Final Examination of 5010 STAT TONY SHI JIASHENG.

1. True / False, give explanation.

① Th 6.12 of Lehmann (Ch 1).

② $\hat{F}_n(x) = \frac{1}{n}\sum_{i=1}^{n} 1\{X_i \le x\}$, $X_1, \cdots, X_n$ sample from P (all pdf).

Then $\hat{F}_n(x)$ is UMVUE of $F(x)$.

③ p is exponential family admits one-parameter canonical form.

then it has monotone likelihood function.

2. $X_1, \cdots, X_n$ iid $\sim U(0, \theta_1)$, $Y_1 \cdots Y_n$ iid $\sim U(0, \theta_2)$

① Find complete sufficient statistic for (θ_1, θ_2) $(X_{(n)}, Y_{(n)})$

② Find UMVUE of $\frac{\theta_1}{\theta_2}$. $c(n) \cdot \frac{X_{(n)}}{Y_{(n)}}$.

3. $X = YZ$, $Y \sim N(\theta, 1)$, $P(Z=1) = P(Z=-1) = -\frac{1}{2}$.

① $T(X) = X$ is sufficient but not complete.

② If there exists a complete sufficient statistic for θ.

4. $f_\theta(x) = e^{-(x-\theta)} 1\{x > \theta\}$. (Refer to Problem 4.7.1 of Keener).

① Find complete sufficient statistic for θ. $(T_n = \sum_{i=1}^{n} X_i)$

② $\phi = P(X_1 > x)$. Find Fisher info $I(\phi)$.

③ State Lehmann-Scheffé Theorem ties together with sufficiency, completeness, uniqueness, unbiased estimation.

Show that $\tilde{\phi}_n = 1\{T_n > x\} \cdot (1 - \frac{x}{T_n})^{n-1}$ is UMVUE of ϕ.

5. $(X_1, Y_1), \cdots, (X_n, Y_n)$ are iid, $X_i \sim N(0,1)$, $Y_i | X_i = x \sim N(\theta x, 1)$.

① Find MLE of θ

② Find limiting dist of $\sqrt{n}(\hat{\theta} - \theta)$ (MLE 极限分布) $\Rightarrow N(0, \frac{1}{I(\theta)})$

③ Construct $1-\alpha$ Asymptotic confidence interval for θ based on $I(\hat{\theta})$, $I(\theta)$ is Fisher info based on one observation (X_1, Y_1).

④ Give the exact dist of $\sqrt{\sum_{i=1}^{n} X_i^2}\,(\hat{\theta}-\theta)$.

6. $H_0: \theta = \theta_0$ v.s. $H_1: \theta = \theta_1$, Find the UMPT (uniform most powerful) of size α ~~for~~ $f_X(x) = e^{-(x-\theta)} \cdot 1\{x>\theta\}$.