

```
In[1]:= Needs["VectorAnalysis`"]  
SetCoordinates[Cartesian]  
Element[{Ttheta, Pphi}, Reals]  
Element[{ $\phi$ ,  $\theta$ }, Reals]
```

```
Out[2]= Cartesian[Xx, Yy, Zz]
```

```
Out[3]= (Ttheta | Pphi)  $\in$  Reals
```

```
Out[4]= ( $\phi$  |  $\theta$ )  $\in$  Reals
```

```

In[5]:= {r, th, ph} =
  Simplify[CoordinatesFromCartesian[{Xx, Yy, Zz}, Spherical]]

Out[5]:=  $\left\{ \sqrt{Xx^2 + Yy^2 + Zz^2}, \text{ArcCos}\left[\frac{Zz}{\sqrt{Xx^2 + Yy^2 + Zz^2}}\right], \text{ArcTan}[Xx, Yy] \right\}$ 

In[6]:= (* Setup some functions for Matlab equivalent spherical
  harmonics. Note: matlab is off by sqrt(2) for normalization,
  AND the Ttheta are [0,pi] (should be [-pi/2,pi/2]) *)
sphFull[l_, m_] := ComplexExpand[
  Re[Sqrt[2] * SphericalHarmonicY[l, m, Ttheta, Pphi]]] // Simplify
(*Give the Spherical Harmonic in Cartesian Coordinates.
  NOTE: MATLAB USES "angle(...)" instead of "Arg[...]"*)
sphFullCart[l_, m_] :=
  ComplexExpand[Re[Sqrt[2] * SphericalHarmonicY[l, m, th, ph]],
    TargetFunctions -> {Re, Im}] // FullSimplify
sphFullCart[l1_, m1_, l2_, m2_] :=
  sphFullCart[l1, m1] + sphFullCart[l2, m2]
sphFull[l1_, m1_, l2_, m2_] :=
  ComplexExpand[Re[Sqrt[2] * SphericalHarmonicY[l1, m1, Ttheta, Pphi] +
    Sqrt[2] * SphericalHarmonicY[l2, m2, Ttheta, Pphi]]] // Simplify

sphLapl[l_, m_] :=
  Laplacian[sphFull[l, m], Spherical] /. Rr -> 1 // Simplify
sphLapl[l1_, m1_, l2_, m2_] :=
  Laplacian[sphFull[l1, m1, l2, m2], Spherical] /. Rr -> 1 // Simplify
sphLaplCart[l_, m_] :=
  Laplacian[sphFullCart[l, m], Cartesian] // FullSimplify
sphLaplCart[l1_, m1_, l2_, m2_] :=
  Laplacian[sphFullCart[l1, m1, l2, m2], Cartesian] // FullSimplify

sphGradCart[l_, m_] :=
  Grad[sphFullCart[l, m], Cartesian] // FullSimplify
sphGradCart[l1_, m1_, l2_, m2_] :=
  Grad[sphFullCart[l1, m1, l2, m2], Cartesian] // FullSimplify

In[16]:= Clear[U, G, H, Qx, Px, Mm, Nn, Pp, Ff, Gg, Hh]

In[17]:= (* Let Qx be the curl matrix which guarantees that Div(Q * Grad(g(x,y,z))) =
  0. where g(x,y,z) is ANY function (i.e., Spherical Harmonics) *)

In[18]:= Qx := {{0, -Zz, Yy}, {Zz, 0, -Xx}, {-Yy, Xx, 0}}

In[19]:= (* Projection Matrix to Constrain To Sphere *)
Px := {{1 - Xx^2, -Xx * Yy, -Xx * Zz},
  {-Xx * Yy, 1 - Yy^2, -Yy * Zz}, {-Xx * Zz, -Yy * Zz, 1 - Zz^2}}

In[20]:= (* Choose a function g(x,y,z) and pressure *)
G := 8 * sphFullCart[3, 2] - 3 * sphFullCart[10, 5] + sphFullCart[20, 20]
Pressure := sphFullCart[10, 5]

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In[22]:= (* Get our projected gradient function *)  
H := Px.Grad[G, Cartesian]
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In[23]:= (* THIS IS THE MANUFACTURED SOLUTION *)  
U = Qx . H // FullSimplify
```

