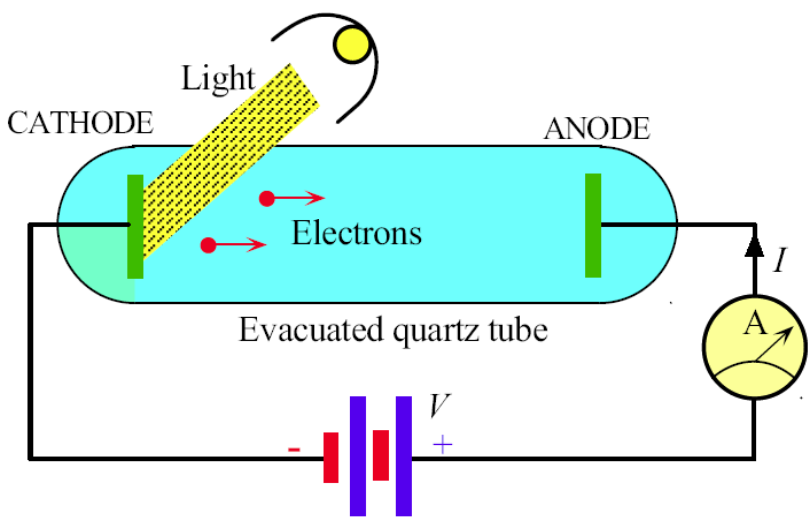
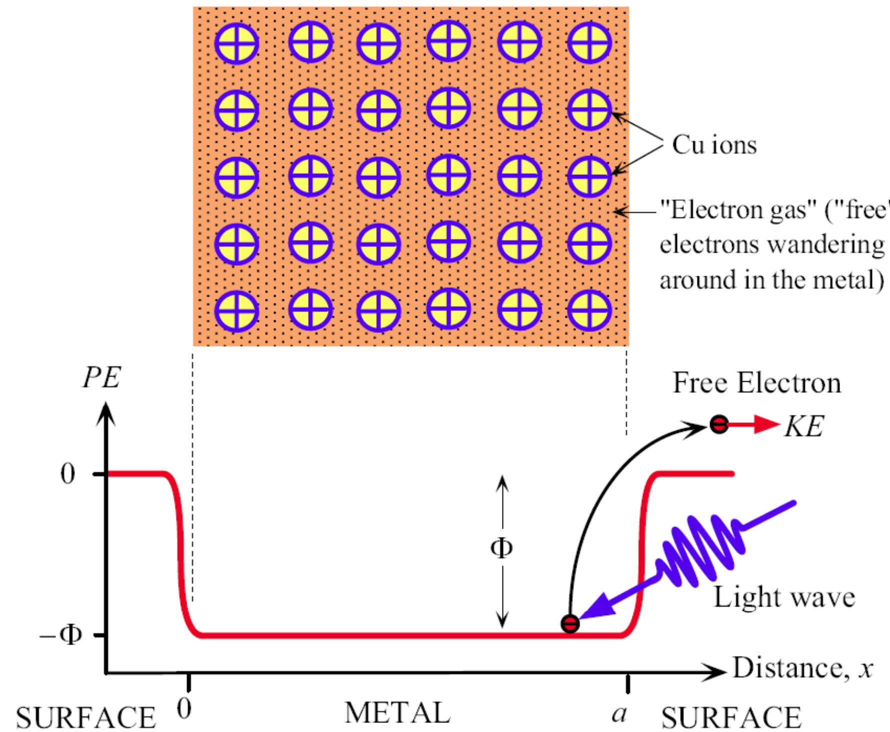
Lecture 10

Wave/Particle Duality: Compton Scattering, Blackbody Radiation, & the Nature of Confined Electrons

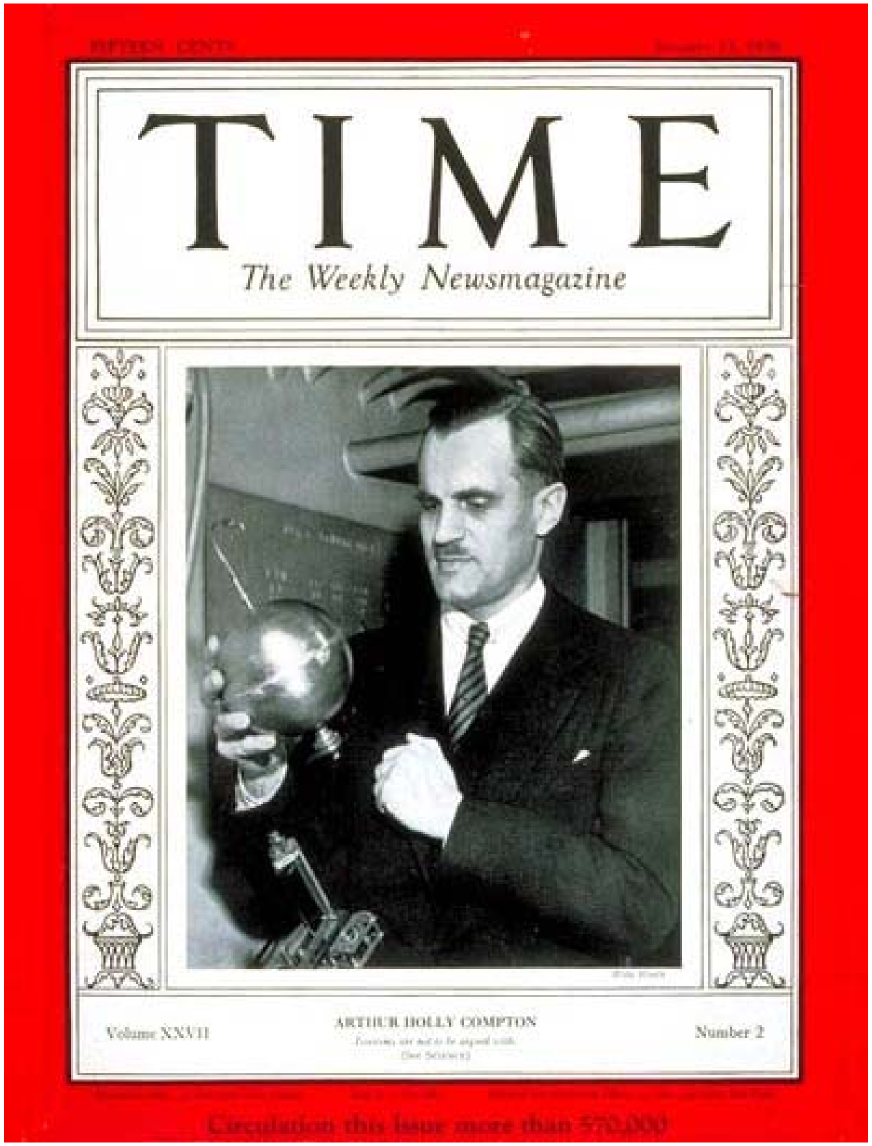
Photoelectric effect: Light is a particle with energy





