

STAT 5020 : Topics in Multivariate Analysis  
Assignment 2 (Due date: 22-Mar-2023)  
*Academic year 22/23, 2nd term*

1. Consider a non-linear SEM defined as follows:

$$\begin{aligned}y_{i1} &= \mu_1 + a_1 * c_i + \eta_i + \epsilon_{i1}, \\y_{i2} &= \mu_2 + a_2 * c_i + \lambda_{21} * \eta_i + \epsilon_{i2}, \\y_{i3} &= \mu_3 + a_3 * c_i + \lambda_{31} * \eta_i + \epsilon_{i3}, \\y_{i4} &= \mu_4 + a_4 * c_i + \xi_{i1} + \epsilon_{i4}, \\y_{i5} &= \mu_5 + a_5 * c_i + \lambda_{52} * \xi_{i1} + \epsilon_{i5}, \\y_{i6} &= \mu_6 + a_6 * c_i + \lambda_{62} * \xi_{i1} + \epsilon_{i6}, \\y_{i7} &= \mu_7 + a_7 * c_i + \xi_{i2} + \epsilon_{i7}, \\y_{i8} &= \mu_8 + a_8 * c_i + \lambda_{83} * \xi_{i2} + \epsilon_{i8}, \\y_{i9} &= \mu_9 + a_9 * c_i + \lambda_{93} * \xi_{i2} + \epsilon_{i9}, \\ \eta_i &= b * d_i + \gamma_1 * \xi_{i1} + \gamma_2 * \xi_{i2} + \gamma_3 * \xi_{i1} * \xi_{i2} + \gamma_4 * \xi_{i2}^2 + \delta_i,\end{aligned}$$

*multinorm.*

where the notations follow the lecture notes. Please use WinBUGS or the *R2WinBUGS* package to conduct Bayesian analysis:

- a. Set true values for the model parameters. Generate data from the model and conduct Bayesian analysis on the basis of 10 replications.
- b. Demonstrate how to check convergence of the model.
- c. Use Bias and RMSE to summarize the estimation results.
- d. Show your prior inputs and check whether the Bayesian analysis is sensitive to the inputs.

2. Continue to Q1, use Bayesian model comparison statistics, including Bayes factor and DIC, and the 10 datasets generated in Q1 to answer the following questions:

- a. Compare the non-linear SEM in Q1 with its linear SEM counterpart.
- b. Consider a new non-linear SEM by modifying the structural equation in Q1 as

$$\eta_i = b * d_i + \gamma_1 * \xi_{i1} + \gamma_2 * \xi_{i2} + \gamma_3 * \xi_{i1} * \xi_{i2} + \gamma_4 * \xi_{i1}^2 + \gamma_5 * \xi_{i2}^2 + \delta_i.$$

Compare the non-linear SEM in Q1 with this new model.