Assignment - 6

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1 a) Given constants

$$4 : Int,$$
$$\mathcal{E}_0 = \{ y : Ref\ Int \}$$
$$y := 4$$

b) $\mathcal{E}_0 = \{ 3 : Int, \quad 4 : Int, \quad func : Int \to Bool \to Int \}$

$$\lambda \left(argv : Bool \times Int \times (Int \to Bool \to Int) \right).$$
$$(proj_3 (argv)) \ proj_1 (<3, 4, func>)$$

c) $for\ y\ B\ then\ \{M\}\ else\ \{N\}$

$\Rightarrow$ M & N should be same type

$\Rightarrow$ B has to be Bool

so, $\mathcal{E}_0 = \{ 2 : Int, \quad 4 : Int \}$,

$$\lambda (b : Bool) . (if\ b\ then\ \{2\}\ else\ \{4\})$$

d) $\mathcal{E}_0 = \{ r : \{ I_1 : Int, I_2 : Float, I_3 : Double \} \}$

$$\lambda (f : Double \to Int, \ g : Int \to Double) . (f(g(r.I_1)))$$

e) In a case expression, every individual expression has to be of the same type

→ $\mathcal{E}_0 = \{ M : Double + Bool + Int \}$

$\cup$

$\{ x : Double, y : Bool, z : Int \}$

→ Case M

$x : Double$ then 1

$y : Bool$ then 2

$z : Int$ then 3

2. a) $succ : Int \to Int$,

$4 : Int$,

$x : Ref\ Bool$,

$True : Bool$

$succ\ 4; x := true;$

⊕ $\mathcal{E}_0 \cup \{ succ\ 4 : Int \} = \mathcal{E}_1$

→ Type deduced from func. application

⊕ $\dfrac{x : Ref\ Bool; true : Bool}{\mathcal{E}_1 \vdash \{ x := true \} : command}$

b) $\mathcal{E}_0 : \{ M : Int + Bool \}$

Case M of

$x : Int$ then $succ\ x$

⊕ $\dfrac{\mathcal{E}_0 \cup \{ x := Int \}, \mathcal{E}_0 \vdash \{ succ : Int \to Int \}}{\mathcal{E}_1 = \mathcal{E}_0 \cup \{ succ\ x : Int \}}$ {func. app}

④ $\dfrac{\mathcal{E}_1 \cup \{b: Bool\} \cup \{a: Int\} \cup \{6: Int\}}{\mathcal{E}_2 = \mathcal{E}_1 \cup (if\ b\ then\ a\ else\ 6):\ Int}$

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(fore assignment)

④ case M of ($x: Int$ then E_1
$\qquad\qquad\qquad b: Bool$ then E_2

$\dfrac{\mathcal{E}_1 \vdash (M: Int + Bool)\ \mathcal{E}_1 \cup \{E_1: Int\},\ \mathcal{E}_1 \cup \{E_2: Int\}}{\mathcal{E}_1: \ case\ M \dots \dots \dots : Int}$

(assignment)

$\Rightarrow$ | Resultant type is Int |

c) $\lambda(f: Float \to float) \cdot \lambda(x: float) \cdot (f x)$

$\Rightarrow \dfrac{\mathcal{E}_0 \{ f: float \to float\ ;\ x: float \}}{\mathcal{E}_0 \vdash (f x): Float}$ { fun. application}

$\Rightarrow \dfrac{x: float,\ (f x): float}{\mathcal{E}_0 \vdash \lambda(x: float) \cdot (f x): Float \to Float}$ { abstraction}

$\Rightarrow \dfrac{\emptyset\ f: float \to float,\ \lambda(x: float) \cdot (f x): Float \to float}{\mathcal{E}_0 \vdash \lambda(f: float \to float) \cdot \lambda(x: float) \cdot (f x): float \to float}$

(float → float)

→ $\mathcal{E}_{0}$

→ $\langle 1, myop, 9.9 \rangle : Int \times (Float \to float) \times float$

$$\overline{\mathcal{E}_0 \vdash proj_2 (\langle 1, myop, 999 \rangle) : float \to float}$$

$$\{projection\}$$

→
with above results
$\mathcal{E}_0 :-$ ans : Float → float

3 a) pred: Int → Int, True: Bool, false: Bool, 1: Int

$\mathcal{E}_0 = \{\}$

→ $\dfrac{\cancel{pred} \quad pred: Int \to Int, \quad 4 : Int}{\mathcal{E}_0 \vdash (pred~4) : Int} \longrightarrow ①$ $\{func.~application\}$

→ $\dfrac{True: Bool, \quad false: Bool}{\mathcal{E}_0 \vdash if~true~then~false~else~false : Bool} \longrightarrow ②$ $\{condition\}$

→ $\dfrac{① , ②}{\mathcal{E}_0 \vdash \{pred~4, \quad if~true~then~false~else~false\}}$ $\{Solution\}$

$: \{I_1 : Int, \quad I_2 : Bool\}$

Evaluation : (pred 4) will be selected

$$= \underline{\underline{3}}$$

b) $(\lambda y : \text{Int}) \cdot (\text{if false then } ((\lambda f : \text{Bool} \to \text{Bool}) \cdot (f (\text{iszero } y))))$
   $\text{else } ((\lambda g : \text{Bool} \to \text{Bool}) (g (3.8 > y))))) 6$

$\mathcal{E}_0 = \{ \text{false, True} : \text{Bool}, \; y : \text{Int}, \; \text{iszero} : \text{Int} \to \text{Bool},$
$\qquad => : \text{Float} \to \text{Int} \to \text{Bool}$
$\qquad\qquad 3.8 : \text{Float}$
$\qquad\qquad 6 : \text{Int} \}$

$$\therefore \quad \xrightarrow{} \quad \dfrac{}{\mathcal{E}_0 \vdash (\text{iszero } y) : \text{Bool}} \qquad \{ \text{func. application} \}$$

$$\mathcal{E}_1 = \mathcal{E}_0 \cup \{ (\text{iszero } y) : \text{Bool} \}$$

$$\textcircled{1} \quad \dfrac{f : \text{Bool} \to \text{Bool}}{\mathcal{E}_1 \vdash f (\text{iszero } y) : \text{Bool}} \qquad \textcircled{6} \quad \{ \text{func. application} \}$$

$$\textcircled{2} \quad \Longrightarrow_{\mathcal{E}_1} \lambda (f : \text{Bool} \to \text{Bool}) \cdot (f (\text{iszero } y))$$
$$\qquad\qquad\qquad : (\text{Bool} \to \text{Bool}) \to \text{Bool}$$

$$\longrightarrow \quad \text{mlly}$$
$$\qquad \mathcal{E}_1 \vdash \lambda (g : \text{Bool} \to \text{Bool}) \cdot g (3.8 > y)$$
$$\qquad\qquad\qquad : (\text{Bool} \to \text{Bool}) \to \text{Bool}$$

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⇒ if false then M else N ∅

: (Bool → Bool) → Bool

{ given M = N : (Bool → Bool) →
                        Bool }

{ conditional }

∴ | Final expression
       type = (Int) ⇒ ((Bool → Bool) → Bool)

{ function abstraction }

Evaluation

     if false then M else N = N

     $N = \lambda(y : Bool \to Bool) \cdot (y (3 \cdot 8 > y))$

     Assuming, the actual meaning of '>'

     $N = \lambda(y : Bool \to Bool) \cdot (y \ false)$