INSTRUCTION MANUAL

Minimization of connected Mealy & Moore machines

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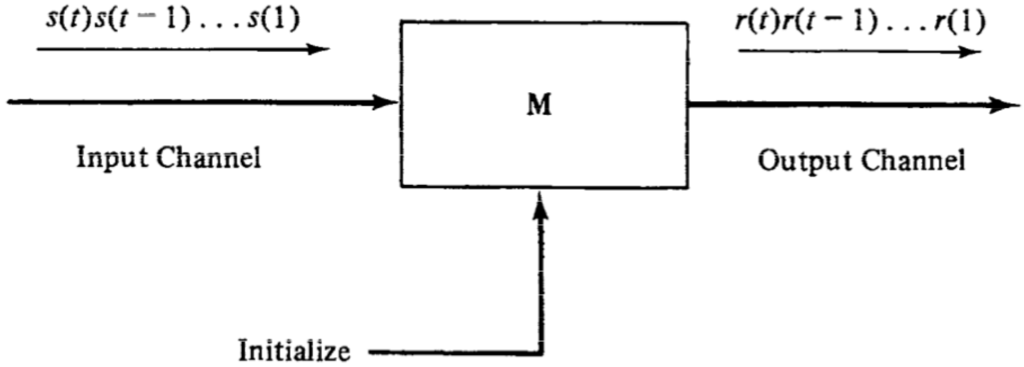
FINITE STATE MACHINES

Mathematical description:

A finite state machine M consists of:

- Finite sets S, R y Q, where S is a finite input alphabet, R is a finite output alphabet and Q is a set of states.
- A state transition function f that gives the next state of M in terms of the current state and the next input symbol.
- An output function g that gives the next output symbol of M in terms of the current state and the next input symbol.
- A predetermined initial state q(0) = q1, where $q_1 \in Q$, in which M is placed prior to instant t = 0.



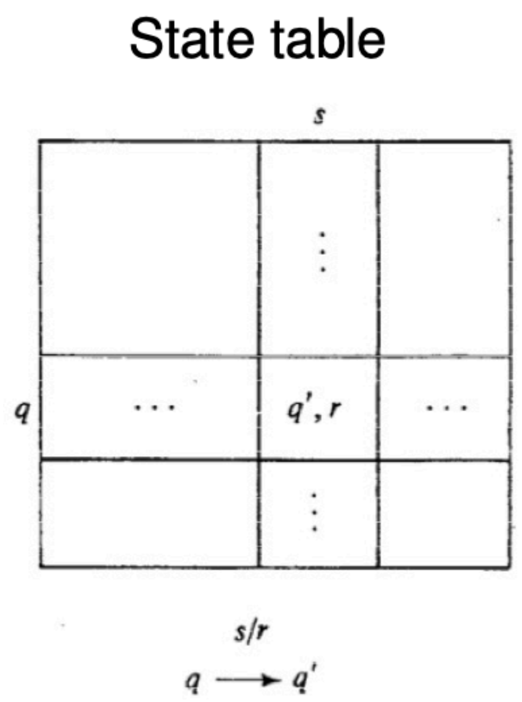
MEALY MACHINE

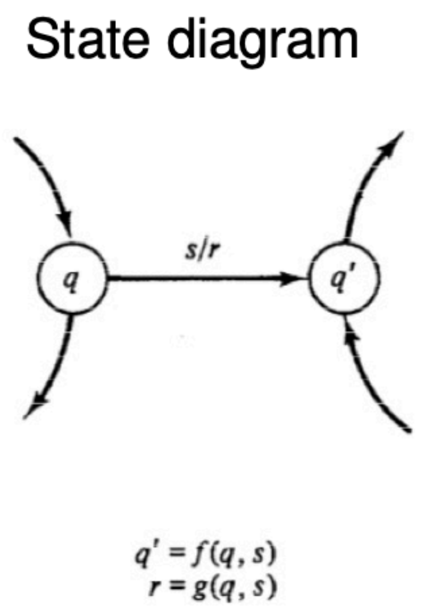
Definition of a Mealy machine

A transition assigned finite-state machine is 6−tuple where:

- Q is a finite set of internal states.
- S is a finite input alphabet.
- R is a finite output alphabet.
- f is the state transition function f: Q × S −→ Q
- g is the output function g: Q × S −→ R
- q_1 ∈ Q is the initial state.

