

Table 1: A1a: corrected Hall voltage, magnet current, and field

$U_H/mV$	$I_B/A$	$B/mT$
$9.0 \pm 0.1$	$0.498 \pm 0.009$	$72 \pm 6$
$10.21 \pm 0.07$	$0.58 \pm 0.01$	$84 \pm 6$
$11.36 \pm 0.08$	$0.66 \pm 0.02$	$94 \pm 6$
$12.48 \pm 0.08$	$0.72 \pm 0.02$	$103 \pm 6$
$13.71 \pm 0.08$	$0.8 \pm 0.6$	$110 \pm 70$
$14.95 \pm 0.09$	$0.87 \pm 0.02$	$124 \pm 6$
$16.34 \pm 0.09$	$0.95 \pm 0.02$	$135 \pm 6$
$17.55 \pm 0.09$	$1.03 \pm 0.02$	$144 \pm 6$
$18.9 \pm 0.1$	$1.10 \pm 0.02$	$155 \pm 6$
$20.3 \pm 0.1$	$1.18 \pm 0.02$	$166 \pm 6$
$21.4 \pm 0.2$	$1.25 \pm 0.02$	$175 \pm 6$
$22.7 \pm 0.2$	$1.33 \pm 0.02$	$185 \pm 6$
$24.1 \pm 0.2$	$1.40 \pm 0.02$	$196 \pm 6$
$25.3 \pm 0.2$	$1.48 \pm 0.02$	$206 \pm 6$
$26.6 \pm 0.2$	$1.55 \pm 0.07$	$220 \pm 10$

Table 2: A1b: source current vs corrected Hall voltage

$U_H/mV$	$I_S/mA$
$26.5 \pm 0.2$	$20.0 \pm 0.5$
$25.7 \pm 0.2$	$19.0 \pm 0.5$
$23.8 \pm 0.1$	$18.0 \pm 0.5$
$22.8 \pm 0.2$	$17.0 \pm 0.5$
$21.12 \pm 0.09$	$16.0 \pm 0.5$
$18.4 \pm 0.2$	$14.0 \pm 0.5$
$16.98 \pm 0.07$	$13.0 \pm 0.5$
$14.08 \pm 0.06$	$11.0 \pm 0.5$
$12.42 \pm 0.05$	$10.0 \pm 0.5$
$10.91 \pm 0.05$	$9.0 \pm 0.5$
$9.02 \pm 0.04$	$8.0 \pm 0.5$
$8.76 \pm 0.08$	$7.0 \pm 0.5$
$6.94 \pm 0.03$	$6.0 \pm 0.5$
$5.67 \pm 0.03$	$5.0 \pm 0.5$
$2.76 \pm 0.06$	$3.0 \pm 0.5$
$0.98 \pm 0.02$	$2.0 \pm 0.5$

Table 3: Diameters of the pinholes and the calculated minimum diameter for each.

names	<i>diameter of pinholes (mm)</i>	<i>d<sub>min</sub> (mm)</i>
<i>A</i> <sub>1</sub>	0.2	0.22
<i>A</i> <sub>2</sub>	1.0	0.044
<i>B</i> <sub>1</sub>	0.3	0.15
<i>B</i> <sub>2</sub>	0.6	0.073
<i>B</i> <sub>3</sub>	0.4	0.11

Table 4: First,  $t$  is fixed at 15 cm while  $x'$  is varied to find  $x_{\text{best}}$ ; then  $x'$  is fixed and  $t$  is varied. Magnification is shown from both theory and experiment.

$t$ (cm)	$\bar{x}$ (cm)	$x$ (cm)	$G$	$B$	$\beta_{ob}$	$t/f$	$\Gamma_{th}$	$\Gamma_{ex}$
$15.0 \pm 0.1$	$16.9 \pm 0.1$	$12.0 \pm 0.2$	$3.0 \pm 0.5$	10.0	$3.3 \pm 0.6$	$3.75 \pm 0.03$	$27.2 \pm 0.2$	$24 \pm 5$
	$18.3 \pm 0.1$	$13.4 \pm 0.2$	$2.5 \pm 0.5$	10.0	$4.0 \pm 0.8$	$3.75 \pm 0.03$	$27.2 \pm 0.2$	$29 \pm 6$
	$21.0 \pm 0.1$	$16.1 \pm 0.2$	$1.5 \pm 0.5$	10.0	$7 \pm 3$	$3.75 \pm 0.03$	$27.2 \pm 0.2$	$50 \pm 20$
$20.0 \pm 0.1$	$18.3 \pm 0.1$	$13.4 \pm 0.2$	$1.5 \pm 0.5$	10.0	$7 \pm 3$	$5.00 \pm 0.03$	$36.2 \pm 0.2$	$50 \pm 20$
$30.0 \pm 0.1$	$18.3 \pm 0.1$	$13.4 \pm 0.2$	$5.0 \pm 0.5$	50.0	$10 \pm 1$	$7.50 \pm 0.03$	$54.4 \pm 0.2$	$72 \pm 8$

Table 5: Carrier density  $n$  and mobility  $\mu$  for selected materials.

ID	Transport	
Material	$n$ (cm <sup>-3</sup> )	$\mu$ (cm <sup>2</sup> /Vs)
Al	$(27.1 \pm 1.0)e18$	$1500 \pm 50$
Cu	$(84.9 \pm 5.0)e21$	
Si	$(20.0 \pm 1.0)e9$	
Ge	$(23.0 \pm 1.0)e12$	

Table 6: Two runs with per-block uncertainties (block errors override header errors).

Run A		Config
$T$ (K)	$R$ ( $\Omega$ )	Bias
$300.0 \pm 0.2$	$10.010 \pm 0.050$	
$320.0 \pm 0.2$	$9.560 \pm 0.050$	
$340.0 \pm 0.2$	$9.120 \pm 0.040$	
$360.0 \pm 0.2$	$8.790 \pm 0.020$	1
$380.0 \pm 0.2$	$8.510 \pm 0.020$	

**Claim.**  $\sum_{k=0}^{N-1} \mathbf{v}_k = \mathbf{0}, \quad \mathbf{v}_k = R(\cos \phi_k \hat{\mathbf{x}} + \sin \phi_k \hat{\mathbf{y}}), \quad N \geq 2.$

**Proof 1 (roots of unity).** Identify  $\mathbf{v}_k$  with the complex number  $Re^{i\phi_k} = Re^{i\phi_0}\omega^k$ , where  $\omega = e^{2\pi i/N}$ . Then

$$\sum_{k=0}^{N-1} \mathbf{v}_k = Re^{i\phi_0} \sum_{k=0}^{N-1} \omega^k = Re^{i\phi_0} \frac{1 - \omega^N}{1 - \omega} = 0,$$

since  $\omega^N = 1$  and  $\omega \neq 1$  for  $N \geq 2$ . Thus the vector sum is  $\mathbf{0}$ .

**Proof 2 (rotation symmetry).** Let  $S = \sum_{k=0}^{N-1} \mathbf{v}_k$ . Rotate every vector by  $2\pi/N$  (which permutes the set), so the sum is unchanged:  $S$  maps to  $e^{i(2\pi/N)}S$  but must still equal  $S$ . Hence  $(1 - e^{i2\pi/N})S = \mathbf{0}$ . For  $N \geq 2$ ,  $e^{i2\pi/N} \neq 1$ , so  $S = \mathbf{0}$ .

□