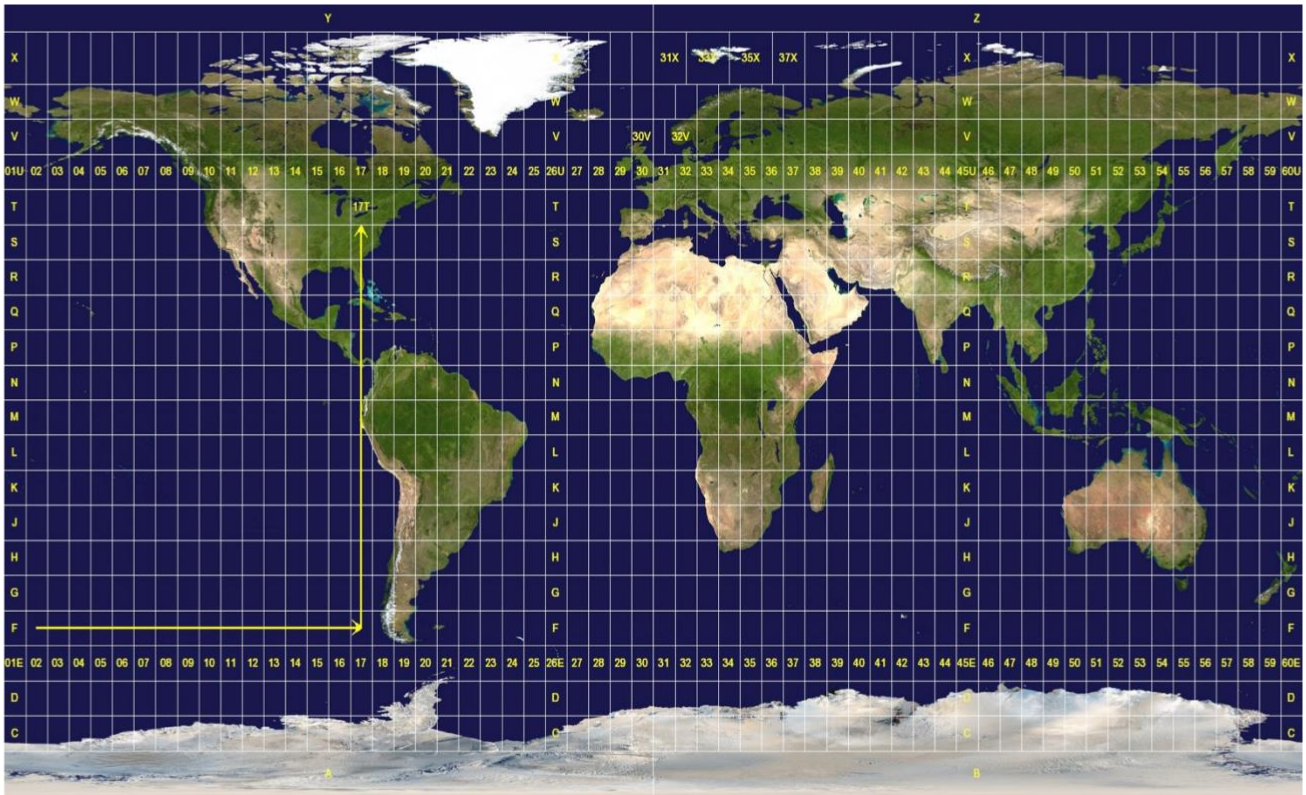
Project 3: Periodic Models



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TU874-1

Introduction:

As a means for us to construct periodic models describing sunrise, sunset and day length at three locations we had to choose three different cities:

- Northern Hemisphere - Dublin.
- Close to the Equator- Singapore
- Close to North Pole-Finnmark

The latitudes of these cities are;

- Dublin with a latitude of 53°
- Singapore with a latitude of 1°
- Finnmark with a latitude of 70°

We chose a variety of cities to get a wide range of results to analyze.

