y_{11}}{Y_{21}} = \frac{Z_{22}}{Z_{21}}$$

<u>For Port M :-</u>

$|Z| = |Z_{11}Z_{22} - Z_{12}Z_{21}| = 1130.637\times10^{3}$.

$|Y| = |Y_{11}Y_{22} - Y_{12}Y_{21}| = 9.66\times10^{-6}$.

- $A = \underline{1.6197}$, $\frac{Z_{11}}{Z_{21}} = \frac{42.75}{26.393} = \underline{1.6197}$
- $B = \underline{-413.103}$, $\frac{-1}{Y_{21}} = \frac{-1}{2.421\times10^{-3}} = \underline{-413.053}$
- $C = \underline{3.789\times10^{-5}}$, $\frac{1}{Z_{21}} = \frac{1}{26.393\times10^{3}} = \underline{3.789\times10^{-5}}$
- $D = \underline{-0.0159}$, $-Y_{11}/Y_{21} = -(3.839\times10^{-5})/(2.421\times10^{-3}) = \underline{-0.159}$

} Hence, verified

2. On connecting 2 networks M and N in cascade, transmission matrix of overall network is given as: $T = T_M . T_N$.

We have,

$$T_M = \begin{bmatrix} 1.6197 & -413.103 \\ 3.789\times10^{-5} & -0.0159 \end{bmatrix} \quad T_N = \begin{bmatrix} 2.983 & -996.67 \\ 8.222\times10^{-5} & -0.03167 \end{bmatrix}$$

$$T = \begin{bmatrix} 8.371 & -2995 \\ 2.448\times10^{-4} & -0.09 \end{bmatrix}$$

$$T_M \circ T_N = \begin{bmatrix} 4.7976 & -1601.223 \\ 1.117\times10^{-4} & -0.0373 \end{bmatrix}$$

3. Establish: $AD - BC = 1$

Network	AD−BC
M	1.322
N	1.0266
MN	1.4927

Values are not so exact due to experimental errors.

Discrepancies/Precautions:-

* Avoid loose connections.
* Reading should be taken carefully.
* Avoid series connections of voltmeters & parallel connection ammeter
* Power Supply should be switched off.
* Make connections acc. to the circuit diagram.

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EXPERIMENT - 5

SHEET NO.

PART - 2

OBJECTIVE :-

To determine the transient response of an RLC network in terms of the parameters σ, ξ, ω, ω_n and initial conditions $i_L(0^-)$, $v_c(0^-)$.

APPARATUS REQUIRED :-

S.No.	APPARATUS NAME	QUANTITY	SPECIFICATIONS
1.	DC Voltage Source	2	12 V
		1	24 V
2.	Resistor	1	1 Ω
		1	86 Ω
		2	100k Ω
3.	Capacitor	1	20 μF
4.	Inductor	1	20 mH
5.	Relay Switch	1	-

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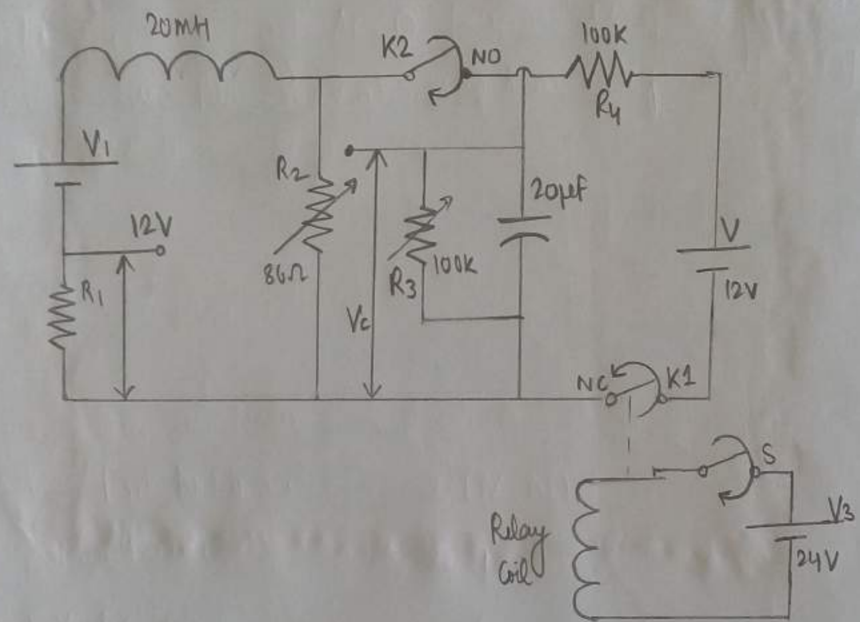
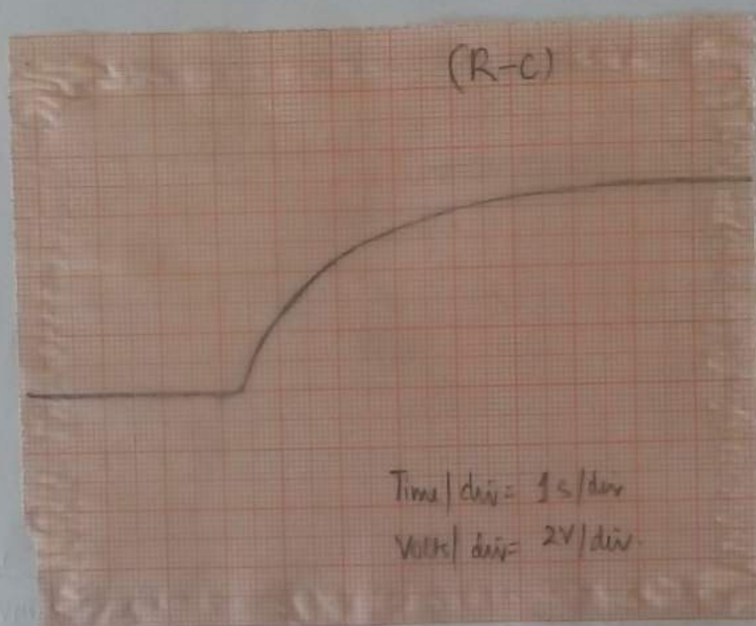




(R-C)
Time/div= 1s/div
Volts/div= 2V/div

REPORT :-

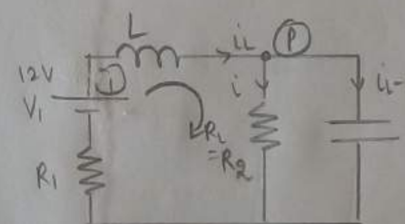
Determine initial conditions $V_C(0^-)$ and $V_C'(0^-)$

From circuit diagram we want to calculate the equivalent circuit and the initial conditions.