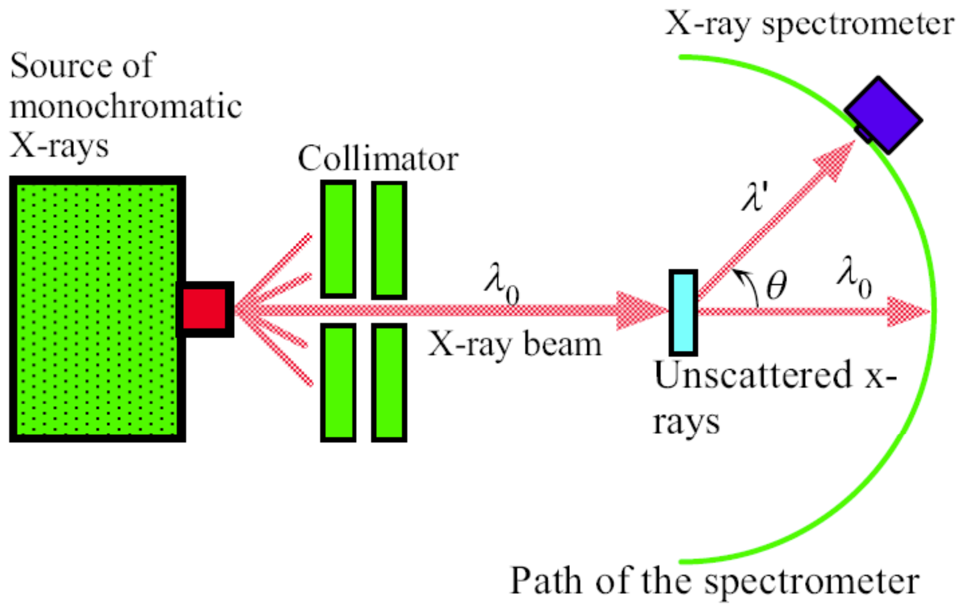
- Red photons – no current; blue photons – measured current
- Light = energy packets (photons) with energy $E=h\nu$
- Photoemission only occurs when $E >$ workfunction $\varnothing$
- $\varnothing=hc/e\lambda_o$, where λ_o is the longest wavelength for photoemission
- Work function of a metal keeps the electron in the material

Compton effect: Light also has momentum

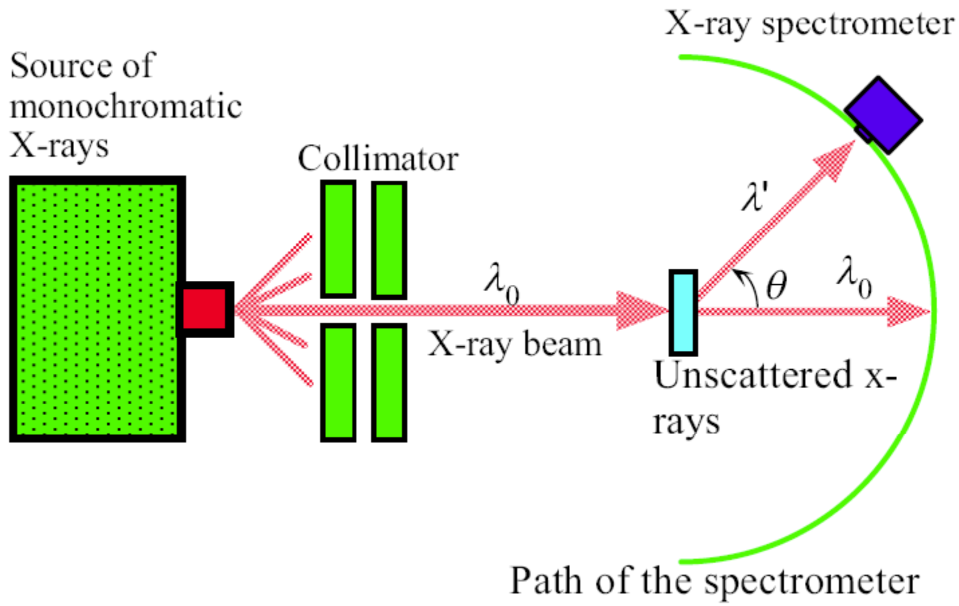


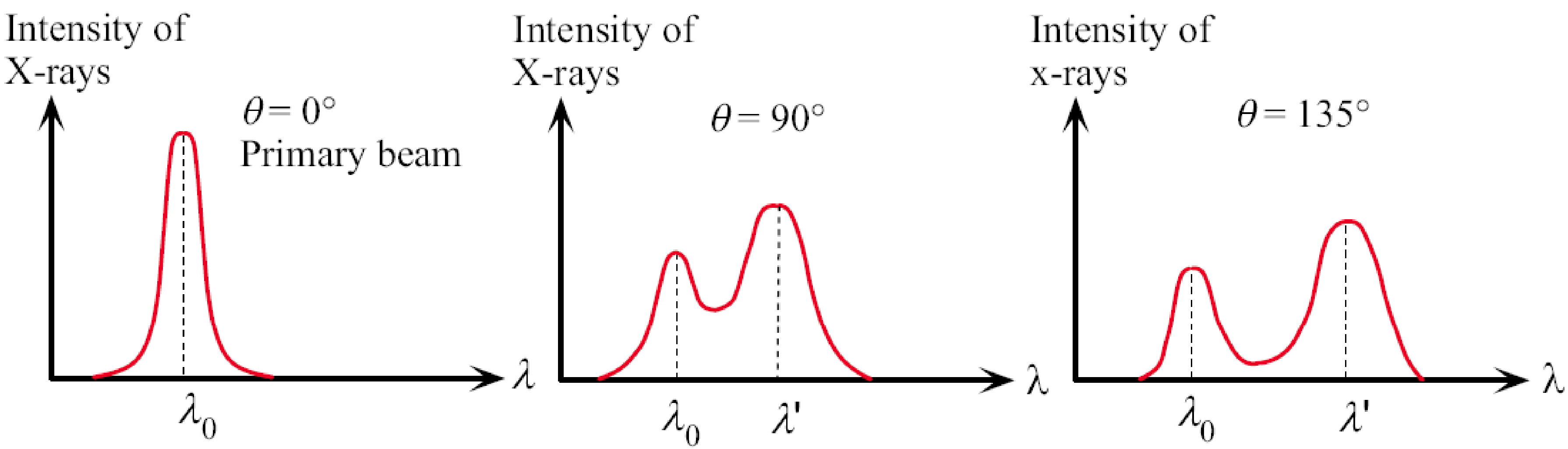
- 1927 Nobel prize
- Holly Compton found that X-ray wavelengths increase due to scattering of the photon by free electrons in the material → further demonstrates the particle nature of light

