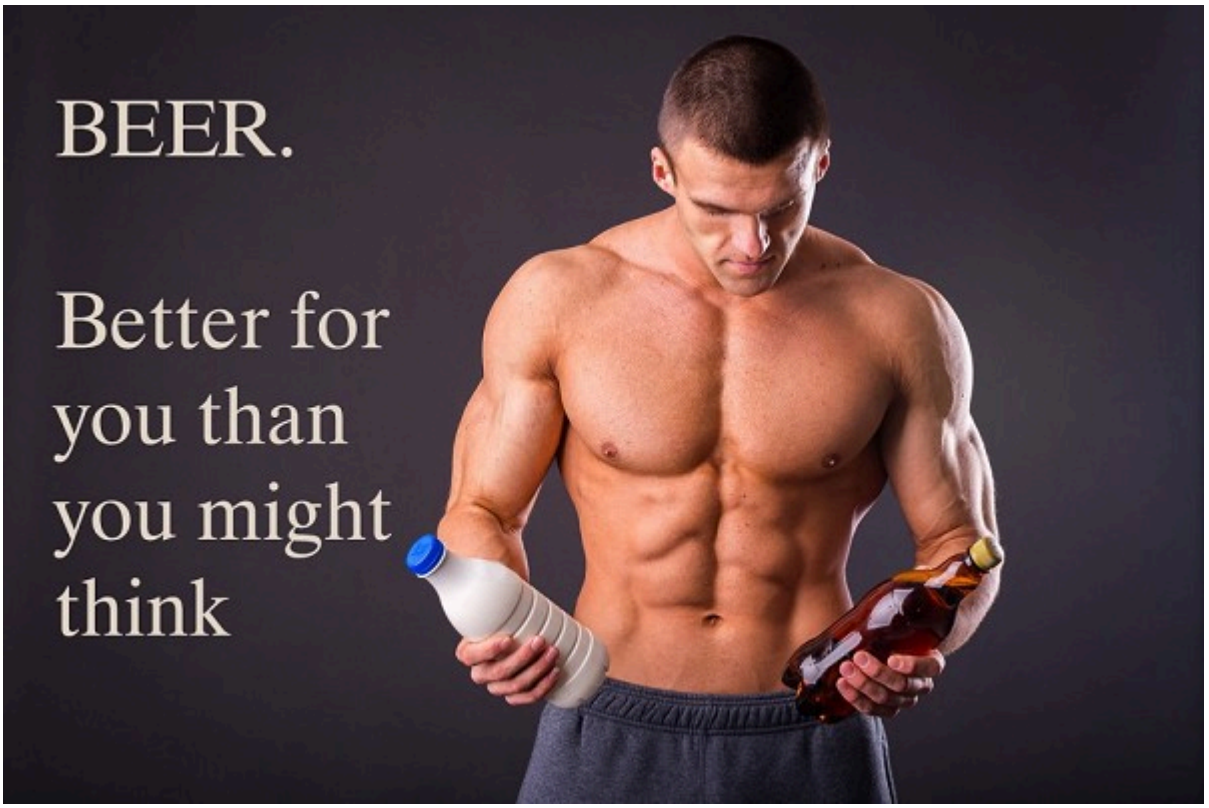
Alcohol Modelling using Differential Equations



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Introduction

When alcohol is consumed, it enters the gastrointestinal (GI) tract, where it is absorbed into the bloodstream and distributed to various tissues and fluids. Most alcohol absorption occurs in the small intestine, so drinking with food, especially those rich in fat and protein, can significantly slow this process. Once alcohol enters the bloodstream, it travels to the liver, where it is metabolized and eliminated from the body.

Men and women process alcohol differently due to differences in body composition, with men having a higher water content and lower body fat. The blood alcohol concentration (BAC) is measured in grams of alcohol per 100 ml of blood. For example, 0.1 grams per 100 ml corresponds to a BAC of 0.1. In some countries, this is also expressed as 0.1%, or 1 gram of alcohol per liter of blood.

