No.

Date 2018/6/8 qualify exam: (类似期末)

probability: 1. ① $y_n \xrightarrow{d} 0$, $\xi_n \xrightarrow{d} \xi$, $y_n + \xi_n \xrightarrow{d} \xi + y$? 加什么条件?

② $P(X_k = \pm k) = \frac{1}{2k^2}$, $P(X_k = \pm 1) = \frac{1}{2}(1 - \frac{1}{k^2})$

$\begin{cases} S_n / (\sum_1^n X_i^2) \to ? & \text{(limiting distribution)} \\ S_n / \sqrt{n \log n} \to 0 \text{ a.s.} \end{cases}$

2. X_i iid $U(-1,1)$

(a). limiting distribution $W_n = \frac{1}{n} \sum_{1 \le i < j \le n} X_i X_j$

(b) $S_n / \sqrt{n \log n} \to 0$ a.s.

(c). $P(|S_n| > \sqrt{n} t) \le 2\exp(-\frac{t^2}{2})$ [类似 self-normalized inequality]

3. martingale inequality. S_n submartingale.

①. $ES_{n \wedge T} \le ES_n$.

②. $xP(\max_{0 \le m \le n} S_m^+ \ge x) \le E S_n^+ 1\{\max_{0 \le m \le n} S_m^+ \ge x\}$.

③. $E(\max_{0 \le m \le n} S_m^+)^2 \le 4 \sum_1^n EX_i^2$

$S_n - S_{n-1} = X_n$..

SEM: Song

1. posterior $\begin{cases} \Sigma \text{ known} \\ \Sigma \text{ unknown} \quad IW. \end{cases}$

2. ①. Assumption of Mixture Model

②. What challenge will meet in Mixture model? How to solve the problem?

③. How to estimate & model selection

④. Use real example to illustrate Mixture model & linear regression?

linear model.

1. $\begin{pmatrix} Y_1 \\ Y_2 \\ Y_3 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 2 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{pmatrix}$

a. $\hat{\theta}_1, \hat{\theta}_2$?

b. $\hat{\sigma}^2$? by (Y_1, Y_2, Y_3) & $(\hat{\theta}_1, \hat{\theta}_2)$

c. Test $\theta_1 = 2\theta_2$, derive test statistic & rejection region with significance level α.

2. prove $\hat\beta_{ridge}$ $\exists \lambda$, s.t. $MSE(\hat\beta^{ridge}) < MSE(\hat\beta^{ls})$.

Inference 4選3

1. $U(0,\theta)$? $U(0,\theta)$.

① $E[X_1/X_{(n)}]^k$ compute.

②. $\sum(X_i - X_{(1)}) \perp\!\!\!\perp X_{(1)}$

2. (X_i, Y_i) from poisson with mean $(e^{\lambda_i}, e^{\lambda_i + \beta W_i})$, W_i covariate.
prove MLE of β is consistant and asymptotic normal.

3. ① $X_i \overset{iid}{\sim} U(\theta - \tau, \theta + \tau)$. UMVUE for EX_1. 汁 $\exists$

②. $X_i \overset{iid}{\sim}$ unknown $F(\cdot)$ UMVUE for EX_1. 汁 $\exists$

③. UMVUE in ① don't ~~suit~~ apply to ②

[different group]

4. $X_i \sim N(\mu_i, \sigma^2)$, σ^2 known, UMP of level. α

H_0: $\mu_i = \cdots = 0$ vs. H_1. $\mu_i = \theta_{i0}, \cdots$

find test statistics, rejection region.