$K = X(X'X)^- X'$ $KX = X$

Problem from STAT5030

1. Consider the model

$$y_{ij} = \mu + \tau_i + \epsilon_{ij}, \qquad i = 1, 2, 3, 4, \quad j = 1, 2, 3, 4,$$

where ϵ_{ij} are independently distributed as $N(0, \sigma^2)$.

(a) Let $\beta = (\mu, \tau_1, \tau_2, \tau_3, \tau_4)'$. Find a set of 4 linearly independent estimable functions of β.

(b) Derive a test to test the null hypothesis $H_0 : \tau_1 - \tau_2 = \tau_3 - \tau_4$.

(c) Is $\tau_1 + 2\tau_2$ estimable? Why?

2. (a) Let $A_{m\times m}$ and $B_{n\times n}$ be two nonsingular matrices. Further, assume that the matrices U and V are $m \times n$ and $n \times m$ respectively. Prove that

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$$(A + UBV)^{-1} = \left(A^{-1} - A^{-1}UB(B + BVA^{-1}UB)^{-1}BVA^{-1}.\right)$$

$(A + UBV)$

(b) Consider a regression model,

$$Y = X\beta + \epsilon,$$

$H \cdot$

$X(X'X)^- X'$

where X, $n \times p$, is full column rank. $Y = (Y_1, \ldots, Y_n)'$. Further, assume that the $Var(\epsilon) = \sigma^2 I$. Let e_i be the ith residual and h_i be the ith diagonal element of the hat matrix. Let $\hat{\beta}$ and $\hat{\beta}_{(i)}$ be the least squares estimate of β with and without the ith case included in the data respectively.

i. Show that

$$(X'_{(i)}X_{(i)})^{-1} = (X'X)^{-1} + \frac{(X'X)^{-1}x_i x_i'(X'X)^{-1}}{1 - h_i},$$

where $X_{(i)}$ denotes the regression matrix with the ith row (x_i') deleted. (Hint: $X'X = X'_{(i)}X_{(i)} + x_i x_i'$)

$X' = X_{(i)} + (0, \ldots, 0, X_i, 0 \ldots 0)$

ii. Prove that

$$\hat{\beta} - \hat{\beta}_{(i)} = \frac{(X'X)^{-1}x_i e_i}{1 - h_i}.$$

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