$$\begin{aligned}
\text{Out[23]} = & \left\{ \frac{1}{262144} \sqrt{\frac{7}{\pi}} \left( \frac{524288 \sqrt{15} \text{Yy} (\text{Xx}^2 - \text{Yy}^2 + 2 \text{Zz}^2)}{(\text{Xx}^2 + \text{Yy}^2 + \text{Zz}^2)^{3/2}} + \right. \right. \\
& 15 \sqrt{286} \left( \frac{1}{(\text{Xx}^2 + \text{Yy}^2 + \text{Zz}^2)^{9/2}} 3072 \text{Yy} (\text{Xx}^2 + \text{Yy}^2)^{3/2} \sqrt{\frac{1}{\text{Xx}^2 + \text{Yy}^2 + \text{Zz}^2}} \right. \\
& \left. \left( 3 (\text{Xx}^2 + \text{Yy}^2)^3 - 111 (\text{Xx}^2 + \text{Yy}^2)^2 \text{Zz}^2 + 364 (\text{Xx}^2 + \text{Yy}^2) \text{Zz}^4 - 168 \text{Zz}^6 \right) \text{Cos}[5 \text{ArcTan}[\text{Xx}, \text{Yy}]] - \right. \\
& \frac{\sqrt{156835045} \text{Yy} (\text{Xx}^2 + \text{Yy}^2)^9 \text{Zz} \text{Cos}[20 \text{ArcTan}[\text{Xx}, \text{Yy}]]}{(\text{Xx}^2 + \text{Yy}^2 + \text{Zz}^2)^{10}} + \text{Xx} \text{Zz} \left( \frac{1}{(\text{Xx}^2 + \text{Yy}^2 + \text{Zz}^2)^{9/2}} \right. \\
& 3072 (\text{Xx}^2 + \text{Yy}^2)^{3/2} \text{Zz} \sqrt{\frac{1}{\text{Xx}^2 + \text{Yy}^2 + \text{Zz}^2}} \left( 15 (\text{Xx}^2 + \text{Yy}^2)^2 - 140 (\text{Xx}^2 + \text{Yy}^2) \text{Zz}^2 + 168 \text{Zz}^4 \right) \\
& \left. \left. \left. \text{Sin}[5 \text{ArcTan}[\text{Xx}, \text{Yy}]] + \frac{\sqrt{156835045} (\text{Xx}^2 + \text{Yy}^2)^9 \text{Sin}[20 \text{ArcTan}[\text{Xx}, \text{Yy}]]}{(\text{Xx}^2 + \text{Yy}^2 + \text{Zz}^2)^{10}} \right) \right) \right), \\
& \frac{1}{262144} \sqrt{\frac{7}{\pi}} \left( - \frac{524288 \sqrt{15} \text{Xx} (\text{Xx}^2 - \text{Yy}^2 - 2 \text{Zz}^2)}{(\text{Xx}^2 + \text{Yy}^2 + \text{Zz}^2)^{3/2}} + 15 \sqrt{286} \right. \\
& \left( - \frac{1}{(\text{Xx}^2 + \text{Yy}^2 + \text{Zz}^2)^{9/2}} 3072 \text{Xx} (\text{Xx}^2 + \text{Yy}^2)^{3/2} \sqrt{\frac{1}{\text{Xx}^2 + \text{Yy}^2 + \text{Zz}^2}} \right. \\
& \left. \left( 3 (\text{Xx}^2 + \text{Yy}^2)^3 - 111 (\text{Xx}^2 + \text{Yy}^2)^2 \text{Zz}^2 + 364 (\text{Xx}^2 + \text{Yy}^2) \text{Zz}^4 - 168 \text{Zz}^6 \right) \text{Cos}[5 \text{ArcTan}[\text{Xx}, \text{Yy}]] + \right. \\
& \frac{\sqrt{156835045} \text{Xx} (\text{Xx}^2 + \text{Yy}^2)^9 \text{Zz} \text{Cos}[20 \text{ArcTan}[\text{Xx}, \text{Yy}]]}{(\text{Xx}^2 + \text{Yy}^2 + \text{Zz}^2)^{10}} + \\
& \text{Yy} \text{Zz} \left( \frac{1}{(\text{Xx}^2 + \text{Yy}^2 + \text{Zz}^2)^{9/2}} 3072 (\text{Xx}^2 + \text{Yy}^2)^{3/2} \text{Zz} \sqrt{\frac{1}{\text{Xx}^2 + \text{Yy}^2 + \text{Zz}^2}} \right. \\
& \left. \left( 15 (\text{Xx}^2 + \text{Yy}^2)^2 - 140 (\text{Xx}^2 + \text{Yy}^2) \text{Zz}^2 + 168 \text{Zz}^4 \right) \text{Sin}[5 \text{ArcTan}[\text{Xx}, \text{Yy}]] + \right. \\
& \left. \left. \left. \frac{\sqrt{156835045} (\text{Xx}^2 + \text{Yy}^2)^9 \text{Sin}[20 \text{ArcTan}[\text{Xx}, \text{Yy}]]}{(\text{Xx}^2 + \text{Yy}^2 + \text{Zz}^2)^{10}} \right) \right) \right), - \frac{1}{262144 (\text{Xx}^2 + \text{Yy}^2 + \text{Zz}^2)^{21/2}} \sqrt{\frac{7}{\pi}} \\
& \left( \frac{46080 \sqrt{286} (\text{Xx}^2 + \text{Yy}^2)^{5/2} \text{Zz} (15 (\text{Xx}^2 + \text{Yy}^2)^2 - 140 (\text{Xx}^2 + \text{Yy}^2) \text{Zz}^2 + 168 \text{Zz}^4) \text{Sin}[5 \text{ArcTan}[\text{Xx}, \text{Yy}]]}{\left( \frac{1}{\text{Xx}^2 + \text{Yy}^2 + \text{Zz}^2} \right)^{11/2}} \right. \\
& + \sqrt{5} \left( 2097152 \sqrt{3} \text{Xx} \text{Yy} \text{Zz} (\text{Xx}^2 + \text{Yy}^2 + \text{Zz}^2)^9 + \right. \\
& \left. \left. 15 \sqrt{8970964574} (\text{Xx}^2 + \text{Yy}^2)^{10} \sqrt{\text{Xx}^2 + \text{Yy}^2 + \text{Zz}^2} \text{Sin}[20 \text{ArcTan}[\text{Xx}, \text{Yy}]] \right) \right\}
\end{aligned}$$

```
In[24]:= (* Prove that our divergence of this field is zero: *)
Div[U] // FullSimplify
```

```
Out[24]= 0
```

```
In[25]:= (* Use projection matrix to get Laplacian of our solution in each U,V,W direction. *)
(* Note: to get nth component of U: U[[n]] *)
GradRHS :=
{Px.Grad[U[[1]], Cartesian], Px.Grad[U[[2]], Cartesian], Px.Grad[U[[3]], Cartesian]}
(* THIS IS A MANUFACTURED RHS *)
LaplRHS := Px.{Div[GradRHS[[1]], Cartesian],
Div[GradRHS[[2]], Cartesian], Div[GradRHS[[3]], Cartesian]}
GradP := Px.Grad[Pressure, Cartesian]
```

```
In[28]:= RHS = -LaplRHS + GradP // FullSimplify
```

