

FACULDADE DE ENGENHARIA DA UNIVERSIDADE DO PORTO

Development of a Lap Time Simulation for a Formula Student Car

Gabriel Wiechert



FEUP FACULDADE DE ENGENHARIA
UNIVERSIDADE DO PORTO

Master's in Mechanical Engineering - Machine Design

Supervisor: Prof. António Ramos Silva

Co-Supervisor: Prof. Filipe Alexandre de Sousa Pereira

July 29, 2023

Development of a Lap Time Simulation for a Formula Student Car

Gabriel Wiechert

Master's in Mechanical Engineering - Machine Design

Abstract

Computational simulators actually are massively used in dynamics analysis, mainly for the ease of virtual systems modelling. This applicability can be done in several areas, including vehicle dynamics analysis. The Formula Student, being an engineering competition, is a place in which this type of approach must be used in order to obtain the best solutions.

In this context, this work consisted of the development of a Lap Time Simulation, a vehicle dynamics simulation type, for the prediction of Formula Student vehicles. The main goal of the simulator is the possibility to create a virtual vehicle with several different parameters, easily configured, to be studied in different tracks and conditions.

For this, the software was developed in MATLAB using a Quasi Steady-State approach, with a Single Track model and the inclusion of Load Transfers and Tire Sensitivity, intending to obtain more variability for the models. The states of the vehicle are calculated through a Non-Linear Constrained Optimization methodology to ensure the Optimal Speed Profile.

The solution appeared to be solid and well-founded in physical terms, being successful when compared with other developed solutions and giving expected and relevant results with a good level of reliability. Moreover, the capabilities of the software in the vehicle's behaviour prediction appear to be very relevant for the Formula Student context, whether in the planning or design stages.

Resumo

Atualmente, simuladores computacionais são massivamente utilizados na análise de sistemas dinâmicos, principalmente pela facilidade na modelação de sistemas virtuais. Essa aplicabilidade pode ser feita em várias áreas, incluindo a análise de dinâmica veicular. O Formula Student, sendo uma competição de engenharia, é um lugar em que esse tipo de abordagem precisa ser utilizado para se obter as melhores soluções.

Nesse contexto, este trabalho consistiu no desenvolvimento de um Lap Time Simulation (simulador de tempo de volta), um tipo de análise de dinâmica de veículos, para a previsão de veículos de Formula Student. O principal objetivo do simulador é a possibilidade de ser criado um veículo virtual com vários e diferentes parâmetros, facilmente configurável, para ser estudado em diferentes pistas e condições.

Para isso, o software foi desenvolvido em MATLAB usando uma abordagem Quasi Steady-State com um modelo Single-Track e a inclusão das transferências de carga e sensibilidade à carga do pneu, com a intenção de obter mais variabilidade nos modelos. Os estados do veículo são calculados através de uma metodologia de Otimização com Restrições Não Lineares para garantir um perfil de velocidades ótimo.

A solução aparentou ser sólida e bem fundamentada em termos físicos, sendo bem sucedida quando comparada a outras soluções desenvolvidas e com resultados relevantes que possuem um bom nível de confiança. Além disso, as capacidades do software na previsão do comportamento do veículo aparentam ser muito relevantes para o contexto do Formula Student, quer seja nas fases de planeamento ou design.

Acknowledgements

Agradeço a Deus, o Criador e Sustentador de todas as leis naturais estudadas neste trabalho, pela cognoscibilidade a mim concedida, para que compreendesse as Tuas obras e contemplasse a Tua glória.

Agradeço à minha família, por todo o amor, graça, cuidado, carinho e sustento desde pequeno até agora. Em especial, dedico este trabalho à minha mãe, que nos últimos anos me acompanhou em todas as frias madrugadas de inverno ao sair de casa. Mãe, o meu curso foi feito na força do teu café.

Agradeço à minha noiva, Marcela, pelo teu amor despejado em mim durante os últimos meses. Obrigado por me acompanhar, ensinar e motivar. Teu apoio foi motivador para continuar e, agora, vais ter que aguentar um engenheiro ao teu lado para sempre.

Agradeço a todos os amigos que me acompanharam durante os últimos anos, me ensinaram e foram instrumentos para que eu me tornasse quem sou hoje. Posso não discernir as relações causais, mas certamente sou devedor a vós pelo que fiz aqui.

Agradeço à FSFEUP, principal razão para que eu desenvolvesse este trabalho. Sou grato por todas as pessoas que trabalharam comigo no projeto, que me ensinaram e me deram a oportunidade de desenvolver as competências aqui apresentadas. Entrego este trabalho a vós para vosso proveito. Que ele sirva de ombros para verem cada vez mais longe.

Por fim, agradeço ao meu orientador, Professor António Ramos Silva, por toda a orientação, apoio e ajudas dadas, desde Setembro até agora. Teus feedbacks foram cruciais para a qualidade do trabalho realizado e para o meu amadurecimento como estudante e engenheiro.

Gabriel Wiechert

“Science is the process of thinking God’s thoughts after Him.”

Johannes Kepler

Contents

1	Introduction	1
1.1	Context	1
1.2	Objectives	2
1.3	Methodology	2
1.4	Structure	2
2	State of the Art	5
2.1	What is Formula Student?	5
2.1.1	Acceleration Event	6
2.1.2	Skidpad	7
2.1.3	Autocross	7
2.1.4	Endurance	8
2.1.5	Formula Student FEUP	8
2.2	Historical Development	9
2.3	Characterizing a Lap Time Simulation	11
2.3.1	Simulation Methodology	12
2.3.2	Physical Model	18
2.3.3	Path Model	22
2.3.4	Control Model	25
2.3.5	Conclusion about Lap Time Characterization	26
2.4	Possibilities and Results	26
2.5	Application in Formula Student	29
2.6	Conclusion	30
3	Quasi Steady-State	31
3.1	Introduction	31
3.2	Vehicle Model	32
3.2.1	Chosen Approach	33
3.2.2	Planar Motion	33
3.2.3	Non-Linear Tire Model	36
3.2.4	Out-of-plan motion	39
3.2.5	Steering System	44
3.2.6	Joining the Effects	45
3.2.7	Acceleration Model	47
3.2.8	Brake Model	49
3.3	Track Model	50
3.4	Computational Implementation	50
3.4.1	Optimization Remarks	51

3.4.2	Definition of the limit velocity profile and apex points	51
3.4.3	Definition of the forward profile	53
3.4.4	Definition of the backward profile	55
3.4.5	Calculating the Grip Limit	57
3.4.6	Joining the Speed Profiles	58
3.4.7	Modifications	59
3.5	Conclusion	61
4	Validation	63
4.1	Model Validation	63
4.1.1	Slip Calculations	64
4.1.2	Tire Model	65
4.1.3	Dynamics Model	65
4.1.4	Load Transfers Model	65
4.1.5	Simulink Implementation	65
4.1.6	Comparison to the Quasi Steady-State Model	68
4.1.7	Results and Discussion	68
4.2	Repeatability Tests	75
4.3	Comparison to other software	76
4.4	Conclusion	79
5	Results	81
5.1	Simulation Conditions	81
5.2	State Variables	81
5.2.1	Speed	81
5.2.2	Longitudinal Acceleration	82
5.2.3	Lateral Acceleration	83
5.2.4	Yaw Rate	84
5.2.5	Lateral Velocity	84
5.2.6	Body Slip Angle	84
5.2.7	Slip Angles and Normal Forces	86
5.2.8	Powertrain Model Data	89
5.2.9	GG Diagram	91
5.3	Derived Results	92
5.3.1	Energy Consumption Estimation	93
5.3.2	Average Electric Quantities	94
5.4	Resume	94
5.5	Sensitivity Analysis	94
5.5.1	Mass Study	96
5.5.2	Gear Ratio Study	99
6	Discussion	103
6.1	Mass Analysis	104
6.2	Gear Ratio Analysis	106
6.3	Other Analyses	108
7	Conclusions and Future Works	109
7.1	Satisfaction of the Goals	109
7.2	Future Works	110

References	113
A Car Parameters	117
B Sensitivity Analysis - Other Parameters	119
B.1 Center of Gravity Height	119
B.2 Center of Gravity Longitudinal Position	120
B.3 Aerodynamics Kit	121
B.4 Anti-Roll Bar Diameters	123

List of Figures

2.1	Acceleration Track.	6
2.2	Skidpad Track.	7
2.3	QSS simulation representation.	14
2.4	Workflow of the "forward/backward plus" solver.	15
2.5	NTA Surface (Left) and GG Diagram (Right).	17
2.6	14 DoF Vehicle Model.	19
2.7	3 DoF Planar Models	21
2.8	Comparison between trajectory obtained by direct integration of raw telemetry data (blue) and with the filtered yaw rate (orange).	23
2.9	Free path track model example.	23
2.10	Example of a Speed Profile.	27
2.11	Acceleration Plots Example.	27
2.12	Example of a GG Diagram.	28
2.13	Example of chassis roll angles and displacement.	28
2.14	Example of sensitivity analysis results.	29
3.1	Bicycle Model.	34
3.2	Tire Radial Deformation.	36
3.3	Tire Lateral Deformation.	37
3.4	Tire Side-Slip.	37
3.5	Non-Linear Tire Plot.	38
3.6	Longitudinal and Lateral Vehicle Models.	39
3.7	Lateral Vehicle Model.	41
3.8	Delft DUT 17 Car.	43
3.9	Static Toe Representation.	45
3.10	Parallel (a); Anti-Ackermann (b); Ackermann (c).	46
3.11	Non-linear vehicle model scheme	46
3.12	EMRAX 228 Combined Cooled Curves	48
3.13	EMRAX 228 Combined Cooled Efficiencies	48
3.14	Corner Representation.	52
3.15	Limit Speed Profile	53
3.16	Apex Points	53
3.17	Forward and Limit Speed Profiles	54
3.18	Forward Speed Profile	55
3.19	Backward and Limit Speed Profiles	56
3.20	Backward Speed Profile	57
3.21	Forward and Backward Speed Profiles	58
3.22	Final Speed Profile	59

3.23 False Apex Points	59
3.24 Explanations False Apex Points	60
3.25 Final Results	61
4.1 Simulink 3DoF	67
4.2 Steering Wheel Input - Case 1	69
4.3 Results - Case 1	70
4.4 Steering Wheel Input - Case 2	71
4.5 Results - Case 2	71
4.6 Understeer (Left); Oversteer (Right).	72
4.7 Steering Wheel Input - Case 3	72
4.8 Results - Case 3	73
4.9 Results - Case 4	74
4.10 Slip Angles - Case 3	74
4.11 Slip Angles - Case 4	75
4.12 Repeatability Test - Curvature	76
4.13 Repeatability Test Input	76
4.14 Lateral Acceleration - Repeatability Test	77
4.15 Motor Curve - OptimumLap	77
4.16 Velocity Comparison - Developed Software and OptimumLap	78
4.17 Velocity Comparison - Developed Software and OptimumLap	79
4.18 Longitudinal Acceleration Comparison - Developed Software and OptimumLap	80
4.19 Lateral Acceleration Comparison - Developed Software and OptimumLap	80
5.1 FSG Autocross 2012 Track	82
5.2 Results - Speed Distribution	82
5.3 Results - Longitudinal Acceleration Distribution	83
5.4 Results - Lateral Acceleration Distribution	83
5.5 Results - Yaw Rate Distribution	84
5.6 Results - Lateral Velocity Distribution	85
5.7 Results - Body Slip Angle Distribution	85
5.8 Results - Nose-in/Nose-out behaviour	86
5.9 Results - Tire Behaviour and Normal Forces - 0-200m	87
5.10 Results - Tire Behaviour and Normal Forces - 0-75m	88
5.11 Results - Electric Current	89
5.12 Results - Electric Voltage	90
5.13 Results - Electric Power	90
5.14 Results - GG Diagram	91
5.15 Comparison between length and time reference	92
5.16 Results - Energy Consumption	93
5.17 Results - State of Charge	94
5.18 FSG Endurance 2012 Track	95
5.19 Sensitivity Analysis - Mass - Autocross	96
5.20 Sensitivity Analysis - Mass - Endurance	97
5.21 Sensitivity Analysis - Mass - Skidpad	98
5.22 Sensitivity Analysis - Mass - Acceleration	99
5.23 Sensitivity Analysis - Gear Ratio - Autocross	100
5.24 Sensitivity Analysis - Gear Ratio - Endurance	100
5.25 Sensitivity Analysis - Gear Ratio - Skidpad	101

5.26 Sensitivity Analysis - Gear Ratio - Acceleration	102
6.1 Mass Analysis	104
6.2 Overall Dynamic Scoring - Mass Analysis	105
6.3 Events Scoring - Gear Ratio Analysis	106
6.4 Events Position - Gear Ratio Analysis	106
6.5 Overall Dynamic Scoring - Gear Ratio Analysis	107
B.1 Sensitivity Analysis - CoG Height - Autocross	119
B.2 Sensitivity Analysis - CoG Height - Skidpad	120
B.3 Sensitivity Analysis - CoG Longitudinal Position - Autocross	120
B.4 Sensitivity Analysis - CoG Longitudinal Position - Skidpad	121
B.5 Sensitivity Analysis - Aerodynamics Kits - Autocross	121
B.6 Sensitivity Analysis - Aerodynamics Kits - Skidpad	122
B.7 Sensitivity Analysis - Aerodynamics Kits - Acceleration	122
B.8 Sensitivity Analysis - Anti-Roll Bar Diameters - Autocross	123
B.9 Sensitivity Analysis - Anti-Roll Bar Diameters - Skidpad	124

List of Tables

2.1	Formula Student Germany Scoring.	6
4.1	OptimumLap Software Input Parameters	78
5.1	Resume of the Simulation	95
5.2	Endurance Parameters Changing	96
A.1	Vehicle Parameters	117
A.2	Suspension Parameters	118
B.1	Aerodynamics Kits	121

Symbols and Abbreviations

CoG	Center of Gravity
CoP	Center of Pressure
CV	Internal Combustion Vehicle Class
DC	Driverless Competition
DV	Driverless Vehicle Class
DoF	Degrees of Freedom
EF	Efficiency Factor
EV	Electric Vehicle Class
F1	Formula One
FSFEUP	Formula Student FEUP Team
GG	Lateral and Longitudinal Accelerations Diagram
GGV	Lateral and Longitudinal Accelerations and Velocity Diagram
GPS	Global Positioning System
LTS	Lap Time Simulation
NTA	Normal Tangential Accelerations Diagram
QSS	Quasi Steady-State
<i>A</i>	Car Frontal Area
<i>a_i</i>	Distance from Axle i to CoG
<i>a_x</i>	Longitudinal Acceleration
<i>a_y</i>	Lateral Acceleration
<i>BB</i>	Brake Bias
<i>C_d</i>	Aerodynamic Drag Coefficient
<i>C_l</i>	Aerodynamic Downforce Coefficient
<i>C_{α_i}</i>	Tire Cornering Stiffness in Axle i
<i>F^{drag}</i>	Aerodynamic Drag
<i>F_x</i>	Pacejka Model Longitudinal Force
<i>F_y</i>	Pacejka Model Lateral Force
<i>h</i>	Center of Gravity Height
<i>h_{CP}</i>	Center of Pressure Height
<i>i</i>	Gear Ratio
<i>I_{motor}</i>	Motor Current
<i>I_z</i>	Vehicle's Inertia in Z-axis
<i>k_φ</i>	Roll Stiffness
<i>k_{φ_i}</i>	Roll Stiffness in Axle i
<i>k_{φ_i}^p</i>	Tire Roll Stiffness in Axle i
<i>L</i>	Wheelbase

m	Total Mass
m_s	Sprung Mass
$m_{u,front}$	Unsprung Front Mass
$m_{u,rear}$	Unsprung Rear Mass
N_{motor}	Motor Speed
P_{motor}	Electric Power
q^b	Roll Center Height
q_i	Roll Center Height in Axle i
r	Yaw Rate
R	Corner Radius
r_{tire}	Tire Radius
t_i	Track Width of Axle i
$T_{constant}$	Motor Torque Constant
T_{max}	Modified Best Event Time
T_{motor}	Motor Torque
T_{team}	Team's Event Time
V_x	Speed
V_y	Lateral Velocity
V_{car}	Car's Velocity
V_i	Velocity in Axle i
V_{ij}	Velocity in Tire ij
V_{motor}	Motor Electric Voltage
V_{tire}	Tire's Velocity
x_{CP}	Center of Pressure Longitudinal Position
X_i	Longitudinal Force in Axle i
X_{ij}	Longitudinal Force in Tire ij
Y_i	Lateral Force in Axle i
Y_{ij}	Lateral Force in Tire ij
Z_i	Normal Force in Axle i
Z_{ij}	Normal Force in Tire ij
α	Slip Angle
β	Body Slip Angle
δ	Steering Angle
δ^0	Static Steering Angle
δ_v	Steering Angle Wheel
Δs	Track Segment Length
ΔZ	Longitudinal Load Transfer
ΔZ_i	Lateral Load Transfer in Axle i
$\Delta Z^{downforce}$	Aerodynamic Downforce
ε_1	Ackermann Coefficient
η_{mec}	Mechanical Efficiency
η_{motor}	Motor Efficiency
κ	Corner Curvature
ρ	Air Density
τ_1	Steering Ratio
ω_{tire}	Tire's Rotational Velocity

Chapter 1

Introduction

This dissertation presents the work developed by Gabriel Wiechert to obtain the degree of Master in Mechanical Engineering - Machine Design. The work was developed with the orientation of Prof. António Ramos Silva and the co-orientation of Prof. Filipe Alexandre de Sousa Pereira.

1.1 Context

The analysis of dynamic systems has the objective of giving information about systems that have time-variable states. The interest and the requirements in these analyses depend on the application of the system and the expected results. In early times, dynamic analysis depended on practical tests, experiments and measurements. But more recently, with the possibility of computational simulation, this methodology turned into a great ally for engineers to understand the characteristics of systems and mechanisms that can be modelled in a virtual environment, without the necessity of its existence in the real world. This join is actually named "dynamics simulation" and can be applied in several areas, from structural dynamics to fluid dynamics.

Within the areas of investigation, this work is being developed in the vehicle dynamics context, more specifically in the motorsport application. The interest is to develop a simulator to predict the behaviour of a Formula Student vehicle, in order to understand and study the built vehicle, test different tuning conditions and help in the design choices of future iterations.

Formula Student is an engineering competition for graduate and undergraduate students in which the participants must design, build, test and run a Formula-type car for dynamic and static events. Besides the objective of a successful car in running, there's the great objective of proofing the engineering knowledge below the design of the car. The work developed here intends to serve all these requirements, giving good data to design choices and delivering good engineering reasons for the decisions taken by the team.

1.2 Objectives

Within the context explained the main objective of the work is to develop a "Lap Time Simulation" software, which must provide an overall result of the car in all the events that compose the Formula Student competition. However, to prove the software's fidelity and reliability, this report will try to give all the justifications for the steps taken and the choices made.

For this, it is necessary to show the theoretical background to find solid assumptions for the software, finishing through the justification about the taken way. After, it is necessary to present the development of the work, with the physical assumptions and computational implementation, assuming the limitations and delineating the boundaries. The next step is to prove that the obtained solution is physically valid and reliable, submitting the simulator to tests and analysing the results. Finally, it is necessary to show the results of the developed work, aiming to prove its value and relevance for the presented context.

1.3 Methodology

The work involved initial theoretical research to find other developed simulators, understand the types of simulation and extract methodologies to create a new one. This was done through articles, vehicle dynamics books and internet research, including coding analysis. After, the requirements were delineated, whether for the software or for the physical model inside it. With these requirements, the software itself was developed in MATLAB, trying to fulfil the expectations and give reliable solutions. The validity of the software involved several tests comparing it to other software (OptimumLap) and comparing the model with a transient model, developed in Simulink. The results extracting part involved the definition of several simulation conditions and the massive simulation, followed by data treatment and presentation.

1.4 Structure

This report is divided into the Introduction (this chapter) and the other 6 chapters, including the last, which is the conclusion. Chapter 2 is the State of the Art, presenting briefly the Formula Student competition and the team from this faculty. After, the chapter brings information about the historical background of vehicle dynamics simulation and has a characterization of Lap Time Simulation, investigating several choices, methodologies and respective authors. The chapter ends with an explanation of the Lap Time Simulation in the Formula Student context, which provides a basis for the development of the software. Chapter 3, continuing the theme, shows the requirements, choices and methodologies used in the Lap Time Simulation development, including physical models and computational implementation. The work developed in Chapter 3 is proved in Chapter 4, which shows three different ways to validate the developed solution and assure the

validity of the software. In Chapter 5, the main physical results are presented, showing the capacities of the simulator and the sensitivity from parameters changing. Chapter 6 has the objective of interpreting the results from Chapter 5 and showing their meaning and relevance for the Formula Student competition, whether in the design or in the planning stages. Finally, Chapter 7 is the conclusion, showing the final remarks about the work, delineating some limitations, pointing to some improvements and giving a starting point for future works to be developed based on this.

Chapter 2

State of the Art

2.1 What is Formula Student?

Formula Student is an engineering competition in which the main objectives are to design, manufacture and assemble a race car to participate in static and dynamic events. The static events consist of the theoretical background of the car building (engineering choices, design costs, rule compliance) and the dynamic events consist in run the car on real tracks and compute its performance. The static events are divided into Engineering Design, Cost and Manufacturing and Business Plan. The last two are tied more with project management while the first is completely tied with the engineering design.

The dynamic events are separated into Acceleration, Skidpad, Autocross, and Endurance. The first consists in evaluate the longitudinal acceleration of the car (and, clearly, the most relevant part is the powertrain system). The second consists in evaluate the lateral dynamics of the car, giving more emphasis to the vehicle dynamics understanding employed in the car's design. The third event consists of a fast lap in the circuit. This lap is done with maximum performance and for this, the best configuration is a combination of the various systems of the car. Finally, endurance is a long-run race, in which the car must complete 22km in the best time. However, because of the energy storage, the main constraint on this event is to manage the energy to assure that it is possible to complete the endurance. Besides the "performance" evaluation, the endurance event is concerned with efficiency evaluation. This means that a car with good performance can waste more energy and, for this reason, there's a penalization on the overall result.

The overall scoring of the competition is divided in the following way:

Table 2.1: Formula Student Germany Scoring. Taken from [1]

Static Events:	CV & EV	DC
Business Plan Presentation	75 points	-
Cost and Manufacturing	100 points	-
Engineering Design	150 points	150 points
<hr/>		
Dynamic Events:		
Skid Pad	50 points	-
DV Skid Pad	75 points	75 points
Acceleration	50 points	-
DV Acceleration	75 points	75 points
Autocross	100 points	-
DV Autocross	-	100 points
Endurance	250 points	-
Efficiency	75 points	-
Trackdrive	-	200 points
Overall	1000 points	600 points

In which the categories CV and EV are Internal Combustion and Electrical Vehicles, respectively. The DC is the Driverless competition. The events followed by DV are also for driverless cars. Each one of the dynamic events will be explained in the following.

2.1.1 Acceleration Event

The acceleration event consists in accelerating the car on a straight track with a total length of 75m. Basically, the fastest car wins the event. An example can be seen in Figure 2.1. For this



Figure 2.1: Acceleration Track. Taken from [2]

event, the scoring is calculated through Equation 2.1 (obtained from [1]), in which the T_{max} is 1.5

times the best time and T_{team} is the time of the team.

$$\text{ACCELERATION_SCORE} = 46.5 \left(\frac{\frac{T_{max}}{T_{team}} - 1}{0.5} \right) \quad (2.1)$$

2.1.2 Skidpad

As written above, the Skidpad is the event to evaluate the lateral dynamics of the car. Basically, the track layout is an eight-shape track, as shown in Figure 2.2. The Skidpad time is the fastest of

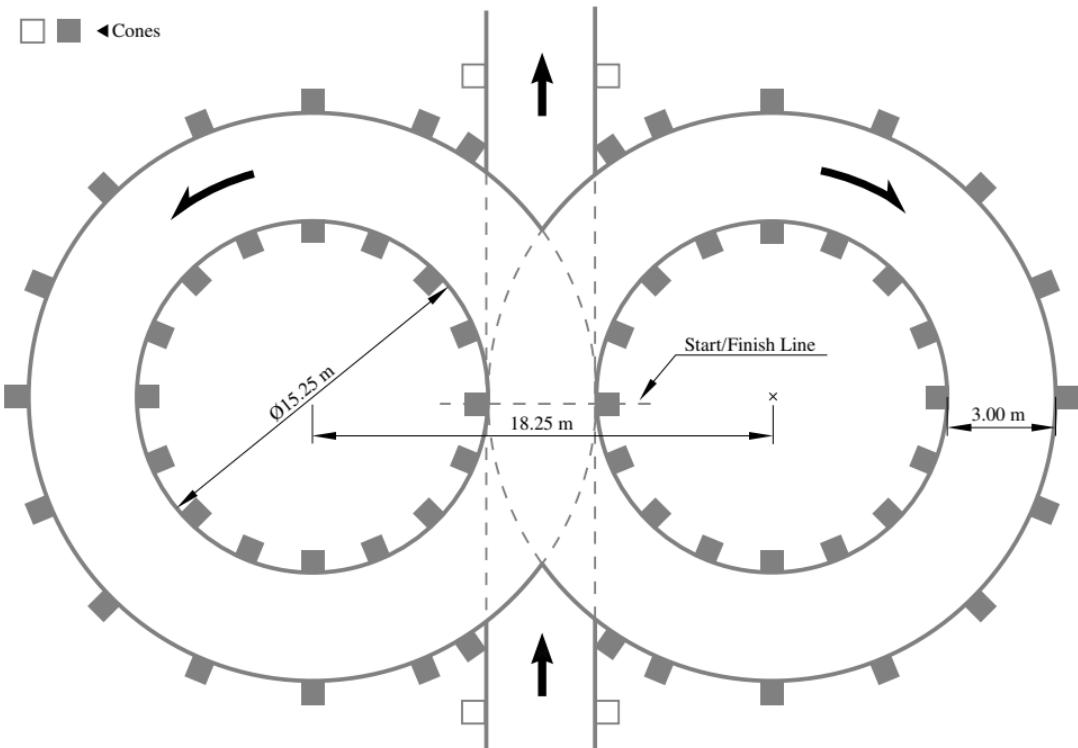


Figure 2.2: Skidpad Track. Taken from [1]

the 4 turns that a driver does (2 for each side). The final scoring is calculated through Equation 2.2 (obtained from [1]), in which the T_{max} is 1.25 times the best time and T_{team} is the time of the team.

$$\text{SKIDPAD_SCORE} = 46.5 \left(\frac{\left(\frac{T_{max}}{T_{team}} \right)^2 - 1}{0.5625} \right) \quad (2.2)$$

2.1.3 Autocross

The autocross, as explained, is a circuit in which the objective of the driver is to do a fast lap. The competition rules describe the characteristics of the track:

- Straights: No longer than 80m
- Constant Turns: up to 50m diameter

- Hairpin Turns: Minimum of 9m outside diameter (of the turn)
- Slaloms: Cones in a straight line with 7.5m to 12m spacing
- Miscellaneous: Chicanes, multiple turns, and decreasing radius turns. The minimum track width is 3m.

Finally, the length of the lap is approximately 1.5km. The scoring of this event is calculated in the following way (Equation 2.3, obtained from [1]). T_{max} is 1.25 times the best time and T_{team} is the time of the team.

$$AUTOCROSS_SCORE = 95.5 \left(\frac{\frac{T_{max}}{T_{team}} - 1}{0.25} \right) \quad (2.3)$$

2.1.4 Endurance

The Endurance is the resistance event of the Formula Student. The objective is to complete the 22km on a track that follows the characteristics of the Autocross track. The scoring for this event is calculated as shown in Equation 2.4 (obtained from [1]). In this equation, T_{max} is 1.333 times the best time and T_{team} is the time of the team. Associated with the Endurance event, there's the Efficiency Scoring (Equation 2.5).

$$ENDURANCE_SCORE = 300 \left(\frac{\frac{T_{max}}{T_{team}} - 1}{0.333} \right) \quad (2.4)$$

$$EFFICIENCY_SCORE = 75 \left(2 - \frac{EF_{team}}{EF_{min}} \right) \quad (2.5)$$

For the efficiency scoring calculation, EF_{min} is the minimum Efficiency Factor and EF_{team} is the Efficiency Factor of the team. This factor is calculated based on the equation:

$$EF = T^2 E \quad (2.6)$$

In which T is the driving time and E is the used energy (in the EV case). For CV, E is the fuel volume.

2.1.5 Formula Student FEUP

Formula Student FEUP (FSFEUP) is the Formula Student team from the Faculty of Engineering of the University of Porto, Portugal. The team is building its first car, an EV (Electric Vehicle) with one rear motor, tubular chassis and without an aerodynamics kit. These characteristics were chosen based mainly on literature review and the concept is simple. However, the next step is to leave just the theoretical background for a simulation/experimental background to take decisions. Being the first car, the objective of the team with a simulator is to have some predictions about the car's dynamics and validate some choices through simulations (besides the literature support). Optimization processes can be thought of for the next car, but only for the parameters (and not for

the track, for example), since the team hasn't the capacity to obtain the travelled path data before building the second car.

2.2 Historical Development

The beginning of Vehicle Dynamics simulation started more than 60 years ago. One of the first known works was done by Mercedes-Benz in the 50s [3]. Other works began to emerge in race context [4], with the main objective of understanding a car's behaviour during a lap. Stirling Moss [5], quoted in [6], told that the Vehicle Analysis departments in F1 teams began to have tools to evaluate "[...] *the speed in every point calculated together with the required gear in every section*". This means that teams began to do Vehicle Dynamics simulation. And the goal becomes clear in the following: "*So if you started by saying that on a certain corner you were coming out at 5.500 r.p.m. and you would prefer 5300 r.p.m., in next to no time the gear ratio would be changed so that without altering your road speed this is how it would be*". [5]. In other words, the teams wanted to understand the car and simulate how it would be if some parameter was changed. This type of analysis is what is called "Lap time simulation".

Advancing in history, some academic works began to appear with methodologies developed to do these simulations. For example, a well-known approach called "GGV Diagram" was cited and used in [7]. This procedure basically consists in describe the car handling properties in a surface (or diagram), in which the limits of the dynamics (lateral and longitudinal acceleration) are related to the velocity. Some other authors developed this concept, such as Milliken and Milliken [8], which wrote one entire chapter about the theme, that will be visited ahead in this work.

In the midst of the development of simulators, one of the most important advances is tyre analysis. The tyre is a very complex part of the car, but in vehicle dynamics, one of the most important. Its importance is a consequence of its function: balance all the loads applied in a car (through friction). Physically, friction is often treated with Coulomb's model ($F_{fr} = \mu F_n$). However, in the tires, the application of this theory is very simple and limited. Tires are, in reality, very non-linear. So, some different models were developed to represent the tyre behaviour. The most important, known and used is the "Magic Formula", developed for Pacejka [9]. Basically, this Formula relates the maximum lateral/longitudinal force developed for a tyre as a function of tyre parameters, normal force and slip angles, concepts to be described after. This remark is important to further development.

Returning to the different methodologies, the simulations done with the GGV diagram are, entering another type of classification, steady-state or quasi steady-state simulations. This means that the physical analysis done in each segment of the simulation neglect the dynamic (transient) behaviour. The reason for dealing with these simplified simulations is that the computational cost for this is also very small compared with transient simulations. Hence, some work was done in

these conditions (with GGV diagrams or not). Gadola [10] used a simplified car model (linear chassis model, springs, anti-roll bars, radial tyre effects, chassis torsion, non-linear tyre model) taking into account the lateral load transfer to evaluate the maximum velocity in a mid-corner point of a specific curve. The simulation was done firstly trying to find the "optimal" path using a genetic algorithm that maximizes the minimum radius in each corner of the circuit. The result of this optimization process is the following points: turn-in, brake release, throttle application, and exit. After this, the velocity at each point (segment) is calculated using the GGV diagram. The results show that the model was reliable for the type of car used in the validation, with differences of 1-2% to the real data (in terms of lap time).

In the 2000s, a very known article compared steady-state, quasi steady-state and transient simulations in terms of approaches, complexity and results [11]. After this, some remarkable works were developed, mainly in order to increase the complexity of the models. One of the most important was developed by Casanova [6] in a complete PhD Thesis. The author developed a lap time simulation with a transient vehicle model (7 Degrees of Freedom) and optimal control in this work. The result is a simulator that can give the optimal theoretical path to a car in a specific circuit. To reach this result, Casanova showed and developed the Optimal Control Problem mathematically, built the track from telemetry data, developed the driver model, and applied the optimisation process to the track. The software developed can present the results to a simple "path following" (when the path is given); it can determine the optimal path, and also it can study the impact of some input parameters of the vehicle in the lap time (what is called "sensitivity analysis"). The cost of this work was the computational complexity. In some runs in Barcelona Circuit, it takes 60 hours to complete the simulation.

In 2004, Brayshaw [12] developed a Quasi Steady-State simulation with a 7DoF vehicle and constrained optimization to achieve a balanced result in computational cost and model accuracy. He mentioned Casanova [6] and Hendrikx to frame his work, arguing that the computational cost wasn't a problem of the mathematical model but of computational technology. For this reason, he attempted to create a simpler model in a simple methodology (with much less computational cost). Physically, the assumption that the suspension has limited deflections on an open-wheel car allows the change from transient to quasi steady-state simulations. Brayshaw developed his model and, using a published vehicle, compared the work with Casanova [6]. The final result was acceptable to the author. The lap time differed by approximately 2s between the two approaches, but the main point for the author was *the capacity that a QSS simulator has to estimate the performance changes due to the design changes* and, for this goal, the result was good comparing with the transient. For comparison, the track studied in this work was simulated by Casanova in 39.8h, while Brayshaw simulated it in 16 minutes. In terms of results and capabilities, his work has the capacity to study the parameters change and the thesis describes these studies for six parameters: vehicle mass, yaw inertia, tire properties, engine, longitudinal centre of gravity, and aerodynamic downforce.

In recent years, the development of Lap Time Simulators become much more segmented. Because of the big time necessary to calculate the transient simulations developed in the 2000s (for example Casanova [6] and Kelly [13]), the recent authors work in a way to achieve more reasonable simulation times maintaining a good accuracy and approximation with reality. Different physical models were developed (3DoF, 7DoF, 10DoF and 14DoF), accounting for or neglecting specific physical effects depending on the objective of each simulator. However, besides the complexity of the physical model, another manner to deal with big simulation times is to find better mathematical approaches to deal with the optimization problem. A lot of advances were done in two ways: Optimal Control Problem (OCP) and Non-Linear Programming (NLP). Sometimes, the two approaches are applied together, as in [14], that converted the OCP in NLP to perform the optimization. Other methodologies for optimization are also cited: Convex Optimization and Pontryagin's Minimum Principle.

2.3 Characterizing a Lap Time Simulation

Lap Time Simulation (LTS) is a simulation methodology used to evaluate a vehicle's dynamic behaviour and performance. This name is used because its applicability is very common in a competition context, in which the main performance parameter is the lap time. In other words, the car that runs a lap in smaller time is better. However, this methodology can be applied to the evaluation of vehicle dynamics in any context. So, other names can be found in the literature, such as "minimum time manoeuvre", "lap time optimisation", "minimum lap time", and "minimum time trajectory".

A Lap Time Simulation isn't a simple algorithm. There are several ways to achieve the results and a lot of different combinations can be done. For example, different vehicle models can be applied; the simulation may be Quasi Steady-State or Transient; the model can be calculated repeatedly during the simulation or can be discretized in an envelope; the path can be just followed or the optimal path may be found; the program can do sensitivity analysis and design optimization studies or not. All these variations arise over the years and, as said before, the academic contributions were dispersed and segmented in all these sub-categories.

For this reason, the objective of this section is to characterize systematically an Lap Time Simulation, explain the main parts of it and present the main approaches for each "category", pointing to recent works and, maybe, comparing them. A systematic structure of a Lap Time Simulation can be the following:

- Simulation Methodology
- Physical Model
- Path Model

- Control Model

2.3.1 Simulation Methodology

Basically, the simulation methodology is the way that we want to deal with the motion and vehicle equations. The question here is: *I want to study the car in Steady-State condition or consider transient effects?* The answer is not simple and depends on each application, objective and condition of the simulation. Simulation methodologies are divided into **Steady-State**, **Quasi Steady-State** and **Transient**.

Before the overview, it is useful to think in the practical experience about the physical effects applied in a car that is in motion. Firstly, the car is submitted to tractive forces. These forces are applied by the motor/engine in the tires. In terms of motion, it is the "first cause" for the motion of a car. Together with this group, there are inertial forces that arise as consequences of accelerations of the car and these accelerations may be caused by tractive forces or by circular motion. Other effects can be considered, such as wind forces or track effects, but the two cited groups are, generally, the most important to this initial analysis.

When a car accelerates, there's a tractive force applied to the wheels and, because of inertia, there's an opposite force that arises from the car's own mass. If a car runs with constant acceleration, this inertia force is also constant. However, there's a transient finite instant, at the beginning of the acceleration, when the force isn't constant. In other words, the forces are in "development". This occurs until its derivatives (in order to time) go to 0. The same case occurs at the finish of an acceleration run. When the car stops the acceleration (or begins to brake), exists a decrease in the tractive force and a consequent decrease in the inertia forces (the derivative in order to time is negative). So, this effect is also transient and finite and occurs until the derivatives (of the applied forces) go to 0.

The empirical analysis can be done also with the curvilinear motion. When a car runs in a corner (with constant radius and constant velocity), there's a lateral acceleration and, consequently, a lateral inertia force applied to the car. This force is constant and can be easily calculated by knowing the velocity and the radius. However, when a car begins to curve, there's a transient instant in which the radius isn't constant and, for this reason, the lateral acceleration isn't constant. This effect occurs until its derivatives go to 0 (when the radius becomes constant). Obviously, this also occurs when a corner finishes. But the cornering in a real case is even more complex because another parameter can change: the velocity. A driver can accelerate or brake during a corner and, for this reason, even when the radius is constant, the lateral acceleration isn't. So, this is another case in which the transient effect can be understood and felt. Much less empirical but greatly relevant, a car curving with variable velocity (cause) has also a variable radius of curvature (consequence). This means that the transient effect during a curve is even more complex to compute.

In a real case, and looking specifically at the race context, all these effects are superposed and occur simultaneously. When a person is driving, they are perceived and calculated by the brain, which can respond intuitively with the steering wheel or the pedal. But a mathematical approach needs to deal with this objectively and for this reason, it is necessary to define which effects will be considered, which will be simplified and which will be neglected. Considering or neglecting one or more of these effects will define the methodology of simulation.

2.3.1.1 Steady-State

Steady-State is a condition in which all the transient effects are neglected. This means that the instants in which the forces are being "developed" are simply skipped and the simulator considers just the forces fully developed. So, for this reason, a steady-state simulation considers the straights as being big segments with constant accelerations. Each corner is considered as a single segment with a constant radius and travelled with constant velocity. The variations (straight-curve; curve-straight) are neglected and the car is every time considered fully balanced.

The physical theory used in steady-state simulations can be useful for theoretical calculations or specific cases (for example in a Skidpad track). Guiggiani, in [15], says that it is usual to use Steady-State analysis for initial investigations of the dynamics of a vehicle. However, this only can be done with accuracy if very particular cases or specific manoeuvres are studied. For a full-circuit simulation, the results of a pure steady-state simulation don't correspond to reality, as seen in [11]. For this reason, there are no many relevant works with this simulation methodology in the actual context. However, exist another very known category, based on the steady-state condition, that is much more realistic: Quasi Steady-State Simulations.

2.3.1.2 Quasi Steady-State

The Quasi Steady-State (QSS) simulation methodology involves the splitting of the track into a lot of different segments. With this, the same calculations done in Steady-State condition are applied, but for a very small part of the track. Between two segments, it is possible to have an increase in the velocity or change in curvature. This means that a corner before calculated with constant radius and constant velocity can be, now, calculated with the variation of these parameters. Siegler and Crolla, in [11], say that a corner can be divided into more than 50 segments, giving a more real result than the steady-state. The following works are presented in order to outline the Quasi Steady-State simulation in an academic context. The firsts are based on fixed-trajectory approaches, while the last look to the possibility of applying the optimal control techniques to transform a QSS problem into a free-trajectory (path-finding) problem. Some other variations can be found depending on the work, as in vehicle modelling and discretization. Some simulators use direct equations solving, while others work with surfaces, such as GGV or NTA surfaces.

