Fourier formula

Evolution Core

Geometric Series

$$(a)\quad \sum_{n=0}^{N-1}\alpha^n = \begin{cases}\dfrac{1-\alpha^N}{1-\alpha} & \alpha \neq 1\\ N & \alpha = 1\end{cases}$$

$$(b)\quad \sum_{n=0}^{\infty}\alpha^n = \frac{1}{1-\alpha} \qquad |\alpha| < 1$$

$$(c)\quad \sum_{n=k}^{\infty}\alpha^n = \frac{\alpha^k}{1-\alpha} \qquad |\alpha| < 1$$

Euler's Formula

cosθ = $(e_{j\theta}+e_{-j\theta})/2$ = sin(θ+(π/2))

sinθ = $(e_{j\theta}-e_{-j\theta})/2j$ = cos(θ-(π/2))

X= sin(at)+cos(bt)

Tx =lcm(1/a.1/b)

<u>Parseval Identity</u>

DTFS: $\frac{1}{N}\sum_{n=0}^{N-1}|x[n]|^2 = \sum_{n=0}^{N-1}|C_k|^2$

DTFT: $\sum_{n=-\infty}^{\infty}|x[n]|^2 = \int_0^{2\pi}|X(e^{j\omega})|^2 d\omega$

Z-transform

Causal: n<=0 Anti-Causal: n>0

Non-Causal: n>0&&n<=0

Abs Sum: $\sum_n|x[n]| < \infty$

Square Sum: $\sum_n |x[n]|^2 < \infty$

Energy: $\sum_{n=-\infty}^{\infty}|x[n]|^2$

Power: $\frac{1}{2L+1}\sum_{n=-L}^{n=+L}|x[n]|^2$

P.Power: $\frac{1}{N}\sum_{n=0}^{N-1}|x[n]|^2$

Power(Freq): $\sum_{n=0}^{N-1}|C_k|^2$

Stable System:

1) BIBO ($|x[n]|<B_x<\infty$)

2) h[n] abs Summable

Trigo Identities

PFE

Down sampling	Up sampling

Filter Specifications

Cut-off Freq : $\omega_c = 0.5A$ (A = amplitude)

Cut-off Attenuation: $20\log_{10}|H(e^{j\omega c})| = -3dB$

Passband Edge : ω_p = Freq which $(1+\delta_p) > A > (1-\delta_p)$

Passband Attenuation: $20\log_{10}(1+\delta_p)/(1-\delta_p)$

Stopband Edge : ω_s = Freq where $A < \delta_s$

Stopband Attenuation: $20\log_{10}\delta_s$

Non-Int Conversion Ratio

Upsample (Interpolator) then Downsample (Decimator) (i.e. L = 100, M = 101, T' = 1.01T)

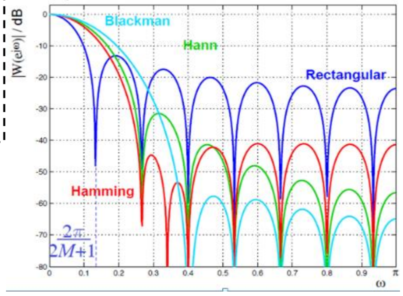
LPF Cut-off = $\min(\frac{\pi}{M}, \frac{\pi}{L})$, Gain = L

Increasing Window Length

1) Main Lobe Reduced
2) Height of first Side Lobe same
3) Narrower transition band

$$|H(\omega)| = \frac{1}{N}\left|\frac{\sin(\frac{N\omega}{2})}{\sin(\frac{\omega}{2})}\right|$$

$$\angle H(\omega) = -\frac{(N-1)\omega}{2}$$



Finite Impulse Response (FIR) Filter Design(Zero Terms)

$$h_i(n \neq 0) = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} H(\omega)e^{j\omega n}d\omega \qquad h_i(n = 0) = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} H(\omega)e^{j\omega 0}d\omega$$

$$h_i(n \neq 0) = \frac{\sin(n\omega_c)}{\pi n} \qquad h_i(n = 0) = \frac{\omega_c}{\pi}$$

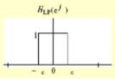
Hamming = 0.54 + 0.46 cos (n π / N) – N is order, = L-1

Hanning = 0.5(1-cos(2 πn/N))

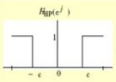
Filter Length Estimation:

Kaiser : $N \cong \frac{-10\log_{10}(\delta_p\delta_s)-13}{14.6\frac{(\omega_s-\omega_p)}{2\pi}}$

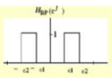
Bellanger : $N \cong \frac{-2\log_{10}(10\delta_p\delta_s)}{3(f_s-f_p)} - 1$



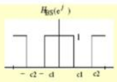
$$h_{LP}[n] = \frac{\sin\omega_c n}{\pi n}, \quad -\infty \le n \le \infty$$



$$h_{HP}[n] = \begin{cases} 1-\frac{\omega_c}{\pi}, & n = 0 \\ -\frac{\sin(\omega_c n)}{\pi n}, & n \neq 0 \end{cases}$$



$$h_{BP}[n] = \begin{cases} \frac{\sin(\omega_{c2}n)}{\pi n} - \frac{\sin(\omega_{c1}n)}{\pi n}, & n \neq 0 \\ \frac{\omega_{c2}}{\pi} - \frac{\omega_{c1}}{\pi}, & n = 0 \end{cases}$$



$$h_{BS}[n] = \begin{cases} 1-\frac{(\omega_{c2}-\omega_{c1})}{\pi}, & n = 0 \\ \frac{\sin(\omega_{c1}n)}{\pi n} - \frac{\sin(\omega_{c2}n)}{\pi n}, & n \neq 0 \end{cases}$$

$$H(e^{j\omega}) = e^{-j\alpha\omega}\{H_r(e^{j\omega})\} = e^{-j\theta(\omega)}\{Hr(e^{j\omega})\}$$

Phase shift   Real part

Since $H_r(e^{j\omega})$ is real, its inverse DTFT is even and symmetric

Since $\theta(\omega) = \alpha\omega$ is linear phase, so it merely shifts the corresponding even and symmetric magnitude in the time domain, so the resulting impulse response is symmetric.

Hr(ejw) is imaginary then IDTFT is odd symmetric.

The frequency responses of the four types of FIR filters are summarized below:

$$H(e^{j\omega}) = e^{-j[(N/2)\omega]}\left\{h\left(\frac{N}{2}\right) + 2\sum_{n=1}^{N/2} h\left[\frac{N}{2}-n\right]\cos(n\omega)\right\}$$

for type I

$$H(e^{-j\omega}) = e^{-j[(N/2)\omega]}\left\{2\sum_{n=1}^{(N+1)/2} h\left[\frac{N+1}{2}-n\right]\cos\left(\left(n-\frac{1}{2}\right)\omega\right)\right\}$$

for type II

$$H(e^{-j\omega}) = e^{-j[(N\omega-\pi)/2]}\left\{2\sum_{n=1}^{N/2} h\left[\frac{N}{2}-n\right]\sin(n\omega)\right\}$$

for type III

$$H(e^{-j\omega}) = e^{-j[(N\omega-\pi)/2]}\left\{2\sum_{n=1}^{(N+1)/2} h\left[\frac{N+1}{2}-n\right]\sin\left(\left(n-\frac{1}{2}\right)\omega\right)\right\}$$

for type IV