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1.1 We have

$$C_i(Q_i) = \frac{1}{2} Q_i^2, \quad i = 1, 2$$

$$P = 10 - Q, \quad Q = Q_1 + Q_2$$

Marginal cost of each firm

$$MC_i = c'(Q_i) = Q_i \qquad (1)$$

$$\therefore Q = Q_1 + Q_2$$

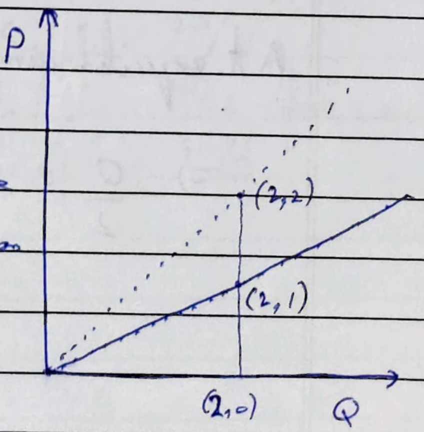
$$\Rightarrow Q = MC + MC$$

$$\Rightarrow MC = \frac{Q}{2}$$

$\therefore$ The market supply curve is given by

$$MC = Q/2 \qquad (2)$$

We see that the slope of the market supply curve given by (2) is lesser than the firm's supply curve given by (1)



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In general, for n firms

$$Q = \sum_{i=1}^{n} Q_i = \sum_{i=1}^{n} MC = n\,MC$$

$$\Rightarrow MC = \frac{Q}{n}$$

Hence, the slope of the supply curve keeps falling as more firms join. In the limiting case,

$$\lim_{n \to \infty} MC = \lim_{n \to \infty} \frac{Q}{n} = 0$$

$\therefore$ If there are infinite firms, the supply curve of the market becomes flat

2 We have,

Market Supply $S = Q/2$
Market Demand $D = P = 10 - Q$

At equilibrium, $S = D$

$$\Rightarrow \frac{Q}{2} = 10 - Q \Rightarrow Q = \frac{20}{3}$$

$\therefore P = 10 - Q \Rightarrow P = 10 - \frac{20}{3} = \frac{10}{3}$

$\therefore$ Market Quantity $= 20/3$
Market Price $= 10/3$

Since the firms have the same cost function and hence the same supply curve, each firm's supply quantity

$$Q_i = Q/2 = \frac{10}{3}$$

$\therefore$ Each firm's cost $= \frac{1}{2} \times \left(\frac{10}{3}\right)^2 = \frac{50}{9}$

$\therefore$ Profit $=$ Revenue $-$ Cost

$$= \frac{10}{3} \times \frac{10}{3} - \frac{50}{9} = \frac{50}{9}$$

$\therefore$ Equilibrium Price $= 10/3$
Equilibrium Market Supply $= 20/3$
Equilibrium Firm Supply $= 10/3$ for each
Equilibrium Firm Profit $= 50/3$

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2.1 Given

- $c(0) = F$
- $c(q)$ is increasing in q
- $MC = c'(q)$ is first decreasing then increasing

We have

$$AVC = \frac{c(q) - c(0)}{q} = \frac{c(q)}{q} - \frac{F}{q}$$

$$AVC' = \frac{c'(q)}{q} - \frac{c(q)}{q^2} + \frac{F}{q^2}$$

$$\therefore AVC' = \frac{MC - AVC}{q}$$

This implies that

When $MC = AVC$, $AVC' = 0$

$MC < AVC$, $AVC' < 0$

$MC > AVC$, $AVC' > 0$

$\therefore$ AVC first decreases, reaches a minimum and then starts rising

$\Rightarrow$ AVC is a convex function

$\Rightarrow$ $AVC''(q) \geq 0 \; \forall q$

Now,

$$AVC' = \frac{MC - AVC}{q}$$

$$\Rightarrow MC(q) = q\,AVC'(q) + AVC(q)$$

$$\Rightarrow MC'(q) = 2\,AVC'(q) + AVC''(q)$$

Let q_0 be the point where MC and AVC meet.

$\therefore AVC'(q_0) = 0$ [Since AVC reaches a minimum at q_0]

also, $AVC''(q_0) \geq 0$

$\therefore MC'(q_0) \geq 0$

$\therefore$ The part where MC & AVC cross can only lie on the upward sloping part of the MC curve

2.2 Let q_1 be the point where MC and AVC meet
Let q_2 be the point where MC and AC meet

From lecture notes and proof in 2.1, we know that

$AC(q_2)$ is the minimum value of AC
$AVC(q_1)$ is the minimum value of AVC

Furthermore,

$$AC(q) = AVC(q) + F/q$$

Consider the below

$$AC(q_2) - AC(q_1) = AVC(q_2) + F/q_2 - [AVC(q_1) + F/q_1]$$

$$AC(q_2) - AC(q_1) = AVC(q_2) - AVC(q_1) + \frac{F}{q_2} - \frac{F}{q_1}$$

Since $AC(q_2)$ is the minimum value of AC, $AC(q_2) - AC(q_1)$ is some negative value N

Similarly, since $AVC(q_1)$ is the minimum value of AVC, $AVC(q_2) - AVC(q_1)$ is some positive value P

$\therefore$ We get

$$N = P + \frac{F}{q_2} - \frac{F}{q_1}$$

$$\Rightarrow N = P + \frac{F}{q_1 q_2}[q_1 - q_2] \quad (1)$$

Now, N is negative, but P & $\frac{F}{q_1 q_2}$ are positive $\Rightarrow$ (1) holds iff $q_1 - q_2$ is negative