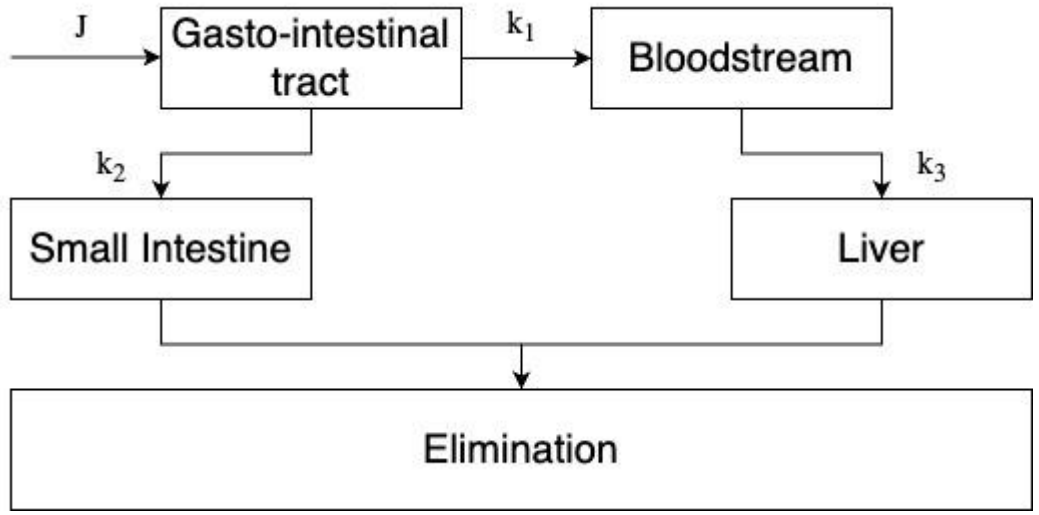
Blood Alcohol Concentration (BAC)	Typical Effects
0.02% - 0.04%	Buzzed - lower inhibitions, relaxed, altered mood, some loss of judgment
0.05% - 0.07%	Impaired judgment, lowered alertness, exaggerated behavior, euphoria
0.08% - 0.10%	Legally impaired - impaired balance, speech, vision, reaction time, hearing, judgment, and self-control
0.12% - 0.14%	Mood shifts, poor coordination, balance disoriented, anxiety or unease, impaired motor skills, severely impaired judgment and perception
0.15% - 0.19%	Lack of muscle control, significant loss of motor skills, nausea and vomiting, difficulty standing and walking, blackout symptoms
0.20% - 0.24%	Loss of consciousness, non-responsive, respiratory distress, cardiac distress, lack of comprehension, severely impaired mental/physical/sensory function
0.25% - 0.39%	Onset of coma, risk of death
0.40% - 0.45%	Lethal dose for most people

The following two differential equations are to be used for modelling the alcohol levels, G(t) is the function of Gut BAC over time and B(t) is the function of Blood BAC over time:

$$\frac{dG}{dt} = J - (k_1 + k_2)G(t)$$

$$\frac{dB}{dt} = k_1 G(t) - \frac{k_3 B(t)}{B(t)+M}$$

Chart, explaining the variables in the equation:



- J — alcohol inflow
- k_1 — alcohol entering the bloodstream
- k_2 — alcohol entering the small intestine (elimination)
- k_3 — alcohol entering the liver (elimination)
- M — randomly picked number

There are 2 types of modelling: continuous and non-continuous. For the non-continuous we set the J to zero and use conditions for time and amount of alcohol consumption. For the continuous one we set $J = \frac{alc.}{10 * v} * \frac{60}{mins}$, where v is body fluid volume, alc is alcohol consumed in grams and mins means that the alcohol will be consumed every x minutes.

Task 1: Realistic Scenario

Introduction

In this task there will be 5 different scenarios of drinking pattern, gender, and body state. The reason for this is to determine which of the different scenarios will Sufis as the legal drinking limit to drive.