Use KCL and KVL,

We will get the differential equation indicating capacitance voltage as :-

$$V_1 = LC\frac{d^2V_C(t)}{dt^2} + \frac{L}{R}\frac{d}{dt}V_C(t) + V_C(t)$$



Applying KVL at loop 1

$$V_1 - L\frac{di_L}{dt} - iR - i_L R_1 = 0 \quad -(1)$$

Applying KCL at node P and KVL at loop 2

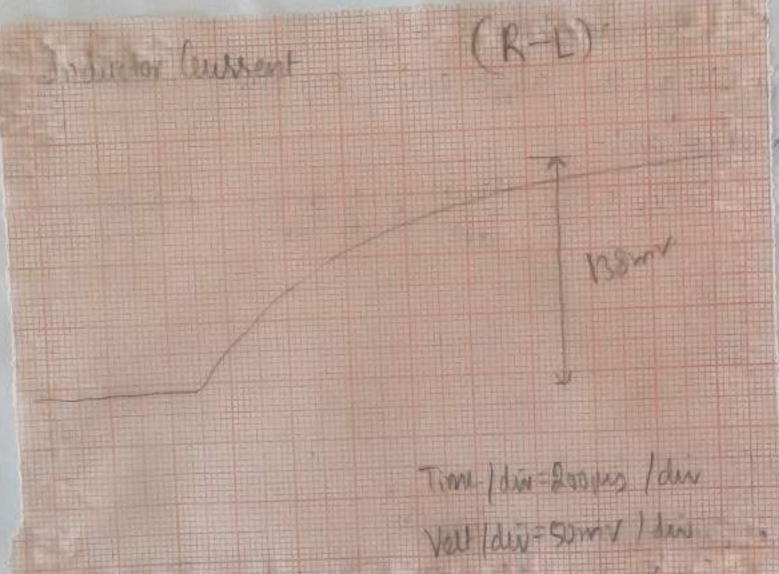
$$-iR + q/_C = 0$$

$$V_C = iR \text{ or } q/_C = iR$$

$$\Rightarrow \frac{i_L - i}{C} = R\frac{di}{dt}$$

$$\Rightarrow i = -RC\frac{di}{dt} + i_L \quad -(2)$$

$$\Rightarrow \boxed{I_L = i + RC\frac{di}{dt}}$$



Inductor Current
(R-L)
138mV
Time/div=200µs/div
Volt/div=50mV/div

But $i = \frac{v_c}{R}$ and $\frac{di}{dt} = \frac{1}{R}\frac{dv_c}{dt}$ — (3)

$$\Rightarrow \frac{di_L}{dt} = \frac{1}{R}\frac{dv_c}{dt} + C\frac{d^2v_c}{dt} \quad \text{— (4)}$$

Using (3), (4) and (1)

$$V_1 - \frac{L}{R}\frac{dv_c}{dt} - LC\frac{d^2v_c}{dt^2} - V_c - R_1\left(\frac{v_c}{R} + C\frac{dv_c}{dt}\right) = 0$$

Since $R_1 << R$, $R_1 \leq 1$, $R_1\left[\frac{v_c}{R} + C\frac{dv_c}{dt}\right]$ can be neglected.

$$\therefore V_1 = LC\frac{d^2v_c(t)}{dt^2} + \frac{L}{R}\frac{dv_c(t)}{dt} + v_c(t). \quad \text{— (5)}$$

Now current in R_3 at $t=0$ is $\left\{\frac{V_2}{(R_3+R_4)}\right\}$

Potential difference across $R_3 = V_c(0^-) = \frac{R_3}{R_3+R_4}V_2$.

$$= \frac{V_2}{2} = 6V = V_c(0^-).$$

Since voltage across capacitor does not change instantaneously,

So $V_c(0^+) = V_c(0^-) = 6V$.

$\therefore V_c'(0) = 0V$.

For Inductor:

Current $= i_L$

We have $V_1 - L\frac{di}{dt} - i_L R_1 = V_c$

$$\Rightarrow -L\frac{di_L^2}{dt^2} - R\frac{di_L}{dt} = \frac{dv_c}{dt}$$


(R-L+C)
Time/div = 2ms/div
Volt/div = 2V/div

Since $C\frac{dv_c}{dt} = i_L - \frac{V_c}{R}$

$$\Rightarrow -L\frac{d^2i_L}{dt^2} - R_1\frac{di_L}{dt} = \frac{1}{C}\left[i_L - \frac{1}{R}\left(V_1 - L\frac{di_L}{dt} - i_LR_1\right)\right]$$

$$\Rightarrow \frac{V_1}{R} = LC\frac{d^2(i_L(t))}{dt} + \left(R_1C \cancel{d} + \frac{L}{R}\right)\frac{di_L}{dt} + \left(1+\frac{R_1}{R}\right)i_L$$

Since, $R_1C \ll L/R$ and $\frac{R_1}{R} \ll 1$ those can be neglected.

$$\Rightarrow \frac{V_1}{4} = RLC\frac{d^2i_L(t)}{dt^2} + L\frac{di_L}{dt} + Ri_L \quad \text{— (6)}$$

$$\Rightarrow i_L(0^-) = \frac{V_1}{R_1 + (R_2 \| R_3)} = \frac{V_1}{1+(86 \| 100k)} = \frac{12}{86.93} = 138\,mA$$

Since current through inductor does not change instantaneously,

$$i_L(0^+) = i_L(0^-) = 138\,mA. \text{ and } \frac{d(i_L(0))}{dt} = 0$$

These are the initial conditions for $i_L(t)$.

Solution for $V_c(t)$:- (from eqn (5):-)

$$V_1 = LC\frac{d^2V_c(t)}{dt^2} + \frac{L}{R}\frac{dV_c(t)}{dt} + V_c(t).$$

Applying Laplace Transform:-

$$\frac{V_1(s)}{s} = LC\left[s^2V_c(s) - sV_c(0) - V_c'(0)\right] + \frac{L}{R}\left[sV_c(s) - V(0)\right] + V_c(s)$$

Since $V_c'(0) = 0$ and $V_c(0) = 6$ (from part a)

$$\Rightarrow \frac{V_1}{s} = LC(s^2V_c(s) - 6s) + \frac{L}{R}(sV_c(s) - 6) + V_c$$

$$\Rightarrow V_c\left(1 + \frac{Ls}{R} + LCs^2\right) = \frac{V_1}{s} + 6sLC + \frac{6L}{R}$$

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$$\Rightarrow V_c(s) = \frac{6RLCs^2 + 6L\cancel{C}s + V_1 R}{RLC} \times \frac{1}{s\left(s^2 + \frac{s}{RC} + \frac{1}{LC}\right)}$$

Using Partial fraction,

$$V_c(s) = \frac{A}{s} + \frac{Bs+C}{s^2+\frac{s}{RC}+\frac{1}{LC}} = \frac{(A+B)RLCs^2 + \left(\frac{A}{RC}+C\right)RLCs + AR}{RLCs\left(s^2+\frac{s}{RC}+\frac{1}{LC}\right)}$$

$$= \frac{6RLCs^2 + 6Ls + V_1 R}{RLCs\left(s^2+\frac{s}{RC}+\frac{1}{LC}\right)}$$

$$\Rightarrow A+B = 6, \quad \frac{A}{LC} = \frac{V_1}{LC}, \quad \frac{A}{RC} + C = \frac{6}{RC}$$

$$\Rightarrow A = V_1, \quad B = 6 - V_1, \quad C = \frac{6-V_1}{RC}$$

$$V_c(s) = \frac{A}{s} + \frac{Bs+C}{s^2+\frac{s}{RC}+\frac{1}{LC}} = \frac{A}{s} + \frac{Bs+C}{\left(s+\frac{1}{2RC}\right)^2 + \left(\sqrt{\frac{1}{LC}-\frac{1}{4R^2C^2}}\right)^2}$$

