



# **Dynamically Stable Walking For Humanoid Bipedal Robots Based On Walking Patterns**

**Bachelors's Thesis  
of**

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# 1 Introduction

Nearly one hundred years of Science-Fiction have established the firm image of a mechanical humanoid servant with super human capabilities, with the term *Robot*. Today we live in a world where large parts of our production circles are already dominated by robots. Yet, widespread adoption of humanoid robots is still nowhere in evidence. One core problem of humanoid robots is their inherent complexity. Vision, cognition, manipulation and locomotion all need to be combined in one mechanism. While each of these problems is under active research, one has been especially resilient: Locomotion. There are multiple reasons for this. For one, locomotion is inherently complex, as a large number of joints need to be controlled precisely. The dynamics for such complex systems are hard to control, thus severe simplifications need to be made to control them. And last but not least, the strategies to solve bipedal walking are very much tied to specific problem and not easily transferable to robots with other leg configurations. Some robots, e.g. the ARMAR III robots, solve it by replacing the legs with a stable base on wheels. While this yields convenient research platform, the navigation in human environments is not as natural as for full humanoid robots.

Over the last two decades a lot of progress has been made in humanoid walking. Every one is familiar with the famous ASIMO robot developed by Honda or the HRP robots developed by AIST which demonstrate stable walking. More recently ATLAS by Boston Dynamics shows great stability even under disturbances. Sadly some of these platforms are closed and it is not known exactly which models are used in each robot to derive stable walking.

As the goal for bipedal walking is to be as human-like as possible, research on human walking is a great inspiration for robotic walking. While the human gait can be divided into many phases, of primary interest for the stability is the number of feet that are in contact with the ground. For walking the ground contact alternates between both feet and one foot. These phases are called dual and single support respectively. Each step starts with a dual support phase, that shifts the center of mass to the foot that supports the weight in the next step. The non-supporting foot (the *swing foot*) is then moved to the next foot hold. (See figure 1.1) The area that is in contact with the ground is not constant in each phase. Most notably at the end of the single support phase the heel is lifted. Consequently the contact changes from full sole contact to the toes. The swing foot hits the ground heel first and rotates to meet the ground while the center of mass is shifted forward.

Early robots were only able to emulate this walking style to varying degrees. A common simplification is to use a foot without movable toes. The result is a characteristic walking style, where the foot

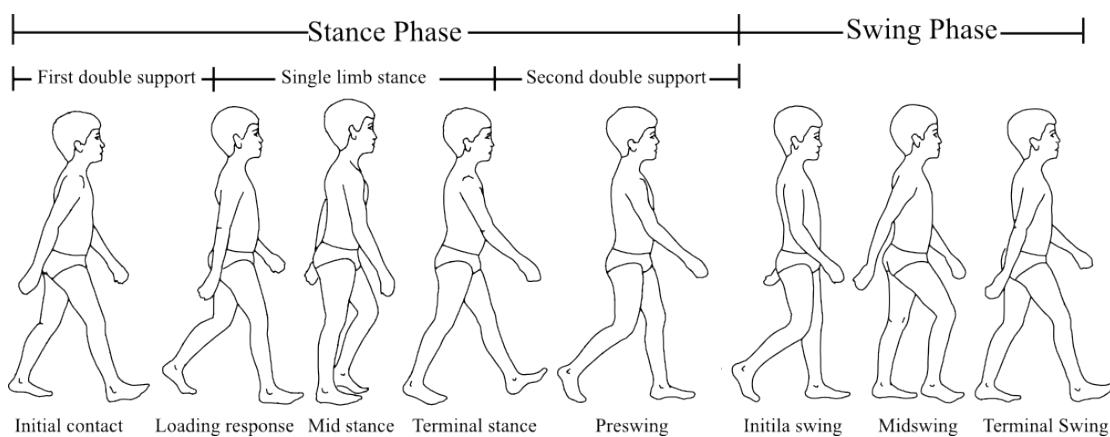


Figure 1.1: Phases of human gait. Source: Dynamics of Human Gait [1], page 9

sole is always parallel to the ground. The most prominent example is the early ASIMO robot. However today human-like walking using the toes has been successfully demonstrated e.g. by the WABIAN-2R robot [2].

Besides the need for an adequate kinematic structure, deriving trajectories that are stable is not trivial. When talking about stability, one needs to consider two cases: *static* and *dynamic* stability. Static stability is concerned with the stability of objects at rest (e.g. standing), while dynamical stability is concerned with the stability of objects in motion (e.g. walking). Deriving a stability criteria for the *static* case is rather simple. If the projection of center of mass to the ground is inside the support polygon, the pose is said to be *statically* stable. The support polygon is the convex hull of all points of the foot that are in contact with the ground. However dynamic stability deals with the stability of objects (e.g. in this case robots) in motion. In that case more elaborate methods need to be derived using a description of the dynamics of the robot.

There are two basic approaches to derive a trajectory for bipedal walking. On the one side are approaches based on direct imitation of a human motion. First a human test subject is equipped with markers. The marker movements are recorded using motion capturing and mapped to corresponding markers on a humanoid model. Since the kinematic structure of a human does not map directly to the kinematic structure of a robot, the resulting trajectory needs to be adapted to the target robot. In general only adapting the motion based on the kinematic structure will not yield a dynamically stable motion. Different weights and inertia values of the links also need to be considered. However this might cause significant modifications of the original trajectory.

On the other side there are completely synthetic approaches. Based on simplified models of the dynamics of a humanoid robot, stability conditions are derived. Using desired foot positions or step length and walking speed as input, corresponding trajectories are derived that satisfy the given stability conditions. Most notably in this category are approaches based on the Zero Moment Point using a so called Pattern Generator. In section 2 the ZMP is derived for different simplified dynamic models. Fundamental for this approach is the insight, that the dynamics of bipedal walking can be approximated sufficiently by an inverted pendulum. The model views the center of mass as the head of the pendulum, while the base is attached to the support foot. Thus the task of walking can be formalized as moving the base of the pendulum while keeping it upright.

In reality executing a trajectory that was obtained by either method is not easy. For one, the models to approximate the dynamics of a complex robot might be inaccurate. Also the environment the robot operates in might not match the assumptions completely. For example a convenient assumption is that the ground is completely flat. This is rarely the case and needs to be controlled for. Thus a stabilizer is needed to adapt the trajectory to the disturbances. Most stabilizer for established robotic platforms are closed source and very robot specific. Notably various iterations of stabilizers based on modifying an already dynamically stable pattern where proposed by Kajita and Miura et al. ([3], [4] and [5])

While a stabilizer can deal with continuous small disturbances, a robot may encounter short but severe disturbances. These disturbances can be viewed as pushes. Pratt et al. established the concept of a Capture Point, that can be used to recover from such disturbances.

This thesis presents a software framework that implements rudimentary support for each component. They are integrated into a dynamic simulation environment and evaluated.

## 2 Models for humanoid walking

### 2.1 The Linear Inverted Pendulum Model

A simple model for describing the dynamics of a bipedal robot during single support phase is the 3D inverted pendulum. We reduce the body of the robot to a point-mass at the center of mass and replace the support leg by a mass-less telescopic leg which is fixed at a point on the supporting foot. Initially this will yield non-linear equations that will be hard to control. However by constraining the movement of the inverted pendulum to a fixed plane, we can derive a linear dynamic system. This model called the 3D *linear* inverted pendulum model (short *3D-LIPM*).

#### 2.1.1 The inverted pendulum

To describe the dynamics of the inverted pendulum we are mainly interested in the effect a given actuator torque has on the movement of the pendulum.

For simplicity we assume that the base of the pendulum is fixed at the origin of the current Cartesian coordinate system. Thus we can describe the position inverted pendulum by a vector  $c = (x, y, z)$ . We are going to introduce an appropriate (generalized) coordinate system  $q = (\theta_x, \theta_y, r)$  to get an easy description of our actuator torques: Let  $m$  be the mass of the pendulum and  $r$  the length of the telescopic leg.  $\theta_y$  and  $\theta_x$  describe the corresponding roll and pitch angles of the pose of the pendulum.

Now we need to find a mapping between forces in the Cartesian coordinate system and the generalized forces (the actuator torques). Let  $\Phi : \mathbb{R}^3 \rightarrow \mathbb{R}^3, (\theta_x, \theta_y, r) \mapsto (x, y, z)$  be a function that maps the generalized coordinates to the Cartesian coordinates. Then the Jacobian  $J_\Phi = \frac{\partial p}{\partial q}$  maps the *generalized velocities* to *Cartesian velocities*. Furthermore we know that the transpose  $J_\Phi^T$  maps *Cartesian forces*  $F = m(\ddot{x}, \ddot{y}, \ddot{z})$  to *generalized forces*  $\Gamma = (\tau_x, \tau_y, f)$ .

We write  $x$ ,  $y$  and  $z$  in terms of our generalized coordinates to compute the corresponding Jacobian  $J_\Phi$ . From the fact that the  $\theta_y$  is the angle between the projection of  $c$  onto the  $xz$ -plane and  $c$  and  $\theta_x$  the angle between  $c$  and the projection onto the  $yz$  plane we can derive the following equations [6]. We use  $s_i = \sin(\Theta_i)$  and  $c_i = \cos(\Theta_i)$  with  $i \in \{x, y\}$  as a shorthand notation.

$$\begin{aligned} x &= r \cdot \sin \theta_y &=: r \cdot s_y \\ y &= -r \cdot \sin \theta_x &=: -r \cdot s_x \\ z &= \sqrt{r^2 - x^2 - y^2} = r \cdot \sqrt{1 - s_y^2 - s_x^2} \end{aligned} \quad (2.1)$$

From which we can compute the Jacobian by partial derivation:

$$J = \frac{\partial p}{\partial q} = \begin{pmatrix} 0 & r \cdot c_y & s_y \\ -r \cdot c_x & 0 & s_y \\ \frac{2 \cdot r \cdot s_y c_y}{\sqrt{1 - s_y^2 - s_x^2}} & \frac{2 \cdot r \cdot s_x c_x}{\sqrt{1 - s_y^2 - s_x^2}} & \sqrt{1 - s_y^2 - s_x^2} \end{pmatrix} \quad (2.2)$$

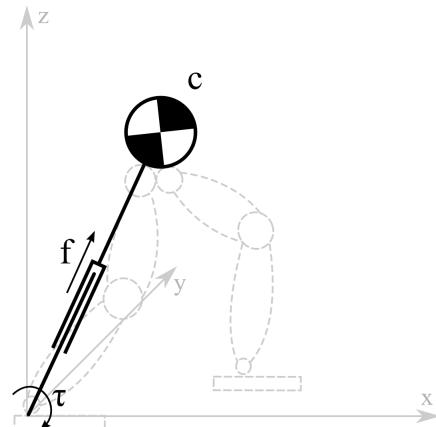


Figure 2.1: The 3D-LIMP

Using the gravity force  $f_g = (0, 0, -m \cdot g)^T$  and the equation of motion as given by

$$\begin{aligned} F &= (J^T)^{-1} \Gamma + f_g \\ m \cdot \begin{pmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{pmatrix} &= (J^T)^{-1} \begin{pmatrix} \tau_x \\ \tau_y \\ f \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -m \cdot g \end{pmatrix} \end{aligned} \quad (2.3)$$

and equations 2.2 and 2.1 we can derive the following equations:

$$m(-z\ddot{y} + y\ddot{z}) = \frac{\sqrt{1 - s_y^2 - s_x^2}}{c_x} \cdot \tau_x + mgy \quad (2.4)$$

$$m(z\ddot{x} - x\ddot{z}) = \frac{\sqrt{1 - s_y^2 - s_x^2}}{c_y} \cdot \tau_y + mgx \quad (2.5)$$

Observe that the terms of the left-hand side are not linear. To remove that non-linearity we are going to use the *linear* inverted pendulum model.

## 2.1.2 Linearization

In a man-made environment it is fair to assume that the ground a robot will walk on can be approximate by a slightly sloped plane. In most cases it can even assumed that there is no slope at all.

The basic assumption in the next section will be that the CoM will have a *constant displacement* with regard to our ground plane. Thus we can constrain the movement of the CoM to a plane that is parallel to the ground plane. Note that this assumption is, depending on the walking speed, only approximately true for human walking as shown by Orendurff et al. [7]. For slow to fast walking (0.7 m/s and 1.6 m/s respectively) the average displacement in  $z$ -direction was found to be between 2.7 cm and 4.81 cm. While the walking patterns generated based on the LIP-model will guarantee dynamic stability, they might not look natural with regard to human walking.

We are going to constrain the  $z$  coordinate of our inverted pendulum to a plane with normal vector  $(k_x, k_y, -1)$  and  $z$ -displacement  $z_c$ :

$$z = k_x \cdot x + k_y \cdot y + z_c \quad (2.6)$$

Subsequently the second derivative of  $z$  can be described by:

$$\ddot{z} = k_x \cdot \ddot{x} + k_y \cdot \ddot{y} \quad (2.7)$$

Substituting 2.6 and 2.7 into the equations 2.4 and 2.5 yields the following equations:

$$\ddot{x} = \frac{g}{z_c} x + \frac{k_y}{z_c} (x\ddot{y} - \ddot{x}y) + mz_c \cdot \tau_y \cdot \frac{\sqrt{1 - s_y^2 - s_x^2}}{c_y} \quad (2.8)$$

$$\ddot{y} = \frac{g}{z_c} y - \frac{k_x}{z_c} (x\ddot{y} - \ddot{x}y) - mz_c \cdot \tau_x \cdot \frac{\sqrt{1 - s_y^2 - s_x^2}}{c_x} \quad (2.9)$$

The term  $x\ddot{y} - \ddot{x}y$  that is part of both equations is still causing the equations to be non-linear. To make this equations linear we will assume that our ground plane has no slope, thus  $k_x = k_y = 0$  and the non-linear terms will vanish.