BAC table & Different situations

Drink Type	Standard Vol	Alcohol %	BAC per kg for men	BAC per kg for women
Beer	500ml	5%	0.00042	0.00058
Vodka	50ml	40%	0.00034	0.00048
Tequila Rose	100ml	15%	0.00039	0.00054
Cider	500ml	4.5%	0.00038	0.00076

Boris: 546ml of beer per hour for a 70kg man, continuous drinking, full stomach,

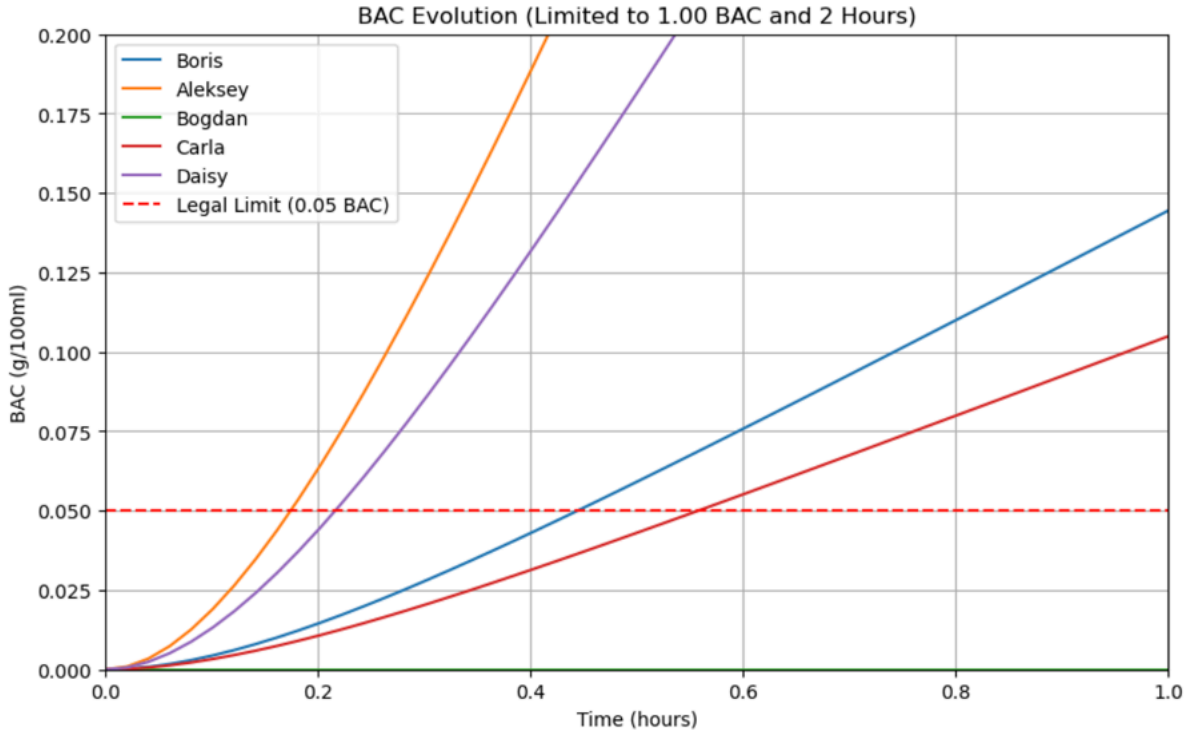
Aleksey: 3500ml of beer per hour for a 109kg man, continuous drinking, full stomach,

Bogdan: 150ml of vodka per hour for an 84kg man, 3 shots quickly, empty stomach,

Carla: 100ml of tequila rose per hour for a 61kg woman, two shots, full stomach,

Daisy: 1100ml of cider per hour for a 52kg woman, continuous drinking, full stomach.

Evaluation



Aleksey has the biggest spike which can mean that the type of drink he consumes alongside obesity does significantly affect him. Daisy exceeds the limit after approximately 15 minutes of drinking, since cider affects women faster than it does for men. Boris exceeds the limit after around 26 minutes, there are mainly 2 reasons why Boris took a longer time to exceed the limit than Aleksey, first is the amount of beer consumed and second is the weight. Carla drank 2 quick shots of tequila rose on a full stomach hence exceeding the limit of legally driving after 34 minutes. Lastly Bogdan did not exceed the limit despite drinking 3 quick shots of Vodka, proving that if a man drinks on an empty stomach, he will not exceed that legal limit. Lastly, the table above proves that gender does indeed have different effects for different types of alcohol.