Applying Inverse Laplace Transform:-

$$v_c(t) = L^{-1}\left[V_c(s)\right] = A + e^{-t/RC}\left[B\cos\left(\sqrt{\frac{1}{LC}-\frac{1}{4R^2C^2}}\,t\right) + \left(\frac{C - B/2RC}{\sqrt{\frac{1}{LC}-\frac{1}{4R^2C^2}}}\right)\sin\left(\sqrt{\frac{1}{LC}-\frac{1}{4R^2C^2}}\,t\right)\right]$$

Substituting A, B, C and $V_1 = 12V$.

$$V_c(t) = \underset{V_1}{\overset{}{12}} - 6e^{-t/RC}\left[\cos\left(\sqrt{\frac{1}{LC}-\frac{1}{4R^2C^2}}\,t\right)\right] - \left[\frac{3e^{-t/RC}}{\sqrt{\frac{R^2C}{L}-\frac{1}{4}}}\sin\left(\sqrt{\frac{1}{LC}-\frac{1}{4R^2C^2}}\,t\right)\right]$$

Steady state solution (at $t \to \infty$) $V_{css}(t) = 12V$.

Transient Solution:-

$$V_{c,t}(t) = -6e^{-t/2RC}\cos\omega t - \frac{3}{RC\omega}\sin\omega t\, e^{-t/RC}$$

where $\omega^2 = \frac{1}{LC} - \frac{1}{4R^2C^2}$

and $R_1 = 5.86$, $R_2 = \frac{2.93}{RC\omega} = 1.094$.

$i_L(max) = 0.14$.

Finding Parameters:-

$$\omega_n = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{20\times10^{-3}\times20\times10^{-6}}} = \frac{\sqrt{10^9}}{20} \simeq 1580\,rad/s.$$

$$\sigma = \frac{1}{2RC} = \frac{10^6}{2\times20\times85.9} = 291\ rad/s.$$

$$\xi = \frac{\sigma}{\omega_n} = 0.184$$

$$\omega = \omega_n\sqrt{1-\xi^2} = 1580\sqrt{1-0.084} = 1526.5\ rad/s.$$

$k_1 = -6$

$k_2 = \frac{-3}{RC\omega} = 1.12$

$V_c(max) = \cancel{85}.9/86.9 \times 12 = 11.89V$

→ Complete Solution for i_L:-

We have,

$$V_1 = RLC\frac{d^2}{dt^2}i_L(t) + L\frac{di_L}{dt} + Ri_L$$

or

$$\frac{V_1}{R} = LC\frac{d^2i_L}{dt^2} + \frac{L}{R}\frac{di_L}{dt} + i_L \quad -(7)$$

Comparing (7) with eqn (5) & (6), we get:-

$$i_L(t) = A + e^{-t/2RC}\left[B\cos\omega t + \frac{G - B/2RC}{\omega}\sin\omega t\right]$$

where $\omega = \sqrt{\frac{1}{LC} - \frac{1}{4R^2C^2}}$ and $V_1 \rightarrow \frac{V_1}{R}C$

$A = \frac{V_1}{R} = 0.14A$ $\quad B = \frac{6 - V_1}{R} = 5.86$ $\quad G = \frac{\frac{6-V_1}{R}}{RC} = \frac{5.86}{RC}$

So steady state solution $i_{L,ss} = 0.14A$.

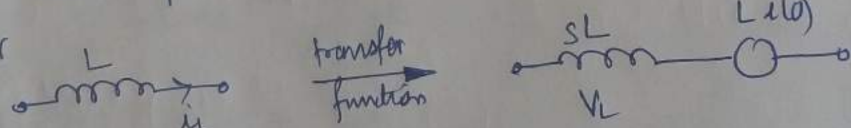
Transient Solution:-

$$i_{L,t}(t) = 5.86\ e^{-t/2RC}\cos(\omega t) + \frac{2.93}{RC\omega}\sin(\omega t)$$

Since $\omega^2 = \frac{1}{LC} - \frac{1}{4R^2C^2}$.

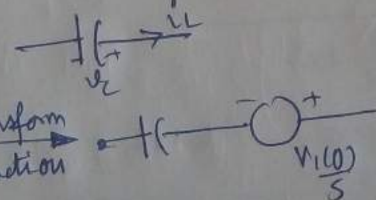
Equivalent circuit in transfer domain and determination of input $Z_1(s)$ and output $Z_2(s)$.

For inductor



$V_L = L\frac{di_L}{dt}$ $\qquad \frac{V_L(s)}{L} = sI(s) - I(0)$

For capacitor,



$i_c = C\frac{dv_c}{dt}$

$\frac{I_c}{C} = sV_1(s) - V(0)$ $\qquad \Rightarrow i_L(0) = 0.138A \quad V_c(0) = 6V.$

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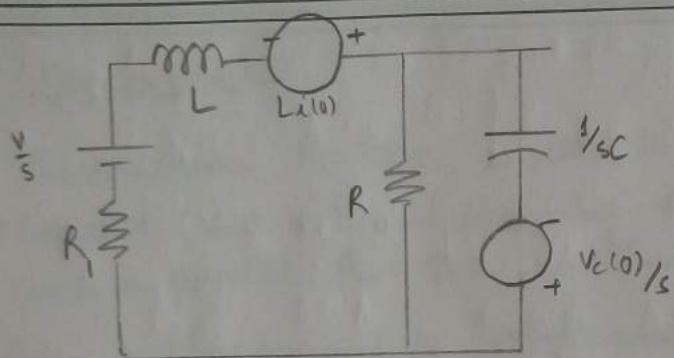

$$Z_{in} = (R_1 + sL) + \left(R \parallel \frac{1}{sC}\right) = R_1 + sL + \frac{R}{RCs+1}$$

$$Z_0 = \frac{1}{sC} \parallel R \parallel (R_1 + sL) = \frac{RR_1 + RsL}{RR_1sC + s^2RLC + R_1 + R_1 + sL.}$$

In transferdomain apply Thevenin's theorem to determine current through inductor. Initial conditions are $i(0) = 0.1A$, $C = 20\mu F$,

$V_c(0) = 5V$, $L = 20mH$.

$$V_{TH} = \frac{12}{s} + L\cdot i_0 - \frac{V_c/s}{R + \frac{1}{sC}} \times R$$

$$R_{TH} = R_1 + R \parallel \frac{1}{cs} = R_1 + \frac{R}{1+sRC}$$

$$i_L = \frac{V_{TH}}{R_{TH}} = \frac{12/s + Li_0 - \frac{V_c(0)RC}{RCs+1}}{R_1 + \frac{R}{1+sRC}}$$

$$= \frac{\frac{12}{s} + 2\times10^{-3} - \frac{5\times86\times20\times10^{-6}}{86\times5\times10^{+}}}{1 + \frac{86}{1.1586}}$$

P.R.E.

