

```
In[95]:= Needs["VectorAnalysis`"]  
SetCoordinates[Cartesian]  
Element[{Ttheta, Pphi}, Reals]  
Element[{ $\phi$ ,  $\theta$ }, Reals]
```

```
Out[96]= Cartesian[Xx, Yy, Zz]
```

```
Out[97]= (Ttheta | Pphi)  $\in$  Reals
```

```
Out[98]= ( $\phi$  |  $\theta$ )  $\in$  Reals
```

```

In[99]:= {r, th, ph} =
  Simplify[CoordinatesFromCartesian[{Xx, Yy, Zz}, Spherical]]

Out[99]:=  $\left\{ \sqrt{Xx^2 + Yy^2 + Zz^2}, \text{ArcCos}\left[\frac{Zz}{\sqrt{Xx^2 + Yy^2 + Zz^2}}\right], \text{ArcTan}[Xx, Yy] \right\}$ 

In[100]:= (* Setup some functions for Matlab equivalent spherical
  harmonics. Note: matlab is off by sqrt(2) for normalization,
  AND the Ttheta are [0,pi] (should be [-pi/2,pi/2]) *)
sphFull[l_, m_] := ComplexExpand[
  Re[Sqrt[2] * SphericalHarmonicY[l, m, Ttheta, Pphi]]] (*//Simplify*)
(*Give the Spherical Harmonic in Cartesian Coordinates.
  NOTE: MATLAB USES "angle(...)" instead of "Arg[...]"*)
sphFullCart[l_, m_] :=
  ComplexExpand[Re[Sqrt[2] * SphericalHarmonicY[l, m, th, ph]],
    TargetFunctions -> {Re, Im}] // FullSimplify
sphFullCart[l1_, m1_, l2_, m2_] :=
  sphFullCart[l1, m1] + sphFullCart[l2, m2]
sphFull[l1_, m1_, l2_, m2_] := ComplexExpand[
  Re[Sqrt[2] * SphericalHarmonicY[l1, m1, Ttheta, Pphi] + Sqrt[2] *
    SphericalHarmonicY[l2, m2, Ttheta, Pphi]]] (*//Simplify*)

sphLapl[l_, m_] := Laplacian[sphFull[l, m], Spherical] /. Rr -> 1
(*//Simplify *)
sphLapl[l1_, m1_, l2_, m2_] :=
  Laplacian[sphFull[l1, m1, l2, m2], Spherical] /. Rr -> 1
(*//Simplify*)
sphLaplCart[l_, m_] := Laplacian[sphFullCart[l, m], Cartesian]
(*//FullSimplify*)
sphLaplCart[l1_, m1_, l2_, m2_] :=
  Laplacian[sphFullCart[l1, m1, l2, m2], Cartesian] (*//FullSimplify*)

sphGradCart[l_, m_] := Grad[sphFullCart[l, m], Cartesian]
(*//FullSimplify*)
sphGradCart[l1_, m1_, l2_, m2_] :=
  Grad[sphFullCart[l1, m1, l2, m2], Cartesian] (*//FullSimplify*)

In[110]:= Clear[U, G, H, Qx, Px, Mm, Nn, Pp, Ff, Gg, Hh]

In[111]:= (* Let Qx be the curl matrix which guarantees that Div(Q * Grad(g(x,y,z))) =
  0. where g(x,y,z) is ANY function (i.e., Spherical Harmonics) *)

In[112]:= Qx := {{0, -Zz, Yy}, {Zz, 0, -Xx}, {-Yy, Xx, 0}}

In[113]:= (* Projection Matrix to Constrain To Sphere *)
Px := {{1 - Xx^2, -Xx * Yy, -Xx * Zz},
  {-Xx * Yy, 1 - Yy^2, -Yy * Zz}, {-Xx * Zz, -Yy * Zz, 1 - Zz^2}}

(* Choose a function g(x,y,z) and pressure *)
G := 8 * sphFullCart[3, 2] - 3 * sphFullCart[10, 5] + sphFullCart[20, 20]
Pressure := sphFullCart[6, 4]

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In[116]:= (* Get our projected gradient function *)  
H := Px.Grad[G, Cartesian]
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In[117]:= (* THIS IS THE MANUFACTURED SOLUTION *)  
U = Qx.H // FullSimplify
```

