Qualify Exam. August 10, 2012

Problem from STAT5010

1. Let μ and Σ be the mean and covariance matrix from a bi-variate normal distribution. Let $X_1, X_2, \cdots, X_n$ be an i.i.d. sample from a truncated bi-variate normal distribution, or for $\mathrm{x} \in R^2$, the density function of X_1 is

$$f(\mathrm{x}|\mu, \Sigma, \gamma) = \begin{cases} C\frac{1}{\sqrt{\det(\Sigma)}}\exp\left\{-\frac{1}{2}(\mathrm{x}-\mu)'\Sigma^{-1}(\mathrm{x}-\mu)\right\}, & \text{for } (\mathrm{x}-\mu)'\Sigma^{-1}(\mathrm{x}-\mu) \leq \gamma^2 \\ 0, & \text{otherwise} \end{cases}$$

   (a) Find constant C and show that C only depends on γ.

   (b) Find minimum sufficient statistic for (μ, Σ) if γ is known.

   (c) Find minimum sufficient statistic for γ if (μ, Σ) is known.

   (d) Find UMVUE for (μ, Σ) if γ is known and prove your answer.

   (e) Find UMVUE for γ if (μ, Σ) is known and prove your answer.

   (f) If all μ, Σ and γ are unknown, how do you estimate all three parameters?

2. Suppose that $X_1, \cdots, X_{n_1}$ and $Y_1, \cdots, Y_{n_2}$ are two samples from the population $N(\mu_1, \sigma^2)$ and $N(\mu_2, \sigma^2)$. Assume σ^2 is unknown.

   (a) Show that the α-level likelihood ratio test for

$$H_o : \mu_2 \leq \mu_1 \text{ VS. } H_a : \mu_2 > \mu_1 \text{ is the usual two sample t-test.}$$

   (b) Can you show that the test is an uniformly most powerful test?