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BSCS_F19_M_63

Analysis of Algorithm

Question #1 - Part #3

Submitted to:

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2nd July, 2021

Final Parenthesization

Given:

- $A_{12 \times 35}$, $A_{35 \times 42}$, $A_{42 \times 60}$, $A_{60 \times 89}$ and $A_{89 \times 110}$

To Find:

- Optimal Parenthesization for Minimal Calculations

Solution:

- For the Calculations of Matrices, according to the laws of Dynamic Programming, we'll use the methods of **Memoization** and **Tabulation**.
- For that, we'll use **M** and **S** Tables to keep track of the calculations (Memoization) and will utilize them as needed.
- Below are the tables of **M** and **S**, with the table of Calculations we have to perform to get the Optimal Results.
 - ◆ **D1, D2, D3, D4, D5** and **D6** are the dimensions of the Matrices, which are getting multiplied. If they aren't listed, then Memoized Results are used

| M | 1 | 2 | 3 | 4 | 5 | S | 1 | 2 | 3 | 4 | 5 |
|---|---|--------|--------|---------|---------|---|---|---|---|---|---|
| 1 | 0 | 17,640 | 47,880 | 111,960 | 229,440 | 1 | | 1 | 2 | 3 | 4 |
| 2 | | 0 | 88,200 | 275,100 | 617,760 | 2 | | | 1 | 2 | 3 |
| 3 | | | 0 | 224,280 | 635,460 | 3 | | | | 1 | 2 |
| 4 | | | | 0 | 557,400 | 4 | | | | | 1 |
| 5 | | | | | 0 | 5 | | | | | |

| Multiplications | D1 | D2 | D3 | Memoized Value | Total Calculations |
|-------------------|----|----|-----|------------------|--------------------|
| 1 : 2 | 12 | 35 | 42 | | 17,640 |
| 2 : 3 | 35 | 42 | 60 | | 88,200 |
| 3 : 4 | 42 | 60 | 89 | | 224,280 |
| 4 : 5 | 60 | 89 | 110 | | 587,400 |
| 1 : (2 : 3) | 12 | 42 | 60 | 17,640 | 47,880 |
| (1 : 2) : 3 | 12 | 35 | 60 | 88,200 | 113,400 |
| 2 : (3 : 4) | 35 | 42 | 89 | 224,280 | 355,110 |
| (2 : 3) : 4 | 35 | 60 | 89 | 88,200 | 275,100 |
| (3 : 4) : 5 | 42 | 89 | 110 | 224,280 | 635,460 |
| 3 : (4 : 5) | 42 | 60 | 110 | 587,400 | 864,600 |
| 1 : (2 : 3 : 4) | 12 | 35 | 89 | 275,100 | 312,480 |
| (1 : 2) : (3 : 4) | | | | 17,640 + 224,280 | 241,920 |
| (1 : 2 : 3) : 4 | 12 | 60 | 89 | 47,880 | 111,960 |
| 2 : (3 : 4 : 5) | 35 | 42 | 110 | 635,460 | 797,160 |
| (2 : 3) : (4 : 5) | | | | 88,200 + 587,400 | 675,600 |
| (2 : 3 : 4) : 5 | 35 | 89 | 110 | 275,100 | 617,750 |

| Multiplications | D1 | D2 | D3 | D4 | D5 | D6 | Memoized Value | Total Calculations |
|-----------------------|----|----|-----|----|----|-----|------------------|--------------------|
| 1 : (2 : 3 : 4 : 5) | 12 | 35 | 110 | | | | 617,750 | 663,950 |
| (1 : 2) : (3 : 4 : 5) | | | | | | | 17,640 + 635,460 | 653,100 |
| 1 : 2 : (3 : 4 : 5) | 12 | 35 | 110 | 35 | 42 | 110 | 635,460 | 843,360 |
| (1 : 2 : 3) : (4 : 5) | | | | | | | 47,880 + 587,400 | 635,280 |
| (1 : 2 : 3) : 4 : 5 | 12 | 60 | 89 | 12 | 89 | 110 | 47,880 | 229,440 |

- Hence, Minimal Parenthesization will be: (((A1 x A2) x A3) x A4) x A5