$$\begin{aligned}
\text{Out[117]} = & \left\{ \frac{1}{262144} \sqrt{\frac{7}{\pi}} \left(\frac{524288 \sqrt{15} Y Y (Xx^2 - Y Y^2 + 2 Z Z^2)}{(Xx^2 + Y Y^2 + Z Z^2)^{3/2}} + \right. \right. \\
& 15 \sqrt{286} \left(\frac{1}{(Xx^2 + Y Y^2 + Z Z^2)^{9/2}} 3072 Y Y (Xx^2 + Y Y^2)^{3/2} \sqrt{\frac{1}{Xx^2 + Y Y^2 + Z Z^2}} \right. \\
& \left(3 (Xx^2 + Y Y^2)^3 - 111 (Xx^2 + Y Y^2)^2 Z Z^2 + 364 (Xx^2 + Y Y^2) Z Z^4 - 168 Z Z^6 \right) \cos[5 \text{ArcTan}[Xx, Y Y]] - \\
& \frac{\sqrt{156835045} Y Y (Xx^2 + Y Y^2)^9 Z Z \cos[20 \text{ArcTan}[Xx, Y Y]]}{(Xx^2 + Y Y^2 + Z Z^2)^{10}} + Xx Z Z \left(\frac{1}{(Xx^2 + Y Y^2 + Z Z^2)^{9/2}} \right. \\
& 3072 (Xx^2 + Y Y^2)^{3/2} Z Z \sqrt{\frac{1}{Xx^2 + Y Y^2 + Z Z^2}} \left(15 (Xx^2 + Y Y^2)^2 - 140 (Xx^2 + Y Y^2) Z Z^2 + 168 Z Z^4 \right) \\
& \left. \left. \left. \sin[5 \text{ArcTan}[Xx, Y Y]] + \frac{\sqrt{156835045} (Xx^2 + Y Y^2)^9 \sin[20 \text{ArcTan}[Xx, Y Y]]}{(Xx^2 + Y Y^2 + Z Z^2)^{10}} \right) \right) \right\}, \\
& - \frac{1}{262144} \sqrt{\frac{7}{\pi}} \left(- \frac{524288 \sqrt{15} Xx (Xx^2 - Y Y^2 - 2 Z Z^2)}{(Xx^2 + Y Y^2 + Z Z^2)^{3/2}} + 15 \sqrt{286} \right. \\
& \left(- \frac{1}{(Xx^2 + Y Y^2 + Z Z^2)^{9/2}} 3072 Xx (Xx^2 + Y Y^2)^{3/2} \sqrt{\frac{1}{Xx^2 + Y Y^2 + Z Z^2}} \right. \\
& \left(3 (Xx^2 + Y Y^2)^3 - 111 (Xx^2 + Y Y^2)^2 Z Z^2 + 364 (Xx^2 + Y Y^2) Z Z^4 - 168 Z Z^6 \right) \cos[5 \text{ArcTan}[Xx, Y Y]] + \\
& \frac{\sqrt{156835045} Xx (Xx^2 + Y Y^2)^9 Z Z \cos[20 \text{ArcTan}[Xx, Y Y]]}{(Xx^2 + Y Y^2 + Z Z^2)^{10}} + Y Y Z Z \left(\frac{1}{(Xx^2 + Y Y^2 + Z Z^2)^{9/2}} \right. \\
& 3072 (Xx^2 + Y Y^2)^{3/2} Z Z \sqrt{\frac{1}{Xx^2 + Y Y^2 + Z Z^2}} \left(15 (Xx^2 + Y Y^2)^2 - 140 (Xx^2 + Y Y^2) Z Z^2 + 168 Z Z^4 \right) \\
& \left. \left. \left. \sin[5 \text{ArcTan}[Xx, Y Y]] + \frac{\sqrt{156835045} (Xx^2 + Y Y^2)^9 \sin[20 \text{ArcTan}[Xx, Y Y]]}{(Xx^2 + Y Y^2 + Z Z^2)^{10}} \right) \right) \right\}, \\
& - \frac{1}{262144 (Xx^2 + Y Y^2 + Z Z^2)^{21/2}} \sqrt{\frac{7}{\pi}} \left(\frac{1}{\left(\frac{1}{Xx^2 + Y Y^2 + Z Z^2} \right)^{11/2}} 46080 \sqrt{286} (Xx^2 + Y Y^2)^{5/2} Z Z \right. \\
& \left(15 (Xx^2 + Y Y^2)^2 - 140 (Xx^2 + Y Y^2) Z Z^2 + 168 Z Z^4 \right) \sin[5 \text{ArcTan}[Xx, Y Y]] + \\
& \sqrt{5} \left(2097152 \sqrt{3} Xx Y Y Z Z (Xx^2 + Y Y^2 + Z Z^2)^9 + \right. \\
& \left. \left. 15 \sqrt{8970964574} (Xx^2 + Y Y^2)^{10} \sqrt{Xx^2 + Y Y^2 + Z Z^2} \sin[20 \text{ArcTan}[Xx, Y Y]] \right) \right\}
\end{aligned}$$

In[118]:= **(* Prove that our divergence of this field is zero: *)**
(*Div[U] // FullSimplify*)

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In[119]:= (* Use projection matrix to get Laplacian of our solution in each U,V,W direction. *)
(* Note: to get nth component of U: U[[n]] *)
GradRHS :=
{Px.Grad[U[[1]], Cartesian], Px.Grad[U[[2]], Cartesian], Px.Grad[U[[3]], Cartesian]}
(* THIS IS A MANUFACTURED RHS *)
LaplRHS := Px.{Div[GradRHS[[1]], Cartesian],
Div[GradRHS[[2]], Cartesian], Div[GradRHS[[3]], Cartesian]}
GradP := Px.Grad[Pressure, Cartesian]

In[122]:= RHS = -LaplRHS + GradP // FullSimplify

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$$\begin{aligned}
\text{Out[122]} = & \left\{ \frac{1}{65536} 3 \sqrt{\frac{7}{\pi}} \left(\frac{1267200 \sqrt{286} Xx^9 Yy \sqrt{\frac{1}{Xx^2 + Yy^2 + Zz^2}}}{(Xx^2 + Yy^2 + Zz^2)^{11/2}} - \right. \right. \\
& \frac{10137600 \sqrt{286} Xx^7 Yy^3 \sqrt{\frac{1}{Xx^2 + Yy^2 + Zz^2}}}{(Xx^2 + Yy^2 + Zz^2)^{11/2}} - \frac{17740800 \sqrt{286} Xx^5 Yy^5 \sqrt{\frac{1}{Xx^2 + Yy^2 + Zz^2}}}{(Xx^2 + Yy^2 + Zz^2)^{11/2}} + \\
& \frac{6336000 \sqrt{286} Xx Yy^9 \sqrt{\frac{1}{Xx^2 + Yy^2 + Zz^2}}}{(Xx^2 + Yy^2 + Zz^2)^{11/2}} - \frac{15206400 \sqrt{286} Xx^7 Yy Zz^2 \sqrt{\frac{1}{Xx^2 + Yy^2 + Zz^2}}}{(Xx^2 + Yy^2 + Zz^2)^{11/2}} + \\
& \frac{390297600 \sqrt{286} Xx^5 Yy^3 Zz^2 \sqrt{\frac{1}{Xx^2 + Yy^2 + Zz^2}}}{(Xx^2 + Yy^2 + Zz^2)^{11/2}} + \frac{177408000 \sqrt{286} Xx^3 Yy^5 Zz^2 \sqrt{\frac{1}{Xx^2 + Yy^2 + Zz^2}}}{(Xx^2 + Yy^2 + Zz^2)^{11/2}} - \\
& \frac{228096000 \sqrt{286} Xx Yy^7 Zz^2 \sqrt{\frac{1}{Xx^2 + Yy^2 + Zz^2}}}{(Xx^2 + Yy^2 + Zz^2)^{11/2}} - \frac{141926400 \sqrt{286} Xx^5 Yy Zz^4 \sqrt{\frac{1}{Xx^2 + Yy^2 + Zz^2}}}{(Xx^2 + Yy^2 + Zz^2)^{11/2}} - \\
& \frac{946176000 \sqrt{286} Xx^3 Yy^3 Zz^4 \sqrt{\frac{1}{Xx^2 + Yy^2 + Zz^2}}}{(Xx^2 + Yy^2 + Zz^2)^{11/2}} + \frac{709632000 \sqrt{286} Xx Yy^5 Zz^4 \sqrt{\frac{1}{Xx^2 + Yy^2 + Zz^2}}}{(Xx^2 + Yy^2 + Zz^2)^{11/2}} + \\
& \left. \frac{283852800 \sqrt{286} Xx^3 Yy Zz^6 \sqrt{\frac{1}{Xx^2 + Yy^2 + Zz^2}}}{(Xx^2 + Yy^2 + Zz^2)^{11/2}} - \frac{283852800 \sqrt{286} Xx Yy^3 Zz^6 \sqrt{\frac{1}{Xx^2 + Yy^2 + Zz^2}}}{(Xx^2 + Yy^2 + Zz^2)^{11/2}} + \right\}
\end{aligned}$$

