MCEN 5228, Spring 2014

Homework 3 due in class on 02/28

In your write-ups, please provide clear explanations of the models chosen, of the equations used, and of the findings, with figures where necessary.

Datasets are available at http://spot.colorado.edu/~henzed/MCEN5228_s2014/hw.html.

1. For this problem we will solve a variation on problem 3.5 from Aster. Data values $\mathbf{G}$, $\mathbf{d}$, and $\mathbf{m}^0$ are available online.

   (a) Estimate $\mathbf{m}$ using 1st order Tikhonov regularization, finding your value of α^2 from an L-curve. Note: you may have to explore a very large range of α's to find the corner for this problem

   (b) Assume that you have a prior estimate of $\mathbf{m}^0$ with a constant standard deviation of 2000, and assume the errors are uncorrelated. Find $\mathbf{m}$ from minimizing

   $$\mathcal{J} = ||\mathbf{Gm} - \mathbf{d}||_2^2 + ||(\mathbf{m} - \mathbf{m}^0)^T \mathbf{C}_{m0}^{-1/2}||_2^2$$

   (c) Repeat part (b), but now include strong correlations across groups of $\mathbf{m^0}$ as a surrogate for aggregation, using a 4x4 block diagonal format for $\mathbf{C}_{m0}$, i.e.,

   $$\mathbf{C}_{m0} = 2000^2 \begin{bmatrix} 1 & 0.9 & 0.9 & 0.9 & 0 & 0 & 0 & 0 & \\ 0.9 & 1 & 0.9 & 0.9 & 0 & 0 & 0 & 0 & \\ 0.9 & 0.9 & 1 & 0.9 & 0 & 0 & 0 & 0 & \\ 0.9 & 0.9 & 0.9 & 1 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 1 & 0.90 & 0.9 & 0.9 & \\ 0 & 0 & 0 & 0 & 0.9 & 1 & 0.9 & 0.9 & \\ 0 & 0 & 0 & 0 & 0.9 & 0.9 & 1 & 0.9 & \\ 0 & 0 & 0 & 0 & 0.9 & 0.9 & 0.9 & 1 & \\ & & & & & & & & \ddots \end{bmatrix}$$

   (d) Make a plot showing $\mathbf{m}$ estimated from parts (a), (b), and (c), along with $\mathbf{m}^0$.

2. In this problem the goal is to fit a 2D surface to a set of observed data points (p2_obs.txt) at locations p2_xy.txt. We will use a thin plate spline interpolation to fit the data. This method minimizes the following function

   $$J = \alpha||G(\mathbf{m}) - \mathbf{d}||_2^2 + (1 - \alpha)R(\mathbf{m})$$

   where G is the thin plate spline function using coefficients $\mathbf{m}$, R is a roughness measure related to the partial derivatives of $G(\mathbf{m})$, and α is a regularization parameter that has a value between 0 and 1.

   Use leave-one-out cross validation to find the optimal thin plate spline fit to the data. Make a plot of $f(\alpha)$ vs α. Also make plots of the optimal surface, the surface corresponding to the $\alpha = 0$ solution and the surface corresponding to the $\alpha = 1$ solution. In each of these plots, show the surface as well as the data.

   The MATLAB function for finding the thin plate spline function is

```
st = tpaps(xy, d, p)
```

This function takes as inputs the $x - y$ coordinate matrix (2 rows, m columns), the m column data vector, and the regularization parameter, respectively. It returns an object, `st`, that contains the $\mathbf{m}$ which minimizes the equation above. To find the values of $G(\mathbf{m})$ at coordinate x_1, y_1, use

```
fnval(st,[x1; y1])
```

To plot the surface, use `fnplt(st)`. To plot the data,

```
plot3(xy(1,:),xy(2,:),d)
```

Another MATLAB command that might come in handy is that you can eliminate a column of a matrix, for example the third column, using

```
 A(:,3) = []
```

3. Aster 6.1, part (a), and write the Kaczmarz's code yourself.