Kobayashi, in [16], created a QSS simulator to improve the dynamic performance of hybrid cars. So, the goal of this simulator is to study the optimal gear ratio for the vehicle. In this work, a simplified vehicle model described through a GGV diagram was used. The simulation was done for a predetermined fixed track with path data obtained from GPS (radius and distance). The algorithm of simulation is known as "forward-backward algorithm":

- A critical speed profile is calculated through the maximum lateral acceleration at each point.
- As a consequence, the vehicle will be ever in its dynamic limit, but there's no capability to have longitudinal acceleration. To solve this, a "backward" algorithm goes from point n to point 1 calculating the necessary longitudinal acceleration to maximize the velocity at each point (taking into account the total friction of the tires). Figure 2.3
- The same procedure is done in the direction "forward". In this case, the acceleration is positive.
- The final velocity and acceleration profiles are the intersection of the two calculated above.

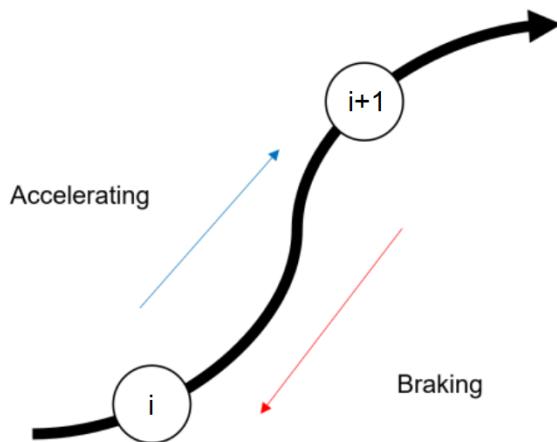


Figure 2.3: QSS simulation representation. Adapted from [16]

For this procedure, the calculation time was lesser than the 30s. The results were very acceptable and good for the necessities and objectives, giving an optimal result of gear ratio to the model. One of the future advances pointed out by the author is the inclusion of non-linear effects and drivetrain differential.

The same procedure (forward-backward) was used by Doyle in [17] to develop its Quasi Steady-State simulator to a Formula Student car with a GGV diagram to evaluate the car behaviour. In the conclusion, the author compares the result with the free tool OptimumLap and the results differed by approximately 1-2% in Lap Time. This difference was argued to be expected due to the existence of weight transfer and rotational inertia and was said as relevant to the Formula Student context.

Another type of QSS simulation was done by Heilmeyer in [18]. Different from the methodology used by Kobayashi, Heilmeyer used a forward-backward "plus" algorithm, in which the speed profile is mainly calculated in the forward direction, but is "backward" when necessary (Figure 2.4). The work does not explain much about the vehicle model, but basically, the steady-state

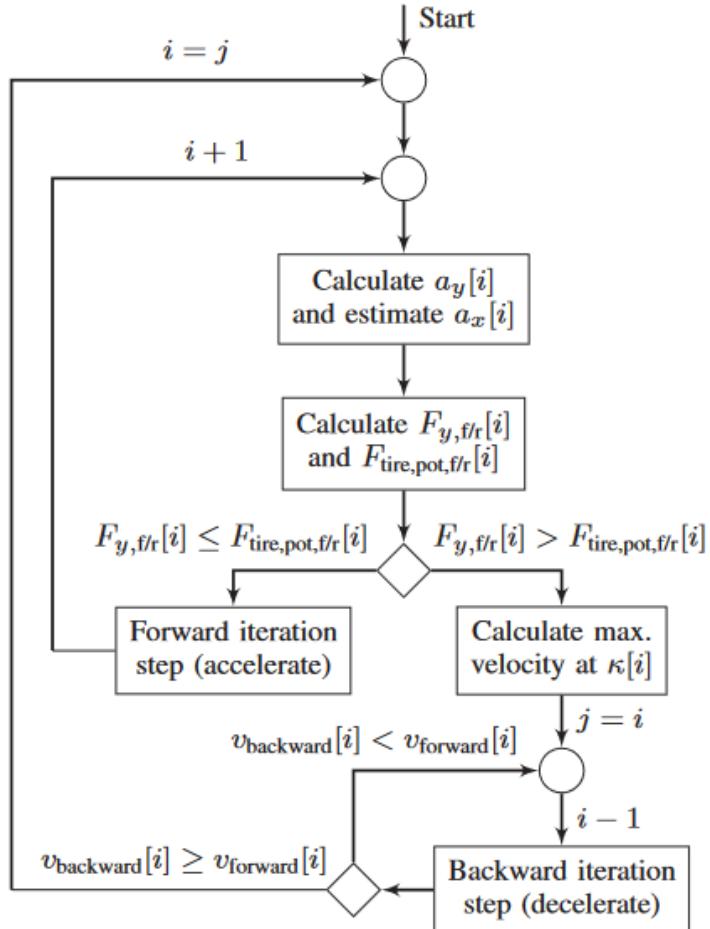


Figure 2.4: Workflow of the "forward/backward plus" solver. Taken from [18]

equilibrium equations are solved for each segment to respect the limitations imposed in the cycles. This approach increase the computational efficiency relative to the "forward-backward" and resulted in a simulation time of 1.3s for the Shanghai circuit (discretized with 5m elements). In terms of accuracy, the simulator followed more or less generally the behaviour of a real F1 car on this track but had some discrepancies in the velocity profile and lap time. The author argued that these discrepancies can be explained by differences between the parameterized and real cars, including tire data. In addition, some simplifications in the powertrain system and gear changing can influence the results. However, with the assumption that an LTS must provide information about relative changes in the parameters (sensitivity analysis), it was argued that this LTS provides a good balance between computational time and accuracy.

Lenzo and Rossi, in [19] used a very similar approach (without call with this name), in which, quoting the authors, the simulation is done with the following methodology:

- Calculate the state of motion at time step i (or, at the beginning of a simulation, use the initial conditions).
- Calculate all of the future vehicle states of motion assuming that the driver is braking, until $k(\lambda)$ has a maximum.
- Check whether the speed satisfies $k(\lambda)v^2 \leq a_{y,max}$ at any future time step j .
- If the condition above is always met, it means that the driver can accelerate at step i , and then the state of motion at step $i + 1$ can be calculated. Otherwise, go back one step and impose that the driver is braking at step $i - 1$.

The main difference, pointed by the authors, is that this method can be applied evaluating the curvature in order of any parameter λ . But the low computational cost is a key advantage of this formulation.

Massaro, in [20], in order to develop a free-trajectory simulator to determine the optimal path and, consequently, the minimum lap time, proposed a different approach. The simulation is done using a vehicle model associated with Optimal Control Problem using cost functions. In this case, the optimal controls are the longitudinal and lateral velocities with the equations of motion replaced by acceleration constraints. The author propose a technique to evaluate the maximum performance of the vehicle using a called "Polar Representation of the GG diagram". This procedure evaluates two parameters $\rho = \frac{1}{g} \sqrt{a_x^2 + a_y^2}$ and $\alpha = \arctan(a_x, a_y)$ and, using the boundary of GG diagram, the achievable performance is calculated. For solve this algorithm, the author used Non-Linear-Programming using IPOPT. 12000 equations were solved simultaneously with a mesh of 0.2m between 2 track points (on corners) and 25m (on straights). The controls used were the longitudinal and lateral jerks (derivative of acceleration).

The same approach was used in [21] to compare the QSS method with transient simulation. As said before by [20], this approach involves the use of a GG diagram to calculate the dynamics limits of the vehicle in each time. However, Tucker [21] provides another method to evaluate the dynamic capabilities of the car. The authors say that QSS simulations often assume the assumption that the body slip (angle between velocity vector and car heading direction) is zero. This means that the accelerations in x and y direction are treated as equal to the tangential and normal accelerations, respectively. However, this assumption is only a simplification (in real life, the car has some body slip) and results in a wrong evaluation of the dynamic capability, since the vector used to intersect the GG diagram ($\alpha = \arctan(a_x, a_y) = \arctan(a_t, a_n)$) isn't the real. For this reason, the envelope proposed is the NTA (Normal-Tangential-Acceleration) surface (Figure 2.5). This surface gives a relation between the velocity V , the acceleration magnitude ρ_a and acceleration orientation α_a , being $\rho_a = \sqrt{a_t^2 + a_n^2}$.

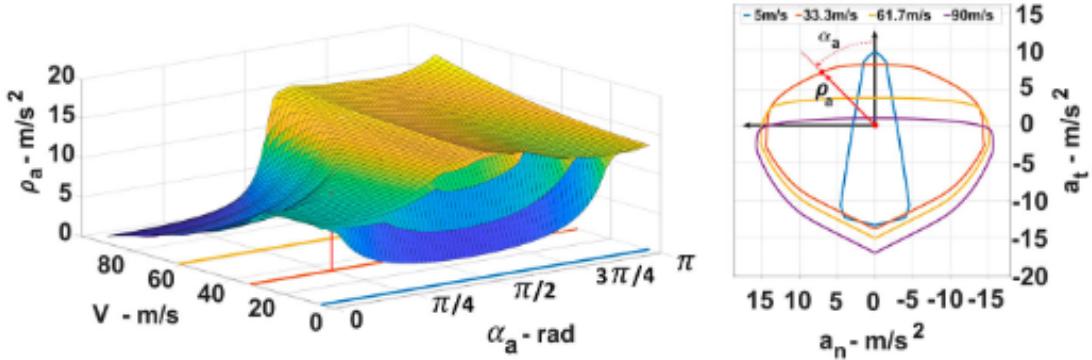


Figure 2.5: NTA Surface (Left) and GG Diagram (Right). Taken from [21]

The procedure to obtain this surface is calculated with an offline optimization process, whose objective is to maximize the acceleration magnitude. To mimic the inertial effect, jerk limitation was implemented. The procedure was compared with QSS with a GGV diagram and a transient simulation for a left corner, double-lane-change and full circuit. The results showed that this new procedure (NTA surface) increased the result accuracy relative to the common GGV surface, however, both yet present some discrepancies with the transient simulation, mainly in normal accelerations (in some corners) and some differences in the tangential accelerations. Clearly, these discrepancies result in different velocities and travelled distances in these corners.

2.3.1.3 Transient

Transient simulations, as the name says, take in account the whole transient behaviour of the car. So, the car is modelled as a dynamic system, through its Differential Equations of Motion. The number of equations (and Degrees of Freedom) determines the complexity and also the accuracy of the simulation. The solution is given by solving the system of equations, in which the DoFs may be coupled or not. In terms of differences with other methodologies, transient simulations have the possibility of detecting with more reality the behaviour of the system, but can also detect the transient instants referred to in the introduction of this section (as variations on state variables of the car). This means that it results in more reliable simulations, but that are more computationally expensive. Academically, this type of simulation is the most known and researched, since the old technology limitations (as argued by Brayshaw) [12] isn't a big problem at this moment. The transient simulators can have vehicle models easily implemented through software (MATLAB/Simulink or others). These models work in a simple way. Basically, given the control variables (often velocity/torque and steering input), the system responds along the time. In some perspective, this implementation can be seen as a "simulator", since it is possible to study small inputs and to know how the car responds. However, in the Lap Time Simulation, the goal is to study the car behaviour on a track, a specific path. So, it is necessary to control the car in this path and, more than this, it is desired that the car be controlled in a specific way (on the limits of its dynamic, for example). For this reason, the vehicle model isn't sufficient for this case. It is

necessary to find some control/solving methodologies to turn the model into a real simulator.

The main problem for transient simulations is that different from the QSS case, it is necessary to implement a solver that takes into account the "history" of the simulation. This occurs because a input in time t_i has effect in t_{i+1} . So, isn't possible to split the track into several segments and calculate each one separately, as done in the previous case. For this reason, one of the main research areas is the control model to solve this problem. These methodologies will be briefly explained in Section 2.3.4, but basically consist of control methods for transient systems joined with optimization scripts, that assure the car on its performance limit. In terms of implemented works, the next sections and subsections will show the different parts of the Transient simulators, with different levels of abstraction.

2.3.2 Physical Model

The physical model is the complexity of the representation of the car. The question here is: *Which is the realism that I need to have and which parameters and effects do I need to study?* More time, the answer depends, but there's a lot of research that points to the results of each type of model. Some limits in the achieved results exist and need to be taken into account before developing a simulator. The most known and utilized physical models have **3, 7, 10 or 14 Degrees of Freedom**. This section will be shown in the opposite way, beginning with the most complete to the simple.

2.3.2.1 14 Degrees of Freedom

The State of the Art of vehicle model in terms of Lap Time Simulation is the 14DoF model. This model represents a double-track car (i.e. a car with 4 wheels). This approach has the capacity to evaluate the behaviour of the suspension accounting for all the effects that it is subjected to: bounce, rolling, pitch and even the damping resistances associated with these motions. Basically, a 14DoF vehicle begins to be what is called "multi-body dynamics", since the car is divided into 5 different bodies (chassis + 4 wheels) connected by springs and dampers (Figure 2.6). As shown in the figure, the 14DoF are longitudinal, lateral and vertical motion of the body; yaw, pitch and roll rotations of the body; vertical motion of the wheels; rotation motion of the wheels.

Lot, in [23], developed a 14DoF model with these generalized coordinates in order to evaluate the performance of a GT class sports car. The model has the influence of aerodynamic forces, non-linear tire model (Pacejka), radial deflection of the tires, differential with torque distribution and constant brake bias. The control variables are the steering input and total torque (throttle or/and brake). Some simplifications were done. For example, with the assumption of suspension small deflections (<40mm), the expressions accounting the chassis rotation variables were approximated to the first order. For the suspension system, was accounted the springs, dampers and anti-roll bars effects. For the unsprung masses (wheels), the spin inertias were considered and for the sprung mass (chassis), all the inertias and masses were considered. The tire vertical forces were calculated

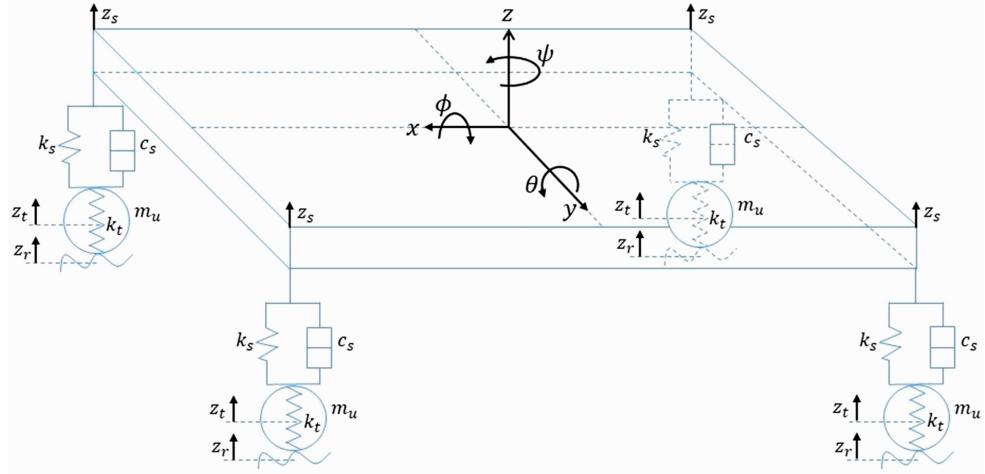


Figure 2.6: 14DoF Vehicle Model. Taken from [22]

through the tire deflection and the overall behaviour of the longitudinal/lateral forces was modelled with first-order relaxation equations, applied to slip. The computational time for the simulation of this model in a transient approach was approximately 1700 s in an Intel Core 2-Quad Q9300 CPU. The model is capable to describe very well the behavior of the car and some studies were done on the parameters. Were studied the longitudinal position of the centre of the mass and the stiffness of the front suspension. The results for the 14DoF were the baseline for some comparisons done by the author, being the 10DoF equally acceptable and accurate, while the 7DoF resulted in different values of optimal parameters (interpreted as wrong or sub-optimal).

Yu, in [24], also developed a simulator with this vehicle model. The main focus of this simulator is to study the parameters related to the powertrain system of an electric vehicle. The same generalized coordinates were used and the approach was very similar to [23]. However, in this case, the torque control variable is changed by the torques in each wheel (that can be controlled individually in order to optimize the behaviour). It resulted in a very consistent simulator, that was validated with the commercial software "VI-CarRealTime". Besides, it was possible to obtain good results with the sensitivity analysis, in which one of the conclusions was that "The lap time performance is very sensitive to the rotational inertia, which can be reduced by adjusting the shape of the wheels and propulsion system" [24].

It is possible to conclude that the modelling of a 14 DoF vehicle gives a very consistent simulator but with some computational cost. The next alternatives have the main focus in simplify the analysis and maintaining a good accuracy in the simulation.

2.3.2.2 10 Degrees of Freedom (with Wheel Inertias)

The 10DoF model is a simplification of the 14DoF model. In this case, the simplification is done on the suspension vertical travel. This means that the vertical motion just accounts for the

body/chassis. Physically speaking, the model cannot take into account the dynamic effects due to the local track bump (in just one or two wheels). Besides, the model cannot take into account local height variations on the track (as small vibrations, acting just in half or a quarter of the car), since there is just one variable related to the vertical motion, accounting for the whole body motion.

Guiggiani, on [25], presents a 10DoF model. In reality, the model presented has 9DoF, because if the model is evaluated on a flat track, the vertical motion is just subjected to aerodynamic downforce. However, this force is applied for a very long time, so the transient (dynamic) behaviour of this degree of freedom can be neglected. However, if there are track height variations, this assumption cannot be done. In this model, the 5 chassis variables (longitudinal, lateral and three rotations) and the 4 wheel spin variables are used. It is also taken into account the roll inertias (pitch, roll and yaw, wheels) and the effect of damping and stiffness on the pitch and roll motions.

Lot, on [23], compared the 10DoF with the 14DoF model. Doing a frequency analysis, the author pointed that the most important motions on the dynamic analysis are the wheel spin and wheel hop. However, the natural frequencies of the wheel spin are bigger (45 Hz vs. 20-30 Hz) and, for this reason, the author tried to simplify its model neglecting the wheel hop dynamics. It is interesting to note that this conclusion accords with Yu (already cited) that the lap time is very sensitive for the wheel spin inertias.

2.3.2.3 7 Degrees of Freedom

The 7 Degrees of Freedom model is another simplification, but in this case, the following degrees of freedom are neglected: pitch angle, roll angle and vertical motion. Guiggiani also has a 7DoF model that doesn't consider the roll, pitch, and vertical motions as degrees of freedom. In reality, the pitch is neglected and the roll is related directly to the lateral applied force and the suspension characteristics, being a direct and simple algebraic relation.

Besides Guiggiani, several other authors worked with this vehicle model, as [21] and [23]. However, each one of them accounted for different effects acting on the car. For example, Guiggiani develop a model that accounts for the suspension geometry, limited slip differential, steering geometry and configuration. Tucker [21] introduces the possibility of having motors in all wheels (even controlled together). These specific internal changes can be introduced since the main equations of the motion be respected. One of the main characteristics of this model is that, despite some authors put an algebraic relation between the roll effect and the lateral acceleration, the planar motion of the car can be decoupled from the out-of-plan motion. This means that the load transfers can be calculated without the planar motion case and, for this reason, some changes can be done in this part (for example introduce the suspension geometry and characteristic).

2.3.2.4 3 Degrees of Freedom

The final degree of simplification of a vehicle model is the 3DoF model. For this step, it is necessary to neglect the wheel spin and look just for the planar motion dynamics. The implementation of this model is very known in Vehicle Dynamics literature since the main part of the understanding of the vehicle behaviour is through the planar motion. For this reason, and for simplification terms, it is very common to find the 3DoF model described and developed as a Single-Track model (or bicycle model). This variation basically condenses each axle in one tire. In the most simple cases, the load transfers are neglected and the tire is modelled through a coefficient that describes more or less its behaviour. However, it is perfectly possible to implement a double-track model (Figure 2.7). The difference isn't great in terms of results, but the description of the real effects isn't completely consistent using the single-track model.

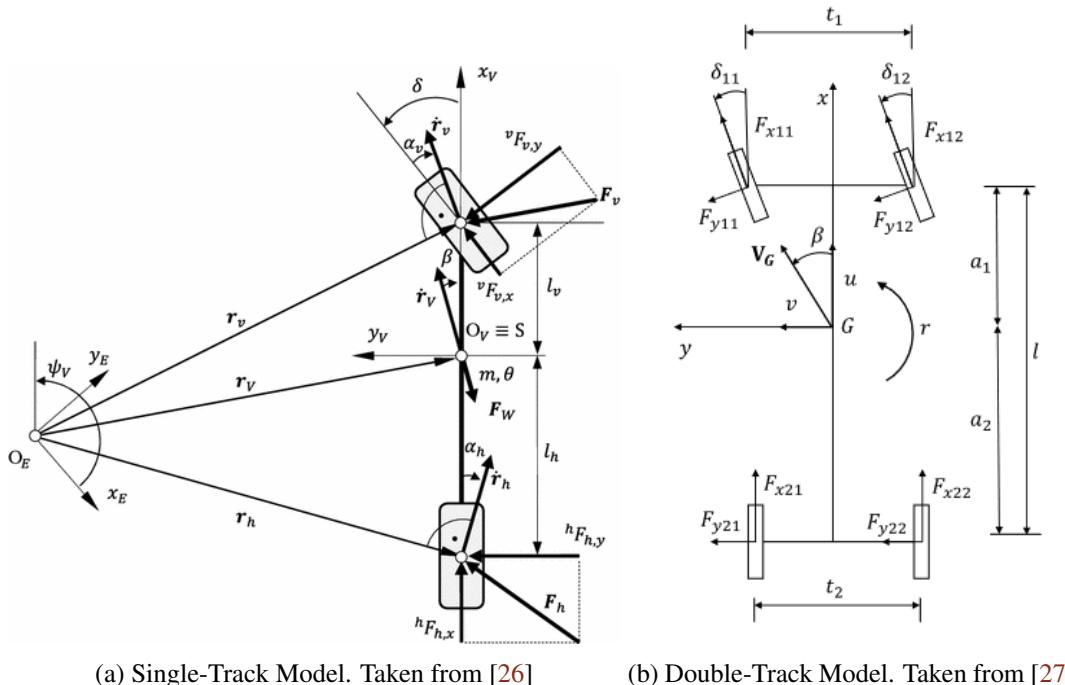


Figure 2.7: 3 DoF Planar Models

2.3.2.5 Conclusion

The chosen vehicle model needs to be consistent with the objectives of the simulator. Quasi Steady-State simulators by definition neglect the transient behaviour of the model. This means that the inertial terms aren't taken into account and, for this reason, the only vehicle model that can be used is the 3DoF with some simplifications. However, as will be developed in the next chapter, this doesn't mean that the model isn't sufficiently real. It is possible to study several parameters depending on the algebraic relations that are taken into account. However, it is clear that the transient simulators, mainly with more degrees of freedom, are more reliable and can detect more effects (such as inertia and damping). If the methodology chosen to develop a simulator

is transient, the 10DoF appears to be an excellent option, since it doesn't take into account the wheel's vertical motion (which simplifies significantly the equations), and has good results. However, it is clear that the computational cost is relevant and, as will be explained at the final of this chapter, the balance of usability-accuracy needs to be studied.

2.3.3 Path Model

The path model is the way that is used to represent the track/path in terms of lengths, corners, coordinates, or radius. This category in the development of the software is very related to the control model since the driver will follow a path and the way that it will follow depends on the track representation. The question here is: *How will I create a path representation and which is its complexity?* This discussion involves some parts: track data acquisition, track reconstruction, track parameterization, and track evaluation.

2.3.3.1 Track Data Acquisition

Track data acquisition is basically the process of obtaining information about the path. This is important in Lap Time Simulation because the main goal isn't just to evaluate the car's behaviour in theoretical or generic manoeuvres but in a real track. This means that the track model needs to be realistic. For this reason, there are several ways to obtain this data. In very known tracks, it is possible to find data easily on the internet. However, mainly when the idea is to tune and simulate a specific car in specific tracks, obtaining the data through GPS acquisition is a very common approach. This data basically can be provided by GPS and, through treatment/conversion, it is possible to have X-Y Coordinates, that will be used for the track reconstruction. Another known way to obtain track data is from Telemetry Data. Gabicci [27] says that a telemetry system can provide the run time, the travelled distance, the longitudinal speed, accelerations, vehicle slip angle, yaw rate, steer angle, and pedal positions. With all this information, it is possible to integrate the information (mainly the velocity profiles) and obtain the coordinates X-Y and the yaw angle.

2.3.3.2 Track Reconstruction

Once the data is obtained, some other treatment processes can be done. One of the main problems is seen in closed tracks. For the existence of small errors, it is common to obtain tracks in which the end doesn't intersect the beginning. For this reason, some mathematical algorithms can easily remake the path, correcting these discrepancies. Another problem is the presence of raw obtained data. Gabicci [27] compares the raw and filtered yaw data (from telemetry) to obtain the path. The difference between them is justly the offset in the obtained positions (relative to the real circuit). After these corrections (Figure 2.8), an X-Y matrix can be obtained. Depending on the necessities, it is possible to obtain the global angle of the trajectory and, with this data, different parameterization can be done.

However, there's another variation on the track model. If the car is allowed to run in a free path (finding the optimal path, for example), besides the reference path, it is necessary to define the

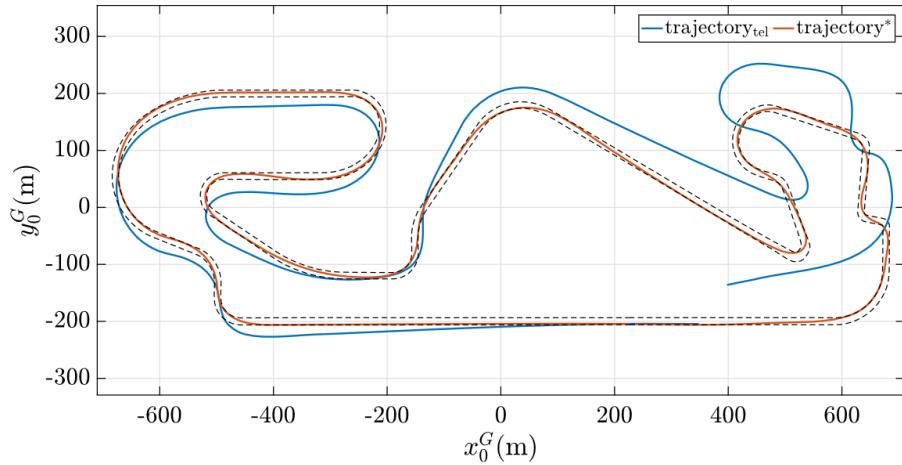


Figure 2.8: Comparison between trajectory obtained by direct integration of raw telemetry data (blue) and with the filtered yaw rate (orange). Taken from [27]

width of the track (or, in mathematical terms, the lateral boundaries of the car's motion). Figure 2.9 shows an example of this type of track model. Besides the information about the centre line (that comes from the coordinates), it is important to know the lateral maximum displacement and the track heading angle.

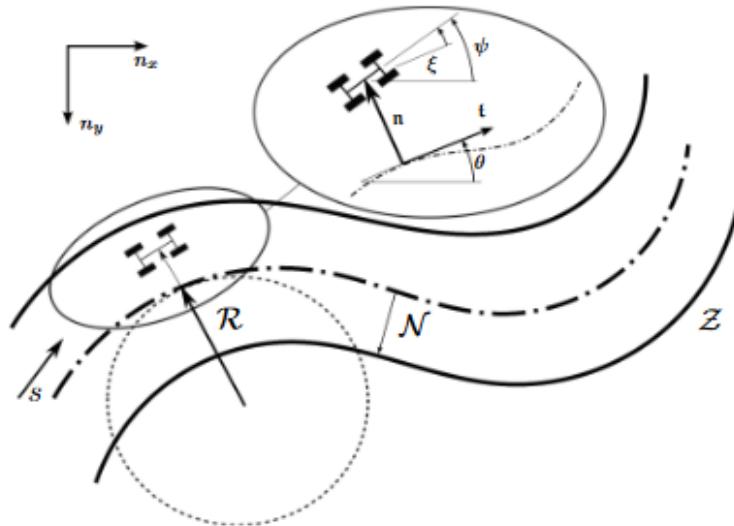


Figure 2.9: Free path track model example. Taken from [14]

2.3.3.3 Track Parameterization

Different simulators can read the track data in different ways. For this reason, several works present a parameterization process to the coordinates. Being a free path or fixed path, a reference circuit need to be given to the software. As seen above, this information can be provided

through the X-Y coordinates. Still, depending on the type of evaluation, frequently, the coordinates need to be transformed into Curvature/Radius-Length data. One of the methods used for this transformation can be seen in [28]. The following equation can be written:

$$\kappa = \frac{x'y'' - y'x''}{(x'^2 + y'^2)^{\frac{3}{2}}} \quad (2.7)$$

In which κ is the curvature of the segment, x', x'', y', y'' are the first and second derivatives of x and y coordinates (relative to the length of the segment), being:

$$x' = \frac{dx}{ds} \quad (2.8)$$

$$x'' = \frac{d^2x}{ds^2} \quad (2.9)$$

$$y' = \frac{dy}{ds} \quad (2.10)$$

$$y'' = \frac{d^2y}{ds^2} \quad (2.11)$$

The length variation ds for each segment can be calculated:

$$ds = \sqrt{(x_i - x_{i-1})^2 + (y_i - y_{i-1})^2} \quad (2.12)$$

And, with κ and ds , it is possible to consider the reference trajectory in several simulators. Other methodology is seen in [14], that state the following relations:

$$dx = ds \cos(\theta) \quad (2.13)$$

$$dy = ds \sin(\theta) \quad (2.14)$$

$$\frac{dx}{dy} = \tan(\theta) \quad (2.15)$$

$$C = \frac{d\theta}{ds} = \frac{d}{ds} \left(\arctan \frac{dy}{dx} \right) \quad (2.16)$$

In this methodology, the X-Y coordinates are directly related to the heading angle θ and to the curvature, which is described as C (equivalent to the already presented κ).

2.3.3.4 Track Evaluation

The evaluation of the track data is the last aspect of the track model to be discussed. This is directly tied to the simulator's control model. The track evaluation combines the car's real path (determined by the controller) with the track reference path to assess the deviation. Continuing the development from Perantoni, the author joins the track data with the vehicle motion data in an index named "Performance Index":

$$J = \int_{s_0}^{s_f} (r\tilde{u}^2 + (x_c - x)^2 + (y_c - y)^2) ds \quad (2.17)$$

x_c and y_c are values from track model while \tilde{u}, x and y are values derived from the car motion. This index will be minimized in the control model, which will be described later.

2.3.4 Control Model

As said before, the solution for the Transient problem is found using some method for the control. The track, in this case, can be fixed or free (and the optimizer can choose the best path within the limits of the track). Moreover, even the quasi steady-state model can be solved using these optimization methodologies, since the track model is free-path. Quoting the same author cited earlier, we can understand the necessity of a control model for this problem. [14]:

The minimum-lap-time optimal control problem includes non-linear and nonsmooth dynamics, as well as state and control constraints. Care is required when formulating these problems in order that a reliable solution is obtained in a reasonable time. Key steps include selecting the optimization strategy to be used, selecting a simple vehicle model that captures all the important problem features, finding an efficient and numerically stable track centre line description, scaling the problem correctly and developing techniques for ‘smoothing out’ non-smooth features of the problem so that the gradients required by the optimization algorithm are properly defined.

In other words, the author says that the control model needs to be capable to deal with the non-linear dynamics from the transient simulators, accounting control and state constraints, and, more than this, optimizing the solution in an efficient way. So, thinking to fulfil these requirements, the control model used in these problems is the Optimal Control Problem (OCP). In a simple way, the mathematical formulation of this problem is:

$$\min_{\{p, \tilde{u}(\cdot), \tilde{x}(s_0), \tilde{x}(s_f)\}} \int_{s_0}^{s_f} l(s, p, \tilde{x}(s), \tilde{u}(s)) ds \quad (2.18)$$

subject to the constraints:

$$\begin{aligned} \frac{dx}{ds} - f(s, p, \tilde{x}(s), \tilde{u}(s)) &= 0 \\ g(s, p, \tilde{x}(s), \tilde{u}(s)) &= 0 \\ h(s, p, \tilde{x}(s), \tilde{u}(s)) &\leq 0 \\ g_b(s, p, \tilde{x}(s), \tilde{u}(s)) &= 0 \end{aligned} \quad (2.19)$$

Following the definitions given by the author, s is the travelled distance along the track, (s_0 and s_f are the initial and final distances, respectively), p is a vector containing the vehicle parameters to be optimized (this vector cannot exist if the goal is just to study the behaviour of a car without parameters optimization), $\tilde{x}(s)$ and $\tilde{u}(s)$ are the state and control vectors, respectively. In terms of functions, f describes the vehicle dynamics, g and h describe the equality and inequality constraints, being the subscript b (in g_b) the boundary constraints. Finally, the function l is a stage cost and is a function of the state, the controls, and the parameters. These functions and equations can be modelled in several ways depending on the models adopted, but in simple terms, the OCP consists of an optimization problem for an entire run (one or several laps) trying to minimize the run

time (in the basic case) varying the described path and control inputs, respecting the performance and path constraints.

2.3.5 Conclusion about Lap Time Characterization

As explained above, there are several solutions and different variations to build a Lap Time Simulation. Each simulator can be built looking for different necessities and this balance need to be found before the beginning. It is obvious that simple vehicle models cannot give specific information about suspension damping, for example. Fixed path track models cannot give information about optimal paths (to suggest to the driver). But this characteristic may not be a problem depending on the necessities of each simulator. In terms of complexity, works that join a 14DoF model with free path tracks and an Optimal Control Problem solver are the best, but the ruler cannot stop at this moment. More specific simulations are needed for structural analysis of the suspension, for example. For this type of investigation, multi-body simulation can be used, to consider the suspension parts as flexible. But on the other hand, maybe it is not so advantageous to develop a Lap Time Simulation with this approach since the computational cost of this is big. Moreover, there are several simulation environments for the investigation of suspension dynamics for the worst-case scenarios (that are important for structural development), as MSC ADAMS, OptimumDynamics and already developed models in MATLAB/Simulink. So, each simulator needs to be thought thinking about the necessities and expected results, that will be taught in the next section.

2.4 Possibilities and Results

During the description of the State of the Art, several works try to give different types of results through the Lap Time Simulation. This clearly is a criterion that depends on the objectives of the software, but it is important to see what these works can give to a dynamics or race engineer. As said in the last section, it is clear that a 14DoF model has more information to give than a 3DoF model (for example the individual wheel displacement). However, some types of data are common to any type of simulator. These data are the state variables directly related to the car's motion, such as the accelerations and velocities.

The main result that a Lap Time Simulation can give is the speed. This parameter is directly related to the "lap time" and can describe how fast the car is in each sector of the track. Figure 2.10 show an example. Other very known and common results are the accelerations (Figure 2.11). This information determines, at some level, the dynamic limits of the car. Longitudinally, on the positive part, it is possible to know the acceleration capability of the car (directly tied to the motor). On the negative part, it is possible to know the deceleration capability, related just to the dynamics characteristics of the car. Laterally, the boundary shows the capability of the car to deal with cornering manoeuvres. These two data, when joined in a plot, result in the named "GG Diagram", which shows the overall dynamics capability of a car (Figure 2.12). This diagram can show something like an envelope which determines the limits of the dynamic in the combined

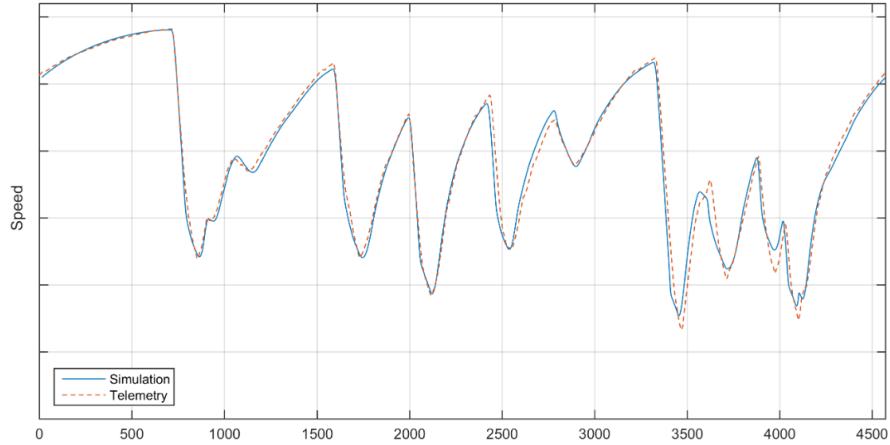


Figure 2.10: Example of a Speed Profile. Taken from [29]

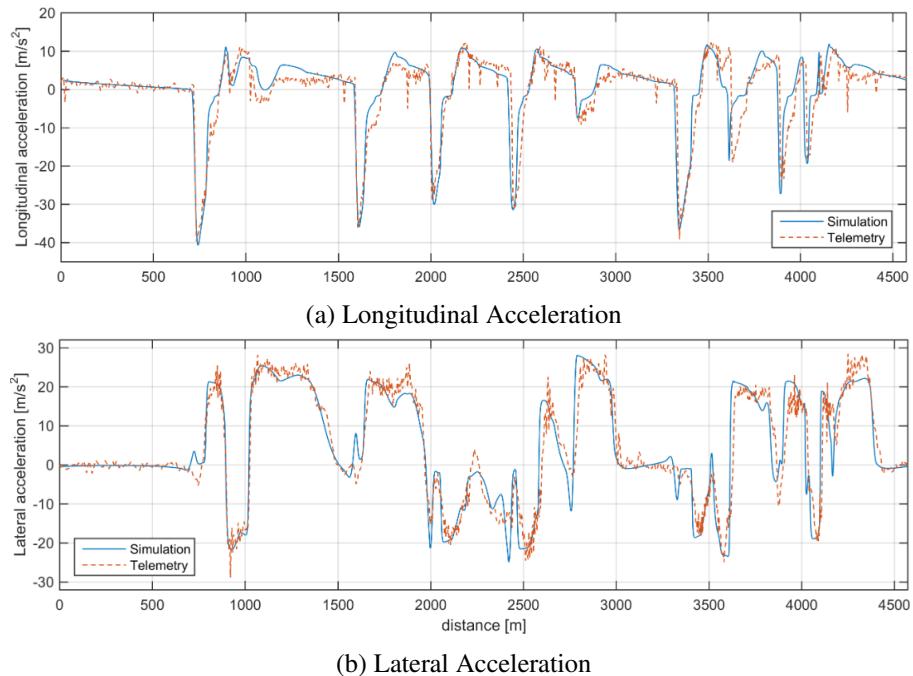


Figure 2.11: Acceleration Plots Example. Taken from [29]

cases (cornering with braking/accelerating commands). Moreover, this diagram is one of the most known and important data for vehicle dynamics analysis. However, despite these common data, it is possible to obtain information about specific parameters of the car depending on the vehicle model chosen. The same author cited in these images presents information about two of these parameters: Chassis roll and pitch angles; chassis vertical displacement (Figure 2.13). The first two are directly related to the lateral and longitudinal forces applied to the car, while the last one is related to the vertical motion (for example bump on the tracks), but mainly with the aerodynamics downforce.

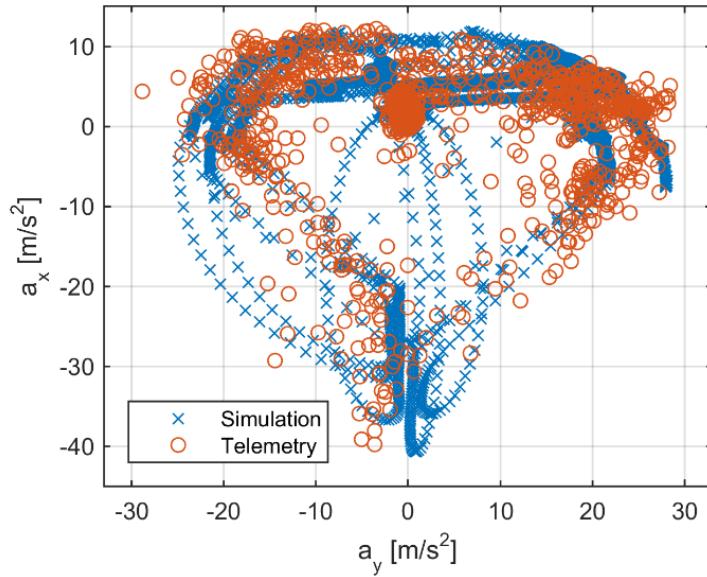


Figure 2.12: Example of a GG Diagram. Taken from [29]

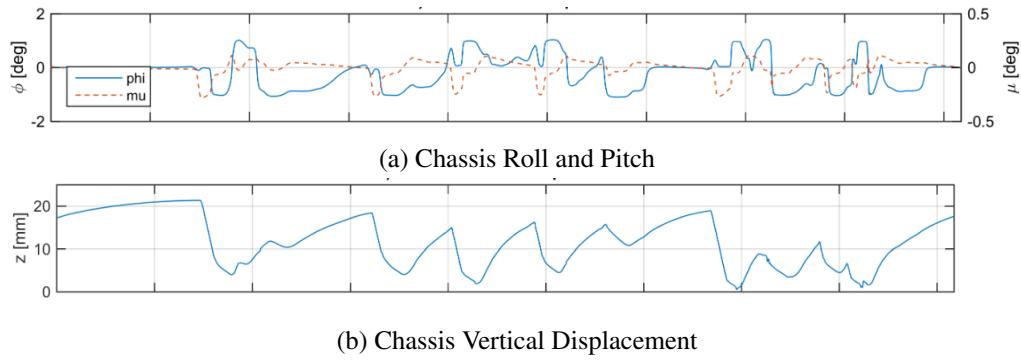


Figure 2.13: Example of chassis roll angles and displacement. Taken from [29]

Finally, one of the main results that a Lap Time Simulation need to be capable to give is a clear parametric study. This is named "sensitivity analysis" and consists in predict how the variation of a parameter impacts the final result. More time, the capability to variate parameters and study their impact on the car's performance depends on the vehicle model. Practically all simulation software can be adapted to do this type of study, but the application of the Optimal Control Problem in the solving can help, since the parameters (to be studied) can be put in the optimization function and the solver will return automatically the optimal result for the parameter. Figure 2.14 shows an example of a result obtained through the Optimal Control Problem. Using this methodology, [23] could achieve the optimal solution. By the nature of the solving methodology, it is possible to think that the optimal path for two different parametric configurations can be different.