Compton's experiment and results

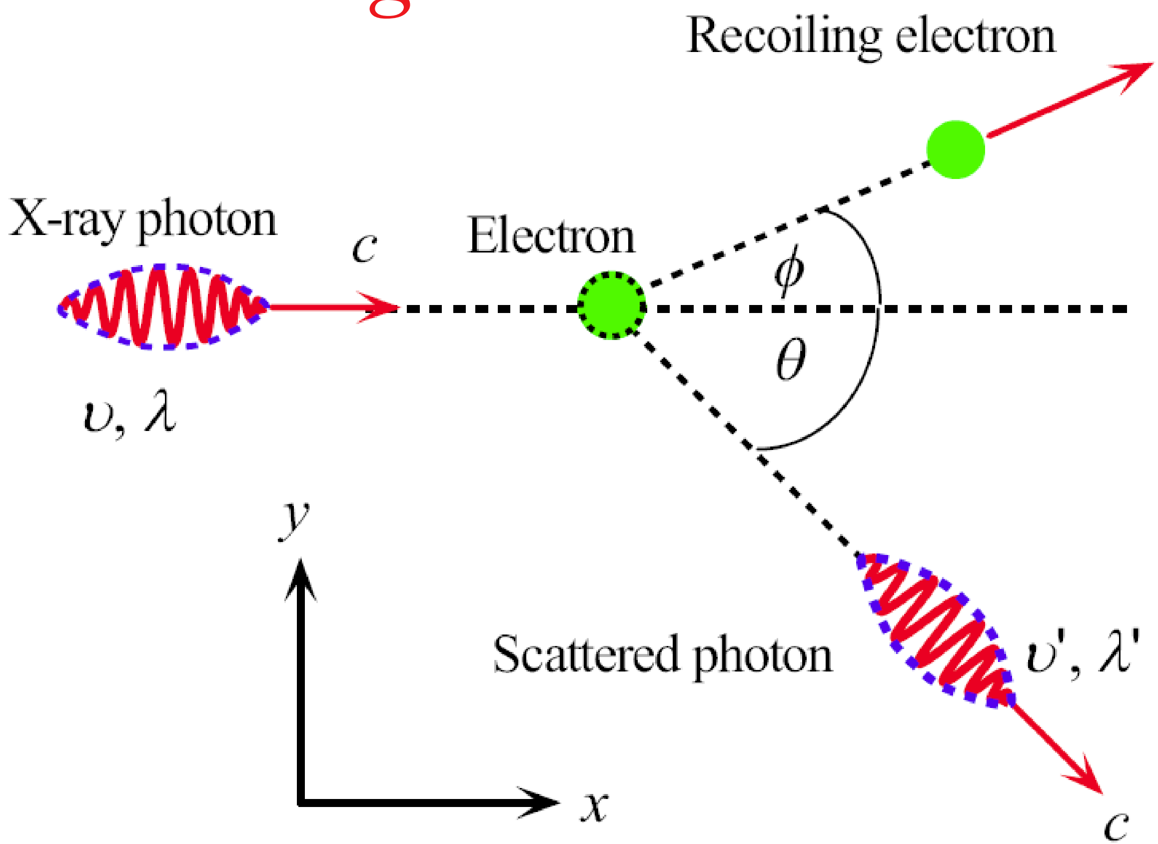


Compton's experiment and results





Compton scattering



Scattering of an X-ray photon by a “free” electron in a conductor. Since the electron has a momentum, by conservation of momentum, the x-ray must also have momentum.

$$KE_m = h\upsilon - h\upsilon'$$

$$p = \frac{h}{\lambda}$$

Summary equations: light as a particle & wave

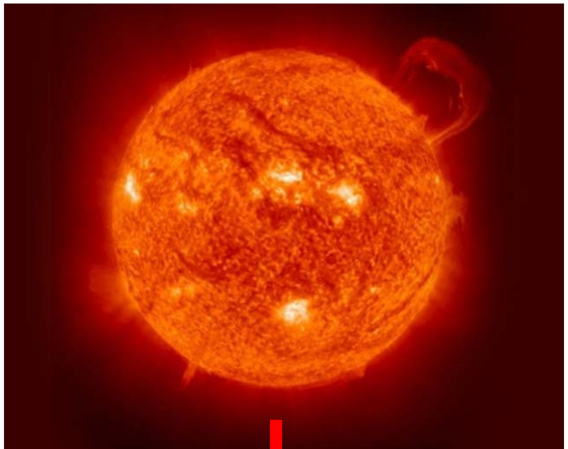
Energy: $E = h\upsilon$

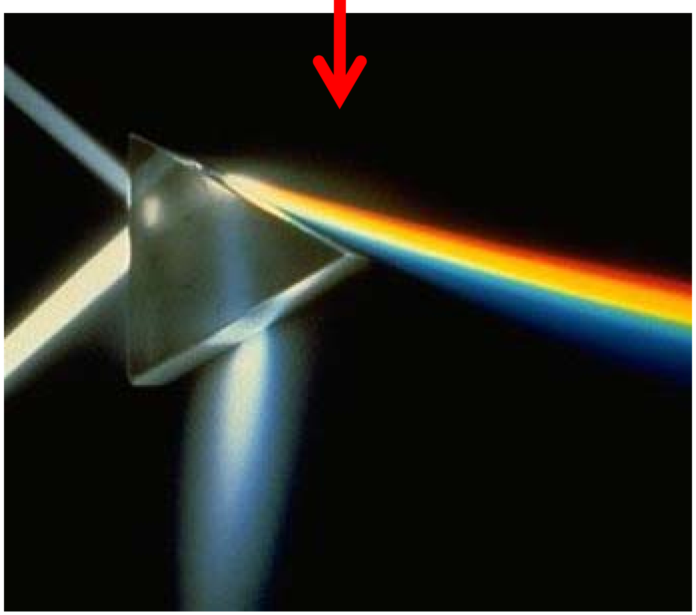
Wavevector: $k = \dfrac{2\pi}{\lambda}$

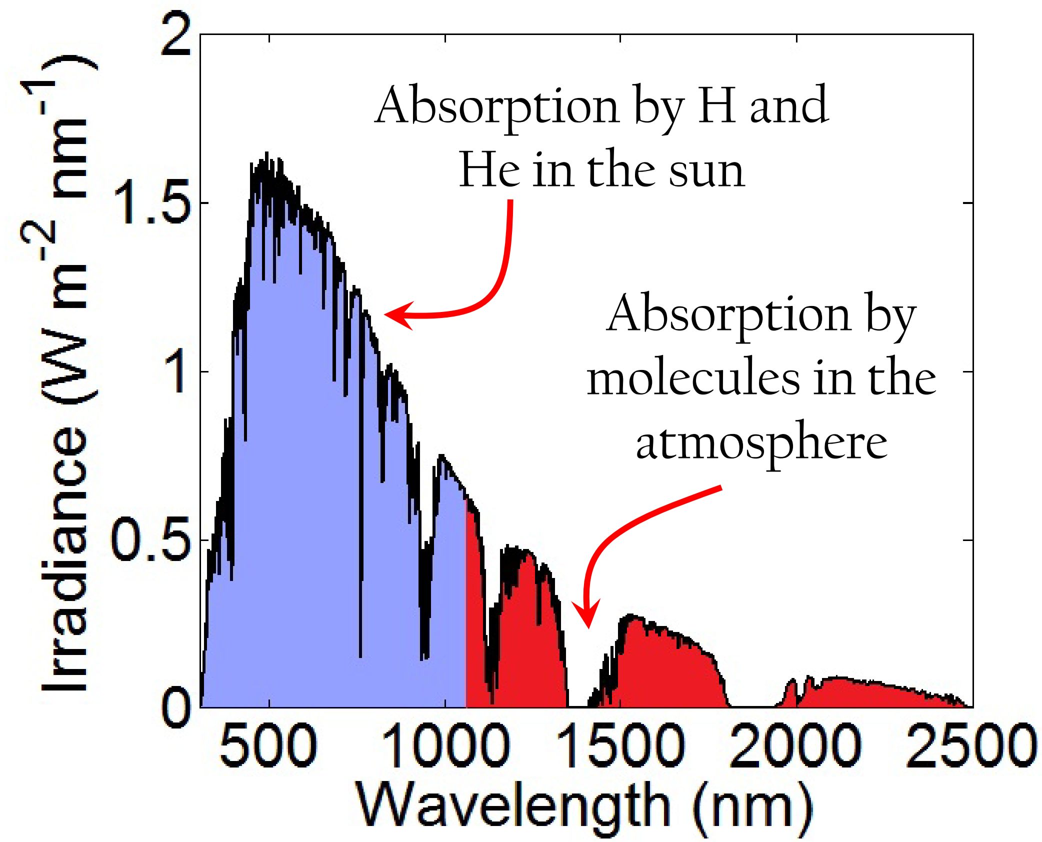
Momentum: $p = \dfrac{h}{\lambda} = \hbar k$