Another problem is that the actuator torques  $\tau_x$  and  $\tau_y$  both have non-linear factors  $\frac{\sqrt{1 - s_y^2 - s_x^2}}{c_x}$  and  $\frac{\sqrt{1 - s_y^2 - s_x^2}}{c_y}$  respectively. This can be solved by substituting with the following *virtual inputs*:

$$\tau_x \cdot \frac{\sqrt{1 - s_y^2 - s_x^2}}{c_x} = u_x \quad (2.10)$$

$$\tau_y \cdot \frac{\sqrt{1 - s_y^2 - s_x^2}}{c_y} = u_y \quad (2.11)$$

Which yields our final description of the dynamics:

$$\ddot{x} = \frac{g}{z_c}x + \frac{u_x}{mz_c} \quad (2.12)$$

$$\ddot{y} = \frac{g}{z_c}y - \frac{u_y}{mz_c} \quad (2.13)$$

As outlined in [6] the inputs  $u_y$  and  $u_x$  are generally set to zero. Thus the 3D-LIMP has no input torque. This is desirable, as the torque that can be applied on the ankle joints is limited. Thus it makes sense to reserve the torque for correcting disturbances.

## 2.2 The Zero Moment Point

The Zero Moment Point is virtual point on the floor that can be used to derive dynamically stable walking. If the support foot (or support feet) have flat ground contact, the walking trajectory is dynamically stable if the ZMP is strictly inside the support polygon. If the ZMP is on the border of the support polygon, the trajectory can be dynamically unstable. By deriving a control scheme that constrains the ZMP to be strictly inside the support polygon, we can use the ZMP to generate dynamically stable trajectories.

For flat ground contact of our support foot with the floor the ZMP corresponds with the position of the center of pressure (CoP). Indeed, some author (notably Pratt) prefer to use the term CoP instead of ZMP. In the context of this thesis, we will use the term ZMP.

The CoP (and in flat ground contact the ZMP) of an object in contact with the ground can be computed as the sum of all contact points  $p_1, \dots, p_n$  weighted by the forces in  $z$ -direction  $f_{1z}, \dots, f_{nz}$  that is applied:

$$p := \begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix} = \frac{\sum_{i=1}^N p_i f_{iz}}{\sum_{i=1}^N f_{iz}} \quad (2.14)$$

An important fact (and the origin of its name) is that there are no torques around the  $x$  and  $y$  axis at the ZMP:

$$\tau = \sum_{i=1}^N (p_i - p) \times f_i \quad (2.15)$$

Splitting that up into each component using the definition of the cross product yields:

$$\tau_x = \sum_{i=1}^N (p_{iy} - p_y) f_{iz} - \underbrace{(p_{iz} - p_z)}^{=0} f_{iy} \quad (2.16)$$

$$\tau_y = \sum_{i=1}^N \underbrace{(p_{iz} - p_z)}^{=0} f_{ix} - (p_{ix} - p_x) f_{iz} \quad (2.17)$$

$$\tau_z = \sum_{i=1}^N (p_{ix} - p_x) f_{iy} - (p_{iy} - p_y) f_{ix} \quad (2.18)$$

Since we have flat ground contact, all contact points have the same  $z$ -coordinate as the ZMP, thus we can simplify  $\tau_x$  and  $\tau_y$  to:

$$\tau_x = \sum_{i=1}^N (p_{iy} - p_y) f_{iz} = \sum_{i=1}^N (p_{iy} f_{iz}) - (\sum_{i=0}^N f_{iz}) \cdot p_y \quad (2.19)$$

$$\tau_y = \sum_{i=1}^N -(p_{ix} - p_x) f_{iz} = \sum_{i=1}^N -(p_{ix} f_{iz}) + (\sum_{i=0}^N f_{iz}) \cdot p_x \quad (2.20)$$

Furthermore we can use the corresponding components  $p_x$  and  $p_y$  from the definition of the ZMP 2.14 and substitute in the equations 2.19 and 2.20.

This will yield:  $\tau_x = \tau_y = 0$ .

Please note that  $\tau_z$  will in general not be zero, nonetheless in case of straight walking it is often assumed to be zero as well.

## 2.3 The cart-table model

The cart-table model is used to compute the resulting ZMP from an CoM motion. The model consists of an infinitely large mass-less table of height  $z_c$ , while the foot of the table has the shape of the support polygon. Given a frictionless cart with mass  $m$  at position  $c = (c_x, c_y, c_z)$  that moves on the table we can compute the resulting ZMP  $p = (p_x, p_y, p_z)^T$  in the support foot. Please note that the 3D-dimensional model is equivalent to having two independent tables with two carts each in the  $xz$  and  $yz$ -plane respectively. First of all, lets compute the torque  $\tau_x$  and  $\tau_y$  around the x-axis and y-axis at the ZMP on the support foot.

$$\tau_y = \underbrace{-mg(c_x - p_x)}_{\text{torque due to gravity}} + \underbrace{m\ddot{x} \cdot z_c}_{\text{torque due to acceleration of cart}} \quad (2.21)$$

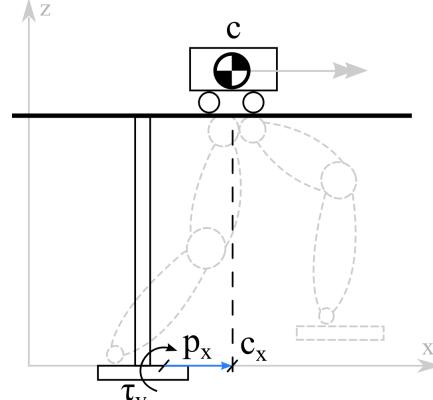


Figure 2.2: The Cart-Table model.

$$\tau_x = -mg(c_y - p_y) + m\ddot{y} \cdot z_c \quad (2.22)$$

Please note the similarity to the equations 2.13 and 2.12 when assuming the base of the pendulum is located at  $p$ . If we now use the property of the ZMP that the torque around the  $x$  and  $y$ -axis is zero, we can solve for the ZMP position  $p$ :

$$p_x = c_x - \frac{z_c}{g} \ddot{c}_x \quad (2.23)$$

$$p_y = c_y - \frac{z_c}{g} \ddot{c}_y \quad (2.24)$$

## 2.4 Multi-Body methode to calculate the ZMP

Besides the simplified cart-table model, there also exists an exact method to calculate the resulting ZMP from the movement from several connected rigid bodies.

Let  $c_i$  be the CoM position and  $m_i$  the mass of the  $i$ -th body ( $i \in \{1, \dots, k\}$ ). Then, as derived in [8] (p. 374), the total linear momentum  $\mathcal{P}$  can be calculated by:

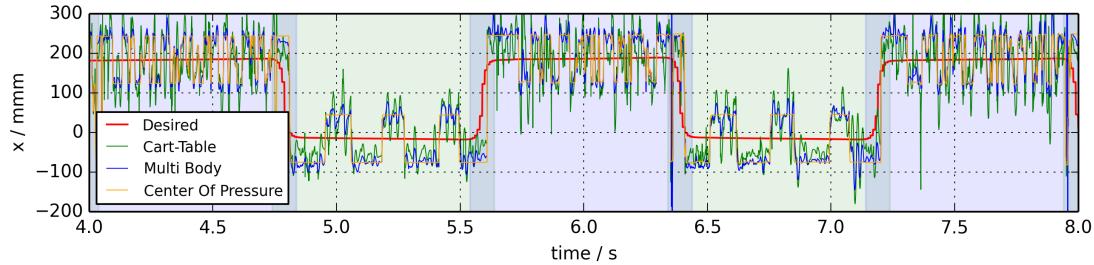


Figure 2.3: Comparison of the Cart-Table and Multi-Body to estimate the realized ZMP during walking simulation.

$$\mathcal{P} = \sum_{j=1}^k m_j \cdot \dot{c}_j \quad (2.25)$$

If  $\omega_i$  the angular momentum and  $R_i$  is the rotational part of the reference frame of the  $i$ -th body and  $I_i$  the inertia tensor in that reference frame, the total angular momentum  $\mathcal{L}$  can be calculated by:

$$\mathcal{L} = \sum_{j=1}^k c_j \times (m_j \dot{c}_j) + R_j I_j R_j^T \omega_j \quad (2.26)$$

If we denote the total mass of the robot with  $M$  and the gravity vector with  $g$  we can express the change of linear momentum if a force  $f$  is applied to the body as:

$$\dot{\mathcal{P}} = Mg + f \quad (2.27)$$

And subsequently the change in angular momentum if a torque  $\tau$  is applied:

$$\dot{\mathcal{L}} = c \times Mg + \tau \quad (2.28)$$

To calculate the resulting torque  $\tau_{ZMP}$  around the ZMP located at  $p$  we can use:

$$\tau_{ZMP} = \tau + (0 - p) \times f = \tau - p \times f \quad (2.29)$$

If solve equation 2.27 for  $f$  and 2.28 for  $\tau$  and substitute them in 2.29 this yields the following equation:

$$\tau_{ZMP} = \dot{\mathcal{L}} - c \times Mg - p \times (\dot{\mathcal{P}} - Mg) \quad (2.30)$$

Since we know that the torque around the ZMP is zero around the  $x$  and  $y$  axis we can apply the definition of the cross product and solve for the ZMP position:

$$p_x = \frac{Mgx + p_z \dot{\mathcal{P}}_x - \dot{\mathcal{L}}_y}{Mg + \dot{\mathcal{P}}_z} \quad (2.31)$$

$$p_y = \frac{Mgy + p_z \dot{\mathcal{P}}_y - \dot{\mathcal{L}}_x}{Mg + \dot{\mathcal{P}}_z} \quad (2.32)$$

Both equations are dependent on  $p_z$ . If we assume the robot walks on a flat floor, we can set  $p_z = 0$ .

See figure ?? to get an idea how much the Multi-Body ZMP derives from the estimation using the Cart-Table model.

## 3 Pattern generator

To generate a walking pattern for a bipedal robot two basic approaches are common:

1. Generate (or modify) foot trajectories that realize a prescribed trajectory of the CoM
2. Generate a CoM trajectory for prescribed foot trajectories

The first approach is generally used for implementing pattern generators solely based on the 3D-LIP model. [6] The second approach is more versatile, since it is easy to incorporate constraints of our environment (e.g. only limited foot holds) in the input of the pattern generator. However care must be taken while choosing adequate step width and step length parameters for the foot trajectory, so that they can actually be realized by the robot. The pattern generator proposed by Kajita et al. [9] based on Preview Control realizes the second approach. We will discuss the theoretical background of this pattern generator here in more detail, since all pattern that we used were generated that way.

### 3.1 Computing the CoM from a reference ZMP

As we saw in the section 2.3 it is easy to compute the resulting ZMP given the CoM and its acceleration. However for generating the walking pattern, we want to compute the CoM trajectory from a given ZMP. If you rearrange the equations 2.23 and 2.24 you see that we have to solve a second order differential equations:

$$c_x = \frac{z_c}{g} \cdot \ddot{c}_x + p_x \quad (3.1)$$

$$c_y = \frac{z_c}{g} \cdot \ddot{c}_y + p_y \quad (3.2)$$

There are several ways to solve this differential equations, for example by transforming them to the frequency-domain. This however would mean that the ZMP trajectory needs to be transformed to the frequency domain as well, e.g. using Fast Fourier Transformation. This has two main problems:

1. It has a significant computational overhead. (For FFT the additional runtime would be in  $O(n \log n)$ )
2. We need to know the whole ZMP trajectory in advance.

Instead Kajita et al. chose to define a dynamic system in the time domain that describes the CoM movement.

#### 3.1.1 Pattern generation as dynamic system

For simplicity we will only focus on the dynamic description of one dimension, as the other one is analogous. To transform the equations to a strictly proper dynamical system, we need to determine the state vector of our system. For the cart-table model it suffices to know the position, velocity and acceleration of the cart, i.e. the CoM. Thus the state-vector is defined as  $x = (c_x, \dot{c}_x, \ddot{c}_x)$ . We can define the evolution of the state vector as follows:

$$\frac{d}{dt} \begin{pmatrix} c_x \\ \dot{c}_x \\ \ddot{c}_x \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}}_{=:A_0} \cdot \begin{pmatrix} c_x \\ \dot{c}_x \\ \ddot{c}_x \end{pmatrix} + \underbrace{\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}}_{=:B_0} u \quad (3.3)$$

As you can see the jerk of the CoM was introduced as an input  $u_x = \frac{d}{dt} \ddot{c}_x$  into the dynamic system.