$$\begin{aligned}
& \frac{524\,288\sqrt{15}\,Xx^2\,Yy}{\left(Xx^2+Yy^2+Zz^2\right)^{5/2}} - \frac{524\,288\sqrt{15}\,Yy^3}{\left(Xx^2+Yy^2+Zz^2\right)^{5/2}} + \frac{1\,048\,576\sqrt{15}\,Yy\,Zz^2}{\left(Xx^2+Yy^2+Zz^2\right)^{5/2}} + \frac{1}{\left(Xx^2+Yy^2+Zz^2\right)^{11}} \\
& \sqrt{13}\left(9975\sqrt{3\,450\,370\,990}\,Xx^{18}\,Yy\,Zz - 508\,725\sqrt{3\,450\,370\,990}\,Xx^{16}\,Yy^3\,Zz + \right. \\
& \quad 6\,104\,700\sqrt{3\,450\,370\,990}\,Xx^{14}\,Yy^5\,Zz - 26\,453\,700\sqrt{3\,450\,370\,990}\,Xx^{12}\,Yy^7\,Zz + \\
& \quad 48\,498\,450\sqrt{3\,450\,370\,990}\,Xx^{10}\,Yy^9\,Zz - 39\,680\,550\sqrt{3\,450\,370\,990}\,Xx^8\,Yy^{11}\,Zz + \\
& \quad 14\,244\,300\sqrt{3\,450\,370\,990}\,Xx^6\,Yy^{13}\,Zz - 2\,034\,900\sqrt{3\,450\,370\,990}\,Xx^4\,Yy^{15}\,Zz + \\
& \quad 89\,775\sqrt{3\,450\,370\,990}\,Xx^2\,Yy^{17}\,Zz - 525\sqrt{3\,450\,370\,990}\,Yy^{19}\,Zz - 4096\,Xx^{19}\left(8\,Yy^2+13\,Zz^2\right) + \\
& \quad 4096\,Xx\,Yy^2\left(Yy^2+Zz^2\right)^7\left(8\,Yy^4-85\,Yy^2\,Zz^2-60\,Zz^4\right) - 4096\,Xx^{17}\left(56\,Yy^4-3\,Yy^2\,Zz^2+71\,Zz^4\right) - \\
& \quad 28\,672\,Xx^{11}\left(Yy^2+Zz^2\right)^2\left(16\,Yy^6-350\,Yy^4\,Zz^2-500\,Yy^2\,Zz^4-35\,Zz^6\right) - \\
& \quad 28\,672\,Xx^{13}\left(Yy^2+Zz^2\right)\left(32\,Yy^6-220\,Yy^4\,Zz^2-280\,Yy^2\,Zz^4+5\,Zz^6\right) + \\
& \quad 4096\,Xx^3\left(Yy^2+Zz^2\right)^6\left(56\,Yy^6-445\,Yy^4\,Zz^2-250\,Yy^2\,Zz^4+20\,Zz^6\right) + \\
& \quad 28\,672\,Xx^7\left(Yy^2+Zz^2\right)^4\left(32\,Yy^6-4\,Yy^4\,Zz^2+176\,Yy^2\,Zz^4+47\,Zz^6\right) + \\
& \quad 28\,672\,Xx^9\left(Yy^2+Zz^2\right)^3\left(16\,Yy^6+238\,Yy^4\,Zz^2+448\,Yy^2\,Zz^4+61\,Zz^6\right) + \\
& \quad 4096\,Xx^5\left(Yy^2+Zz^2\right)^5\left(160\,Yy^6-764\,Yy^4\,Zz^2-104\,Yy^2\,Zz^4+127\,Zz^6\right) - \\
& \quad \left. 4096\,Xx^{15}\left(160\,Yy^6-356\,Yy^4\,Zz^2-416\,Yy^2\,Zz^4+133\,Zz^6\right)\right), \frac{1}{65\,536} \\
& 3\sqrt{\frac{7}{\pi}}\left(\frac{137\,625\,600\sqrt{44\,854\,822\,870}\,Xx^{19}\,Zz}{\left(Xx^2+Yy^2+Zz^2\right)^{11}} - \frac{653\,721\,600\sqrt{44\,854\,822\,870}\,Xx^{17}\left(Xx^2+Yy^2\right)\,Zz}{\left(Xx^2+Yy^2+Zz^2\right)^{11}} + \right. \\
& \quad \frac{1\,307\,443\,200\sqrt{44\,854\,822\,870}\,Xx^{15}\left(Xx^2+Yy^2\right)^2\,Zz}{\left(Xx^2+Yy^2+Zz^2\right)^{11}} - \\
& \quad \frac{1\,430\,016\,000\sqrt{44\,854\,822\,870}\,Xx^{13}\left(Xx^2+Yy^2\right)^3\,Zz}{\left(Xx^2+Yy^2+Zz^2\right)^{11}} + \\
& \quad \left. \frac{929\,510\,400\sqrt{44\,854\,822\,870}\,Xx^{11}\left(Xx^2+Yy^2\right)^4\,Zz}{\left(Xx^2+Yy^2+Zz^2\right)^{11}} - \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{365\,164\,800 \sqrt{44\,854\,822\,870} \, Xx^9 \left(Xx^2 + Yy^2 \right)^5 Zz}{\left(Xx^2 + Yy^2 + Zz^2 \right)^{11}} + \\
& \frac{84\,268\,800 \sqrt{44\,854\,822\,870} \, Xx^7 \left(Xx^2 + Yy^2 \right)^6 Zz}{\left(Xx^2 + Yy^2 + Zz^2 \right)^{11}} - \\
& \frac{10\,533\,600 \sqrt{44\,854\,822\,870} \, Xx^5 \left(Xx^2 + Yy^2 \right)^7 Zz}{\left(Xx^2 + Yy^2 + Zz^2 \right)^{11}} - \\
& \frac{6\,758\,400 \sqrt{286} \, Xx^6 \sqrt{\frac{1}{Xx^2 + Yy^2 + Zz^2}} \left(3 \left(Xx^2 + Yy^2 \right)^2 - 96 \left(Xx^2 + Yy^2 \right) Zz^2 + 224 Zz^4 \right)}{\left(Xx^2 + Yy^2 + Zz^2 \right)^{11/2}} + \\