$$\text{Out[28]= } \left\{ \frac{1}{65536 \sqrt{Xx^2 + Yy^2} (Xx^2 + Yy^2 + Zz^2)^{21/2}} \right. \\ \left. 3 \sqrt{\frac{7}{\pi}} \sqrt{\frac{1}{Xx^2 + Yy^2 + Zz^2}} \left( \frac{524288 \sqrt{15} Yy \sqrt{Xx^2 + Yy^2} (Xx^2 - Yy^2 + 2 Zz^2)}{\left(\frac{1}{Xx^2 + Yy^2 + Zz^2}\right)^{17/2}} + \right. \right. \\ \left. 5 \sqrt{286} \left( 256 (Xx^2 + Yy^2)^2 (330 Yy + Xx Zz) (Xx^2 + Yy^2 + Zz^2)^5 \right. \right. \\ \left. \left( 3 (Xx^2 + Yy^2)^3 - 111 (Xx^2 + Yy^2)^2 Zz^2 + 364 (Xx^2 + Yy^2) Zz^4 - 168 Zz^6 \right) \cos[5 \text{ArcTan}[Xx, Yy]] + \right. \\ \left. Zz \left( -105 \sqrt{156835045} Yy (Xx^2 + Yy^2)^{19/2} \sqrt{\frac{1}{Xx^2 + Yy^2 + Zz^2}} \sqrt{Xx^2 + Yy^2 + Zz^2} \cos[ \right. \right. \\ \left. 20 \text{ArcTan}[Xx, Yy]] - 256 (Xx^2 + Yy^2)^2 (Xx^2 + Yy^2 + Zz^2)^5 (Xx^2 Yy + Yy^3 - 330 Xx Zz + Yy Zz^2) \right. \\ \left. \left( 15 (Xx^2 + Yy^2)^2 - 140 (Xx^2 + Yy^2) Zz^2 + 168 Zz^4 \right) \sin[5 \text{ArcTan}[Xx, Yy]] + 105 \sqrt{156835045} \right. \\ \left. Xx (Xx^2 + Yy^2)^{19/2} \sqrt{\frac{1}{Xx^2 + Yy^2 + Zz^2}} \sqrt{Xx^2 + Yy^2 + Zz^2} \sin[20 \text{ArcTan}[Xx, Yy]] \right) \left. \right) \left. \right) , \\ - \frac{1}{65536 \sqrt{Xx^2 + Yy^2} (Xx^2 + Yy^2 + Zz^2)^{21/2}} 3 \sqrt{\frac{7}{\pi}} \sqrt{\frac{1}{Xx^2 + Yy^2 + Zz^2}}$$

