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DBAS101 Homework 2

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1.1 Case 1: Firm uses uniform price in both markets

Let the common price be P

$\therefore$

$$Q_A = D_A - d_A P, \quad P \le D_A/d_A$$
$$Q_B = D_B - d_B P, \quad P \le D_B/d_B$$
$\Big\}$ Q can't be -ve

Here, $D_A, D_B, d_A, d_B > 0$

Without any loss of generality, we can assume that

$$\frac{D_A}{d_A} \le \frac{D_B}{d_B} \qquad (A)$$

$\therefore$ $MC = c, \quad 0 < c < D_A/d_A$

Now, $Q = Q_A + Q_B$

$$\cancel{\therefore Q = \begin{cases} D_B/d_B - \end{cases}}$$

$$Q = \begin{cases} D_B - d_B P, & D_A/d_A < P \leq D_B/d_B \\ (D_A + D_B) - (d_A + d_B) P, & P \leq D_A/d_A \end{cases}$$

$\Rightarrow$

$$P = \begin{cases} D_B/d_B - Q/d_B, & Q \leq D_B - \dfrac{d_B D_A}{d_A} \\ \dfrac{D_A + D_B}{d_A + d_B} - \dfrac{Q}{d_A + d_B} & Q > D_B - \dfrac{d_B D_A}{d_A} \end{cases}$$

$\therefore$ The revenue $R = PQ$

$$R = \begin{cases} Q D_B/d_B - Q^2/d_B, & Q \leq \alpha \\ \dfrac{Q(D_A + B_A)}{d_A + d_B} - \dfrac{Q^2}{d_A + d_B} & Q > \alpha \end{cases}$$

Where, $\alpha = D_B - \dfrac{d_B D_A}{d_A}$

$$\therefore MR = \begin{cases} D_B/d_B - 2Q/d_B, & Q \leq \alpha \\ \dfrac{D_A + D_B}{d_A + d_B} - \dfrac{2Q}{d_A + d_B}, & Q > \alpha \end{cases}$$

Setting $MR = MC$,

$$D_B/d_B - \frac{2Q_E}{d_B} = c, \quad Q_E \le \alpha \qquad (1)$$

$$\frac{D_A + D_B}{d_A + d_B} - \frac{2Q_E}{d_A + d_B} = c, \quad Q_E > \alpha \qquad (2)$$

Case 1 Solving for (1),

$$\cancel{Q = \frac{D_B}{d_B} - \frac{2Q}{d_B}} \qquad Q_E = \frac{D_B - d_B c}{2}$$

Now, from assumption (A)

$$c < \frac{D_A}{d_A} \le \frac{D_B}{d_B}$$

$$\Rightarrow \quad d_B c < \frac{d_B D_A}{d_A} \le D_B$$

$$\Rightarrow \quad -d_B c > -\frac{d_B D_A}{d_A} \ge -D_B$$

$$\Rightarrow \quad D_B - d_B c > D_B - \frac{d_B D_A}{d_A} \ge 0$$

$$\Rightarrow \quad Q_E > \frac{\alpha}{2}$$

$\therefore$ Q_E is true iff

$$\frac{\alpha}{2} < Q_E \le \alpha$$

$$\Rightarrow \frac{D_B}{2} - \frac{d_B D_A}{2d_A} < \frac{D_B - d_B c}{2} \leq D_B - \frac{d_B D_B}{d_A}$$

$$\Rightarrow D_B - \frac{d_B D_A}{d_A} < D_B - d_B c \leq 2D_B - \frac{2 d_B D_A}{d_A}$$

$$\Rightarrow -\frac{D_A}{d_A} < -c \leq \frac{D_B}{d_B} - \frac{2D_A}{d_A}$$

$$\Rightarrow \frac{2D_A}{d_A} - \frac{D_B}{d_B} \leq c < \frac{D_A}{d_A}$$

$\therefore$ Q_E is valid iff

$$\min\left\{\frac{2D_A}{d_A} - \frac{D_B}{d_B}, 0\right\} \leq c < \frac{D_A}{d_A}$$

$$\therefore Q_E = \frac{D_B - d_B c}{2}$$

$$P_E = \frac{D_B}{d_B} - \frac{Q_E}{d_B} = \frac{D_B}{d_b} - \frac{D_B}{2d_B} + \frac{c}{2} = \frac{D_B + d_B c}{2 d_B}$$

