



Problem Solving Part 3

Like, and not like, it used to be ...



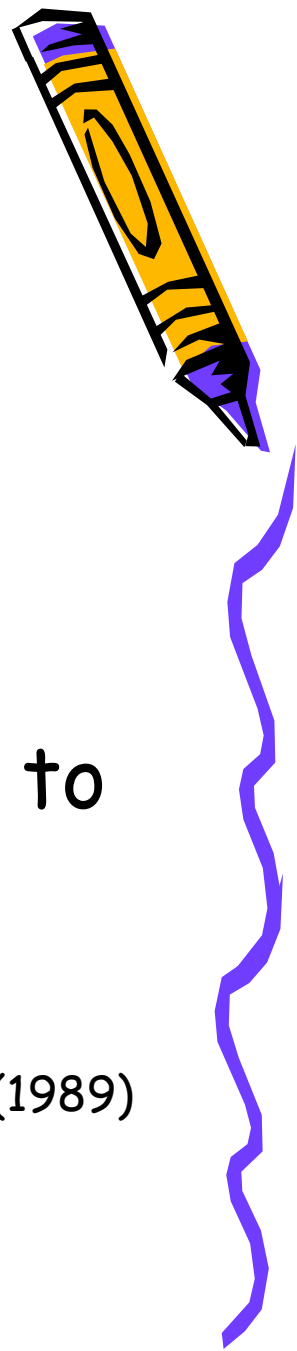
Mathematics curriculum should

" . . . include numerous and varied experiences with **problem solving** as a method of inquiry and application."

NCTM, Curriculum and Evaluation Standards (1989)



Mathematical power includes



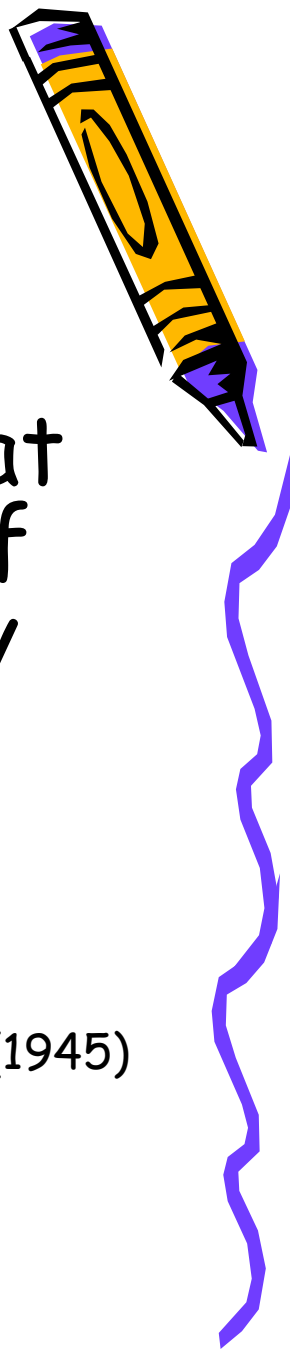
" . . . the ability to use a variety of mathematical methods effectively to **solve** non-routine **problems**."

NCTM, Curriculum and Evaluation Standards (1989)



" A great discovery solves a great problem, but there is a grain of discovery in the solution of any problem."

G. Polya, How to Solve It, (1945)



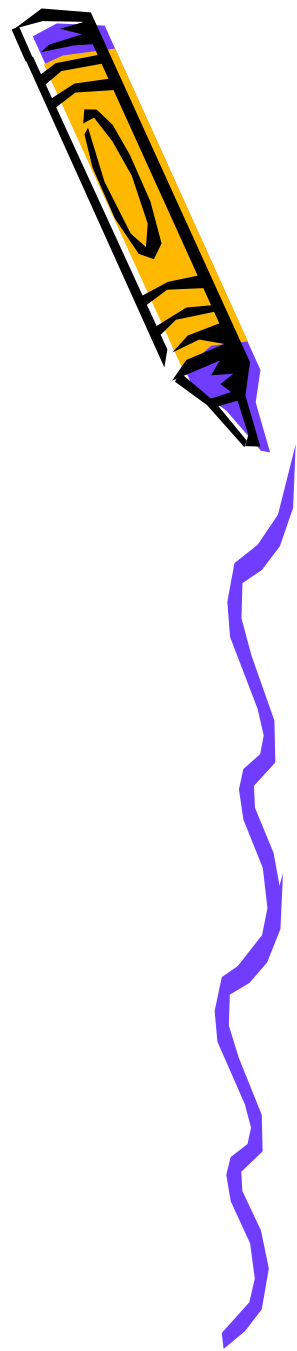


"A problem is not necessarily solved because the correct answer has been made. A problem is not truly solved unless the learner understands . . ."