& 4 \sqrt{5} \, Xx^3 \left(\frac{149\,625 \sqrt{8\,970\,964\,574} \left(Xx^2 + Yy^2 \right)^8 Zz}{\left(Xx^2 + Yy^2 + Zz^2 \right)^{11}} - \frac{262\,144 \sqrt{3}}{\left(Xx^2 + Yy^2 + Zz^2 \right)^{5/2}} \right) + \\
& \sqrt{5} \, Xx \left(- \frac{99\,75 \sqrt{8\,970\,964\,574} \left(Xx^2 + Yy^2 \right)^9 Zz}{\left(Xx^2 + Yy^2 + Zz^2 \right)^{11}} - \frac{524\,288 \sqrt{3} \, Xx^2}{\left(Xx^2 + Yy^2 + Zz^2 \right)^{5/2}} - \right. \\
& \quad \left. \frac{524\,288 \sqrt{3} \, Yy^2}{\left(Xx^2 + Yy^2 + Zz^2 \right)^{5/2}} + \frac{1\,048\,576 \sqrt{3}}{\left(Xx^2 + Yy^2 + Zz^2 \right)^{3/2}} \right) + 2048 \sqrt{13} \, Xx^4 \\
& \left(- \frac{799\,425 \sqrt{22} \, Zz^6 \sqrt{\frac{1}{Xx^2 + Yy^2 + Zz^2}}}{\left(Xx^2 + Yy^2 + Zz^2 \right)^{11/2}} + \frac{1\,472\,625 \sqrt{22} \, Zz^4 \sqrt{\frac{1}{Xx^2 + Yy^2 + Zz^2}}}{\left(Xx^2 + Yy^2 + Zz^2 \right)^{9/2}} + \frac{32 \, Yy}{\left(Xx^2 + Yy^2 + Zz^2 \right)^3} + \right. \\
& \quad \left. \frac{12\,375 \sqrt{22} \sqrt{\frac{1}{Xx^2 + Yy^2 + Zz^2}}}{\left(Xx^2 + Yy^2 + Zz^2 \right)^{5/2}} + Zz^2 \left(- \frac{528 \, Yy}{\left(Xx^2 + Yy^2 + Zz^2 \right)^4} - \frac{408\,375 \sqrt{22} \sqrt{\frac{1}{Xx^2 + Yy^2 + Zz^2}}}{\left(Xx^2 + Yy^2 + Zz^2 \right)^{7/2}} \right) \right) + \\
& 512 \sqrt{13} \, Zz^2 \left(\frac{266\,475 \sqrt{22} \, Zz^8 \sqrt{\frac{1}{Xx^2 + Yy^2 + Zz^2}}}{\left(Xx^2 + Yy^2 + Zz^2 \right)^{11/2}} - \frac{673\,200 \sqrt{22} \, Zz^6 \sqrt{\frac{1}{Xx^2 + Yy^2 + Zz^2}}}{\left(Xx^2 + Yy^2 + Zz^2 \right)^{9/2}} - \frac{104 \, Yy}{\left(Xx^2 + Yy^2 + Zz^2 \right)^2} + \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{12\,375\sqrt{22}\sqrt{\frac{1}{Xx^2+Yy^2+Zz^2}}}{(Xx^2+Yy^2+Zz^2)^{3/2}} + 66\,Zz^4 \left(-\frac{4\,Yy}{(Xx^2+Yy^2+Zz^2)^4} + \frac{8475\sqrt{22}\sqrt{\frac{1}{Xx^2+Yy^2+Zz^2}}}{(Xx^2+Yy^2+Zz^2)^{7/2}} \right) + \\
& 8\,Zz^2 \left(\frac{46\,Yy}{(Xx^2+Yy^2+Zz^2)^3} - \frac{20\,625\sqrt{22}\sqrt{\frac{1}{Xx^2+Yy^2+Zz^2}}}{(Xx^2+Yy^2+Zz^2)^{5/2}} \right) + \\
& 512\sqrt{13}\,Xx^2 \left(\frac{799\,425\sqrt{22}\,Zz^8\sqrt{\frac{1}{Xx^2+Yy^2+Zz^2}}}{(Xx^2+Yy^2+Zz^2)^{11/2}} - \frac{168\,300\sqrt{22}\,Zz^6\sqrt{\frac{1}{Xx^2+Yy^2+Zz^2}}}{(Xx^2+Yy^2+Zz^2)^{9/2}} - \frac{64\,Yy}{(Xx^2+Yy^2+Zz^2)^2} - \right. \\
& \left. \frac{12\,375\sqrt{22}\sqrt{\frac{1}{Xx^2+Yy^2+Zz^2}}}{(Xx^2+Yy^2+Zz^2)^{3/2}} + 66\,Zz^4 \left(-\frac{32\,Yy}{(Xx^2+Yy^2+Zz^2)^4} - \frac{14\,625\sqrt{22}\sqrt{\frac{1}{Xx^2+Yy^2+Zz^2}}}{(Xx^2+Yy^2+Zz^2)^{7/2}} \right) + \right. \\
& \left. Zz^2 \left(\frac{1536\,Yy}{(Xx^2+Yy^2+Zz^2)^3} + \frac{346\,500\sqrt{22}\sqrt{\frac{1}{Xx^2+Yy^2+Zz^2}}}{(Xx^2+Yy^2+Zz^2)^{5/2}} \right) \right) \Bigg/ \frac{1}{65\,536\sqrt{Xx^2+Yy^2}\,(Xx^2+Yy^2+Zz^2)^{11}} \\
& 3\sqrt{\frac{7}{\pi}} \left(4096\sqrt{13}\,(Xx^2+Yy^2)^{5/2}\,Zz\,(13\,(Xx^2+Yy^2)-20\,Zz^2)\,(Xx^2+Yy^2+Zz^2)^7 \cos[4\,\text{ArcTan}[Xx, Yy]] - \right. \\
& 422\,400\sqrt{286}\,(Xx^2+Yy^2)^3\,Zz\sqrt{\frac{1}{Xx^2+Yy^2+Zz^2}}\,(Xx^2+Yy^2+Zz^2)^{11/2} \\
& \left(15\,(Xx^2+Yy^2)^2 - 140\,(Xx^2+Yy^2)\,Zz^2 + 168\,Zz^4 \right) \sin[5\,\text{ArcTan}[Xx, Yy]] - \\
& \sqrt{5}\sqrt{Xx^2+Yy^2}\left(2\,097\,152\sqrt{3}\,Xx\,Yy\,Zz\,(Xx^2+Yy^2+Zz^2)^{17/2} + \right. \\
& \left. 525\sqrt{8\,970\,964\,574}\,(Xx^2+Yy^2)^{10}\sin[20\,\text{ArcTan}[Xx, Yy]] \right) \Bigg\}
\end{aligned}$$