We use equation 2.23 to calculate the actual output of the dynamic system the resulting ZMP, that will be controlled:

$$p_x = \begin{pmatrix} 1 & 0 & \frac{-z_c}{g} \end{pmatrix} \cdot \begin{pmatrix} c_x \\ \dot{c}_x \\ \ddot{c}_x \end{pmatrix} \quad (3.4)$$

Using this formulation of the dynamic system we need to derive the evolution of our state vector using the state-transition matrix. Since our input ZMP trajectory will consist of discrete samples at equal time intervals  $T$  we define the discrete state as  $x[k] := x(k \cdot T)$ . Please note that this system is a linear time-invariant system (LTI), and both matrices  $A_0$  and  $B_0$  are constant. We can therefore use the standard approach to solve this system using the equation:

$$x(t) = e^{A_0 \cdot (t-\tau)} x(\tau) + \int_{\tau}^t e^{A_0 \cdot (t-\lambda)} B_0 u(\lambda) d\lambda \quad (3.5)$$

In our discrete case that becomes:

$$x[k+1] = e^{A_0 \cdot ((k+1)T - kT)} x[k] + \int_{kT}^{(k+1)T} e^{A_0 \cdot ((k+1)T - \lambda)} B_0 u(\lambda) d\lambda \quad (3.6)$$

$$= e^{A_0 \cdot T} x[k] + \left( \int_{kT}^{(k+1)T} e^{A_0 \cdot ((k+1)T - \lambda)} d\lambda \right) \cdot B_0 u[k] \quad (3.7)$$

$$= e^{A_0 \cdot T} x[k] + \left( \int_T^0 e^{A_0 \cdot \lambda} d\lambda \right) \cdot B_0 u[k] \quad (3.8)$$

Keep in mind that  $u(\lambda) = u[k], \lambda \in (kT, (k+1)T)$  so we can move it outside of the integral. Let us first compute a general solution for the matrix exponential  $e^{A_0 t}$ . It is easy to see that  $A_0$  is nilpotent and  $A_0^3 = 0$ , thus the computation simplifies to the following:

$$e^{A_0 t} := \sum_{i=0}^{\infty} \frac{(A_0 \cdot t)^i}{i!} = I + A_0 \cdot t + A_0^2 \cdot \frac{t^2}{2} + 0 = \begin{pmatrix} 1 & t & \frac{t^2}{2} \\ 0 & 1 & t \\ 0 & 0 & 1 \end{pmatrix} \quad (3.9)$$

Using this solution computing the integral in 3.6 is quite easy:

$$\int_T^0 e^{A_0 \cdot \lambda} d\lambda = - \int_0^T \begin{pmatrix} 1 & t & \frac{t^2}{2} \\ 0 & 1 & t \\ 0 & 0 & 1 \end{pmatrix} dt = - \left( \begin{pmatrix} t & \frac{t^2}{2} & \frac{t^3}{6} \\ 0 & t & \frac{t^2}{2} \\ 0 & 0 & t \end{pmatrix} \right) \Big|_0^T = \begin{pmatrix} T & \frac{T^2}{2} & \frac{T^3}{6} \\ 0 & T & \frac{T^2}{2} \\ 0 & 0 & T \end{pmatrix} \quad (3.10)$$

Substituting the results in 3.6 yields:

$$x[k+1] = \underbrace{\begin{pmatrix} 1 & T & \frac{T^2}{2} \\ 0 & 1 & T \\ 0 & 0 & 1 \end{pmatrix}}_{=:A} x[k] + \underbrace{\begin{pmatrix} \frac{T^3}{6} \\ \frac{T^2}{2} \\ T \end{pmatrix}}_{=:B} \cdot u_x[k] \quad (3.11)$$

### 3.1.2 Controlling the dynamic system

To control this dynamic system we need to determine an adequate control input  $u_x$  to realize the reference ZMP trajectory. A performance index  $J_x$  for a given control input  $u_x$  is needed to formalize what a “good”

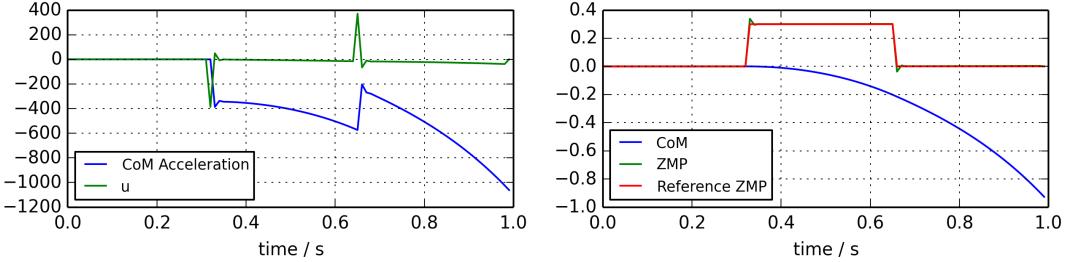


Figure 3.1: ZMP control based on the performance index described by equation 3.12.

control input would be. A naive performance index could be:

$$J_x[k+1] := (p_x^{ref}[k+1] - p_x[k+1])^2 \quad (3.12)$$

To minimize it, we need to find  $u_x$  for which  $p_x = p_x^{ref}$ . By substituting  $p_x[k+1]$  with 3.4 and  $x[k+1]$  with 3.11 this yields:

$$u_x[k] = \frac{p_x^{ref}[k+1] - C \cdot A \cdot x[k]}{C \cdot B} = \frac{p_x^{ref}[k+1] - (1, T, \frac{1}{2}T^2 - \frac{z_c}{g}) \cdot x[k]}{\frac{1}{6}T^3 - \frac{z_c}{g}T} = \frac{p_x^{ref}[k+1] - p_x[k] - T\dot{c}_x[k] - \frac{1}{2}T^2\ddot{c}_x[k]}{\frac{1}{6}T^3 - \frac{z_c}{g}T} \quad (3.13)$$

To analyze the behavior of this control law for  $u_x$  we simulate the rapid change of reference ZMP when changing the support foot.

As you can see the reference ZMP is perfectly tracked. However, the CoM does not behave as expected. To achieve the required ZMP position the CoM will be *accelerated indefinitely* in the opposite direction. Clearly this is not desired and will lead to falling on a real robot. A more sophisticated performance index is needed. To eventually reach a stable state at which the CoM comes to rest, the performance index should include a state feedback. Also note the large jerk that is applied to the system when the reference ZMP position changes rapidly. In a real mechanical system large jerks will lead to oscillations, which will disturb the system. Thus the performance index should also try to limit the applied jerk.

Another problem is caused by the very nature of a controller: The controller starts to act *after* we have a deviation from our reference ZMP trajectory. Trying to reduce this lag as much as possible can lead to very high velocities, which might not be realizable by motors of a robot. However we have at least limited knowledge of the future reference trajectory. This knowledge can be leveraged by using Preview Control, which considers the next  $N$  time steps for computing the performance index.

Kajita et al. use a performance index proposed by Katayama et al. [10] to solve all of the problems above:

$$J_x[k] = \sum_{i=k}^{\infty} Q_e e[i]^2 + \Delta x[i]^T Q_x \Delta x[i] + R \Delta u_x[i]^2 \quad (3.14)$$

$Q_e$  is the error gain,  $Q_x$  a symmetric non-negative definite matrix (typically just a diagonal matrix) to weight the components of  $\Delta x[i]$  differently and  $R > 0$ . Conveniently Katayama also derived an optimal controller for this performance index, which is given by:

$$u[k] = -G_i \sum_{i=0}^k e[k] - G_x x[k] - \sum_{j=1}^N G_p p_x^{ref}[k+j] \quad (3.15)$$

The gains  $G_i, G_x, G_p$ , can be derived from the parameters of the performance index. Since the calculation is quite elaborate we refer to the cited article by Katayama p. 680 for more details.

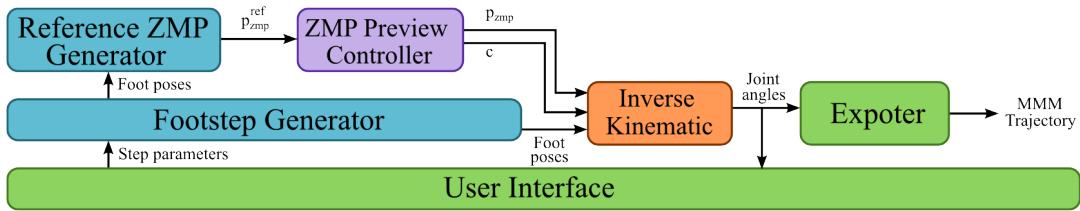


Figure 3.2: Architecture of the pattern generator

## 3.2 Implementation

To generate walking patterns based on the ZMP preview control method, the approach from Kajita was implemented in LIBBIPEDAL a shared library. A front-end was developed to easily change parameters, visualize and subsequently export the trajectory to the MMM format. The implementation was build on a previous implementation, which was refactored, extended and tuned with respect to results from the dynamics simulation.

The pattern generator makes extensive usage of SIMOX VIRTUALROBOT, for providing a model of the robot and the associated task of computing the forward- and inverse kinematics.

Generating a walking pattern consists of multiple steps. First the foot positions are calculated. These are used to derive the reference ZMP trajectory which is feed into the ZMP preview controller. From that the CoM trajectory is computed. The CoM trajectory and feet trajectories are then used to compute the inverse kinematics. The resulting joint trajectory is displayed in the visual front-end and can be exported. Each step is contained in dedicated modules that can be easily replaced, if needed. We will outline the implementation of each module separately.

### 3.2.1 Generating foot trajectories

To generate the foot trajectories several parameters are needed:

**Step height**  $h$  Maximum distance between the foot sole and the floor

**Step length**  $l$  Distance in anterior direction (y-Axis) between the lift-off point and the touch-down point

**Step width**  $w$  Distance in lateral direction (x-Axis) between both TCP on the feet

**Single support duration**  $t_{ss}$  Time the weight of the robot is only support by exactly one foot

**Dual support duration**  $t_{ds}$  Time the weight of the robot is supported by both feet

See figure ?? for the values used to generate the trajectories using a model of the ARMAR IV robot.

#### Walking straight

Since the foot trajectories of a humanoid walking have a cyclic nature, we only need three different foot trajectories that can be composed to arbitrarily long trajectories: Two transient trajectories for the first and last step respectively and a cyclic motion that can be repeated indefinitely. We can use the same trajectories for both feet, as they are geometrically identical. Each foot trajectory starts with swing

$h$	0.1 m
$l$	0.3 m
$w$	0.2 m
$t_{ss}$	0.7 s

Figure 3.3: Default parameters used for generating a walking trajectory.

phase and a resting phase. The trajectory in  $y$  and  $z$  direction is computed by a 5th order polynomial that assures the velocities and accelerations are approaching zero at the lift-off and touch-down point. The first and last step only have half of the normal step length, since the trajectory is starting and ending from a dual support stance, where both feet are placed parallel to each other. Each trajectory is encoded as a  $6 \times N$  matrix, where  $N$  is the number of samples. A sample period of  $T = 10ms$  was used. Each column contains Cartesian coordinates and roll-pitch-yaw angles.

### Walking on a circle

Much of the general structure of the foot trajectory remains the same as for walking straight. However instead of specifying the step length, it is implicitly given by the segment of the circle that should be traversed and the number of steps. So extra care needs to be taken to specify enough steps so that the generated foot positions are still reachable. Each foot needs to move on a circle with radius  $r_{inner} = r - \frac{w}{2}$  or  $r_{outer} = r + \frac{w}{2}$  depending which foot lies in the direction of the turn. The movement in  $z$ -direction remains unaffected. However the movement in the  $xy$ -plane is transformed to follow the circle for the specific foot. The same polynomial that was previously used for the  $y$ -direction is now used to compute the angle on the corresponding circle and the  $x$  and  $y$  coordinates are calculated accordingly. The foot orientation is computed from the tangential ( $y$ -Axis) and normal ( $x$ -Axis) of circle the foot follows.

### Balancing on one foot

To test push recovery from single support stance a special pattern was needed. Another footstep planer was implemented that generates a trajectory for standing on one foot. Starting from dual support stance, the swing leg is moved in vertical direction until the usual step height is achieved. Additionally the foot is moved in lateral direction to half the step width. This reduces the necessary upper body tilt to compensate the imbalance. For the last step the inverse movement is performed to get back into dual support stance. This method could be extended to walk by setting the next support foot in a straight line before the current support foot. The swing foot would need to be moved in an arc in lateral direction to avoid self-collisions.

### 3.2.2 ZMP reference generation

As an input for the ZMP preview control, we need a reference ZMP movement that corresponds with the foot trajectory. The reference generator receives a list of intervals associated with the desired support stance and foot positions as input. In single support phase, the reference generator places the ZMP in the center of the support polygon of the corresponding foot. Since the support polygon is convex, the center is the furthest point away from the border of the polygon. This should guarantee a maximum stability with regard to possible ZMP errors. In dual support phase, the reference generator shifts the ZMP from the previous support foot to the next support foot. Kajita et al. suggest using a polynomial to interpolate the ZMP positions between the feet. However a simple step function  $\sigma(t) = \begin{cases} p_1 & t \leq t_0 \\ p_2 & t > t_0 \end{cases}$  seems to suffice as well. Since the touch-down of the swing foot might have a small lag, it is important that  $t_0$  is the middle of the dual support phase. This assures we do not start to move the ZMP too early.

### 3.2.3 ZMP Preview Control

This module implements the method described by Kajita et al. and uses the method outlined by Katayama et al. to compute the optimal control input  $u[k]$ . Since it is computational feasible, the preview period consists of the full reference trajectory. For an online usage of this method, this could be reduced to a much smaller sample size, e.g. only preview one step ahead. Using the system dynamics described by 3.11 the CoM trajectory, velocity and acceleration can be computed. The implementation makes heavy use of EIGEN, a high performance linear algebra framework that uses SIMD instructions to speed up

calculations. Thus thus a calculation time of 6.2s could be achieved to calculate ten steps, including the inverse kinematics.