$$\begin{aligned}
& \left( \frac{524\,288\sqrt{15}\,Xx\sqrt{Xx^2+Yy^2}\,(Xx^2-Yy^2-2\,Zz^2)}{\left(\frac{1}{Xx^2+Yy^2+Zz^2}\right)^{17/2}} + \right. \\
& 5\sqrt{286}\left( 256\,(Xx^2+Yy^2)^2\,(330\,Xx-Yy\,Zz)\,(Xx^2+Yy^2+Zz^2)^5 \right. \\
& \left. \left( 3\,(Xx^2+Yy^2)^3 - 111\,(Xx^2+Yy^2)^2\,Zz^2 + 364\,(Xx^2+Yy^2)\,Zz^4 - 168\,Zz^6 \right) \cos[5\,\text{ArcTan}[Xx, Yy]] + \right. \\
& Zz\left( -105\sqrt{156\,835\,045}\,Xx\,(Xx^2+Yy^2)^{19/2}\sqrt{\frac{1}{Xx^2+Yy^2+Zz^2}}\sqrt{Xx^2+Yy^2+Zz^2}\cos[20\,\text{ArcTan}[ \right. \\
& \quad Xx, Yy]] - 256\,(Xx^2+Yy^2)^2\,(Xx^2+Yy^2+Zz^2)^5\left( 15\,(Xx^2+Yy^2)^2 - 140\,(Xx^2+Yy^2)\,Zz^2 + \right. \\
& \quad \left. 168\,Zz^4 \right) (Xx^3 + 330\,Yy\,Zz + Xx\,(Yy^2+Zz^2))\sin[5\,\text{ArcTan}[Xx, Yy]] - 105\sqrt{156\,835\,045} \\
& \quad \left. \left. \left. Yy\,(Xx^2+Yy^2)^{19/2}\sqrt{\frac{1}{Xx^2+Yy^2+Zz^2}}\sqrt{Xx^2+Yy^2+Zz^2}\sin[20\,\text{ArcTan}[Xx, Yy]] \right) \right) \right) \Bigg), \\
& - \frac{1}{65\,536\sqrt{Xx^2+Yy^2}\,(Xx^2+Yy^2+Zz^2)^{21/2}} 3\sqrt{\frac{7}{\pi}}\sqrt{\frac{1}{Xx^2+Yy^2+Zz^2}} \\
& \left( \frac{2\,097\,152\sqrt{15}\,Xx\,Yy\sqrt{Xx^2+Yy^2}\,Zz}{\left(\frac{1}{Xx^2+Yy^2+Zz^2}\right)^{17/2}} + \right. \\
& 5\sqrt{286}\left( 256\,(Xx^2+Yy^2)^3\,(Xx^2+Yy^2+Zz^2)^5 \right. \\
& \left. \left( 3\,(Xx^2+Yy^2)^3 - 111\,(Xx^2+Yy^2)^2\,Zz^2 + 364\,(Xx^2+Yy^2)\,Zz^4 - 168\,Zz^6 \right) \cos[5\,\text{ArcTan}[Xx, Yy]] + \right. \\
& 15\left( 5632\,(Xx^2+Yy^2)^3\,Zz\,(Xx^2+Yy^2+Zz^2)^5\left( 15\,(Xx^2+Yy^2)^2 - 140\,(Xx^2+Yy^2)\,Zz^2 + 168\,Zz^4 \right) \right. \\
& \quad \left. \sin[5\,\text{ArcTan}[Xx, Yy]] + 7\sqrt{156\,835\,045}\,(Xx^2+Yy^2)^{21/2} \right. \\
& \quad \left. \left. \left. \sqrt{\frac{1}{Xx^2+Yy^2+Zz^2}}\sqrt{Xx^2+Yy^2+Zz^2}\sin[20\,\text{ArcTan}[Xx, Yy]] \right) \right) \right) \Bigg) \Bigg\}
\end{aligned}$$