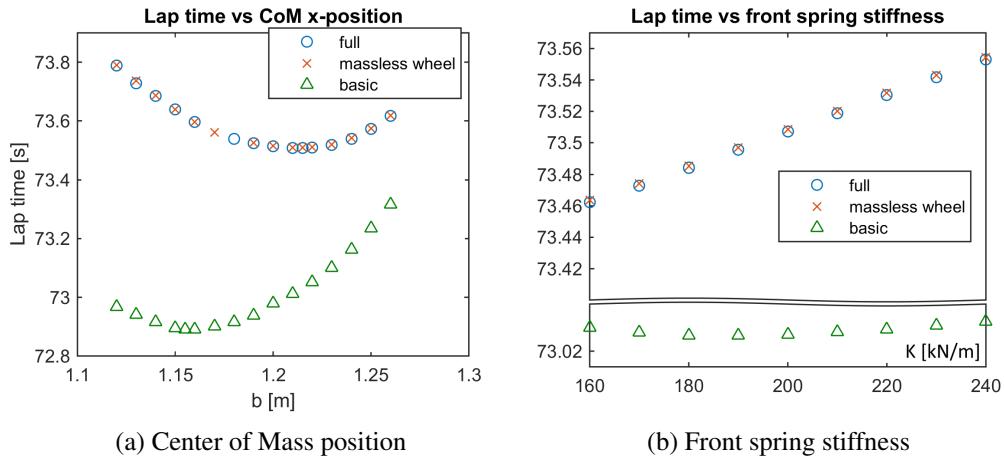


Figure 2.14: Example of sensitivity analysis results. Taken from [23]

2.5 Application in Formula Student

In the Formula Student context, and specifically in FSFEUP, the simulation can be applied everywhere during the process. When the design of the car begins, it is necessary to have a direction to go in terms of design choices. Frequently the decision begins with powertrain decisions: 1, 2 or 4 motors; gear ratio choice; differential type (if applicable). Other decisions about battery pack (size and type of cells) need to be done and, through this selection, some constraints and predictions about the car's weight can be reached.

However, as the competition involves different events, all these variations and combinations need to be studied for all the dynamic events cited earlier. The best overall result can be not the best result for a specific event. This focus needs to be decided by the team in the design concept phase. But independently of the strategy, this balance just can be done through simulations. Once the main specifications are predicted by simulations and initial choices, optimization processes can be done in the suspension system in order to create a balanced car with specific dynamics characteristics. Wheelbase, track width and tire radius can be chosen through sensitivity analysis. Moreover, some goals for weight distribution and CoG height can be defined for the design parts phase. Finally, one of the most important parts is the tires, The choice of them can be made iteratively with the suspension parts.

The result of these simulations doesn't affect just the dynamic behaviour (and consequently the dynamic result), but the Engineering Event also. The engineering decisions need to be founded on solid theoretical backgrounds or reliable simulations. With this, it is possible to define the requirements of a Lap Time Simulation to be developed for a Formula Student context and the necessities of a team with its first car.

2.6 Conclusion

The literature review pointed to a subjective task of finally choosing the strategy for the Lap Time Simulation developed. It was shown, in an overall approach, the different variations that can be employed in simulation methodology, vehicle model, path model and control model. These variations need to fulfil the requirements about model correspondence with reality and the types of studies to be done by the software. The chosen approach will be shown in the next chapters of this work.

Chapter 3

Quasi Steady-State

This chapter shows the development of a Quasi Steady-State Lap Time Simulation. This methodology was chosen among all exposed in the last chapter. So, the exposition begins with an introduction, stating the most important requirements for the software. Once these requirements are stated, the following part is the exposition of the vehicle model and track model. Finally, the chapter ends with a section explaining the computational implementation, touching on some optimization processes used and difficulties in the development.

3.1 Introduction

As argued at the end of the last chapter, the choice of an approach to a simulator need to be done from the necessities or expected results/conditions of the simulator. For this reason, the following requirements are given for this application:

- The simulator needs to be capable to give information about a real car, showing its dynamics capabilities and characteristics.
- The simulator needs to be capable to study the performance impact of parameters changing.
- The simulator needs to be capable to give initial design inputs in the beginning stage of the development of a Formula Student car.
- It is necessary to assure that all these analyses and studies can be done in different tracks and conditions, without the necessity of deep code changes to change tracks or operating conditions.
- The simulator needs to be capable to make fast simulations to be used massively and iteratively in the stage of parameters studying.
- The driving configuration needs to assume the performance limits of the car (i.e. a perfect driver).

- It is not necessary to find the optimal path for different tracks, since the actual stage of the team is just to analyze different configurations and correlate the simulations with a real path (that can be given as input to the simulator).

For this reason, a Quasi Steady-State methodology was chosen for the development of this simulator. This choice was done for the ease of implementation of this methodology (it doesn't need the application of Optimal Control theory), the low computational cost (compared to the transient solutions), and the capability to deal easily with parameters changing or tracks changing. Moreover, in terms of Vehicle Model, the following requirements are imposed:

- The model needs to have an electric powertrain system, including the motor, the transmission (gear ratio, efficiencies), and battery constraints (power limit, maximum current, maximum voltage).
- The suspension model needs to be sensible to spring stiffness and anti-roll bars.
- The model needs to have a non-linear tire model, that takes into account the slip angles and normal forces.
- The steering model must have information about Ackermann geometry (percentage), toe-in/out, and steering ratio.
- The aerodynamics can be considered in a simple way: drag, downforce, and CoP position.
- Other general parameters need to be easily changed (for studies), such as the masses and general dimensions.

To have all these requirements into account, the model chosen is a Non-Linear 3DoF. The meaning of "non-linear" will be explored later, but the presented approach involves the most important effects of a race car in motion.

3.2 Vehicle Model

The main concern of the vehicle model, as seen in the last chapter, is to know how to describe accurately the behavior of a race car in physical terms. For the model developed in this part, the following physical effects were considered:

- Longitudinal Load Transfer
- Lateral Load Transfer
- Aerodynamic Downforce
- Aerodynamic Drag
- Tire Sensitivity (Non-Linear Model)

Besides the interest on the physical realism in the longitudinal/lateral dynamics of the car, it is necessary to describe accurately the physics of the powertrain system. The following aspects were considered:

- Motor Simplified Model
- Battery Voltage Limitation
- Powertrain Efficiencies (Chain Drive + Differential)

3.2.1 Chosen Approach

QSS simulators frequently have single-point mass vehicles, in which the lateral dynamics are defined easily through the circular motion theory. However, this approach neglects some important effects of the car motion, such as the existence of yaw rate, lateral velocity and the effect of the tires on the dynamic behaviour of the car. So, for this work, the model chosen is the called "bicycle model" or "single-track model". This model is the most known in Vehicle Dynamics applied literature since it is more intuitive and simple to implement. However, by definition, a "bicycle model" cannot take into account lateral and longitudinal load transfers directly, despite the influence of the tire coefficients. So, as will be shown, the model used in this work was modified and in reality is a "double-track" model transcript to the "single-track" theory. This modification means that the model is a non-linear single track. To make this transformation, it is necessary to assure that the planar motion of the car isn't affected by the out-of-plan motion (as longitudinal and load transfers).

In reality, in Guiggiani's book, [30] and [25], it is possible to compare the double-track model (Chapter 3) with the double-track model with roll and pitch inertias (Chapter 9). The planar degrees of freedom are coupled with the out-of-plan degrees of freedom only in the inertia accounting model. This means that, since the body rotations inertias (except yaw) are neglected, it is possible to divide the planar motion from the roll motion (lateral loads) and from the pitch motion (longitudinal loads). This approach will be explained better in the following.

3.2.2 Planar Motion

Beginning with the planar motion, to describe the equations, it is useful to look at what is the bicycle model in physical terms. Figure 3.1 shows that the two wheels of each axle are transformed into an equivalent wheel. The motion of the body is described by the longitudinal displacement x , lateral displacement y and rotational displacement (yaw) ψ , which defines a coordinate system with origin point C . The control parameters are the longitudinal velocity V_x and the steering angle δ . Some other state parameters are included. The body absolute velocity V , the body yaw rate r , the body side-slip angle β , the front and rear tire-slip angles α_1 and α_2 , the front and rear body side-slip angles β_1 and β_2 . So, the idea is that to a pair of input parameters (velocity and steering angle), the state variables will react in a specific way.

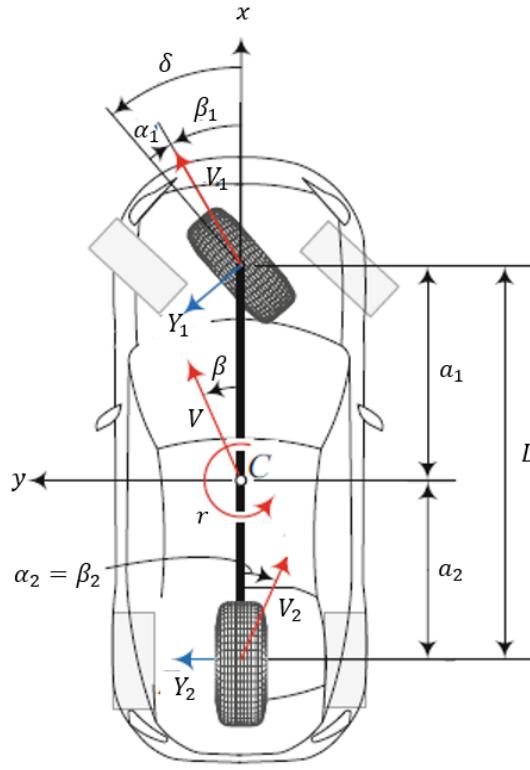


Figure 3.1: Bicycle Model. Taken from [31]

The planar motion equations come from the Newton-Euler equations of motion (Equations 3.1-3.3). However, since we are concerned only with lateral dynamics, the two last equations will be developed. The lateral forces can be calculated as shown in Equation 3.4 and the moment is calculated in 3.5. In these equations, some additional parameters appear, such as the vehicle inertia I_z and the Tire Cornering Stiffness for each axle $C_{\alpha i}$.

$$X = m(\dot{V}_x - rV_y) \quad (3.1)$$

$$Y = m(\dot{V}_y + rV_x) \quad (3.2)$$

$$M_z = \dot{r}I_z \quad (3.3)$$

$$Y = Y_1 + Y_2 = -C_{\alpha 1} \left(\frac{V_{y1}}{V_x} - \delta \right) - C_{\alpha 2} \frac{V_{y2}}{V_x} \quad (3.4)$$

$$M_z = a_1 Y_1 - a_2 Y_2 = -a_1 C_{\alpha 1} \left(\frac{V_{y1}}{V_x} - \delta \right) + a_2 C_{\alpha 2} \frac{V_{y2}}{V_x} \quad (3.5)$$

These equations must be combined with the kinematic analysis (Equation 3.6) and, consequently, they are rearranged (Equations 3.7 and 3.8).

$$V_{y1} = V_y + ra_1 \quad V_{y2} = V_y - ra_2 \quad (3.6)$$

$$Y = \left(-\frac{a_1}{V_x} C_{\alpha 1} + \frac{a_2}{V_x} C_{\alpha 2} \right) r - (C_{\alpha 1} + C_{\alpha 2}) \frac{V_y}{V_x} + C_{\alpha 1} \delta \quad (3.7)$$

$$M_z = \left(-\frac{a_1^2}{V_x} C_{\alpha 1} - \frac{a_2^2}{V_x} C_{\alpha 2} \right) r - (a_1 C_{\alpha 1} - a_2 C_{\alpha 2}) \frac{V_y}{V_x} + a_1 C_{\alpha 1} \delta \quad (3.8)$$

It is possible to define β (Equation) and simplify the expressions obtained (Equations 3.10 and 3.11).

$$\beta = \frac{V_y}{V_x} \quad (3.9)$$

$$Y = C_r r + C_\beta \beta + C_\delta \delta \quad (3.10)$$

$$M_z = D_r r + D_\beta \beta + D_\delta \delta \quad (3.11)$$

Finally, Milani ([31]) defines these six coefficients as "Force System Coefficients", that serve to simplify the representation of the equations. The same author explains and exposes, in this development, the Steady-State Responses for these equations of motion:

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{C_\beta}{mV_x} & \frac{C_r}{m} - V_x \\ \frac{D_\beta}{I_z V_x} & \frac{D_r}{I_z} \end{bmatrix} \begin{bmatrix} (V_y)_{ss} \\ (r)_{ss} \end{bmatrix} + \begin{bmatrix} \frac{C_\delta}{m} \\ \frac{D_\delta}{I_z} \end{bmatrix} \delta \quad (3.12)$$

The obtained matrices can be manipulated (Equation 3.13) and the final quantities can be defined (Equations 3.14-3.16).

$$\begin{bmatrix} (V_y)_{ss} \\ (r)_{ss} \end{bmatrix} = \begin{bmatrix} \frac{C_\beta}{mV_x} & \frac{C_r}{m} - V_x \\ \frac{D_\beta}{I_z V_x} & \frac{D_r}{I_z} \end{bmatrix}^{-1} \begin{bmatrix} \frac{-C_\delta}{m} \\ \frac{-D_\delta}{I_z} \end{bmatrix} \delta \quad (3.13)$$

$$(V_y)_{ss} = \frac{D_\delta(C_r - mV_x) - D_r C_\delta}{D_r C_\beta - C_r D_\beta + mV_x D_\beta} V_x \delta \quad (3.14)$$

$$(r)_{ss} = \frac{C_\delta D_\beta - C_\beta D_\delta}{D_r C_\beta - C_r D_\beta + mV_x D_\beta} \delta \quad (3.15)$$

$$(a_y)_{ss} = (r)_{ss} V_x \quad (3.16)$$

Then, it is possible to see that the system has two control inputs and for each pair, it is possible to discover the dynamic response (in this case, in steady-state condition) for the other state variables. However, as said before, the model developed in this work is an improvement of the single-track (bicycle) model. The improvement was done in the tire coefficients, $C_{\alpha 1}$ and $C_{\alpha 2}$, that differently of [31], aren't constant, but are dependent on the state of the car. To understand it, it is necessary to investigate the tire model that is utilized in the model.

3.2.3 Non-Linear Tire Model

A very simple way to determine the friction force of a body is with the friction coefficient μ . The already cited "Coulomb Theory" says that the friction force is dependent on the friction coefficient and the normal force

$$F_{friction} = \mu F_n \quad (3.17)$$

But on reality, the friction behavior of a tire is much more complex. This occurs for several reasons, but the first is the constructive solution of a tire.

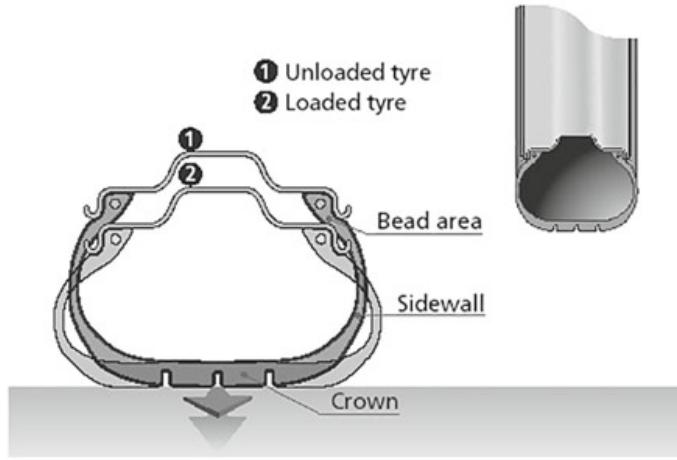


Figure 3.2: Tire Radial Deformation. Taken from [32]

Actually, a tire has a carcass made with rubber and, for this reason, the tire mechanical behavior is very different of a rigid body; the carcass can deform. To complicate more, the tires are inflated with compressed air. This means that the deformation of a specific tire depends on its characteristics and, obviously, on the internal pressure. Continuing, the deformation of the tire affects what is known as the "contact patch", that is the region of the tire that is in contact with the ground. A more loaded tire has a bigger contact patch (Figure 3.2) and, for this reason, more superficial interaction and, for this reason, more friction. But the tires are not elastic (or visco-elastic) just in the radial direction. This behaviour also occurs in the lateral direction. But laterally, some effects occur simultaneously. The first, and more intuitive, is the lateral deformation due to the inertial effect from the car mass (Figure 3.3). The other, which shows one of the most important concepts in vehicle dynamics, is the (already cited) side-slip effect on the tire (Figure 3.4). So, the final resume is that the tire has a highly non-linear behaviour and, for this reason, it is important, to have a more accurate result, and develop a model that can detect the non-linearities on the tire. The most known model in Vehicle Dynamic was developed by Hans Pacejka, known as "Magic Formula". This formula can be represented in the following way (Taken from [32]):

$$y(x) = D \sin\{C \arctan[Bx - E(Bx - \arctan(Bx))]\} \quad (3.18)$$

B is the "stiffness factor", C is the "shape factor", D is the "peak value" and E is the "curvature

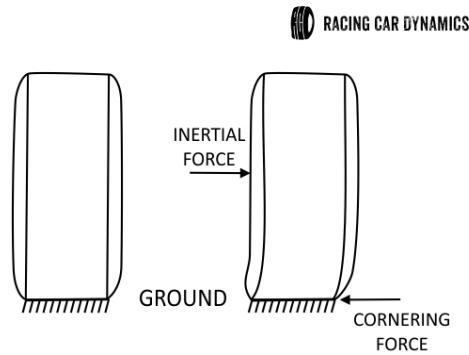


Figure 3.3: Tire Lateral Deformation. Taken from [33]

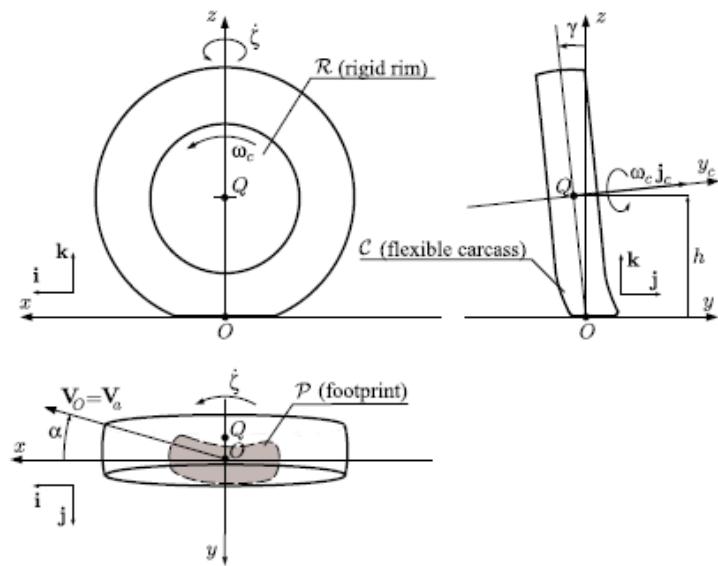


Figure 3.4: Tire Side-Slip. Taken from [32]

factor". In the equation, the y is the Lateral or Longitudinal Forces and x is the respective slip quantity (slip ratio for longitudinal and slip angle for lateral). The equation won't be explored so much here, but the most important conclusion is that the lateral and longitudinal forces that the tire applies to the road are a function of the following parameters:

$$F_x = F_x(F_z, \gamma, \kappa, \alpha) \quad (3.19)$$

$$F_y = F_y(F_z, \gamma, \kappa, \alpha) \quad (3.20)$$

In which F_z is the normal force on the tire, γ is the camber angle, κ is the slip ratio and α is the side-slip angle. The camber angle is the angle between the tire and the vertical axis (Figure 3.4) and is a kinematic quantity that depends on the suspension geometry of each car. The slip ratio is the ratio of the tangential slipping of a tire, it can be defined through Equation 3.21. Finally, the slip angle is the ratio between the lateral velocity and the car velocity (Equation 3.22). Combining

these parameters, the plot of Figure 3.5 can be obtained.

$$\kappa = \frac{V_{car} - V_{tire}}{V_{car}}, \quad V_{tire} = \omega_{tire} r_{tire} \quad (3.21)$$

$$\alpha = \arctan \left(\frac{V_{car,y}}{V_{car,x}} \right) \quad (3.22)$$

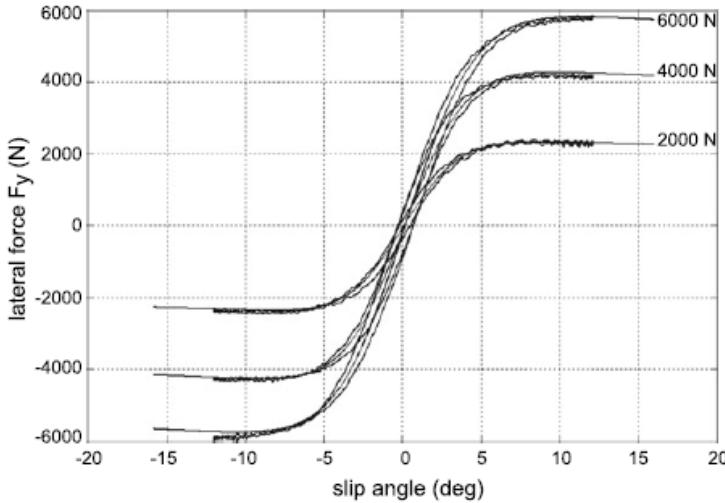


Figure 3.5: Non-Linear Tire Plot. Taken from [32]

With this figure, it is possible to understand that, the capacity that a tire has to support lateral forces is dependent on two main parameters: normal force and slip angles (it depends of others, such as camber angle, slip ratio, pressure and temperature, but the main are these two). For this reason, this behaviour of the tire is named "tire sensitivity". In other words, the tire is sensitive to vertical loads and slip angles. These parameters need to be taken into account to reach an accurate result about the tire behaviour in each moment.

But now, returning to the vehicle model that is shown here, it is necessary to relate the Pacejka Non-Linear Model with the coefficients described in the vehicle model. Basically, the coefficients $C_{\alpha 1}$ and $C_{\alpha 2}$ are called "Tire Cornering Stiffness". This definition has some differences. Milani [31] relates these coefficients with the lateral forces (Equation 3.23). Assuming that the tire behaves in a linear way in respect to the slip angles (the other variables can change), Equation 3.24 can be achieved.

$$Y_i = \alpha_i C_{\alpha i} \quad (3.23)$$

$$C_{\alpha i} = \frac{Y_i}{\alpha_i} \quad (3.24)$$

However, as the tire lateral force isn't linear with the slip angle, a more specific definition can

be given (Equation 3.25).

$$C_{\alpha_i} = \frac{\partial Y_i}{\partial \alpha_i} \Big|_{\alpha_i=0} \quad (3.25)$$

That said, the chosen approach for the inclusion of the tire model is to make the Tire Cornering Stiffness variable (depending on the normal forces variation). However, if this is assumed, it is created an interdependence between the state and the tire behaviour. This interdependence makes the model non-linear. Basically, the state of the car depends on the tire stiffness, but the tire stiffness depends on the state of the car. To include the sensitivity of the normal forces on the Tire Cornering Stiffness, it is necessary look to at the Out-of-Plan motion, which defines the Lateral and Longitudinal Load Transfers.

3.2.4 Out-of-plan motion

As said before, the vehicle model developed in this chapter can be calculated separately (it means, the model can be decoupled) in planar motion and out-of-plan motion. The main effects that occur out-of-plan are the lateral and longitudinal load transfers. The first depends on the lateral acceleration and the second depends on the longitudinal acceleration. Clearly, these effects need to be superposed, since the highest performance of the car involves brakes and acceleration on dynamics limit before and after the corner apexes, respectively.

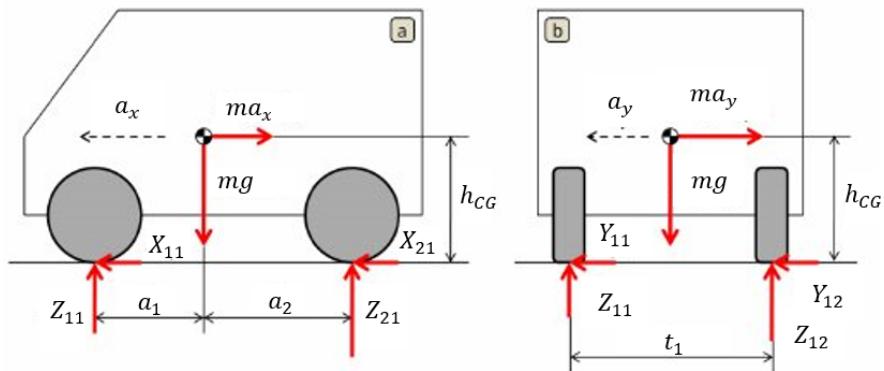


Figure 3.6: Longitudinal and Lateral Vehicle Models. Taken from [34]

Figure 3.6 shows briefly the main forces acting in the car. It is possible to see that, in addition to the gravitational force, there are three types of forces: Frictional Reactions, Inertial Forces and Normal Reactions. The Frictional Reaction forces are simply the forces from the tires. They were generally defined above in Tire Model, but at the same time, these forces are directly related to the inertial forces. The Inertial Forces, which involve most of the theme of this subsection, are developed from the motion of the car. They follow Newton Second's Law $F = ma$ and, for this reason, any acceleration associated with the car motion results in forces applied to the masses. Finally, the Normal Reactions, despite being easy to understand, are directly affected by the Inertial Forces. Basically, by definition, the inertial forces act in the Center of Gravity (CoG) of the bodies. However, since the CoG has a specific height h_{CG} from the ground, the force generates

an equivalent moment. To balance this moment, it is necessary a normal reaction in the tires. this reaction is the load transfer, which occurs for longitudinal and lateral accelerations.

3.2.4.1 Longitudinal Load Transfer

The calculation begins with the balance of the forces assuming the existence of longitudinal acceleration.

$$\sum X = ma_x \quad (3.26)$$

$$\sum Z = mg \quad (3.27)$$

$$\sum M_y|_{x=0,z=0} = Z_2 \times a_2 - m \times a_x \times h_{CG} - Z_1 \times a_1 = 0 \quad (3.28)$$

It is possible to define the vertical loads in its static parts and dynamics (load transfer contribution) parts (Equations 3.29 and 3.30). The static terms are calculated in Equations 3.31 and 3.32.

$$Z_2 = Z_2^{static} + \Delta Z \quad (3.29)$$

$$Z_1 = Z_1^{static} - \Delta Z \quad (3.30)$$

$$Z_2^{static} = \frac{m_{CG} \times a_1}{L} \quad (3.31)$$

$$Z_1^{static} = \frac{m_{CG} \times a_2}{L} \quad (3.32)$$

It is possible to manipulate Equation 3.28 and join it with (3.31) and (3.32), resulting in equations 3.33-3.34).

$$Z_2 \times a_2 - Z_1 \times a_1 = m \times a_x \times h_{CG} \quad (3.33)$$

$$\Delta Z \times a_2 + \Delta Z \times a_1 = m \times a_x \times h_{CG} \quad (3.34)$$

And returning to the wheelbase (L) definition, it is possible to find the Longitudinal Load Transfer (Equations 3.35 and 3.36).

$$L = a_1 + a_2 \quad (3.35)$$

$$\Delta Z^x = \frac{m \times a_x \times h_{CG}}{L} \quad (3.36)$$

The same effect can be seen laterally, but the lateral case isn't so simple by the suspension geometry and characteristics. The front and rear suspensions have different characteristics and stiffness' and this results in a complex effect on the lateral load transfer. The question is: If a certain amount of lateral force is applied to the CoG, what occurs to the lateral load transfer in each axle? It is necessary to remember that the CoG isn't symmetrically distributed from one to another axle and, more than this, the way in which the suspension reacts to the applied force is different in each axle and, for this reason, the evaluation of the lateral load transfer is more complex.

3.2.4.2 Lateral Load Transfer

Guiggiani, in [30], describes a model that shows the effects of lateral loads on the lateral load transfer (Figure 3.7). The suspension geometry in each axle has a point, Q_i , defined as the no-roll

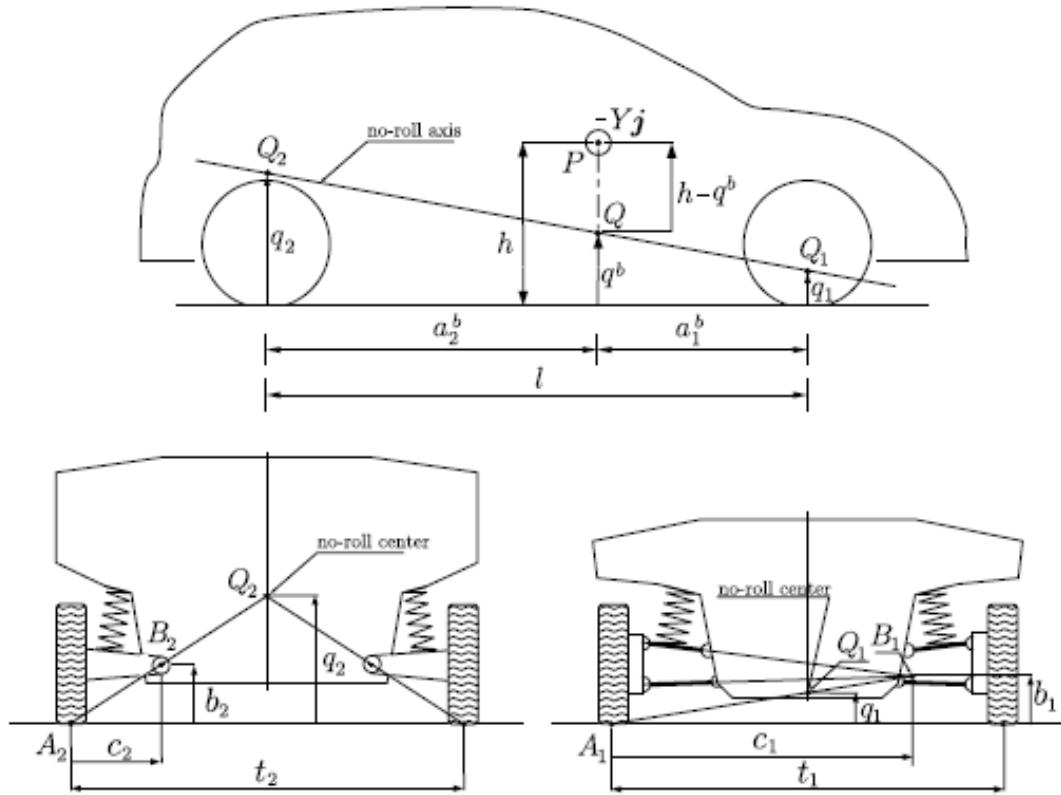


Figure 3.7: Lateral Vehicle Model. Taken from [30]

centre. At this point, the lateral velocity due to the roll motion is zero. The intersection between the no-roll centre of the two axles generates the no-roll axis. The definition of the axis is the same as the point. But more important than this, if a force is applied in the no-roll axis, there's no roll motion generated by the force. Guiggiani explains better [30]:

More precisely, a force (of any direction) applied to the vehicle body and whose line of action goes through the no-roll axis may affect the vehicle roll angle, but only because of tire deflections, with no contribution from the suspensions.

It is for this reason that the lateral load transfer depends on the suspension characteristics. If the CoG was in the no-roll axis, the lateral load transfer wouldn't depend on the suspension. But as there's a difference in these points, it is necessary to take into account the roll motion. To this, it is necessary to relate the lateral forces with the longitudinal reactions. However, the static terms from the gravitational force and the respective normal force can be cancelled, and the expression relates directly to the lateral force with the Δ generated by the lateral force. This is obtained by doing an equilibrium of moments in the direction of the longitudinal axis of the car (Equation

3.37).

$$Yh = \Delta Z_1 t_1 + \Delta Z_2 t_2 \quad (3.37)$$

The roll motion of the car can be separated into two contributions: roll due to the suspension deflection and roll due to the tire deflection:

$$\phi = \phi_1^s + \phi_1^p = \phi_2^s + \phi_2^p \quad (3.38)$$

being ϕ the roll angle and s and p the superscripts pointing to suspension and tire, respectively. Through the definition of "tire roll stiffness" [30], it is possible to relate the tire roll stiffness and deflection with the variation in the normal force (due the load transfer) of each axle:

$$\Delta Z_1 t_1 = k_{\phi_1}^p \phi_1^p \quad (3.39)$$

$$\Delta Z_2 t_2 = k_{\phi_2}^p \phi_2^p \quad (3.40)$$

With (3.39) and (3.40) in (3.37), the lateral force is directly related with the tire roll stiffness and the roll angle of the car. Looking to Figure 3.7, the following equations can be drawed:

$$L^b = Y(h - q^b) \quad (3.41)$$

$$q^b = \frac{a_2 q_1 + a_1 q_2}{a_1 + a_2} \quad (3.42)$$

The individual lateral tire reactions are related to the lateral force:

$$Yq^b = Y_1 q_1 + Y_2 q_2 \quad (3.43)$$

$$Yh = Y(h - q^b) + Y_1 q_1 + Y_2 q_2 \quad (3.44)$$

Guiggiani describes the parcel $L_b = Y(h - q^b)$ as the moment responsible to the suspension roll. The another part is the load transfer transmitted for the rigid bodies. Relating this parcel with (3.37):

$$L^b = Y(h - q^b) = L_1^b + L_2^b = k_{\phi_1}^s \phi_1^s + k_{\phi_2}^s \phi_2^s = \Delta Z_1^L t_1 + \Delta Z_2^L t_2 \quad (3.45)$$

Rearranging and generalizing:

$$\Delta Z_i t_i = (\Delta Z_i^Y + \Delta Z_i^L) t_i = Y_i q_i + k_{\phi_i}^s \phi_i^s = k_{\phi_i}^p \phi_i^p \quad (3.46)$$

The load transfer has two parts, the transferred through the suspension linkage ΔZ_i^Y and the transferred through the suspension springs ΔZ_i^L . Developing it, it is possible to achieve the final expression that defines completely the variation of the normal forces due to the lateral load transfer for each axle (Equations 3.47 and 3.48).

$$\Delta Z_1 = \frac{1}{t_1} \left[\frac{k_{\phi_1}}{k_\phi} Y(h - q^b) + Y_1 q_1 + \frac{k_{\phi_1} k_{\phi_2}}{k_\phi} \left(\frac{Y_2 q_2}{k_{\phi_2}^p} - \frac{Y_1 q_1}{k_{\phi_1}^p} \right) \right] = \frac{k_{\phi_1}^p \phi_1^p}{t_1} \quad (3.47)$$

$$\Delta Z_2 = \frac{1}{t_2} \left[\frac{k_{\phi_2}}{k_\phi} Y(h - q^b) + Y_2 q_2 + \frac{k_{\phi_1} k_{\phi_2}}{k_\phi} \left(\frac{Y_1 q_1}{k_{\phi_1^p}} - \frac{Y_2 q_2}{k_{\phi_2^p}} \right) \right] = \frac{k_{\phi_2^p} \phi_2^p}{t_2} \quad (3.48)$$

3.2.4.3 Aero Downforce Effect

The lateral and longitudinal load transfers are crucial to evaluate correctly the dynamic behaviour with a non-linear tire model. However, there's another important effect which is the Aerodynamic downforce. Basically, it can be calculated through:

$$\Delta Z^{downforce} = \frac{1}{2} \rho C_l A V^2 \quad (3.49)$$

The ρ is the air density, C_l is the lift (or downforce) coefficient, A is the car frontal area and V is the velocity of the car. Assuming a Center of Pressure (CoP) aligned with the longitudinal axis of the car, the effects of the downforce in the front and rear axles can be calculated:

$$\Delta Z_1^{downforce} = \Delta Z^{downforce} \left(\frac{L - x_{CP}}{L} \right) \quad (3.50)$$

$$\Delta Z_2^{downforce} = \Delta Z^{downforce} \left(\frac{x_{CP}}{L} \right) \quad (3.51)$$

Being x_{CP} the distance between the front axle and the CoP of the car.

3.2.4.4 Aero Drag Load Transfer

Finally, the last effect is the load transfer due the drag force. This can be easily forgotten, but is a very important effect, mainly thinking in Formula Student context, in which it is possible to have a car with high wings (Figure 3.8)



Figure 3.8: Delft DUT 17 Car. Taken from [35]

Basically, if the longitudinal load transfer is caused by the inertial effect of the car acceleration applied at the CoG, the aero drag load transfer is similar but related to the drag applied at the CoP

of the car. The drag force can be calculated easily:

$$F^{drag} = \frac{1}{2} \rho C_d A V^2 \quad (3.52)$$

being C_d the drag coefficient. The load transfer can be written similarly as (3.36):

$$\Delta Z^{aero,drag} = \frac{F^{drag} \times h_{CP}}{L} \quad (3.53)$$

being h_{CP} the height of the Center of Pressure. If a car doesn't have an aerodynamic kit, it is possible to neglect this effect, but a complete simulator must consider this. So, resuming, these four effects need to be superposed to obtain the final normal forces and obtain the tire cornering stiffness at each moment, through the following equations:

$$Z_{11} = 0.5(Z_1^{static} + \Delta Z_1^{downforce}) - \Delta Z^x - \Delta Z_1 - \Delta Z^{aero,drag} \quad (3.54)$$

$$Z_{12} = 0.5(Z_1^{static} + \Delta Z_1^{downforce}) - \Delta Z^x + \Delta Z_1 - \Delta Z^{aero,drag} \quad (3.55)$$

$$Z_{21} = 0.5(Z_2^{static} + \Delta Z_2^{downforce}) + \Delta Z^x - \Delta Z_2 + \Delta Z^{aero,drag} \quad (3.56)$$

$$Z_{22} = 0.5(Z_2^{static} + \Delta Z_2^{downforce}) + \Delta Z^x + \Delta Z_2 + \Delta Z^{aero,drag} \quad (3.57)$$

3.2.5 Steering System

Looking for the system that joins the steering wheel with the steering motion of the front tires, several different configurations can be found in Formula Student cars. This type of car must be capable to make small radius corners and, for this reason, the steering angle becomes high. For example, it is very common to find FS cars with 25° of steering angle. However, as can be seen in any theory about steering systems, the relation between the steering wheel input and the steering angle at the tire isn't constant. In reality, looking to (3.58) and (3.59) (extracted from [30]), there are three effects that relate the steering wheel angle and the steering angle.

$$\delta_{11} \simeq -\delta^0 + \tau_1 \delta_v + \epsilon_1 \frac{t_1}{2L} (\tau_1 \delta_v)^2 \quad (3.58)$$

$$\delta_{12} \simeq \delta^0 + \tau_1 \delta_v - \epsilon_1 \frac{t_1}{2L} (\tau_1 \delta_v)^2 \quad (3.59)$$

The steering angle of each wheel δ_{ij} is related with the static-toe δ^0 , the parallel steering $\tau_1 \delta_v$ and the dynamic-toe. The two firsts are simple to understand. The static toe is an "offset" angle defined by the geometry. Is the angle that the tires have when the steering wheel is not rotated (Figure 3.9). Parallel steering is the contribution due to the rigid link (pinion-rack) between the steering column and the steering rack. It is clear that a rotational input must have a respective linear output and consequent wheel rotational output. The ratio between the input angle and the output angle is the "steering ratio" τ_1 .