Representations of a Mealy machine





MOORE MACHINE

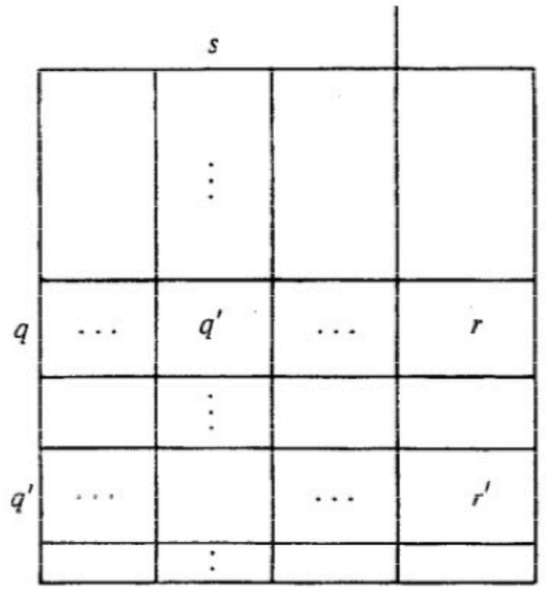
Definition of a Moore machine

A state assigned finite-state machine is 6−tuple where:

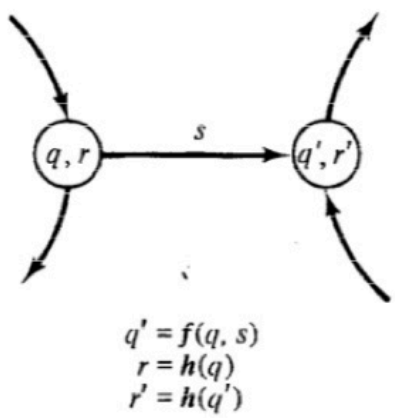
- Q is a finite set of internal states.
- S is a finite input alphabet.
- R is a finite output alphabet.
- f is a state transition function f: Q × S → Q
- h is an output function h: Q → R
- q_1 ∈ Q is the initial state.

Representations of a Moore machine

State table



State diagram



FINITE STATE AUTOMATA MINIMIZATION ALGORITHM

Step 1. Obtain the equivalent connected automata, eliminating all states that are not accessible from the initial state.

Step 2. Perform the partitioning algorithm on the equivalent connected automata produced from the previous step:

> **Step 2a.** Form an initial partition P1 of Q. Grouping together states that are 1-equivalent, that is, states that produce identical outputs for each input symbol:
>
> - For a Mealy automaton: the states q and q′ are in the same block of P_1 if, and only if, for each $s \in S$, $g(q, s) = g(q', s)$.
> - For a Moore automata: the states q and q′ are in the same block of P_1 if, and only if, for each $s \in S$, $h(q) = h(q')$.
>
> **Step 2b.** Obtain P_{k+1} from Pk as follows: states q and q′ are in the same block of P_{k+1} if, and only if,
>
> 1. They are in the same P_k block.
> 2. For each $s \in S$ its successors s $f(q, s)$ and $f(q', s)$ are in the same P_k block.
>
> **Step 2c.** Repeat step 2b until P_{m+1} = Pm for some m. We call PM
>
> to the final partition of Q.