William A Brownell, The Measurement of Understanding (1946)



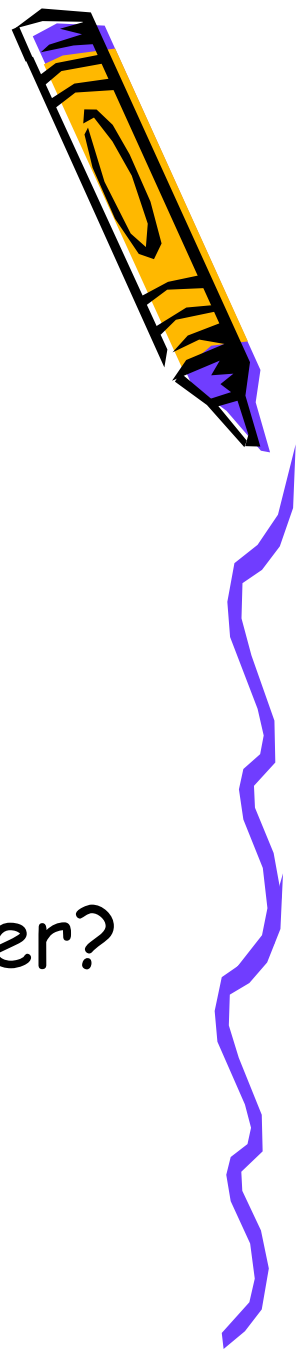
Overall plan



- Understanding the problem
- Devising a Plan
- Carrying Out the Plan
- Looking Back



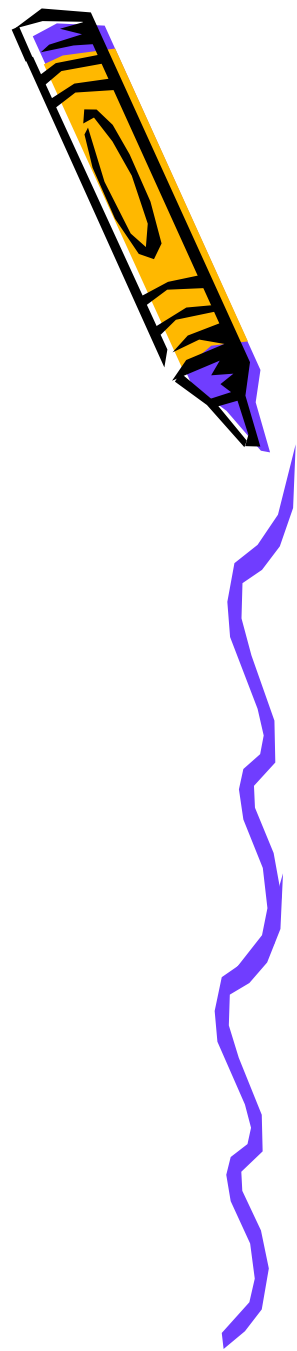
Starter Questions



- What is the unknown?
- What are the data?
- What is the condition?
- Is the problem analogous to another?
- Is the solution reasonable?



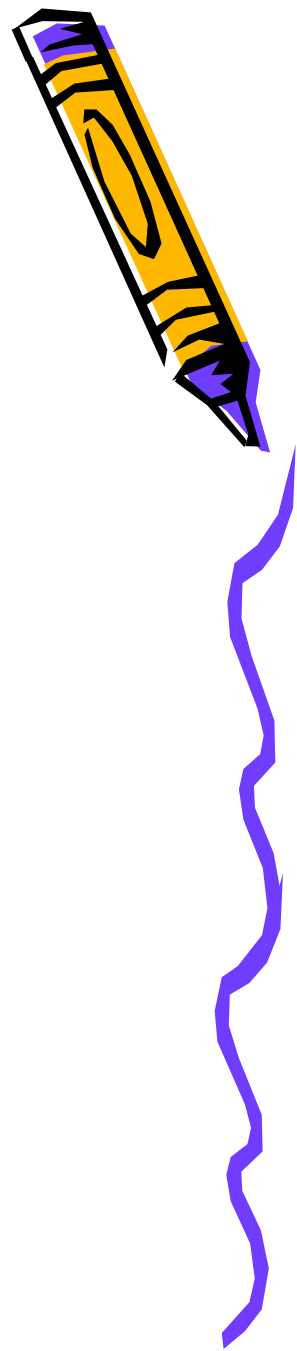
Strategies



- Compute or simplify
- Use a formula
- Make a model or diagram
- Make a table, chart or list
- Guess, check and revise