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DISCUSSION:-

* The resistance $R_1 = 100k$ is used to create initial conditions for capacitor ($V_C(0) = 6V$). and when relay is used, this resistance comes in parallel $R_2 = 86\Omega$ and $R_{eq} = 86\Omega$.
* Relay coil has been used to simultaneously open and close two switches. So relay used to ~~[illegible]~~ determine transient response with initial conditions.
* Current through conductor remains same before and after using relay (steady state value), current value only oscillates for a small duration (~10 ms). then becomes steady state again. This current oscillates because voltage across cap. oscillates
* Without relay there is no oscillation in V_C because it has a large resistance in parallel (100kΩ) but after relay $R_{parallel}$ ~~[illegible]~~ (86Ω). So as R decreases overshoot

PRECAUTIONS:-

* Time scale of oscilloscope must be chosen wisely for transient to be observable.
* While measuring current by multimeter, it should be connected in series, not parallel.
* Triggering value must be ~~added~~ selected about half the steady value for better observation.
* All connections must be tight.

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Exp. No.	EXPERIMENT	PAGE No.	DATE	Teacher's Signature.
1	Active Low Pass Filters	1 - 11	05/08/16	19.8.16
2	Verification of Max. Power Theorem and Reciprocity Theorem	12 - 24	19/08/16	2/9
3	To study Transient & freq response of RLC series ckt.	25 - 32	02/09/16	23/9/16
4	To find frequency response of Negative Impedance Converter Gyrator Ckt.	33 - 47	23/09/16	07.10.16
5	Determination of Fourier Coefficients and Analysis of Non linear elements.	48 - 59	14/10/16	
6.	Determination of parameters of a two port Network and to measure transient responsE of a RLC Ckt.	60 - 76	28/10/16	

NEEDS

DATE 05/08/16 | EXPERIMENT No. 1 | SHEET NO 1

ACTIVE LOW PASS FILTERS

OBJECTIVE :-
To familiarize with 2nd order Sallen Key active Low pass filters and to measure their feqluency responses

APPARATUS REQURED :-

NAME	QUANTITY	SPECIFICATIONS
Breadboard	1	–
Function Generator	1	–
Cathode Ray Oscilloscope	1	–
Resistance	3	10kΩ
Resistance	1	2.7kΩ
Resistance	1	4.7kΩ
Capacitor	2	1000pf
Operational amplifier – OPAMP	1	OPAMP-741

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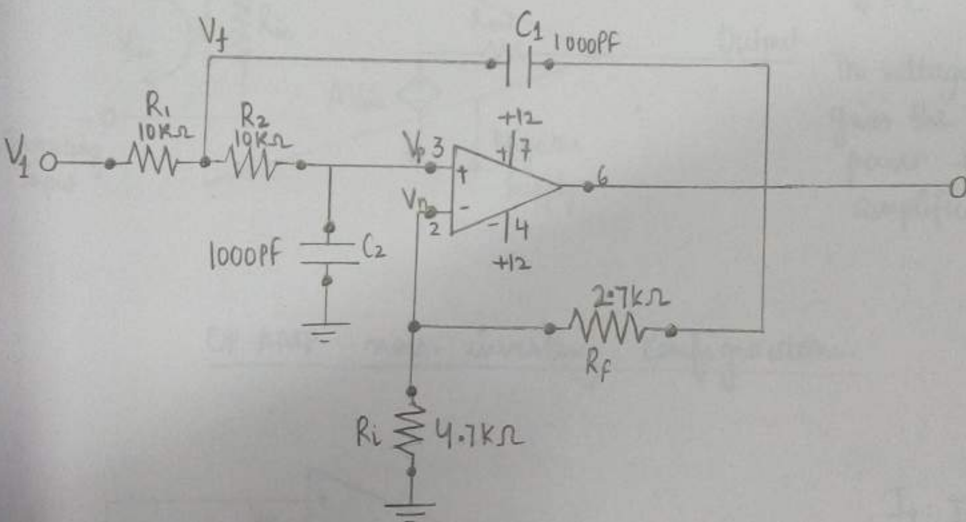
SHEET NO 2

CIRCUIT DIAGRAM



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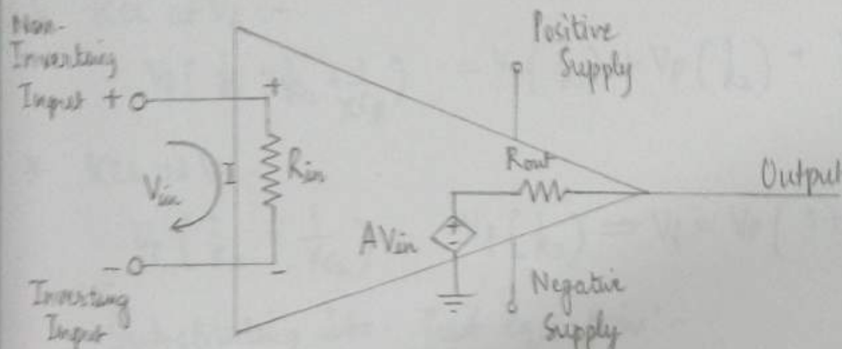
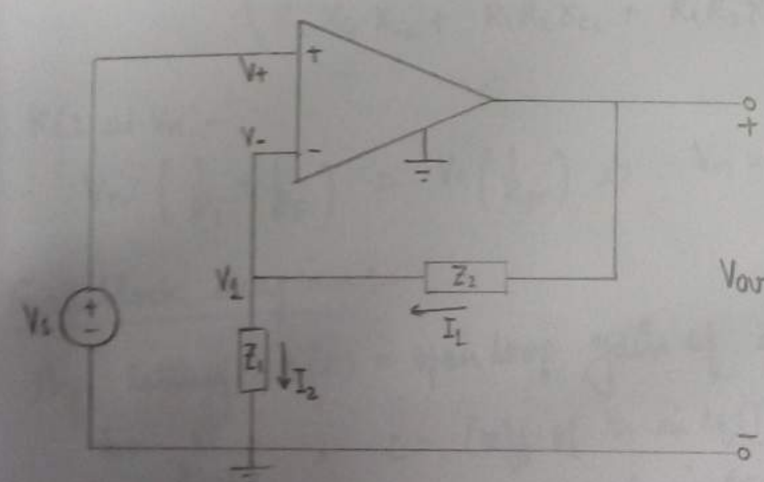
THEORY :-



For ideal OPAMP
$R_{in} \to \infty$ & $I \to 0$
$V_+ = V_-$

The voltage supply gives the necessary power for amplification.

OP-AMP non-inverting configuration.



