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BSCS-F19-M63

Cal-II

Assignment # 1

Due Date : 22-4-20

1- Integrate $\int \frac{t^3 dt}{\sqrt{9-4t^4}}$

2- Integrate $\int \frac{x^3+x^2}{x^2+x-2} dx$

3- Integrate $\int (x+1)^2 e^x dx$

4- Find a vector 5 units long in ~~to~~ the direction opposite ~~as~~ to the direction of $\vec{v} = \frac{3}{5} i + \frac{4}{5} k$

5. write $\vec{u}$ as ~~a~~ ~~vector~~ the sum of a vector parallel to $\vec{v}$ and a vector orthogonal to $\vec{v}$.

$\vec{v} = 2i + j - k, \quad \vec{u} = i + j - 5k$

Date 18th April, 2020

ASSIGNMENT

* During Lock-Down .

By Muhammad Ammar
BSCS-F19-M-63

To Sir Mubashar
of Mathematics (calculus)

① Integrate the following :-

i) $\int \frac{x^3}{\sqrt{9-4x^4}} dx$

Solution:

$$I = \int \frac{x^3}{\sqrt{9-4x^4}} . dx$$

$$= \frac{1}{16} \int \frac{-16x^3}{\sqrt{9-4x^4}} . dx \quad , \text{Multiplying with } {}^{-16}/_{-16}$$

$$= \frac{1}{16} \frac{(9-4x^4)^{1/2}}{1/2} + c \quad , \because \int f'(x).f(x) = \frac{f(x)^{n+1}}{n+1}$$

$$= \frac{-\sqrt{9+4x^4}}{8} + c$$

ii) $\int \frac{x^3+x^2}{x^2+x-2} . dx$

Solution:

$$I = \int \frac{x^2+x^3}{x^2+x-2} dx$$

$$x^2+x-2 \overline{) \, x^3+x^2 \,} \quad x$$
$$\underline{-x^3 \pm x^2 \mp 2x}$$
$$2x$$

$$\Rightarrow I = \int x + \frac{2x}{x^2+x-2} dx$$

$$I = \int x + \frac{2x}{(x-1)(x+2)} . dx$$

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$$I = \frac{x^2}{2} + 2\int \frac{x}{x^2+x-2}\,.dx + c \quad \dots (F)$$

We take, $\dfrac{x}{x^2+x-2} = \dfrac{x}{(x-1)(x+2)} = \dfrac{A}{x-1} + \dfrac{B}{x+2}$

Multiplying with L.C.M.

$$x = A(x+2) + B(x-1)$$

If, $(x-1) = 0 \Rightarrow x = 1$

$\Rightarrow 1 = 3A \Rightarrow A = 1/3$ ✓

If, $(x+2) = 0 \Rightarrow x = -2$

$-2 = -3B \Rightarrow B = 2/3$ ✓

Now, (F) will be,

$$I = \frac{x^2}{2} + 2\int \frac{1}{3(x-1)} + \frac{2}{3(x+2)}\,.dx$$

$$= \frac{x^2}{2} + \frac{2\ln|x-1|}{3} + \frac{4\ln|x+2|}{3} + c$$

3). $\int (x+1)^2 \cdot e^x \cdot dx$

Solution:

$$I = \int (x+1)^2 \cdot e^x \cdot dx$$

Integrating by Parts,

$$I = (x+1)^2 \cdot e^x - \int 2(x+1) \cdot e^x \cdot dx + c$$

$$= (x+1)^2 \cdot e^x - 2\left[\cancel{2}(x+1) \cdot e^x - \int e^x\right] + c$$

$$= (x+1)^2 \cdot e^x - 2(x+1)e^x + 2e^x + c$$

$$= \left[x^2 + 2x + 1 - 2x - 2 + 2\right]e^x + c$$

$$= \left[x^2 + 1\right]e^x + c$$ ✓

Tuesday

Assignment Date. 21st April, 2020

4) Given :-

- Let, the required vector is $\vec{u}$.

$\vec{v} = (3/5)i + (4/5)k$, $\hat{v} = (-\hat{u})$

$\vec{u} = ?$, $|\vec{u}| = 5$

Solution :-

- Since, $\vec{u} = |\vec{u}| . \hat{u}$

$\Rightarrow \vec{u} = 5 . (-\hat{v})$

or $\vec{u} = (-5) . \left[\dfrac{\vec{v}}{|\vec{v}|} \right]$ ... (i)

- Now, Magnitude of $\vec{v}$ will be,

$\therefore \vec{v} = \dfrac{(3/5)i + (4/5)j}{\sqrt{(3/5)^2 + (4/5)^2}} = 1$

- Substituting the values in eq # (i),

$\vec{u} = (-5) . \left[\dfrac{3i}{5} + \dfrac{4k}{5} \right]$

$= -3i + (-4)k$

5) $\vec{v} = 2\vec{i} + \vec{j} - \vec{k}$, $\vec{u} = \vec{i} + \vec{j} - 5\vec{k}$

To Find:-

Express $\vec{u}$ as sum of vector parallel to $\vec{v}$ and a vector orthogonal to $\vec{v}$.

Solution:

- Since, $\vec{u} = 2\vec{i} + \vec{j} - \vec{k}$, $\vec{v} = \vec{i} + \vec{j} - 5\vec{k}$

$\Rightarrow \vec{u}.\vec{v} = (2\times 1)\cancel{\vec{i}} + \cancel{\vec{j}} + (-1 . -5)\cancel{\vec{k}}$

$= 2\cancel{\vec{i}} + 1\cancel{\vec{j}} + 5\cancel{\vec{k}} = 8$ units.

- Now, their magnitudes will be,

$|\vec{u}| = \sqrt{2^2 + 1 + 1} = \sqrt{6}$

$|\vec{v}| = \sqrt{1 + 1 + 25} = \sqrt{27}$

• Now,

$$\text{Proj}_v\,\vec{u} = \frac{\vec{u}\cdot\vec{v}}{|\vec{v}|^2}\,\vec{v}$$

$$= \frac{48}{36}\left(2\vec{i}+\vec{j}-\vec{k}\right) = \frac{4}{3}\left(2\vec{i}+\vec{j}-\vec{k}\right)$$

$$= (8/3)\vec{i} + (4/3)\vec{j} - (4/3)\vec{k}$$

$$\Rightarrow \vec{u} - \text{Proj}_v\,\vec{u} = \left(\vec{i}+\vec{j}-5\vec{k}\right) - \left[\frac{8\vec{i}}{3} + \frac{4\vec{j}}{3} - \frac{4\vec{k}}{3}\right]$$

$$= (-5/3)\vec{i} - (1/3)\vec{j} - (11/3)\vec{k}$$

• Now, the required form of $\vec{u}$ is,

$$\vec{u} = \text{Proj}_v\,\vec{u} + \left(\vec{u} - \text{Proj}_v\,\vec{u}\right)$$

$$= \left[\frac{8\vec{i}}{3} + \frac{4\vec{j}}{3} - \frac{4\vec{k}}{3}\right] + \left[\frac{-5\vec{i}}{3} - \frac{\vec{j}}{3} - \frac{11\vec{k}}{3}\right]$$

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