

Walking pattern generation for humanoid bipedal robots

Bachelors's Thesis
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1 Introduction

motivation, and a bit of overview of humanoid walking. I recommend to leave it for later, start with the sections that you feel its easier to write (usually, the ones that have more content).

2 Models for humanoid walking

2.1 Humanoid walking

- Support phases
- ...

2.2 Static and dynamic walking

2.3 The linear inverted pendulum model

A simple model for describing the dynamics of a bipedal robot during single support phase is the 3D inverted pendulum. We reduce the body of the robot to a point-mass at the center of mass and replace the support leg by a mass-less telescopic leg which is fixed at a point on the supporting foot. Initially this will yield non-linear equations that will be hard to control. However by constraining the movement of the inverted pendulum to a fixed plane, we can derive a linear dynamic system. This model called the 3D *linear* inverted pendulum model (short *3D-LIPM*).

Use different name for CoM, p will be rather used for the ZMP, maybe c ?
picture of 3D-LIPM

2.4 The inverted pendulum

To describe the dynamics of the inverted pendulum we are mainly interested in the effect a given actuator torque has on the movement of the pendulum.

For simplicity we assume that the base of the pendulum is fixed at the origin of the current cartesian coordinate system. Thus we can describe the position inverted pendulum by a vector $p = (x, y, z)$. We are going to introduce an appropriate (generalized) coordinate system $q = (\theta_R, \theta_P, r)$ to get an easy description of our actuator torques: Let m be the mass of the pendulum and r the length of the telescopic leg. θ_P and θ_R describe the corresponding roll and pitch angles of the pose of the pendulum.

Now we need to find a mapping between forces in the cartesian coordinate system and the generalized forces (the actuator torques). Let $\Phi : \mathbb{R}^3 \rightarrow \mathbb{R}^3, (\theta_R, \theta_P, r) \mapsto (x, y, z)$ be a function that maps the generalized coordinates to the cartesian coordinates. Then the jacobian $J_\Phi = \frac{\partial p}{\partial q}$ maps the *generalized velocities* to *cartesian velocities*. Furthermore we know that the transpose J_Φ^T maps *cartesian forces* $F = m(\ddot{x}, \ddot{y}, \ddot{z})$ to *generalized forces* (τ_r, τ_p, f) .

We write x , y and z in terms of our generalized coordinates to compute the corresponding jacobian J_Φ . From the fact that the θ_P is the angle between the projection of p onto the xz -plane and p and θ_R the angle between p and the projection onto the yz plane we can derive the following equations :

add image with angles here

$$\begin{aligned} x &= r \cdot \sin \theta_P & &=: r \cdot s_P \\ y &= -r \cdot \sin \theta_R & &=: -r \cdot s_R \\ z &= \sqrt{r^2 - x^2 - y^2} = r \cdot \sqrt{1 - s_P^2 - s_R^2} \end{aligned} \quad (2.1)$$

From which we can compute the jacobian by partial derivation:

$$J = \frac{\partial p}{\partial q} = \begin{pmatrix} 0 & r \cdot c_P & s_P \\ -r \cdot c_R & 0 & s_P \\ \frac{2 \cdot r \cdot s_P c_P}{\sqrt{1 - s_P^2 - s_R^2}} & \frac{2 \cdot r \cdot s_R c_R}{\sqrt{1 - s_P^2 - s_R^2}} & \sqrt{1 - s_P^2 - s_R^2} \end{pmatrix} \quad (2.2)$$

reference paper

Using the equation of motion as given by

$$F \cdot \begin{pmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{pmatrix} = (J^T)^{-1} \begin{pmatrix} \tau_R \\ \tau_P \\ f \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -m \cdot g \end{pmatrix} \quad (2.3)$$

and equations 2.2 and 2.1 we can derive the following equations:

$$m(-z\ddot{y} + y\ddot{z}) = \frac{\sqrt{1-s_P^2-s_R^2}}{c_R} \cdot \tau_R + mgy \quad (2.4)$$

$$m(z\ddot{x} - x\ddot{z}) = \frac{\sqrt{1-s_P^2-s_R^2}}{c_P} \cdot \tau_P + mgx \quad (2.5)$$

Observe that the terms of the left-hand side are not linear. To remove that non-linearity we are going to use the *linear* inverted pendulum model.