$I_1 = I_2$

$\Rightarrow \frac{V_1}{Z_1} = \frac{V_{OUT} - V_1}{Z_2}$

where $V_S = V_+$

$\Rightarrow V_{out} = \left(1 + \frac{Z_2}{Z_1}\right) V_1$

For the generalized Sallen-key circuit,

* KCL at V_f:-

$$V_f\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{X_{C_1}}\right) = V_1\left(\frac{1}{R_1}\right)+V_P\left(\frac{1}{R_2}\right)+V_0\left(\frac{1}{X_{C_1}}\right)$$

* KCL at V_p:-

$$V_P\left(\frac{1}{R_2}+\frac{1}{X_{C_2}}\right) = V_f\left(\frac{1}{R_2}\right) \Rightarrow V_f = V_P\left(1+\frac{R_2}{X_{C_2}}\right)$$

Substituting into first equation:-

$$V_p = V_1\left(\frac{R_2X_{C_1}X_{C_2}}{R_2X_{C_1}X_{C_2}+R_1R_2X_{C_1}+R_1R_2X_{C_2}+R_2^2X_{C_1}+R_2^2R_1}\right)+$$

$$V_0\left(\frac{R_1R_2X_{C_2}}{R_2X_{C_1}X_{C_2}+R_1R_2X_{C_1}+R_1R_2X_{C_2}+R_2^2X_{C_1}+R_2^2R_1}\right) \quad (1)$$

* KCL at V_n:-

$$V_n\left(\frac{1}{R_i}+\frac{1}{R_F}\right) = V_0\left(\frac{1}{R_F}\right) \Rightarrow V_n = V_0\left(\frac{R_i}{R_i+R_F}\right)$$

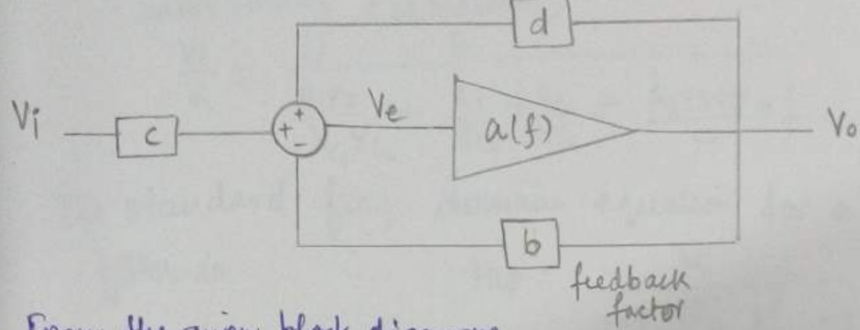
Gain block diagram:-

By letting $a(f)$ = open loop gain of amplifier,

$b = \frac{R_i}{R_i+R_f}$, C = Coeff. of V_1 in eq (1),

D = Coeff of V_0 in eq (2)

and taking $V_e = V_p - V_n$, the generalized Sallen Key filter circuit can be represented in gain block.



From the given block diagram,

$$V_o = a(f) V_e$$

$$(d-b) V_o = V_e - cV_i$$

$$\Rightarrow V_e = cV_i + dV_o - bV_o$$

Solving for the generalised transfer function

$$\frac{V_o}{V_i} = \left(\frac{c}{b}\right)\left[\frac{1}{1+\frac{1}{a(f)\, b} - \frac{d}{b}}\right]$$

Assuming $a(f)b$ is very large over the frequency of operation, $\frac{1}{a(f)b} \approx 0$, the ideal transfer function from gain block analysis becomes

$$\frac{V_o}{V_i} = \left(\frac{c}{b}\right)\left[\frac{1}{1-d/b}\right]$$

By letting $1/b = K$, $c = \frac{N_1}{D}$ and $d = \frac{N_2}{D}$, where N_1, N_2 and D are the numerators & denominator of the expression we derived in eq (1)

$$\frac{V_o}{V_i} = \left[\frac{K}{\frac{D}{N_1} - K\frac{N_2}{N_1}}\right]$$

Substituting the values,

$$\frac{V_o}{V_i} = \frac{K}{\frac{R_1R_2}{X_{C_1}X_{C_2}} + \frac{R_1}{X_{C_2}} + \frac{R_2}{X_{C_2}} + \frac{R_1(1-K)}{C} + 1}$$

The standard freq. domain equation for a 2nd order lowpass filter is

$$H_{LP} = \frac{K}{-\left(\frac{f}{f_c}\right)^2 + \frac{jf}{Qf_c} + 1}$$

where f_c is the corner frequency (breakpoint between pass band & stop band) and Q is the quality factor.

i) $f << f_c \Rightarrow H_{LP} \approx K \rightarrow$ signal multiplied by gain factor K.

ii) $f = f_c \Rightarrow H_{LP} = -jKQ \rightarrow$ signals are phase shifted $90°$ and enhanced by Q-factor.

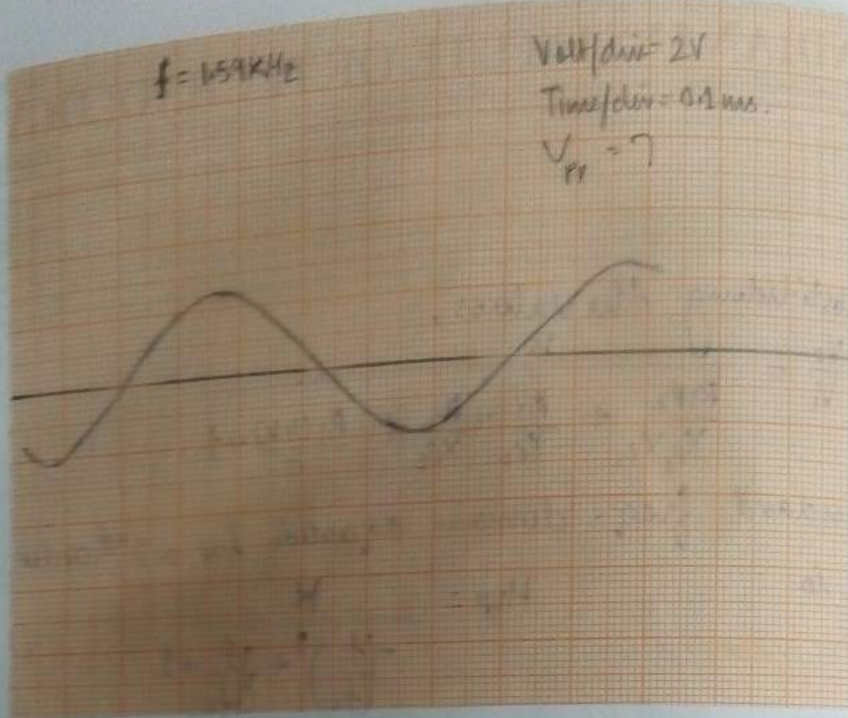
iii) $f >> f_c \Rightarrow H_{LP} = -K\left\{\frac{f_c}{f}\right\}^2 \rightarrow$ Signals are phase shifted by $180°$ and attenuated by sq. of frequency.