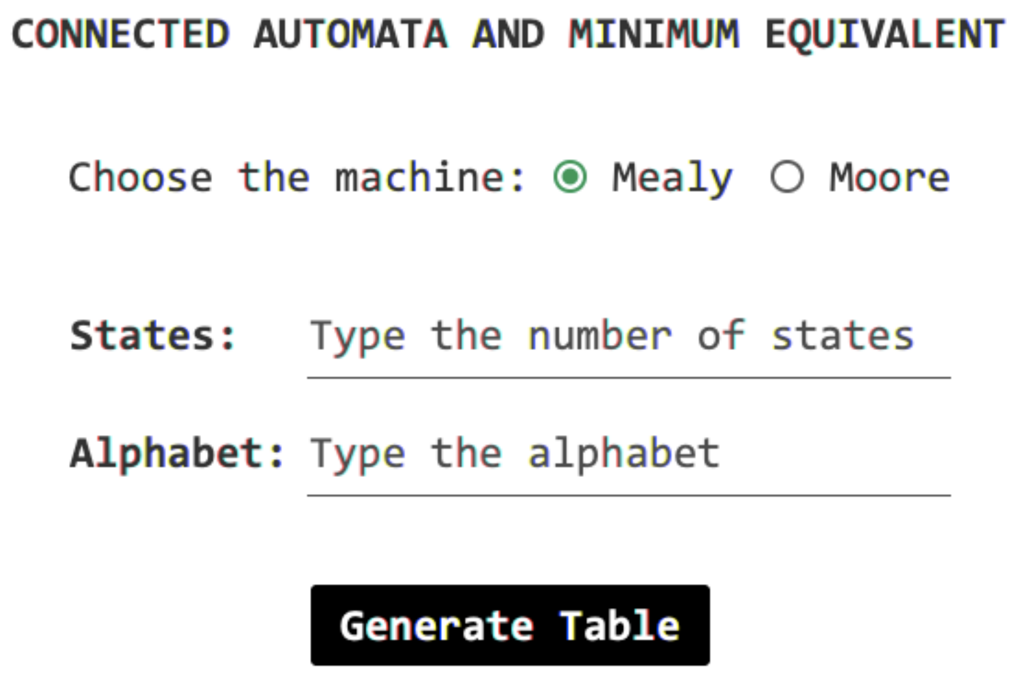
Step 3. Each one of the blocks of the final partition P_f, produced by the previous step, would correspond to an equivalent minimal automata state. The initial state of this new automata will be the block that contains the initial state of the original automata. From these new states, its corresponding table of states is obtained by applying the following rules:

1. To find the successor s of the state q′ in M′ select any state in the block of the partition P_f corresponding to q′ and find the block containing its successor s; the state corresponding to M′ is the successor s of q′.

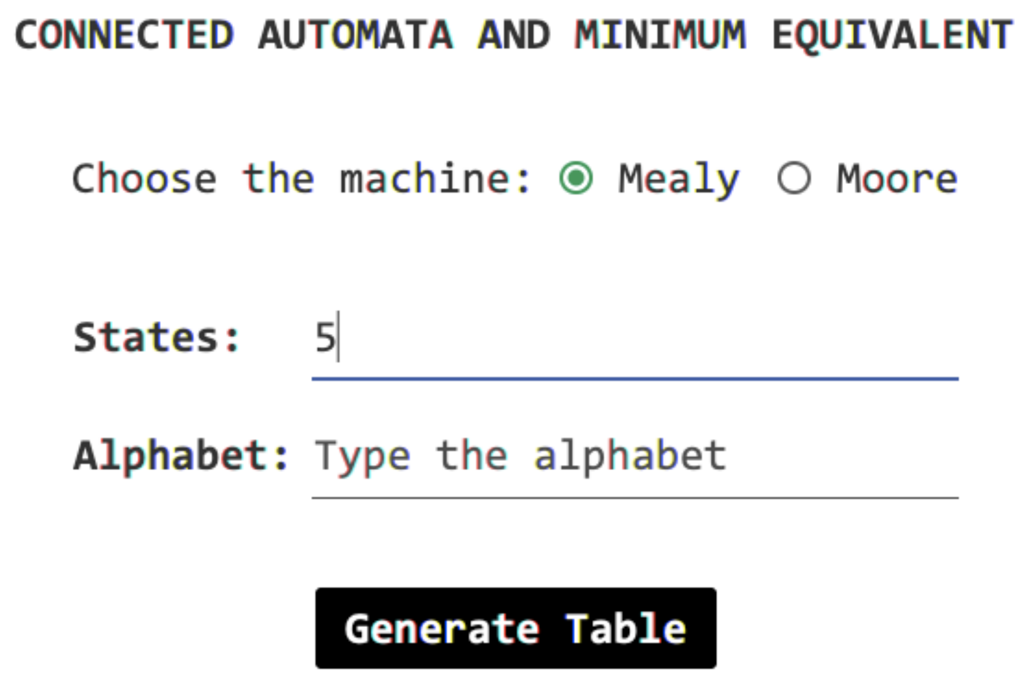
2. The output for a transition s from state q′ of M′ is the output for a transition s for any state in the block corresponding to q′.