2.5 Linear Inverted Pendulum Model

In a man-made environment it is fair to assume that the ground a robot will walk on can be approximate by a slightly sloped plane. In most cases it can even assumed that there is no slope at all.

The basic assumption in the next section will be that the CoM will have a *constant displacement* with regard to our ground plane. Thus we can constrain the movement of the CoM to a plane that is parallel to the ground plane. Note that this assumption is, depending on the walking speed, only approximately true for human walking as shown by Orendurff et. al. For slow to fast walking (0.7 m/s and 1.6 m/s respectively) the average displacement in z -direction was found to be between 2.7cm and 4.81 cm. While the walking patterns generated based on the LIP-model will guarantee dynamic stability, they might not look natural with regard to human walking.

We are going to constrain the z coordinate of our inverted pendulum to a plane with normal vector $(k_x, k_y, -1)$ and z -displacement z_c :

$$z = k_x \cdot x + k_y \cdot y + z_c \quad (2.6)$$

Subsequently the second derivative of z can be described by:

$$\ddot{z} = k_x \cdot \ddot{x} + k_y \cdot \ddot{y} \quad (2.7)$$

Substituting 2.6 and 2.7 into the equations 2.4 and 2.5 yields the following equations:

$$\ddot{y} = \frac{g}{z_c} y - \frac{k_x}{z_c} (x\ddot{y} - \ddot{x}y) - m_{z_c} \cdot \tau_R \cdot \frac{\sqrt{1-s_P^2-s_R^2}}{c_R} \quad (2.8)$$

$$\ddot{x} = \frac{g}{z_c} x + \frac{k_y}{z_c} (x\ddot{y} - \ddot{x}y) + m_{z_c} \cdot \tau_P \cdot \frac{\sqrt{1-s_P^2-s_R^2}}{c_P} \quad (2.9)$$

The term $x\ddot{y} - \ddot{x}y$ that is part of both equations is still causing the equations to be non-linear. To make this equations linear we will assume that our ground plane has no slope, thus $k_x = k_y = 0$ and the non-linear terms will vanish.

Another problem is that the actuator torques τ_R and τ_P both have non-linear factors $\frac{\sqrt{1-s_P^2-s_R^2}}{c_R}$ and $\frac{\sqrt{1-s_P^2-s_R^2}}{c_P}$ respectively. This can be solved by substituting with the following *virtual inputs*:

$$\tau_P \cdot \frac{\sqrt{1 - s_P^2 - s_R^2}}{c_P} = u_P \quad (2.10)$$

$$\tau_R \cdot \frac{\sqrt{1 - s_P^2 - s_R^2}}{c_R} = u_R \quad (2.11)$$

Which yields our final description of the dynamics:

$$\ddot{y} = \frac{g}{z_c} y - \frac{u_R}{m z_c} \quad (2.12)$$

$$\ddot{x} = \frac{g}{z_c} x + \frac{u_R}{m z_c} \quad (2.13)$$

2.6 The Zero Moment Point

A very popular approach to humanoid walking are schemes based on the Zero Moment Point. One reason for that might be that it is very simple to describe constraints for dynamic stability using this reference point. As long as the following condition is met we will have full ground contact of our support foot and thus can realize dynamically stable walking: *The ZMP is strictly inside the support polygon of the support foot.*

For flat ground contact of our support foot with the floor the ZMP corresponds with the position of the center of pressure (CoP). Indeed, some author (notably Pratt) prefer to use the term CoP instead of ZMP.