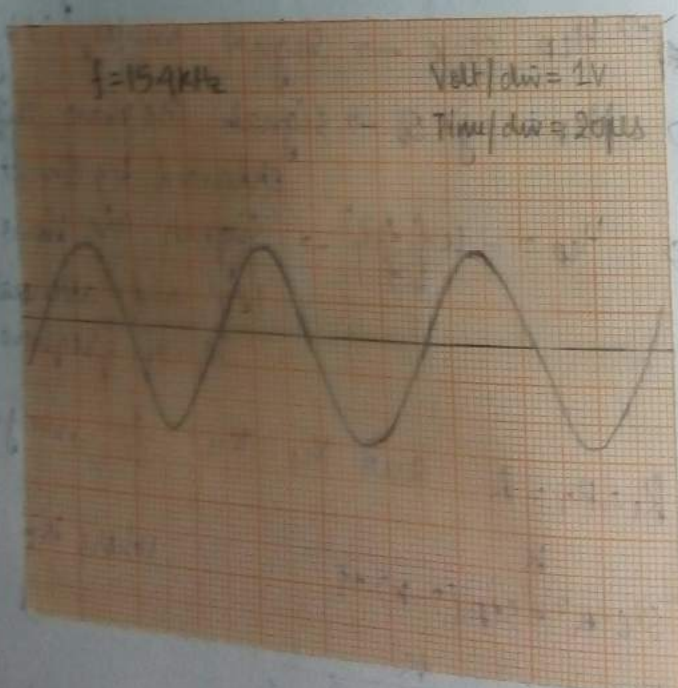
By putting $R_1 = R_2 = R$ and $X_{C_1} = X_{C_2} = \frac{1}{sC}$, we get

$$\frac{V_o}{V_i} = \frac{K}{s^2C^2R^2 + sCR(3-K) + 1} \qquad \text{where } K = 1 + \frac{R_f}{R_1}$$

Comparing $\frac{V_o}{V_i} = \frac{K}{-(f/f_c)^2 + \frac{jf}{Qf_c} + 1}$

we get $f_c = \frac{1}{2\pi RC}$ and $Q = \frac{1}{3-K}$.





INDIAN I

OBSERVATION

Input voltage

FREQUENCY (
0.1
0.2
0.3
0.5
1
1.59
3
6
8
10
15.9
25
30
31.8
50
80
100
120
159
200

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DATE | SHEET NO 7

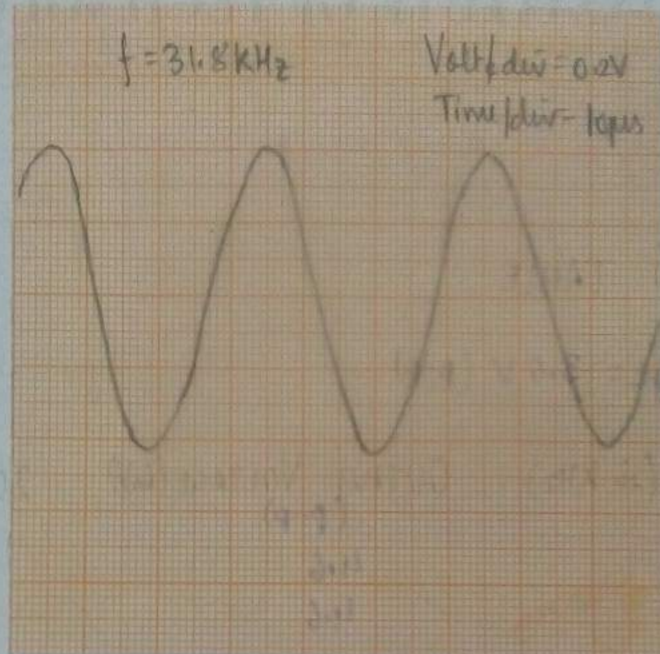
OBSERVATION TABLE

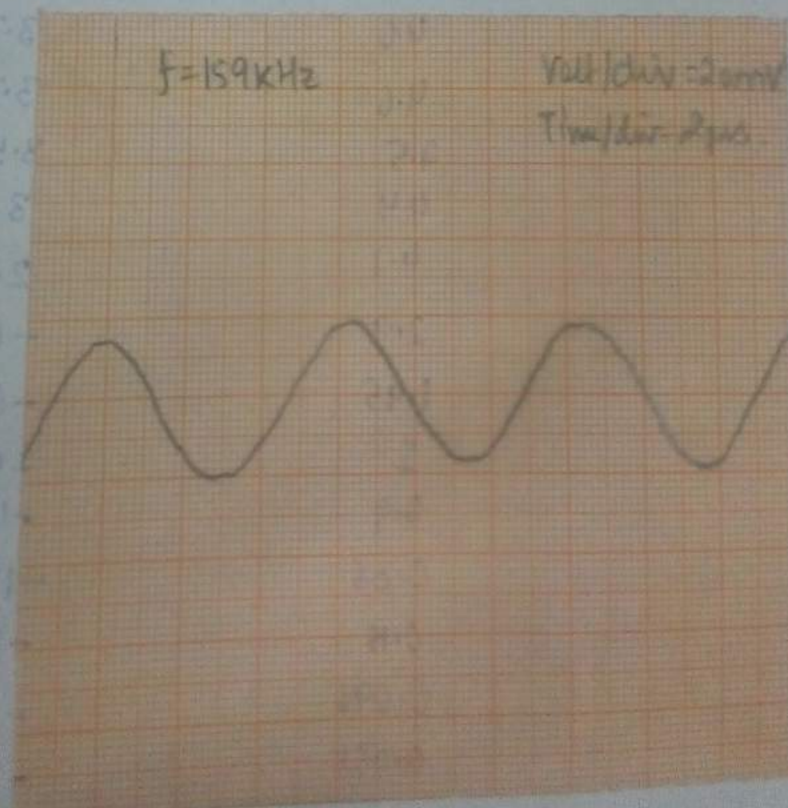
Input Voltage = 3.0 V (p-p)