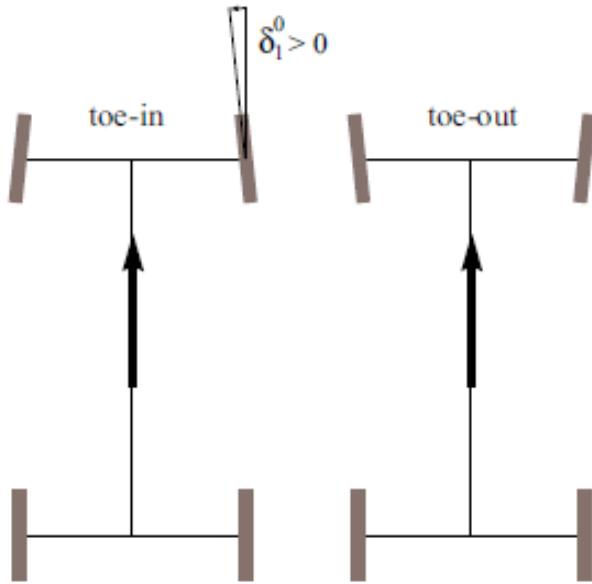


Figure 3.9: Static Toe Representation. Taken from [30]

In the third term, a more complex effect is shown. Looking to (3.58) or (3.59), it is possible to see the square of the parallel steering effect related to some car dimensions (track width and wheelbase) and with a term ε_1 , that is the "Ackermann coefficient". This value is directly related to the difference in the steering angle between the two tires. If $\varepsilon_1 = 0$, the configuration is named "parallel steering" and, for this reason, there's no difference between the steered angle from one to another tire (Figure 3.10a). If $\varepsilon_1 = -1$, the configuration is named Anti-Ackermann, since the outer wheel steers more than the inner (Figure 3.10b). Finally, if $\varepsilon_1 = 1$, the configuration is "Ackermann" or "Full Positive-Ackermann", in which the inner wheel steers more than the outer (Figure 3.10c).

The steering geometry can be defined and included in the vehicle model through the coefficients described above. It is very common for Formula Student cars to have Positive Ackermann geometries $\varepsilon_1 > 0$ and this parameter need to be changed to study different steering geometries.

3.2.6 Joining the Effects

In the last sections, the different effects acting on a race car were explained separately, but it is necessary to remember that all these effects occur simultaneously during the ride and, for this reason, need to be joined. Moreover, these effects are a function of the control variables of the car (V_x and δ) and also of the state variables (a_x, a_y). But as developed, the superposition of these effects affects the characteristics of the tires and this affects the state of the car. This means that the model developed in this chapter is a non-linear model. For this reason, it is important to join the effects, understand how each part of the model is related to others and create a logical scheme of the model.

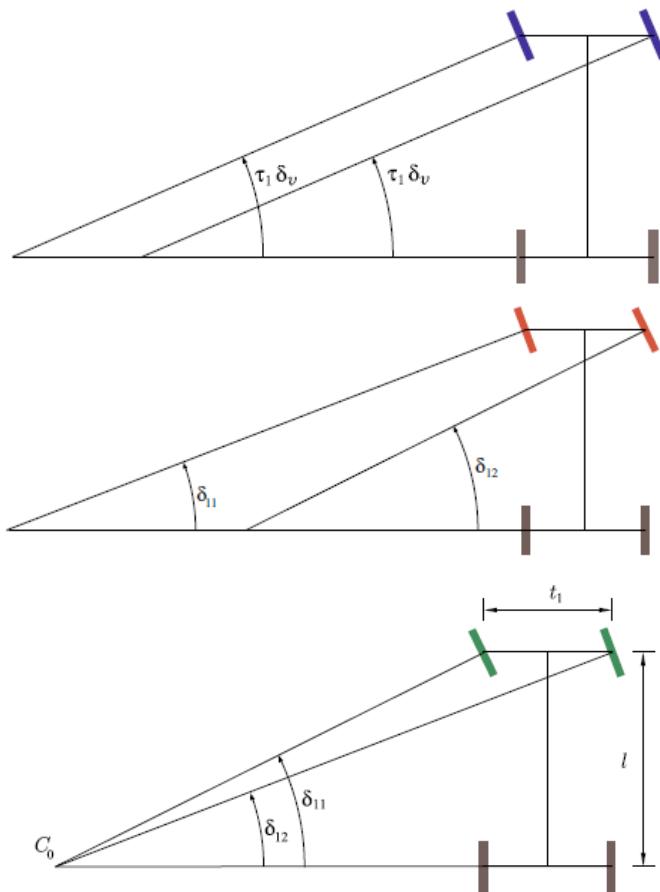


Figure 3.10: Parallel (a); Anti-Ackermann (b); Ackermann (c). Taken from [30]

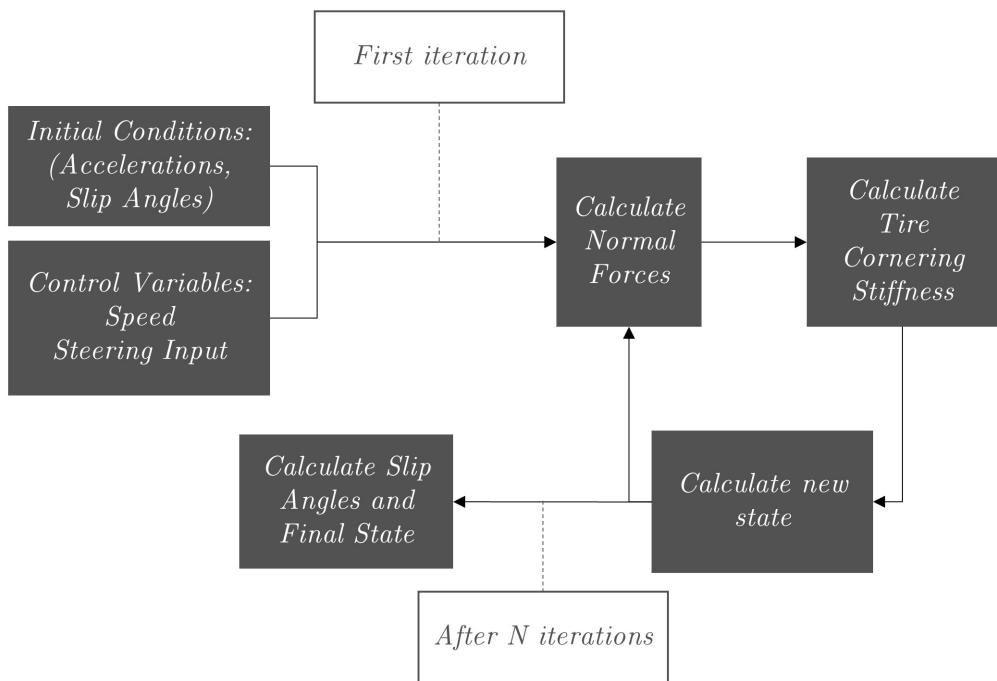


Figure 3.11: Non-linear vehicle model scheme

For each input of the control variables, this calculation is done N times. After some tests, it was seen that the results for $N \geq 2$, the results remain constant. However, an initial condition to the variable model needs to be assumed. In this case, the initial condition is the lateral acceleration. Other parameters remain constant during the calculation of the state, such as the speed and longitudinal acceleration, as well as the steering angle.

The estimation of the initial condition to the acceleration is founded in a theoretical approximation. Assuming the handling of a neutral car, the steering angle for this specific case is named "Ackermann steer angle" and is defined by:

$$\delta = \frac{L}{R} \quad (3.60)$$

This simplification neglects the effect of the slip angle on the steering, but it is a good starting point to define the initial condition:

$$a_y^{initial} = \frac{V_x^2}{R^{initial}} \quad (3.61)$$

$$R^{initial} = \frac{L}{\delta} \quad (3.62)$$

So, for the first iteration, there's a value of lateral acceleration (to impose an initial lateral load transfer) that will be corrected after the two firsts iterations.

3.2.7 Acceleration Model

Was presented the dynamic model of the car, including the relation of the control variables with the state and so on. However, there's a state variable that wasn't treated until now because it was omitted from the equations of motion. Here we're talking about the acceleration. The reason for this omission is that a Quasi Steady-State planar model neglects the longitudinal behaviour since the derivatives of the velocities are neglected. This means that the application of the acceleration in the model (for example in the out-of-plan model) is just virtual. However, a race car obviously needs to account for some type of acceleration model, since it is directly tied to the motor. Basically, the relation between the acceleration and the already shown speed is the following:

$$V_{x,i} = \sqrt{V_{x,i-1}^2 + 2a_{x,i}\Delta s} \quad (3.63)$$

For this simulator, the used approach is simple and it is directly related to the type of motor to be used in the car. The chosen motor for the actual version is the EMRAX 228 Medium Voltage, which has the torque and power curves as shown in Figure 3.12. The efficiencies can be seen in Figure 3.13.

Moreover, the motor datasheet has the following information:

- Maximum Torque = 230 Nm

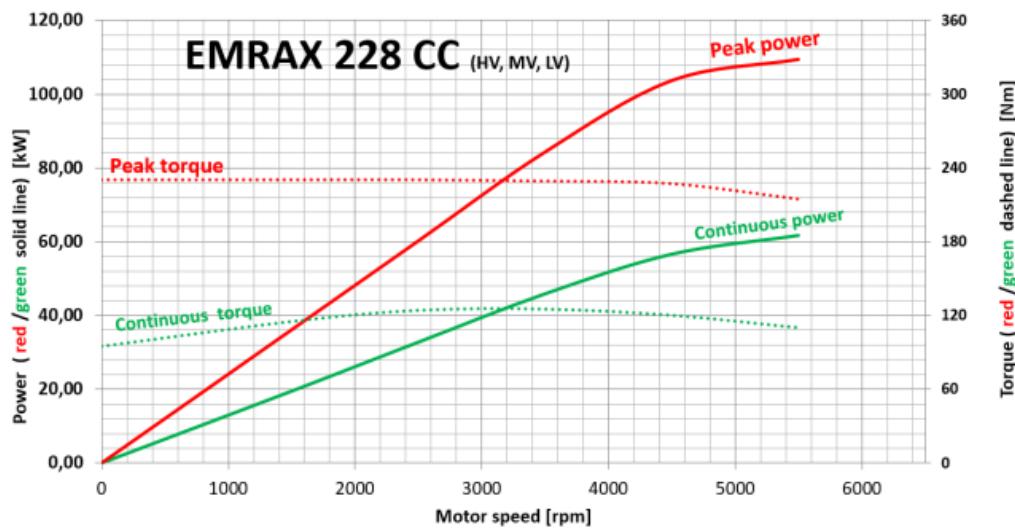


Figure 3.12: EMRAX 228 Combined Cooled Curves

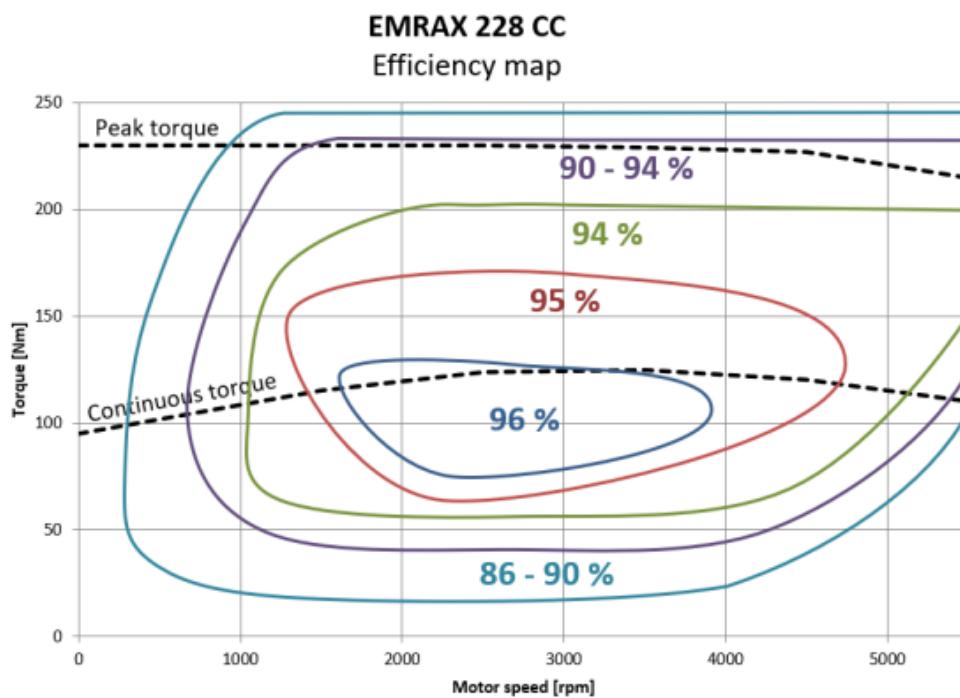


Figure 3.13: EMRAX 228 Combined Cooled Efficiencies

- Maximum Velocity = 5500 RPM
- Maximum Voltage = 500 V
- Maximum Current = 350 Arms
- Torque Constant = 0.75 Nm/Arms
- Speed Constant = 11 RPM/V

- Maximum Power = 109 kW

However, there are two parameters that need to be changed. The first is the maximum voltage. This constraint actually isn't imposed by the motor but by the battery, which has 454 V. This means that the maximum velocity of the motor is limited by this value. Moreover, the Formula Student competition doesn't allow electric power bigger than 80 kW, so this needs to be taken into account in the motor model.

With all this, the calculation of the acceleration in each instant is done in the following way:

$$a_{x,i} = \frac{T_{motor} \times \eta_{mec} \times i \times r_{tire} - F^{drag}}{m} \quad (3.64)$$

being T_{motor} the Torque Motor, η_{mec} the mechanic global efficiency (transmission system), i the gear ratio, r_{tire} the tire radius and the already cited drag force. The torque of the motor is related to the electrical characteristics:

$$T_{motor} = \min \left(I_{motor,max} \times T_{constant} \times \eta_{motor}, \frac{P_{max}}{V_{motor}} \times T_{constant} \times \eta_{motor} \right) \quad (3.65)$$

In which the maximum power P_{max} is the power limitation, V_{motor} is the motor voltage (depending on the velocity), $T_{constant}$ is the torque constant and η_{motor} is the motor efficiency (Figure 3.13). The maximum torque is calculated through the maximum current, $I_{motor,max}$, the torque constant and the efficiency. The efficiencies can be calculated by an efficiency map depending on the actual motor speed and torque. And with this, it is possible to have an approximation of the maximum acceleration through the motor information and accounting for aerodynamic drag.

Finally, since the tires are submitted by torque, it is necessary to account for the longitudinal forces of the tire. This information will be used by the solver to determine if a determinate state is possible. Case the applied torque makes the car to have skidding, the solution needs to be rejected. For this calculation, there are just two simple equations:

$$X_1 = 0 \quad (3.66)$$

$$X_2 = ma_x \quad (3.67)$$

The equations shown before can be modified when necessary. For example, when the car is in its dynamic limit, the acceleration to be used isn't the maximum available by the motor, but the maximum held by the tires. In this case, the torque, current, voltage and power are calculated through this acceleration value.

3.2.8 Brake Model

Finally, to calculate the cases in which the car is braking, a simple brake model was created. This model doesn't assume directly the brake parts, but only the effect of the brakes on the car. This

means that brake disks, pistons, callipers and pressures aren't accounted for. The main reason for this simplification is that by the rules of the competition, the car needs to be capable to block all the tires. So, the limit of the braking capacity isn't the limitation of the system, but just the dynamic limitation of the car. As the limit is just dynamic (and the best braking performance will be justly in the dynamic limit), the brake model just serves to calculate the tire saturations.

$$X_1 = ma_x BB \quad (3.68)$$

$$X_2 = ma_x(1 - BB) \quad (3.69)$$

X_1 and X_2 are the longitudinal forces applied at the front and rear axle and BB is the brake bias, a design parameter (adjustable) that defines the percentage of braking that occurs in the front axle. Curiously, one of the best ways to help in the choice of this parameter is through some simulations done through the developed simulator.

3.3 Track Model

Among the several models that were shown in the last chapter, the chosen track is a fixed path without height variation. In other words, it is a bi-dimensional track with the path already specified. This choice was done by its ease and simplicity in use. However, it is necessary to justify these characteristics. Firstly, the Formula Student tracks don't have much higher height variations, which means that neglecting this effect isn't a problem for a Lap Time Simulation. Moreover, the necessity of an optimal path isn't so important in the first car, since the objective isn't to give the best path to the driver as input. Even if the software can find this information, actually isn't possible to extract real data from the car to determine if the driver is contouring the best path. So, the possibility of the optimal path wasn't chosen. The path information is given to the software through the length and the curvature (as explained in the last chapter). However, it is possible to find this information once there's access to the X-Y coordinates. The method was explained earlier in Equations 2.7 and 2.12.

Through the internet and other software, some tracks were found. In the library of Optimum-Lap, it was found the Endurance and Autocross tracks from Formula Student Germany 2012 and Formula Student Austria 2012. Moreover, in a GitHub repository, it was found more recent tracks, such as Formula Student Austria 2015 Autocross Track.

3.4 Computational Implementation

As shown in the last section, the vehicle model has several components that must be joined in a script to create a simulator. This section will explain the chosen methodologies for this implementation. Firstly, as this simulator will be used by a Formula Student team, one of the main concerns is the ease of the set-up of a simulation. The vehicle model chosen involves the set-up of several

parameters, but the idea is that once there's a file with these parameters and another file with the track data, someone with basic knowledge in MATLAB can run the simulator and extract results. This requirement pointed to several decisions in terms of pre-processing and software optimization. It was preferable to sacrifice computational performance in order to maintain the utilization easily. It was chosen a "Forward-Backward" methodology for the simulator, with the following sequence:

- Definition of the limit velocity profile.
- Definition of the apex points.
- Definition of the forward profile from the apex points.
- Definition of the backward profile from the apex points.
- Join the velocity profiles and state matrices.

There are some changes in this sequence that will be explained later.

3.4.1 Optimization Remarks

Before starting to show the methodology used for the state matrices calculation, it is important to talk about the optimization process involved in all the following steps. As said earlier, a QSS simulator can be decoupled into several segments that are calculated separately. However, to this calculation, and remembering the vehicle model utilized, the result is a function of two different parameters, δ and V_x . This means that the state of the car in each segment needs to be found for each pair (steering and velocity). However, as stated before, the objective of the simulator is to find the optimal control variables in each segment of the track, in order to optimize the result. In other words, it is necessary to find the result that maintains the car within the limit of its performance.

For this objective, the software uses a function existent in MATLAB named *fmincon*. This function is used to solve optimization problems with linear and non-linear constraints. In the different parts shown in the following, the approach of the simulator is to call the function and it, in some iterations, can find the optimal solution respecting the constraints. Each constraint about dynamics limitation or described path is an output of the vehicle model. This means that the optimization function *fmincon* in each iteration uses the vehicle model, the constraints are changed and after some iterations, the optimal result is found.

3.4.2 Definition of the limit velocity profile and apex points

The first part consists in the find the limit velocity profile. In other words, the first step is to discover the maximum velocity that the car can have in each segment of the track. This evaluation is done assuming that the car isn't accelerating or braking, being a pure lateral case. Once the profile is defined, it is possible to search and find the local minimum points of this profile. By definition,

these points are known as apex points. An apex point (Figure 3.14) is a transition between the braking and the acceleration of the car. Is the point at which the pilot reaches the minimum velocity during the cornering and, despite the transient effects, the longitudinal acceleration is null. For

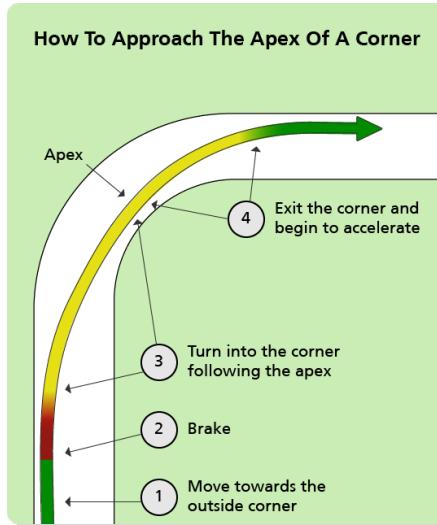


Figure 3.14: Corner Representation. Taken from [36]

this, the following problem was solved with the following methodology:

$$\begin{aligned}
 & \text{Maximize} && V_x \\
 & \text{subjected to:} && \kappa_{ref} = \kappa_{car} \\
 & && Y_1 < Y_{1,max} \\
 & && Y_2 < Y_{2,max} \\
 & && V_x < 30m/s \\
 & && -2.36 < \delta < 2.36
 \end{aligned} \tag{3.70}$$

In which κ_{ref} is the path curvature, κ_{car} is the curvature described by the car, Y_1 and Y_2 are the lateral forces applied to the tires of each axle (F and R) and $Y_{1,max}$, $Y_{2,max}^R$ are the maximum forces that can be applied at each axle. These constraints basically say that the combination of velocity and steering angle needs to respect the reference trajectory maintaining the car within the limit of its dynamic, including its maximum velocity. So, by doing this, it is possible to obtain the Limit Speed Profile (Figure 3.15). By the definition written above, an apex point is an instant in which the car is at its minimum local speed. This occurs justly because it is the point between the braking and acceleration commands. For this reason, with the velocity profile calculated, it is possible to use a native MATLAB function to find the minimum local points. These points result in a vector with all the apex points.

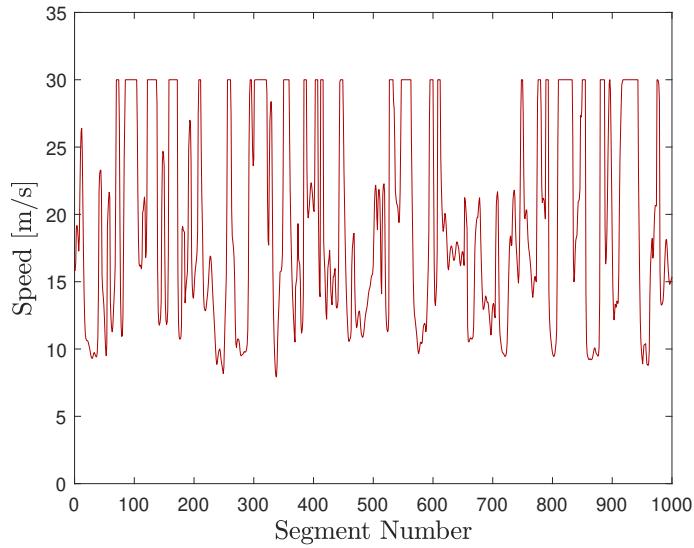


Figure 3.15: Limit Speed Profile

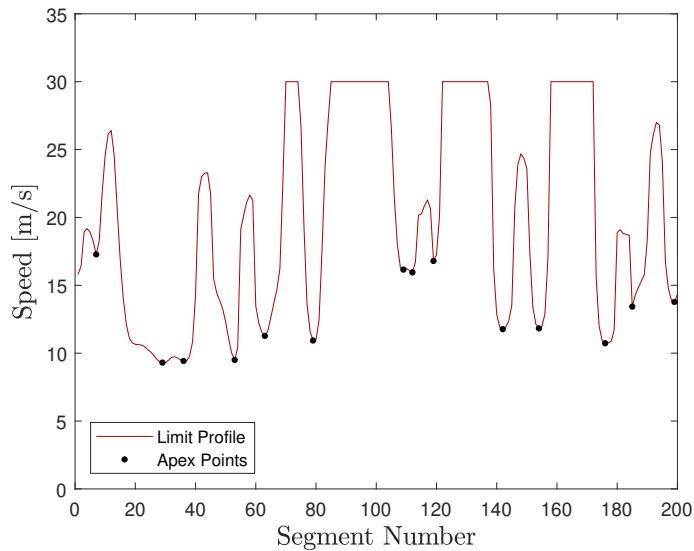


Figure 3.16: Apex Points

3.4.3 Definition of the forward profile

Once the apex points are found, it is possible to start to define the forward and backward profiles (acceleration and braking). The approach chosen started with the forward profile to correct some "false" apex points that can exist. These false apexes points are some points generated by the procedure shown above but that, in reality, the car cannot achieve. This specific effect and the correction will be explained later. For the definition of the forward profile, it is necessary to do the following logic:

- Objective: From the *apex point "k"*, calculate the speed profile until *apex point "k+1"*.

- If $segment = apex\ point\ "k" + 1$: Start with speed from the apex point to calculate the velocity in the point.
- Else: Use the speed of the last point to calculate the velocity in the point.
- Stopping Condition: $segment = apex\ point\ "k+1"$;

To understand better the procedure, it is useful to look at the result obtained for the initial part of a track (Figure 3.17) It is possible to see that the forward profile is defined through the apex

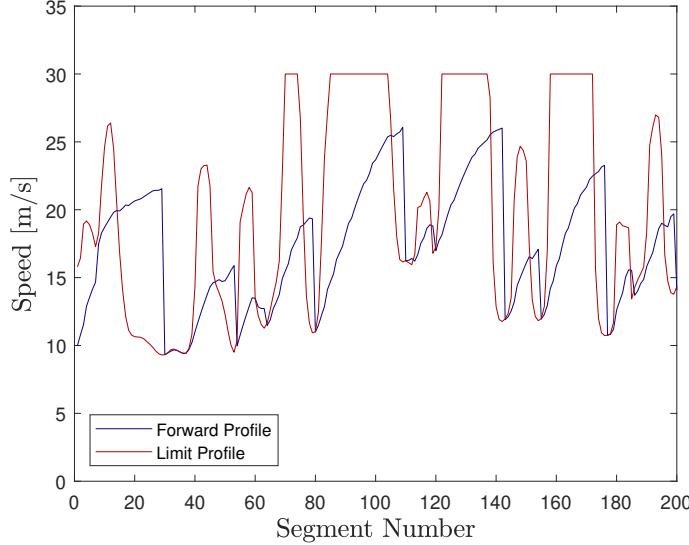


Figure 3.17: Forward and Limit Speed Profiles

points (Figure 3.16) determined by the limit profile. The calculation stops when the solver reaches the next apex point. For this case, the optimization problem has the approach shown in Equation 3.71.

$$\begin{aligned}
 & \text{Maximize} && a_x \\
 & \text{subjected to:} && \kappa_{ref} = \kappa_{car} \\
 & && a_x \leq a_{x,max} \\
 & && Y_1 < Y_{1,max} \\
 & && Y_2 < Y_{2,max} \\
 & && X_1 < X_{1,max} \\
 & && X_2 < X_{2,max} \\
 & && N_{motor} < N_{motor,max} \\
 & && P_{motor} < P_{motor,max} \\
 & && V_{x,i} = \sqrt{V_{x,i-1}^2 + 2a_{x,i}\Delta s} \\
 & && 2.36 < \delta < 2.36 \\
 & && -10 < a_x < 20
 \end{aligned} \tag{3.71}$$

Maximizing the acceleration means, in this case, maximizing the velocity of the car. But compared to (3.70), the difference appears in more constraints that are introduced. X_1 and X_2 are the applied longitudinal forces on the front and rear axles, respectively, while $X_{1,max}$ and $X_{2,max}$ are the maximum force that can be applied. This introduction is necessary because, different to the last case, the forward profile involves the acceleration of the car and, for this reason, it is necessary to assure that the tangential force generated by the tires doesn't overpass the maximum grip forces that the tire can support. Moreover, one more restriction was added to the optimization problem. P_{motor} and $P_{motor,max}$ are the actual power and the maximum power of the motor, respectively. This constraint is important because the acceleration is dependent on the applied torque and this is directly related to the power. As the tractive system has a power limit, it is necessary to assure that the optimal solution chosen for a segment is realistic within the vehicle constraints. As a result of this calculation, it is obtained the forward speed profile (Figure 3.18).

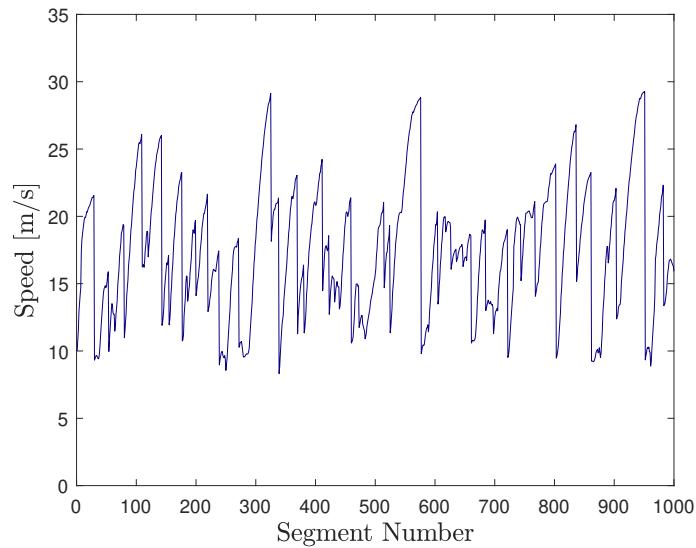


Figure 3.18: Forward Speed Profile

3.4.4 Definition of the backward profile

For the definition of the backward profile, the same approach used in the last case is applied here, with some little differences. The backward profile uses a backward loop, going from the final to the beginning. The logic scheme is:

- Objective: From the *apex point "k"*, calculate the speed profile until *apex point "k-1"*.
 - If *segment = apex point "k" - 1*: Start with speed from the apex point to calculate the velocity in the point
 - Else: Use the speed of the last ($i + 1$) point to calculate the velocity in the point.
- Stopping Condition: *segment = apex point "k-1"*;

It is possible to understand that the procedure seems the used early. The result is also similar and can be seen in Figure 3.19. The conditions of the optimization problem are also very similar, but as

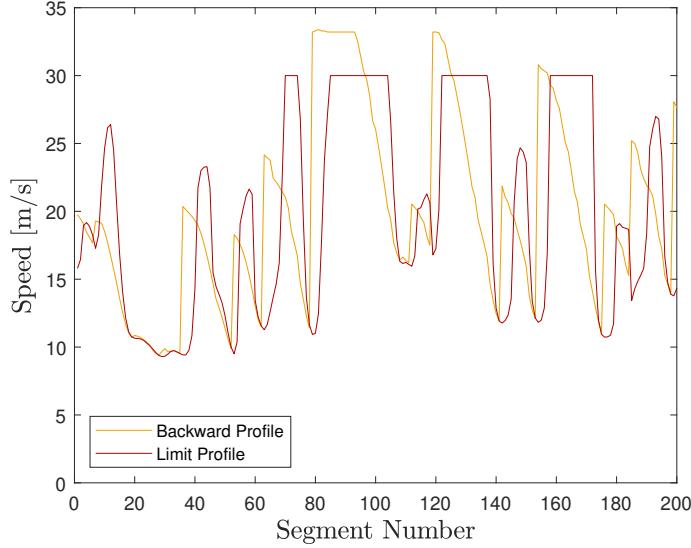


Figure 3.19: Backward and Limit Speed Profiles

said, there are some differences (Equation 3.72). As the objective is to minimize the acceleration, the velocity in each point, calculated in a backward way, will successively increase. When seen in the "normal way" (forward), in reality, the speed is decreasing (the car is braking). The final result can be seen in Figure 3.20.

$$\begin{aligned}
 & \text{Minimize} && a_x \\
 & \text{subjected to:} && \kappa_{ref} = \kappa_{car} \\
 & && Y_1 < Y_{1,max} \\
 & && Y_2 < Y_{2,max} \\
 & && X_1 < X_{1,max} \\
 & && X_2 < X_{2,max} \\
 & && N_{motor} < N_{motor,max} \\
 & && P_{motor} < P_{motor,max} \\
 & && V_{x,i} = \sqrt{V_{x,i+1}^2 - 2a_{x,i}\Delta s} \\
 & && 2.36 < \delta < 2.36 \\
 & && -20 < a_x < 10
 \end{aligned} \tag{3.72}$$

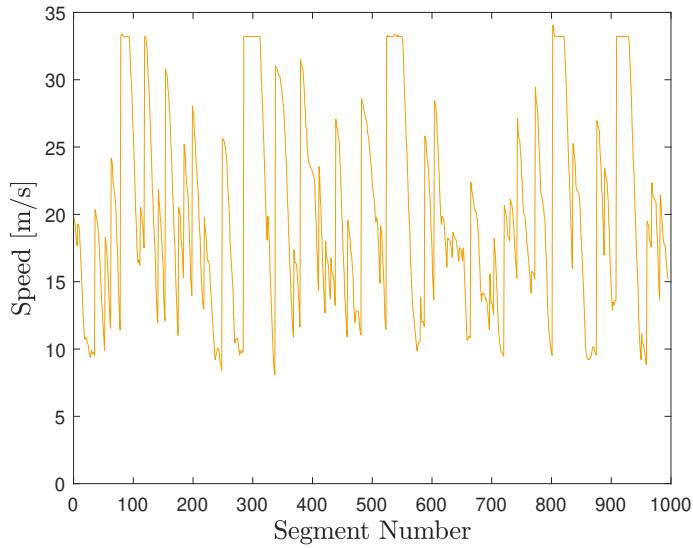


Figure 3.20: Backward Speed Profile

3.4.5 Calculating the Grip Limit

The four grip limits described in the optimization conditions depend on the Pacejka Tire Model. Basically, the maximum value can be obtained through the following equations:

$$Y_{i,max} = Y_{i1,max} + Y_{i2,max} \quad (3.73)$$

$$Y_{ij,max} = \max(F_y(Z_{ij}, \gamma = 0, \kappa = 0, \alpha = 0 : 0.1 : 10)) \quad (3.74)$$

These equations were presented at the beginning of the chapter, which basically Y is the lateral force obtained through the Pacejka model, γ is the camber angle, neglected in this case, and κ is the slip ratio, also neglected (slip ratio = 0 is the best case for the lateral forces). The reason for neglecting the slip ratio is that the wheel dynamics aren't considered, so, there's difficulty on calculate it correctly. The idea of these equations is to find the maximum lateral force for a specific normal force, which occur for some slip angle between 0 and 10 degrees. The assumed slip angle isn't the one calculated through the vehicle model because the assumption used in that case was the lateral cornering stiffness, which assumed the linear part of the tire curve. However, the maximum force isn't in the linear part. In reality, the tire will skid a little bit more until reach its saturation. After this, the maximum value for the longitudinal forces were calculated, with the difference that the used slip angle, α , is the one obtained for the maximum case in Equation 3.74:

$$X_{i,max} = X_{i1,max} + X_{i2,max} \quad (3.75)$$

$$X_{ij,max} = \max(F_x(Z_{ij}, \gamma = 0, \kappa = 0 : 0.01 : 0.3, \alpha)) \quad (3.76)$$

In this case, the longitudinal force is calculated for the best slip ratio, following the same principle as the lateral case, that the tire will have a little increase in the slip ratio until reaches its maximum force (even if it is not quantified). Through these equations, the optimization problem

is completely defined.

3.4.6 Joining the Speed Profiles

After calculating each speed profile, the next step is to join them to create a final result. It is important to remember that, besides the speed profiles, all the state variables are calculated with the control variables. The focus here is on speed because it is the factor that is used to calculate the final speed profile of a car. Once calculated the forward and backward profiles, Figure 3.21 shows how is their superposition at the beginning of a track.

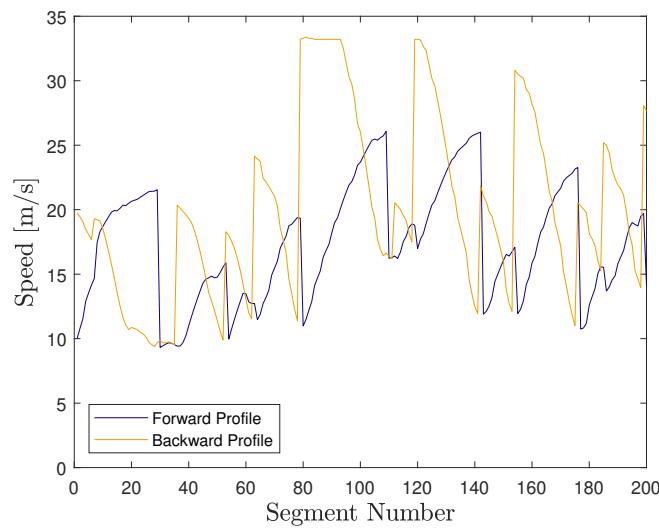


Figure 3.21: Forward and Backward Speed Profiles

To join the curves, it is necessary to implement a "finder" of intersection points. To make this, it was stated that between two apex points, there's one intersection point. The logic scheme for this is:

- Objective: Between two apex points (including before the first apex point and after the last), find the intersection point between the curves.
- After finding the intersection points:
 - If the segment is between the last apex point and the intersection point, the forward state and control matrices are used.
 - Else: The backward state and control matrices are used.

The achieved result (for the first 200 segments) is shown in Figure 3.22

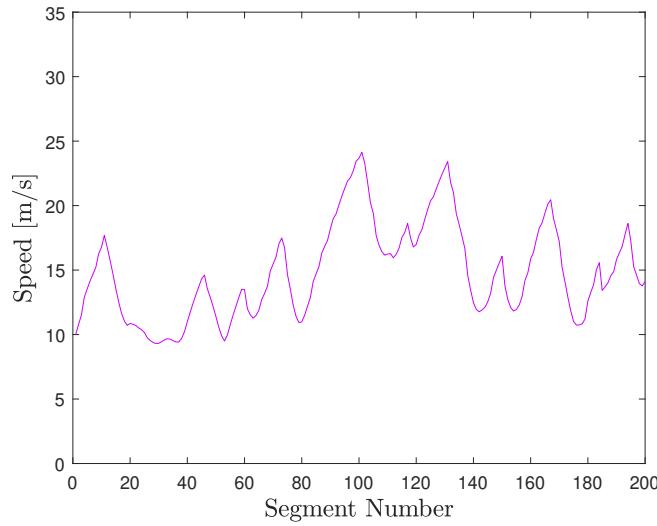


Figure 3.22: Final Speed Profile

3.4.7 Modifications

The methodology shown here is simple and can be calculated in simple steps. With some code modifications, the calculation performance can increase considerably using parallel computing. The final result (as said before, optimal) can be obtained in less than 60 seconds for a track with 1000 segments. However, after some tests, some errors were detected. Figure 3.23 shows an example of this error occurring. This plot shows the speed profile between segments 700 and 800 of a track.

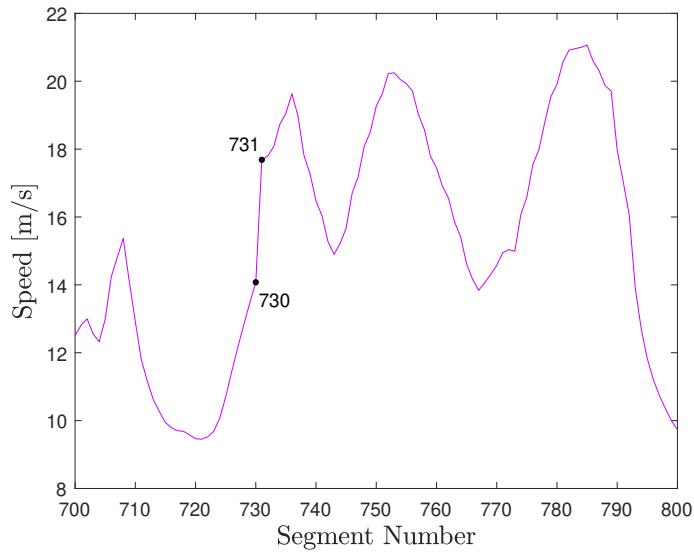


Figure 3.23: False Apex Points

Between two segments (730 and 731) there's an increase of 4m/s in the velocity. It is almost a "step". This represents a discontinuity in the speed profile. The explanation of the effect is seen in Figure 3.24. Basically, segment 731 was accounted as an apex point. However, the last apex

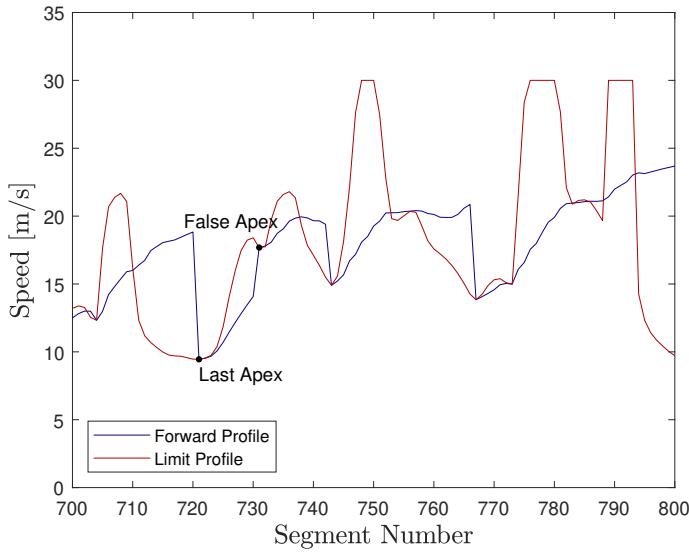


Figure 3.24: Explanations False Apex Points

isn't distant from that and, for this reason, the forward profile (that shows the acceleration of the car) cannot achieve the velocity of the "apex" (segment 731). For this reason, this point is a false apex. A real car never could achieve this segment at this velocity. For this reason, it is necessary to implement some method that automatically can detect and neglect these false apex points, independent of the track used.