Intensity: $I = \Gamma h\upsilon = \dfrac{1}{2} c\varepsilon E_0^2$

The solar spectrum





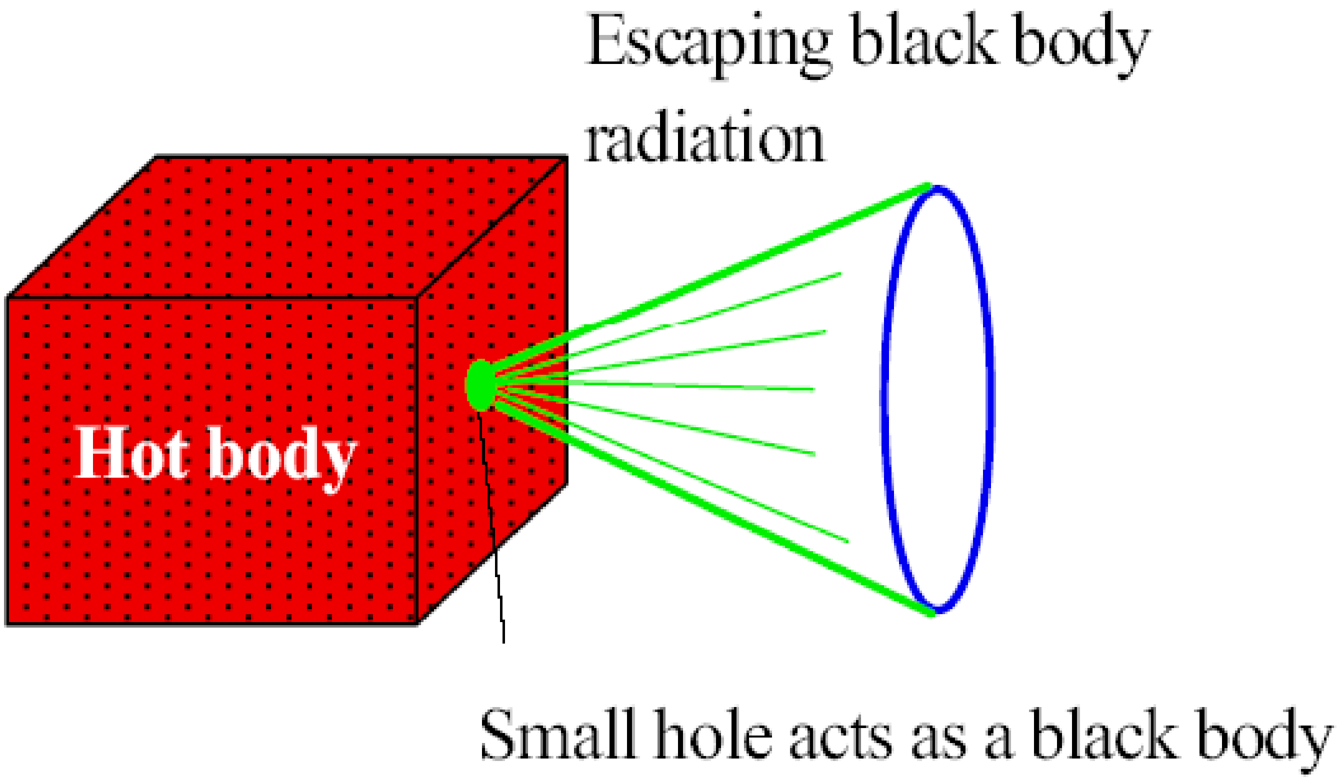


What causes the overall shape?

Blackbody radiation

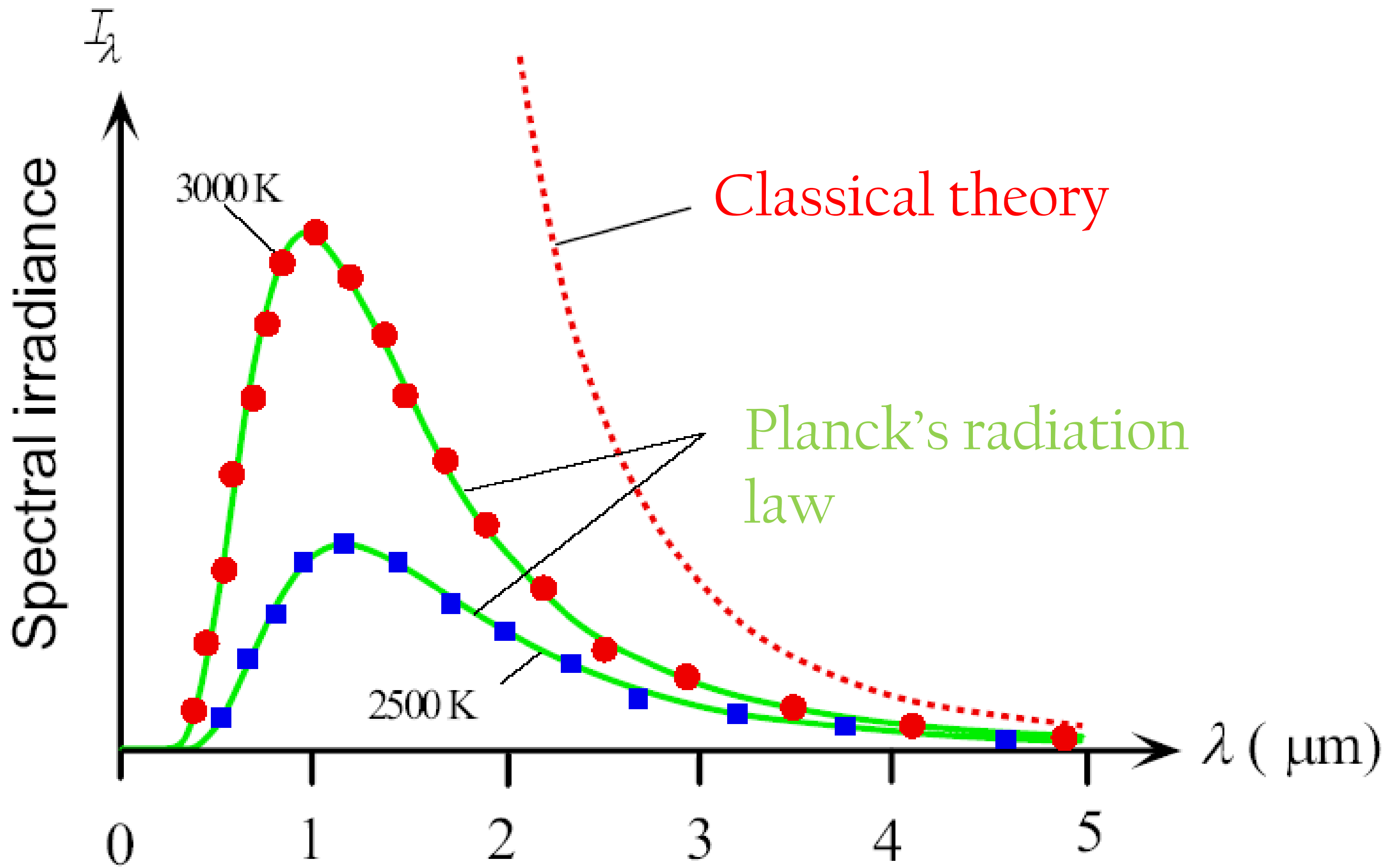
- Blackbodies absorb all electromagnetic radiation
- Appear black when cold
- As they heat up, begin to emit radiation
- Color depends on temperature – the hotter the temperature, the higher the emitted photon energy

Blackbody radiation (1900, Max Planck)



Schematic illustration of black body radiation

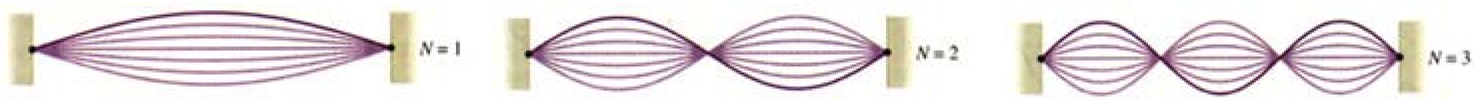
Blackbody radiation



Spectral irradiance vs. wavelength at two temperatures (3000K is about the temperature of the incandescent tungsten filament in a light bulb.)

