Qualify Exam. Decemeber 2013

Problem from STAT5010

1. Let $X_1, \cdots, X_0$ be a random sample from Uniform $(\theta, \theta+1)$ distribution. To test $H_0 : \theta = \theta_0$ Vs $H_1 : \theta > \theta_0$, use the test

$$\text{reject } H_0 \text{ if } Y_n \geq 1+\theta_0 \text{ or } Y_1 \geq k+\theta_0$$

where k is a constant, $Y_1 = \min\{X_1, \cdots, X_n\}, Y_n = \max\{X_1, \cdots, X_n\}$

(a) Determin k so that the test will have size α.

(b) Find the power function of the test in a).

(c) Prove that the test is UMP size α test.

(d) Find values of n and k so that the UMP $\alpha = .05$ level test will have power at least .8 if $\theta > \theta_0 + 1$.

(e) Obtain an $1-\alpha$ level confidence interval by inverting the above test.

2. Let $X_1, X_2, \cdots, X_n$ be an sequence of i.i.d. r.v.s from $f(x|\beta) = \frac{1}{\beta}\exp(-\frac{x}{\beta}), x > 0$. We also know that $Y_1, Y_2, \cdots, Y_n$ be an sequence of i.i.d. r.v.s from a known population with density $g(y)$. Suppose that the following sample is observed $\min(X_i, Y_i), 1_{[X_i \leq Y_i]}, i = 1, \cdots, n$.

(a) Find a MLE of β based on the observed sample.

(b) Find a minimal suficient statistics for β.

(c) Is the statistics in a) an UMVUE? Prove or disprove your answer.