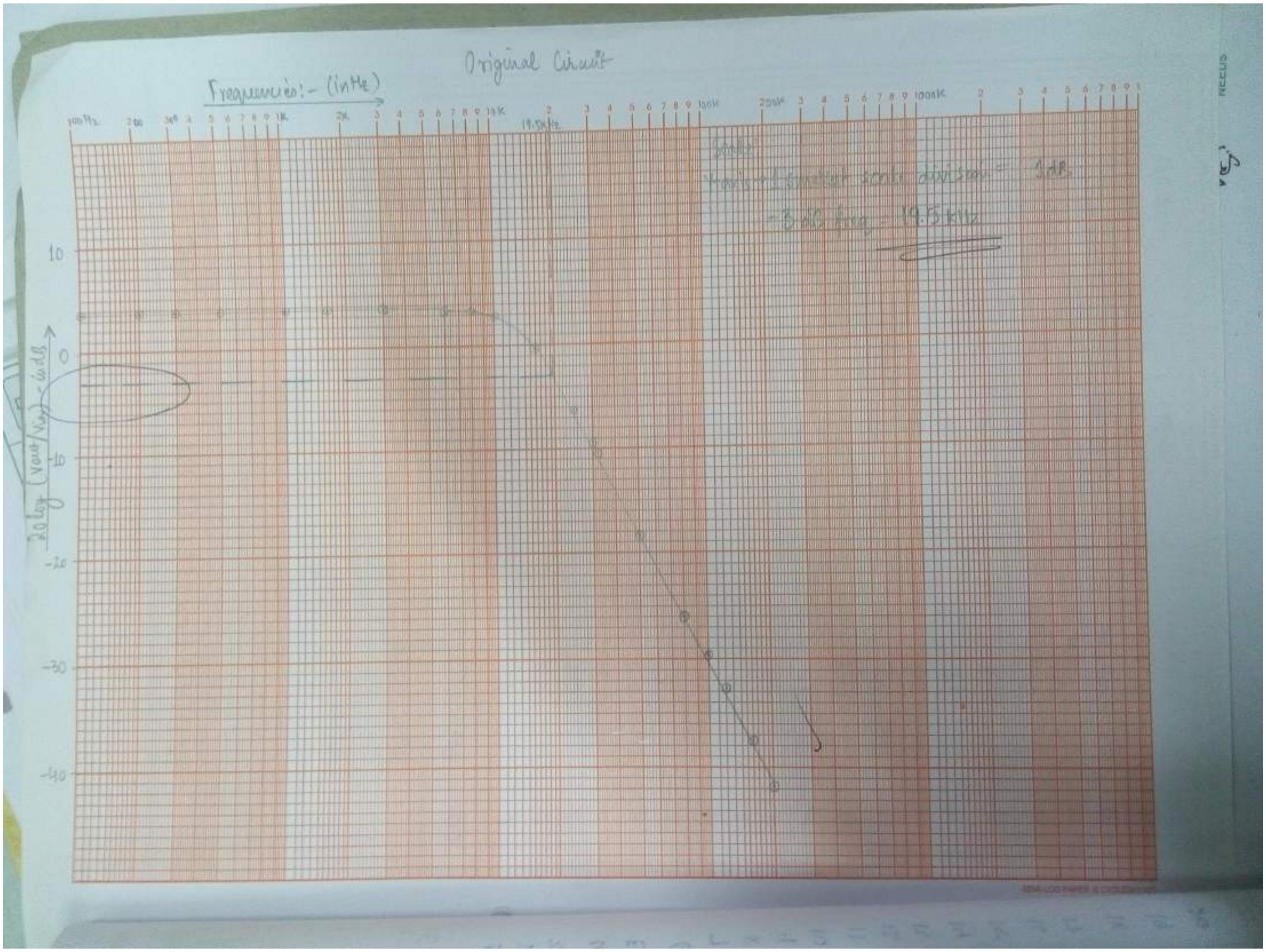
Original ckt ($R_1 = 4.7k\Omega$)

S.No.	FREQUENCY (in KHz)	OUTPUT VOLTAGE (in V) (p-p)	$20\log(V_{out}/V_{in})$ (in dB)
1	0.1	4.6	3.713
2	0.2	4.6	3.713
3	0.3	4.6	3.713
4	0.5	4.6	3.713
5	1	4.6	3.713
6	1.59	4.6	3.713
7	3	4.6	3.713
8	6	4.5	3.522
9	8	4.4	3.327
10	10	4.1	2.713
11	15.9	2.9	-0.294
12	25	1.45	-6.315
13	30	1	-9.542
14	31.8	0.9	-10.457
15	50	0.36	-18.416
16	80	0.15	-26.021
17	100	0.096	-29.897
18	120	0.068	-32.892
19	159	0.04	-37.501
20	200	0.024	-41.938

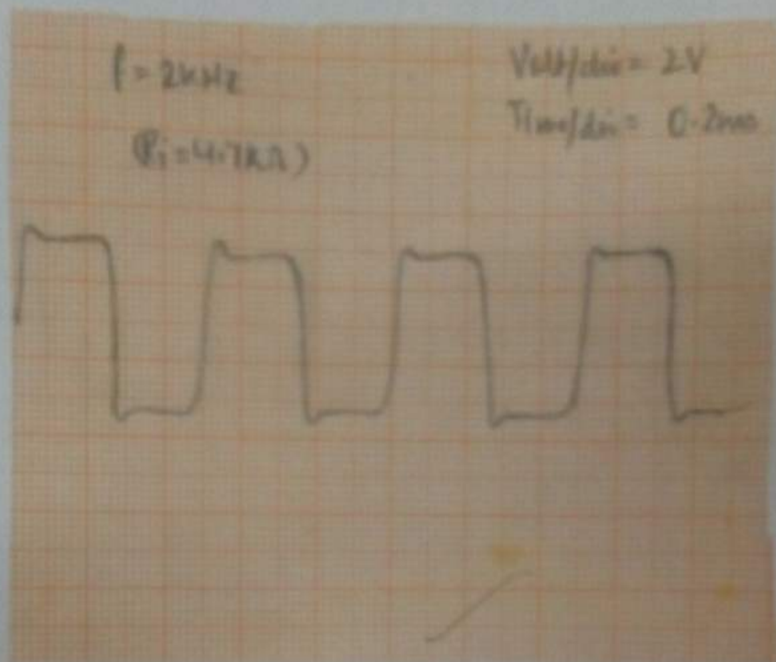
8/2/16


f = 31.8 kHz
Volt/div = 0.2V
Time/div = 10µs


f = 159 kHz
Volt/div = 20mV
Time/div = 2µs



Original Circuit
Frequencies:- (in Hz)
-3 dB freq - 19.5 kHz
19.5kHz
20 log (Vout/Vin) - in dB
10
0
-10
-20
-30
-40



