Mathematical Model for Employee Seat Allocation

Sets:

1. T: Set of teams.
2. S: Set of special teams, where $S \subseteq T$.
3. J: Set of floors in the building.
4. K: Set of working days, $\{M, T, W, Th, F\}$.
5. C: Set of seats in the building, representing the capacity of the building.

Parameters:

1. $|C|$: Capacity of the building.
2. $\text{CP}[i][k]$: Collaboration pattern matrix indicating the required collaboration days for each team. It's a binary matrix where $\text{CP}[i][k] = 1$ if team i is scheduled to come to the office on day k, and $\text{CP}[i][k] = 0$ otherwise.

Decision Variables:

1. X_{ijk}: Integer variable representing the number of members of team i assigned to floor j on day k.
2. y_{ijk}: Binary variable indicating if team i is assigned a seat on floor j on day k.
3. $y_{\text{new}_{jk}}$: Binary variable indicating if floor j is being used on day k.

Objective Function:

$$
\begin{aligned}
\text{Minimize} \quad & 10 \times \sum_{j=1}^{|J|} \sum_{k=1}^{5} y_{\text{new}_{jk}} \\
& + \sum_{i=1}^{|T \setminus S|} \sum_{j=1}^{|J|} \sum_{k=1}^{5} y_{ijk} \\
& + 100 \times \sum_{i=|T \setminus S|+1}^{|T|} \sum_{j=1}^{|J|} \sum_{k=1}^{5} y_{ijk}
\end{aligned}
$$

Constraints:

1. If there are no members of team i assigned to floor j on day k ($X_{ijk} = 0$), then y_{ijk} must also be zero:

$$X_{ijk} \leq |C| \times y_{ijk}$$

2. $X_{ijk} \geq 0$ for all i, j, k.

3. If floor j is not being used on day k by any team (i.e., $\sum_{i=0}^{|T|} X_{ijk} = 0$), then $y_{\text{new}_{jk}}$ must also be zero:

$$\sum_{i=0}^{|T|} X_{ijk} \leq |C| \times y_{\text{new}_{jk}}$$

4. Ensure that the number of employees sitting on floor j on day k doesn't exceed its capacity:

$$\sum_{i=0}^{|T|} X_{ijk} \leq |C|$$

5. If the collaboration pattern indicates that team i does not come to the office on day k, then y_{ijk} must be zero:

$$y_{ijk} = 0 \text{ for } CP[i][k] = 0 \text{ for all } i, j, k$$

6. Ensure that each team is allocated at least one seat on any day they are scheduled to come to the office:

$$\sum_{i=0}^{|T|} y_{ijk} \geq 1 \text{ for } CP[i][k] = 1 \text{ for all } j, k$$

7. Ensure that the total number of employees from each team assigned to floor j on day k is equal to the team strength specified by the collaboration pattern:

$$\sum_{j=1}^{|J|} X_{ijk} = \text{team\_strength}[i] \text{ for all } i, k \text{ where } CP[i][k-1] = 1$$

Necessary Assumptions:

1. Each team requires a minimum allocation of seats on any given day to maintain productivity.

2. Collaboration patterns are fixed and known in advance.

3. The building has a fixed number of floors and seats.

4. Teams can be moved to different floors or days if their requirements cannot be met initially.

5. Social distancing norms are maintained while allocating seats.

6. Night shift employees are not considered in the model as they occupy spaces vacated by day shift employees.

Here's how you can add the provided information to the LaTeX document: "'latex

Special Team Adjustment Options:

If the model fails to provide all special team's demand that all their members could get the same floor, the leader will be given three options:

1. Move to another floor.

2. Reduce their team size.

3. Change one of their working days to a different day.

Here's how it can be achieved:

Option 1: Move to Other Floor

The above model will output the most optimal floor to move.

Option 2: Reduce Team Size

Objective Function:

$$\text{Minimize} \quad 10\times\sum_{j=1}^{|J|}\sum_{k=1}^{5} y_{\text{new}_{jk}}+\sum_{i=1}^{|T\setminus S|}\sum_{j=1}^{|J|}\sum_{k=1}^{5} y_{ijk}+100\times\sum_{i=|T\setminus S|+1}^{|T|}\sum_{j=1}^{|J|}\sum_{k=1}^{5} y_{ijk}-1000\times\sum_{i\in T'}\alpha_i$$

where T' represents the set of teams that could not get the same floor for each of their members, and α_i is a binary variable indicating if team i should have its size reduced.

Constraints:

1. $X_{ijk} \leq |C| \times y_{ijk}$ for all i, j, k.

2. $X_{ijk} \geq 0$ for all i, j, k.

3. $\sum_{i=0}^{|T|} X_{ijk} \leq |C| \times y_{\text{new}_{jk}}$ for all j, k.

4. $\sum_{i=0}^{|T|} X_{ijk} \leq \frac{|C|}{|J|}$ for all j, k.

5. $y_{ijk} = 0$ for $\text{CP}[i][k] = 0$ for all i, j, k.

6. $\sum_{i=0}^{|T|} y_{ijk} \geq 1$ for $\text{CP}[i][k] = 1$ for all j, k.

7. $\sum_{j=1}^{|J|} X_{ijk} = \alpha_i \times \text{team\_strength}[i]$ for all i, k where $\text{CP}[i][k-1] = 1$, and $0.6 \leq \alpha_i \leq 1$.

Option 3: Change Working Day

For each team i in T', generate all possible collaboration pattern (CP) matrix swaps and evaluate the objective function for each. Choose the CP matrix swap that minimizes the objective function and assign the corresponding values of X_{ijk} and y_{ijk}.

These additional constraints aim to ensure that special teams either get the same floor for all their members or adjust their collaboration patterns to achieve this goal, thus optimizing floor allocation and promoting team cohesion. ‘‘‘

You can insert this additional content after the original content in your LaTeX document. Compile the document again to include these additions in the output PDF.

Additional Notes:

1. **Fixed Seat Allocation:** Note that this can easily be accommodated in the model by adding a team_0. In this case, we will not minimize the y_{ijk} corresponding to this team, and the model will decide how many members to send on which floor according to the original LP's objective.

2. **Social Distancing:** Even social distancing can be easily incorporated into the model by simply multiplying a fraction to the $|C|$ capacity.

3. **Codes:** I have added an ipynb file in this repo in which I have coded the first part.