Conclusion

To conclude, the main factor that can lengthen the time for a person to exceed the driving limit is the emptiness of the stomach, as the graph above demonstrated that only Bogdan did not exceed the limit. Nonetheless there are two other important factors that should be taken

into place, first gender which is clear in Daisys' case, second is the amount consumed which can be clear in Alekseys' case.

Task 2: Tourist Death Case

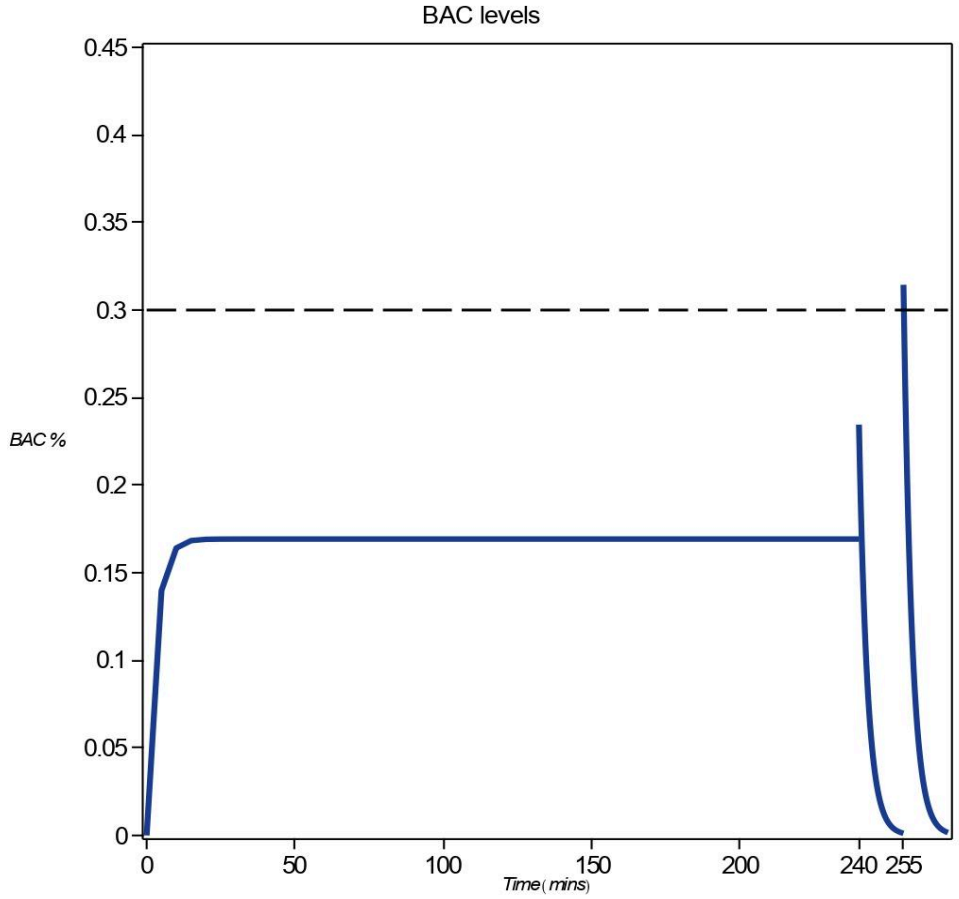
Introduction

In 2008 a tourist Graham Parish, celebrating his birthday in an Irish hotel tragically passed away due to acute alcohol poisoning. Our task was to simulate his BAC levels during his last hours.

Given information:

- He had around 10 pints of Guinness over the course of 3-4 hours, after which he encouraged a couple drinking competitions, one of which included a Guinness with 2 vodka shots and the latter a mixed drink with 8 shots of 4 different beverages.
- He was an Englishman, 42 years of age.
- Let's assume his weight at 84kg.
- Let's assume 25 mins between each pint and 15 mins between competitions.
- Following the table from the introduction we also assume that the death threshold for this person was 0.3% BAC.
- Other input values:
  - 0.82 volume of body fluids to weight ratio
  - $k_1 = 0.1$
  - $k_2 = 0.25$
  - $k_3 = \frac{8}{10*v}$, where v is volume of body fluids
  - $M = 0.005$

Results:



We notice a sharp increase when he began drinking and a steady upholding of the level as we used the continuous formula. At the 240th minute we see the first spike which is when Graham drank the Guinness accompanied by two vodka shots, which let the BAC levels reach around 0.24% and a very fast decrease afterwards, which is so sudden because of the use of the non-continuous formula. After 15 minutes we see the second spike, this time over the 0.3 limit, putting the poor celebrant in a condition he could not return from.

