6010 qualify.

1. $X_i \overset{iid}{\sim} U(0,1)$; (1) 求 $E\left(\frac{X_1}{X_{(n)}}\right)^k$ $\forall k \in \mathbb{N}_+$. (2) $\Sigma(X_i - X_{(1)})$ 与 $X_{(1)}$ independent.

2. (X_i, Y_i) ind pairs of ind poi with mean $(e^{\lambda_i}, e^{\lambda_i + \beta w_i})$ λ_i, β 未知, w_i 为 observed covariates.
求 $\hat{\beta}^{mle}$, it is consistent and asymptotic normal.

3. (1) $X_i \overset{iid}{\sim} U(\theta - \tau, \theta + \tau)$ $\theta \in R$, $\tau > 0$ 未知, 求 EX_1 的 UMVUE 并证.
(2) $X_i \sim^{iid} F$ 未知, 求 EX_1 UMVUE.
(3) (1) is in (2), why (1) 中 UMVUE isn't UMVUE of (2).

4. UMPT with size α, $H_0: \theta_1 = \cdots = \theta_n = 0$ $H_1: \theta_i = \theta_i > 0$
$X_i \sim N(\theta_i, \sigma^2)$, σ^2 已知. Identify test statistic and rejection region.

5005 qualify.

1 (a) Slutsky's Th (b) $P(X_n = \pm 1) = \frac{1}{2}(1 - \frac{1}{n^2})$ $P(X_n = k) = \frac{1}{2n^2}$ 求 lim dist of $\frac{\Sigma X_n}{(\Sigma X_n^2)^{\frac{1}{2}}}$.

2. $X_i \sim U(-1, 1)$ (a) 求 dist of $W_n = \frac{1}{n}\sum_{1 \le i < j \le n} X_i X_j$; (b) $\frac{S_n}{\sqrt{n}\log n} \to 0$ a.s.
(c) $P(|S_n| > x\sqrt{n}) \le 2 \cdot e^{-\frac{x^2}{2}}$ (hint: $e^t + e^{-t} \le 2e^{t^2}$). Doob's inequality

3. ① $E(S_{n \wedge \tau}) \le ES_n$ ② EX ③ $E(\max_{1 \le i \le n} S_n^+)^2 \le 4\sum_{i=1}^n EX_i^2$;