f = 2kHz
(R1 = 4.7kΩ)
Volt/div = 2V
Time/div = 0.2ms

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DATE | | SHEET NO 8

OBSERVATION TABLE

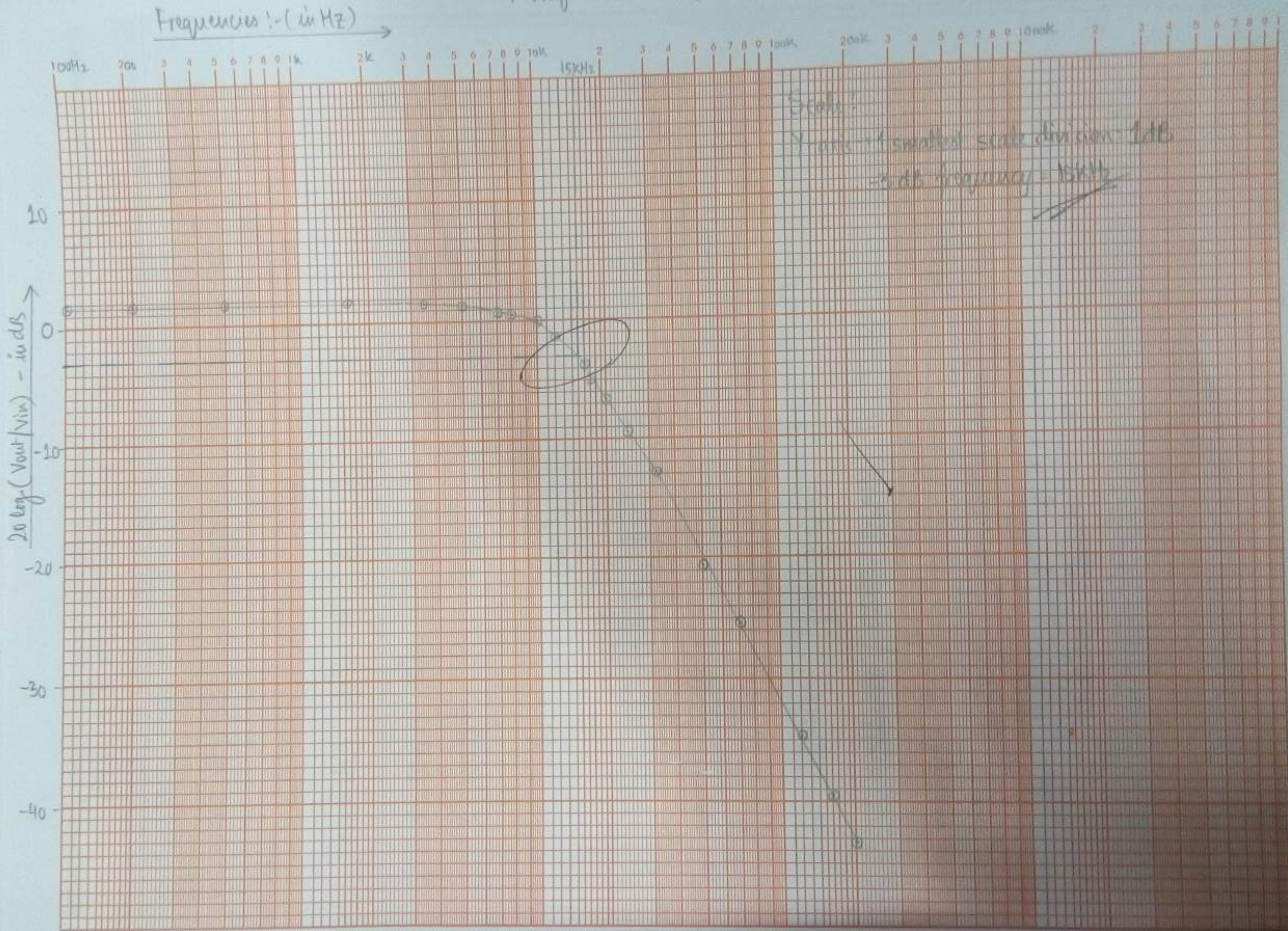
Input Voltage = 3.0V (p-p) Modified ckt (Ri = 10kΩ)

S.NO.	FREQUENCY (in KHz)	OUTPUT VOLTAGE (in V) (p-p)	20 log (Vout/Vin) (in dB)
1	0.1	3.7	1.822
2	0.2	3.7	1.822
3	0.5	3.7	1.822
4	1.59	3.7	1.822
5	3.5	3.6	1.584
6	5	3.5	1.339
7	7	3.3	0.828
8	8	3.2	0.560
9	10	3.0	0
10	12	2.6	-1.243
11	15.9	1.9	-3.967
12	17	1.7	-4.933
13	20	1.4	-6.620
14	25	1.0	-9.542
15	31.8 KHz	0.68	-12.892
16	50KHz	0.28	-20.600
17	70KHz	0.16	-25.460
18	120KHz	0.056	-34.578
19	159KHz	0.032	-39.439
20	200KHz	0.021	-43.098

S.S.

NEEDS

Modified Circuit (4.7kΩ replaced by 10kΩ)



RESULT :-

Calculated value of $f_0 = \frac{1}{2\pi RC}$

$R = 10k\Omega = 10^4\Omega$

$C = 1000pF$

$\therefore f_0 = \frac{1}{2\pi(10^4)(10^{-9})}$ Hz

$= 15915.5$ Hz

$\boxed{f_0 = 15.9 \text{ KHz}}$

For normal circuit :-

$f_0 = 15.9$ kHz (Calculated value)

$f_0 = 19.5$ kHz (from graph)

$\therefore$ deviation / error $= 3.6$ KHz.

For modified circuit :-

$f_0 = 15.9$ KHz (Calculated value)

$f_0 = 15$ kHz (from graph)

$\therefore$ deviation / ~~Error~~ $= 4.9$ kHz.

DISCUSSION:-

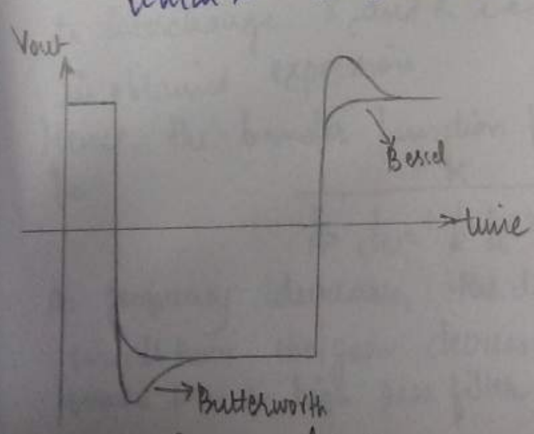
Q1. Derive the expression for transfer function.

Ans. Transfer function:- $\frac{V_o(s)}{V_i(s)} = \frac{K}{1+(3-K)RCs+R^2C^2s^2}$, $K = 1+\frac{R_f}{R_1}$

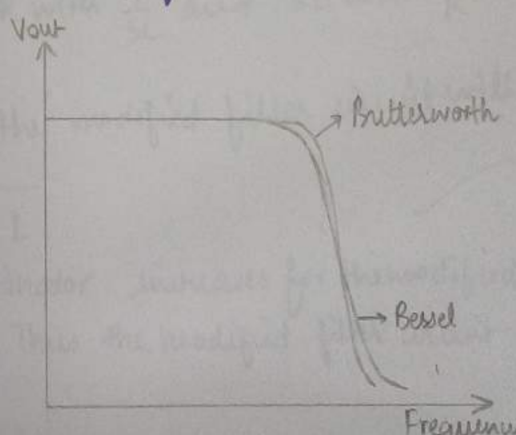
This expression has already been derived in theory.

Q2. Identify the type of filter in the modified ckt. Comment on its Performance compared to Butterworth filter.

Ans. When we change the 4.7kΩ resistance with a 10kΩ resistance, the Q factor changes from 0.71 to 0.58, which is characteristic of a Bessel filter.



Transient Response for a Square Wave form.



FREQUENCY RESPONSE

Butterworth filter	Bessel filter.
(i) Q-factor = 0.71	(i) Q-factor = 0.58
(ii) It is critically damped.	(ii) It is over damped.
(iii) It has maximum pass band flatness, but it overshoots slightly in response to pulse input.	(iii) It has constant group delay, and no overshoot with pulse input.
(iv) Moderate rate of attenuation above fc.	(iv) Slow rate of attenuation above fc.

Q3. If you interchange R_1, R_2 with C_1, C_2 what type of filter characteristics would you get?

Ans. If the resistance & capacitor are interchanged, we need to interchange X_C and R i.e R with $\frac{1}{SC}$ and SC with $\frac{1}{R}$ in obtained expression.

Hence, the transfer function for the modified filter circuit will be

$$\frac{K}{\frac{1}{R^2}\cdot\frac{1}{(SC)^2}+\frac{1}{R}\cdot\frac{1}{SC}(3-K)+1}$$

As frequency decreases, the denominator increases for the modified circuit, hence the gain decreases. Thus the modified filter circuit would be a high pass filter.

19.8.16