Classical Theory ("Rayleigh-Jeans law")

Thermal vibrations and rotations give rise to radiated electromagnetic waves that will interfere with each other, giving rise to many standing electromagnetic waves in the "oven"



Each standing wave contributes ~kT of energy (from kinetic molecular theory)

By calculating the number of standing waves, find irradience

$$I_\lambda \sim T \sim 1/\lambda^4$$

Planck's Theory

Assumed radiation in oven involved emission and absorption of discrete amounts of light energy by the oscillation of molecules.

Assumed the probability of a molecule (an oscillator) possessing an energy nhν (n an integer) was proportional to the Boltzmann factor (think kinetic molecular theory)

$$I_\lambda = \frac{2\pi hc^2}{\lambda^5\left[\exp\left(\frac{hc}{\lambda kT}\right) - 1\right]}$$

Stefan's Law

Integrating the irradiance over all wavelengths yeilds the total radiated power P_S emitted by a blackbody per unit surface area at a temperature T:

$$P_S = \sigma_S T^4$$

Stefan's constant

$$\sigma_S = \frac{2\pi^5 k^4}{15 c^2 h^3} = 5.670 \times 10^{-8}\ \mathrm{W\,m^{-2}\,K^{-4}}$$

Stefan's law for real surfaces
Electromagnetic radiation emitted from a hot surface

$P_{\text{radiation}}$ = total radiation power emitted (W = J s^{-1})

$$P_{\text{radiation}} = S\varepsilon\sigma_S[T^4 - T_0^4]$$

σ_S = Stefan's constant, W m^{-2} K^{-4}

ε = emissivity of the surface

ε = 1 for a perfect black body

ε < 1 for other surfaces

S = surface area of emitter (m^2)

Temperature of a lightbulb filament

100 Watt lightbulb
emissivity $\varepsilon = 0.35$.
Filament length of 57.9cm, diameter of 31.7 microns.

$$S = 2\pi(31.7\times10^{-6}\,m)(0.579\,m) = 1.15 \times 10^{-4}\,m^2$$

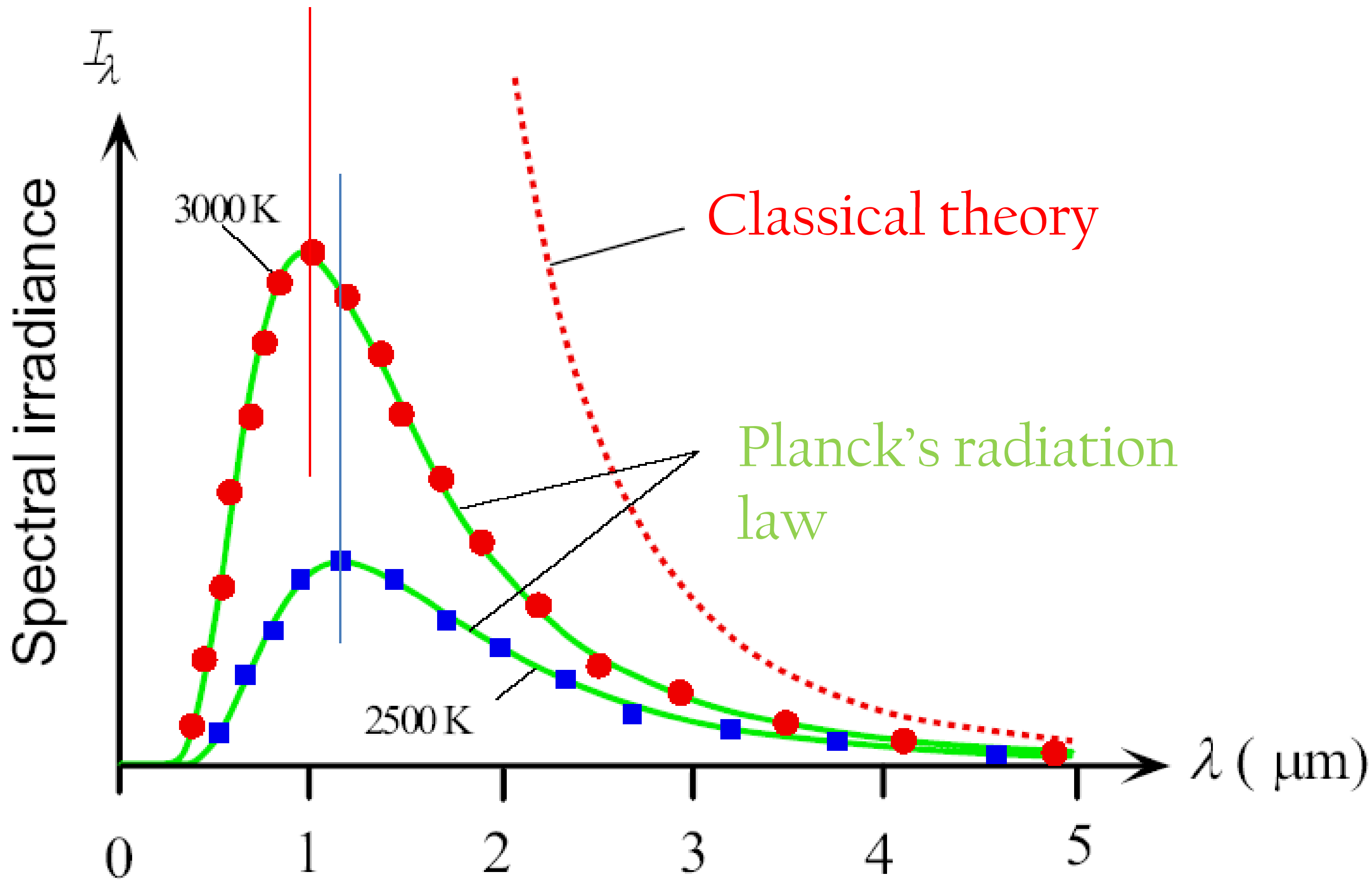
$$P_s = 100\ W = S\varepsilon\sigma_S(T_F^4 - T_0^4)$$

$$T_0 = 300K$$

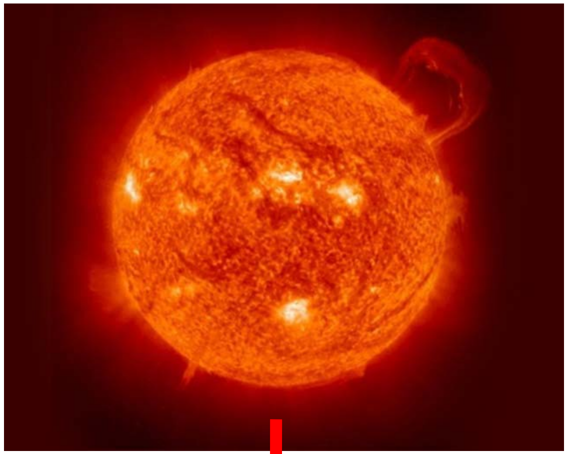
$$\rightarrow T_F = 2573\ K = 2300^o\ C$$

