Qualify Exam. (Topics in Multivariate Analysis), December 2013

1. Suppose we have a collection of binary responses y_i, $i = 1, \cdots, n$, and associated k-dimensional predictor variables x_i. Define the latent variable y_i^* as

$$y_i^* = \mathrm{x}_i^T \beta + \epsilon_i, \quad i = 1, \cdots, n,$$

where the ϵ_i are independent mean-zero errors having cumulative distribution function F, and β is a k-dimensional regression parameter. Consider the model

$$y_i = \begin{cases} 0, & \text{if } y_i^* \geq 0 \\ 1, & \text{if } y_i^* < 0 \end{cases}.$$

(a) Specify a conjugate prior distribution for β.

(b) Under (a), find the full conditional distributions for β and y_i^*, $i = 1, \cdots, n$.

(a) $\beta \sim N(\gamma_0, \Sigma_0)$ $\quad p(\beta \mid y_1, y_2, \cdots, y_n)$ $\quad p(y_i = 0 \mid \beta) = p(x_i'\beta + \varepsilon_i \geq 0 \mid \beta)$

$\varepsilon_i = y_i^* - x_i'\beta$ $\quad \propto p(y_1, y_2, \cdots, y_n \mid \beta)\, p(\beta)$ $\quad = p(\varepsilon_i \geq -x_i'\beta \mid \beta)$

$= 1 - F(-x_i'\beta)$ $\quad p(y_i \mid \beta) = \Phi(x_i'\beta)$.

$= \Phi(x_i'\beta)$

2. In Bayesian model comparison:

(a) Use a concrete example to illustrate how to implement the path sampling procedure for computing Bayes factor in the context of mixture structural equation models (SEMs).

(b) Discuss other Bayesian model comparison statistics in the comparison of mixture SEMs.