There are several issues with this model:

- The changes in BAC are too sharp and the plot is non-continuous.
- Continuous consumption is taken too literally, and it is therefore as if the person is drinking 0.379ml of Guinness every second for 4 hours.

Conclusion

We finalise that this was a poor choice of model for this case. Some of the input data was fixed and our team put high effort into picking the most appropriate numbers for the rest of input values.

Task 3: Model Comparison

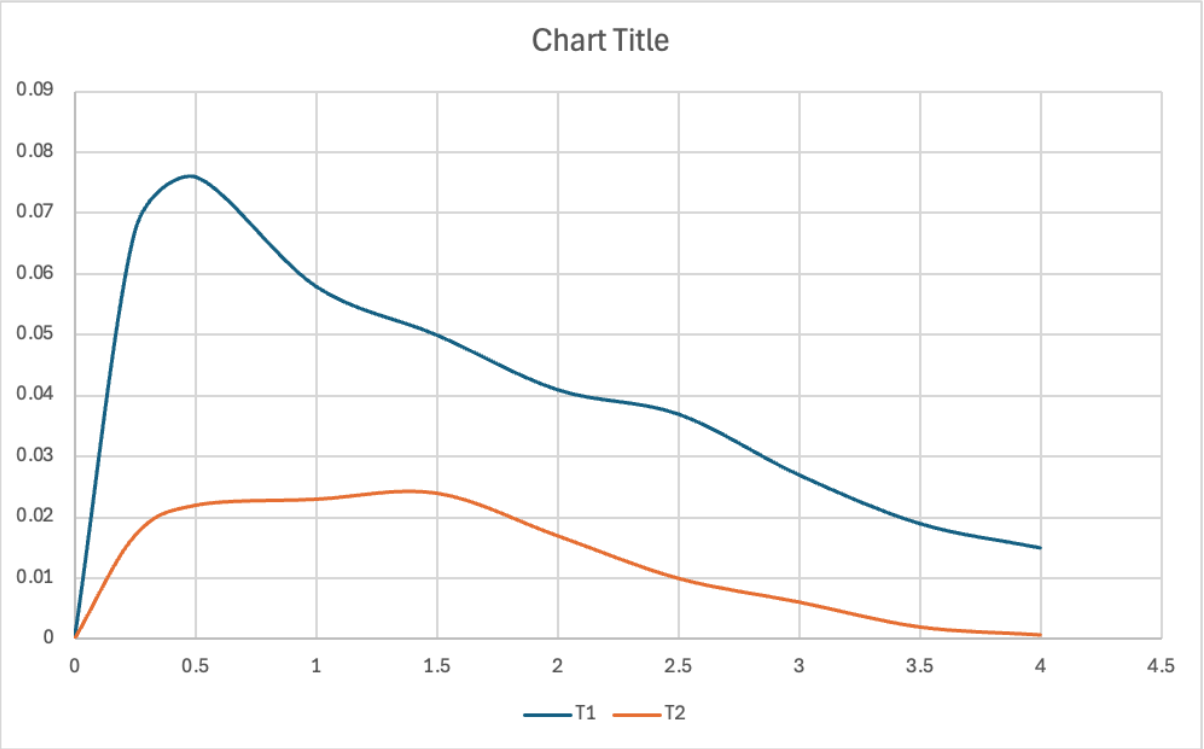
Introduction

The following task requires us to recreate an experiment with certain conditions using our model. The experiment was conducted on a healthy 29 year old male, weighing 75 kilograms. The man received 45 ml of 95% alcohol in a total volume of 150 ml made up of orange juice. The only difference was in Test 1 (T1) he was under fasting conditions, whereas Test 2 (T2) followed a steak meal. His BAC levels were then measured over the next few hours. By comparing these values we can see if our model is a good fit for the measured BAC levels (shown in the table below).