$\therefore q_1 - q_2 < 0 \Rightarrow q_1 < q_2 \Rightarrow MC(q_2) > MC(q_1)$
(As MC is increasing)

Hence, the point where MC crosses AC is greater than the point where MC crosses AVC

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2.3 We know that

$$AVC = \frac{c(q) - c(0)}{q} = \frac{c(q)}{q} - \frac{F}{q}$$

$$\therefore AVC' = \frac{c'(q)}{q} - \frac{c(q)}{q^2} + \frac{F}{q^2}$$

$$= \frac{c'(q)}{q} - \frac{1}{q}\left[\frac{c(q) - F}{q}\right]$$

$$= \frac{c'(q)}{q} - \frac{AVC}{q}$$

$$\therefore AVC'(q) = \frac{MC - AVC}{q}$$

$\therefore$ When $MC > AVC$, $AVC'(q) > 0$

$MC = AVC$, $AVC'(q) = 0$

$MC < AVC$, $AVC'(q) < 0$

$\therefore$ Before MC & AVC meet, MC is below AVC and AVC is decreasing
After meeting, MC is greater than AVC and AVC is increasing

$\therefore$ When AVC & MC meet, AVC is minimum

3 We have,

$$\hat{\beta}_1 | x_1, x_2, \ldots x_n = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n} (x_i - \bar{x})^2}$$

$$Var(\hat{\beta}_1 | x_1, x_2, \ldots x_n) =$$

$$= Var\left[\frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sum_i (x - x_i)^2}\right]$$

Substituting $y_i = \beta_0 + \beta_1 x_i + e_i$

$$= Var\left[\frac{\sum (x_i - \bar{x})(\beta_0 + \beta_1 x_i + e_i - \bar{y})}{\sum (x - x_i)^2}\right]$$

$$= \cancel{Var}$$

$$= \frac{1}{[\sum (x - x_i)^2]^2} Var\Big[\beta_0 \sum (x_i - \bar{x}) + \cancel{\beta_1 \bar{x}} + \beta_1 \sum x_i (x_i - \bar{x}) - \bar{y} \sum (x_i - \bar{x}) + \sum e_i (x_i - \bar{x})\Big]$$

Of the four terms, only the last term contains a random variable (e_i)

$\therefore$ The first three terms have 0 variance as they are constants

$$\therefore Var(\hat{\beta}_1 | x_1, x_2, \ldots x_n) = \frac{Var\left(\sum e_i (x_i - \bar{x})\right)}{[\sum (x - x_i)^2]^2}$$

$$= \frac{\left[\Sigma(x_i - \bar{x})\right]^2 Var(e_i)}{\left[\Sigma(x_i - \bar{x})^2\right]^2}$$

$$= \frac{Var(e_i)}{\Sigma(x_i - \bar{x})^2} = \frac{\sigma^2}{\Sigma(x_i - \bar{x})^2}$$

$$\therefore Var(\hat{\beta}_1 | x_1, x_2 \dots x_n) = \frac{\sigma^2}{\hat{\sum\limits_{i=1}}(x_i - \bar{x})^2}$$

Where σ^2 is the variance of the error term e_i

Now,

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

$$\Rightarrow Var(\hat{\beta}_0) = Var(\bar{y}) + (\bar{x})^2 Var(\hat{\beta}_1) + 2\bar{x}\, Cov(\bar{y}, \hat{\beta}_1)$$

$$Cov(\bar{y}, \hat{\beta}_1) = Cov\left[\frac{1}{n}\Sigma y_i, \frac{\Sigma(x_j - \bar{x})y_j}{\Sigma(x_i - \bar{x})^2}\right]$$

$$= \frac{1}{n\,\Sigma(x_i - \bar{x})^2} Cov\left[\Sigma y_i, \Sigma(x_j - \bar{x})y_j\right]$$

$$= \frac{1}{n\,\Sigma(x_i - \bar{x})^2} \times \Sigma(x_j - \bar{x})\, Cov(y_i, y_j) = 0$$

$\because \Sigma(x_i - \bar{x})$ is 0

$$\therefore Var(\hat{\beta}_0) = Var(\bar{y}) + \bar{x}^2 Var(\hat{\beta}_1)$$

$$= \frac{\sigma^2}{n} + \frac{\bar{x}^2 \sigma^2}{\Sigma(x_i - \bar{x})^2}$$

$$= \frac{\sigma^2 \left[\Sigma(x_i^2 + \bar{x}^2 - 2x_i\bar{x})\right] + n\bar{x}^2\sigma^2}{\Sigma(x_i - \bar{x})^2}$$

$$= \frac{\sigma^2 \Sigma x_i^2 + n\bar{x}^2\sigma^2 - \sigma^2 \bar{x} \Sigma x_i + n\bar{x}^2\sigma^2}{\Sigma(x_i - \bar{x})^2}$$

$$= \frac{\sigma^2 \Sigma x_i^2 + 2n\bar{x}^2\sigma^2 - 2n\bar{x}^2\sigma^2}{\Sigma(x_i - \bar{x})^2} \quad \left(\text{Since } \Sigma x_i = n\bar{x}\right)$$

$$= \frac{\sigma^2 \Sigma x_i^2}{\Sigma(x_i - \bar{x})^2}$$