In[29]:= **( \* TRANSLATE SOLUTION TO MATLAB \*)**

**FortranForm[U[[1]]]**

Out[29]/FortranForm=

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(Sqrt(7/Pi))*((524288*Sqrt(15)*Yy*(Xx**2 - Yy**2 + 2*Zz**2))/(Xx**2 + Yy**2 + Zz**2)**
- 15*Sqrt(286)*((3072*Yy*(Xx**2 + Yy**2)**1.5*Sqrt(1/(Xx**2 + Yy**2 + Zz**2)))*
- (3*(Xx**2 + Yy**2)**3 - 111*(Xx**2 + Yy**2)**2*Zz**2 + 364*(Xx**2 + Yy**2)*
- (Sqrt(156835045)*Yy*(Xx**2 + Yy**2)**9*Zz*Cos(20*ArcTan(Xx,Yy)))/(Xx**2 + Yy**
- Xx*Zz*((3072*(Xx**2 + Yy**2)**1.5*Zz*Sqrt(1/(Xx**2 + Yy**2 + Zz**2)))*(15*(Xx**
- (Xx**2 + Yy**2 + Zz**2)**4.5 + (Sqrt(156835045)*(Xx**2 + Yy**2)**9*Ssin(20*

```

In[30]:=

**FortranForm**[U[[2]]]

Out[30]//FortranForm=

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(Sqrt(7/Pi)*((-524288*Sqrt(15)*Xx*(Xx**2 - Yy**2 - 2*Zz**2))/(Xx**2 + Yy**2 + Zz**2)*
- 15*Sqrt(286)*((-3072*Xx*(Xx**2 + Yy**2)**1.5*Sqrt(1/(Xx**2 + Yy**2 + Zz**2))*
- (3*(Xx**2 + Yy**2)**3 - 111*(Xx**2 + Yy**2)**2*Zz**2 + 364*(Xx**2 + Yy**2)*
- (Sqrt(156835045)*Xx*(Xx**2 + Yy**2)**9*Zz*Cos(20*ArcTan(Xx,Yy)))/(Xx**2 + Yy**
- Yy*Zz*(3072*(Xx**2 + Yy**2)**1.5*Zz*Sqrt(1/(Xx**2 + Yy**2 + Zz**2)))*(15*(Xx**
- (Xx**2 + Yy**2 + Zz**2)**4.5 + (Sqrt(156835045)*(Xx**2 + Yy**2)**9*Sin(20*Ar

```

In[31]:=

**FortranForm**[U[[3]]]

Out[31]//FortranForm=

```

-(Sqrt(7/Pi)*((46080*Sqrt(286)*(Xx**2 + Yy**2)**2.5*Zz*(15*(Xx**2 + Yy**2)**2 - 140*(
- (1/(Xx**2 + Yy**2 + Zz**2))**5.5 + Sqrt(5)*(2097152*Sqrt(3)*Xx*Yy*Zz*(Xx**2 + Y
- 15*Sqrt(8970964574)*(Xx**2 + Yy**2)**10*Sqrt(Xx**2 + Yy**2 + Zz**2)*Sin(20*Ar