PROGRAM OPERATION

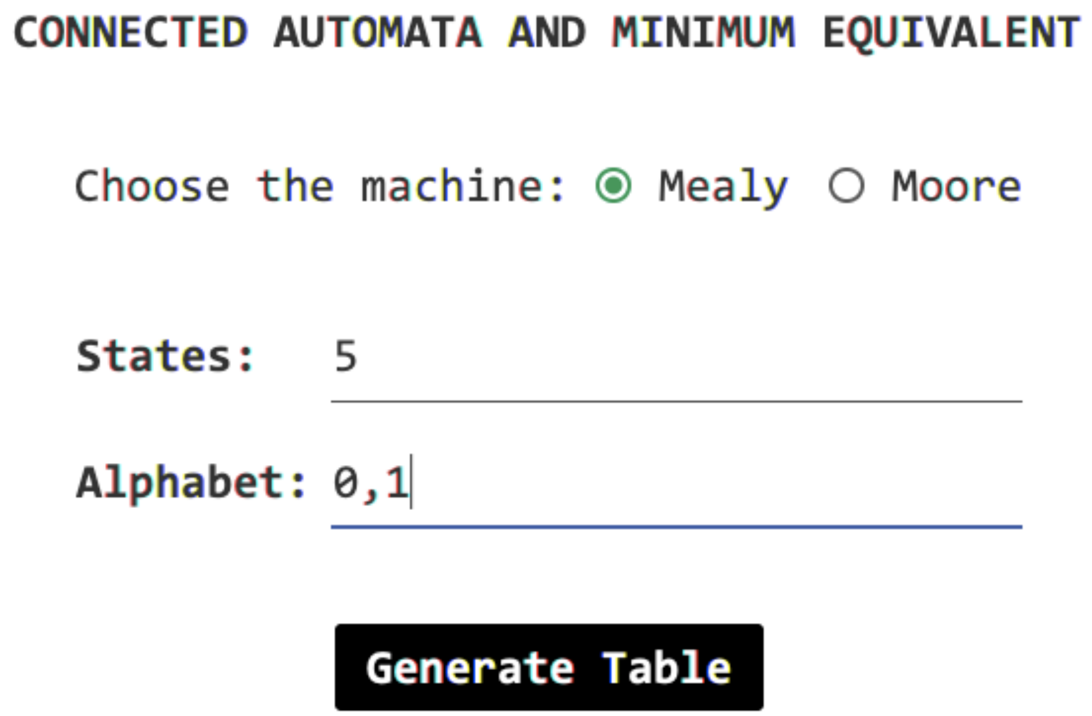
Step 1 – Choose the type of machine: Mealy or Moore.



Step 2 – Enter the number of states.

