Problem 3 Solution:

- **Given:**
  - Plane Φ containing two parallel lines.
  - Image plane Π intersecting Φ at the horizon line h.
  - Pinhole O.
- **To Prove:** The projections of the parallel lines onto Π converge on the horizon line h.

  Let l be the line of intersection between Φ and Π.

  Consider any two parallel lines l_1 and l_2 on Φ, parallel to l. Their projections, l_1' and l_2', on the image plane Π will also be parallel and parallel to l.

  We observe that the farther line on Φ from l will be closer to the horizon line on Π.

  Thus, the projection of a line parallel to l on Φ, extending to infinity, will approach the horizon line without intersecting it.

  Additionally, any plane joining O and a line above the horizon line, parallel to it, will never intersect Φ.

  Hence, the plane Φ' joining O and the horizon line is the first instance where it doesn't intersect with Φ. Since this is a continuous process, Φ' must be parallel to Φ.

  Therefore, the horizon line on Π, formed by the intersection of Π with Φ', represents the convergence point for the projections of the parallel lines on Φ onto the image plane Π.

Problem 4 Solution:

- **Given:**
  - Image plane Π is a distance f away from the pinhole O.
  - Virtual plane Π' is d away from O.

  Consider a point $P(X, Y, Z)$ in space, with the projections being $p(x, y)$ on Π and $p'(x', y')$ on Π'.

  As p' is a projection of P formed by the pinhole O, $\overrightarrow{OP}$ and $\overrightarrow{Op'}$ are collinear. Thus, for some $\lambda \in \mathbb{R}$,

$$\overrightarrow{Op'} = \lambda \overrightarrow{OP}$$

In our coordinate system, for p on Π, $z = f$, and for p' on Π', $z' = -d$. This means,

$$x' = \lambda X$$
$$y' = \lambda Y$$
$$z' = \lambda Z$$
$$\implies \lambda = \frac{z'}{Z} = \frac{-d}{Z}$$

This gives

$$x' = -d\left(\frac{X}{Z}\right); y' = -d\left(\frac{Y}{Z}\right)$$

which are the perspective equation projections for a virtual image located at a distance d in front of the pinhole.

Problem 5: Visual Illusions
Mask of Love:

- **Why it works**: The "Mask of Love" illusion plays with our brain's tendency to interpret ambiguous images and its reliance on Gestalt principles of perception, particularly figure-ground relationship.
- **Explanation**: When viewed from a certain distance, the two faces in the image merge into a single face. This occurs because our brains naturally seek to organize visual information into meaningful patterns and forms. The contrast between the faces and the negative space between them allows our brains to interpret the image in multiple ways.
- **Effect**: From a distance, the contrast between the faces is reduced, leading our brains to perceive them as a single, merged image. This illusion demonstrates how our brains interpret ambiguous visual information to create a coherent perception, resulting in the impression of a singular face.

Pulsating Heart:

- **Why it works**: The "Pulsating Heart" illusion likely exploits principles of contrast, color perception, and motion perception to create the impression of movement.
- **Explanation**: The alternating colors and shapes in the image create a dynamic visual pattern that our brains interpret as movement. Additionally, the arrangement of the shapes may create an illusion of depth or perspective, further enhancing the perception of pulsation.
- **Effect**: The illusion creates the impression of a pulsating heart, as if it's expanding and contracting. However, there is no actual movement; it's a trick of perception. This illusion demonstrates how our brains interpret visual stimuli based on various cues such as patterns, contrasts, and shapes, often leading us to perceive motion or depth where there is none.