```

In[38]:= **CForm**[U[[1]]]

Out[38]//CForm=

```

(Sqrt(7/Pi)*((524288*Sqrt(15)*Yy*(Power(Xx,2) - Power(Yy,2) + 2*Power(Zz,2)))/Power(Power(Xx,
15*Sqrt(286)*((3072*Yy*Power(Power(Xx,2) + Power(Yy,2),1.5)*Sqrt(1/(Power(Xx,2) + Powe
(3*Power(Power(Xx,2) + Power(Yy,2),3) - 111*Power(Power(Xx,2) + Power(Yy,2),2)*P
Cos(5*ArcTan(Xx,Yy)))/Power(Power(Xx,2) + Power(Yy,2) + Power(Zz,2),4.5) -
(Sqrt(156835045)*Yy*Power(Power(Xx,2) + Power(Yy,2),9)*Zz*Cos(20*ArcTan(Xx,Yy)))/Po
Xx*Zz*((3072*Power(Power(Xx,2) + Power(Yy,2),1.5)*Zz*Sqrt(1/(Power(Xx,2) + Power(Yy
(15*Power(Power(Xx,2) + Power(Yy,2),2) - 140*(Power(Xx,2) + Power(Yy,2))*Powe
Power(Power(Xx,2) + Power(Yy,2) + Power(Zz,2),4.5) + (Sqrt(156835045)*Power(Pow
Power(Power(Xx,2) + Power(Yy,2) + Power(Zz,2),10)))))/262144.

```

In[39]:= **CForm**[U[[2]]]

Out[39]//CForm=

```

(Sqrt(7/Pi)*((-524288*Sqrt(15)*Xx*(Power(Xx,2) - Power(Yy,2) - 2*Power(Zz,2)))/Power(Power(Xx
15*Sqrt(286)*((-3072*Xx*Power(Power(Xx,2) + Power(Yy,2),1.5)*Sqrt(1/(Power(Xx,2) + Pow
(3*Power(Power(Xx,2) + Power(Yy,2),3) - 111*Power(Power(Xx,2) + Power(Yy,2),2)*P
Cos(5*ArcTan(Xx,Yy)))/Power(Power(Xx,2) + Power(Yy,2) + Power(Zz,2),4.5) +
(Sqrt(156835045)*Xx*Power(Power(Xx,2) + Power(Yy,2),9)*Zz*Cos(20*ArcTan(Xx,Yy)))/Po
Yy*Zz*((3072*Power(Power(Xx,2) + Power(Yy,2),1.5)*Zz*Sqrt(1/(Power(Xx,2) + Power(Yy
(15*Power(Power(Xx,2) + Power(Yy,2),2) - 140*(Power(Xx,2) + Power(Yy,2))*Powe
Power(Power(Xx,2) + Power(Yy,2) + Power(Zz,2),4.5) + (Sqrt(156835045)*Power(Pow
Power(Power(Xx,2) + Power(Yy,2) + Power(Zz,2),10)))))/262144.

```

In[40]:= **CForm**[U[[3]]]

Out[40]//CForm=

```

-(Sqrt(7/Pi)*((46080*Sqrt(286)*Power(Power(Xx,2) + Power(Yy,2),2.5)*Zz*
(15*Power(Power(Xx,2) + Power(Yy,2),2) - 140*(Power(Xx,2) + Power(Yy,2))*Power(Zz,
Power(1/(Power(Xx,2) + Power(Yy,2) + Power(Zz,2)),5.5) + Sqrt(5)*
(2097152*Sqrt(3)*Xx*Yy*Zz*Power(Power(Xx,2) + Power(Yy,2) + Power(Zz,2),9) +
15*Sqrt(8970964574)*Power(Power(Xx,2) + Power(Yy,2),10)*Sqrt(Power(Xx,2) + Power(Y
(262144.*Power(Power(Xx,2) + Power(Yy,2) + Power(Zz,2),10.5))

```

In[32]:= **( \* RHS IN MATLAB \* )****FortranForm**[RHS[[1]]]