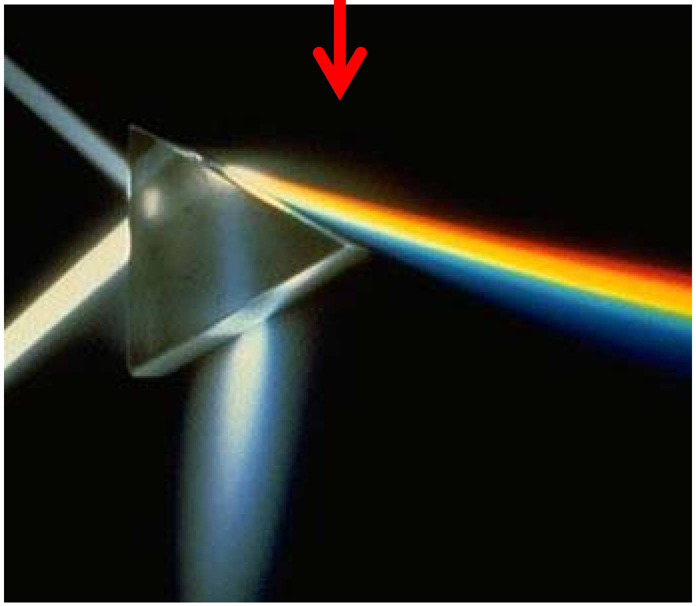
Wein's displacement law

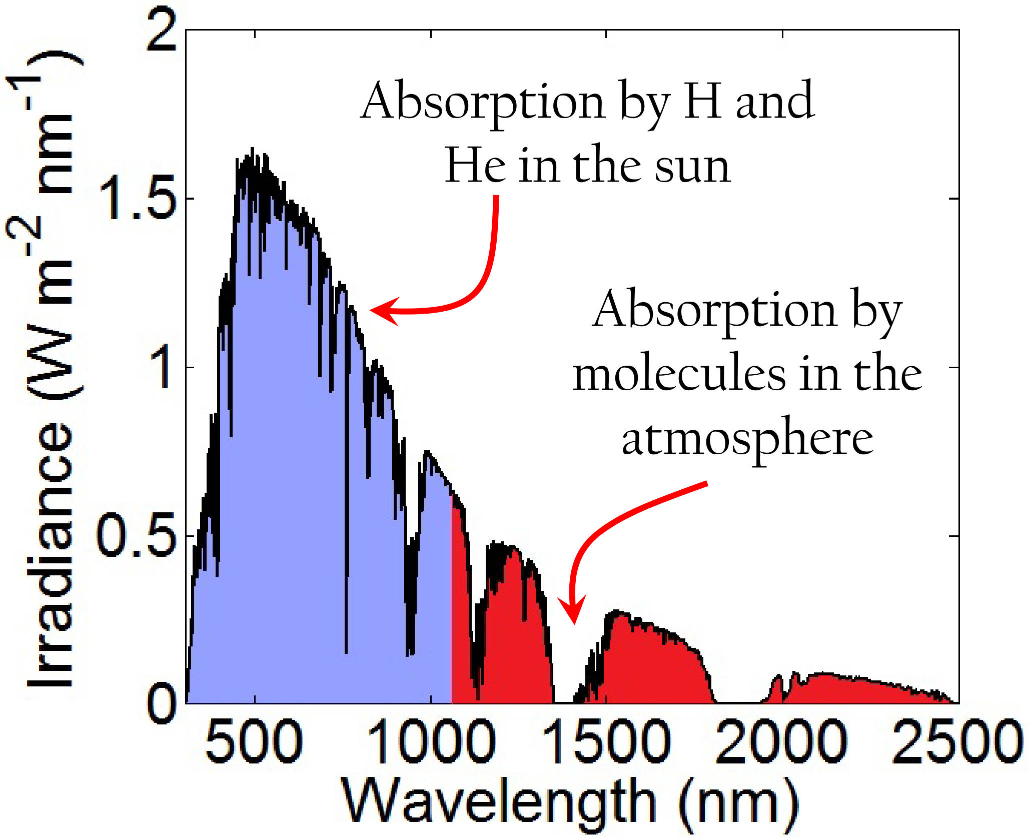
As the temperature of a blackbody increases, the peak emission shifts to shorter wavelengths: $\lambda * T = 2.89 \times 10^{-3}$ m*K



The solar spectrum

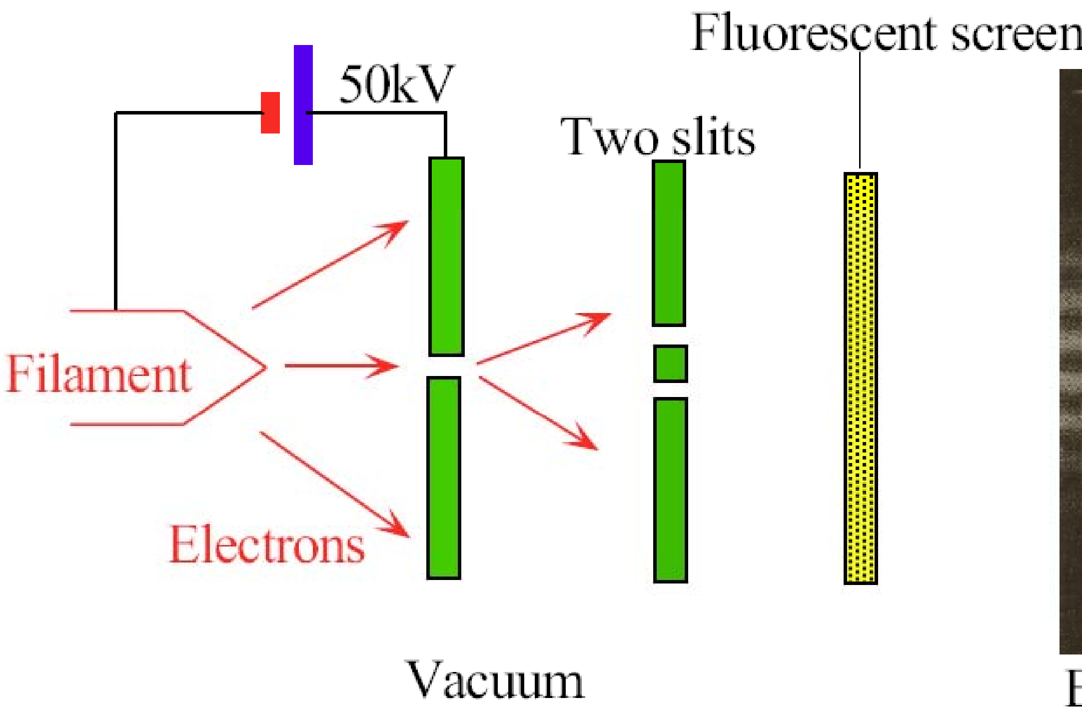


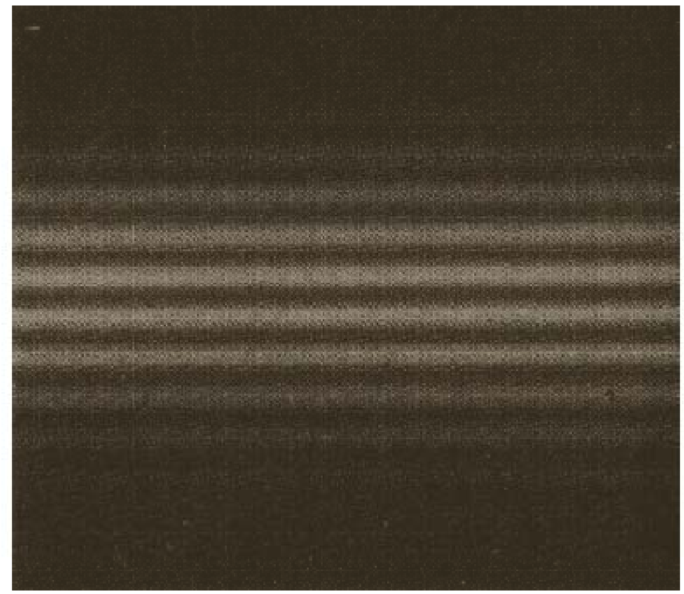




What causes the overall shape?
Blackbody emission at ~6000K!

