Solution 1

Given: We have a point $p(6, 12, 4) \in \mathbb{P}^2$.
In $\mathbb{R}^2$, this point transforms to $\left(\frac{6}{4}, \frac{12}{4}\right)$, which simplifies to $\left(\frac{3}{2}, 3\right)$.
Hence, the point $p(6, 12, 4) \in \mathbb{P}^2$ corresponds to the point $\left(\frac{3}{2}, 3\right)$ in the 2D plane.

Solution 2

Given: We're provided with a point $(4, 1) \in \mathbb{R}^2$.
In $\mathbb{P}^2$, this point can be represented as any point of the form:

$$\begin{aligned}&(4\lambda, \lambda, \lambda), \text{ where } \lambda \in \mathbb{R}\\ &\lambda \neq 0\end{aligned}$$

Solution 3

Suppose $\tilde{p}_1(5, 6)$ represents a point in $\mathbb{R}^2$ in the image I_1. A possible representation of this point is $p_1(5, 6, 1)$ in $\mathbb{P}^2$.
Given: We know the homography matrix H from image I_1 to I_2.
Suppose p_2 is the corresponding point of p_1 in I_2. Then,

$$\begin{aligned}p_2 &= Hp_1\\ &= \begin{pmatrix} 1 & 3 & 5 \\ 4 & 3 & 7 \\ 5 & -4 & 2 \end{pmatrix} \begin{pmatrix} 5 \\ 6 \\ 1 \end{pmatrix} = \begin{pmatrix} 28 \\ 45 \\ 3 \end{pmatrix}\end{aligned}$$

Hence, $p_2 = (28, 45, 3)$ is the projective point in I_2.
Thus, $\tilde{p}_2\left(\frac{28}{3}, 15\right)$ is the point corresponding to $\tilde{p}_1(5, 6)$ in I_2.

Solution 4

Let $\mathbf{x}$ denote the intersection.

$$\begin{aligned}\mathbf{x} &= l_1 \times l_2\\ &= (3, 2, 1) \times (3, 2, 6)\\ &= \begin{vmatrix} i & j & k \\ 3 & 2 & 1 \\ 3 & 2 & 6 \end{vmatrix} = (10, -15, 0)\end{aligned}$$

Therefore, the intersection of l_1 and l_2 is $(10, -15, 0)$ in $\mathbb{P}^2$

Solution 5

Consider lines $l_1(3, 2, 1)$ and $l_2(3, 2, 6)$ in $\mathbb{P}^2$. Since these lines are parallel (as the first two coordinates are equal), their intersection will be an ideal point.

Solution 6

Let $\tilde{L}$ represent the transformation of $L_\infty = (0,0,1)$ with respect to H. The transformed line passes through the points Hx lying on L_∞, i.e., $x^T L_\infty = 0$. Hence,

$$(Hx)^T \tilde{L} = 0$$
$$\implies x^T H^T \tilde{L} = 0$$

From the above two equations,

$$L_\infty = H^T \tilde{L}$$
$$\implies \tilde{L} = (H^T)^{-1} L_\infty$$

Since $H = \begin{pmatrix} 2 & 3 & -4 \\ -4 & 3 & 2 \\ 3 & 1 & 1 \end{pmatrix}$, its inverse, $H^{-1} = \frac{1}{84} \begin{pmatrix} 1 & -7 & 18 \\ 10 & 14 & 12 \\ -13 & 7 & 18 \end{pmatrix}$

Thus,

$$\tilde{L} = (H^T)^{-1} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \frac{1}{84} \begin{pmatrix} 1 & 10 & -13 \\ -7 & 14 & 7 \\ 18 & 12 & 18 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$
$$= \frac{1}{84} \begin{pmatrix} -13 \\ 7 \\ 18 \end{pmatrix}$$

Thus, the vanishing line in the transformed space is $\tilde{L} = \left(\frac{-13}{84}, \frac{1}{12}, \frac{3}{14}\right)$