CS41001 : Theory of Computation
Autumn 2020
Due Date - 21/10/2020 (EOD)

Assignment 4

Guidelines: Marks for each question are mentioned alongside the questions.

1. NAE−3SAT takes as input a 3−CNF formula and determines if there is a satisfying assignment such that in each clause at least one literal is true and at least one literal is false. Show that this problem is **NP**-complete. **[5]**

2. (a) A Hamiltonian path/cycle in an undirected graph is a path/cycle that visits every vertex in the graph exactly once. Let

$$\mathsf{HAMPATH} = \{\langle G\rangle : \text{undirected graph G contains a Hamiltonian path}\},$$
$$\mathsf{HAMCYCLE} = \{\langle G\rangle : \text{undirected graph G contains a Hamiltonian cycle}\}.$$

   If $\mathsf{HAMPATH}$ is **NP**-complete, prove that $\mathsf{HAMCYCLE}$ is **NP**-complete. **[5]**

   (b) Show that the travelling salesman problem (TSP) defined as

$$\mathsf{TSP} = \{\langle V, w, k\rangle : \text{Vertex Set } V \text{ with weight } w : V \text{x} V \to N \text{ contains a cycle visiting every vertex exactly once with total weight} \leq k\},$$

   is **NP**-complete. Here, total weight is the sum of weights on the edges that form the cycle. **[5]**