In[123]:= **(* TRANSLATE SOLUTION TO MATLAB *)**

FortranForm[U[[1]]]

Out[123]/FortranForm=

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(Sqrt(7/Pi))*((524288*Sqrt(15)*Yy*(Xx**2 - Yy**2 + 2*Zz**2))/(Xx**2 + Yy**2 + Zz**2)**
- 15*Sqrt(286)*((3072*Yy*(Xx**2 + Yy**2)**1.5*Sqrt(1/(Xx**2 + Yy**2 + Zz**2)))*
- (3*(Xx**2 + Yy**2)**3 - 111*(Xx**2 + Yy**2)**2*Zz**2 + 364*(Xx**2 + Yy**2)*
- (Sqrt(156835045)*Yy*(Xx**2 + Yy**2)**9*Zz*Cos(20*ArcTan(Xx,Yy)))/(Xx**2 + Yy**
- Xx*Zz*((3072*(Xx**2 + Yy**2)**1.5*Zz*Sqrt(1/(Xx**2 + Yy**2 + Zz**2)))*(15*(Xx**
- (Xx**2 + Yy**2 + Zz**2)**4.5 + (Sqrt(156835045)*(Xx**2 + Yy**2)**9*Ssin(20*

```


In[124]:=

FortranForm[U[[2]]]

Out[124]//FortranForm=

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      (Sqrt(7/Pi)*((-524288*Sqrt(15)*Xx*(Xx**2 - Yy**2 - 2*Zz**2))/(Xx**2 + Yy**2 + Zz**2)*
-      15*Sqrt(286)*((-3072*Xx*(Xx**2 + Yy**2)**1.5*Sqrt(1/(Xx**2 + Yy**2 + Zz**2))*
-      (3*(Xx**2 + Yy**2)**3 - 111*(Xx**2 + Yy**2)**2*Zz**2 + 364*(Xx**2 + Yy**2)*
-      (Sqrt(156835045)*Xx*(Xx**2 + Yy**2)**9*Zz*Cos(20*ArcTan(Xx,Yy)))/(Xx**2 + Yy**
-      Yy*Zz*(3072*(Xx**2 + Yy**2)**1.5*Zz*Sqrt(1/(Xx**2 + Yy**2 + Zz**2)))*(15*(Xx**
-      (Xx**2 + Yy**2 + Zz**2)**4.5 + (Sqrt(156835045)*(Xx**2 + Yy**2)**9*Sin(20*

```

In[125]:=

FortranForm[U[[3]]]

Out[125]//FortranForm=

```

      -(Sqrt(7/Pi)*((46080*Sqrt(286)*(Xx**2 + Yy**2)**2.5*Zz*(15*(Xx**2 + Yy**2)**2 - 140*(
-      (1/(Xx**2 + Yy**2 + Zz**2))**5.5 + Sqrt(5)*(2097152*Sqrt(3)*Xx*Yy*Zz*(Xx**2 + Y
-      15*Sqrt(8970964574)*(Xx**2 + Yy**2)**10*Sqrt(Xx**2 + Yy**2 + Zz**2)*Sin(20*Ar

```

In[126]:= **FortranForm**[Pressure]

Out[126]//FortranForm=

```

      (-3*Sqrt(91/Pi)*(Xx**2 + Yy**2)**2*(Xx**2 + Yy**2 - 10*Zz**2)*Cos(4*ArcTan(Xx,Yy)))/(32.*(Xx*

```

In[127]:= **CForm**[U[[1]]]

Out[127]//CForm=

```

      (Sqrt(7/Pi)*((524288*Sqrt(15)*Yy*(Power(Xx,2) - Power(Yy,2) + 2*Power(Zz,2)))/Power(Power(Xx,
-      15*Sqrt(286)*((3072*Yy*Power(Power(Xx,2) + Power(Yy,2),1.5)*Sqrt(1/(Power(Xx,2) + Powe
-      (3*Power(Power(Xx,2) + Power(Yy,2),3) - 111*Power(Power(Xx,2) + Power(Yy,2),2)*P
-      Cos(5*ArcTan(Xx,Yy)))/Power(Power(Xx,2) + Power(Yy,2) + Power(Zz,2),4.5) -
-      (Sqrt(156835045)*Yy*Power(Power(Xx,2) + Power(Yy,2),9)*Zz*Cos(20*ArcTan(Xx,Yy)))/Po
-      Xx*Zz*((3072*Power(Power(Xx,2) + Power(Yy,2),1.5)*Zz*Sqrt(1/(Power(Xx,2) + Power(Yy
-      (15*Power(Power(Xx,2) + Power(Yy,2),2) - 140*(Power(Xx,2) + Power(Yy,2))*Powe
-      Power(Power(Xx,2) + Power(Yy,2) + Power(Zz,2),4.5) + (Sqrt(156835045)*Power(Pow
-      Power(Power(Xx,2) + Power(Yy,2) + Power(Zz,2),10)))))/262144.

