LOGSPACE, RandomWalks on Graphs and Universal Traversal Sequences

Seminar “Gems of Theoretical Computer Science”

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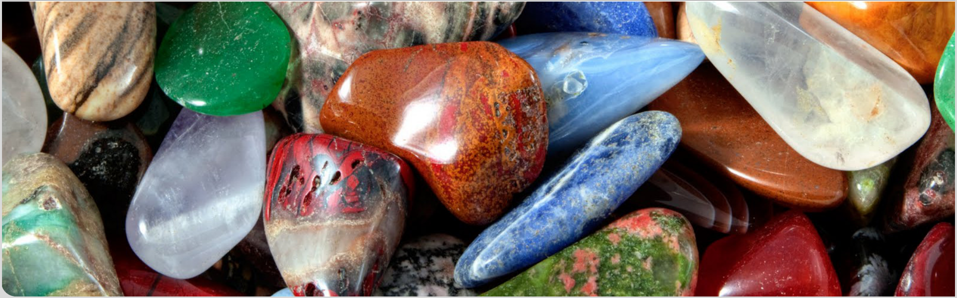

(Image source: http://therainbowfarmer.wordpress.com/tag/gemstone-healing-benefits)

Outline

Path finding as decision problems

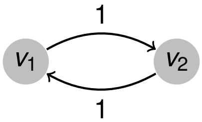

Figure: A simply cyclic markov chain. For a definition of the term cyclic we refer to appropriate literature.

- Example graphs + animation
- Interesting result: UPATH *seems* to be easier (note: PATH *could* be just as easy if $L = NL$)

Random Walk

- Example animation (dice as random visualization)

Universal Traversal Sequence

- Example for graph of size 3 with d=2 (there are only 3)

Foo

Definition

- Turing machine model (picture)

Examples

- What can we do with poly/log/constant space

$NLSPACE \subseteq P$

- Proof

PATH is NL-complete

- Informal definition of NL-completeness:
  - Transform instance of decision problem A to instance for B and use B to solve it
  - Restrict power of transformation function
- More precisely cover the transformation: TM -> configuration graph
  - only log-bounded space needed to encode current configuration
  - look at possible next configurations and write to output tape
- Applying PATH to that should be trivial

- Use RandomWalk to construct randomized decider for *UPATH*

- Show that it can not work for *PATH*

Proofing a bound on the number of steps

- Short example of Markov-Chain
- Relation to RandomWalk: Important connected graph
- RandomWalk can be modeled as Markov-Chain (Markov Property)
- Derive expected length of a random walk starting at any node *a* to reach any other node *b*: We proof an upper bound: Expected number of steps to visit all nodes in G.
- Overview:
- First compute P_v and thus $E(v, v)$
- Compute upper bound for $E(u, v)$
- Compute upper bound for $E(a, G)$
- Apply Markov-Inequality to compute probaboility that we need more than $8e$n steps.

d-regular graphs

- Definition
- Number of d-regular graphs of size n
- Traversal sequences and random walks

Probability amplification

- Conduct m random walks, probability of not finding a path is 2^{-m}, but size only $8en \cdot m$
- Make m big enough that the probability of a given traversal sequences to work for all graphs is bigger than 0:
  - Again markov inequality: $E(F) = g_{n,d} \cdot 2^{-m} < 1$, thus: $1 - Pr[F < 1] = Pr[F \geq 1] \leq E(F)/1 < 1$ which results in $0 < Pr[F < 1]$

References