### 3.2.4 Inverse Kinematics

Using the foot trajectories and CoM trajectory the actual resulting joint angles need to be calculated. Since the kinematic model that is used has a total of 35 degrees of freedom, we need to reduce the number of joints that are used to a sensible value. For walking only the joints of the legs and both the torso roll and pitch joints are used. All other joints are constrained to static values that will not cause self-collisions (e.g. the arms are slightly extended and do not touch the body). For computing the IK additional constraints where added, to make sure the robot has a sensible pose: The chest should always have an upright position and the pelvis should always be parallel to the floor. To support non-straight walking, the pelvis and chest orientation should also follow the walking direction. Thus the following method to compute the desired chest and pelvis orientation is used:

1. Compute walking direction  $y'$  as mean of y-Axis of both feet and normalize:  $y' := \frac{y_{left} + y_{right}}{|y_{left} + y_{right}|}$
2. Both should have an upright position  $z' := (0, 0, 1)^T$
3. Compute  $x'$  as the normal to both vectors:  $x' := y' \times z'$
4. Pose  $R'$  is given by  $R' = (x', y', z')$

A special property of the model that was used for computing the inverse kinematics, is that TCP of the left leg was chosen as root node. Since we can specify the root position freely, that removes the need of solving for the left foot pose. Thus the following goals need to be satisfied by the inverse kinematics:

1. Chest orientation
2. Pelvis orientation
3. CoM position
4. Right foot pose

A hierarchical solver was used to solve the inverse kinematics for that goals in the given order. It was observed that specifying a good target height for the CoM is of utmost importance for the quality of the IK. For the model of ARMAR IV that used here, a height of 0.86 m yielded the best results.

### 3.2.5 Trajectory Export

The trajectory was exported in open MMM trajectory format. The format was extended to export additional information useful for debugging and controlling the generated trajectory. That means besides the joint values and velocities the trajectory also includes the CoM and ZMP trajectory that was used to derive them. Also information about the current support phase is saved. For convenience the pose of chest, pelvis, left and right foot are exported as homogeneous matrices as well. This was done to save the additional step of computing them again from the exported joint trajectory for the stabilizer and also eliminate an additional error source while debugging.

# 4 Stabilizing a trajectory

While executing a trajectory there are several sources of errors that will make it necessary to correct the trajectory. We can divide them in about three main classes:

**Disturbances of the environment:** Pattern generator make some assumptions about the environment they operate in. Most prominently the 3D-LIMP assumes the floor is completely flat and has no slope. Also we assume we can navigate without colliding with other object. Any environment that deviates from this assumption can be seen as a disturbance.

**Disturbances due to simulation errors:** Physical simulations often make a trade-off between speed and simulation accuracy. Thus the simulation might not always behave as it was modeled during calculating the pattern, or as it would behave in reality.

**Disturbances due to errors of the method:** Often pattern generators use simplified models of the dynamics involved to derive generation scheme. For example the pattern generator that was used here assumes the ZMP behaves as the cart-table-model predicts. However the real ZMP calculated from the multi-body dynamics can substantially deviate.

## 4.1 Controlling a deviation

When using a ZMP based control scheme to derive a walking pattern it seems natural to check for deviations of the actual ZMP from the goal ZMP. However a deviation from the reference ZMP does not necessarily mean we will see any disturbance. As long as the ZMP remains inside the support polygon the trajectory can be executed as planned. Also we saw before, it is entirely possible to realize the reference ZMP while being in an overall state that deviates significantly from the state we assumed while generating the pattern. Thus we also need to check for a deviation in the trajectory of our CoM. A common approach to correct for CoM position is to control the pose of the chest frame of the robot. This only works if the majority of the mass of a robot is located in the upper body and arms. Luckily for most humanoid robots this is the case.

## 4.2 Stabilizer

We chose a stabilizer proposed by Kajita et al. in their 2010 paper. [3] The stabilizer only needs the joint trajectory of the walking pattern augmented with a desired ZMP trajectory. This allows the stabilizer to use patterns that were generated synthetically, e.g. by a pattern generator, or patterns that are the results of (adapted) motion capturing. The method proposed by Kajita does not need a torque controlled robot, but works with position control. This was very important for the selection of this stabilizer. The underlying simulation engine BULLET only supports velocity controlled motors, consequently the torque can not be controlled directly.

The controller works by attaching control frames to specific points on the robot. The reference position of these frames can be calculated from the input trajectory using forwards kinematics. To compensate a disturbance the orientation of a reference frame is modified. The modified reference frames are then converted to the modified joint angles by the inverse kinematics.

In the remainder of this chapter we will use the superscript  $d$  to denote reference values and the subscript  $*$  to denote modified values.

For this approach four control frames were selected. The chest to modify the body posture, the feet to modify the ankle torque, and the pelvis to modify the difference between contact forces of the two feet.

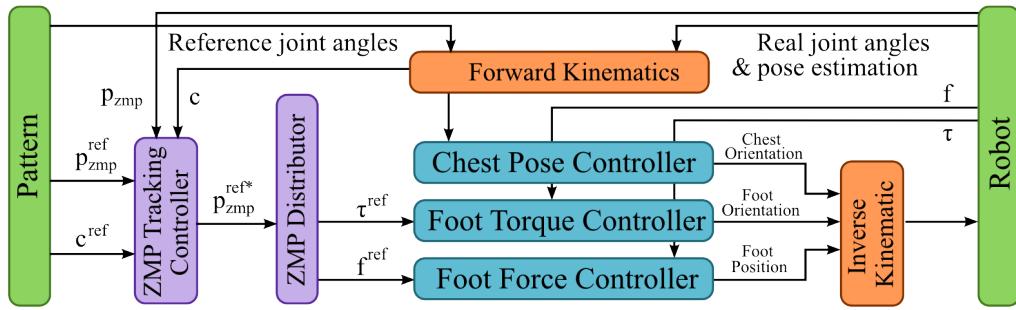


Figure 4.1: Architecture of the stabilizer

#### 4.2.1 Reference coordinate system

To control the ZMP and CoM it is desirable to have a reference system that is static with respect to the ground in each support phase. It is convenient to place the reference coordinate system in the center of the respective support polygon. That means we place the ground frame at the TCP of the respective foot in single support phase. As the ground frame should be aligned with the floor, we use the projection of the foot poses to the floor plane. In dual support phase, we calculate the pose from the position by  $p = \frac{1}{2} \cdot (p_{left} + p_{right})$  and the y-axis by  $y = \frac{1}{2} \cdot (y_{left} + y_{right})$ . The z-axis is the normal of the floor plane:  $z = (0, 0, 1)$ . See figure 4.2 for an example. The resulting reference frame is called the *ground frame*.

#### 4.2.2 Controlling the body posture

The control strategy of the chest pose is straightforward: Given the reference roll angle  $\phi^d$  and reference pitch angle  $\theta^d$  compute the differences to the actual angles  $\phi$  and  $\theta$ . The main problem in a real robot is to obtain the actual global pose of this frame. The proposed method is to use a Kalman filter to estimate the pose from the joint position and accelerometers. We did not implement this method in simulation, as it is easy to obtain the exact pose from the simulator. To prevent rapid movements of the chest that cause large accelerations, a dampening controller is used. The angles  $\Delta\phi$  and  $\Delta\theta$  can be calculated by the following equations:

$$\Delta\dot{\phi}_C = \frac{1}{D_C}(\phi_C^d - \phi_C) - \frac{1}{T_C} \cdot \Delta\phi_C \quad (4.1)$$

$$\Delta\dot{\theta}_C = \frac{1}{D_C}(\theta_C^d - \theta_C) - \frac{1}{T_C} \cdot \Delta\theta_C \quad (4.2)$$

$D_C$  describes the damping gain.  $T_C$  is the constant that describes how long it will take to reach the normal positions  $\Delta\phi = 0$  and  $\Delta\theta = 0$  respectively if there is no error.

The modified reference frame  $R_C^{d*}$  can be calculated by rotating the reference frame by the additional angles:

$$R_C^{d*} = R^d \cdot R_{RPY}(\Delta\phi_C, \Delta\theta_C, 0) \quad (4.3)$$

To get an idea how this controller compensates CoM inaccuracies consider the case where the upper

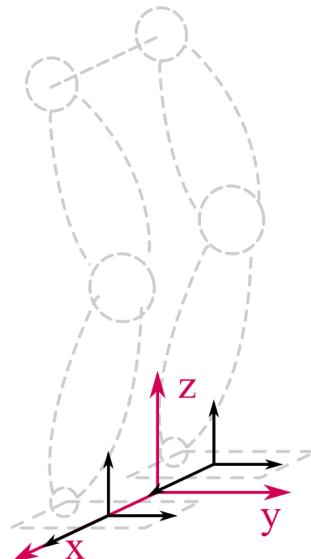


Figure 4.2: Ground frame in dual support

body is bent forward. Since our reference trajectory specifies an upright upper body pose we can assume that  $\phi_C^d = 0$ . Since the upper body is bent forward the roll angle  $\phi$  will be below zero. Depending on  $D_C$  we will eventually reach  $\Delta\phi_C \approx |\phi_C|$ , thus the reference frame will be modified to bent backwards to compensate the wrong pose.

### 4.2.3 Controlling the ankle torques

Since the stabilizer only has the joint trajectory and desired ZMP trajectory as input, we need a way to compute the desired actuation torques on the ankles. The canonical way to do this would be to solve the inverse dynamics of the robot. However for this we need an accurate model of the robot, including correct masses and moments of inertia for each link. This model is not always easy to obtain and calculating the inverse dynamics of a robot with many degrees of freedom is rather slow. For this reason a simple heuristic is proposed to yield approximate torques given a reference ZMP position. However in the single support phase it is easy to calculate the *exact* actuation torque on the ankle, that is required to realize the given reference ZMP.

First we need to calculate the force in  $z$ -direction applied on the foot at the ankle  $p_{ankle}$  which we name  $f_g$  by:

$$f_g = M \cdot g \quad (4.4)$$

Where  $g$  is the gravity vector and  $M$  the mass of the robot. Given  $f_g$  acting on the ankle position  $p_{ankle}$  we can obtain the ankle torque in single support phase easily using the fact, that the torque around the ZMP is zero:

$$\begin{aligned} \tau_{zmp} &= (p_{ankle} - p_{zmp}^d) \times f_g + \tau_{ankle}^d \\ 0 &= (p_{ankle} - p_{zmp}^d) \times f_g + \tau_{ankle}^d \\ \tau_{ankle}^d &= -(p_{ankle} - p_{zmp}^d) \times f_g \end{aligned} \quad (4.5)$$

In dual support phase however that matter is more complicated. Since both feet are in contact with the ground, the weight of the robot is distributed between them. If we take the forces  $f_R$  and  $f_L$  which act on the right ankle  $p_R$  and left ankle  $p_L$  respectively we know that  $f_R + f_L = f_g$ . Thus there exists  $\alpha \in [0, 1]$  for which:  $f_R = \alpha \cdot f_g$  and  $f_L = (1 - \alpha) \cdot f_g$ . A heuristic for computing this alpha is the *ZMP distributor*.

The idea is to calculate the nearest points  $p_{L\#}$  and  $p_{R\#}$  from the ZMP to the support polygons of the feet. If the ZMP falls inside one of the support polygons set  $\alpha = 1$  or  $\alpha = 0$  respectively. If it is outside of both support polygons the ZMP is projected onto line from  $p_{L\#}$  to  $p_{R\#}$  yielding the point  $p_\alpha$ .

We can then set  $\alpha$  to:

$$\alpha = \frac{|p_\alpha - p_{L\#}|}{|p_{R\#} - p_{L\#}|} \quad (4.6)$$

If  $\tau_L$  and  $\tau_R$  are the torques in the left and right ankle respectively, we can calculate the torque around the ZMP as:

$$\tau_{zmp} = (p_R - p_{zmp}^d) \times f_R + (p_L - p_{zmp}^d) \times f_L + \tau_L^d + \tau_R^d \quad (4.7)$$

As before, we assume that  $\tau_{zmp} = 0$  which lets us solve 4.7 for  $\tau_0 := \tau_L^d + \tau_R^d$ :

$$\tau_0 = (p_R - p_{zmp}^d) \times f_R + (p_L - p_{zmp}^d) \times f_L \quad (4.8)$$

We now again apply a heuristic using the  $\alpha$  computed before to distribute  $\tau_0$  to each ankle. First we need to transform  $\tau_0$  from the global coordinate system to a local coordinate system described by the *ground frame*. We mark all vectors in the local coordinate system with  $'$ . The heuristic applied is: The torque around the  $x$ -Axis in each ankle is approximately proportional to the force applied at that ankle. Thus:

$$\tau_{Rx}^{d'} = \alpha \tau'_{0x} \quad (4.9)$$

$$\tau_{Lx}^{d'} = (1 - \alpha) \tau'_{0x} \quad (4.10)$$

The torque around the  $y$ -Axis depends on the direction of the total torque  $\tau'_{0y}$ . If the total torque acts in clockwise direction (negative sign), we can assume it will only be applied to the left foot. If the torque acts in anti-clockwise direction (positive sign), we assume it will only be applied to the right foot.

$$\tau_{Ry}^{d'} = \begin{cases} \tau'_{0y}, & \tau_{0y} > 0 \\ 0, & \text{else} \end{cases} \quad (4.11)$$

$$\tau_{Ly}^{d'} = \begin{cases} \tau'_{0y}, & \tau_{0y} < 0 \\ 0, & \text{else} \end{cases} \quad (4.12)$$

We can now transform the torques from our local coordinate system to the coordinate system of the corresponding foot yielding  $\tau_L^d$  and  $\tau_R^d$ .