```

In[128]:= **CForm**[U[[2]]]

Out[128]//CForm=

```

      (Sqrt(7/Pi)*((-524288*Sqrt(15)*Xx*(Power(Xx,2) - Power(Yy,2) - 2*Power(Zz,2)))/Power(Power(Xx
-      15*Sqrt(286)*((-3072*Xx*Power(Power(Xx,2) + Power(Yy,2),1.5)*Sqrt(1/(Power(Xx,2) + Pow
-      (3*Power(Power(Xx,2) + Power(Yy,2),3) - 111*Power(Power(Xx,2) + Power(Yy,2),2)*P
-      Cos(5*ArcTan(Xx,Yy)))/Power(Power(Xx,2) + Power(Yy,2) + Power(Zz,2),4.5) +
-      (Sqrt(156835045)*Xx*Power(Power(Xx,2) + Power(Yy,2),9)*Zz*Cos(20*ArcTan(Xx,Yy)))/Po
-      Yy*Zz*((3072*Power(Power(Xx,2) + Power(Yy,2),1.5)*Zz*Sqrt(1/(Power(Xx,2) + Power(Yy
-      (15*Power(Power(Xx,2) + Power(Yy,2),2) - 140*(Power(Xx,2) + Power(Yy,2))*Powe
-      Power(Power(Xx,2) + Power(Yy,2) + Power(Zz,2),4.5) + (Sqrt(156835045)*Power(Pow
-      Power(Power(Xx,2) + Power(Yy,2) + Power(Zz,2),10)))))/262144.

```

In[129]:= **CForm**[U[[3]]]

Out[129]//CForm=

```

      -(Sqrt(7/Pi)*((46080*Sqrt(286)*Power(Power(Xx,2) + Power(Yy,2),2.5)*Zz*
-      (15*Power(Power(Xx,2) + Power(Yy,2),2) - 140*(Power(Xx,2) + Power(Yy,2))*Power(Zz,
-      Power(1/(Power(Xx,2) + Power(Yy,2) + Power(Zz,2)),5.5) + Sqrt(5)*
-      (2097152*Sqrt(3)*Xx*Yy*Zz*Power(Power(Xx,2) + Power(Yy,2) + Power(Zz,2),9) +
-      15*Sqrt(8970964574)*Power(Power(Xx,2) + Power(Yy,2),10)*Sqrt(Power(Xx,2) + Power(Y
-      (262144.*Power(Power(Xx,2) + Power(Yy,2) + Power(Zz,2),10.5))

```

In[130]:= **CForm**[Pressure]

Out[130]//CForm=

```

      (-3*Sqrt(91/Pi)*Power(Power(Xx,2) + Power(Yy,2),2)*(Power(Xx,2) + Power(Yy,2) - 10*Power(Zz,2)

```

In[131]:= **(* RHS IN MATLAB *)**

FortranForm[RHS[[1]]]

Out[131]//FortranForm=

```
(3*Sqrt(7/Pi))*((1267200*Sqrt(286)*Xx**9*Yy*Sqrt(1/(Xx**2 + Yy**2 + Zz**2)))/(Xx**2 +
- (10137600*Sqrt(286)*Xx**7*Yy**3*Sqrt(1/(Xx**2 + Yy**2 + Zz**2)))/(Xx**2 + Yy**2 +
- (17740800*Sqrt(286)*Xx**5*Yy**5*Sqrt(1/(Xx**2 + Yy**2 + Zz**2)))/(Xx**2 + Yy**2 +
- (6336000*Sqrt(286)*Xx**Yy**9*Sqrt(1/(Xx**2 + Yy**2 + Zz**2)))/(Xx**2 + Yy**2 + Zz**
- (15206400*Sqrt(286)*Xx**7*Yy**Zz**2*Sqrt(1/(Xx**2 + Yy**2 + Zz**2)))/(Xx**2 + Yy**
- (390297600*Sqrt(286)*Xx**5*Yy**3*Zz**2*Sqrt(1/(Xx**2 + Yy**2 + Zz**2)))/(Xx**2 +
- (177408000*Sqrt(286)*Xx**3*Yy**5*Zz**2*Sqrt(1/(Xx**2 + Yy**2 + Zz**2)))/(Xx**2 +
- (228096000*Sqrt(286)*Xx**Yy**7*Zz**2*Sqrt(1/(Xx**2 + Yy**2 + Zz**2)))/(Xx**2 + Yy**
- (141926400*Sqrt(286)*Xx**5*Yy**Zz**4*Sqrt(1/(Xx**2 + Yy**2 + Zz**2)))/(Xx**2 + Yy**
- (946176000*Sqrt(286)*Xx**3*Yy**3*Zz**4*Sqrt(1/(Xx**2 + Yy**2 + Zz**2)))/(Xx**2 +
- (709632000*Sqrt(286)*Xx**Yy**5*Zz**4*Sqrt(1/(Xx**2 + Yy**2 + Zz**2)))/(Xx**2 + Yy**
- (283852800*Sqrt(286)*Xx**3*Yy**Zz**6*Sqrt(1/(Xx**2 + Yy**2 + Zz**2)))/(Xx**2 + Yy**
- (283852800*Sqrt(286)*Xx**Yy**3*Zz**6*Sqrt(1/(Xx**2 + Yy**2 + Zz**2)))/(Xx**2 + Yy**
- (524288*Sqrt(15)*Yy**3)/(Xx**2 + Yy**2 + Zz**2)**2.5 + (1048576*Sqrt(15)*Yy**Zz**2
- (Sqrt(13)*(9975*Sqrt(3450370990)*Xx**18*Yy**Zz - 508725*Sqrt(3450370990)*Xx**16*Yy
- 26453700*Sqrt(3450370990)*Xx**12*Yy**7*Zz + 48498450*Sqrt(3450370990)*Xx**10
- 14244300*Sqrt(3450370990)*Xx**6*Yy**13*Zz - 2034900*Sqrt(3450370990)*Xx**4*Y
- 525*Sqrt(3450370990)*Yy**19*Zz - 4096*Xx**19*(8*Yy**2 + 13*Zz**2) + 4096*Xx*
- 4096*Xx**17*(56*Yy**4 - 3*Yy**2*Zz**2 + 71*Zz**4) - 28672*Xx**11*(Yy**2 + Zz
- 28672*Xx**13*(Yy**2 + Zz**2)*(32*Yy**6 - 220*Yy**4*Zz**2 - 280*Yy**2*Zz**4 +
- 4096*Xx**3*(Yy**2 + Zz**2)**6*(56*Yy**6 - 445*Yy**4*Zz**2 - 250*Yy**2*Zz**4
- 28672*Xx**7*(Yy**2 + Zz**2)**4*(32*Yy**6 - 4*Yy**4*Zz**2 + 176*Yy**2*Zz**4 +
- 28672*Xx**9*(Yy**2 + Zz**2)**3*(16*Yy**6 + 238*Yy**4*Zz**2 + 448*Yy**2*Zz**4
- 4096*Xx**5*(Yy**2 + Zz**2)**5*(160*Yy**6 - 764*Yy**4*Zz**2 - 104*Yy**2*Zz**4
- 4096*Xx**15*(160*Yy**6 - 356*Yy**4*Zz**2 - 416*Yy**2*Zz**4 + 133*Zz**6)))/(X
```