Out[32]//FortranForm=

```

(3*Sqrt(7/Pi)*Sqrt(1/(Xx**2 + Yy**2 + Zz**2))*((524288*Sqrt(15)*Yy*Sqrt(Xx**2 + Yy**2
- 5*Sqrt(286)*(256*(Xx**2 + Yy**2)**2*(330*Yy + Xx*Zz)*(Xx**2 + Yy**2 + Zz**2)**5*
- (3*(Xx**2 + Yy**2)**3 - 111*(Xx**2 + Yy**2)**2*Zz**2 + 364*(Xx**2 + Yy**2)*Zz
- Zz*(-105*Sqrt(156835045)*Yy*(Xx**2 + Yy**2)**9.5*Sqrt(1/(Xx**2 + Yy**2 + Zz**2
- 256*(Xx**2 + Yy**2)**2*(Xx**2 + Yy**2 + Zz**2)**5*(Xx**2*Yy + Yy**3 - 330*X
- (15*(Xx**2 + Yy**2)**2 - 140*(Xx**2 + Yy**2)*Zz**2 + 168*Zz**4)*Sin(5*ArcT
- 105*Sqrt(156835045)*Xx*(Xx**2 + Yy**2)**9.5*Sqrt(1/(Xx**2 + Yy**2 + Zz**2))
- (65536.*Sqrt(Xx**2 + Yy**2)*(Xx**2 + Yy**2 + Zz**2)**10.5)

```

In[33]:=

**FortranForm[RHS[[2]]]**

Out[33]//FortranForm=

```
(-3*Sqrt(7/Pi)*Sqrt(1/(Xx**2 + Yy**2 + Zz**2)))*((524288*Sqrt(15)*Xx*Sqrt(Xx**2 + Yy**
5*Sqrt(286)*(256*(Xx**2 + Yy**2)**2*(330*Xx - Yy*Zz)*(Xx**2 + Yy**2 + Zz**2)**5*
- (3*(Xx**2 + Yy**2)**3 - 111*(Xx**2 + Yy**2)**2*Zz**2 + 364*(Xx**2 + Yy**2)*Zz
Zz*(-105*Sqrt(156835045)*Xx*(Xx**2 + Yy**2)**9.5*Sqrt(1/(Xx**2 + Yy**2 + Zz**2)
- 256*(Xx**2 + Yy**2)**2*(Xx**2 + Yy**2 + Zz**2)**5*(15*(Xx**2 + Yy**2)**2 -
Sin(5*ArcTan(Xx,Yy)) - 105*Sqrt(156835045)*Yy*(Xx**2 + Yy**2)**9.5*Sqrt(1/
- (65536.*Sqrt(Xx**2 + Yy**2)*(Xx**2 + Yy**2 + Zz**2)**10.5)
```

In[34]:=

**FortranForm[RHS[[3]]]**

Out[34]//FortranForm=

```
(-3*Sqrt(7/Pi)*Sqrt(1/(Xx**2 + Yy**2 + Zz**2)))*((2097152*Sqrt(15)*Xx*Yy*Sqrt(Xx**2 +
5*Sqrt(286)*(256*(Xx**2 + Yy**2)**3*(Xx**2 + Yy**2 + Zz**2)**5*(3*(Xx**2 + Yy**2)
- Cos(5*ArcTan(Xx,Yy)) + 15*(5632*(Xx**2 + Yy**2)**3*Zz*(Xx**2 + Yy**2 + Zz**2)
- Sin(5*ArcTan(Xx,Yy)) + 7*Sqrt(156835045)*(Xx**2 + Yy**2)**10.5*Sqrt(1/(Xx**
- (65536.*Sqrt(Xx**2 + Yy**2)*(Xx**2 + Yy**2 + Zz**2)**10.5)
```