However, this modification implies a new calculation of forward and backward velocities without the false apexes. Moreover, this effect can occur also in the backward direction, when a car hasn't the capacity to brake sufficiently to start the braking (in a false apex) with that speed. A simple way to deal with this problem is manually, seeing the occurrences, noting the false apexes and creating a condition to neglect them between the real apexes. However, this would need manual intervention in the script every time that the track is changed and the goal of having an automatic and flexible simulator would be lost.

To overpass this problem, a finder for false apexes was implemented through the calculated forward profile and the limit profile. When it is detected an apex point (from the limit profile) with a speed lesser than the speed calculated from the last apex point (through the forward profile; remember the logic sequence from Section 3.4.3), the apex point is defined as false. After, the script calculates new speed profiles (forward and backward) and the loop tries to find another false apex. However, the problem doesn't finish at this moment. As the speed profile is re-calculated in some parts (that causes a decrease in the forward and backward speeds), it is possible that new false apexes occur. So, the script needs to do this procedure sometimes until there are no errors along the track.

Clearly, these modifications make the calculation longer, but the increase isn't so big. Finally,

it is possible to calculate a lap (in the same track said above) in approximately 2 min. The final result can be seen in Figure 3.25.

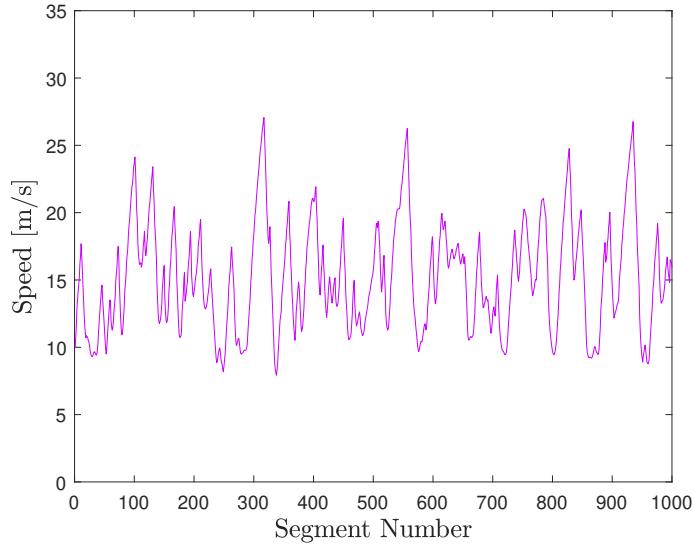


Figure 3.25: Final Results

3.5 Conclusion

It was explained all the different parts of the developed software, including the vehicle model, path model, optimization processes and computational implementation. The implementation tests and results pointed to a solid simulator, that can deal with the requirements stated in the beginning. The following chapters will focus on this analysis and will show the reliability of the choices. Moreover, a simple addition can be done in this model. The software, developed as explained, can easily simulate the car in Autocross and Endurance conditions. However, it is possible to make a simple change and add the Acceleration and Skidpad simulations, through files that read the track data for these two events. This final modification was done to have an overall perspective of the performance and the results will be shown later.

Chapter 4

Validation

After the presentation of the software developed, one of the most important parts is the validation of the developed solution. This part is important to understand if the software has the capability of representing really the car (vehicle model) and if the simulated values have any correlation with the real situations. For the validation, a very useful method is the instrumentation of a real car and the comparison between the simulated value and the measured value for a specific manoeuvre. Ideally, this procedure would be done. However, because of the actual stage of the car's development, it is not possible to implement a Data Acquisition System in a real car to validate this software. Despite this, it is possible to think in another manner to validate the simulator. The objective of this chapter is to present these methodologies and their respective results. By doing this, it is possible to understand the limits of the developed solution. The validation plan will involve the comparison between the utilized model and another (more realistic); repeatability tests; comparison between the software and other known software.

4.1 Model Validation

The first part of the validation is the verification of the vehicle model utilized. As explained in the last chapter, the vehicle model is Quasi Steady-State with 3DoF. A similar transient model can be drawn, following the next equations:

$$mA_x = m(\dot{V}_x - V_y r) = X_1 + X_2 - \frac{1}{2} \rho C_d A V_x^2 \quad (4.1)$$

$$mA_y = m(\dot{V}_y + V_x r) = Y_1 + Y_2 \quad (4.2)$$

$$I_z \ddot{r} = Y_1 a_1 - Y_2 a_2 + \Delta X_1 t_1 + \Delta X_2 t_2 \quad (4.3)$$

The X_1, X_2, Y_1 and Y_2 are the longitudinal and lateral forces at each axle, respectively. These forces come directly from the non-linear tire model (that, in this case, doesn't need to be transformed in the lateral cornering stiffness). Expanding them:

$$X_1 = X_{11} + X_{12} \quad (4.4)$$

$$X_2 = X_{21} + X_{22} \quad (4.5)$$

$$Y_1 = Y_{11} + Y_{12} \quad (4.6)$$

$$Y_2 = Y_{21} + Y_{22} \quad (4.7)$$

$$\Delta X_1 = \frac{X_{12} - X_{11}}{2} \quad (4.8)$$

$$\Delta X_2 = \frac{X_{22} - X_{21}}{2} \quad (4.9)$$

These values can be expanded in order to the individual forces (in each tire):

$$X_1 = (F_{x_{11}} \cos(\delta_{11}) + F_{x_{12}} \cos(\delta_{12})) - (F_{y_{11}} \sin(\delta_{11}) + F_{y_{12}} \sin(\delta_{12})) \quad (4.10)$$

$$X_2 = F_{x_{21}} + F_{x_{22}} \quad (4.11)$$

$$Y_1 = (F_{y_{11}} \cos(\delta_{11}) + F_{y_{12}} \cos(\delta_{12})) + (F_{x_{11}} \sin(\delta_{11}) + F_{x_{12}} \sin(\delta_{12})) \quad (4.12)$$

$$Y_2 = F_{y_{21}} + F_{y_{22}} \quad (4.13)$$

Despite the existence of three equations of motion, the lateral dynamics of the car can be described by the two lasts (lateral direction and yaw). As the objective of this transient model is to compare with the QSS, the longitudinal motion (that is neglected in the static case) can be neglected. The individual tire forces come from the tire model. This model was described briefly in equations (3.19) and (3.20). It is possible to see that the forces from the tires are tied to the state of the vehicle (normal forces, slip ratio, and slip angle). So, once obtained the state variables, it is possible to find the forces to compute the new state variables (non-linear model). Different from the approach used in the first version, the transient model can be easily modeled in Simulink, since the solver is inherently transient and non-linear. Moreover, the block diagrams can be directly connected and create a This block diagram can be divided into some parts: Slip Calculations, Tire Model, Dynamics Model, and Load Transfers Model. All these models are connected between them and will be explained better in the following.

4.1.1 Slip Calculations

It is useful to start in some place to a logical progression of the work. It is a little bit difficult in vehicle models since the slip are a function of the state and the state is a function of the slips. However, it is possible to start in the slip, since it depends directly on the velocities of the vehicle (given as initial conditions). The slip quantities are slip angle α_{ij} and the slip ratio κ_{ij} . The first is related to lateral dynamics and the latter to longitudinal. As the longitudinal dynamics are being neglected, the slip ratio will be neglected at this moment (it would be very important if the model have the wheel spin dynamics). The following equations describe the calculation of slip ratios and

slip angles.

$$\kappa_{ij} = \frac{V_{tire,ij}}{V_x} \quad (4.14)$$

$$\alpha_{11} = -\arctan\left(\frac{V_y + ra_1}{V_x - r\frac{t_1}{2}}\right) + \delta_{11} \quad (4.15)$$

$$\alpha_{12} = -\arctan\left(\frac{V_y + ra_1}{V_x + r\frac{t_1}{2}}\right) + \delta_{12} \quad (4.16)$$

$$\alpha_{21} = -\arctan\left(\frac{V_y - ra_2}{V_x - r\frac{t_2}{2}}\right) \quad (4.17)$$

$$\alpha_{22} = -\arctan\left(\frac{V_y - ra_2}{V_x + r\frac{t_2}{2}}\right) \quad (4.18)$$

4.1.2 Tire Model

Once obtained the slip quantities, it is possible to go to the tire model. The Pacejka Tire Model (already implemented in the QSS model) was introduced in Simulink through a "MATLAB Function" block since the script is made in MATLAB. This block, for each tire, has 4 inputs: slip angle, slip ratio, camber and normal force. The output is simply the lateral/longitudinal force.

4.1.3 Dynamics Model

The dynamics model is just the already presented equations of motion that link the forces (became from the tire model) with the state of the vehicle. These states are inputs from the Load Transfers Model, to be seen in the following.

4.1.4 Load Transfers Model

The load transfer calculation, despite being part of the "vehicle model", is the already explained "Out-of-plan motion" shown in the Quasi Steady-State simulator. It was seen that this motion is unattached to the planar motion model, so, despite the planar model is the same, this part is exactly the same used in the last chapter, just implemented in Simulink environment.

4.1.5 Simulink Implementation

Figure 4.1 shows the model implemented in Simulink. This model is composed by (from left to right):

- Slip Model and Tire Model
- Forces in the Tires —> Forces in the Axles
- Equations of Motion (Lateral and Yaw). From these equations, most of the state was calculated.

- In the bottom, there's a wheel model (not explored in this implementation, but it serves as kinematic relation between the state and the slips).
- In the extreme right, there's a model of the out-of-plane motion, which has the calculation of the static, aero and load transfers).

Inside these blocks, the same equations shown in this work were implemented.

4.1 Model Validation

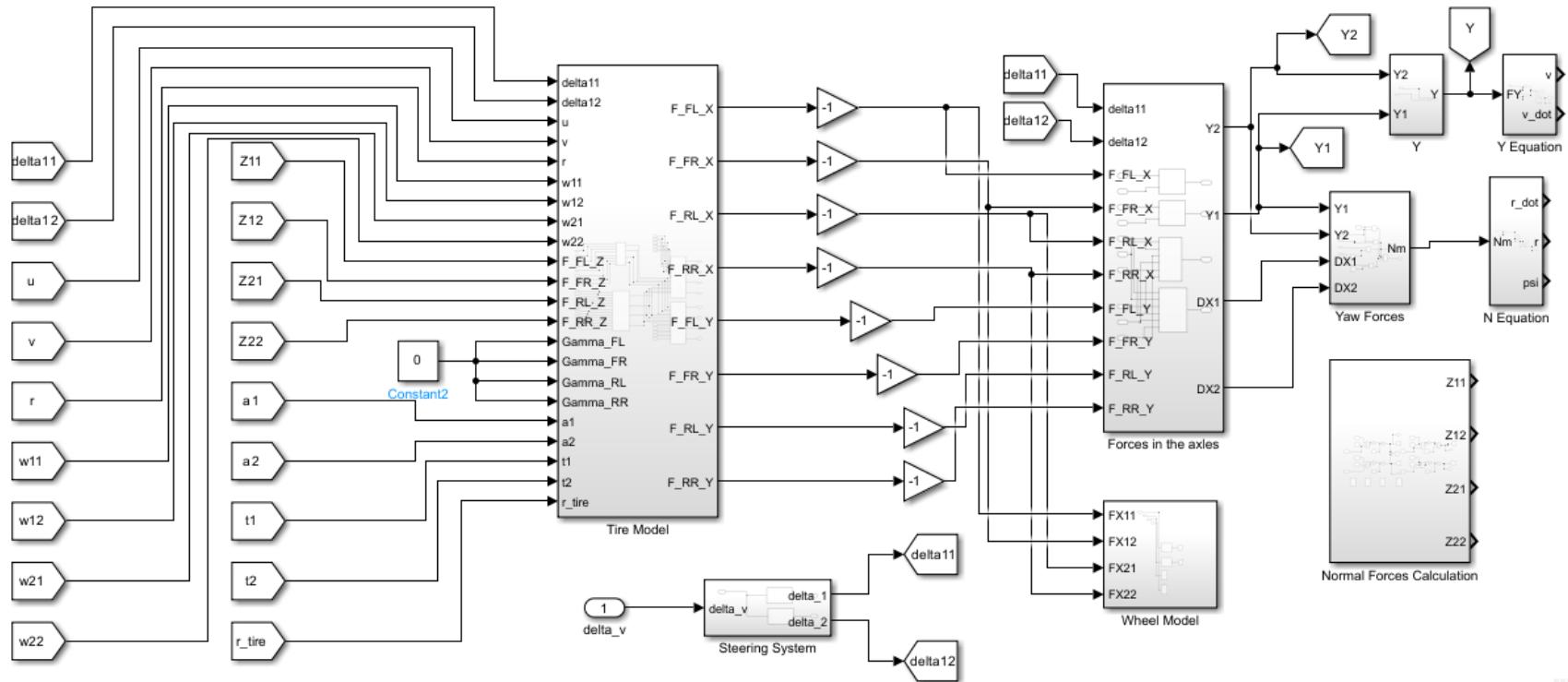


Figure 4.1: Simulink 3DoF

4.1.6 Comparison to the Quasi Steady-State Model

Several parts of the model developed in this section are already shown in the Quasi Steady-State model. So, it is important to make a qualitative/characteristic comparison between them. Firstly, using the 3DoF model, the only transient part is the planar motion model, accounting for longitudinal, lateral, and yaw motion. This means that the inertial effects are related to these motions (including the yaw inertia, doesn't considered at all in the QSS). However, even the slip ratios are neglected since the wheel inertia wasn't accounted for. Secondly, the Pacejka tire model, despite being considered in the QSS, was simplified to the "Lateral Cornering Stiffness". In the transient case, the tire model is directly coupled with the vehicle model, which can give more reality. Thirdly, in the two cases, the calculation of the slip ratio was simplified. In the Quasi Steady-State case, the lateral force is calculated for a null slip ratio (best case/pure lateral). In the transient, the slip ratio is calculated according to:

$$\kappa_{ij} = \frac{V_x - (\omega_{ij} r_{tire} \pm r \frac{t_1}{2})}{V_x} \quad (4.19)$$

being ω_{ij} the rotational velocity of a tire. However, when the wheel inertia is neglected, isn't possible to have some prediction about the wheel velocity, so the term $\omega_{ij} r_{tire}$ is simplified by V_x . So, in the transient case, the κ_{ij} is calculated, but it is only affected by the yaw motion.

Fourthly, the only parametric difference between the models is the yaw inertia. As the yaw motion dynamics isn't considered in the Quasi Steady-State, this parameter doesn't appear in it. However, in the transient, a value need to be given to this. For simplification, it was considered yaw inertia of 300kgm^2 . The other parameters were the same. Finally, the out-of-plan motion, steering system, and aerodynamic effects used in the transient model are the same used in the QSS model. For this reason, the showing of these aspects was just skipped here.

4.1.7 Results and Discussion

To compare quantitatively the models, some tests were thought and executed. These are simple tests that show simple manoeuvres, but it is possible to see some correlations between them. The list is:

1. Simple Cornering at Medium Velocity.
2. Simple Cornering at High Velocity.
3. Double Lane Change at Medium Velocity.
4. Double Lane Change at Medium Velocity (with longitudinal load transfer).

The two firsts are simple manoeuvres done in different conditions. The first condition is a medium steering wheel angle (0.45 rad) with medium velocity (14 m/s). The second is a low steering wheel

angle (0.15 rad) with high velocity (25 m/s). The importance of this difference will be explained later. The input for the first case can be seen in Figure 4.2. For this input, were extracted results of yaw rate, lateral velocity, lateral acceleration and body slip angle. The results are shown in Figure 4.3.

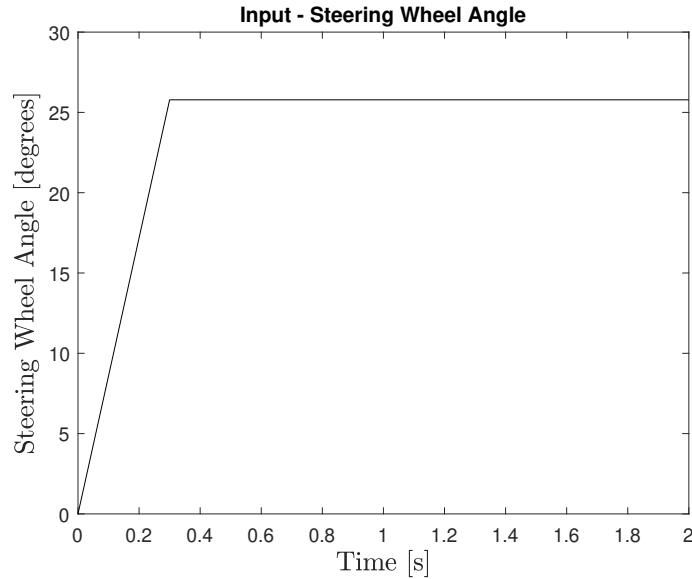


Figure 4.2: Steering Wheel Input - Case 1

Each one of these results is related directly to some dynamic characteristics of the car. The yaw rate is directly tied to the curvature described by the car. The ratio $\frac{r}{V_x}$ is exactly the actual curvature of the car. The lateral velocity is tied to the dynamic response to the solicitation (this will be more clear later). These two parameters have a direct influence on the slip angles since it is calculated through the lateral velocity in each tire (dependent on the lateral velocity of the car and the yaw rate). Moreover, the lateral acceleration is completely tied to the forces applied to the car and its study is important mainly looking to limit cases. Finally, the body slip angle is a derived parameter, calculated through the equation:

$$\beta = \arctan\left(\frac{V_y}{V_x}\right) \quad (4.20)$$

This value is the angle that the car does relative to its actual trajectory. In terms of comparison, it is possible to see that the transient case is much more non-linear than the QSS case. In the Yaw Rate and Lateral Acceleration, it is possible to see that exists a "delay" until the state is developed. In the lateral velocity and the body slip, there's an "overshoot" until the value is stable. These two effects are related to the inertial behaviour of the car. In the yaw rate, this inertia is justly the yaw inertia (described earlier). If this inertia is bigger, there's a tendency for the delay increase. In the lateral velocity and acceleration, the overshoot is a consequence of the lateral inertia of the car. After the "transient" instant, there's stabilization in a constant value. Another point is some difference in the steady-state values of the two cases. This effect can occur for several reasons.

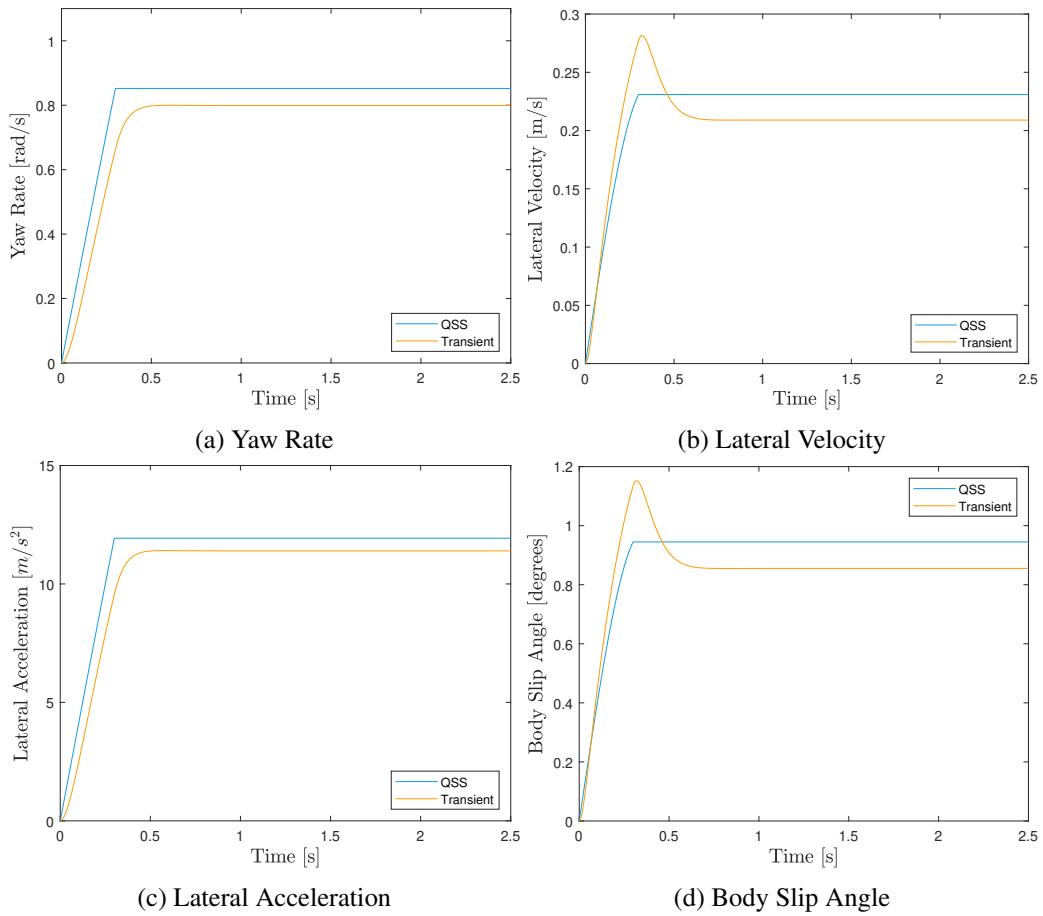


Figure 4.3: Results - Case 1

One of them is the absence of the transients in QSS model. For example, the overshoot in the lateral velocity can change the slip angles in a way that makes the tire behave in a different way. Besides, the transient model has 4 tires and the QSS, has 2. The already-explained lateral cornering stiffness, as a simplification for the QSS model, can lose some precision that a direct 4-wheel model can have. However, it is possible to see that the difference in the steady-state responses is very small, mainly in the lateral acceleration, which is one of the most important parameters. Besides, the yaw rate, related to the described trajectory, is practically the same. This means that the QSS model can be a good approximation, at least in this example.

However, a little difference can occur in the solicitation case. For the second case, the high velocity (25 m/s) with the small steering wheel input (Figure 4.4) will cause a different response in the car. The results are shown in Figure 4.5. It is possible to see that a very similar lateral acceleration is caused (around 10-12 m/s^2), but the yaw rate and the lateral velocity are significantly different. Firstly, the yaw rate has almost half of the value computed for the first case. The same "delayed" behaviour can be seen, but the delay is a little bigger. In the lateral velocity and the body slip angle, the QSS model shows a full negative response, while the transient shows a positive increase in the beginning with a great subsequent decrease until it stabilizes in a negative

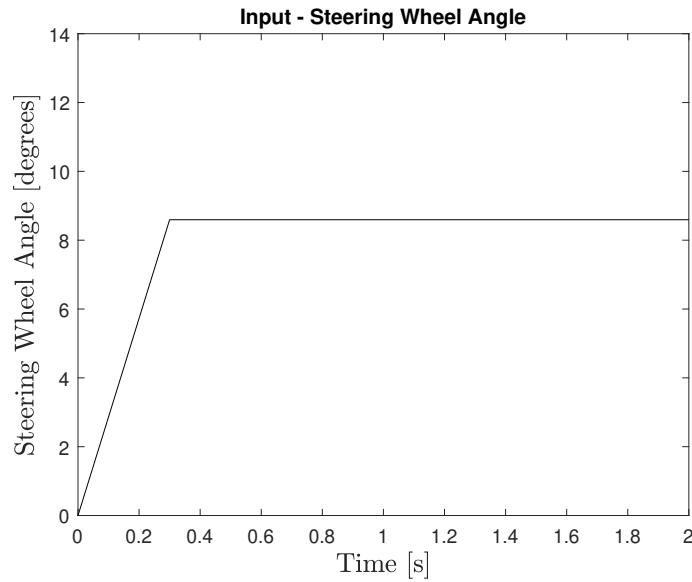


Figure 4.4: Steering Wheel Input - Case 2

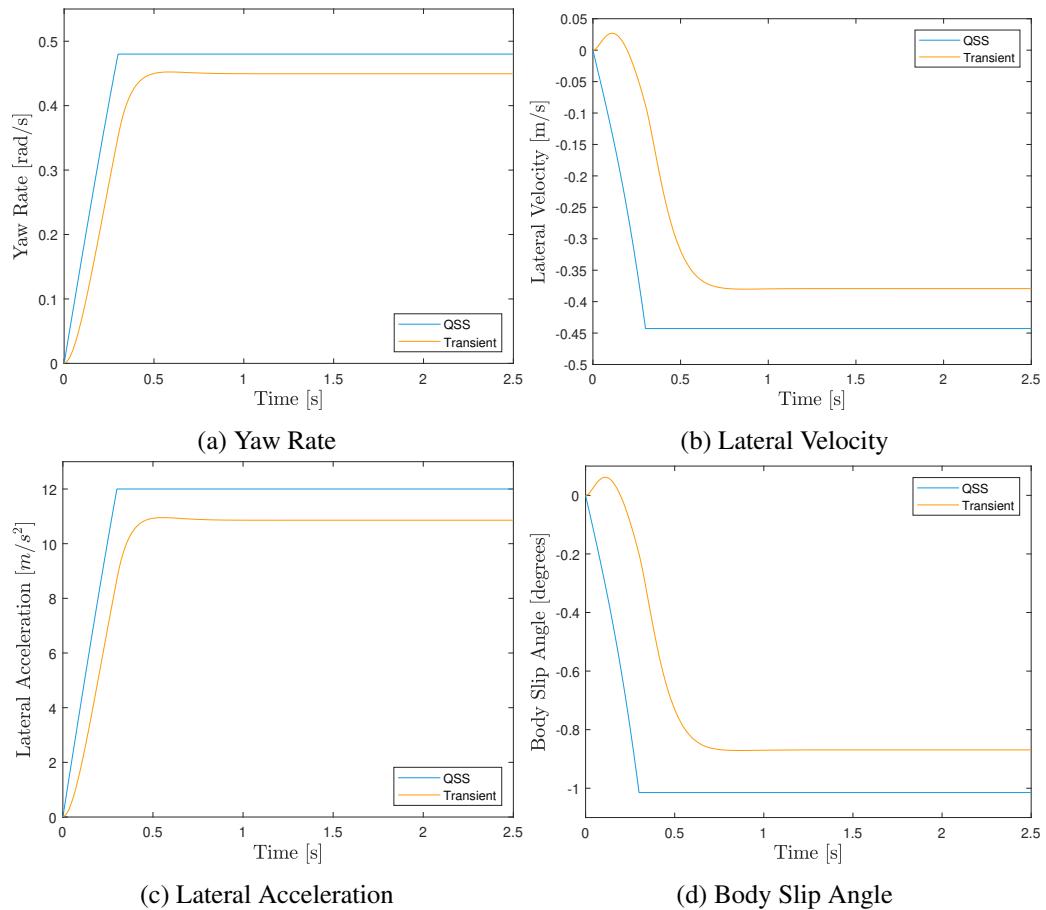


Figure 4.5: Results - Case 2

constant value. Despite the differences, the focus of this analysis is the fact that the lateral velocity

and the body slip angle are negative. This behaviour is known as nose-in (or oversteer). This means that the car is pointing to the inner part of the corner. This effect depends basically on the speed. When the car is at high speed, the slips are developed in a way that the car becomes to have a oversteer condition. Figure 4.6 shows the two conditions that can occur. In terms of comparison

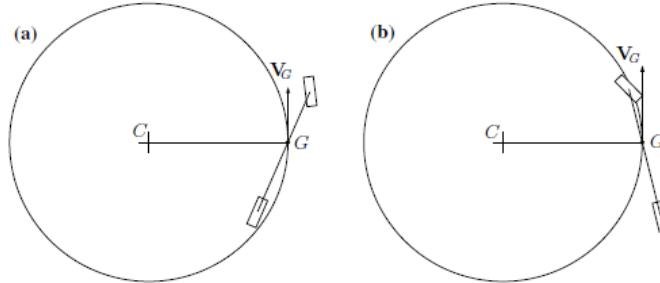


Figure 4.6: Understeer (Left); Oversteer (Right). Taken From [15]

between the models, the discrepancy is a little bigger in this second case, but the results between the QSS and the Transient aren't so different. Besides, the fact that the Quasi Steady-State model is sensitive to the effect of "oversteer/understeer" is interesting and important to evaluate the car's dynamics in several situations during a lap.

The third case has the objective of inducing the car in a more non-linear solicitation. For this, the manoeuvre is a repetition of the first case, but with two subsequent corners. The steering wheel is turned to the left, after to the right and, finally, to zero. The input is shown in Figure 4.7. The results are shown in Figure 4.8.

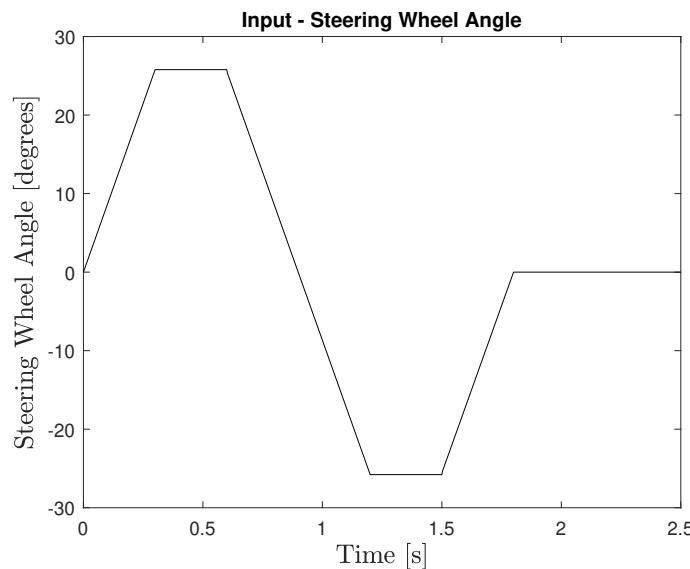


Figure 4.7: Steering Wheel Input - Case 3

With the results, it is possible to see that the responses are very similar to the first case. The importance of this test is just to know if the QSS model can deal with the high dynamics of a

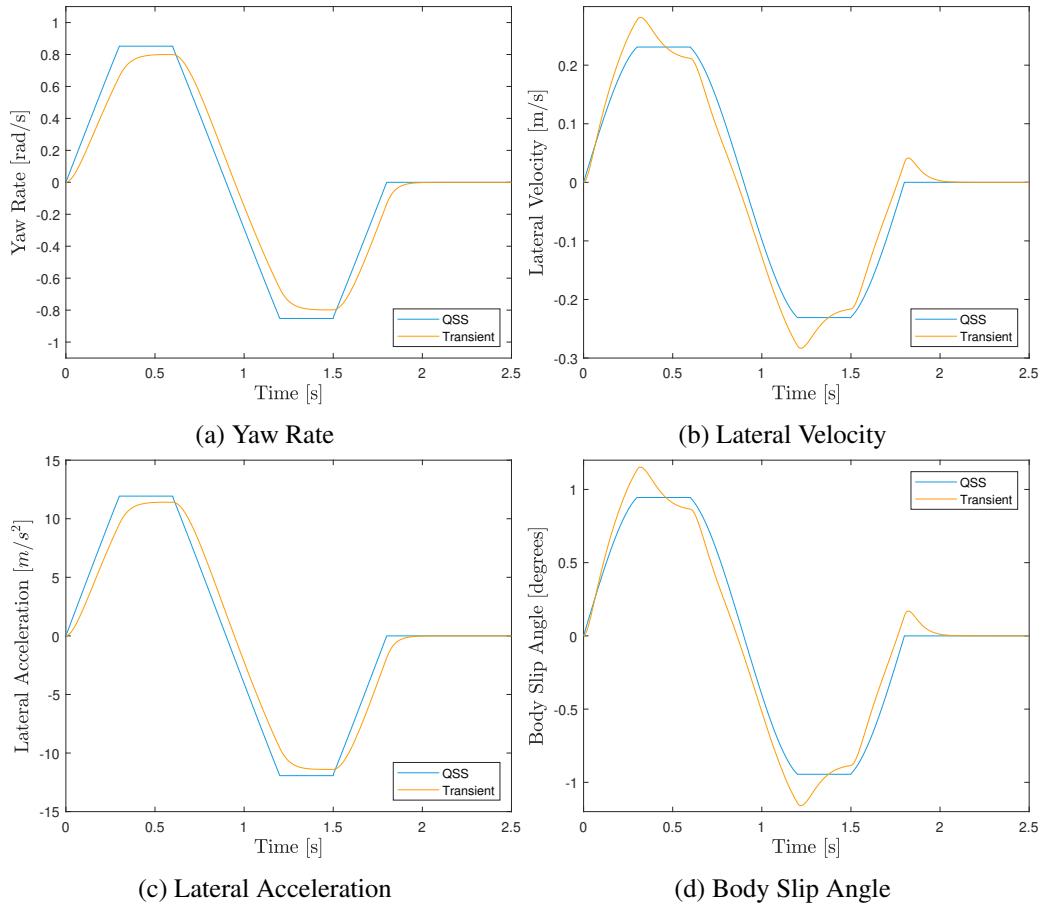


Figure 4.8: Results - Case 3

double lane change in the same way that the transient behaves. The conclusion is that, despite the delay and the offset (of the transient) and the transient regions, the response of the two models is very similar.

Finally, the same manoeuvre was tested with a virtual deceleration applied to the body (4 m/s^2). This deceleration has the only objective of forcing a longitudinal load transfer (in this case, to the front tires). This example is important since the variation of the normal loads in the tires changes significantly the dynamic behaviour. The results are seen in Figure 4.9. Compared to Case 3, it is possible to see that the yaw rate increased a little as well as the lateral acceleration. The lateral velocity and the body slip angle had practically the same values for the transient case but in the QSS, there's a decrease. To understand this difference, it is useful to look at the slip angles in Cases 3 (Figure 4.10) and 4 (Figure 4.11). In the two cases, the value of the front slip angles remains practically the same for the two models. However, the rear slip angles increase when the deceleration is applied. This effect occurs due to the change of the loads in the rear tires (in case 4 there's a normal load reduction) which, consequently, increased the slip angles. However, the change in the rear slip angles (compared to the front) changes significantly the car's behaviour (it can change the properties of the car as oversteer or understeer, for example).

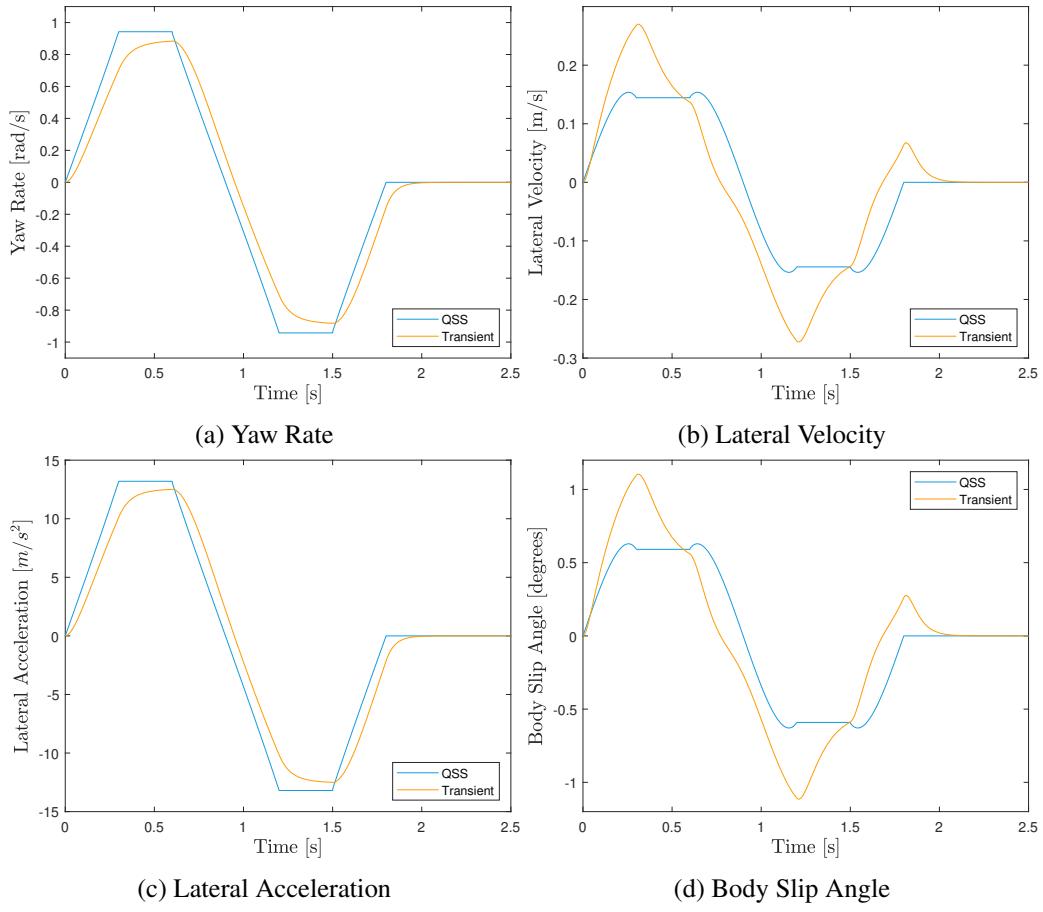


Figure 4.9: Results - Case 4

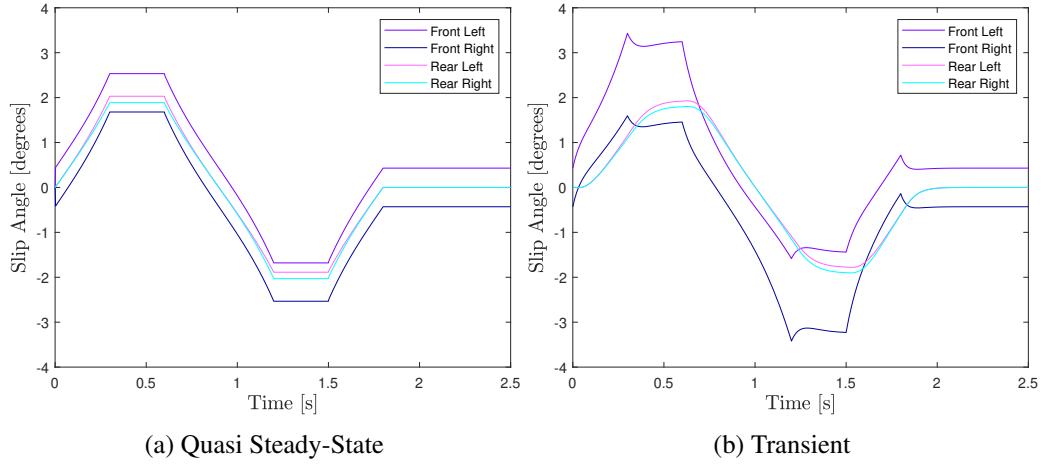


Figure 4.10: Slip Angles - Case 3

However, it is necessary to try to understand the decrease in the lateral velocity for the QSS model, since the transient remained similar. In Figure 4.11, it is possible to see that the values of the rear slip angles are nearer the front values in the QSS than in the transient. This can be explained by the differences in the tire models. As exposed earlier, the QSS model has a

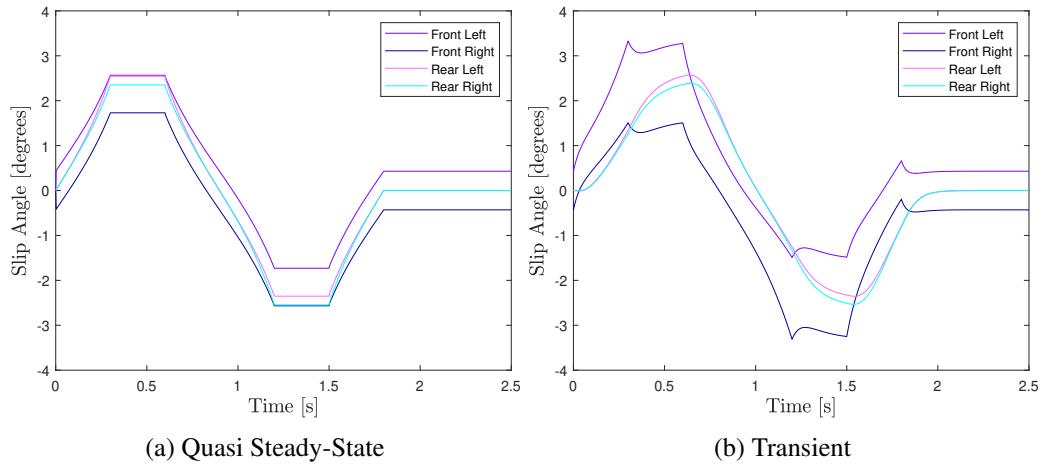


Figure 4.11: Slip Angles - Case 4

Pacejka Tire Model converted to Tire Cornering Stiffness. However, in this conversion, there's no sensitivity on the slip angles (this concept assumes the linear part of the tire curve), but just on the normal forces. So, when the tire begins to have bigger slip angles (and the behaviour begins to be more non-linear), this non-linearity isn't detected. On the other hand, the transient model can detect all these effects and, for this reason, it is possible that some differences between the models can occur when the slip angles begin to increase in a manoeuvre. However, in terms of comparison, the QSS model followed the most important effects of the transient and obtained similar values in all tests. So, the conclusion is that, in the main conditions that a car is submitted, the vehicle model can be approximated to a Quasi Steady-State.