$$\therefore \pi_E = (P_E - c) Q_E = \left[\frac{D_B + d_B c}{2 d_B} - c\right]\left[\frac{D_B - d_B c}{2}\right]$$

$$= \frac{(D_B - d_B c)^2}{4 d_B}$$

Since $Q \leq \alpha$, only market B is active

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Case 2 Solving for (2)

$$Q_E = \frac{D_A - d_A c}{2} + \frac{D_B - d_B c}{2}$$

However, this solution is only valid if

$$0 < c < \max(MR), \text{ where}$$

$$MR = \frac{D_A + D_B}{d_A + d_B} - \frac{2Q}{d_A + d_B}, \quad Q > \alpha$$

Since MR is a negatively sloped linear curve,

$$\max(MR) < \left.\frac{D_A + D_B}{d_A + d_B} - \frac{2Q}{d_A + d_B}\right|_{Q=\alpha}$$

Let this upper bound be β

$$\therefore \beta = \frac{D_A + D_B}{d_A + d_B} - \frac{2\left[D_B - \frac{d_B D_A}{d_A}\right]}{d_A + d_B}$$

$$= \frac{D_A + D_B}{d_A + d_B} - \frac{2 d_A D_B - 2 d_B D_A}{d_A (d_A + d_B)}$$

$$= \frac{d_A D_A + d_A D_B - 2 d_A D_B + 2 d_B D_A}{d_A (d_A + d_B)}$$

$$= \frac{d_A D_A + 2 d_B D_A - d_A D_B}{d_A (d_A + d_B)}$$

$$= \frac{2d_A D_A + 2d_B D_A - d_A D_B - d_A D_A}{(d_B + d_A) d_A}$$

$$= \frac{2D_A(d_B + d_A)}{d_A(d_A + d_B)} - \frac{d_A(D_B + D_A)}{d_A(d_B + d_A)}$$

$$= \frac{2D_A}{d_A} - \frac{D_B + D_A}{d_B + d_A}$$

$\therefore$ The solution is only valid when

$$0 < c < \frac{2D_A}{d_A} - \frac{D_B + D_A}{d_B + d_A}$$

$$\therefore Q_E = \frac{D_A - d_A c}{2} + \frac{D_B - d_B c}{2}$$

$$P_E = \frac{D_A + D_B}{d_A + d_B} - \frac{Q_E}{d_A + d_B}$$

$$= \frac{D_A + D_B + d_A c + d_B c}{2(d_A + d_B)}$$

$$\Pi_E = (P_E - c) Q_E$$

$$= \frac{(D_A + D_B - d_A c - d_B c)^2}{4(d_A + d_B)}$$

Note that there is a certain region

$$\min\left\{\frac{2D_A}{d_A} - \frac{D_B}{d_B}, 0\right\} \leq c < \frac{2D_A}{d_A} - \frac{D_B + D_A}{d_B + d_A}$$

where both the cases are active. In such a scenario, it is intuitive to select the solution for case 2 as it allows the producer to access two markets & hence make greater profits as there is greater ~~marginal~~ total revenue at the same marginal cost and marginal revenue by selling a larger quantity.

For individual market

1.2 We have

$$Q_A = D_A - d_A P_A$$
$$Q_B = D_B - d_B P_B$$

$$\Rightarrow P_A = D_A/d_A - Q_A/d_A$$
$$P_B = D_B/d_B - Q_B/d_B$$

$$\Rightarrow R_A = D_A Q_A/d_A - Q_A^2/d_A$$
$$R_B = D_B Q_B/d_B - Q_B^2/d_B$$

$$\Rightarrow MR_A = D_A/d_A - 2Q_A/d_A$$
$$MR_B = D_B/d_B - 2Q_B/d_B$$

Equating $MR = MC$ for each market separately,

$$\frac{D_A - 2Q_{AE}}{d_A} = c \quad \& \quad \frac{D_B - 2Q_{BE}}{d_B} = c$$

$$\Rightarrow Q_{AE} = \frac{D_A - d_A c}{2}$$

$$Q_{BE} = \frac{D_B - d_B c}{2}$$

$$\therefore P_{AE} = \frac{D_A}{d_A} - \frac{D_A - d_A c}{2 d_A}$$

$$= \frac{D_A + d_A c}{2 d_A}$$

$$P_{BE} = \frac{D_B + 2 d_B c}{2 d_B}$$

$$\therefore \cancel{\pi_{AE} = \left(\frac{D_A + d_A c}{2 d_A}\right)\left(D_A\right.}$$