In[35]:=

**CForm[RHS[[1]]]**

Out[35]//CForm=

```
(3*Sqrt(7/Pi)*Sqrt(1/(Power(Xx,2) + Power(Yy,2) + Power(Zz,2)))*((524288*Sqrt(15)*Yy*Sqrt(Pow
Power(1/(Power(Xx,2) + Power(Yy,2) + Power(Zz,2)),8.5) + 5*Sqrt(286)*
(256*Power(Power(Xx,2) + Power(Yy,2),2)*(330*Yy + Xx*Zz)*Power(Power(Xx,2) + Power(Yy
(3*Power(Power(Xx,2) + Power(Yy,2),3) - 111*Power(Power(Xx,2) + Power(Yy,2),2)*Pow
Cos(5*ArcTan(Xx,Yy)) + Zz*(-105*Sqrt(156835045)*Yy*Power(Power(Xx,2) + Power(Yy,2)
Sqrt(Power(Xx,2) + Power(Yy,2) + Power(Zz,2))*Cos(20*ArcTan(Xx,Yy)) -
256*Power(Power(Xx,2) + Power(Yy,2),2)*Power(Power(Xx,2) + Power(Yy,2) + Power(Z
(15*Power(Power(Xx,2) + Power(Yy,2),2) - 140*(Power(Xx,2) + Power(Yy,2))*Power(
105*Sqrt(156835045)*Xx*Power(Power(Xx,2) + Power(Yy,2),9.5)*Sqrt(1/(Power(Xx,2)
Sin(20*ArcTan(Xx,Yy))))))/(65536.*Sqrt(Power(Xx,2) + Power(Yy,2))*Power(Power(X
```

In[36]:=

**CForm[RHS[[2]]]**

Out[36]//CForm=

```
(-3*Sqrt(7/Pi)*Sqrt(1/(Power(Xx,2) + Power(Yy,2) + Power(Zz,2)))*((524288*Sqrt(15)*Xx*Sqrt(Po
Power(1/(Power(Xx,2) + Power(Yy,2) + Power(Zz,2)),8.5) + 5*Sqrt(286)*
(256*Power(Power(Xx,2) + Power(Yy,2),2)*(330*Xx - Yy*Zz)*Power(Power(Xx,2) + Power(Yy
(3*Power(Power(Xx,2) + Power(Yy,2),3) - 111*Power(Power(Xx,2) + Power(Yy,2),2)*Pow
Cos(5*ArcTan(Xx,Yy)) + Zz*(-105*Sqrt(156835045)*Xx*Power(Power(Xx,2) + Power(Yy,2)
Sqrt(Power(Xx,2) + Power(Yy,2) + Power(Zz,2))*Cos(20*ArcTan(Xx,Yy)) -
256*Power(Power(Xx,2) + Power(Yy,2),2)*Power(Power(Xx,2) + Power(Yy,2) + Power(Z
(15*Power(Power(Xx,2) + Power(Yy,2),2) - 140*(Power(Xx,2) + Power(Yy,2))*Power(
(Power(Xx,3) + 330*Yy*Zz + Xx*(Power(Yy,2) + Power(Zz,2)))*Sin(5*ArcTan(Xx,Yy))
105*Sqrt(156835045)*Yy*Power(Power(Xx,2) + Power(Yy,2),9.5)*Sqrt(1/(Power(Xx,2)
Sin(20*ArcTan(Xx,Yy))))))/(65536.*Sqrt(Power(Xx,2) + Power(Yy,2))*Power(Power(X
```

In[37]:=

**CForm[RHS[[3]]]**

Out[37]//CForm=

```
(-3*Sqrt(7/Pi)*Sqrt(1/(Power(Xx,2) + Power(Yy,2) + Power(Zz,2)))*((2097152*Sqrt(15)*Xx*Yy*Sqr
Power(1/(Power(Xx,2) + Power(Yy,2) + Power(Zz,2)),8.5) + 5*Sqrt(286)*
(256*Power(Power(Xx,2) + Power(Yy,2),3)*Power(Power(Xx,2) + Power(Yy,2) + Power(Zz,2)
(3*Power(Power(Xx,2) + Power(Yy,2),3) - 111*Power(Power(Xx,2) + Power(Yy,2),2)*Pow
Cos(5*ArcTan(Xx,Yy)) + 15*(5632*Power(Power(Xx,2) + Power(Yy,2),3)*Zz*Power(Power(
(15*Power(Power(Xx,2) + Power(Yy,2),2) - 140*(Power(Xx,2) + Power(Yy,2))*Power(
7*Sqrt(156835045)*Power(Power(Xx,2) + Power(Yy,2),10.5)*Sqrt(1/(Power(Xx,2) + Po
Sin(20*ArcTan(Xx,Yy))))))/(65536.*Sqrt(Power(Xx,2) + Power(Yy,2))*Power(Power(X
```