In[132]:=

FortranForm[RHS[[2]]]

Out[132]//FortranForm=

```
(3*Sqrt(7/Pi))*((137625600*Sqrt(44854822870)*Xx**19*Zz)/(Xx**2 + Yy**2 + Zz**2)**11 -
- (653721600*Sqrt(44854822870)*Xx**17*(Xx**2 + Yy**2)*Zz)/(Xx**2 + Yy**2 + Zz**2)**
- (1307443200*Sqrt(44854822870)*Xx**15*(Xx**2 + Yy**2)**2*Zz)/(Xx**2 + Yy**2 + Zz**
- (1430016000*Sqrt(44854822870)*Xx**13*(Xx**2 + Yy**2)**3*Zz)/(Xx**2 + Yy**2 + Zz**
- (929510400*Sqrt(44854822870)*Xx**11*(Xx**2 + Yy**2)**4*Zz)/(Xx**2 + Yy**2 + Zz**2
- (365164800*Sqrt(44854822870)*Xx**9*(Xx**2 + Yy**2)**5*Zz)/(Xx**2 + Yy**2 + Zz**2)
- (84268800*Sqrt(44854822870)*Xx**7*(Xx**2 + Yy**2)**6*Zz)/(Xx**2 + Yy**2 + Zz**2)*
- (10533600*Sqrt(44854822870)*Xx**5*(Xx**2 + Yy**2)**7*Zz)/(Xx**2 + Yy**2 + Zz**2)*
- (6758400*Sqrt(286)*Xx**6*Sqrt(1/(Xx**2 + Yy**2 + Zz**2)))*(3*(Xx**2 + Yy**2)**2 -
- 4*Sqrt(5)*Xx**3*(149625*Sqrt(8970964574)*(Xx**2 + Yy**2)**8*Zz)/(Xx**2 + Yy**2 +
- Sqrt(5)*Xx*((-9975*Sqrt(8970964574)*(Xx**2 + Yy**2)**9*Zz)/(Xx**2 + Yy**2 + Zz**2
- (524288*Sqrt(3)*Yy**2)/(Xx**2 + Yy**2 + Zz**2)**2.5 + (1048576*Sqrt(3))/(Xx**2
- 2048*Sqrt(13)*Xx**4*((-799425*Sqrt(22)*Zz**6*Sqrt(1/(Xx**2 + Yy**2 + Zz**2)))/(Xx
- (1472625*Sqrt(22)*Zz**4*Sqrt(1/(Xx**2 + Yy**2 + Zz**2)))/(Xx**2 + Yy**2 + Zz**2)
- (12375*Sqrt(22)*Sqrt(1/(Xx**2 + Yy**2 + Zz**2)))/(Xx**2 + Yy**2 + Zz**2)**2.5
- Zz**2*((-528*Yy)/(Xx**2 + Yy**2 + Zz**2)**4 - (408375*Sqrt(22)*Sqrt(1/(Xx**2 +
- 512*Sqrt(13)*Zz**2*((266475*Sqrt(22)*Zz**8*Sqrt(1/(Xx**2 + Yy**2 + Zz**2)))/(Xx**
- (673200*Sqrt(22)*Zz**6*Sqrt(1/(Xx**2 + Yy**2 + Zz**2)))/(Xx**2 + Yy**2 + Zz**2)
- (12375*Sqrt(22)*Sqrt(1/(Xx**2 + Yy**2 + Zz**2)))/(Xx**2 + Yy**2 + Zz**2)**1.5
- 66*Zz**4*((-4*Yy)/(Xx**2 + Yy**2 + Zz**2)**4 + (8475*Sqrt(22)*Sqrt(1/(Xx**2 +
- 8*Zz**2*((46*Yy)/(Xx**2 + Yy**2 + Zz**2)**3 - (20625*Sqrt(22)*Sqrt(1/(Xx**2 +
- 512*Sqrt(13)*Xx**2*((799425*Sqrt(22)*Zz**8*Sqrt(1/(Xx**2 + Yy**2 + Zz**2)))/(Xx**
- (168300*Sqrt(22)*Zz**6*Sqrt(1/(Xx**2 + Yy**2 + Zz**2)))/(Xx**2 + Yy**2 + Zz**2)
- (12375*Sqrt(22)*Sqrt(1/(Xx**2 + Yy**2 + Zz**2)))/(Xx**2 + Yy**2 + Zz**2)**1.5
- 66*Zz**4*((-32*Yy)/(Xx**2 + Yy**2 + Zz**2)**4 - (14625*Sqrt(22)*Sqrt(1/(Xx**2
- Zz**2*((1536*Yy)/(Xx**2 + Yy**2 + Zz**2)**3 + (346500*Sqrt(22)*Sqrt(1/(Xx**2 +
```