4.2 Repeatability Tests

One of the best ways to understand if a model is consistent is to force it to repeatability tests. In other words, if the model executes some manoeuvre in a specific way, it is useful to understand if the same manoeuvre executed several times (in a loop) produces ever the same behaviour (or state) in the car. The chosen manoeuvre is a series of straights and corners that combines low and high velocities. This section was extracted from the Formula Student Germany 2012 Autocross track and there are 4 repetitions. The curvature profile of the segment can be seen in Figure 4.12. The extracted results of this analysis are basically the speed and the steering wheel angle (Figure 4.13) and lateral acceleration (Figure 4.14). The two firsts are completely related to the driver's input to the car. It is important to see if the speed and the steering angle are the same in all the repetitions. This would show that the simulator is consistent. The final information, about the lateral acceleration, shows that the vehicle model is working as expected. If the speed profile and the steering input profile are the same (in all repetitions), the expected result for the lateral acceleration is also a repetition. It is possible to see that all the results are periodic. The only discrepancy occurs at the beginning of each one since there's an initial velocity (5 m/s) and, consequently, there's a time until the response stabilizes for a constant repeating value.

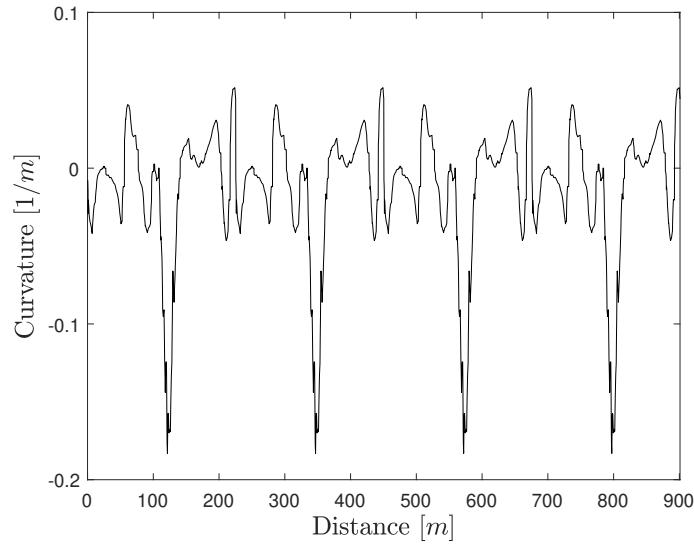


Figure 4.12: Repeatability Test - Curvature

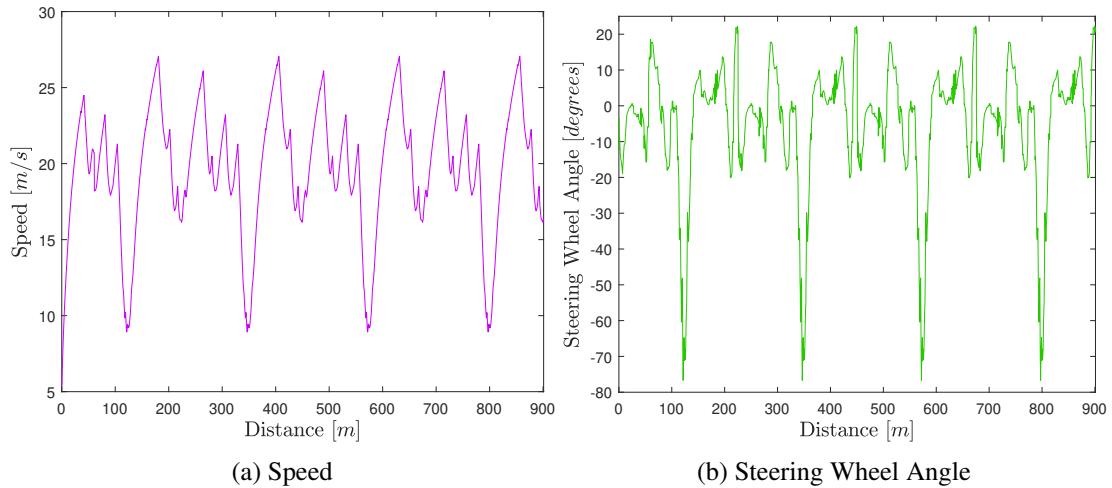


Figure 4.13: Repeatability Test Input

4.3 Comparison to other software

Finally, another way to validate the simulator is to compare it with existing software. For this reason, this final section will show this comparison. OptimumLap is a free software developed by OptimumG company with the objective to provide a simple tool to lap time estimation. As said by the company, it is possible to create a vehicle with just 10 parameters and study it in several tracks and conditions. The model used in this software is a single-point mass Quasi Steady-State. The documentation of the software shows that the model doesn't account for the load transfers, the tire sensitivity, the yaw and the banking. The parameters to be studied are aerodynamic parts, vehicle mass, tire grip, gear ratios and engine power. The simulation strategy is exactly the same used in this work, a forward-backward algorithm.

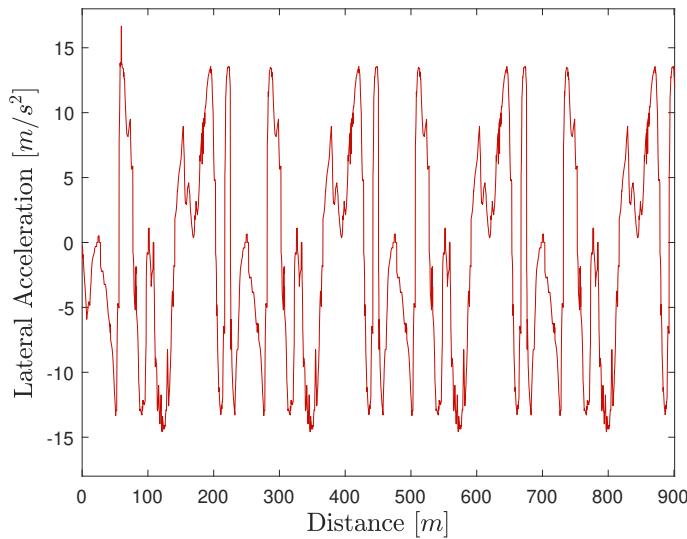


Figure 4.14: Lateral Acceleration - Repeatability Test

Firstly, the vehicle was parameterized in OptimumLap, with a motor curve similar to the one obtained as a result of the simulations in the developed model (Figure 4.15). The curve is already affected by the efficiencies of the motor. The other parameters (aerodynamics, mass, gear ratio, tire friction) were parameterized (Table 4.1) and the vehicle was created. For the tests, the Autocross Track from FSG 2012 was chosen. This track is available in the library of the software. Joining the track with the vehicle, a simulation was done and the results were extracted in a CSV file.

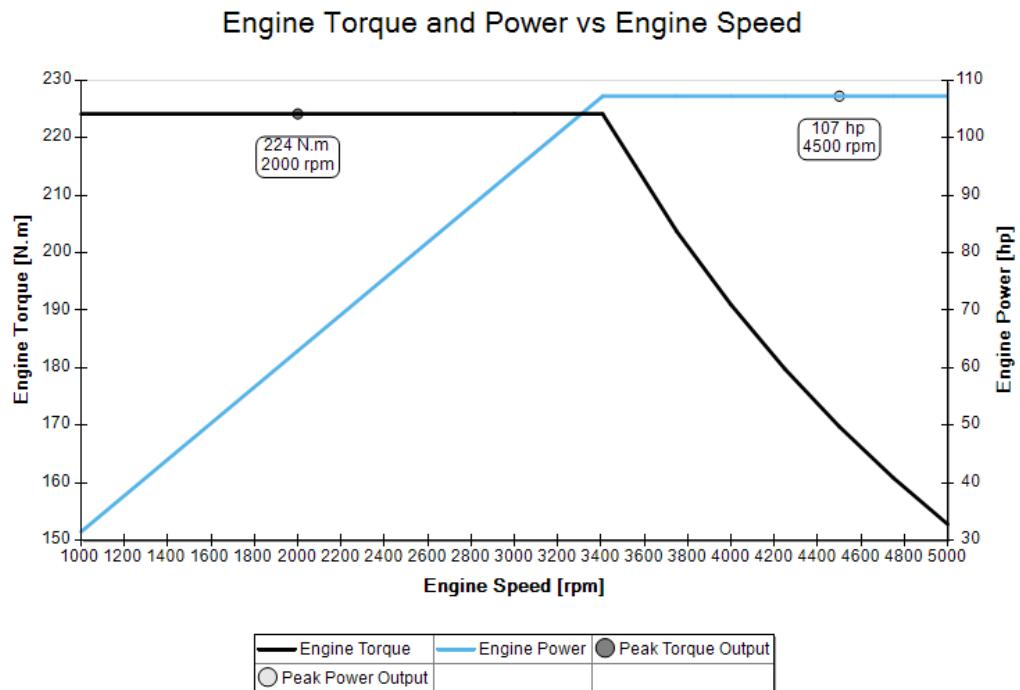


Figure 4.15: Motor Curve - OptimumLap

Table 4.1: OptimumLap Software Input Parameters

Parameter	Value
Vehicle Type	FSAE
Mass [kg]	350
Driven Type	2WD
Drag Coefficient	0.675
Lift Coefficient	0.343
Front Area [m^2]	1
Tire Radius [m]	0.254
Rolling Resistance	0
Longitudinal Friction	1.5
Lateral Friction	1.5
Gear Ratio	4:1
Drive Efficiency [%]	88.5

In the CSV file, some state variables were extracted, such as the longitudinal velocity, longitudinal and lateral accelerations, yaw rate, yaw acceleration and so on. Information about the track also was extracted (X and Y coordinates, segment radius and segment length). With this information, it was possible to input it into the developed software to compare the results. Beginning with the speed comparison, Figures 4.16 and 4.17 show the velocity profiles. The first figure is for the entire lap, while the second focus on a specific segment.

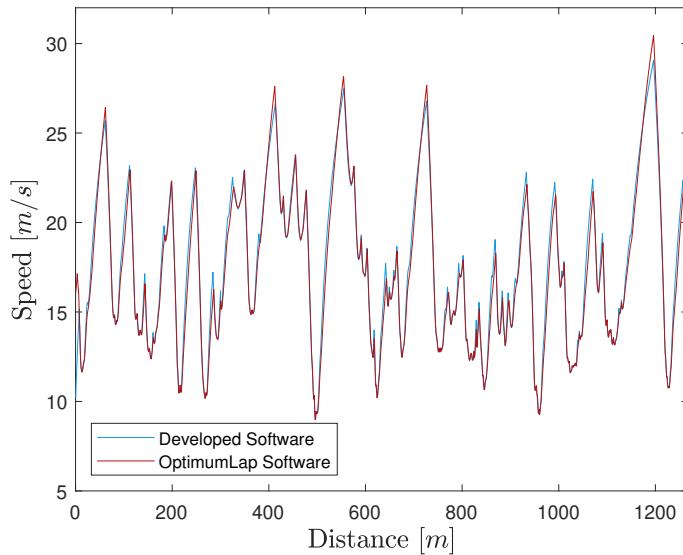


Figure 4.16: Velocity Comparison - Developed Software and OptimumLap

It is interesting to see that in apex points the speed is practically the same which corresponds to an equivalent tire capacity. Some discrepancies can occur mainly for the assumption of linear tire model (constant friction coefficient) from OptimumLap software, while the one assumed in the developed software is the non-linear model. However, there is a main difference in the obtained solutions. It is possible to see that the developed software has a bigger acceleration. This can be

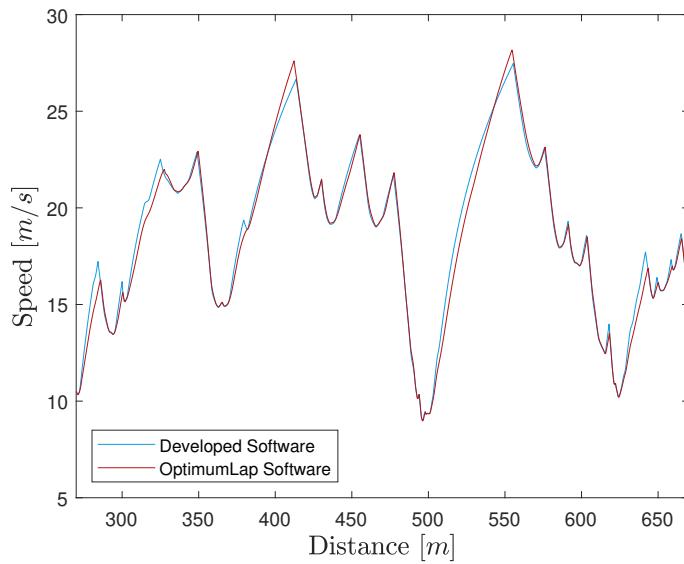


Figure 4.17: Velocity Comparison - Developed Software and OptimumLap

seen through the speed profile when the curve overpass the curve from OptimumLap. This effect it is related to the dynamic model. The OptimumLap doesn't work with load transfers, which means that the longitudinal load transfer (that helps the acceleration of a rear motor car by the increasing normal force in the rear axle) is being neglected for it. This causes a dynamic limit on acceleration capacity. Even if the motor parameters are changed (performance increased), the final result doesn't change.

With this, it is useful to look for the acceleration distribution. Figure 4.18 shows the longitudinal acceleration for the same part of the track. It is possible to see that the two curves follow a similar behaviour. In the developed solution, there's more roughness, but the general behaviour is similar. For the lateral acceleration, it is possible to look to Figure 4.19. More time, the aspect is practically the same, with just some local discrepancies.

4.4 Conclusion

Some validation methods were used to assure that the solution developed in this work is consistent with reality and can deal with the several conditions that a race car is submitted. The conclusion is that for the necessities delineated in this work, the developed simulator is a good alternative to transient models, being sensitive to most of the effects that are perceived by the transient model, such as the manoeuvrability condition changes (due to the load transfers), body slip accounting and difference between nose-in and nose-out conditions. Clearly, there are some limitations, mainly by neglecting the transient effects, which makes the performance limit slightly overrated, since the critical dynamic cases aren't detected by the model (the dynamics of entering or leaving corners, braking points). For this reason, it is necessary to agree that optimal tuning of the suspension

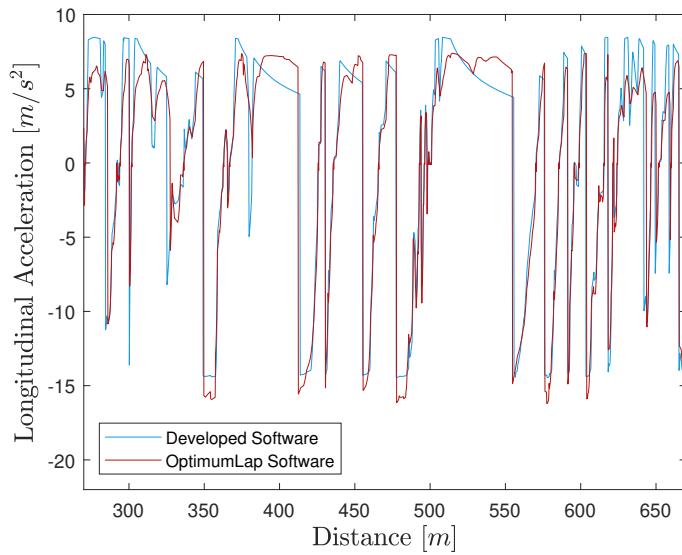


Figure 4.18: Longitudinal Acceleration Comparison - Developed Software and OptimumLap

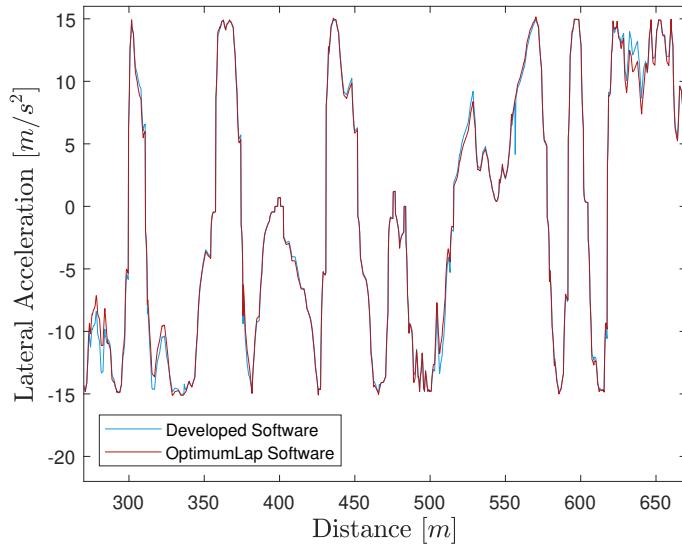


Figure 4.19: Lateral Acceleration Comparison - Developed Software and OptimumLap

parameters could be done better using a transient tool, but this Lap Time Simulation can give, in an overall analysis, a good estimation of the car's performance.

Chapter 5

Results

After the development of all the models, the objective of this chapter is to show the possible results to be extracted from the developed Lap Time Simulation. This exposition will be done in some parts. Firstly, some direct results will be extracted. These are, generally, state variables of the car, calculated for each segment of the track. Moreover, other types of results need to be obtained through post-processing. One of these is the time, which is calculated from the travelled distance and the velocity at each point. However, other electrical parameters, such as energy consumption and mean current need to be calculated also.

5.1 Simulation Conditions

The results presented in this first part are for an Autocross simulation in the Formula Student Germany 2012 Autocross track (Figure 5.1). Remembering the Formula Student presentation, the autocross is a hot lap, which means that the car is at its best performance configuration. The car was simulated with the parameters shown in Appendix A. The start velocity is $10m/s$ and a perfect driver was assumed.

5.2 State Variables

There are state variables related to the dynamics of the car, mainly the velocities and accelerations. Other variables (already presented in the other chapters) can be derived, such as the slip angles, normal forces and even the lateral force applied at each axle. Moreover, these variables can be combined, generating diagrams as the GG, already cited earlier. Examples of these results will be plotted here

5.2.1 Speed

The main result of a Lap Time Simulation is the speed profile (Figure 5.2). This value shows at any point on the track what is the speed of the car. Despite being obvious, the speed profile can show more than just the speed. The form of the profile shows the acceleration capabilities of the

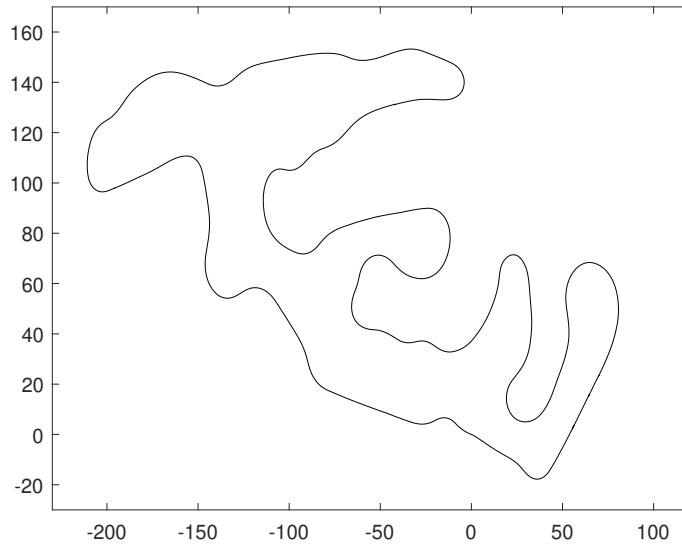


Figure 5.1: FSG Autocross 2012 Track

car (in acceleration or braking). Moreover, the minimum speed points show the already explained apexes, which are points that belong to the corners. So, the speed profile can say much about the car's performance during a lap and about the track itself.

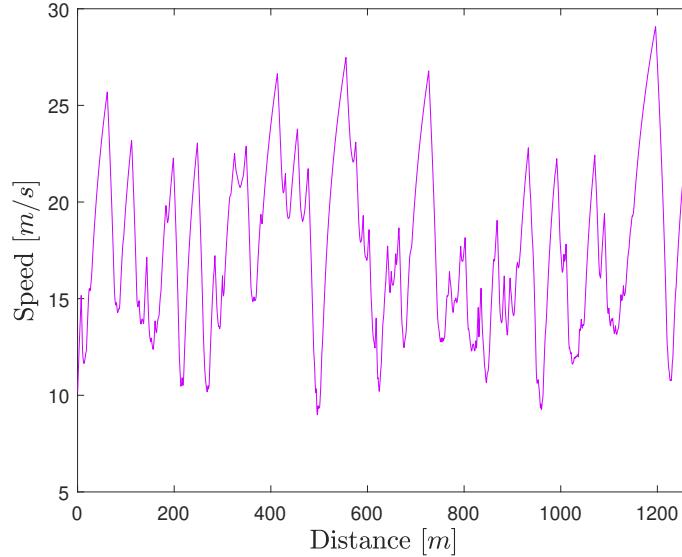


Figure 5.2: Results - Speed Distribution

5.2.2 Longitudinal Acceleration

Another important result is the longitudinal acceleration profile (Figure 5.3). This plot must be analysed with the speed profile since they are joined in the definitions (the acceleration is theoretically the derivative of the velocity), so, the longitudinal acceleration profile quantifies a behaviour that can be seen in the speed profile.

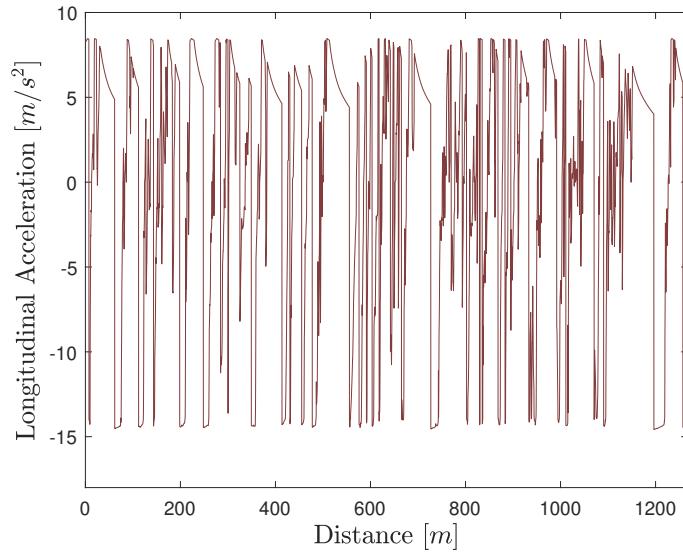


Figure 5.3: Results - Longitudinal Acceleration Distribution

5.2.3 Lateral Acceleration

One of the most important results is the lateral acceleration (Figure 5.4). If the longitudinal acceleration is completely related to the capability of a car to accelerate or brake (motor and brake system performances), the lateral acceleration is related to the whole lateral dynamics of the car, which is affected by the masses, dimensions, suspension geometry and characteristics, and steering system. Basically, a car that can run with more lateral accelerations has better lateral capability (or good handling performance).

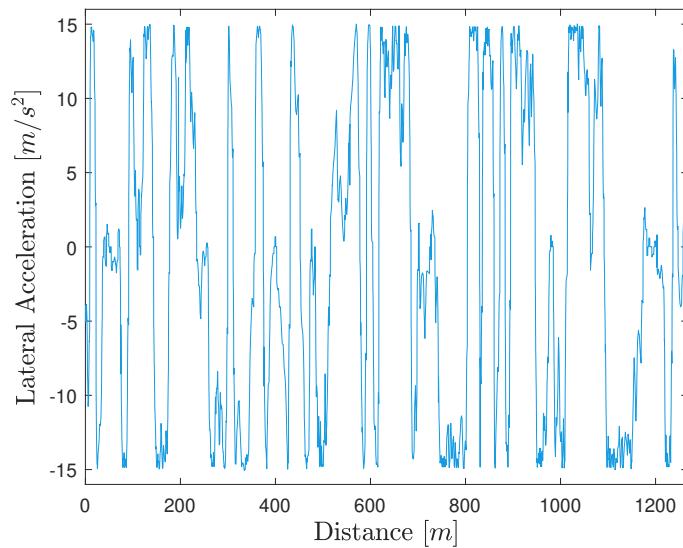


Figure 5.4: Results - Lateral Acceleration Distribution

5.2.4 Yaw Rate

Another state variable is the yaw rate (Figure 5.5). It is the variation rate of the heading angle during the time. This value is related to the velocity of the car and the radius of the curvature:

$$r = \frac{V_x}{R} \quad (5.1)$$

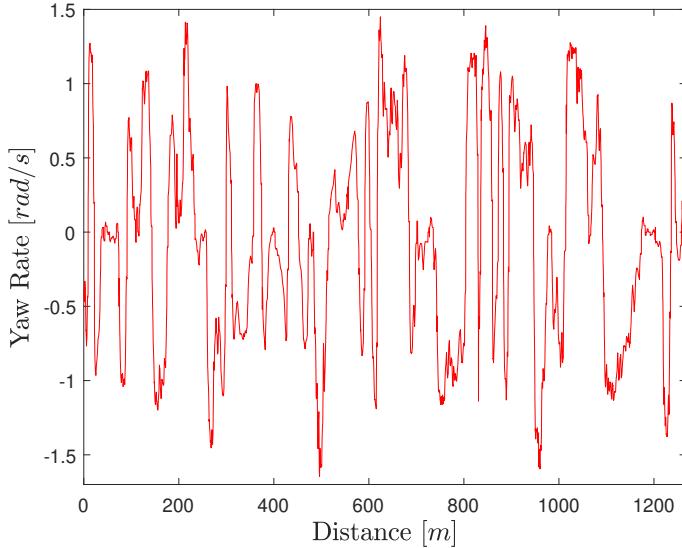


Figure 5.5: Results - Yaw Rate Distribution

5.2.5 Lateral Velocity

The lateral velocity (Figure 5.6) is a state parameter related directly to the dynamic behaviour of the car. Clearly, this parameter depends on the car's parameters and the control inputs. However, for a specific steering angle and tire characteristics, the lateral velocity depends significantly on the speed, which can be verified by looking at the body slip angle in different conditions since the lateral velocity directly affects the body slip angle.

5.2.6 Body Slip Angle

Another parameter that can be extracted is the body slip angle. This parameter is calculated through the lateral velocity and the speed. As already explained, it is a value related to the dynamic behaviour of the car at each time. A high-speed manoeuvre will cause a body slip angle qualitatively different from a low/medium-speed manoeuvre. The result is shown in Figure 5.7 To understand the sensitivity of the software to the differences in this parameter, it was chosen an instant, between 400 and 600 m, to have a better analysis. Figure 5.8 shows the body slip angle in this part of the track. However, the plot also shows the heading direction of the corners. Simply, when the corner is for left, the direction is 1; when the corner is for right, the direction is -1. A reference dash-line was included to mark the zero reference. The idea of this analysis is to look at

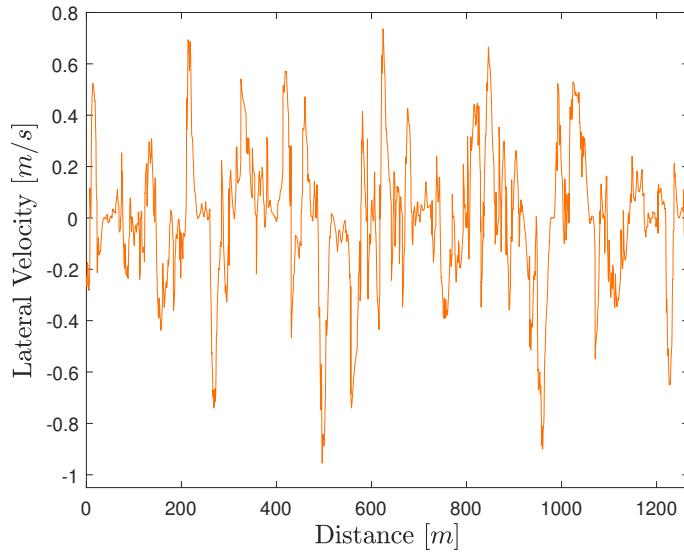


Figure 5.6: Results - Lateral Velocity Distribution

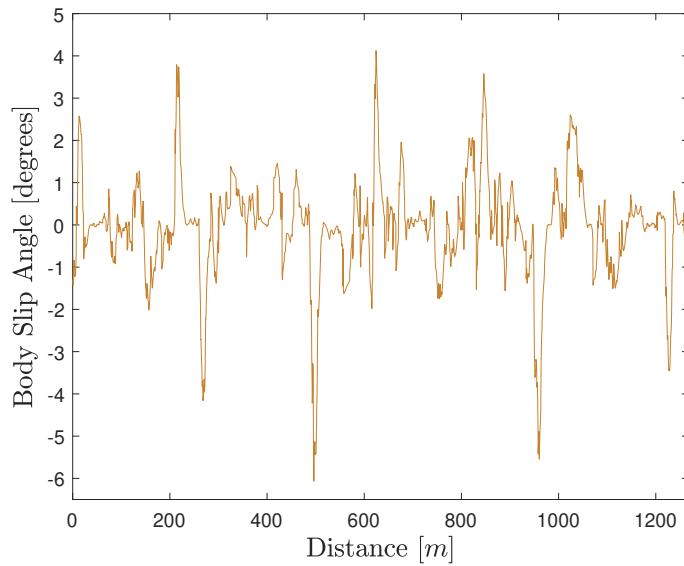


Figure 5.7: Results - Body Slip Angle Distribution

the body slip angle sign and compare it with the direction. When the sign is the same for the two quantities (++ or -), the car works in nose-out (understeer) condition. This means that the car is pointing to the outer part of the corner. When the sign is different (+- or -+), the car is working in nose-in (oversteer) condition, in which the heading direction is for the inner part of the corner. As said in the last chapter, this behaviour depends on the conditions of the cornering. High-speed turns cause the oversteer condition, while low-speed turns cause the understeer condition. It is for this reason that the figure includes a second curve from the speed distribution in this track part. Most time, the car works in an oversteer condition, but when there's a slow corner (approximately in the 500 m), the car has an understeer condition. When it reaches a higher speed (above 20 m/s), the behaviour turns to be oversteer. This information about the body slip angle can be used in

deeper analyses, and the variation of the parameters can change the dynamic behaviour. However, it is a great thing the software is sensitive to this effect. Compared to the theory (remember Figure 4.6), there's validity for the effects obtained in the simulations.

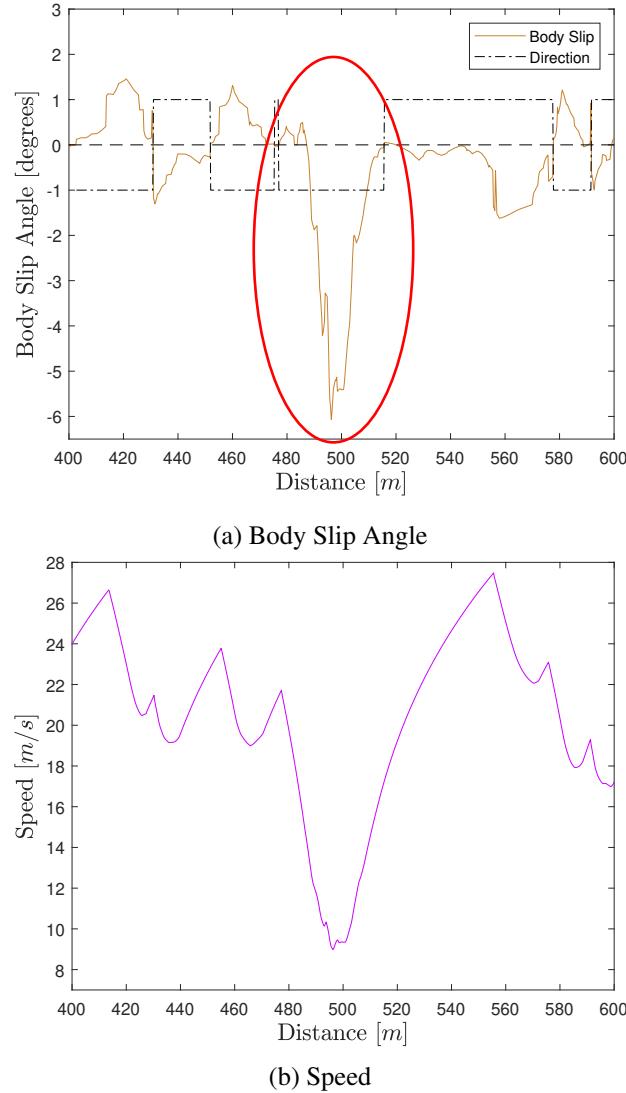


Figure 5.8: Results - Nose-in/Nose-out behaviour

5.2.7 Slip Angles and Normal Forces

Other two results very important to the dynamic analysis of the car are the slip angles and the normal forces (Figure 5.9). These two could be presented separately, but there's a close relation that values to be explained together. The first thing is that the parameters are presented for the 4 tires. The presentation needs to be done in this way since the results for one tire are related to the result for other tires. Looking for the Normal Forces distribution, it is possible to see that there's a "symmetry axis" in the results. This almost "symmetry" occurs because of the load transfers. When there are accelerations applied to the car, they will increase the load in one tire

while decreasing in the other. Moreover, as will be shown more specifically, the less loaded tires ever have more ease to skidding and, for this reason, the slips in these tires are bigger.

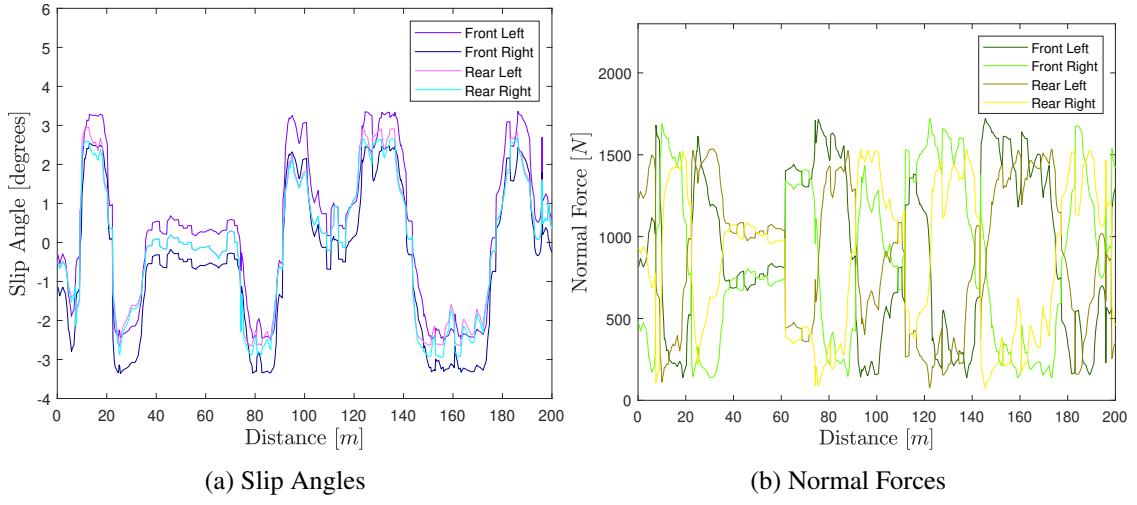


Figure 5.9: Results - Tire Behaviour and Normal Forces - 0-200m

The results were presented for the 0-200 m. It is possible to see in a whole lap, but since there are several plots (for each tire), it is useful to change the visualization to lesser lengths. However, to have a more deep analysis, Figure 5.10 shows three plots: slip angles, normal forces and accelerations.

These three plots need to be analysed together. The reason for this is that the slip angle, normal forces and accelerations are completely tied. As explained before, the slips depend on the state and the state depends on the slips. So, some things about the car can be learned with this analysis. Firstly, the developed model assumes the lateral acceleration as being positive for positive curvatures (and positive steering input). The implication of this can be seen in the Normal Force distribution. In the beginning, there's a little amount of negative lateral acceleration applied to the car and this implies bigger normal forces in the left tires. If the car roll in the outer direction, it is easy to understand that, in this case, the car is rolling to the left and, consequently, the corner is in the right direction. Obviously, the implication of the load transfer is that the inner tires are low solicited and this causes an interesting effect on the slip angles. It is possible to see that the lower-loaded tires almost ever have bigger slip angles. The reason for this is clear. A small load generates less lateral force and, consequently, allows more skidding.

Another interesting effect occurs between 10 m and approximately 13 m. Normally, the front tires have bigger slip angles because of the steering system (there's an increase in the slip because of the steering angle). However, in this interval, the rear slip angles are bigger than the front slips. More time, this needs to be analysed with the acceleration plots. At this moment, a big deceleration is applied to the car, which means that the front tires are very loaded. For the same reason seen in the last paragraph, there's an increase in the slips of the less loaded tires. When the deceleration

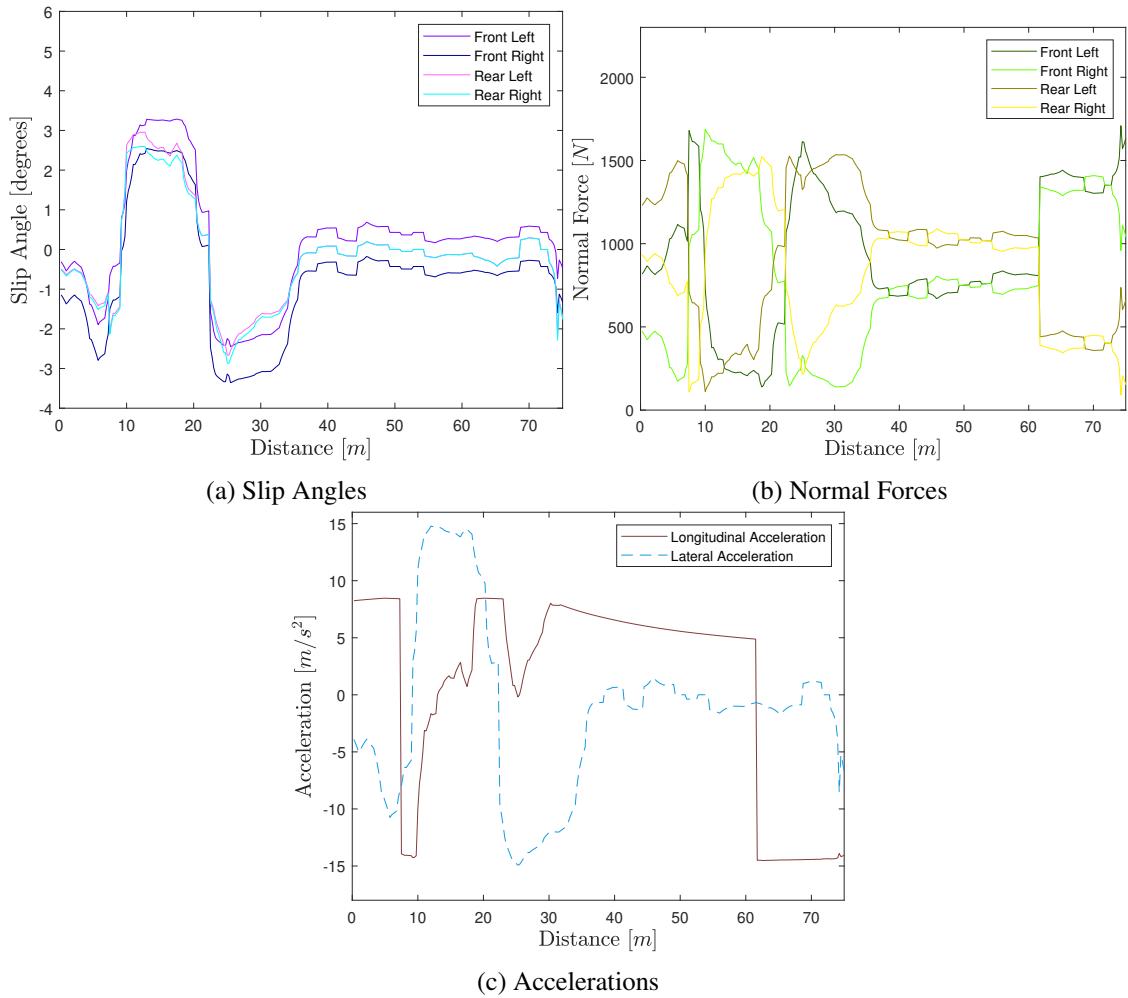


Figure 5.10: Results - Tire Behaviour and Normal Forces - 0-75m

decreases, more loads are turned for the rear tires and the front slip angles begin to be bigger. It is important to see that at this moment, there's a positive lateral acceleration (left corner), meaning that the left tires are less loaded and, consequently, their slips are bigger than the right tires slips.