The CoP of an object in contact with the ground can be computed as the sum of all contact points p_1, \dots, p_n weighted by the forces in z -direction f_{1z}, \dots, f_{nz} that is applied:

$$p := \frac{\sum_{i=1}^N p_i f_{iz}}{\sum_{i=1}^N f_{iz}} \quad (2.14)$$

An important fact (and the origin of its name) is that there are no torques around the x and y axis at the ZMP:

$$\tau = \sum_{i=1}^N (p_i - p) \times f_i \quad (2.15)$$

Splitting that up into each component using the definition of the cross product yields:

$$\tau_x = \sum_{i=1}^N (p_{iy} - p_y) f_{iz} - \overbrace{(p_{iz} - p_z)}^{=0} f_{iy} \quad (2.16)$$

$$\tau_y = \sum_{i=1}^N \overbrace{(p_{iz} - p_z)}^{=0} f_{ix} - (p_{ix} - p_x) f_{iz} \quad (2.17)$$

$$\tau_z = \sum_{i=1}^N (p_{ix} - p_x) f_{iy} - (p_{iy} - p_y) f_{ix} \quad (2.18)$$

Since we have flat ground contact, all contact points have the same z -coordinate as the ZMP, thus we can simplify τ_x and τ_y to:

$$\tau_x = \sum_{i=1}^N (p_{iy} - p_y) f_{iz} = \sum_{i=1}^N (p_{iy} f_{iz}) - \left(\sum_{i=0}^N f_{iz} \right) \cdot p_y \quad (2.19)$$

include pattern generation just based on 3D-LIPM, I don't understand how they derived the controller

$$\tau_y = \sum_{i=1}^N -(p_{ix} - p_x) f_{iz} = \sum_{i=1}^N -(p_{ix} f_{iz}) + \left(\sum_{i=0}^N f_{iz} \right) \cdot p_x \quad (2.20)$$

Furthermore we can use the corresponding components p_x and p_y from the definition of the ZMP 2.14 and substitute in the equations 2.19 and 2.20.

This will yield: $\tau_x = \tau_y = 0$.

Please note that τ_z will in general not be zero, nonetheless in case of straight walking it is often assumed to be zero as well.

2.6.1 The table-cart model

The table-cart model is equivalent to the 3D-LIPM model discussed before, but somewhat more intuitive for computing the resulting ZMP from an CoM motion.

The model consists of an (infinitely) large mass-less table of height z_c , while the foot of the table has the shape of the support polygon. Given a frictionless cart with mass m that moves on the table we can compute the resulting ZMP in the support foot.

First of all, lets compute the torque τ_x and τ_y around the x-axis and y-axis at the ZMP on the support foot.

$$\tau_y = \overbrace{-mg(c_x - p_x)}^{\text{torque due to gravity}} + \overbrace{m\ddot{x} \cdot z_c}^{\text{torque due to acceleration of cart}} \quad (2.21)$$

$$\tau_x = -mg(c_y - p_y) + m\ddot{y} \cdot z_c \quad (2.22)$$

Please note the similarity to the equations 2.12 and 2.13 when assuming the base of the pendulum is located at p .

If we now use the property of the ZMP that the torque around the x and y -axis is zero, we can solve for the ZMP position p :

$$p_x = c_x - \frac{z_c}{g} \ddot{c}_x \quad (2.23)$$

$$p_y = c_y - \frac{z_c}{g} \ddot{c}_y \quad (2.24)$$

2.7 Simulating rigid body dynamics

Maybe describe the full-body methode to compute the ZMP

short introduction how that works and some problems with the approach

3 Pattern generator

Theory how to plan a dynamically stable movement preview controller Implementation Dynamic simulation

4 Controllers to stabilize a trajectory

Theory Implementation Evaluation

5 Push recovery

Theory Implementation Evaluation

6 Results

Make sure that somewhere, either here or in the evaluation sections, you show how you plotted the desired vs. real zmp, even the phantom robot if you want, and the graphics that you generated.

7 Conclusions

things to improve summary of work done and results

Bibliography