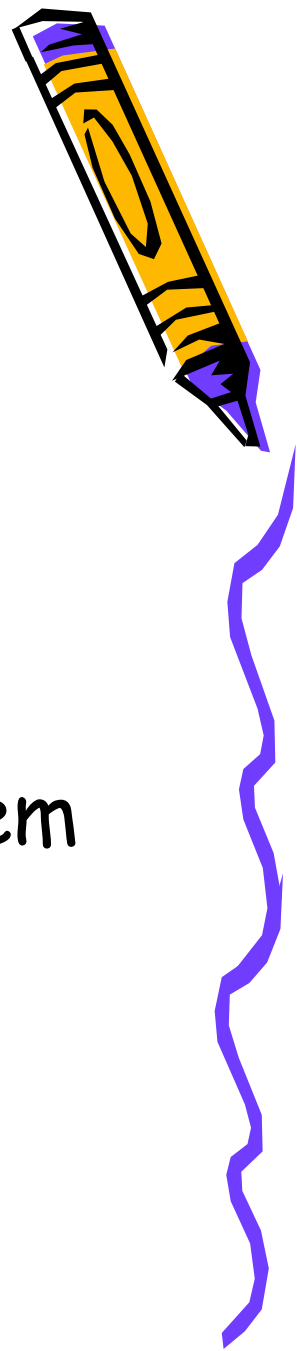
More Strategies



- Consider a simpler case
- Eliminate
- Look for patterns
- Work backwards
- Restate the problem



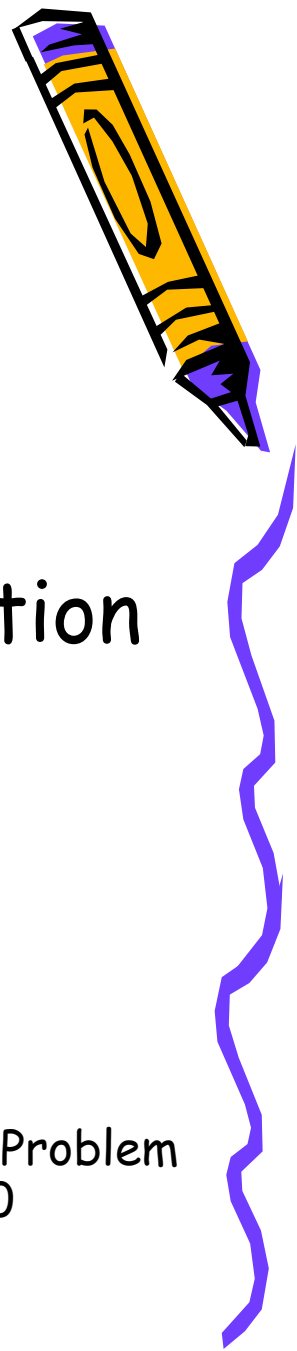
Other Strategies



- Start somewhere
- Talk it over with someone else
- Take a risk
- Make mistakes, and learn from them
- Draw and write
- Stay cool



