Discrete Structure

Practice Set 2

1. Use a Venn diagram to illustrate the set of all months of the year whose names do not contain the letter *R* in the set of all months of the year.
2. Use a Venn diagram to illustrate the relationship *A* ~ *B* andB~C.
3. How many different elements does *A* x *B* have if *A* has *m* elements and *B* has *n* elements?
4. Show that *A* x *B =1= B* x *A,* when *A* and *Bare* nonempty, unless *A* = *B.*
5. Explain why *A* x *B* x C and *(A* x *B)* x C are no~ the same.
6. Explain why *(A* x *B)* x (C x *D)* and *A* x *(B* x C) x *D* are not the same.
7. Translate each of these quantifications into English and determine its truth value.
8. ɏ *x* ϵ R (x2 *=* -1*)*
9. ɏ *x* ϵ Z *(x2* > 0)
10. Ǝ *x* ϵ R (x3 = -1)
11. Ǝ *x* ϵ *Z (X* + 1 > *x)*
12. Let *A* and *B* be sets. Show that

a) *(A* n *B)* С *A.* b) *A* С *(A* U *B).*

c) *A* - *B* С *A.* d) A n *(B - A) =* φ

1. Prove the first De Morgan law by showing that

if *A* and *B* are sets, then (*A* U *B)****’*** *=* A***’*** n *B****’***

1. Draw the Venn diagrams for each of these combinations of the sets *A, B,* and C.

a) A n *(B* - C) b) (A n *B)* U (A n C)

e*)* (A n B**’**) U (A n C**’**)

1. Determine whether each of these functions is a bijection from R to R.

a) *f(x)* = *-3x +4*

b) *f(x)* = *-3x2* + 7

c) *f(x)* = *(x* + *1)/(x* + 2)

d) *f(x)* = *x 5* + I

1. Show that the function *f(x)* = *eX* from the set of real number to the set of real numbers is not invertible, but if the codomain is restricted to the set of positive real numbers, the resulting function is invertible.
2. Prove or disprove each of these statements about the floor (fl) and ceiling(cl) functions.

a) cl(fl(x)) = fl(x) for all real numbersx.

b) fl(2x) = 2fl(x) whenever *x* is a real number.

c) cl(x) + cl(y) – cl(x + y) = 0 or 1 whenever *x* and *y* are real numbers.

1. Show that the polynomial function *f* : Z+ x Z+ 🡪 Z+

with *f(m, n)* = *(m* + *n* - *2)(m* + *n* - 1)/2 + *m* is one to-one and onto.

1. a) Define the domain, codomain, and the range of a function.

b) Let *f(n)* be the function from the set of integers to the set of integers such that

*f(n)* = *n2* + 1. What are the domain, codomain, and range of this function?

16. a) Define the inverse of a function.

b) When does a function have an inverse?

c) Does the function *f(n)* = 10 - *n* from the set of integers to the set of integers have

an inverse? If so, what is it?

17. Let *f* be a one-to-one function from the set A to the set *B.* Let S and *T* be subsets of A. Show that *f(S* n *T) =* *f(S)* n *f(T).*

18. Give an example to show that the equality in Question 17 may not hold if *f* is not one-to-one.

19. Let A and *B* be subsets of the finite universal set *U.*

Show that IA n BI = 1U1-IAI-IBI + IA n BI*.*

20. Let fand g be functions from {I, 2, 3, 4} to *{a, b,* c, *d}* and

from *(a, b,* c, *d}* to {1, 2, 3, 4}, respectively,

suchthat *f(1)* = *d, f(2)* = c, *f(3)* = *a,* and *f(4)* = *b,* and

*g(a)* = 2, *g(b)* =1, *g(c)* = 3, and *g(d)* = 2.

a) Is *f* one-to-one? Is g one-to-one?

b) Is *f* onto? Is g onto?

c) Does either *f* or g have an inverse? If so, find this inverse.