Values Comparison

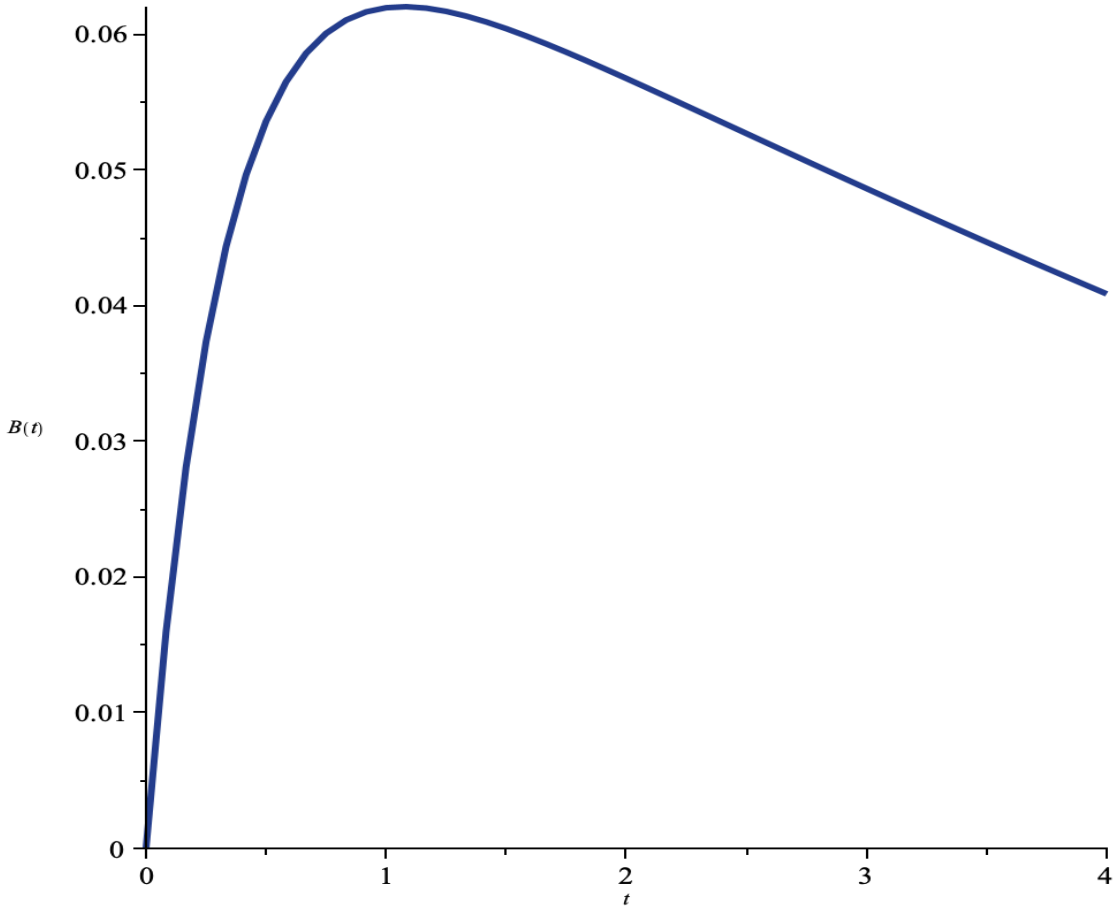
Recorded BAC levels from the experiment :

Time (hours)	T1	T2
0	0	0
0.25	0.067	0.017
0.5	0.076	0.022
1	0.058	0.023
1.5	0.05	0.024
2	0.041	0.017
2.5	0.037	0.01
3	0.027	0.0061
3.5	0.019	0.002
4	0.015	0.0007