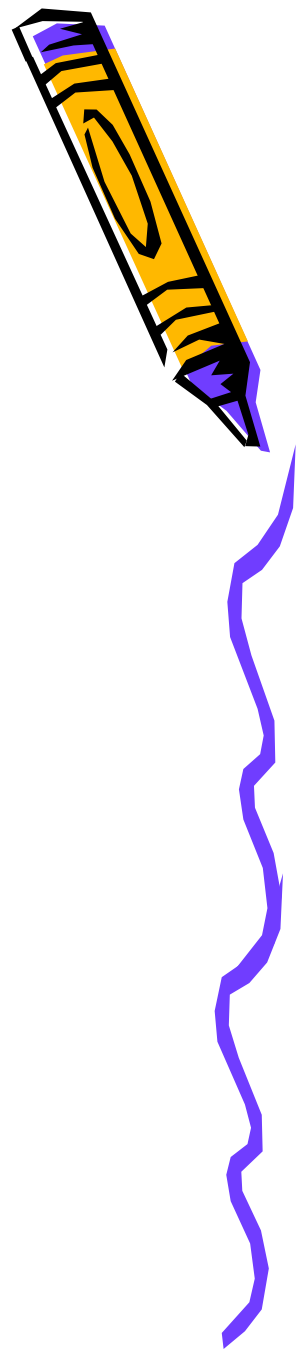
Open-ended problem solving



- Has multiple possible answers
- Has multiple solution methods
- Focuses on method rather than solution
- Is just beyond student skill level
- Is challenging, yet unfamiliar
- Is not insurmountable

» McIntosh and Jarrett, "Teaching Mathematical Problem Solving: Implementing the Vision", NWRel, 2000

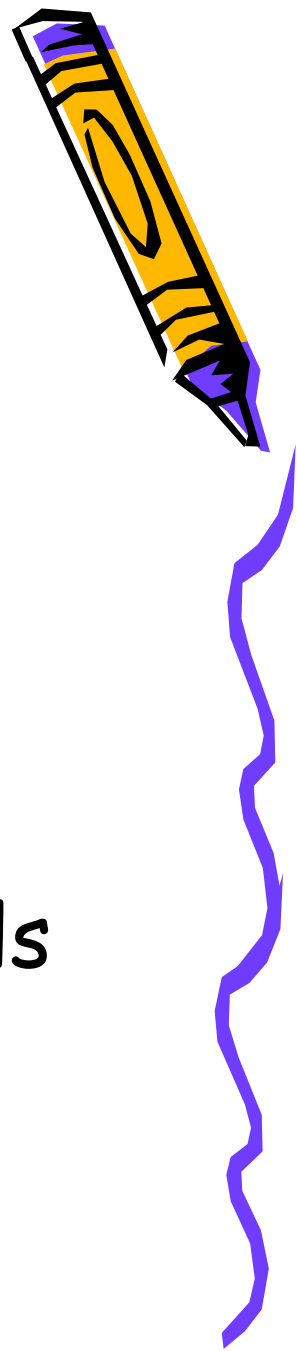




- Addresses important mathematics concepts
- Connects to students' previous learning
- Is meaningful and relevant
- Accommodates diverse learning styles



Problem Solving as context



- Justification for teaching math
- Motivation for learning math
- Recreation for diversion
- Practice for reinforcement of skills and concepts



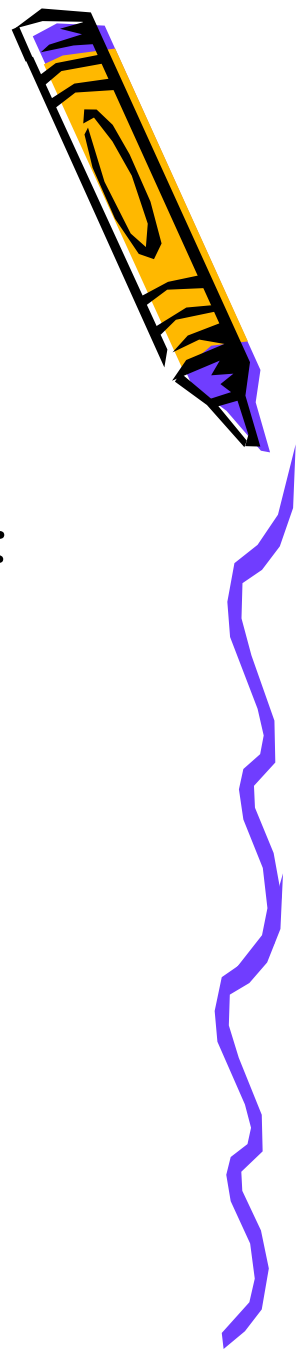
Problem solving as a skill



- Is usually an extra-curricular activity
- Is a set of procedures to be practiced
- Can be infused into curriculum