Objectives:

1. Obtain a sinusoidal model of the form
   $A(t) = a + b \sin \omega (t - t0)$
   for the sunrise time in day for three cities.
2. Obtain a sinusoidal model for the sunset time for each of the three cities.
3. Obtain a sinusoidal model for the day length for each of the three cities.
4. For each of the three locations, compare the sunset, sunrise, and day length models.
5. Use these models to estimate the sunrise time, sunset time, and day length for 14 March 2024 in Dublin.
6. Calculate the errors (the differences between your estimations and the values given in the calendar for that date.

Assumptions:

1. 1^{st} January 2024 is where t = 0.
2. t = 172 on the Summer Solstice 21^{st} June.
3. t = 355 on the Winter Solstice 21^{st} December.
4. A(t) needs to be expressed as the number of hours after midnight.

Calculations

For all, we let:

$$\omega = \frac{2Pi}{366}, a = \frac{(\max + \min)}{2}, b = \frac{(\max - \min)}{2}, t \to a + b \cdot \sin(\omega \cdot (t - t_0))$$

Dublin:

Dublin Daylength:

$$\min = 7 + \frac{29}{60} + \frac{56}{3600}$$

$$\max = 17 + \frac{12}{3600}$$

$$t_0 = solve\left(\omega \cdot (172 - x) = \frac{Pi}{2}, x\right)$$

$$C = t$$

Dublin Sunrise:

min = 4 + $\frac{56}{60}$

max = 9 + $\frac{40}{60}$

$$t_0 = solve\left(\omega \cdot (364 - x) = \frac{Pi}{2}, x\right)$$

$$A = t$$

Dublin Sunset:

$$\min = 17 + \frac{6}{60}$$

$$\max = 21 + \frac{57}{60}$$

$$t_0 = solve\left(\omega \cdot (176 - x) = \frac{Pi}{2}, x\right)$$

$$B = t$$

Finnmark:

<u>Finnmark Daylength:</u>

$$\max \quad = 18 + \frac{29}{60} + \frac{03}{3600}$$

$$\min \quad = 6 + \frac{11}{60} + \frac{03}{3600}$$

$$t_0 = solve\left(\omega \cdot (172 - x) = \frac{Pi}{2}, x\right)$$

$$C = t$$

<u>Finnmark Sunrise</u>

$$\max \quad = 10 + \frac{15}{60}$$

$$\min \quad = 4 + \frac{8}{60} t_0 = solve\left(\omega \cdot (362 - x) = \frac{Pi}{2}, x\right)$$

$$A = t$$

<u>Finnmark Sunset:</u>

$$\max \quad = 22 + \frac{38}{60}$$

$$\min \quad = 16 + \frac{24}{60}$$

$$t_0 = solve\left(\omega \cdot (174 - x) = \frac{Pi}{2}, x\right)$$

$$B = t$$

Singapore:

<u>Singapore Daylength:</u>

$$\max \quad = 12 + \frac{11}{60} + \frac{46}{3600}$$

$$\min \quad = 12 + \frac{3}{60} + \frac{4}{3600}$$

$$t_0 = solve\left(\omega \cdot (172 - x) = \frac{Pi}{2}, x\right)$$

$$C = t$$

<u>Singapore Sunrise:</u>

$$\max \quad = 7 + \frac{16}{60}$$

$$\min \quad = 6 + \frac{46}{60}$$

$$t_0 = solve\left(\omega \cdot (41 - x) = \frac{Pi}{2}, x\right)$$

$$A = t$$

<u>Singapore Sunset:</u>

$$\max \quad = 19 + \frac{21}{60}$$

$$\min \quad = 18 + \frac{50}{60}$$

$$t_0 = solve\left(\omega \cdot (45 - x) = \frac{Pi}{2}, x\right)$$

$$B = t$$

Methods:

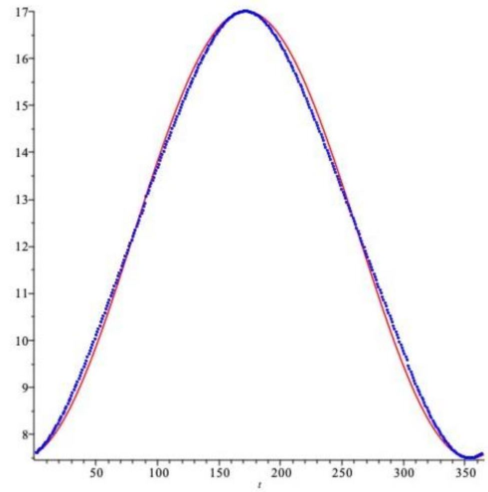
To get a clear view of our data we decided to graph the information we found in our sinusoidal methods against the actual times of the daylength, sunrise and sunset.

Results:

(we rounded all values to 2 significant points in the report for neatness)

Dublin:

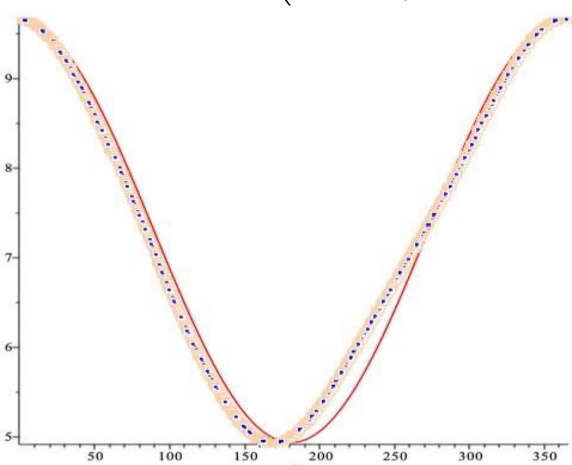
Dublin Daylength:



$$C = 12.25 + 4.75 \cdot \sin\left(0.017 \cdot \left(t - 0.017(172 - x)\right)\right)$$

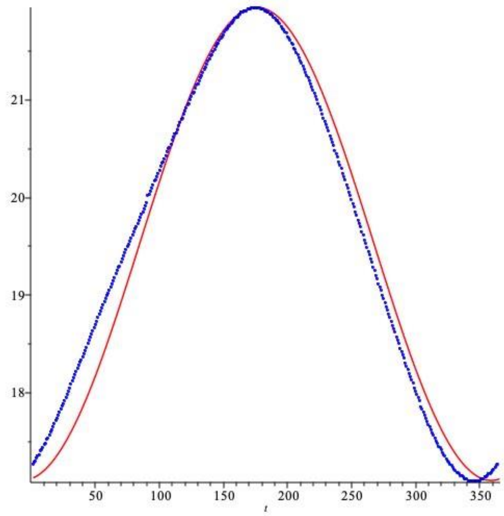
Dublin Sunrise:

$$A = 7.3 + 2.37 \cdot \sin\left(0.017 \cdot \left(t - \left(0.017 \cdot (364 - x)\right)\right)\right)$$



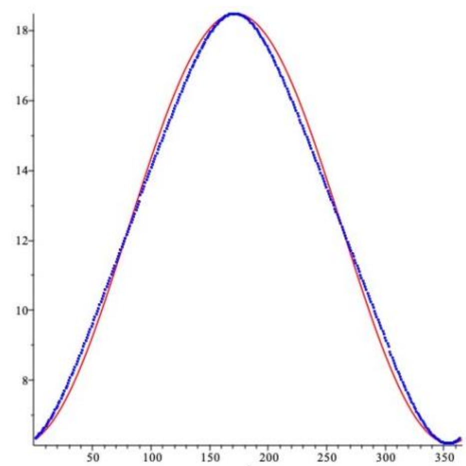
Dublin Sunset:

$B = 19.53 + 2.43 \cdot \sin\left(0.017 \cdot \left(t - 0.017(176 - x)\right)\right)$



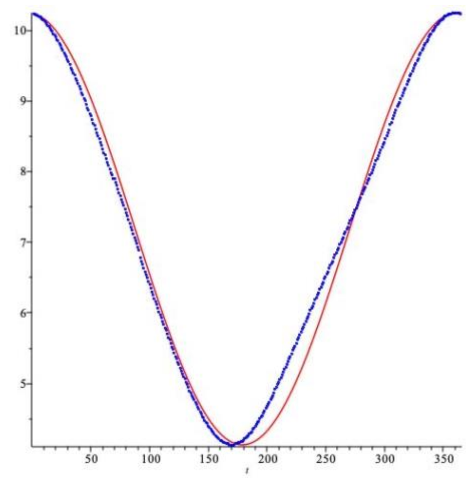
Finnmark:

<u>Finnmark Daylength:</u>



$C = 12.34 + 6.14 \cdot \sin\left(0.017 \cdot \left(t - 0.017(172 - x)\right)\right)$

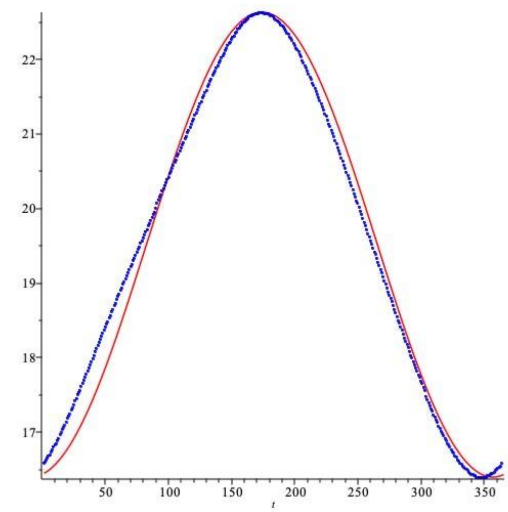
<u>Finnmark Sunrise:</u>



$$A = 7.19 + 3.058 \cdot \sin\left(0.017 \cdot \left(t - 0.017(362 - x)\right)\right)$$

<u>Finnmark Sunset:</u>

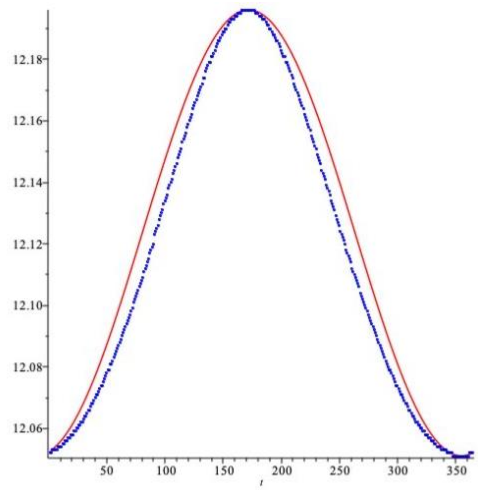
$$B = 19.52 + 3.12 \cdot \sin\left(0.017 \cdot \left(t - 0.017(174 - x)\right)\right)$$



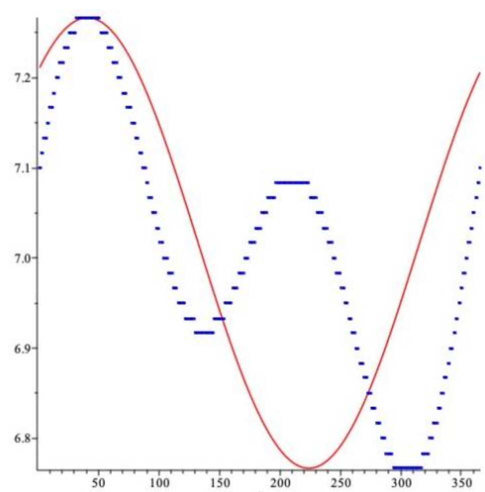
Singapore:

Singapore Daylength:

$$C = 12.12 + 0.073 \cdot \sin\left(0.017 \cdot \left(t - 0.017(172 - x)\right)\right)$$

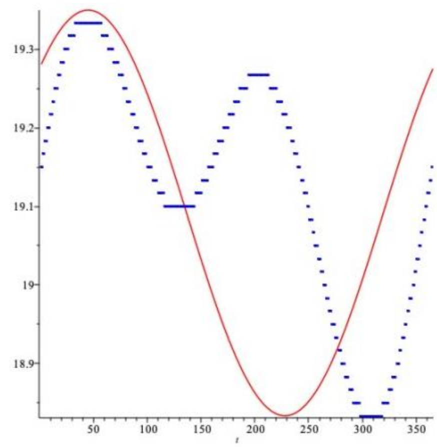


Singapore Sunrise:



$$A = 7.017 + 0.25 \cdot \sin\left(0.017 \cdot \left(t - 0.017(41 - x)\right)\right)$$

Singapore Sunset:



$$B = 19.092 + 0.26 \cdot \sin\left(0.017 \cdot \left(t - 0.017(45 - x)\right)\right)$$

March 14th 2024 (according to sinusoidal formula)

- Sunrise: 7:55:20
- Sunset: 19:05:25
- Day length: 11:43:19

Errors:

Dublin:

Day length

```
for i from 1 to 364 do
 err[i] := evalf(sb[i][2] − A(i))
 end do:
AbsErr := sqrt(add(err[i]^2, i = 1 ..364))
```

$AbsErr := 142.6258326$

$$RelErr := \frac{AbsErr}{\sqrt{add(sb[i][2]^2, i = 1 ..364)}}$$

$RelErr := 0.3809990165$

Sunrise

```
for i from 1 to 364 do
 err[i] := evalf(sa[i][2] − A(i))
 end do:
```

$$AbsErr := \sqrt{add(err[i]^2, i = 1..364)}$$

$$AbsErr := 134.3380827$$

$$RelErr := \frac{AbsErr}{\sqrt{add(sa[i][2]^2, i = 1..364)}}$$

$$RelErr := 0.9465880066$$

Sunset

```
for i from 1 to 364 do
 err[i] := evalf(sc[i][2] − A(i))
 end do:
```

$$AbsErr := \sqrt{add(err[i]^2, i = 1..364)}$$

$$AbsErr := 3.169631594$$

$$RelErr := \frac{AbsErr}{\sqrt{add(sc[i][2]^2, i = 1..364)}}$$

$$RelErr := 0.01307848079$$

Finnmark:

```
for i from 1 to 364 do
 err[i] := evalf(sa[i][2] − A(i))
 end do:
```

$$AbsErr := \sqrt{add(err[i]^2, i = 1..364)}$$

$$AbsErr := 156.9217304$$

$$RelErr := \frac{AbsErr}{\sqrt{add(sa[i][2]^2, i = 1..364)}}$$

$$RelErr := 1.103225723$$

Singapore:

```
for i from 1 to 364 do
 err[i] := evalf(sc[i][2] − A(i))
 end do:
```

$$AbsErr := \sqrt{add(err[i]^2, i = 1..364)}$$

$$AbsErr := 0.1713436953$$

$$RelErr := \frac{AbsErr}{\sqrt{add(sc[i][2]^2, i = 1..364)}}$$

$$RelErr := 0.0007411939015$$

Conclusion:

- As a result our periodic models describing sunrise, sunset and day length at three locations we had to choose three different cities are:
- Dublin and Finnmark had similar results as the graphs for daylength, sunrise and sunset look the same and got us similar results.
- In other hand Singapore gave us an awkward result and graph for sunset and sunrise due to the latitude of the city being 1°