Now that we have obtained the reference torques, we can try to control the torque in each angle using the measured torques  $\tau_R$  and  $\tau_L$ . However since we assume a position controlled robot, the torque differences need to be translated into pose changes. There are three primary cases that need to be considered if we change the reference pose of a foot:

**The foot is not in contact with the ground:** Changing the reference pose will just affect the foot

**The foot is in contact with the ground, but the contact is non-solid:** If the foot has a soft contact surface (e.g. rubber) we can model the contact with the ground as springs that connect the ground with the contact points on the foot. Changing the pose of the foot will relax/compress the springs and change the contact forces accordingly.

**The foot is in solid contact with the ground:** Changing the reference foot pose *will not change the foot pose at all*. Instead, the pose of the rest of the robot is changed. If the foot pose is changed by the angles  $\Delta\phi$  and  $\Delta\theta$  all other frames of the robot will be changed by  $-\Delta\phi$  and  $-\Delta\theta$ .

For a foot with a rubber surface we will start with a non-solid contact and transition to a solid contact, once the rubber is sufficiently compressed. [11]

To get an idea how changing the pose on such a foot with rubber surface affects the torque, consider the case of rotating the foot around its lateral axis ( $x$ -Axis) in anti-clockwise direction. Since the contact with the ground is at first non-solid, we can employ the spring model. The springs at the front of the foot are compressed thus the force applied at the corresponding contact points increases. Accordingly the springs at the back are compressed less, thus the force applied to the corresponding contact points decreases. Resulting we see a increase in torque around the  $x$ -axis.

If the springs are compressed sufficiently, we can assume the contact with the floor is solid. Since the pose of the foot does not change, the increase in the joint angle in the ankle will rotate the upper body backwards. Recall the 3D-LIMP model for a moment, in that model this means our pendulum swings backwards. This will lead to an increase of the torque around the  $x$ -Axis in the base of the pendulum, the ankle joint.

For rotating the foot around the  $y$ -axis the same ideas hold. As a result we see that additional foot rotation long the  $x$ - and  $y$ -Axis have a proportional relationship with the torque around that axis. This motivates the definition of the controller proposed by Kajita et al. The additional rotation angles are can be calculated by the following equations:

$$\Delta\dot{\phi}_i = \frac{1}{D_{ix}}(\tau_{ix}^d - \tau_{ix}) - \frac{1}{T_{ix}} \cdot \Delta\phi_i \quad (4.13)$$

$$\Delta\dot{\theta}_i = \frac{1}{D_{ix}}(\tau_{iy}^d - \tau_{iy}) - \frac{1}{T_{ix}} \cdot \Delta\theta_i \quad (4.14)$$

Where  $i \in \{R, L\}$ . This again utilizes the same concept of a dampening controller that was used previously for controlling the chest frame orientation. We can use the obtained angles  $\Delta\phi_i$  and  $\Delta\theta_i$  to compute the modified reference frames for the feet:

$$R_i^{d*} = R_i^d \cdot R_{RPY}(\Delta\phi_i, \Delta\theta_i, 0) \quad (4.15)$$

#### 4.2.4 Foot force difference controller

In the previous section only the ankle torque were controlled to match the reference values that were derived using the ZMP distributor. However reference values for the gravitational force that each foot exerts on the ground were also obtained. These forces are not necessarily realized. Consider the case of slightly uneven ground. If the pattern assumed a flat ground, the ankle of both feet will have the same altitude. Depending on variation of floor height, that might lead to one foot not touching the ground at all. In the case of feet with rubber soles, slight variations in floor height lead to a different compression of the soles. Both cases cause a different force acting on each foot.

If we assume the mass  $M$  of the robot and the gravity vector  $g$  are correct, we know that the reference force  $f_g^d = M \cdot g$  will exactly match the force in  $z$ -direction  $f_g$  exerted by the support foot in single support. Thus in single support we can guarantee that we realize our reference force. In dual support we know that  $f_L^d + f_R^d = M \cdot g$ . If we apply the same reasoning as above we know that  $f_L^d + f_R^d = f_L + f_R$ . If we can additionally make sure that  $f_L^d - f_R^d = f_L - f_R$  we can deduce that  $f_L^d = f_L$  and  $f_R^d = f_R$ . Equation 4.7 can be used to calculate the ZMP position from the applied forces and torques on the foot. If both reference torques and reference forces on the feet are realized, that will guarantee the reference ZMP is also realized.

Since the  $x$  and  $y$  components of both  $f_L^d - f_R^d$  and  $f_L = f_R$  are zero we only need to control the  $z$  components. As we motivated in the beginning of this section, differences in floor height are the main cause of deviation in the force. To compensate that, the height of the ankle needs to be changed. Thus the difference in ankle height  $z_{ctl}$  was chosen to compensate the difference in forces exerted by the foot. The description of the controller again uses the concept of a dampening controller, that was used in the previous sections.

$$\dot{z}_{ctl} = \frac{1}{D_z} [(f_L^d - f_R^d) - (f_L - f_R)] - \frac{1}{T_z} z_{ctl} \quad (4.16)$$

Two methods were proposed to realize this difference in ankle height. The first method is straight forward change the reference position of the feet in  $z$ -direction:

$$p_R^{d*} = p_R^d + 0.5 \cdot \begin{pmatrix} 0 \\ 0 \\ z_{ctl} \end{pmatrix} \quad (4.17)$$

$$p_L^{d*} = p_L^d - 0.5 \cdot \begin{pmatrix} 0 \\ 0 \\ z_{ctl} \end{pmatrix} \quad (4.18)$$

This can lead to singularities if both legs are already fully stretched, as the edge of their workspace is reached. The second method relies on an additional rotating the pelvis link. For this approach to work, the robot needs a joint that allows rotations around the anterior axis ( $y$ -Axis) to keep the upper body upright. Since the robot model we used does not have this DOF, we only implemented the first approach. To circumvent singularities we chose a CoM height that guarantees knee bending while generating the reference pattern.

#### 4.2.5 Interaction between controllers

While each controller operate independently, their effects are highly coupled. The most important coupling exists between the chest posture controller and the ankle torque controller. Recall that in case of

a solid contact with the ground the ankle torque controller will not rotate the supporting foot but rather the body of the robot. This will however change the posture of chest frame. The chest posture controller compensates that and keeps the body upright. This tight coupling makes tuning the parameters  $D_i$  and  $T_i$  of the controllers difficult, as their performance depends on the other controllers. Best results were observed when the chest posture controller was tuned independently first, disabling the other controllers. Then the foot force controllers were enabled and tuned and finally the ankle torque controller was added and tuned.

#### 4.2.6 CoM and ZMP control

The controllers specified in the previous sections can make sure, that the ZMP that is realized tracks the ZMP that would result from a perfect execution of the input pattern. However depending on how the reference ZMP was predicted, that prediction might have already been wrong. For example the ZMP Preview Control approach uses the cart-table model to predict the ZMP. That prediction can deviate significantly from the real ZMP as the model simplifies the dynamics. Thus to make sure the desired ZMP is tracked accurately, the reference ZMP needs to be adapted as well.

Kajita et al. propose a dynamic system that describes the 3D-LIMP dynamics. To model mechanical lag they introduce a parameter  $T_p$  that should specify the ZMP delay. The state-space description of the dynamic system for the  $x$ -direction is given below. As before, the description of the dynamic system for the  $y$ -direction is analogous.

$$\frac{d}{dt} \begin{pmatrix} c_x \\ \dot{c}_x \\ p_{zmp_x} \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & 1 & 0 \\ \frac{g}{z_c} & 0 & -\frac{g}{z_c} \\ 0 & 0 & -\frac{1}{T_p} \end{pmatrix}}_{=:A} \cdot \begin{pmatrix} c_x \\ \dot{c}_x \\ p_{zmp_x} \end{pmatrix} + \underbrace{\begin{pmatrix} 0 \\ 0 \\ \frac{1}{T_p} \end{pmatrix}}_{=:B} u \quad (4.19)$$

As controller a feedback controller is proposed:

$$p_x^{d*} = u = (k_1, k_2, k_3) \cdot \left[ \begin{pmatrix} c_x^d \\ \dot{c}_x^d \\ p_{zmp_x}^d \end{pmatrix} - \begin{pmatrix} c_x \\ \dot{c}_x \\ p_{zmp_x} \end{pmatrix} \right] + p_{zmp_x}^d \quad (4.20)$$

To derive the corresponding gains  $(k_1, k_2, k_3)$  pole-placement with the poles  $(-13, -3, \sqrt{\frac{g}{c_x}})$  was proposed. The gains can be easily computed from the poles,  $A$  and  $B$  using predefined functions in MATLAB or similar software.

### 4.3 Implementation

To implement the stabilizer above, a number of problems had to be solved. For one, computing the inverse kinematics is challenging. During walking the base of support depends on which foot is in contact with the ground. For example, if the left foot is the support foot, it is considered the base and the right foot is considered the TCP. The reverse is true if the right foot is the support foot. In dual support phase, we actually have two bases of support, which yields a parallel kinematic chain. SIMOX, the framework used to compute the inverse kinematics, describes the kinematic structure of a robot as a directed tree. Solving the inverse kinematics was initially only possible for sub-trees of that tree. Which means it was not possible to choose base and TCP freely, but it was determined by the structure in which the kinematic model was initially described.

As a first approximation, a kinematic model with the left foot as root node was used. In the case of the right foot being the support foot, this approach leads to an increased error. Consider solving the inverse kinematics for both legs using a differential solver in two steps: First from the base (the left foot) to the pelvis, then from the pelvis to the right foot. Lets assume the IK computes a perfect solution to place the pelvis link and only achieves a small pose error of  $e_\alpha = 0.1$  in the pitch angle of the right foot. If

the trajectory is execute, the right foot will achieve its target pose, since it is constrained by the ground contact. However the error in the right foot pose will effect all other frames of the robot. Assuming a offset of  $v = (-0.5, 0, 1)^T m$  from the right foot to the pelvis link, we can compute the realized pelvis offset as  $v' = R_y(-e_\alpha) \cdot v = (-0.49825, 0, 1.00087)^T m$  Which yields 1.76mm error in x-direction and 0.87mm error in y-direction.

To solve this SIMOX was extended by a function `cloneInversed` that can compute a kinematic structure with arbitrary root placement from an existing description. For the parallel kinematic chain, it proofed sufficient to approximate it as a normal kinematic chain and constrain the base and TCP targets accordingly. However since the position of one foot will have small positioning errors, there will be some jitter introduced into the system. Integrating a solver for parallel kinematics might decrease some of the jitter observed in foot contact forces during dual support, as the constraint solver used for the simulation will only amplify this jitter.

As was the case with the components of the pattern generator, the core of the stabilizer is implemented as part of LIBBIPEDAL. A plugin integrates the stabilizer into the dynamic simulation.

## 4.4 Problems

Some problems became immediately clear when testing the stabilizer proposed by Kajita. The ground reaction forces in dual support are oscillating wildly. Instead of a continuous force at about  $0.5 \cdot f_g$  the forces on both feet oscillate between 0 and  $f_g$ , the support foot changes in rapid successions. As outlined in section 6.1 this is a result of the constraint solver method employed by BULLET. Subsequently the measured torques on the ankles did not follow the prediction as well. Please note that this is not merely sensor noise, these are the actual values used by the simulator. Even when adding mean-filters to smoothen the measured torques it was not possible to extract a meaningful control signal. Besides the sequential impulse solver newer versions of BULLET support a solver based on the Featherstone algorithm. Given the scope of this thesis, integrating that solver was out of question. Thus an alternative approach to stabilizing had to be found, but the Featherstone solver should be implemented in future work.

## 4.5 Alternative approach

A simple heuristic, we used the controllers proposed by Kajita as inspiration and replaced the force and torque feedback with the pose error of pelvis and feet frames respectively. The chest controller (Equation 4.1 and 4.2) Kajita et al. proposed for controlling the body posture where adapted to all control frames to provide a feedback on the pose error.

This yields a controller that keeps the feet pose parallel to the ground, which is important when the swing foot touches the ground. Controlling the pelvis and chest pose to follow the reference also keeps the robot upright. It should be noted that it is not feasible to implement this stabilizer in practice. As mentioned in section 4.2 precisely estimating the pose of a robot is not easy. While the dampening controllers can be configured to smoothen a noisy sensor signal, a high level of precision is required to ensure a correct foot posture.

Since the ZMP and CoM trajectory is not adapted, the compensation of environment disturbances is only based on a fast controller reaction to leave the reference trajectory as little as possible and the stability margins the ZMP provides. However as we will discuss in the evaluation section, this simple approach is already surprisingly resilient.

## 5 Push recovery

As we saw in the chapter about Stabilizers, not all disturbances can be compensated. If the disturbance reaches a certain severity, the trajectory can not be executed as planned without falling. Thus the trajectory needs to be changed radically to avoid falling. There is little hope to recover from continuous heavy disturbances, as any attempt to recover will be defeated. Thus we focus on short but severe disturbances, pushes.

The most prominent method to recover from pushes is the Capture Point. The idea is to find a point, that will guarantee that the CoM comes to a rest, if the support foot is instantaneously placed there. The push recovery implemented here uses a simplistic method based on the Capture Point.