$$\pi_{AE} = \left[\frac{D_A + d_A c}{2 d_A} - c\right]\left[\frac{D_A - d_A c}{2}\right]$$

$$= \frac{(D_A - d_A c)^2}{4 d_A}$$

$$\pi_{BE} = \frac{(D_B - d_B c)^2}{4 d_B}$$

$\therefore$ Total Quantity Q_E

$$= \frac{D_A - d_A c}{2} + \frac{D_B - d_B c}{2}$$

Total Profit π_E

$$= \frac{(D_A - d_A c)^2}{4 d_A} + \frac{(D_B - d_B c)^2}{4 d_B}$$

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1.3 Case 1: Only one market is active

$$\Pi_1 = \left(\frac{D_B - d_B c}{4 d_B}\right)^2$$

$$\Pi_2 = \left(\frac{D_A - d_A c}{4 d_A}\right)^2 + \left(\frac{D_B - d_B c}{4 d_B}\right)^2$$

$$= \Pi_1 + \left(\frac{D_A - d_A c}{4 d_A}\right)^2$$

Since $c < D_A/d_A$, $\left(\frac{D_A - d_A c}{4 d_A}\right)^2 > 0$

$\therefore \Pi_2 = \Pi_1 + \gamma$,

where γ is some positive quantity

$\therefore \Pi_2 > \Pi_1$

Case 2: Both markets are active

$$\Pi_1 = \frac{(D_A + D_B - d_A c - d_B c)^2}{4(d_A + d_B)}$$

$$\Pi_2 = \frac{(D_A - d_A c)^2}{4 d_A} + \frac{(D_B - d_B c)^2}{4 d_B}$$

$$\Pi_1 = \frac{\left[(D_A - d_A c) + (D_B - d_B c)\right]^2}{4(d_A + d_B)}$$

$$= \frac{(D_A - d_A c)^2}{4(d_A + d_B)} + \frac{(D_B - d_B c)^2}{4(d_A + d_B)} + \frac{2(D_A - d_A c)(D_B - d_B c)}{4(d_A + d_B)}$$

$\Pi_2 - \Pi_1$

$$= \frac{(D_A - d_A c)^2}{4 d_A} + \frac{(D_B - d_B c)^2}{4 d_B}$$

$$- \frac{(D_A - d_A c)^2}{4(d_A + d_B)} - \frac{(D_B - d_B c)^2}{4(d_A + d_B)} - \frac{2(D_A - d_A c)(D_B - d_B c)}{4(d_A + d_B)}$$

$$= \frac{(D_A - d_A c)^2}{4}\left[\frac{1}{d_A} - \frac{1}{d_A + d_B}\right]$$

$$+ \frac{(D_B - d_B c)^2}{4}\left[\frac{1}{d_B} - \frac{1}{d_A + d_B}\right]$$

$$- \frac{(D_A - d_A c)(D_B - d_B c)}{2(d_A + d_B)}$$

$$= \frac{d_B(D_A - d_A c)^2}{4 d_A (d_A + d_B)} + \frac{d_A(D_B - d_B c)^2}{4 d_B (d_A + d_B)}$$

$$- \frac{2\left[\sqrt{d_B}\,(D_A - d_A c)\right]\left[\sqrt{d_A}\,(D_B - d_B c)\right]}{\left[2\sqrt{d_A}\sqrt{d_A + d_B}\right]\left[2\sqrt{d_B}\sqrt{d_A + d_B}\right]}$$

$$= \left[\frac{\sqrt{d_B}\,(D_A - d_A c)}{2\sqrt{d_A(d_A + d_B)}} - \frac{\sqrt{d_A}\,(D_B - d_B c)}{2\sqrt{d_B(d_A + d_B)}}\right]^2$$

$\therefore\ \pi_2 - \pi_1 \geq 0$ [as it is a square of some number]

Equality occurs when $\frac{D_A}{d_A} = \frac{D_B}{d_B}$ in which case both the markets are identical and price discrimination is not possible. Hence, it is trivial

$\therefore\ \pi_2 > \pi_1$

Hence proved