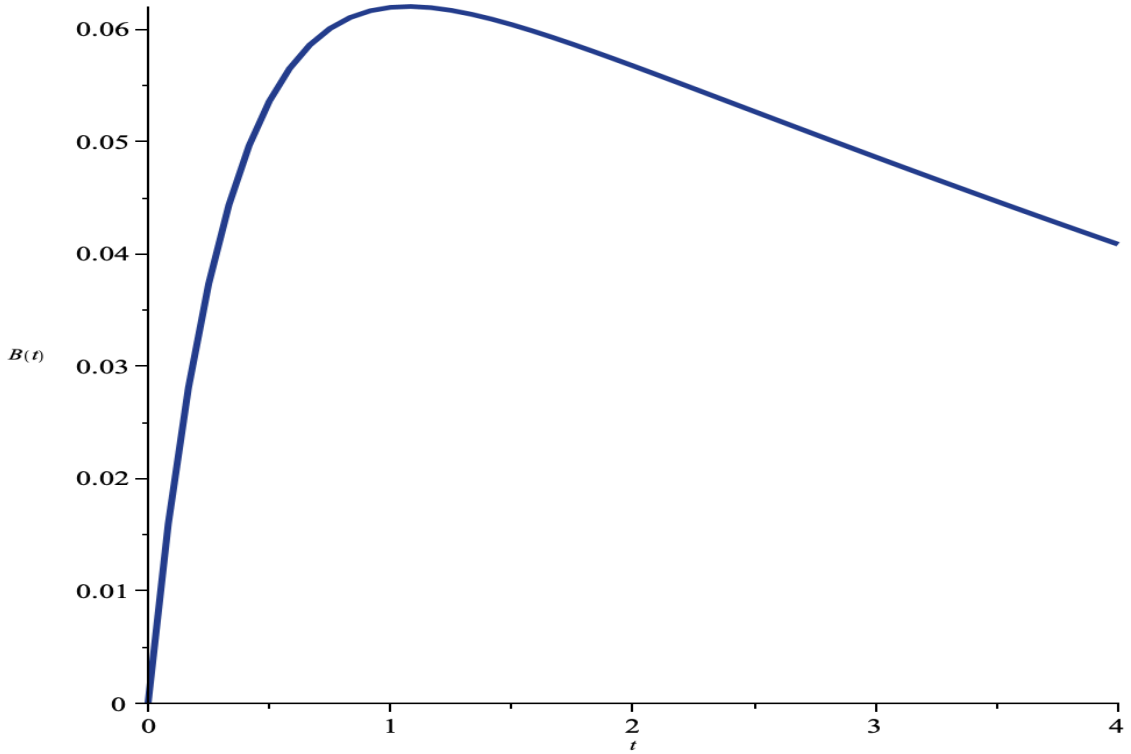
The above is a graph plot of the values in the table above. There is a clear spike in T1 but a gradual decrease afterwards. T2 has a clear surge in the first 30 minutes and gradually increases till it reaches 1.5hours, afterwards it gradually declines until the end.

T1



The model demonstrates a huge spike in comparison to the one given. It starts increasing from beginning to and tops the around the first hour then on forward slowly declines unlike the provided graph.

T2



for T2 a similar result can be seen, where the model values are again higher than the experiment values.

Conclusion

To conclude, our model has provided values higher than the experiment values for both T1 and T2. We can therefore say our model is not the best fit but it could roughly approximate BAC values (just not very accurately). However it would be unfair to assume that our model is a bad fit based on just one experiment. To really give a strong answer we would have to compare it to averages of multiple experiments. Comparing our model to more data will give us a more definite answer to whether or not it is a good fit for accurately approximating BAC levels.

Resources

- Introduction
  - Mathematical Modelling with Case Studies, B. Barnes, G.R. Fulford, CRC Press, 2002.
  - [2] Fasting and Nonfasting Blood Ethanol Concentrations Following Repeated Oral Administra- tion of Ethanol to One Adult Male Subject, P.K. Wilkinson et. al., Journal of Pharmacokinetics and Biopharmaceutics, Vol. 5, No. 1, 1977.
- Task 2
  - average UK weight
  - news article 1
  - news article 2
  - news article 3

Contributions

Vladimir Filatov	Introduction, Task 2, Presenting
Abdullah Alrubian	Task 1, Task 3
Kristian Don	Task 1 , Task 3