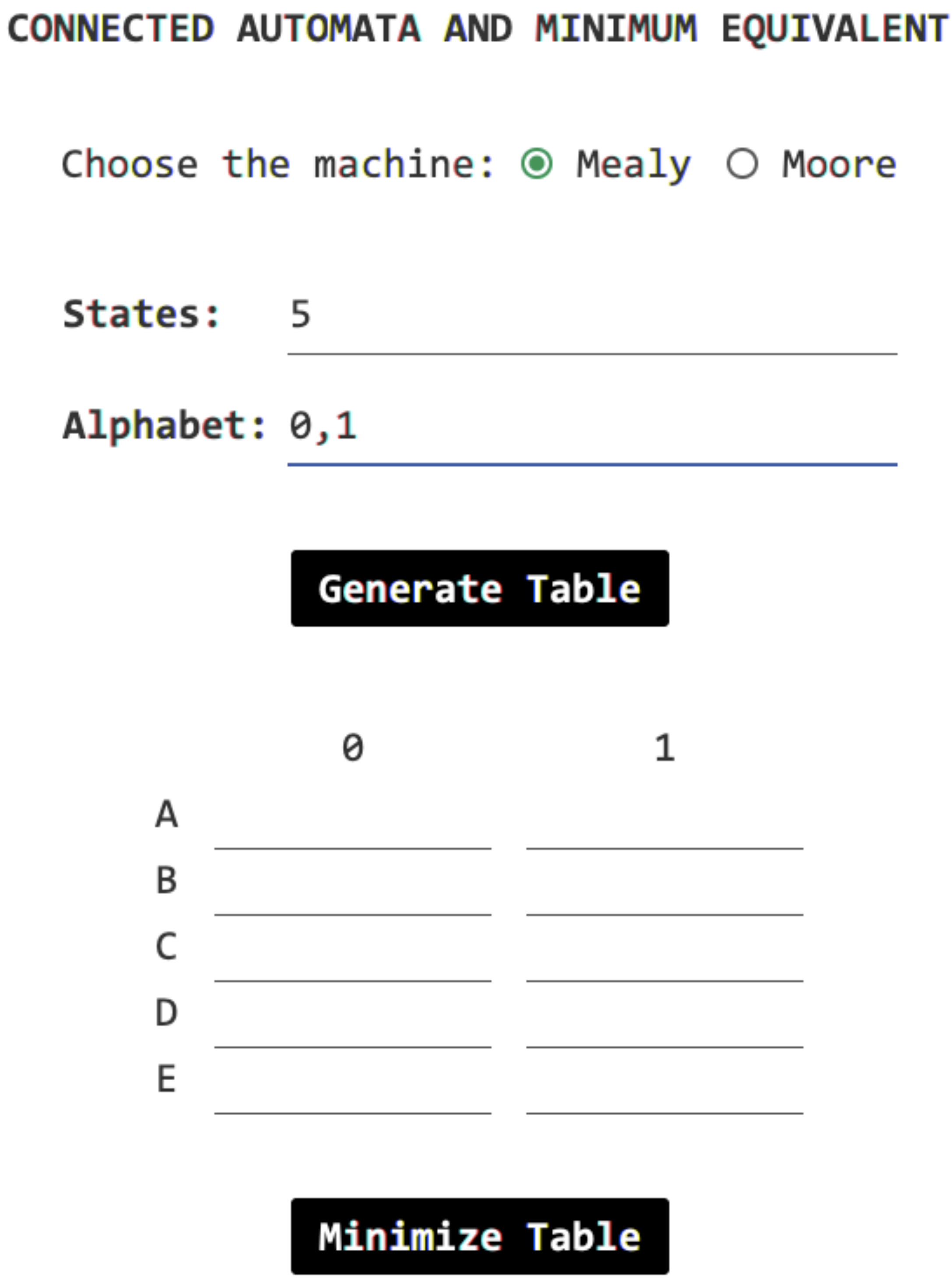


Step 3 – Enter the alphabet: each input symbols must be separated by a coma.