### 5.1 Capture Point

Koolen et al. [12] derive the Capture Point for multiple models based on the 3D-LIPM. The simplest model is the 3D Linear Inverted Pendulum with a point contact and a massless telescopic rod. If we use the LIP equations 2.12 and 2.13 with zero input torque, we can derive that so called *orbital energy* of the pendulum. As we did in the section about the pattern generator, we will derive the equations only for one dimension. The other dimension follows analogous.

The base of the pendulum is assumed to be at the origin of the reference frame in 2.12. Since we want to specify the location of the CoM  $c = (c_x, c_y, c_z)$  freely, we need to substitute  $c_x$  with  $c_x - p_x$  to yield:

$$\ddot{c}_x = \frac{g}{z_c} (c_x - p_x) \quad (5.1)$$

The orbital energy  $E_x$  can be derived by subtracting the potential and kinetic energy:

$$E_x = \underbrace{\frac{1}{2} \dot{c}_x^2}_{\text{kinetic energy}} - \underbrace{\frac{g}{2 \cdot z_c} (c_x - p_x)^2}_{\text{potential energy}} \quad (5.2)$$

For the CoM to come to a rest the orbital energy  $E_x$  must be zero. Thus we can solve 5.2 to yield the  $x$  coordinate of point  $p$  that will achieve this. Since it is a quadratic equation there are two solutions:

$$p_x = x - \sqrt{\frac{z_c}{g}} \dot{c}_x \quad \text{or} \quad p_x = x + \sqrt{\frac{z_c}{g}} \dot{c}_x \quad (5.3)$$

Since it is generally desirable to chose a point that lies in the direction of the CoM motion we define the *Immediate Capture Point* as:

$$p_{ic} := c_x + \sqrt{\frac{z_c}{g}} \dot{c}_x \quad (5.4)$$

Placing the base of the pendulum (the ankle) there *instantaneously* will cause an orbital energy of zero, thus the head of the pendulum (the CoM) will stop at  $p_{ic}$ . In most cases we will not be able to move the base of the pendulum instantaneously. So we are more interested in the Immediate Capture Point in  $\Delta t$  seconds from now. This point can be obtained by 5.5. For a detailed derivation, we recommend the paper by Koolen et al. [12].

$$p_{ic}(\Delta t) = p_{ankle} + (p_{ic}(0) - p_{ankle}) \cdot e^{\frac{g}{z_c} \cdot \Delta t} \quad (5.5)$$

Where  $p_{ankle}$  is the current position of ankle of the support foot (the current base of the pendulum). and  $p_{ic}(0)$  is the current Immediate Capture Point as calculated by equation 5.4.

## 5.2 Fall detection

The first important step to recover from a large disturbance is to detect it. That means we need to detect if we reached a unstable state, from which it is unlikely that we can recover only by means of the stabilizer. We can use the measured ZMP to obtain a heuristic for such states. If the ZMP is outside or on the edge of the support polygon, the Cart-Table model does not guarantee stability. However in normal operations it is likely that the ZMP will touch (or leave) the border of the support polygon for short periods of time. A simple method to filter out this noise is to define a minimum duration the ZMP as to be in an unstable state. Choosing a small value will let us react faster to disturbances, but make the method error prone. Choosing a large value will add an additional detail until we can react, but is much less prone to be triggered by mistake. An experimental evaluation yielded good results with a duration of  $t = 0.35s$ .

## 5.3 Recovery

If a fall is detected, a recovery maneuver needs to be executed. In single support phase, that means we need to place the swing foot at the capture point. In dual support phase we try to move the support foot that is closest to the capture point. Recall that the base of the pendulum coincides with the ZMP in our model view. Thus if the resulting support polygon includes the capture point, the position will be stable. Since the motor velocities are limited, we need a minimal amount of time  $t_{min}$  to place the foot in the desired location. Thus instead of using the Immediate Capture Point, we use the future Immediate Capture Point  $p_{ic}(t_{min})$  to derive the desired location.

## 5.4 Implementation

The stabilizer that is outlined in chapter 4.2 was extended to call the fall detection module in each iteration of the control loop. If an unstable state is reached, the recovery module overrides the normal trajectory of the stabilizer. After completing the recovery maneuver, the robot remains in the given position. Resuming the original pattern requires planning dynamically stable transition trajectory. This is left for future work.

# 6 Dynamic Simulation

To evaluate the generated trajectories a simulator for the dynamics was developed. The simulator was build on the SIMDYNAMICS framework that is part of SIMOX. SIMDYNAMICS uses BULLET PHYSICS as underlying physics framework. A big part of the work on the simulator was spend on configuring the parameters and finding flaws in the physics simulation. Thus the simulator includes a extensive logging framework that measures all important parameters of the simulation. For visualizing and analysing the measurement the Open Source tools IPYTHON, NUMPY and PANDAS where used.

## 6.1 Simulating rigid body dynamics

Physical simulation in general can be divided into discrete methods and continuous methods. Discrete simulators only compute the state of the system at specific points in time, while continuous simulators are able to compute the state of the system at any point in time. While continuous simulation is the more flexible approach, it quickly becomes impractical with the number of constraints involved. Typically a large amount of differential equations need to be solved. Since it is hard to obtain analytical solutions for most differential equations, numerical methods need to be used, which often have a large runtime. In contrast, discrete simulation methods only compute simulation values for specific time steps. This exploits the observation that we will typically query the state of the physics engine only at a fixed rate anyway, e.g. at each iteration of our control loop). Rather than solving the differential equations that describe the physical system in each step, a solution is derived from the previous simulation state.

In a physical system we can typically find two kind of forces: Applied forces and constraint forces. Applied forces are the input forces of the system. Source of applied forces are for example objects like springs or gravity. Constraint forces are fictional forces that arise from constraints we impose on the system: Non-penetration constraints, friction constraints, position constraints of joints or velocity constraints for motors. Mathematically we can express such constraints in the form:  $C(x) = 0$  or  $\dot{C}(x) = 0$  in the case of equality constraints, or as  $C(x) \geq 0$  or  $\dot{C}(x) \geq 0$  in the case of inequality constraints. For example the position constraint of a joint  $p$  connected to a base  $p_0$  with distance  $r_0 = ||p - p_0||$  would be:  $C(p) = ||p - p_0||^2 - r_0^2$ . If  $p$  is moving with a linear velocity  $v$  a constraint force  $F_c$  is applied to  $p$  to maintain this constraint. We can view  $C$  as a transformation from our Cartesian space to the constraint space. Thus by computing the Jacobian  $J$  of  $C$  we can relate velocities in both spaces. Furthermore we can relate constraint space forces  $\lambda$  with Cartesian space forces using the transpose of the Jacobian. Thus if we can find the constraint space force  $\lambda$  that is needed to maintain this constraint we can compute  $F_c$  using  $F_c = J^T \lambda$ . Computing this constraint space forces is the task of the constraint solver.

The constrained solver used by BULLET and thus the constraint solver used for simulating the patterns here is a sequential impulse solver. To make some calculations easier, a SI solver works with impulses and velocities rather than forces and accelerations. Impulses and forces can be easily transformed in each other as  $P = F \cdot T$  where  $P$  is the impulse and  $T$  the time step size. A sequential impulse solver tries to compute the constraint force (in this case rather impulse)  $\lambda$  for each constraint *separately*. For each constraint the following steps are executed:

1. Compute the velocity that results from *applied forces* on the body
2. Calculate constraint force to satisfy the velocity constraint
3. Compute new velocity resulting from constraint force *and* applied force on the body
4. Update position of the body by integrating velocity:  $p[n+1] = p[n] + v \cdot T$

Of course this might not lead to a global solution, as satisfying a constraint might violate a previously solved one. The idea is to repeatedly loop over all constraints, so that a global solution will be reached.

Obviously the quality of this method relies on how often this loop is executed. Consider the case of a kinematic chain where a movement of a link always violates at least one constraint. It is clear that this method needs a lot of iterations to yield good results in this case. It becomes even worse in the case of a parallel kinematic chain that is in contact with the ground, as is the case for a bipedal robot in dual support stance. Solving a non-penetration constrain on either end will invalidate the position constraint of the next link. In turn, the position constraint of each link needs to be updated until the other end of the kinematic chain is reached. If the non-penetration constraint is violated again for this end, the whole process starts again in reverse direction. This leads to oscillations that need a lot more iterations to level off to an acceptable level. Despite these inaccuracies using enough solver iterations this still yields an overall usable systems. However the velocities will still have a small random error in each simulation step. This poses a major problem when trying to measure accelerations, as the random error causes them to accelerate wildly. This circumstance needs to be taken into account when dealing with values derived from the acceleration (e.g. the ZMP), as mean-filters might be necessary.

## 6.2 Practical challenges of physics simulation

While walking only the feet of the robot are in contact with the ground. Thus the stability of the whole robots depends on the contact of the feet with the floor. Especially in single support phase that area is very small with regard to the size of the robot. For that reason the accuracy of ground contact forces and friction is of utmost importance for the quality of the simulation. In general three classes of errors need to be eliminated to get a good simulation:

1. Incorrectly configured parameters, such as fictions coefficients and contact thresholds
2. Numerical errors
3. Inherent errors of the method

As outlined in section 6.1, the physics of the system are formulated as input forces and constraints that need to be solved for the constraint forces. Since BULLET uses the iterative approach described in section 6.1, it is important to use a sufficient amount of iterations for each simulation step. Another important parameter is the time step of each simulation step. Through experimental evaluation, a simulation with 2000 solver iterations and a time step size of 1ms was sufficiently stable. However since the number of iterations is very high and a lot of time steps are calculated during the simulation, numeric errors become significant. That made is necessary to enable using double precision floating point numbers for the values used during simulation.

To decide which contact constraints are active for which points, BULLET must solve for object collisions. Depending on the objects involved different algorithms are used to calculate the contact points. Major gains in accuracy could be observed by replacing the feet and the floor with simple box shapes, instead using mesh based models.

## 6.3 Implementation

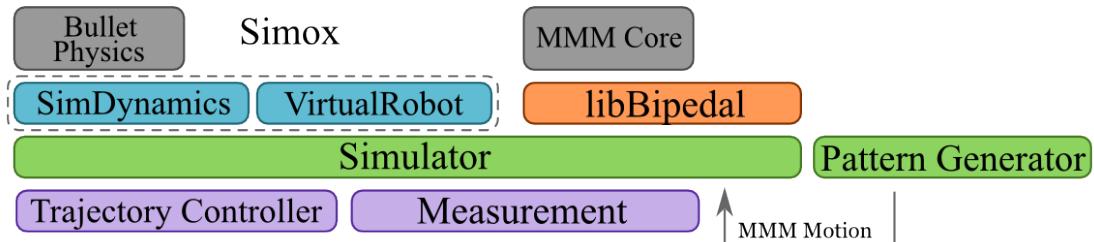


Figure 6.1: Achitechture of the simulator

The simulator was designed to load arbitrary motions in the MMM format and replay them. Additional stabilization algorithms can be applied depending on additional information provided in the MMM motions. These stabilization algorithms are called by real-time plugins implemented by the class `TrajectoryController`, that get invoked in each simulation step.

Currently three controllers are implemented. A controller based on the stabilizer proposed by Kajita (as outlined in section 4.2), a simple heuristic stabilizer (outlined in section 4.5) and a controller that just plays back the specified walking pattern. Each control algorithm is invoked in the same cycle time as the reference walking pattern.

The resulting, possibly modified, joint angles are then interpolated using cubic splines. This ensures a smooth velocity profile and mitigates destabilizing oscillations caused by large velocity changes.

The interpolated angles are send via `SIMDYNAMICS` to the corresponding motors. Since the motor implemented in `BULLET` are velocity controlled, PID based motor controllers were added to `SIMDYNAMICS`. They control the motor velocities to compensate position errors.

The motors implemented in `BULLET` do not limit the motor velocity and acceleration. This is not consistent with real motors, thus limits for velocities and acceleration where introduced to `SIMDYNAMICS`, that can be configured per joint.

The graphical user-interface supports the visualization of measured and desired values for CoM, Immediate Capture Point, ZMP, ankle torque, and ground reaction forces. The best-case trajectory can be visualized as an overlay robot (“phantom robot”). Figure 6.2 shows some of the supported visualizations.

