CS 60047

Indian Institute of Technology Kharagpur

AUTUMN Semester 2020

COMPUTER SCIENCE AND ENGINEERING

CS 60047 Advanced Graph Theory

24-hour Take-Home Test | Date: 17 October 2020

Full Marks: 100 | Credit: 20% | Time: Opens 3 PM, 17-10-2020, Closes 4 PM, 18-10-2020

Instructions: This is a 24-hour Take-Home Test; This question paper has two pages.

Submission of answers: Please create a pdf file including **your name, roll-number**, and your **answers**, and submit it to the CSE Moodle Page by **4:00 PM, Sunday, 18 October 2020.**

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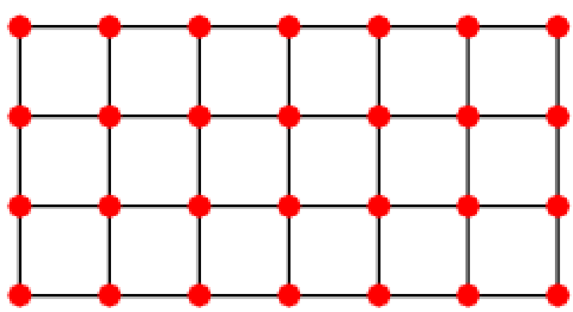
1. (8 points) Draw all non-isomorphic 4-vertex simple, connected, unlabeled graphs. Justify why there are no other classes of such graphs.

2. (8 points) Let S be the set of n points in a plane such that the Euclidean distance between any two points is at least 1 *cm*. Show, using basic graph theory, that there exist at most $3n$ pairs of points in S whose distance is exactly 1 *cm*.

3. (8 points) For the following grid-graph shown below, find the minimum-size dominating set. Justify that the size of your solution is indeed minimum.

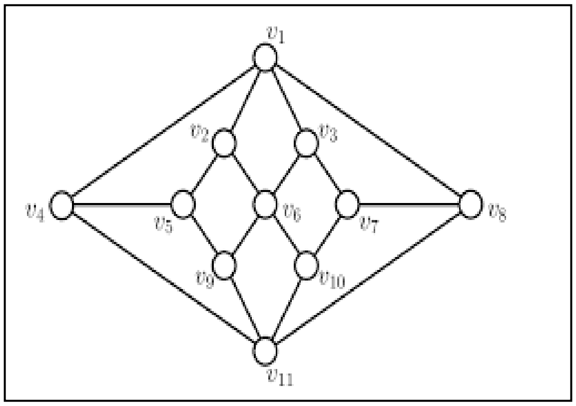


4. (8 points) In the grid graph shown above, assume that all horizontal edges have cost of 3 unit each, and all vertical edges have cost of 1 unit each. A post-office is located at the top-most and left-most corner. What is the optimal cost of a Postman's tour if he has to travel along each edge at least once and return to the post-office from where he had picked up the letters for delivery?

5. (10 points) Consider a hypercube Q_k, $k \geq 3$.
(i) Let $T(k)$ be a spanning tree of Q_k. Estimate the diameter of $T(k)$.
(ii) Let $L(Q_k)$ denote the line graph of Q_k. Determine the diameter of $L(Q_k)$. (5 + 5)

6. (8 points) Twenty-nine employees of a company sit regularly for lunch around a big round table. They sit in such a way that each employee finds two different neighbors every day. How long can such a seating arrangement continue? Formulate it as a graph-theoretic problem and present your solution.

7. (8 points) Does there exist a Hamiltonian cycle in the following graph shown below? If so, show it. Otherwise, justify why it is not possible.



8. (10 points). Two dissections of a bounding rectangle are shown below. In Fig. 8.1, there exists a sequence of vertical and horizontal "through-knife-cuts" (e.g., 1, 2, 4, 7, 3, 5, 6, 8) that finally separate out all small rectangles A, B, C, …, I. In the dissection shown in Fig. 8.2, all small rectangles cannot be separated out individually using such recursive through-knife-cuts.

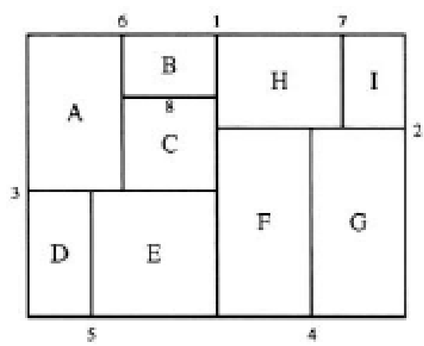


Fig. 8.1

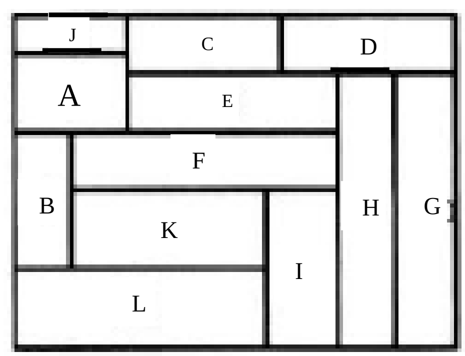


Fig. 8.2

Describe the geometry of dissections using a graph and suggest a method to differentiate between these two types of dissection. For simplicity assume that when a horizontal and a vertical cut meet, they do not cross each other, one just hits upon other as shown in the figures above.

9. (8 points) Let T_k denote a tournament of order $k \geq 3$. Show that T_k cannot have exactly two kings.

10. (8 points) Prove that K_5 (a complete graph with five vertices) is not graceful.

11. (6 points) Draw the tree for which the Prufer Code is {2, 3, 2, 3, 2, 3, 2, 3}.

12. (10 points) Every edge of the graph K_{66} (a complete graph with sixty-six vertices) is colored randomly with four colors: red, blue, green, or yellow. Show that there exists a monochromatic triangle.