

Table 1: A1a: corrected Hall voltage, magnet current, and field

U_H/mV	I_B/A	B/mT
9.0 ± 0.1	0.498 ± 0.009	72 ± 6
10.21 ± 0.07	0.58 ± 0.01	84 ± 6
11.36 ± 0.08	0.66 ± 0.02	94 ± 6
12.48 ± 0.08	0.72 ± 0.02	103 ± 6
13.71 ± 0.08	0.8 ± 0.06	110 ± 70
14.95 ± 0.09	0.87 ± 0.02	124 ± 6
16.34 ± 0.09	0.95 ± 0.02	135 ± 6
17.55 ± 0.09	1.03 ± 0.02	144 ± 6
18.9 ± 0.1	1.10 ± 0.02	155 ± 6
20.3 ± 0.1	1.18 ± 0.02	166 ± 6
21.4 ± 0.2	1.25 ± 0.02	175 ± 6
22.7 ± 0.2	1.33 ± 0.02	185 ± 6
24.1 ± 0.2	1.40 ± 0.02	196 ± 6
25.3 ± 0.2	1.48 ± 0.02	206 ± 6
26.6 ± 0.2	1.55 ± 0.07	220 ± 10

Table 2: A1b: source current vs corrected Hall voltage

U_H/mV	I_S/mA
26.5 ± 0.2	20.0 ± 0.5
25.7 ± 0.2	19.0 ± 0.5
23.8 ± 0.1	18.0 ± 0.5
22.8 ± 0.2	17.0 ± 0.5
21.12 ± 0.09	16.0 ± 0.5
18.4 ± 0.2	14.0 ± 0.5
16.98 ± 0.07	13.0 ± 0.5
14.08 ± 0.06	11.0 ± 0.5
12.42 ± 0.05	10.0 ± 0.5
10.91 ± 0.05	9.0 ± 0.5
9.02 ± 0.04	8.0 ± 0.5
8.76 ± 0.08	7.0 ± 0.5
6.94 ± 0.03	6.0 ± 0.5
5.67 ± 0.03	5.0 ± 0.5
2.76 ± 0.06	3.0 ± 0.5
0.98 ± 0.02	2.0 ± 0.5

Table 3: Diameters of the pinholes and the calculated minimum diameter for each.

names	<i>diameter of pinholes (mm)</i>	d_{min} (mm)
A_1	0.2	0.22
A_2	1.0	0.044
B_1	0.3	0.15
B_2	0.6	0.073
B_3	0.4	0.11

Table 4: First, t is fixed at 15 cm while x' is varied to find x_{best} ; then x' is fixed and t is varied. Magnification is shown from both theory and experiment.

t (cm)	\bar{x} (cm)	x (cm)	G	B	β_{ob}	t/f	Γ_{th}	Γ_{ex}
15.0 ± 0.1	16.9 ± 0.1	12.0 ± 0.2	3.0 ± 0.5	10.0	3.3 ± 0.6	3.75 ± 0.03	27.2 ± 0.2	24 ± 5
	18.3 ± 0.1	13.4 ± 0.2	2.5 ± 0.5	10.0	4.0 ± 0.8	3.75 ± 0.03	27.2 ± 0.2	29 ± 6
	21.0 ± 0.1	16.1 ± 0.2	1.5 ± 0.5	10.0	7 ± 3	3.75 ± 0.03	27.2 ± 0.2	50 ± 20
20.0 ± 0.1	18.3 ± 0.1	13.4 ± 0.2	1.5 ± 0.5	10.0	7 ± 3	5.00 ± 0.03	36.2 ± 0.2	50 ± 20
	30.0 ± 0.1	18.3 ± 0.1	13.4 ± 0.2	5.0 ± 0.5	50.0	10 ± 1	7.50 ± 0.03	54.4 ± 0.2
								72 ± 8

Table 5: Carrier density n and mobility μ for selected materials.

ID	Transport		
	Material	n (cm^{-3})	μ (cm^2/Vs)
Al	$(27.1 \pm 1.0)e18$	1500 ± 50	
Cu	$(84.9 \pm 5.0)e21$		
Si	$(20.0 \pm 1.0)e9$		
Ge	$(23.0 \pm 1.0)e12$		

Table 6: Two runs with per-block uncertainties (block errors override header errors).

Run A		Config
T (K)	R (Ω)	Bias
300.0 ± 0.2	10.010 ± 0.050	
320.0 ± 0.2	9.560 ± 0.050	
340.0 ± 0.2	9.120 ± 0.040	
360.0 ± 0.2	8.790 ± 0.020	1
380.0 ± 0.2	8.510 ± 0.020	

Claim. $\sum_{k=0}^{N-1} \mathbf{v}_k = \mathbf{0}, \quad \mathbf{v}_k = R(\cos \phi_k \hat{\mathbf{x}} + \sin \phi_k \hat{\mathbf{y}}), \quad N \geq 2.$

Proof 1 (roots of unity). Identify \mathbf{v}_k with the complex number $Re^{i\phi_k} = Re^{i\phi_0}\omega^k$, where $\omega = e^{2\pi i/N}$. Then

$$\sum_{k=0}^{N-1} \mathbf{v}_k = Re^{i\phi_0} \sum_{k=0}^{N-1} \omega^k = Re^{i\phi_0} \frac{1 - \omega^N}{1 - \omega} = 0,$$

since $\omega^N = 1$ and $\omega \neq 1$ for $N \geq 2$. Thus the vector sum is $\mathbf{0}$.

Proof 2 (rotation symmetry). Let $S = \sum_{k=0}^{N-1} \mathbf{v}_k$. Rotate every vector by $2\pi/N$ (which permutes the set), so the sum is unchanged: S maps to $e^{i(2\pi/N)}S$ but must still equal S . Hence $(1 - e^{i2\pi/N})S = \mathbf{0}$. For $N \geq 2$, $e^{i2\pi/N} \neq 1$, so $S = \mathbf{0}$.

□