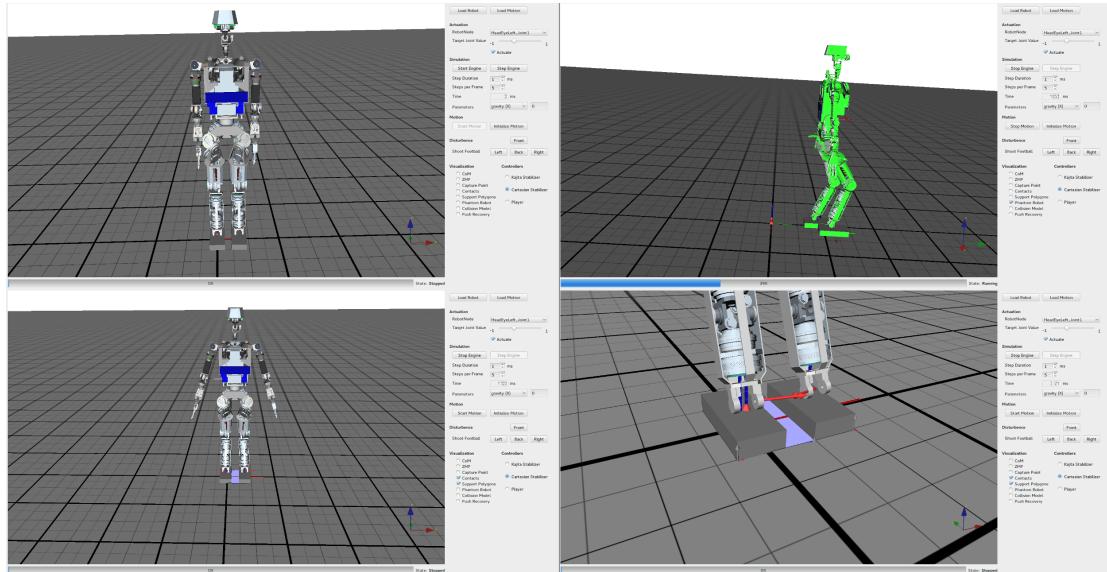


Figure 6.2: GUI of the simulator. Top left: Before starting the physics engine. Top right: Robot with best-case “phantom” overlay. Bottom left: Robot standing with support polygone visualization. Bottom right: Close-up of the support polygone, ankle torques and ground reaction forces.

## 6.4 Simulation analysis

An important part of the simulation is the generation of measurements that can be carefully evaluated offline or displayed in the visualization. For this purpose a modular measurement component was added to the simulator. An important design goal was to keep the measurement component as simple to extend and maintain as possible. Each module measures a specific set of values and writes them, indexed by the corresponding timestamp, to its log file. As output format the well known plain text format CSV was used. The visualization can query the measurement components directly to get the newest values to be displayed. For example the ZMP module measures the actual ZMP and also provides an interface to

query the trajectory ZMP and the reference ZMP that was provided as input for the pattern generator. Thus all three values can be displayed in the visualization and easily compared later by analysing the log file. Since the goal was to keep the component as simple as possible, we use existing well known tools for analyzing the generated log files. Some small helper scripts are provided to make it easier to load the data into the time series analysis framework PANDAS. PANDAS interfaces with the popular plotting framework MATPLOTLIB to display plots of the data. IPYTHON is used to easily run the analysis and display the results in a browser window. All plots of simulated patterns found in this thesis can be generated automatically for every simulation.

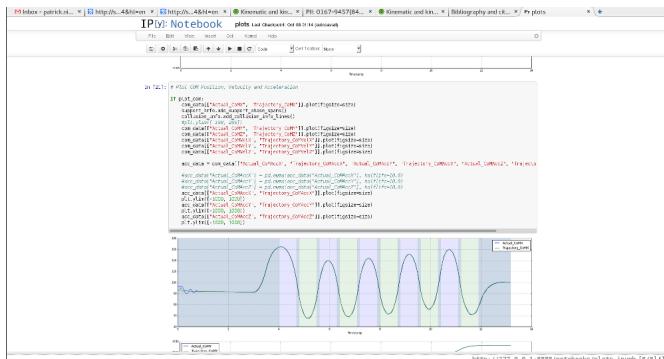


Figure 6.3: Screenshot of ipython showing plots of the CoM obtained by a simulation

## 7 Experimental Evaluation

To test the performance of the stabilizer against simple pattern playback, both walking straight and walking in circle where tested. To simulate further disturbances, the simulation shoots a football at the robot. In the unstabilized case, only the joint trajectory that is generated by the pattern generator is interpolated and replayed. In the stabilized case the alternative stabilizer described in section 4.5 is used.

### 7.1 Undisturbed walking

#### 7.1.1 Walking in a straight line

The first scenario is walking in a straight line for 10 steps. Figure 7.1 shows the simulation of walking straight for 10 steps using no stabilization.

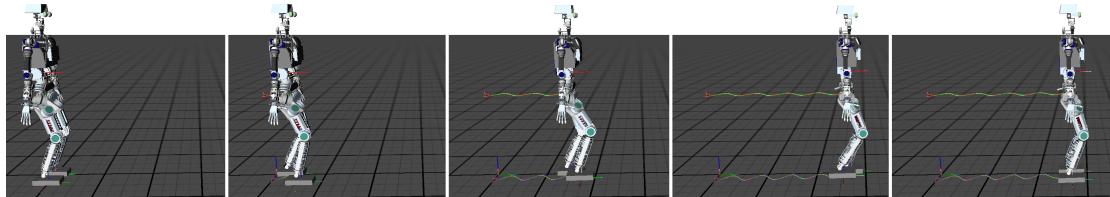


Figure 7.1: Frames of undisturbed straight walking

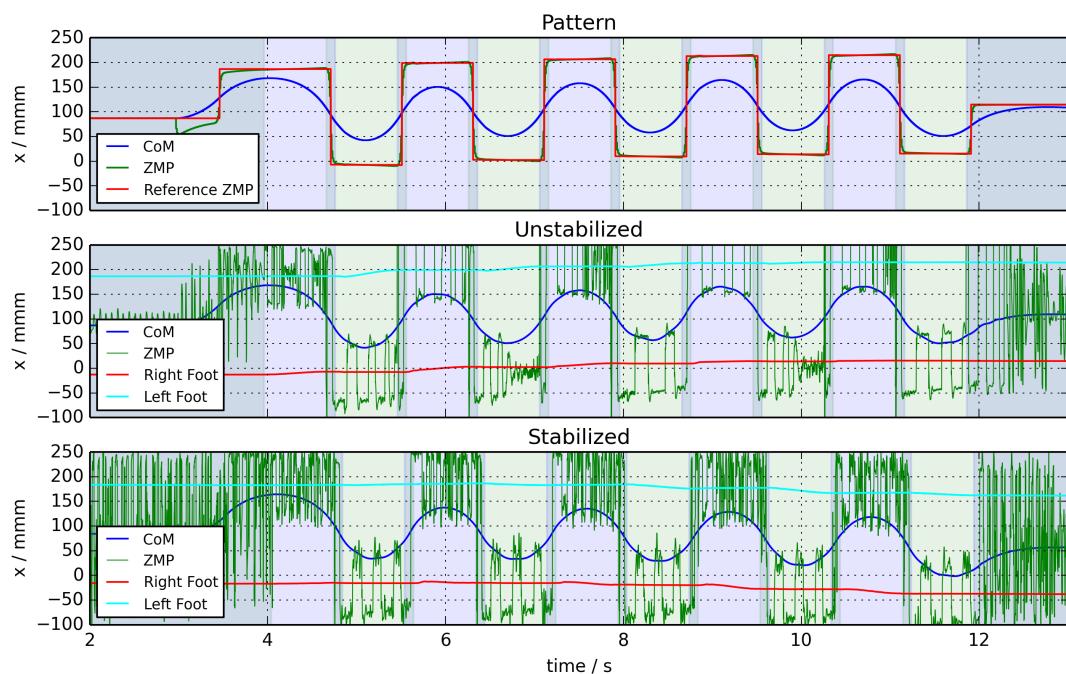


Figure 7.2: CoM and ZMP as specified by the pattern (top) and actual realized values (middle and bottom). All coordinates in the global reference frame.

Figure 7.2 shows the desired and realized CoM and ZMP trajectories in both cases. As you can see the ZMP deviates significantly from the desired trajectory and oscillates between both edges of the support polygon. We believe this is caused by the problems outlined in section 6.1. Consider figure 7.3 which shows the desired CoM acceleration in comparison to the realized acceleration.

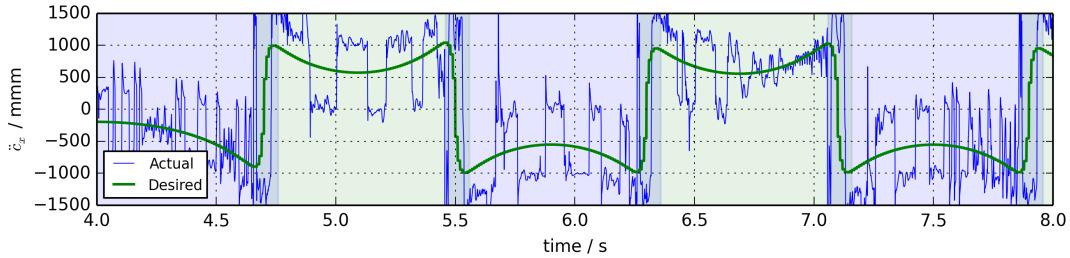


Figure 7.3: Acceleration of the CoM in  $x$ -direction.

However the foot remains in full ground contact and friction forces are applied accurately. Thus dynamically stable walking is realized for both unstabilized and stabilized walking. See figure 7.4 for the contact forces that are applied to each foot while walking. The forces oscillate quite a bit due to simulation error. However the mean values meet the expectations of the gravity force vector.

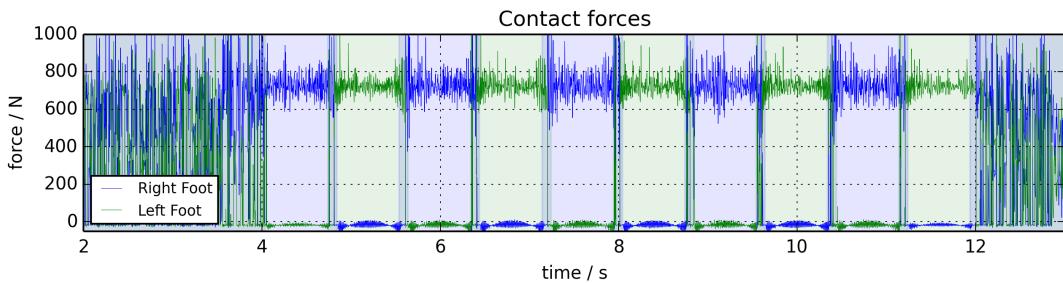


Figure 7.4: Contact forces in  $z$  direction for both feet.

Figure 7.5 depicts the ankle torques that are applied to each feet. Please note the rapid change in sign during single support. We believe this is caused by BULLETS method of applying contact forces. Contact forces are always applied to the contact point with the deepest penetration into the floor. In flat ground contact the penetration depth of contact points on the edge of the support polygon is almost the same. Small errors lead BULLET to alternate between contact points on opposite sides of the support polygon. We hope that a more stable Featherstone constraint solver will produce more accurate contact forces and ankle torques.

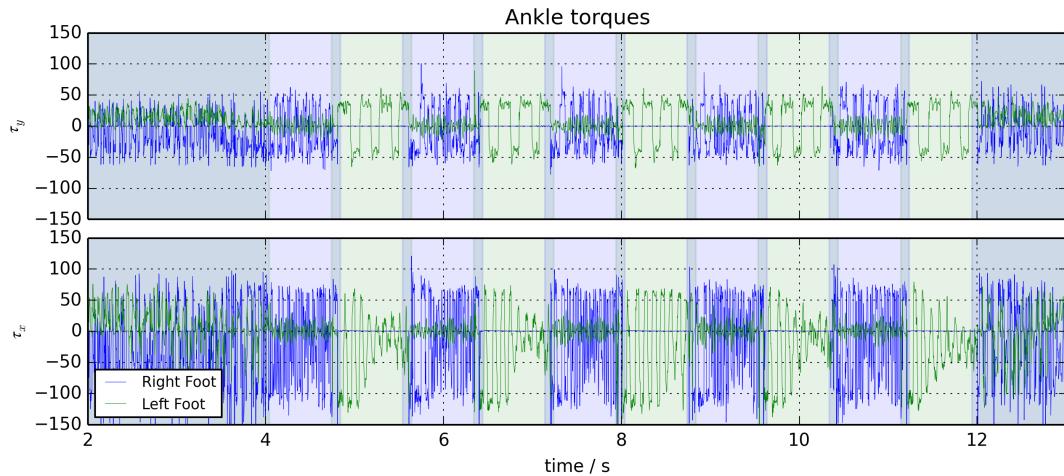


Figure 7.5: Ankle torques along the  $x$  and  $y$  axis for both feet.

An interesting way to compare the trajectory executed by ARMAR 4 to walking in humans is to use the angular momentum around the CoM. See figure 7.6 for the angular momentum trajectories as measured in test subjects by Laturnus [13] in her Bachelor's Thesis. Note the similarities to the angular momentum that is realized in the simulation in figure 7.7

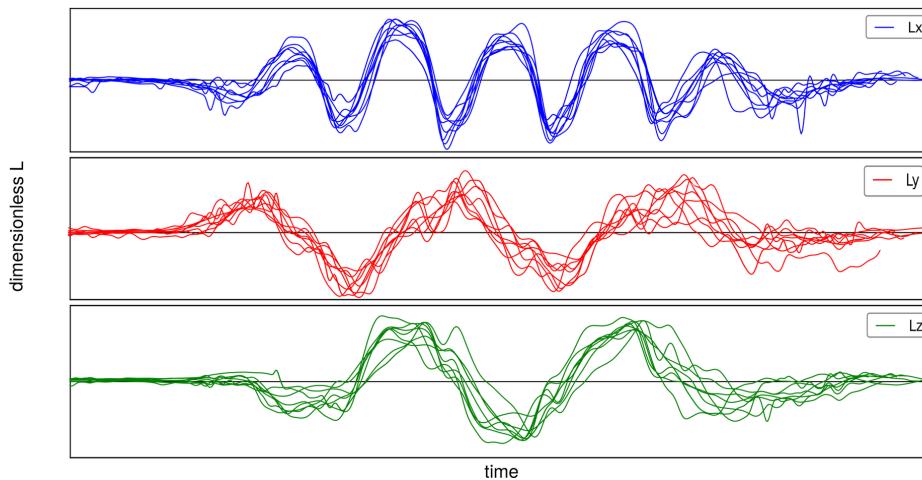


Figure 7.6: The angular momentum as measured on test subjects for straight walking by Laturnus [13]

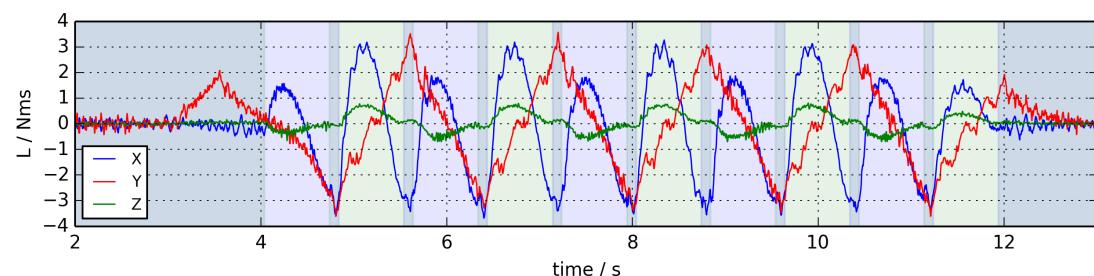


Figure 7.7: The angular momentum as measured in the simulation while walking straight.

