Mathematical Model for Round Robin Match Scheduling

Sets:

- Teams: $\{1, 2, ..., T\}$ (where T is the total number of teams)
- Stadiums: $\{1, 2, ..., S\}$ (where S is the total number of stadiums)
- Days: $\{1, 2, ..., D\}$ (where D is the total number of days)
- Zones: $\{E, W, N, S\}$ (representing geographical zones)

Parameters:

- I_{ij}: Binary value indicating if the match between teams i and j is interesting.
- W_d: Binary value indicating if day d is a weekend.
- A_{ds}: Binary value indicating if stadium s can't be scheduled on day d.
- L_s: Binary value indicating if stadium s is large.
- Z_i: Zone of team i.
- $D_{ss'}$: Distance between stadiums s and s'.
- D_{ij}: Distance between teams i and j (calculated based on stadium distances).
- $Start\_Date$: Start date of the tournament.
- End_Date: End date of the tournament.
- DN_{ij}: Binary value indicating if the match between teams i and j must be scheduled as a day-night match.

Decision Variables:

- $Match_{ijd}$: Binary decision variable indicating if the match between teams i and j takes place on day d.
- $Venue_{ijs}$: Binary decision variable indicating if the match between teams i and j takes place at stadium s.
- $Extra_d$: Real-valued variable representing additional interesting matches on day d.
- $Home_1, Home_2$: Real-valued variables representing the maximum difference between the number of home matches any two teams have.

Objective Function:

Minimize:

$$\sum_{d} Extra_d - \sum_{ijs} Venue_{ijs} I_{ij} L_s$$
$$- \sum_{ijd} Match_{ijd} I_{ij} W_d + Home_2 - Home_1$$

Constraints:

1. Each match takes place on a single day:

$$\sum_{d} Match_{ijd} = 1$$

2. Each match takes place at a single stadium:

$$\sum_{s} Venue_{ijs} = 1$$

3. Only one match can take place on a given day and stadium:

$$\sum_{ijd} Match_{ijd} Venue_{ijs} \leq 1$$

4. Can't schedule a match at a stadium on a particular day:

$$\sum_{ijd} Match_{ijd} Venue_{ijs} A_{ds} = 0$$

5. Constraints on $Extra_d$:

$$Extra_d \geq \sum_{ijd} Match_{ijd} I_{ij} - 1, \quad Extra_d \geq 0$$

6. Fairness constraint on home matches:

$$Home_1 \leq \sum_{j} Venue_{ijs}, \quad Home_2 \geq \sum_{j} Venue_{ijs}$$

7. Fair distribution of matches across zones:

$$\text{for } Z \in \text{Zones}, \sum_{i \in Z} \sum_{j \notin Z} Match_{ijd} = \sum_{i \notin Z} \sum_{j \in Z} Match_{ijd}$$

8. Ensure matches are scheduled within the tournament timeframe:

$$\sum_{d} \sum_{ij} Match_{ijd} \leq End\_Date - Start\_Date + 1$$

9. Ensure day-night matches are scheduled accordingly:

$Match_{ijd} = DN_{ij}$ if $DN_{ij} = 1$

10. Constraints on travel distance:

$$\text{for } i, j \text{ where } i \neq j, D_{ij} = \sum_{s,s'} Venue_{ijs} Venue_{jjs'} D_{ss'}$$

Assumptions:

- The input parameters (I_{ij}, W_d, A_{ds}, L_s, Z_i, $D_{ss'}$, $Start\_Date$, End_Date, DN_{ij}) are provided by the user.
- The number of teams, stadiums, and days are predefined.

- The solution minimizes the number of interesting matches scheduled on the same day, maximizes interesting matches on weekends and in large stadiums, and minimizes the difference in the number of home matches for any two teams.

Additional Notes:

- Matches abandoned due to rain are rescheduled for the next day.
- Consideration for D/N matches if there are constraints around them.

Output:

The model provides a schedule indicating when and where a match between two teams should be scheduled, considering the constraints and objectives outlined.