In[133]:=

FortranForm[RHS[[3]]]

Out[133]//FortranForm=

```
(3*Sqrt(7/Pi))*(4096*Sqrt(13))*(Xx**2 + Yy**2)**2.5*Zz*(13*(Xx**2 + Yy**2) - 20*Zz**2)*
- 422400*Sqrt(286)*(Xx**2 + Yy**2)**3*Zz*Sqrt(1/(Xx**2 + Yy**2 + Zz**2))*(Xx**2 + Y
- (15*(Xx**2 + Yy**2)**2 - 140*(Xx**2 + Yy**2)*Zz**2 + 168*Zz**4)*Sin(5*ArcTan(Xx,
- Sqrt(5)*Sqrt(Xx**2 + Yy**2))*(2097152*Sqrt(3)*Xx*Yy*Zz*(Xx**2 + Yy**2 + Zz**2)**8.
- (65536.*Sqrt(Xx**2 + Yy**2)*(Xx**2 + Yy**2 + Zz**2)**11)
```

In[134]:=

CForm[RHS[[1]]]

Out[134]/CForm=

```

(3*Sqrt(7/Pi)*((1267200*Sqrt(286)*Power(Xx,9)*Yy*Sqrt(1/(Power(Xx,2) + Power(Yy,2) + Power(Zz,2))
(10137600*Sqrt(286)*Power(Xx,7)*Power(Yy,3)*Sqrt(1/(Power(Xx,2) + Power(Yy,2) + Power(Zz,2))
(17740800*Sqrt(286)*Power(Xx,5)*Power(Yy,5)*Sqrt(1/(Power(Xx,2) + Power(Yy,2) + Power(Zz,2))
(6336000*Sqrt(286)*Xx*Power(Yy,9)*Sqrt(1/(Power(Xx,2) + Power(Yy,2) + Power(Zz,2))))/P
(15206400*Sqrt(286)*Power(Xx,7)*Yy*Power(Zz,2)*Sqrt(1/(Power(Xx,2) + Power(Yy,2) + Pow
(390297600*Sqrt(286)*Power(Xx,5)*Power(Yy,3)*Power(Zz,2)*Sqrt(1/(Power(Xx,2) + Power(Y
(177408000*Sqrt(286)*Power(Xx,3)*Power(Yy,5)*Power(Zz,2)*Sqrt(1/(Power(Xx,2) + Power(Y
(228096000*Sqrt(286)*Xx*Power(Yy,7)*Power(Zz,2)*Sqrt(1/(Power(Xx,2) + Power(Yy,2) + Po
(141926400*Sqrt(286)*Power(Xx,5)*Yy*Power(Zz,4)*Sqrt(1/(Power(Xx,2) + Power(Yy,2) + Po
(946176000*Sqrt(286)*Power(Xx,3)*Power(Yy,3)*Power(Zz,4)*Sqrt(1/(Power(Xx,2) + Power(Y
(709632000*Sqrt(286)*Xx*Power(Yy,5)*Power(Zz,4)*Sqrt(1/(Power(Xx,2) + Power(Yy,2) + Po
(283852800*Sqrt(286)*Power(Xx,3)*Yy*Power(Zz,6)*Sqrt(1/(Power(Xx,2) + Power(Yy,2) + Po
(283852800*Sqrt(286)*Xx*Power(Yy,3)*Power(Zz,6)*Sqrt(1/(Power(Xx,2) + Power(Yy,2) + Po
(524288*Sqrt(15)*Power(Xx,2)*Yy)/Power(Power(Xx,2) + Power(Yy,2) + Power(Zz,2),2.5) -
(524288*Sqrt(15)*Power(Yy,3))/Power(Power(Xx,2) + Power(Yy,2) + Power(Zz,2),2.5) +
(1048576*Sqrt(15)*Yy*Power(Zz,2))/Power(Power(Xx,2) + Power(Yy,2) + Power(Zz,2),2.5) +
(Sqrt(13)*(9975*Sqrt(3450370990)*Power(Xx,18)*Yy*Zz - 508725*Sqrt(3450370990)*Power(Xx
26453700*Sqrt(3450370990)*Power(Xx,12)*Power(Yy,7)*Zz + 48498450*Sqrt(3450370990)
39680550*Sqrt(3450370990)*Power(Xx,8)*Power(Yy,11)*Zz + 14244300*Sqrt(3450370990)
2034900*Sqrt(3450370990)*Power(Xx,4)*Power(Yy,15)*Zz + 89775*Sqrt(3450370990)*Pow
4096*Power(Xx,19)*(8*Power(Yy,2) + 13*Power(Zz,2)) + 4096*Xx*Power(Yy,2)*Power(Po
(8*Power(Yy,4) - 85*Power(Yy,2)*Power(Zz,2) - 60*Power(Zz,4)) - 4096*Power(Xx,17
28672*Power(Xx,11)*Power(Power(Yy,2) + Power(Zz,2),2)*(16*Power(Yy,6) - 350*Power
28672*Power(Xx,13)*(Power(Yy,2) + Power(Zz,2))*(32*Power(Yy,6) - 220*Power(Yy,4)*
4096*Power(Xx,3)*Power(Power(Yy,2) + Power(Zz,2),6)*(56*Power(Yy,6) - 445*Power(Y
28672*Power(Xx,7)*Power(Power(Yy,2) + Power(Zz,2),4)*(32*Power(Yy,6) - 4*Power(Yy
28672*Power(Xx,9)*Power(Power(Yy,2) + Power(Zz,2),3)*(16*Power(Yy,6) + 238*Power(
4096*Power(Xx,5)*Power(Power(Yy,2) + Power(Zz,2),5)*(160*Power(Yy,6) - 764*Power(
4096*Power(Xx,15)*(160*Power(Yy,6) - 356*Power(Yy,4)*Power(Zz,2) - 416*Power(Yy,2
Power(Power(Xx,2) + Power(Yy,2) + Power(Zz,2),11)))/65536.

```



```
In[147]:= TeXForm[RHS[[2]]]
```

Out[147]//TeXForm=

[illegible]

```
In[146]:= TeXForm[RHS[[3]]]
```

Out[146]//TeXForm=

[illegible]