Immediately clear are the more pronounced sharp minima and maxima of the  $y$ -component of the angular momentum. This is probably caused by the missing toe/heel lift-off and strike down phases, as

the foot is kept parallel to the floor at all times.

### 7.1.2 Walking in a circle

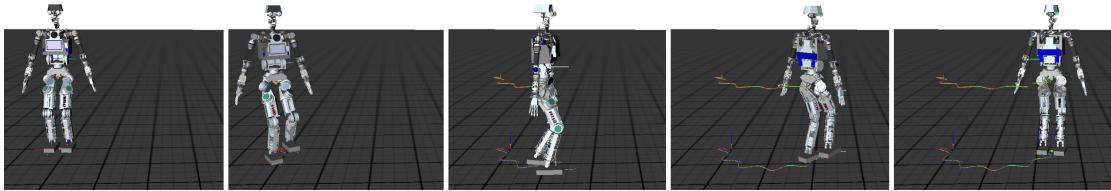


Figure 7.8: ARMAR4 walking in a half circle in 12 steps.

Figure 7.10 shows the simulation of unstabilized walking in a circle. The robot walks in 12 steps in an arc of  $180^\circ$  and 0.5 m radius. As you can see the desired  $180^\circ$  turn is not realized completely. Due to the chest rotation following the tangent of the circle, a torque around the yaw-axis is exerted on the foot. Recall that during pattern generation we assumed that this torque is zero. Since we do not correct this disturbance the trajectory deviates significantly. See figure 7.9 for the realized ZMP distribution and CoM trajectory for both unstabilized and stabilized walking. The unstabilized trajectory will get unstable towards the end of the trajectory. However the stabilized trajectory remains stable.

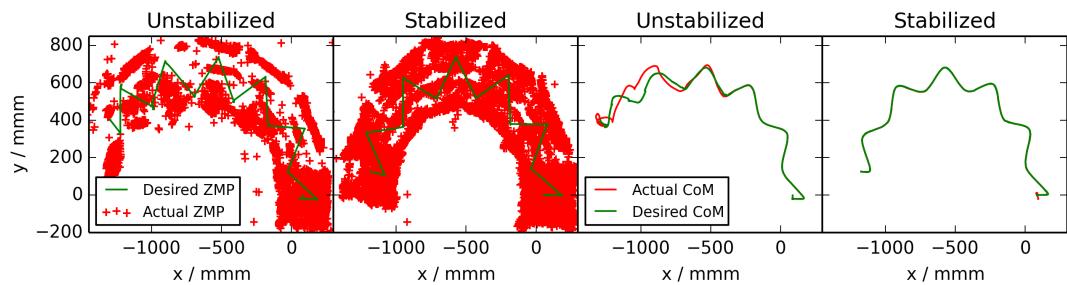


Figure 7.9: Walking in a circle. ZMP (left) and CoM (right) as specified by the pattern and the actually realized values. Each for the unstabilized and stabilized case.

## 7.2 Disturbed walking

The scenario used to test the performance of the stabilizer under disturbance, was applying a short push to the chest. The results are compared with the performance of the unstabilized trajectory.

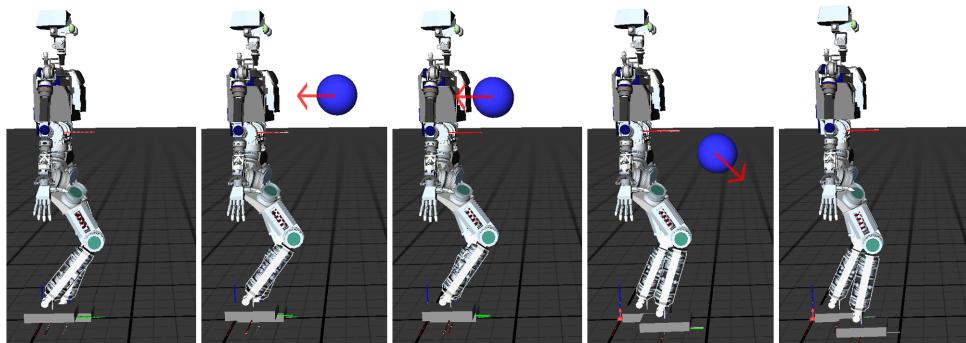


Figure 7.10: ARMAR4 getting hit by a ball at chest height. Red arrows show approximate direction of the ball velocity vector.

To simulate a push, a ball with a radius of 11cm and weight of 450g (FIFA football) is shot from 1 meter distance at the chest. See figure 7.11 for the realized CoM and ZMP trajectories. The point of impact is denoted as red lines. The unstabilized trajectory becomes unstable and leads to a fall at  $t = 9.5s$ . The stabilized trajectory is noticeable disturbed, as the yaw momentum is not compensated, but remains stable.

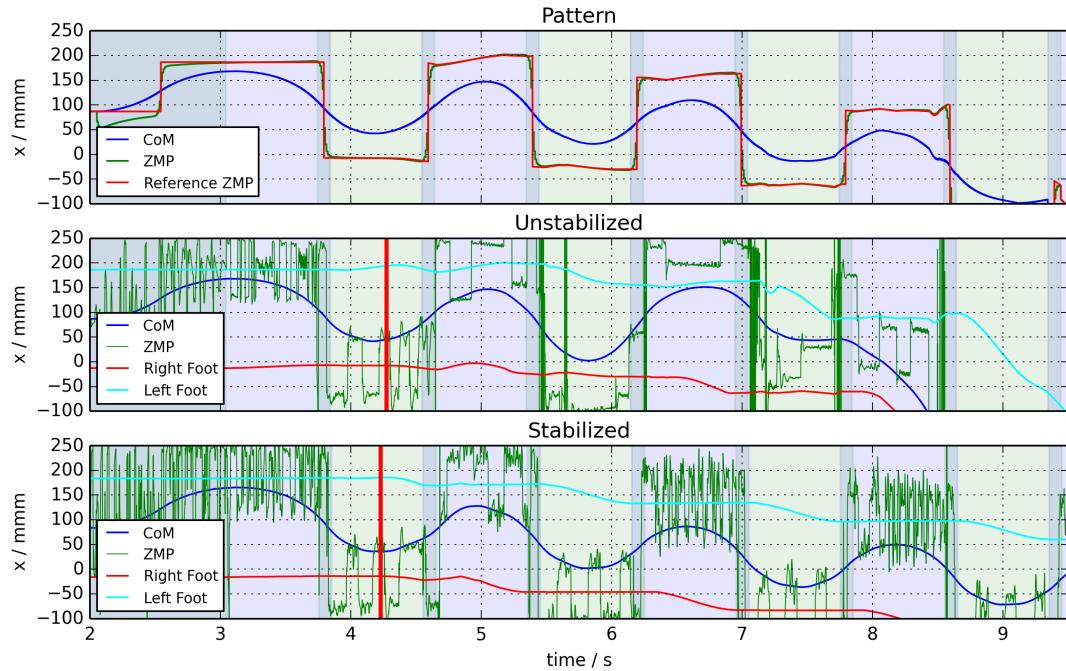


Figure 7.11: CoM and ZMP as specified by the pattern (top) and actual realized values (middle and bottom). All coordinates in the global reference frame. Red lines denote the point of impact.

Figure 7.12 show the impact of the push to the chest on the immediate capture point. The capture point shifts rapidly in the direction of the disturbance. This property will be utilized to derive a capture position for push recovery.

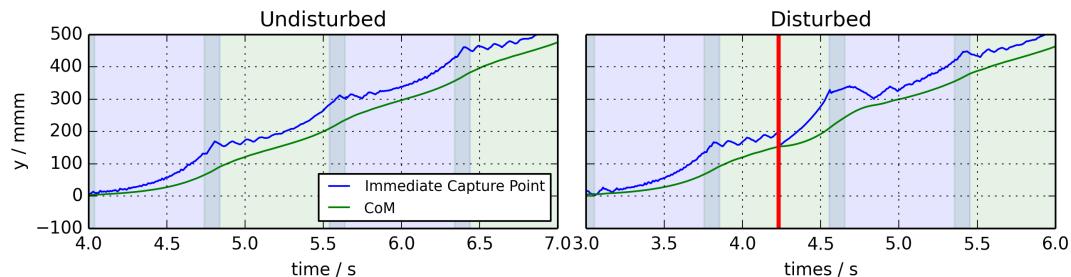


Figure 7.12: Impact of the disturbance on the immediate capture point. Red lines denote the point of impact.

## 7.3 Push recovery

The performance of the push recovery was evaluated with a simple best-case scenario. The robot is balancing on the left foot and is pushed on the right shoulder. Most notably this scenario was used to

demonstrate the push recovery based on the Capture Point implemented in the IHMC/Yobotics Biped. [14]. While the push recovery implemented is able to recover from any position, with the current method it is not reliable in the general case. This has multiple reason, for one the fall detection is not very reliable. Another reason is that the speed limits of the leg joints impose a maximum velocity for the foot movement. So not only needs the target position to be reachable, it also needs to be reachable in a specific amount of time. As above a football is used as source of disturbance. See figure ?? for a push recovery.

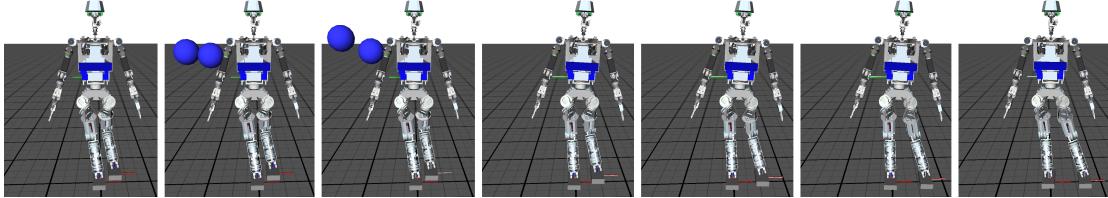


Figure 7.13: ARMAR4 getting hit by two balls at the left shoulder.

## 8 Conclusion and future work

A design and the implementation of a software framework for bipedal humanoid walking were presented. Building on an implementation by Ömer Telemz, a generator for walking pattern using ZMP Preview Control was developed. To test the resulting patterns a dynamic simulation for walking was build. The simulation utilizes the SIMDYNAMICS framework contained in SIMOX. The simulation was extended by real-time controllers for stabilization and push recovery. For easily integration into other software projects all methods presented here are implemented in a shared library LIBBIPEDAL. LIBBIPEDAL is independent of the dynamic simulation and only depends on VIRTUALROBOT for computing forward and inverse kinematics.

Evaluation showed that stable walking could be realized in simulation. By applying a stabilizer the trajectories remained stable, even when faced with disturbances. Falling was avoided by initiating a push recovery mechanism based on the Capture Point.

Since accuracy of the dynamic simulation suffered severely by using the Sequential Impulse Solver method, it should be replaced by a solver based on the Featherstone method. The improved accuracy should make a torque feedback more viable and enable to actually use the stabilizer based described and implemented in 4.2.

The pattern generator and stabilizer only realize basic walking, that is noticeable different to human walking. For one the knees remain bent at all times, the center of mass stays at the same height and the toe joint is not used. A follow up paper [5] extends the methods implemented here to include the toes. The realized walking trajectories seem to be more natural.

As the evaluation of the circular trajectory shows, the yaw moment exerted on the foot can cause severe disturbances. To deal better with trajectories that include turns or arm movements, that torque should be compensated. Kim et al.[15] propose to move the arms around the roll axis to compensate for yaw momentum.

The push recovery implemented is very rudimentary. The placement of the foot to recover from a push does not consider collisions with the environment. Also, after executing a push recovery step, the original trajectory can not be resumed. To enable that, online planning of dynamically stable motions needs to be integrated. Also the foot is placed at the exact location of the future capture point. However it suffices to place the foot in a way to include the capture point. In most cases this would require a lot less foot movement. Moving the leg to the target point also changes the velocity of the CoM, thus changes the location of the capture point. To compensate this, the capture point should be tracked using a controller instead of using a static future capture point.

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