The electron is also a wave





Electron diffraction fringes on the screen

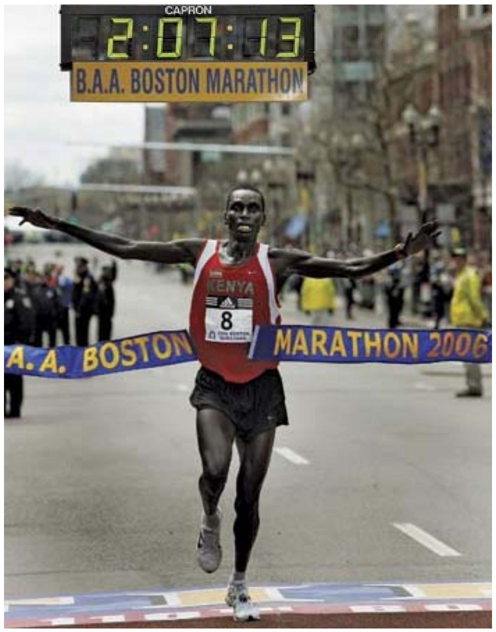
Young's double-slit experiment with electrons involves an electron gun and two slits in a Cathode ray tube (CRT) (hence, in vacuum).
Electrons from the filament are accelerated by a 50 kV anode voltage to produce a beam that Is made to pass through the slits. The electrons then produce a visible pattern when they strike A fluorescent screen (e.g., a TV screen), and the resulting visual pattern is photographed.
SOURCE: Pattern from C. Jonsson, D. Brandt, and S. Hirschi, *Am. J. Physics*, 42, 1974, p.9, figure 8. Used with permission.

De Broglie Relationship

Wavelength of the electron depends on its momentum p
(just like photons!)

$$\lambda = \frac{h}{p}$$

Wavelengths of particle-like objects



Marathon winner (60 kg, 26 miles or 16.2 km)

50 gram golf ball travelling at 20 m/s

Electron accelerated by 100 V

$h = 6.626{\times}10^{-34}$ J·s $\quad$ $e = 1.602{\times}10^{-19}$ C

The Schrodinger equation describes electrons

Steady-state total wave function

$$\Psi(x,t) = \psi(x)\exp\left(-\frac{iEt}{\hbar}\right)$$

Schrodinger's equation for one dimension

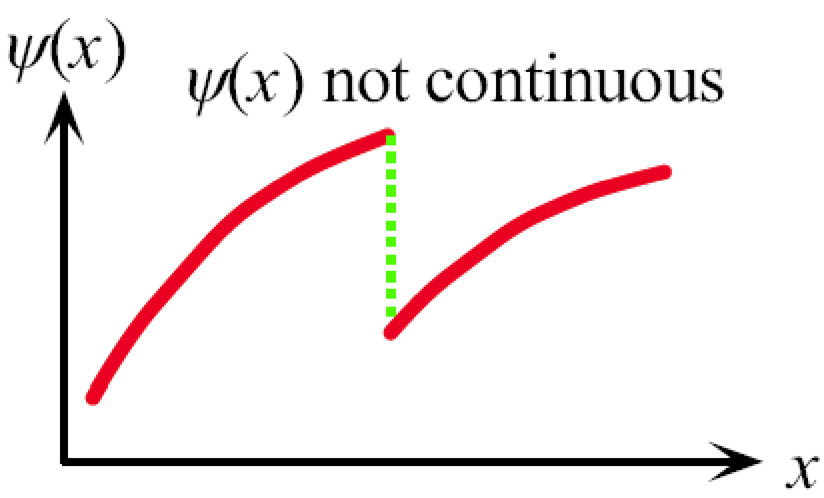
$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E-V)\psi = 0$$

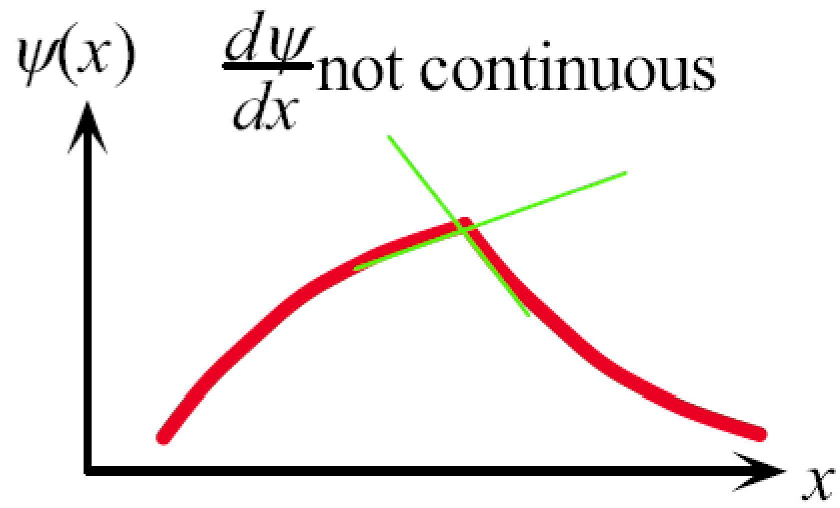
Schrondinger's equation for three dimensions

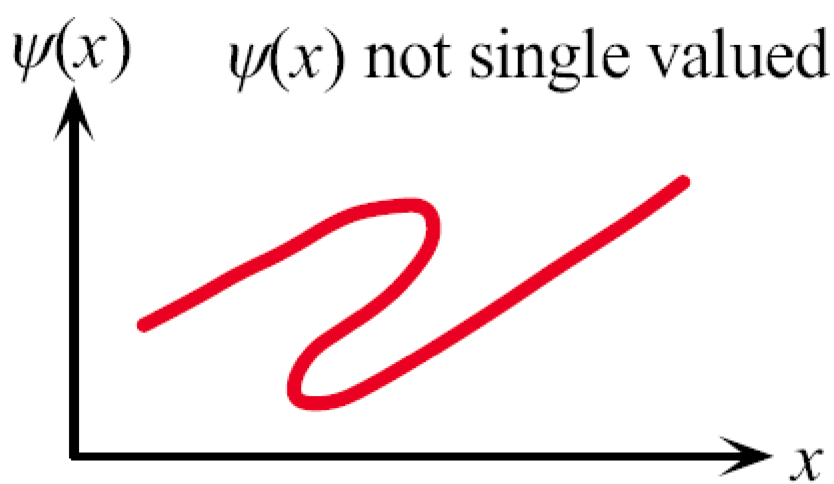
$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} + \frac{2m}{\hbar^2}(E-V)\psi = 0$$

From *Principles of Electronic Materials and Devices, Third Edition*, S.O. Kasap (© McGraw-Hill, 2005)