Another important remark is an effect that can be seen in the slip angle distribution between 35 m and 65 m. The lateral acceleration is very low in this region, which means that the slip angles are almost zero. However, there's an offset between the front left and right slip angles. This offset occurs because of the configuration of the steering system, which has a "static steering angle", called "toe-in". In this case, the car has a toe-in of around 0.4 degrees, which causes this offset. It was seen earlier in Figures 4.10 and 4.11. Finally, a final commentary that can be done is seen near the 70m. In this region, the car is submitted to a big deceleration and there's a small change in the lateral acceleration. This change caused a great impact on the slip angles, which locally went until -3 degrees. This can be a calculation error (a sub-optimal solution from the solver, for example), but this shows that a car with less load in the rear tires (as tested earlier, in Chapter 4) can show more instability to small steering inputs.

5.2.8 Powertrain Model Data

From the powertrain model, it is possible to extract data from the electric part that is directly related to the acceleration capabilities of the car. This information is the electric power, electric current and electric voltage. The calculation of these quantities is done through the following equations:

$$P_{electric} = VI \quad (5.2)$$

$$V = \frac{N_{motor}}{V_{constant}} \quad (5.3)$$

$$I = \frac{T_{motor}}{T_{constant}} \quad (5.4)$$

$$P_{mec} = P_{electric} \times \eta_{mec} \quad (5.5)$$

5.2.8.1 Electric Current

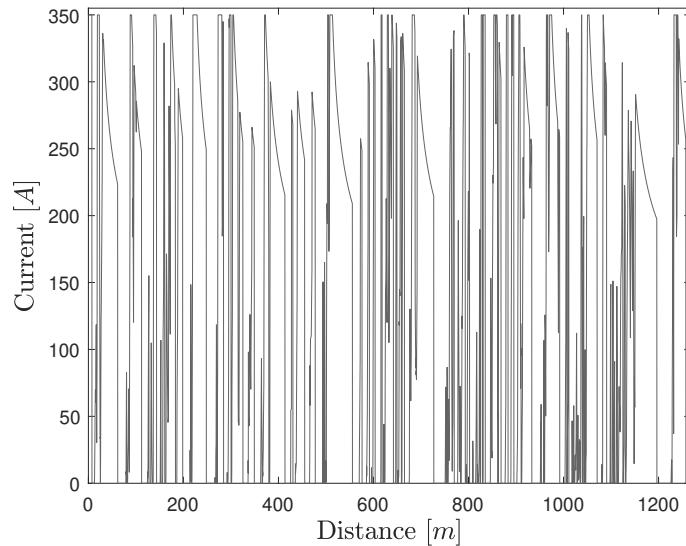


Figure 5.11: Results - Electric Current

This plot has the same form as the positive part of the acceleration plot. The idea is that the current is tied to the torque, which is tied to the acceleration capability. So, when the car is accelerating, there's a torque coming from the motor that involves a current coming from the electric system.

5.2.8.2 Electric Voltage

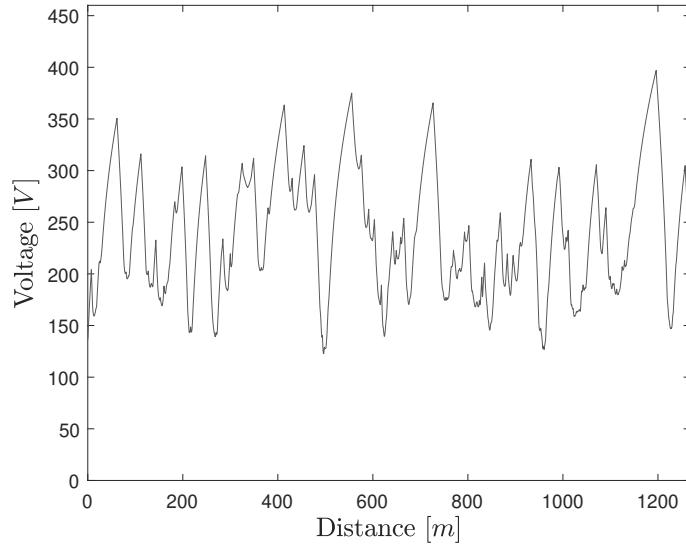


Figure 5.12: Results - Electric Voltage

This plot is completely related to speed. As explained before, the motor speed is proportional to the electric voltage. For this reason, the curve has the same form as the speed curve.

5.2.8.3 Electric Power

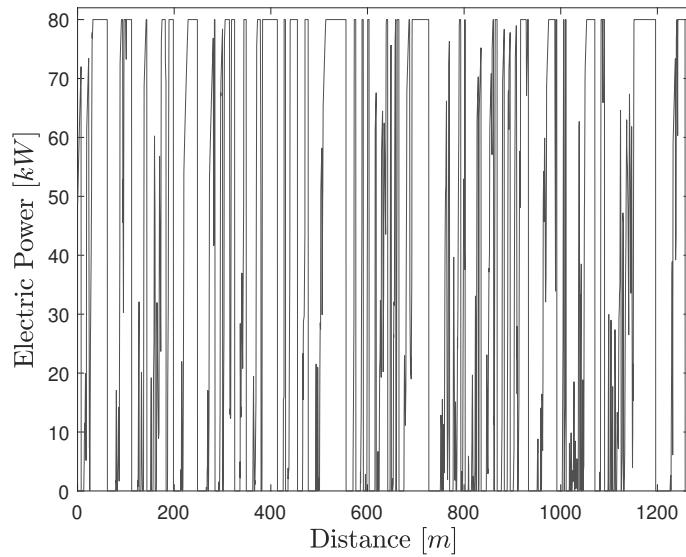


Figure 5.13: Results - Electric Power

Finally, electric power is completely tied to the current and voltage. There's a limit value, in this case, 80kW, that represents the maximum power available in the electric system. This value can be adjusted in the motor controller and is completely related to energy consumption. As will be shown after, in some conditions, it is necessary to decrease the maximum power to assure that

the car can complete some of the events. Moreover, the power limit forces a decrease in the current/torque/acceleration. It is for this reason that, in Figure 5.11, there are regions in which the torque is decreasing along the distance. In these regions, the speed is increasing and the torque is decreasing in the same proportion.

5.2.9 GG Diagram

A good way to condense the information about the overall dynamics of the car is through the GG Diagram. This diagram shows the lateral acceleration and the longitudinal acceleration, creating an "envelope" that determines the dynamic capabilities of the car. In the X axis, it is possible to measure the lateral acceleration and in the Y, the longitudinal. The extreme values for the lateral acceleration occur for pure lateral slip when the longitudinal acceleration is zero. Looking at the track, those are the apex points. When the car begins to brake or accelerate, there's a decrease in the lateral capability. This is a condition in which the tire is submitted to combined slip and, for this reason, all the available slip needs to be divided into lateral and longitudinal directions. This diagram is shown in Figure 5.14.

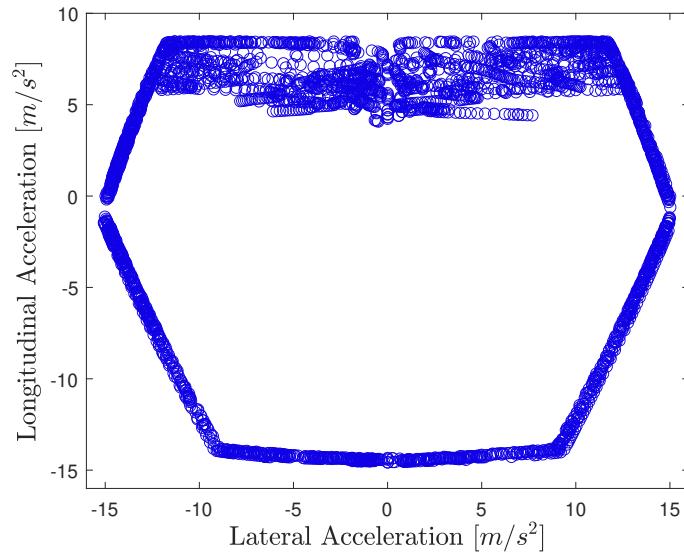


Figure 5.14: Results - GG Diagram

Looking at the positive part of the longitudinal acceleration axis, it is a clear limit. This limit can be defined by two different conditions: motor limitation or grip limitation. In the first condition, the torque of the motor is limited to a specific value that will correspond to the maximum acceleration that the car can achieve. In the second condition, the car is working within its dynamic limit and the tires used for the traction (in this case, the rear tires) are in the limit of the grip. On the negative part, the limit condition is just the grip limitation. but in this case, the grip isn't just from the tractive axle, but from all the tires, since the car, in this case, is braking. Laterally, the negative and positive parts are just related to the direction of the corner. As explained earlier, the positive part represents a left corner.

5.3 Derived Results

The derived results are those calculated after the simulation run. In a Quasi Steady-State simulator, the most important is the time, which is calculated through the track length profile and the speed profile. The calculation is done in the following way:

$$t_i = \frac{2(s_i - s_{i-1})}{v_{i-1} + v_i} \quad (5.6)$$

So, it is possible to have a time vector and, through it, it is possible to plot all the state variables (shown above) along the time. One important effect to verify is that the conversion to the profiles (from length to time) causes a scaling effect on the results. In the regions in which the speed is lower, there's a tendency for the distribution to be wider. In the regions in which the speed is higher, the tendency is for the data to be narrower.

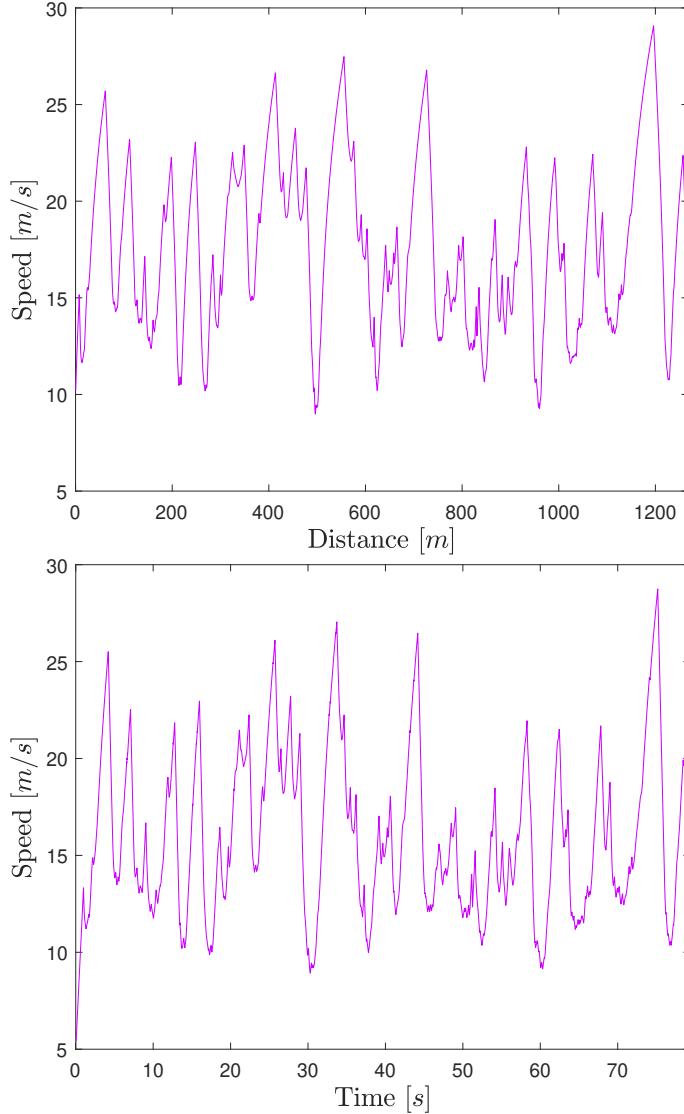


Figure 5.15: Comparison between length and time reference

Through the calculated time, there's possible to calculate other important data, mainly from the electrical system. For example, from the instant power, it is possible to integrate this value and obtain the Energy Consumption. Moreover, some average values can be obtained, such as the average current or voltage, important values for the electrical sizing of some components.

5.3.1 Energy Consumption Estimation

The energy consumption, in this case, is calculated through numerical integration. The equation to calculate the accumulated energy consumption is:

$$E_{consumed,i}^{acc} = \frac{(P_{electric,i} + P_{electric,i-1})(t_i - t_{i-1})}{2} + E_{consumed,i-1}^{acc} \quad (5.7)$$

The results can be seen in Figure 5.16.

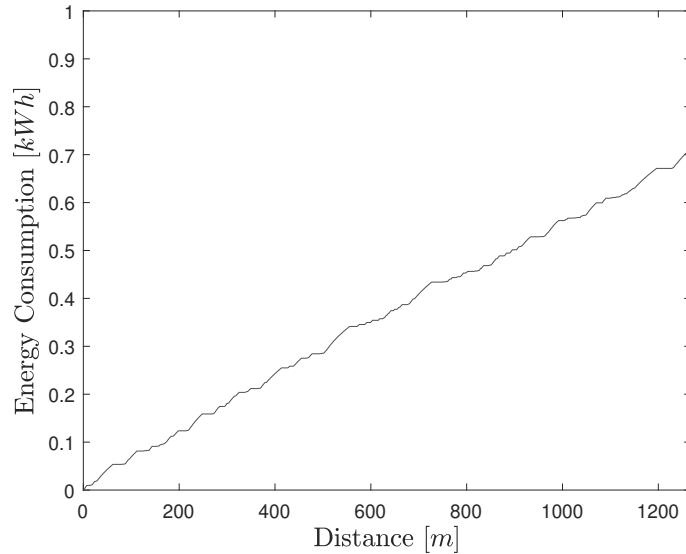


Figure 5.16: Results - Energy Consumption

It is possible also to obtain the instantaneous value, but this isn't so important since the main concern is to know the "State of Charge" for one lap (or several, in the Endurance case). It is possible, also, to display the energy information in the percentage format, calculated through the equation:

$$SoC = \frac{E_{total} - E_{consumed}}{E_{total}} \times 100\% \quad (5.8)$$

The result is shown in Figure 5.17. This estimation is important mainly for the Endurance event, in which the most constraint is energy consumption. So, it is important to simulate the car in several conditions (for example, different power limitations for the tractive system) and see what occurs in the energy consumption.

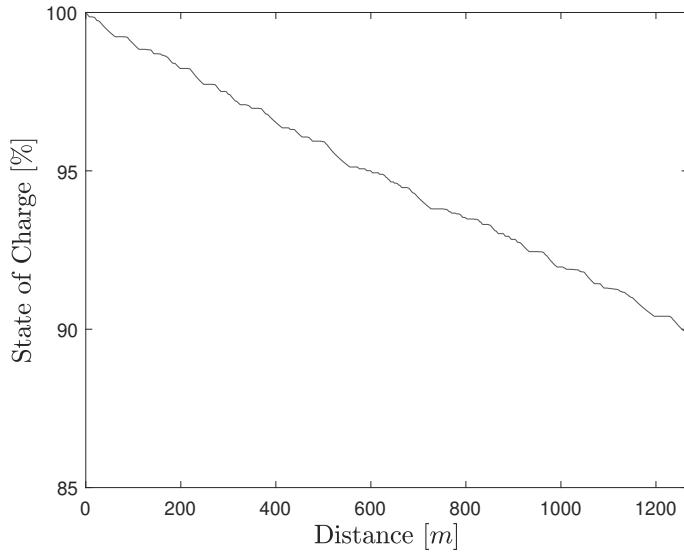


Figure 5.17: Results - State of Charge

5.3.2 Average Electric Quantities

The most important electric quantities are the average voltage, current and power. The voltage can be calculated in the following way:

$$V_{electric}^{avg} = \frac{1}{N} \sum_{i=1}^{i=N} \frac{(V_{electric,i} + V_{electric,i-1})(t_i - t_{i-1})}{2} \quad (5.9)$$

The current and the power can be calculated in the same way.

5.4 Resume

As a final result of a simulation, it is possible to extract dozens of data that can be treated in several ways, depending on the information that an engineer wants to have about the car. However, some values were basically important to understand the performance characteristics of a car. These values can be condensed into a table, as shown in the following: These data can serve as input to other departments (for electrical and mechanical sizing, for example) and serves to characterize better each iteration/configuration of the car. However, more a step can be taken, which will be shown in the following section.

5.5 Sensitivity Analysis

A simple way to compare several iterations or configurations of a car is through sensitivity analysis. Basically, the analysis is done by simulating the car in several configurations (or with different design parameters) and comparing the simulations. The comparison parameter can be any chosen by the engineer. In a race context, this frequently is the lap time, but other parameters can be

Table 5.1: Resume of the Simulation

Parameter	Value
Run Time [s]	76.39
Average Speed [m/s]	16.60
Maximum Speed [m/s]	29.07
Maximum Longitudinal Acceleration [m/s ²]	8.47
Maximum Longitudinal Deceleration [m/s ²]	14.58
Maximum Lateral Acceleration [m/s ²]	15.07
Energy Consumption [Wh]	703.30
Average Current [A]	139.23
Average Voltage [V]	225.61
Average Power [kW]	33.15

chosen (such as maximum velocity in the lap, energy consumption or average electric quantities). Normally this analysis is done in a conceptual stage, but this can be done also looking back at the design choices and developments. Despite the impossibility of changing them, it is important for an engineer to know what impact the design choices had on the car's performance and quantify it.

These simulations were done for the four events: Autocross, Endurance, Skidpad and Acceleration. The Autocross track is the same one used for the already shown simulations. The Skidpad and Acceleration layouts were explained in Chapter 2. For the Endurance simulations, the used track is from Formula Student Germany 2012. It is shown in Figure 5.18.

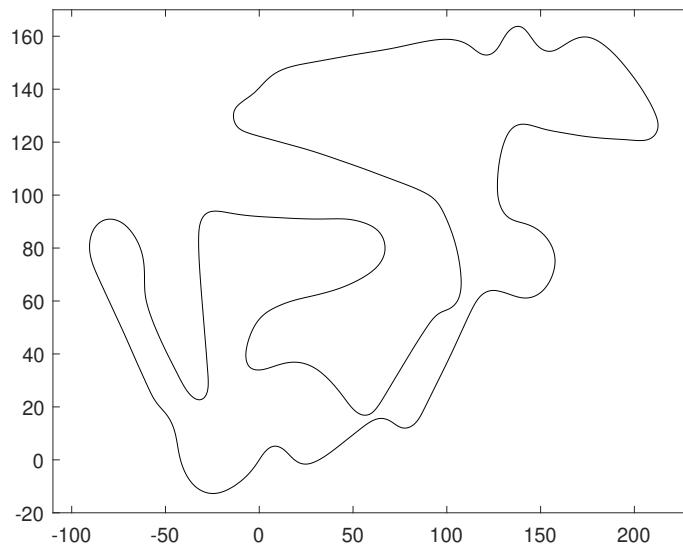


Figure 5.18: FSG Endurance 2012 Track

Moreover, for the Endurance event, some parameters were changed (Table 5.2). This reduction is necessary for several reasons. Firstly, endurance is a resistance event and, for this reason, it is necessary to decrease consumption to assure that a car has the necessary energy to complete the run. Besides, the torque need to be decreased for electrical and temperature reasons. A car working

in peak torque condition in an Endurance would have problems with the electrical components. As the power limit is low, the real impact on the acceleration isn't so big.

Table 5.2: Endurance Parameters Changing

Parameter	Value
Power Limit [kW]	19
Maximum Current [A]	160
Maximum Torque [Nm]	120

5.5.1 Mass Study

The mass analysis was done for the four events: Autocross, Endurance, Skidpad and Acceleration. Despite the changes for the Endurance case, the simulation conditions for Autocross, Skidpad and Acceleration were the same as the results shown earlier, with the mass being the only variable parameter. The study was done for masses from 260 to 350 kg since these boundaries are realistic for a Formula Student car. For the Autocross event, the result is seen in Figure 5.19

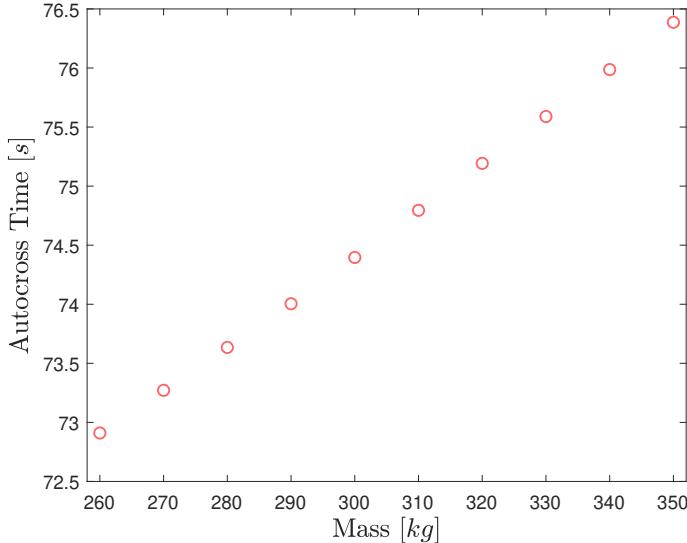


Figure 5.19: Sensitivity Analysis - Mass - Autocross

It is possible to see that the relation between the mass and the run time is practically linear. With this analysis, it is possible to see that an increase of 90 kg in mass would cause an increase of approximately 3.5 s in the Lap Time. This gives an average rate of 0.039 s for each kilogram added to the car (or lost from the car). This rate will be named "Performance Impact" (PI). Continuing, the results for the Endurance are shown in Figure 5.20.

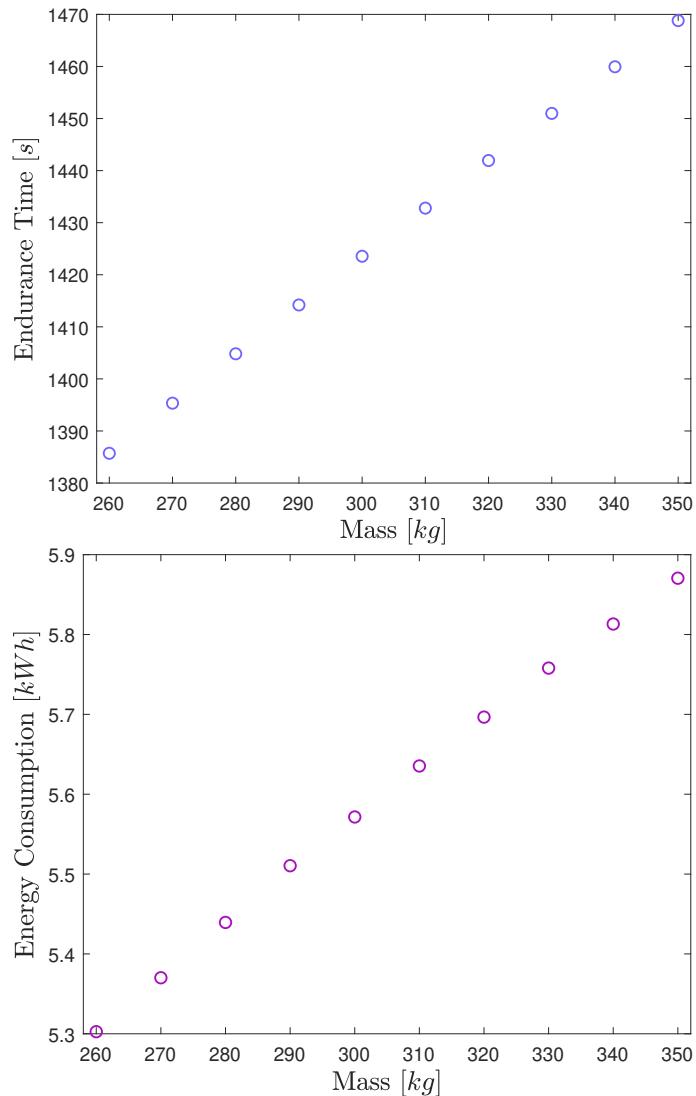


Figure 5.20: Sensitivity Analysis - Mass - Endurance

For Endurance, it was plotted the results of time and Energy Consumption. Looking for the time, the relation between the run time and the car's mass is very similar, being practically linear also. In this case, the impact of 90 kg of mass in the Endurance Time is around 84 s, resulting in a Performance Impact of 0.933 s/kg. It is interesting to compare this rate with the one obtained in the Autocross. The length of this Endurance track is approximately 17x the length of the Autocross track. So, the rate obtained from the Endurance can be converted to a rate equivalent to the one of Autocross:

$$PI_{mass,AX}^{eq} = \frac{PI_{mass,EN}}{17} = 0.0549 \text{ s/kg} > 0.039 \text{ s/kg} = PI_{mass,AX} \quad (5.10)$$

With this analysis, it is possible to see that the mass, in average terms, has more impact on the Endurance (with these electrical conditions) than on the Autocross. In other words, work on mass loss will give more time loss in Endurance than in Autocross.

About Energy Consumption, the relationship between consumption and mass is almost linear also. This consumption is very important to the calculation of the Efficiency Factor (explained in Chapter 2 and shown in Chapter 6), but another tuning can be done. The battery of the FSFEUP car has around 7 kWh of energy. The challenge of Endurance is to assure that the car has the capacity to complete the event. With a lighter car, it is possible to increase the power limitation and it will give an even better result in terms of time. This analysis won't be done here, but it is a good exercise to estimate the best power limitation on the car. But continuing, and looking for the Skidpad analysis, Figure 5.21 shows the results.

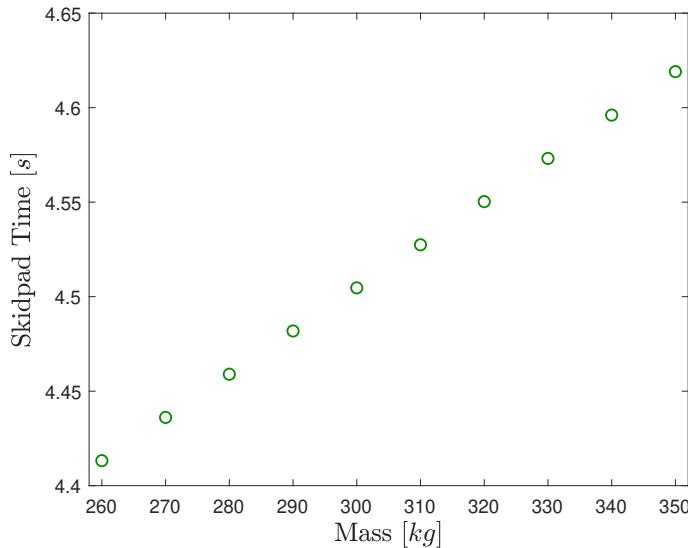


Figure 5.21: Sensitivity Analysis - Mass - Skidpad

Different from the two first analyses, the Skidpad calculation is a simple steady-state calculation (since the radius is constant). However, the impact can be seen in the same way. The PI for this case is approximately 0.0023 s/kg. However, the analysis can be related to other physical quantities. The Skidpad time is a way to quantify the Lateral Acceleration:

$$a_y = \frac{V_x^2}{R} = \frac{(2\pi R)^2}{t_{skidpad}^2 R} = \frac{4R\pi^2}{t_{skidpad}^2} \quad (5.11)$$

Knowing that the radius of this track is 8 m and through the calculated times, it is possible to find the equivalent lateral accelerations and calculate an equivalent rate for it. In this case, each kilogram lost causes an increase of 0.0156 m/s² on the maximum lateral acceleration. The difference in these values between the 260 kg car and the 350 kg car is 1.4061 m/s². To end, the final result is from the Acceleration event, shown in Figure 5.22.

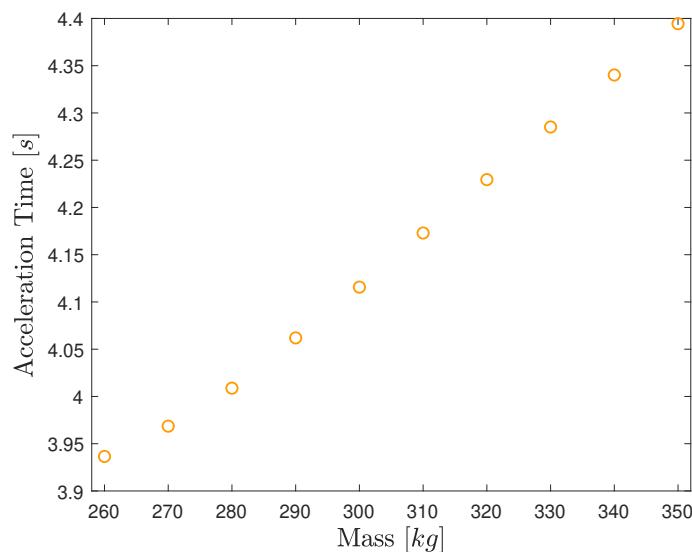


Figure 5.22: Sensitivity Analysis - Mass - Acceleration

This result is, similar to the others, almost linear. However, there's a little difference in the beginning. A non-linear behaviour is seen until 290 kg. A possibility for this behaviour is the difference between the grip limit and the motor limit. For low masses, the car begins to be more susceptible to the grip limit case, when the motor can give more tractive force than the limit of the tires. This effect occurs because, with the decrease of the mass, the normal force on the tires decreases and, with this, the grip capacity decreases also. When it occurs, the change in mass doesn't change so much the results of the acceleration, since the motor will need to work in a limited condition to avoid skidding, which means that a certain amount of torque is being wasted. However, doing the same exercise as the one done in the last cases, the Performance Impact on the mass in the acceleration, for this car, is about 0.00506 s/kg. The explanation of these time values for the Formula Student context will be shown in the next chapter.

5.5.2 Gear Ratio Study

The same simulations were done for the gear ratio. In this case, the mass was fixed at 350 kg and the gear ratio was changed from 3:1 until 7:1. For the Autocross case, Figure 5.23 shows the run time.

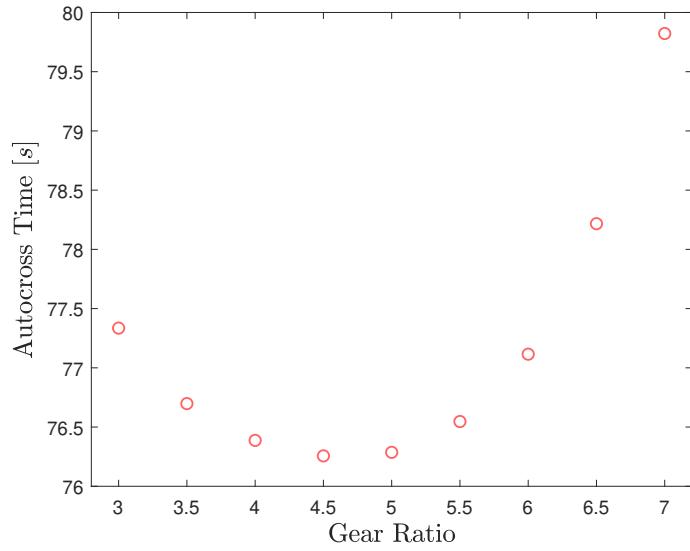


Figure 5.23: Sensitivity Analysis - Gear Ratio - Autocross

The time distribution for the gear ratio analysis is completely different than for the mass analysis. In this case, Instead of linear, the distribution is U-shaped, with a global minimum around the 4.5:1 gear ratio. This value depends on the car and mainly on the motor. This means that if the motor configurations were changed, these simulations need to be remade and possibly the result would be very different. The balance for the best case isn't easy to understand. A car with a lower gear ratio has a bigger maximum speed, but less torque. In tracks with fast corners or long straights, probably these configurations would be better. However, if the track has several braking points and slow corners, it is necessary to have more torque, so, a bigger gear ratio would interest more. However, big gear ratios would make the car achieve its maximum motor speed and, despite the big amount of torque, wouldn't be possible to cross a specific limit on the speed. It is for this reason that this plot is U-shaped. For the Endurance case, it is possible to see the results for the time and energy consumption in Figure 6.1.

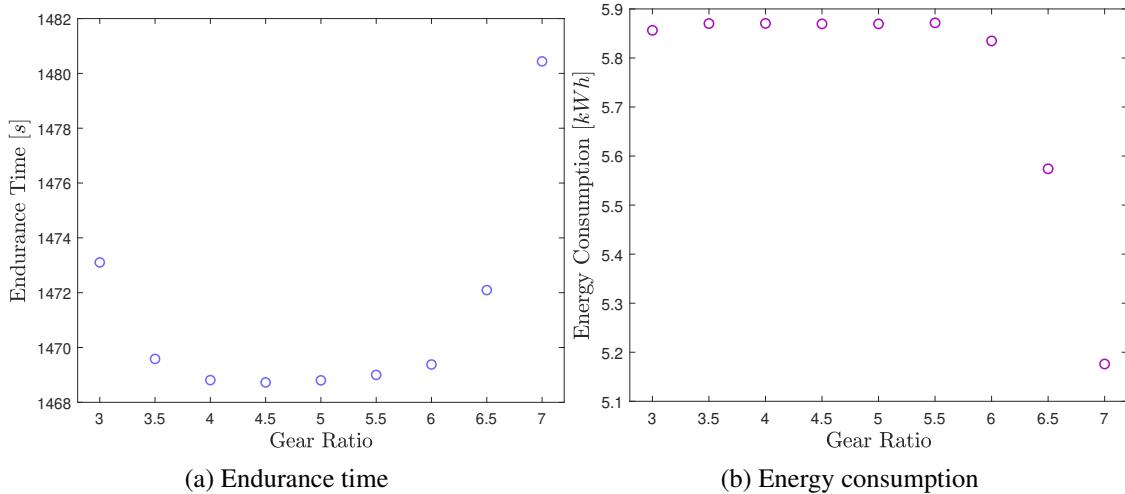


Figure 5.24: Sensitivity Analysis - Gear Ratio - Endurance

The result is very similar to the one obtained in the last figure, but different from the Autocross, the Endurance results don't change so much between the 4:1 and 5:1 gear ratios. Probably this difference occurs by the power limitation of the car. A bigger gear ratio, despite giving more torque, would demand a bigger motor speed to maintain the same car speed, but with the low power limit, this increase in the motor speed makes the power increase also. So, the region between the 4:1 and 5:1 probably causes a trade-off in the electrical system that makes the result practically the same, whether in the time or the Energy Consumption. For gear ratios above 6:1, it is a big increase in the Endurance Time and a decrease in Energy Consumption. This type of effect needs to be investigated mainly in the Efficiency case, but this analysis will be shown after.

Continuing the results, it is possible to see the Skidpad in Figure 5.25. The Skidpad time doesn't change, since there's no impact of the torque in this event (just to accelerate the car until the constant speed). However, it is important to refer that big gear ratios would cause a speed limit that could affect the Skidpad time, but this case is practically inapplicable in reality. For the Acceleration case, the results are shown in Figure 5.26.

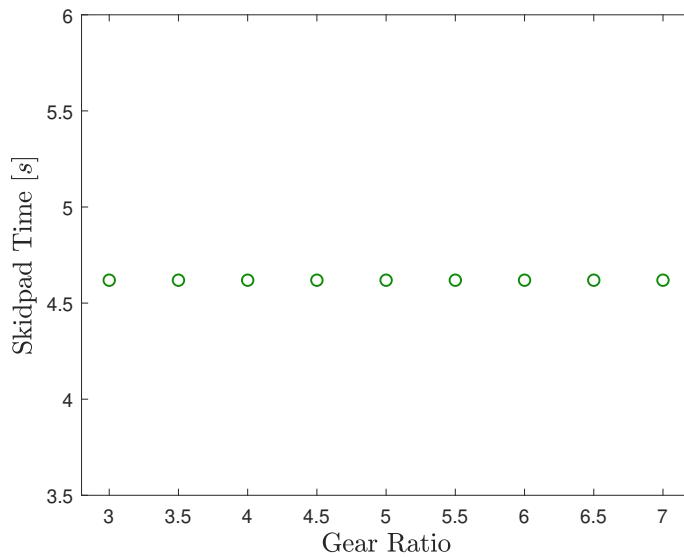


Figure 5.25: Sensitivity Analysis - Gear Ratio - Skidpad

The shape of Figure 5.26 is more similar to the Autocross case. However, there's an emphasis between the 4.5:1 and the 5:1 gear ratios, with a more discrepancy in the other values. It is also interesting to see that, in Figure 5.23, lower gear ratios affect less the final times than the bigger gear ratios, but in this case, lower gear ratios affect more than bigger ones. This points to the importance of torque in this event.

The mass study, despite being very important for the design changes analysis, tries to quantify the gain of performance of a known improvement (reducing the mass is, in the boundaries presented, better every time). However, the gear ratio study gives an answer to a completely unknown choice, that is the gear ratio itself. For a specific car with a specific motor, the different

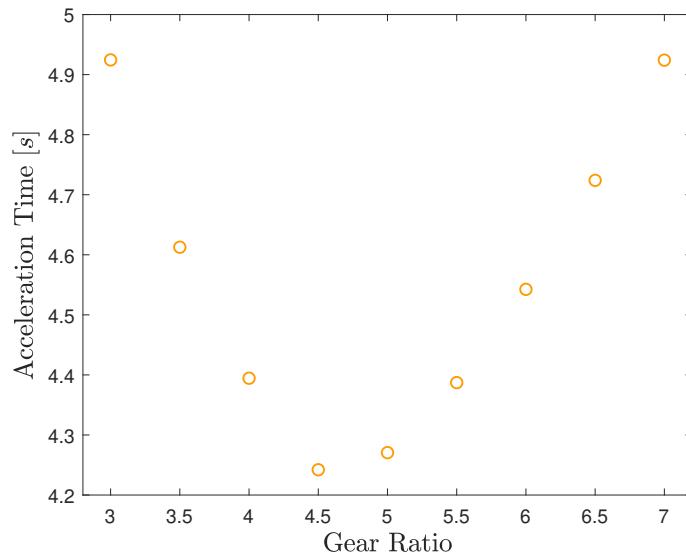


Figure 5.26: Sensitivity Analysis - Gear Ratio - Acceleration

situations that a Formula Student car is submitted to (tracks with slow and fast corners, the acceleration event and efficiency analysis) make this choice very difficult without simulations. So, the value of the software, in this case, is expressed in the fact that it is a guide for the team in the design stage. Now that the physical data is available, it is necessary to investigate if the physical achievements have a real impact on the competition results. This analysis and discussion will be shown in Chapter 6.

Chapter 6

Discussion

After the presentation of the results, the objective of this chapter is to have a brief discussion about the overall results given by this work. In Chapter 5, the results were plotted and explained, mainly the results extracted from the state of the car. The chapter ended with the exposition of some comparative quantities in an analysis named "sensitivity analysis". However, the results obtained, despite with physical meaning, have low value without some interpretation of their significance and, more than this, their applicability to reality. The context of all this work is the Formula Student competition. So, it is necessary to return to this context to evaluate the results and delineate conclusions.

The obtained result can be applied in two main contexts, as explained earlier. One of these is to take decisions in order to optimize the design of the car. The other is to understand what is the actual performance of the car and predict the results of the competition. A third secondary applicability can be found, that is knowing how it would be if some parameters were changed. This is secondary since it belongs to the first case, but in a later moment, when the design is occurring and there's the possibility to change something in order to optimize more.

It is clear that some decisions have just physical reasons. A car with low a centre of gravity height is more stable, for example. The optimal Gear Ratio would be those that give the best result in the Autocross/Endurance/Acceleration events. This is a physical reason. However, an engineer needs to think about "how to achieve this goal" and frequently the answer will be "with time and money". In the Formula Student context, another common answer appears, that is "with experience and know-how". So, mainly for a beginning team, the same engineer needs to think about "it is worth it?". This question is subjective, but the main goal of the Lap Time Simulation is to help to answer it.

Returning to Chapter 2, it was explained the different events that exist in the Formula Student. Together with the explanation, it was shown the different scoring calculations for each one. It is possible to continue the analysis of the parameters change and translate the physical results

to scoring and positions of the competition. For this, the obtained results were compared to the Formula Student Germany 2022 results and the scoring was calculated through them. The only modification was the applicability of Safety Factors to the results since the simulator assumes a perfect driver in all the moments and the track used wasn't exactly the track from FSG 2022. The safety factors are:

$$SF_{AX} = 0.9 \quad (6.1)$$

$$SF_{EN} = 0.8 \quad (6.2)$$

$$SF_{SP} = 0.9 \quad (6.3)$$

$$SF_{AC} = 0.9 \quad (6.4)$$

The subscripts AX, EN, SP and AC are Autocross, Endurance, Skidpad and Acceleration, respectively. So, the corrected times are calculated in the following way:

$$T^{corrected} = \frac{T^{original}}{SF} \quad (6.5)$$

6.1 Mass Analysis

For the results of the Mass Analysis obtained in the last chapter, the conversion to the scoring can be seen in Figure 6.1a. The respective positions are shown in Figure 6.1b.