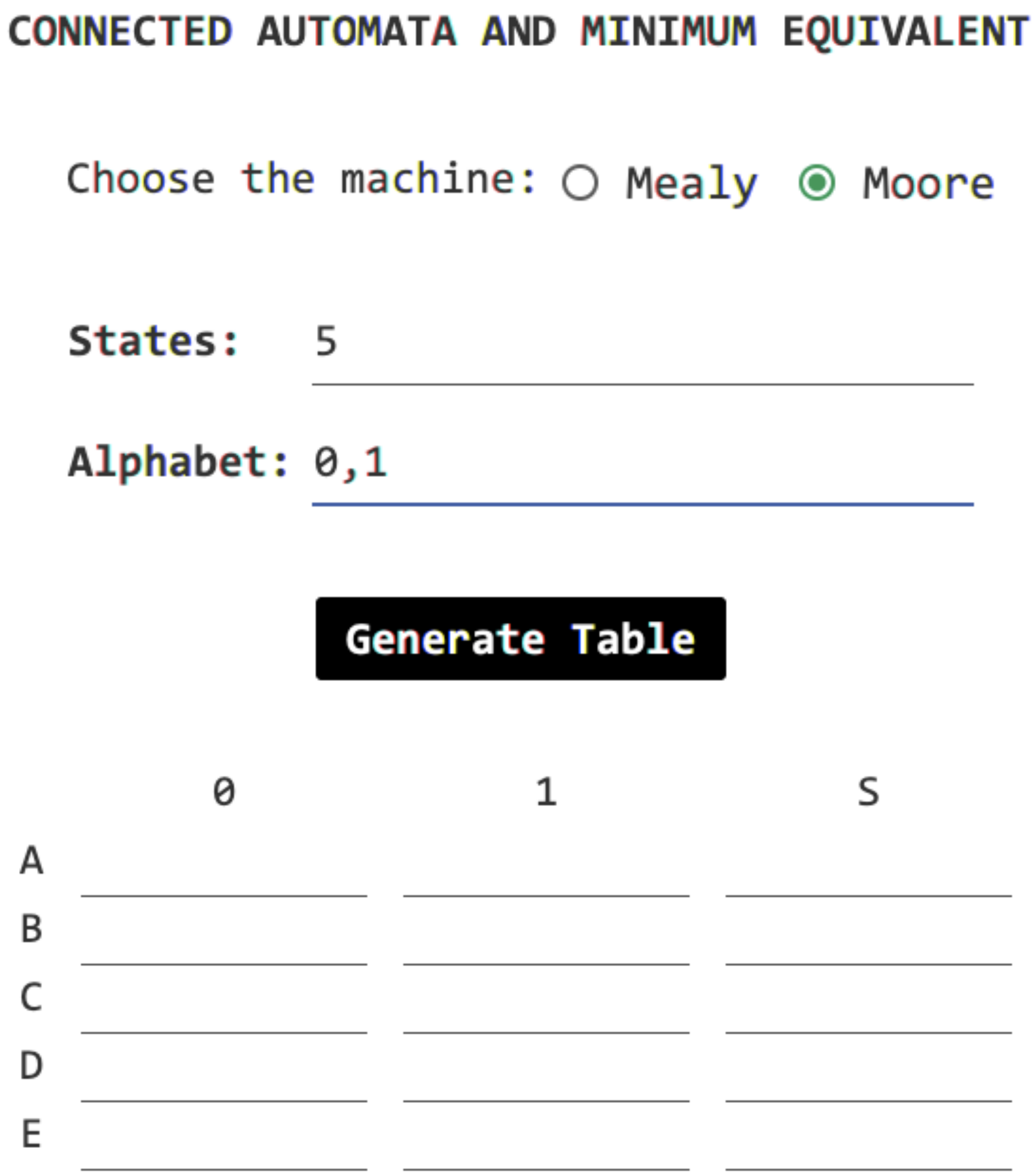


Step 4 – Press the button "Generate Table": an input table M will be generated, according with the type of machine chosen. A button called "Minimize Table" will appear too.

<u>Mealy:</u>



Moore:



Step 5 – Enter the desired values for each state and input symbol. For Mealy machines, in the columns of the input symbol (column 0 and column 1 in this example), first enter the output state, then a coma and finally the output symbol. For Moore machines, just enter the state in the columns of the input symbol; in the column S, enter the output symbols.

In this case, we stayed with the Mealy machine for this example. So, random values were entered in the input table M, and 0 and 1 were also used as the unique output symbols.

CONNECTED AUTOMATA AND MINIMUM EQUIVALENT

Choose the machine: ◉ Mealy ○ Moore

States: 5

Alphabet: 0,1

Generate Table

	0	1
A	B,0	A,0
B	A,0	C,1
C	C,1	A,0
D	B,0	E,1
E	A,1	D,0

Minimize Table

Step 6 – Press the button "Minimize Table": two tables will be generated. One for showing the final blocks obtained after the partitioning algorithm, and another one for the final table M' with the renamed states.

In this case, V corresponds to {A}; W corresponds to {B}; X corresponds to {D}; Y corresponds to {C}; and Z corresponds to {E}. So, in general, the name of a state in M’ in the column “New Names” corresponds to the block {Q’, Q’’, ...} in the same row of the intermediate table.

CONNECTED AUTOMATA AND MINIMUM EQUIVALENT

Choose the machine: ◉ Mealy ○ Moore

States: 5

Alphabet: 0,1

Generate Table

	0	1
A	B,0	A,0
B	A,0	C,1
C	C,1	A,0
D	B,0	E,1
E	A,1	D,0

-->

Blocks	0	1
{A}	{B}, 0	{A}, 0
{B}	{A}, 0	{C}, 1
{D}	{B}, 0	{E}, 1
{C}	{C}, 1	{A}, 0
{E}	{A}, 1	{D}, 0

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New Names	0	1
V	W, 0	V, 0
W	V, 0	Y, 1
X	W, 0	Z, 1
Y	Y, 1	V, 0
Z	V, 1	X, 0

Minimize Table