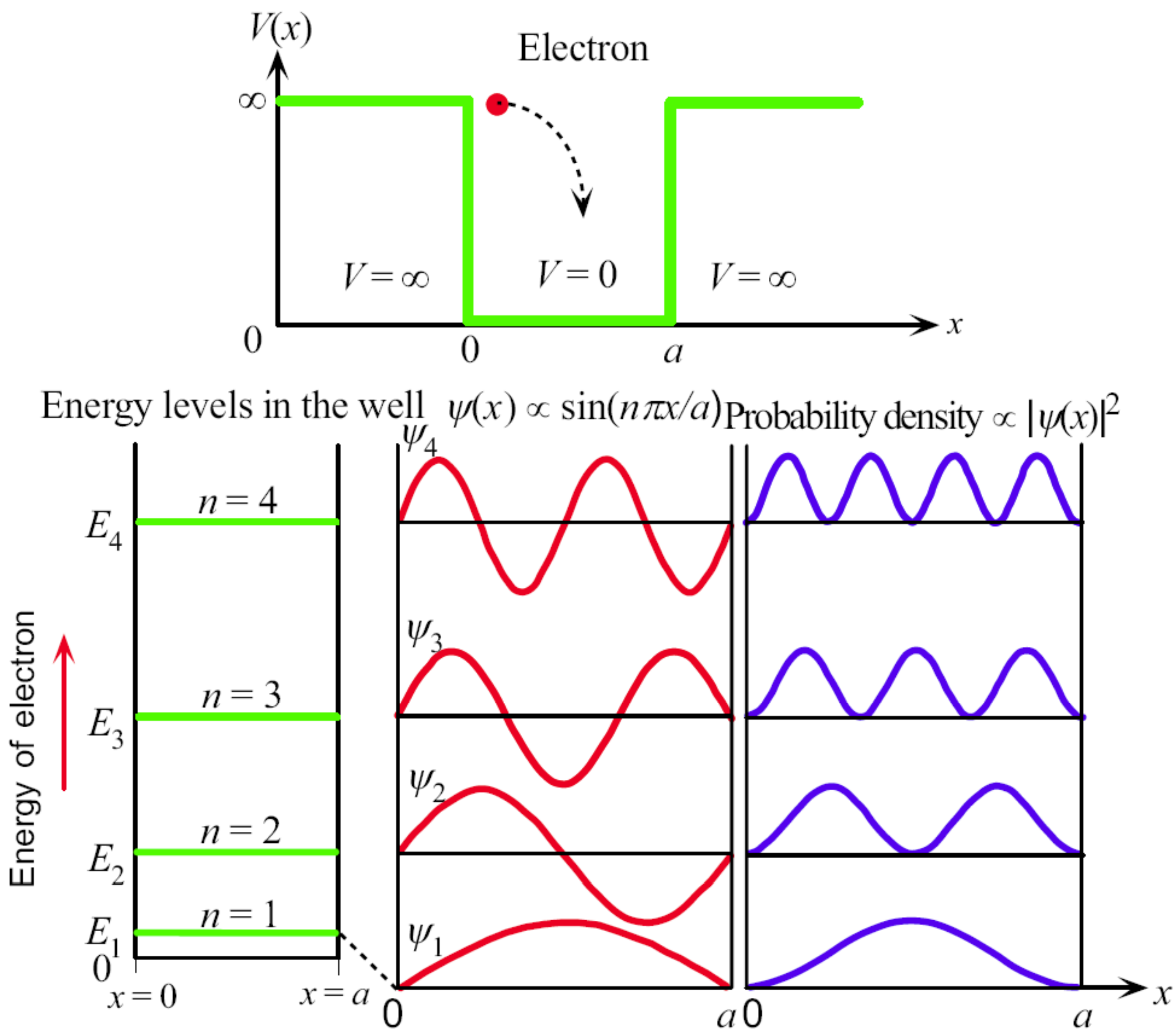
Wavefunctions will NEVER look like...







Wavefunctions of electrons in a 1D box



Infinite Potential Well

Wavefunction in an infinite PE well

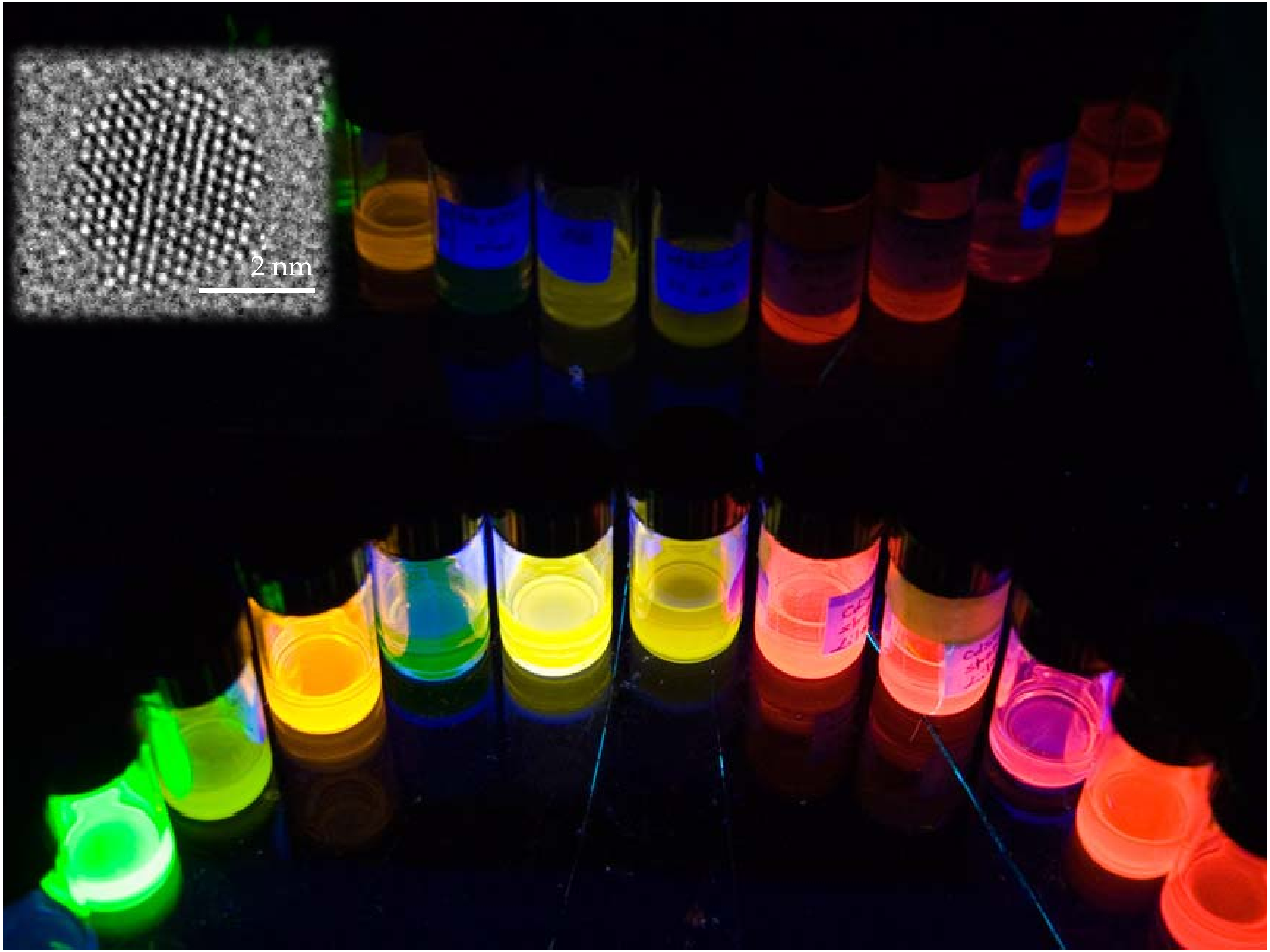
$$\psi_n(x) = 2Ai\sin\left(\frac{n\pi x}{a}\right)$$

Electron energy in an infinite PE well

$$E_n = \frac{\hbar^2(\pi n)^2}{2ma^2} = \frac{h^2n^2}{8ma^2}$$

Energy separation in an infinite PE well

$$\Delta E = E_{n+1} - E_n = \frac{h^2(2n+1)}{8ma^2}$$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