K=X(X'X)-X' KX=X

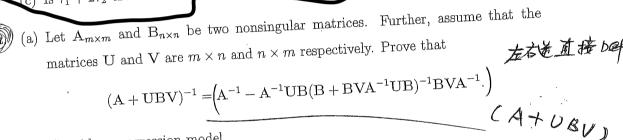
Problem from STAT5030

1. Consider the model

$$y_{ij} = \mu + \tau_i + \epsilon_{ij}, \quad i = 1, 2, 3, 4, \quad j = 1, 2, 3, 4,$$

where ϵ_{ij} are independently distributed as $N(0, \sigma^2)$.

- (a) Let $\beta = (\mu, \tau_1, \tau_2, \tau_3, \tau_4)'$. Find a set of 4 linearly independent estimable functions
- (b) Derive a test to test the null hypothesis $H_0: \tau_1 \tau_2 = \tau_3 \tau_4$.
- c) Is $\tau_1 + 2\tau_2$ estimable? Why?



(b) Consider a regression model,

$$Y = X\beta + \epsilon$$

where X, $n \times p$, is full column rank. Y = $(Y_1, ..., Y_n)'$. Further, assume that $Var(\epsilon) = \sigma^2 I$. Let e_i be the *i*th residual and h_i be the *i*th diagonal element of the hat matrix. Let $\hat{\beta}$ and $\hat{\beta}_{(i)}$ be the least squares estimate of β with and without the *i*th case included in the data respectively.

i. Show that

$$(X'_{(i)}X_{(i)})^{-1} = (X'X)^{-1} + \frac{(X'X)^{-1}x_ix_i'(X'X)^{-1}}{1-h_i},$$

where $X_{(i)}$ denotes the regression matrix with the *i*th row (x_i') deleted. (Hint:

$$X'X = X_{(i)}'X_{(i)} + x_ix_i'$$

$$X' = X_{(i)}^{(i)} + (o_{x}\rho_{x}X_{i}, o...o)$$

ii. Prove that

$$\hat{\beta} - \hat{\beta}_{(i)} = \frac{(X'X)^{-1}x_ie_i}{1 - h_i}.$$