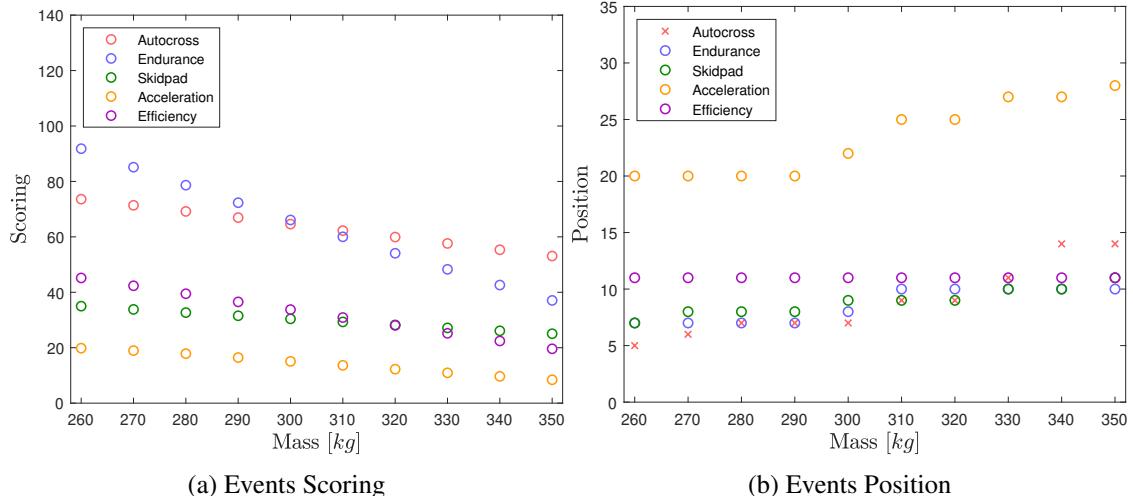


Figure 6.1: Mass Analysis

These plots show very important information to know about the Formula Student context. A relative increase in performance in terms of event time doesn't represent the same increase in terms of results (in this case, scoring and position). Looking for Figure 6.1a, it is possible to see that the difference of the mass in the Skidpad, for example, doesn't cause a big difference in the scoring. But looking for Endurance, the difference between the 260 kg car and the 350 kg car means a

difference of almost 60 points.

Moreover, looking at Figure 6.1b, it is possible to see that a change from 260 kg to 290 kg doesn't change the position for the Acceleration and Skidpad events. This information is crucial to prevent the team from making useless design changes. If a team with a car with 290 kg of mass desires to reduce 20 kg to try to reach the top 15 in the Acceleration event, this reduction cannot be the best option to achieve it. However, this effort may be more interesting for a team that wants to reduce from 310 kg to 290 kg to achieve the top 20.

However, the change in the parameters affects the car in all the events. So, the variation in the overall scoring is shown in Figure 6.2.

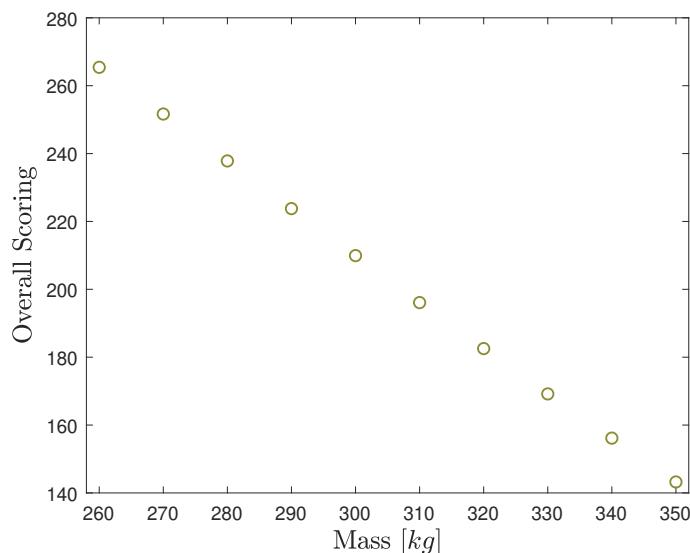


Figure 6.2: Overall Dynamic Scoring - Mass Analysis

It is possible to see that the relation between the scoring and the mass is almost linear. This occurs because, in all the events, the relation between the performance and the mass is linear. In the boundaries treated in this analysis, it is possible to say that more mass is worst for the car. Assuming this behaviour, it is possible to assume a Scoring Impact of approximately 1.37 pts/kg. This means that a loss of 1 kg would give 1.37 more points for the team.

Aiming in objectives to the team, when the goal is the Overall Position, the Overall Scoring is the one that really matters for it. Different from the analysis done above, it is clear that a loss of 20kg, in this case, can change significantly the final result, since despite the individual positions in some events doesn't be changed, the overall position can change. Taking the results for the 350 kg car and assuming the achievement of 70% of the Static Events (see Figure 2.1), the Overall Scoring of the team would be around 370 points, which gives the 17th position in the 2022 competition (among 66 teams). If the mass was 50 kg lesser, the scoring would increase to 437 points, giving the 11th position.

6.2 Gear Ratio Analysis

For the Gear Ratio analysis, the same transformation from time values to scoring can be done, resulting in the Scoring Plot (Figure 6.3) and in the Position Plot (Figure 6.4). These plots were made for each one of the 4 events, accounting also for efficiency.

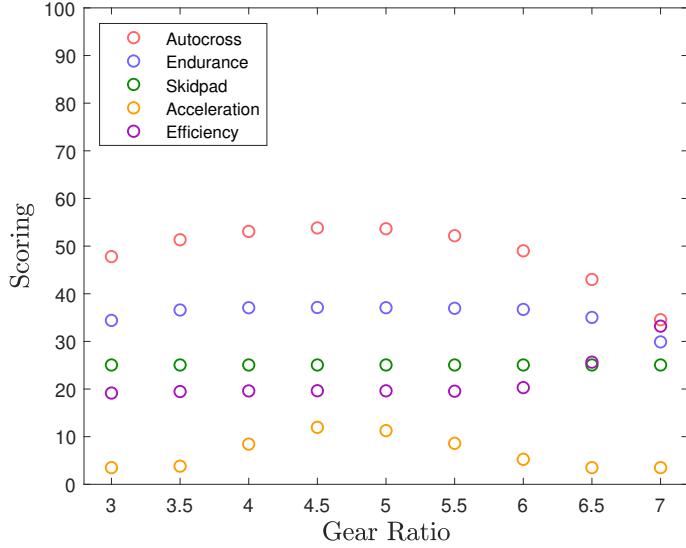


Figure 6.3: Events Scoring - Gear Ratio Analysis

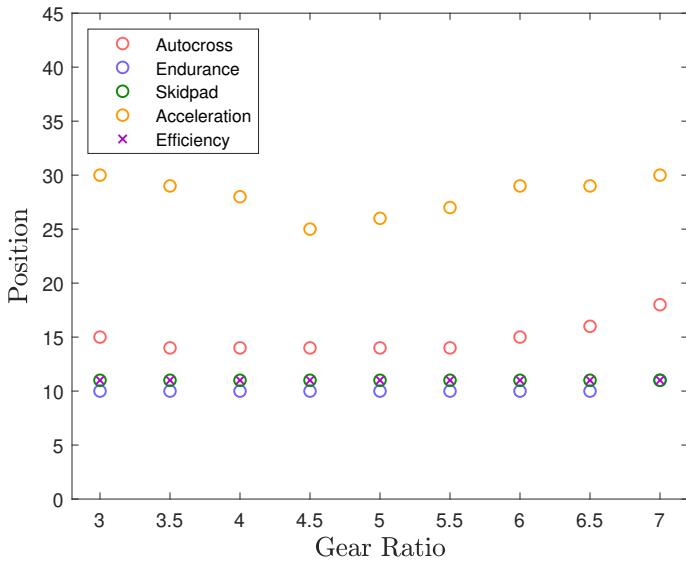


Figure 6.4: Events Position - Gear Ratio Analysis

Figure 6.3 shows that the most relevant event for the overall scoring, in this case, is the Autocross, followed by Endurance and Skidpad. The event that less contribute is the Acceleration, which for gear ratios between 3:1-3.5:1 and 6.5:1-7:1 the scoring is near to the minimum assured by the competition (3.5 points). However, for a Gear Ratio of around 4.5:1, the scoring is near 12 points. Despite the low overall impact, this difference (of approximately 8.5 points) shows that

the Acceleration is relevantly affected by changes in the Gear Ratio. Another relevant event is the Autocross, Which has a difference of approximately 7 points between the Gear Ratios of 3:1; 6:1 and the "optimal" 4.5:1.

Besides, it is possible to see that exists a great sensitivity for Gear Ratios above 6.5:1. This sensitivity occurs because of the great variations seen in Figures 5.23 and 6.1, which show a significant increase in the run times. The only exception is Efficiency, which increases significantly (more than 10 points). This effect is a consequence of the less energy consumption shown in Figure 6.1. In terms of positions, the variations aren't so big. For Autocross there are low changes (which occur for low Gear Ratios and mainly for the ones above 5.5:1) and for the Endurance, the result is almost constant, except for a Gear Ratio of 7:1. Bigger variations can be seen in the Acceleration event, mainly near the "optimal", with a total increase of 5 positions.

However, as analysed earlier, it is important to see what happens to the overall result. With the seen until now, it is safe to argue that a Gear Ratio near 4.5:1 is the best solution, but how distant a different solution is from the best? So, it is interesting to plot the overall scoring, seen in Figure 6.5.

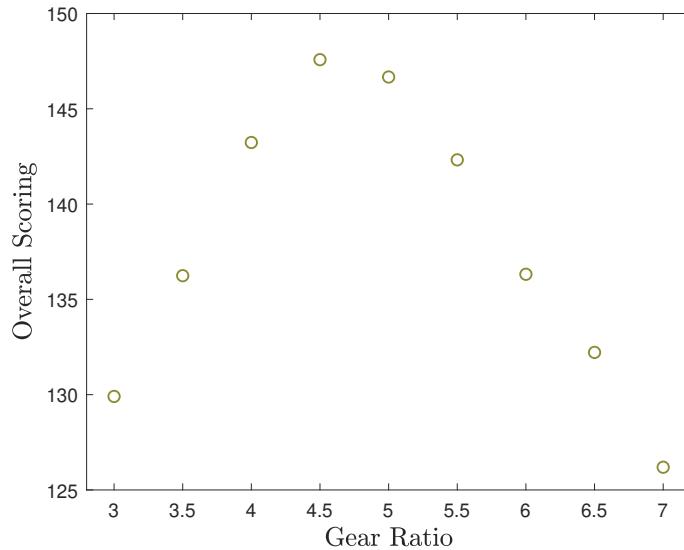


Figure 6.5: Overall Dynamic Scoring - Gear Ratio Analysis

In the overall scoring, the already-known result is consolidated. The best option (in this analysis) is a gear ratio of 4.5:1. It is clear that a new analysis with more intermediate values needs to be done to find a more specific Gear Ratio. However, looking at the configuration with 5:1, it is possible to see that the difference is just around 1 point. This conclusion is important to understand that if there is any design constraint that doesn't allow the specific optimal Gear Ratio, there's a safety margin in which the Gear Ratio can be chosen without a big impact on the overall result.

6.3 Other Analyses

One of the biggest gains of the development of this simulator is the possibility of having great variability in the parameters and, with this, to study the changing of each one. As explained earlier, it is clear that some analyses have limitations since the model isn't transient, and the driver is perfect. However, the sensitivity of the simulator for several different parameters is relevant for its importance and applicability. So, several studies were done for different variations, such as CoG height, CoG longitudinal position, different aerodynamics configurations and anti-roll bars stiffness. The results are shown in Appendix B. For a Formula Student context, this variability doesn't affect just the choice of components and design parameters but helps the team to understand the different effects that occur in the car. There are several books that talk about the influence of the parameters on car behaviour, and the simulations can serve as know-how and practical applicability to solidify these theoretical concepts. With this, the initial motivation for the development of the own team's simulator is satisfied, since there's more than just parameterizing a specific car and running it in a track. The development of the simulator provided a solid understanding of the effects that occur in a run car and this is extremely valued in the competition.

Chapter 7

Conclusions and Future Works

After the presentation of all the parts of the development of a Lap Time Simulation, this chapter will finish the work, evaluating the obtained result according to the initial objectives and delivering a checkpoint for future works to be developed.

7.1 Satisfaction of the Goals

The main goal of the work was the development of solid software to evaluate vehicle dynamics in the Formula Student context. This software needed to be easy to use and flexible, with the vehicle model being changed in a simple way and the possibility of change easily the used track. One of the requirements was the not necessity of deep code changes to change the conditions of the simulation. Moreover, it was necessary that the software give an overall estimation of the four main events of the competition.

Fortunately, all these requirements are satisfied with some limits. The software is easy to use and a MATLAB Basic-User can run simulations and extract plots in an easy way. Besides, the software can be easily changed, with several tracks pre-processed included in it and other tracks can be easily implemented through a *.mat* type file. The results obtained have a good correlation with reality and as argued in the two last chapters, are very relevant to all the parts of the competition, the dynamics or static events.

The limits of the obtained solution belong to some errors and difficulties that can appear when non-physical inputs are given. A studied example was the introduction of a torsional stiffness 16x bigger in the rear axle than the front axle, which caused a non-realistic result (and probably caused many difficulties for the solver to achieve feasible solutions). These limits weren't completely perceived and until now they depend on the analysis of the output data. However, during the use of the software, these limits can be delineated and conditions can be created (such as error messages, and automatic stopping).

Moreover, the utilization of the simulator yet depends on MATLAB Code Environment, as well as the data extraction and treatment (plots). It is useful to create a GUI to facilitate the use of each team member (and not only by the Vehicle Dynamics department) and to extract the plots

in a very simple way. Moreover, the sensitivity analyses could be programmed easily in a graphic environment and the comparison plots could be generated automatically also.

Some technical improvements can be delineated in the same software, as the treatment for track data. It is possible to create some algorithms for the software to separate the straights from the corners and, with this, increase the calculation performance (since for the straights the optimization problem doesn't need to be used, it is just to assume that the car is accelerating). Moreover, the sensitivity analysis could be integrated into another optimization process, in which some parameters could be studied together (such as wheelbase, CoG position and track widths) to achieve the best overall configuration, whether in run time or in overall scoring. However, there are other improvements that can be studied in future works.

7.2 Future Works

The developed solution in this software used the Quasi Steady-State methodology, but the next step is the introduction of a transient model with an Optimal Control Problem Solver. Clearly, it is necessary to compare the solutions (with the same conditions) to delineate the differences. The most interest of a Formula Student team in transient simulators, besides the possibility of having more realistic results, is the connection to the structural design and simulation, since the limit cases (for structural sizing) were justly those in which the transients are present (in the beginning or in the end of corners, for example).

Walking parallel to this, the transient simulator can be improved to an integrated simulator, in which besides the dynamic model of the vehicle, there's the dynamic model of the structural parts of the car. The most relevant situation is for the suspension subsystem. The developed load transfers model assumed rigid parts and the only flexible parts were the springs and the tire. However, it is known that these parts deform during the corners and this can affect the car's dynamics (and can affect the structural design). Another example is the powertrain system since there are some shafts and transmission linkages with flexible joints that can affect the response for a torque input by the motor.

Another design requirement for the Formula Student is the design of the electrical components. These depend on the data extracted from simulators, as shown before. However, for better sizing, and aiming in an integrated simulation condition, it is possible to have a more specific electrical model, accounting for the motor, inverter, controller and battery working in transient conditions. This doesn't need necessary work during the simulation, but a model of this system can give information for the components and for the sizing of cooling systems, for example.

Finally, a more analytic step can be done, trying to obtain and create more methods and parameters for decision-making through the data obtained in these simulations. As argued, a physical result doesn't is the only way to evaluate the validity of a solution, but management variables, such as cost and time need to be taken into account and have much value for the static events on the competition. An example can be the estimation of the financial cost of design improvements to relate to the expected increase in the performance or competition scoring. This would return a

more realistic parameter to look at data and take decisions. To take concrete decisions from budget estimations would be a great starting point for the next iterations and maybe would be one of the most important improvements to the team.

References

- [1] Formula Student Germany Rules, 2023. Accessed on June 13, 2023.
- [2] Formula Student Germany - Resume of Day 4, 2017. Accessed on June 13, 2023.
- [3] H Scherenberg. Mercedes-benz racing cars—design and experience. *SAE Transactions*, pages 414–420, 1958.
- [4] *Computer Simulation of Watkins Glen Grand Prix Circuit Performance*. 1971.
- [5] S. Moss and L.E.W. Pomeroy. *Design and Behaviour of the Racing Car*. W. Kimber, 1963.
- [6] Daniele Casanova. On minimum time vehicle manoeuvring: The theoretical optimal lap. 2000.
- [7] Roy S Rice. Measuring car-driver interaction with the gg diagram. Technical report, SAE Technical Paper, 1973.
- [8] William F Milliken, Douglas L Milliken, and L Daniel Metz. *Race car vehicle dynamics*, volume 400. SAE international Warrendale, 1995.
- [9] Hans B Pacejka and Egbert Bakker. The magic formula tyre model. *Vehicle system dynamics*, 21(S1):1–18, 1992.
- [10] Marco Gadola, David Vetturi, Danilo Cambiaghi, and Luca Manzo. A tool for lap time simulation. Technical report, SAE Technical Paper, 1996.
- [11] Blake Siegler, Andrew Deakin, and David Crolla. Lap time simulation: Comparison of steady state, quasi-static and transient racing car cornering strategies. *SAE transactions*, pages 2575–2581, 2000.
- [12] Damien Brayshaw. *Use of numerical optimisation to determine on-limit handling behaviour of race cars*. PhD thesis, Cranfield University, 2004.
- [13] D.P. Kelly. *Lap time simulation with transient vehicle and tyre dynamics*. PhD thesis, Cranfield University, May 2008.
- [14] G. Perantoni and D.J.N. Limebeer. Optimal control for a Formula One car with variable parameters. *Vehicle System Dynamics*, 52(5):653–678, 2014. Publisher: Taylor and Francis Ltd.
- [15] Massimo Guiggiani and Massimo Guiggiani. Handling of road cars. *The Science of Vehicle Dynamics: Handling, Braking, and Ride of Road and Race Cars*, pages 213–322, 2018.

- [16] I. Kobayashi, K. Ogawa, D. Uchino, K. Ikeda, T. Kato, A. Endo, M.H.B. Peeie, T. Narita, and H. Kato. A Basic Study on Hybrid Systems for Small Race Car to Improve Dynamic Performance Using Lap Time Simulation. *Actuators*, 11(7), 2022. Publisher: MDPI.
- [17] D.A. Doyle, G. Cunningham, G. White, and J. Early. Lap time simulation tool for the development of an electric formula student car. *SAE Technical Papers*, 2019-April(April), 2019. Publisher: SAE International.
- [18] A. Heilmeier, M. Geisslinger, and J. Betz. A quasi-steady-state lap time simulation for electrified race cars. In *2019 14th International Conference on Ecological Vehicles and Renewable Energies, EVER 2019*. Institute of Electrical and Electronics Engineers Inc., 2019.
- [19] B. Lenzo and V. Rossi. A simple mono-dimensional approach for lap time optimisation. *Applied Sciences (Switzerland)*, 10(4), 2020. Publisher: MDPI AG.
- [20] M. Massaro and D.J.N. Limebeer. Minimum-lap-time optimisation and simulation. *Vehicle System Dynamics*, 59(7):1069–1113, 2021. Publisher: Taylor and Francis Ltd.
- [21] K. Tucker, R. Gover, R.N. Jazar, and H. Marzbani. A comparison of free trajectory quasi-steady-state and transient vehicle models in minimum time manoeuvres. *Vehicle System Dynamics*, 2021. Publisher: Taylor and Francis Ltd.
- [22] Arash Hosseiniyan Ahangarnejad and Stefano Melzi. Numerical analysis of the influence of an actively controlled spoiler on the handling of a sports car. *Journal of Vibration and Control*, 24(22):5437–5448, 2018.
- [23] Roberto Lot and Nicola Dal Bianco. The significance of high-order dynamics in lap time simulations. In *Proceedings of the 24th Symposium of the International Association for Vehicle System Dynamics (IAVSD 2015), Graz, Austria*, pages 17–21, 2015.
- [24] H. Yu, F. Cheli, and F. Castelli-Dezza. Optimal design and control of 4-IWD electric vehicles based on a 14-DOF vehicle model. *IEEE Transactions on Vehicular Technology*, 67(11):10457–10469, 2018. Publisher: Institute of Electrical and Electronics Engineers Inc.
- [25] Massimo Guiggiani. *Handling with Roll Motion*, pages 393–415. Springer International Publishing, Cham, 2018.
- [26] Dieter Schramm, Manfred Hiller, and Roberto Bardini. *Single Track Models*, pages 223–253. Springer Berlin Heidelberg, Berlin, Heidelberg, 2014.
- [27] M. Gabiccini, L. Bartali, and M. Guiggiani. Analysis of driving styles of a GP2 car via minimum lap-time direct trajectory optimization. *Multibody System Dynamics*, 53(1):85–113, 2021. Publisher: Springer Science and Business Media B.V.
- [28] A. Heilmeier, A. Wischnewski, L. Hermansdorfer, J. Betz, M. Lienkamp, and B. Lohmann. Minimum curvature trajectory planning and control for an autonomous race car. *Vehicle System Dynamics*, 58(10):1497–1527, 2020. Publisher: Taylor and Francis Ltd.
- [29] N. Dal Bianco, R. Lot, and M. Gadola. Minimum time optimal control simulation of a GP2 race car. *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*, 232(9):1180–1195, 2018. Publisher: SAGE Publications Ltd.
- [30] Massimo Guiggiani. *Vehicle Model for Handling and Performance*, pages 67–168. Springer International Publishing, Cham, 2018.

- [31] Sina Milani, Hormoz Marzbani, Ali Khazaei, and Reza N. Jazar. *Vehicles Are Lazy: On Predicting Vehicle Transient Dynamics by Steady-State Responses*, pages 3–38. Springer International Publishing, Cham, 2020.
- [32] Massimo Guiggiani. *Mechanics of the Wheel with Tire*, pages 7–65. Springer International Publishing, Cham, 2018.
- [33] Rodrigo de Oliveira Santos. The absolute guide to racing tires - part 1 - lateral force, Jul 2021.
- [34] Kiumars Jalali, Thomas Uchida, Steve Lambert, and John McPhee. Development of an advanced torque vectoring control system for an electric vehicle with in-wheel motors using soft computing techniques. *SAE International Journal of Alternative Powertrains*, 2:261–278, 04 2013.
- [35] Eurocircuits - Formula Student Delft Team, Dec 2019. Accessed on June 3, 2023.
- [36] Wikimedia Commons - How to Approach the Apex of a Corner, 2023. Accessed on June 13, 2023.

Appendix A

Car Parameters

Table A.1: Vehicle Parameters

Parameter	Symbol	Value
Sprung Mass [kg]	m_s	290
Front Unsprung Mass [kg]	$m_{u,front}$	30
Rear Unsprung Mass [kg]	$m_{u,rear}$	30
Total Mass [kg]	m	350
Front Track Width [m]	t_1	1.2
Rear Track Width [m]	t_2	1.2
Wheelbase [m]	L	1.6
CoG Height [m]	h	0.28
Front Axle - CoG [m]	a_1	0.77
CoG - Rear Axle [m]	a_2	0.83
Tire Radius [m]	r_{tire}	0.254
Front Roll Center Height [m]	q_1	0.032
Rear Roll Center Height [m]	q_2	0.042
Roll Center Height [m]	q^b	0.0368
Front Roll Stiffness [Nm/rad]	k_{ϕ_1}	60390
Rear Roll Stiffness [Nm/rad]	k_{ϕ_2}	50015
Roll Stiffness [Nm/rad]	k_ϕ	110405
Tire Roll Stiffness [Nm/rad]	$k_{\phi_{1,2}}^P$	134200
Air Density [kg/m^3]	ρ	1.225
Downforce Coefficient	C_l	0.3425
Drag Coefficient	C_d	0.675
Frontal Area [m^2]	A	1
Centre of Pressure Position [m]	x_{CP}	0.8
Centre of Pressure Height [m]	h_{CP}	0
Static Steering [degrees]	δ_0	-0.430
Steering Ratio	τ_1	0.222
Ackermann Coefficient	ε_1	0.6
Gear Ratio	i	4
Mechanical Efficiency	η_{mec}	0.885
Brake Bias	BB	0.67

All the values of stiffness need to be calculated through others.

$$k_{\phi_i}^P = \frac{k_{tire} t_i^2}{2} \quad (\text{A.1})$$

$$k_{\phi_i}^{spring} = \frac{k_{spring} t_i^2}{2MR_i^2} \quad (\text{A.2})$$

$$k_i^{arb} = \frac{d_{i,arb}^4 E \pi}{64(1+\nu)L_{i,arb}} \quad (\text{A.3})$$

$$k_{\phi_i}^{arb} = MR_{i,arb}^2 k_i^{arb} \quad (\text{A.4})$$

$$k_{\phi_i}^s = k_{\phi_i}^{spring} + k_{\phi_i}^{arb} \quad (\text{A.5})$$

$$k_{\phi_i} = \frac{k_{\phi_i}^P k_{\phi_i}^s}{k_{\phi_i}^P + k_{\phi_i}^s} \quad (\text{A.6})$$

Table A.2: Suspension Parameters

Parameter	Symbol	Value
Tire Roll Stiffness [Nm/rad]	$k_{\phi_i}^P$	-
Tire Stiffness [N/m]	k_{tire}	186390
Spring Stiffness [N/m]	k_{spring}	35000
Front Motion Ratio	MR_1	1.45
Front Motion Ratio	MR_2	1.1
Anti-Roll Bar Stiffness [Nm/rad]	k_i^{arb}	-
Front Anti-Roll Bar Diameter [m]	$d_{1,arb}$	0.0150
Rear Anti-Roll Bar Diameter [m]	$d_{2,arb}$	0.0130
Young's Modulus [GPa]	E	210
Poisson Coefficient	ν	0.3
Front Anti-Roll Bar Length [m]	$L_{1,arb}$	0.228
Rear Anti-Roll Bar Length [m]	$L_{2,arb}$	0.426
Anti-Roll Bar Roll Stiffness [Nm/rad]	$k_{\phi_i}^{arb}$	-
Front Motion Ratio ARB	$MR_{1,arb}$	7.46
Rear Motion Ratio ARB	$MR_{1,arb}$	10.5263
Suspension Roll Stiffness [Nm/rad]	$k_{\phi_i}^s$	-

Appendix B

Sensitivity Analysis - Other Parameters

Besides the studies shown in the report, other types of analyses can be done and will be shown in this Appendix.

B.1 Center of Gravity Height

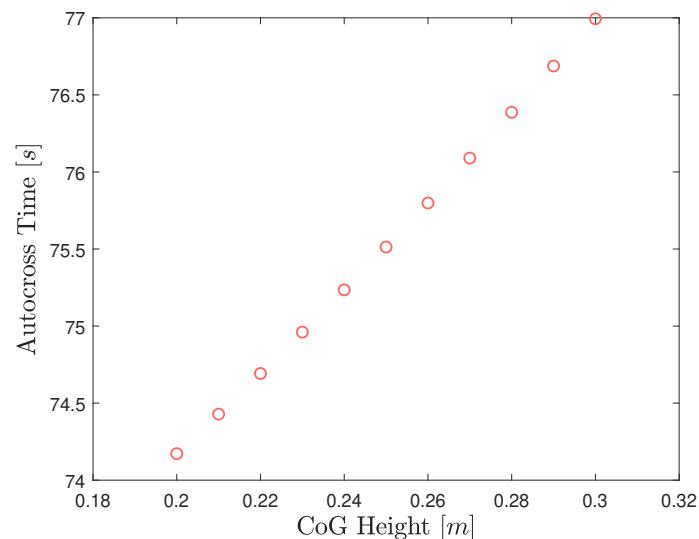


Figure B.1: Sensitivity Analysis - CoG Height - Autocross

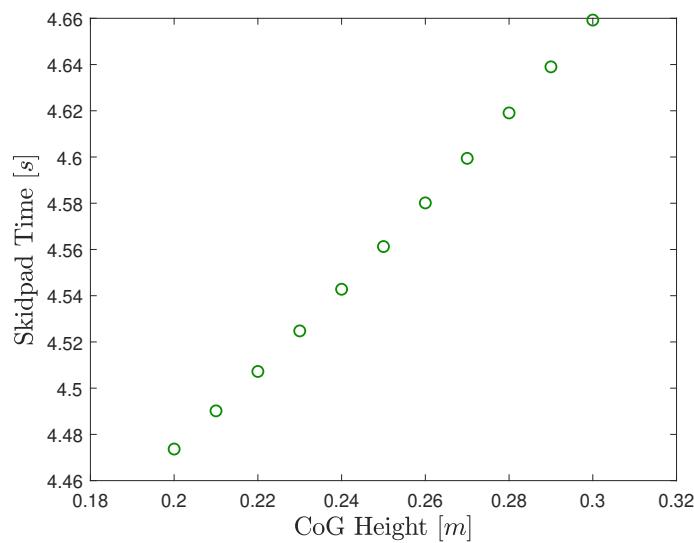


Figure B.2: Sensitivity Analysis - CoG Height - Skidpad

B.2 Center of Gravity Longitudinal Position

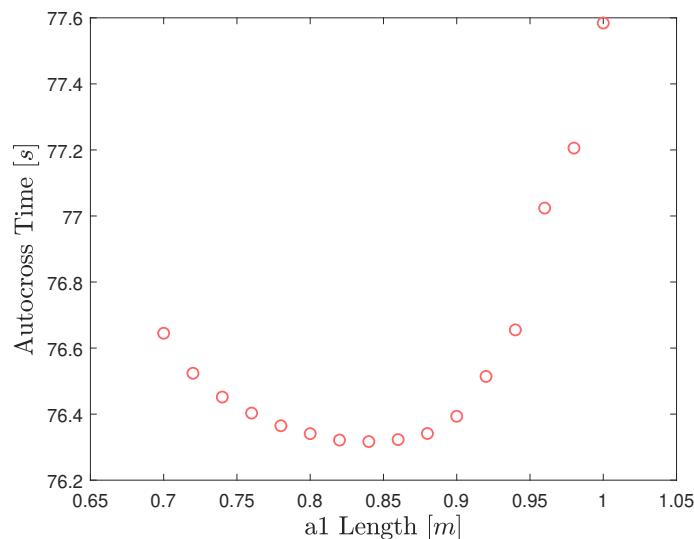


Figure B.3: Sensitivity Analysis - CoG Longitudinal Position - Autocross

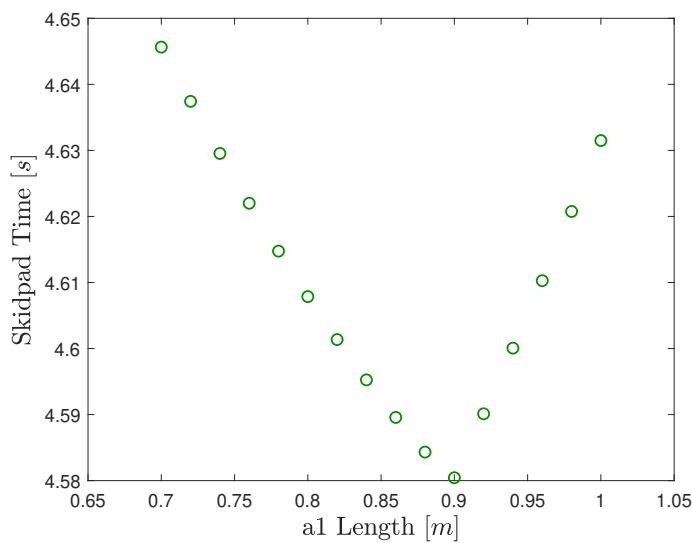


Figure B.4: Sensitivity Analysis - CoG Longitudinal Position - Skidpad

B.3 Aerodynamics Kit

Aero Type	C_d	C_l	Increase in weight (kg)
1 - No Aerodynamic Devices	0.675	0.3425	0
2 - Undertray	0.65	0.79	6
3 - Undertray, wings	0.66	1.28	10
4 - Undertray, wings with side plates	0.68	1.56	11
5 - Undertray, wings with side plates and flaps	0.9	2.34	13
6 - Undertray, wings with multiple elements	0.98	2.56	17

Table B.1: Aerodynamics Kits

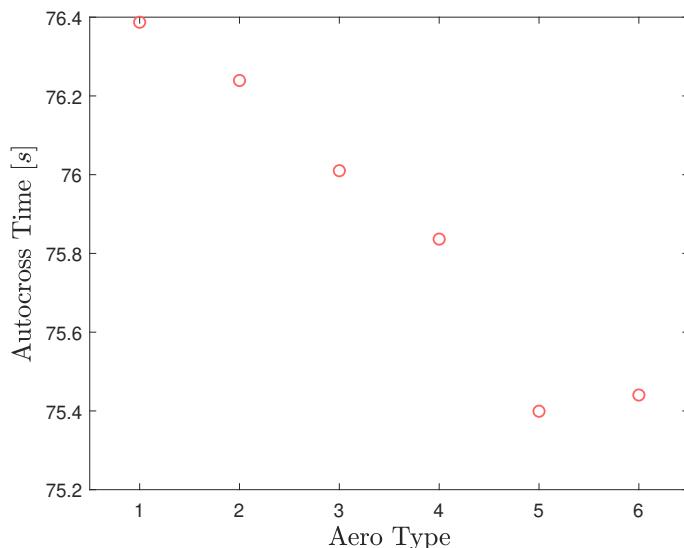


Figure B.5: Sensitivity Analysis - Aerodynamics Kits - Autocross

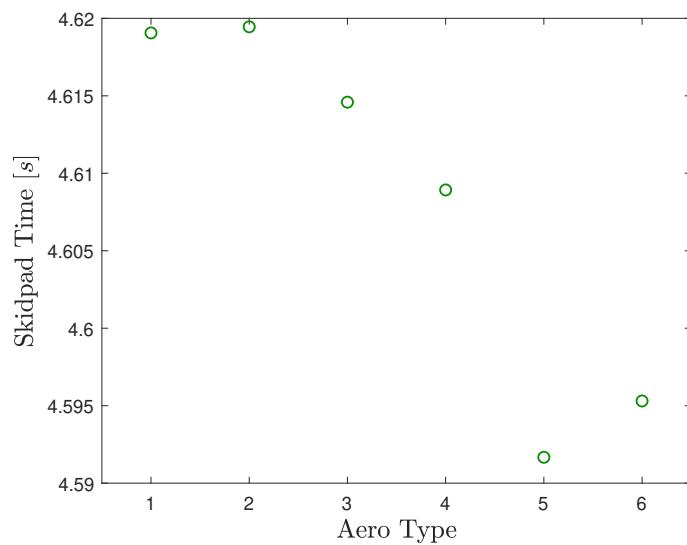


Figure B.6: Sensitivity Analysis - Aerodynamics Kits - Skidpad

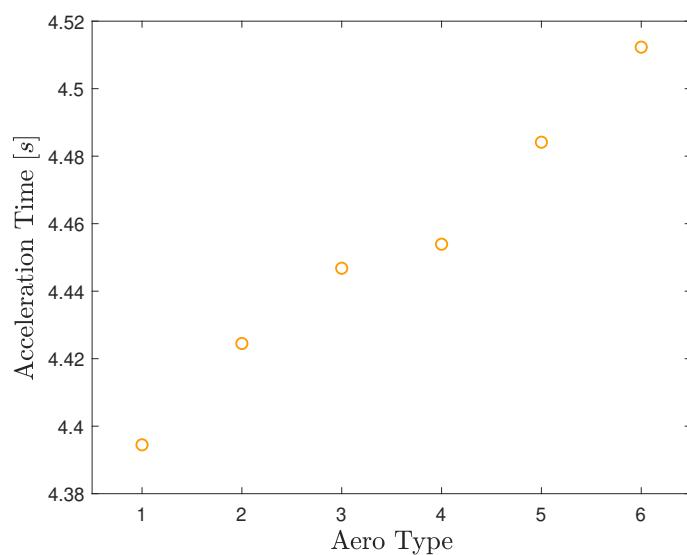


Figure B.7: Sensitivity Analysis - Aerodynamics Kits - Acceleration

B.4 Anti-Roll Bar Diameters

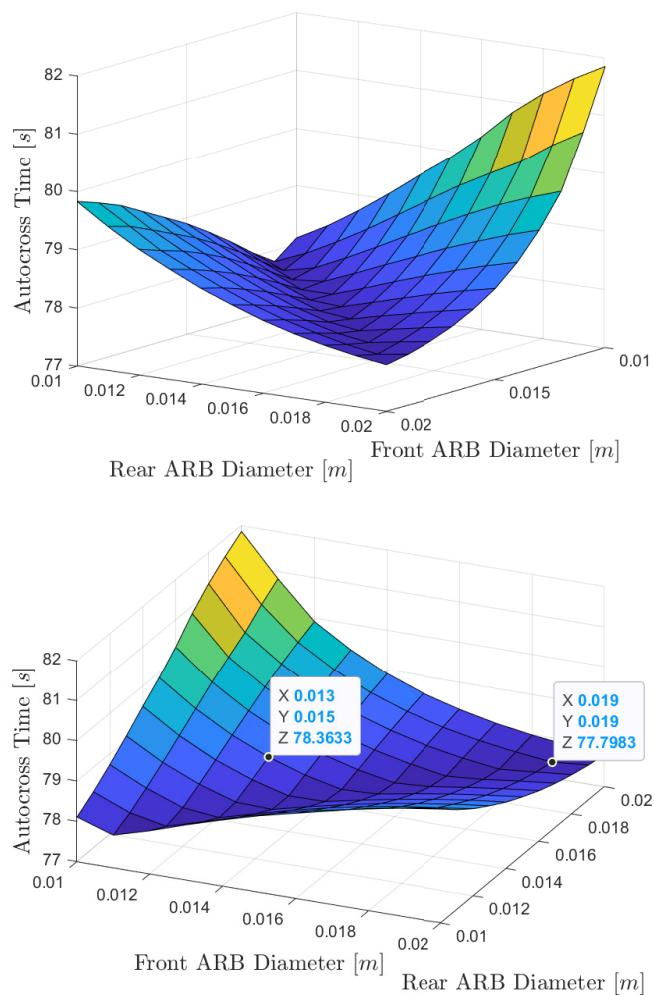


Figure B.8: Sensitivity Analysis - Anti-Roll Bar Diameters - Autocross

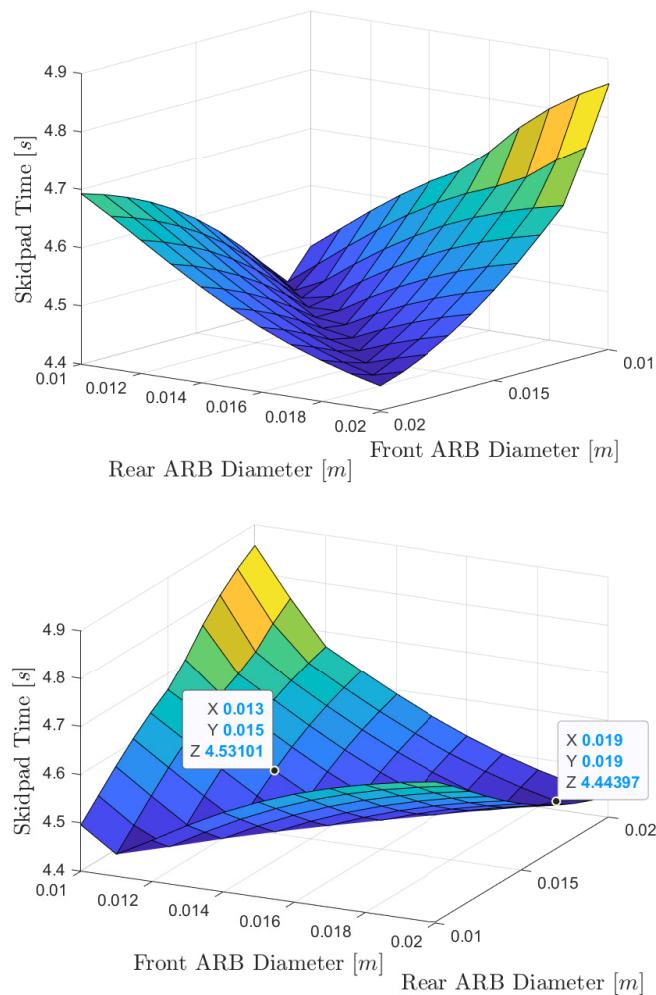


Figure B.9: Sensitivity Analysis - Anti-Roll Bar Diameters - Skidpad