STA 5010

Final Exam. December 10, 2003

1. Suppose that
$$Y_i = \alpha + \beta x_i + \varepsilon_i, \quad i = 1, \cdots, n$$
$\varepsilon_i's$ are i.i.d. with density function
$$f(u) = \exp\{c_2 u^2 + c_1 u + c_0\}$$
where $x_1, \cdots, x_n, c_0, c_1, c_2$ are known constants and α, β are unknown parameters.
   (a) Find a minimal sufficient statistics for (α, β).
   (b) Is your statistic complete?
   (c) Give the UMVUE for (α, β).

2. Continue with the Last problem. Let X_{n+1} and X_{n+2} be two known numbers. What is the UMVUE of $P\{Y_{n+1} > Y_{n+2}\}$, where $Y_{n+i} = \alpha + \beta\ X_{n+i} + \varepsilon_{n+i}$.

3. Let $X_1, \cdots, X_n$ be a random Sample from $N(0, \sigma_X^2)$, and let $Y_1, \cdots, Y_m$ be a random Sample from $N(0, \sigma_Y^2)$, independent of X_s'. Define $\boxed{\lambda = \sigma_Y^2/\sigma_X^2}$.
   (a) Find the level α LRT of $H_0 : \lambda = \lambda_0$ versus $H_1 : \lambda \neq \lambda_0$.
   (b) Express the rejection region of the LRT of part (a) in terms of an F random variable.
   (c) Find a $1 - \alpha$ confidence interval for λ.

4. Let X_1, X_2, X_3 be a random sample of size three from a uniform $(\theta, 2\theta)$ distribution, where $\theta > 0$,
   (a) Find the method of moments estimator of θ.
   (b) Find the MLE, $\hat{\theta}$, and find a constant k such that $E_\theta(k\hat{\theta}) = \theta$.
   (c) Which of the two estimators can be improved by using sufficiency? How?

5. Let $X_1, \cdots, X_n, X_{n+1}, \cdots, X_{n+m}$ be an i.i.d. sample from $f(x) = \frac{1}{\beta}\exp\left(-\frac{x}{\beta}\right), x > 0$. We observe the values of $X_1, \cdots, X_n$ and the event that $X_{n+1} > Y_{n+1}, \cdots, X_{n+m} > Y_{n+m}$ (The exact value of $X_{n+1}, \cdots, X_{n+m}$ are unknown, but $Y_{n+1}, \cdots, Y_{n+m}$ are observed).
   (a) Find a MLE of β.
   (b) Find a minimal sufficient.
   (c) Is you Statistic in a) an UMVUE?

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