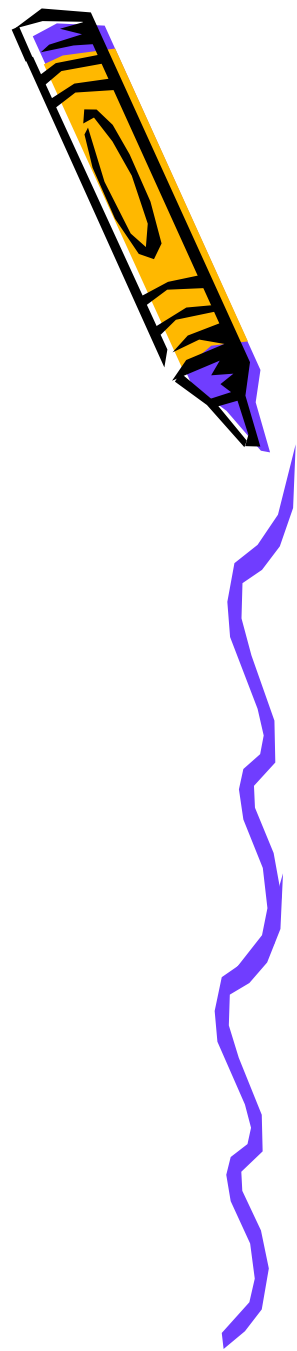
Problem solving as an art



- Is an act of inquiry and discovery
- Assists successful investigation of new problem
- Presents mathematics as an experiment, inductive science
- Promotes independent thinking



Problem Solving as Instructional Strategy

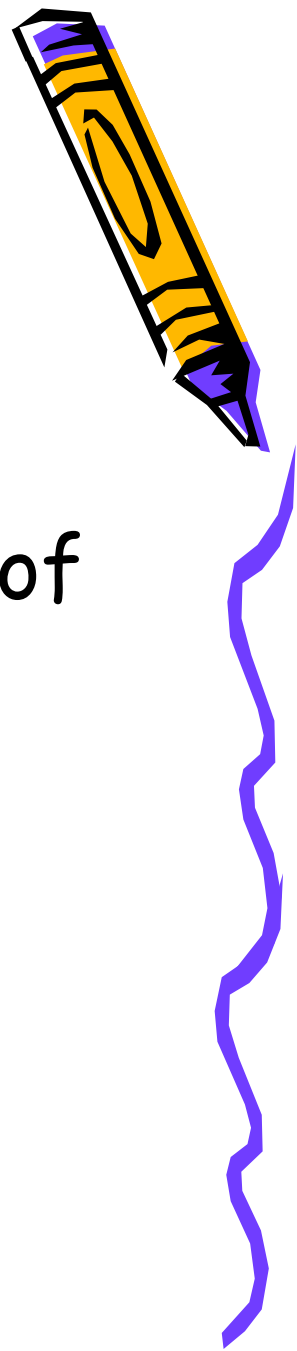


- Conceptual understanding
- Strategies and reasoning
- Communication
- Computation and execution
- Mathematical insights



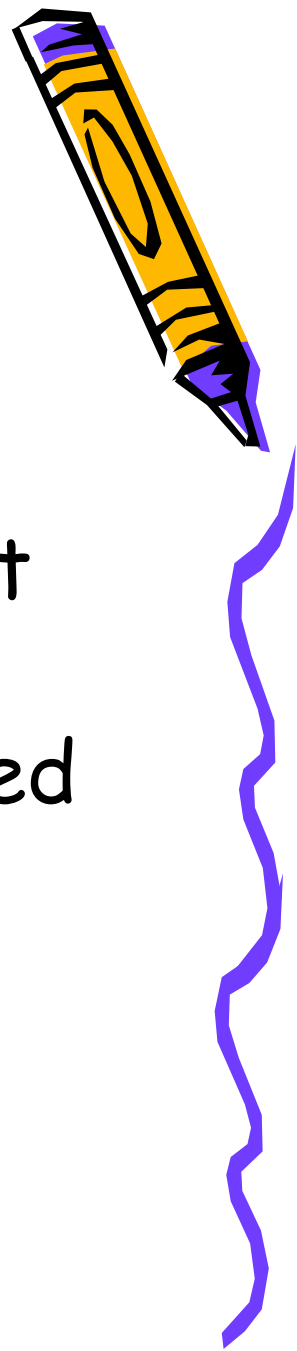
Conceptual Understanding

Does the student's interpretation of the problem, using mathematical representations and procedures, accurately reflect the key mathematical concepts?



Strategies and reasoning

Is there evidence that the student proceeded from a plan, applied appropriate strategies, and followed a logical and verifiable process toward a solution?

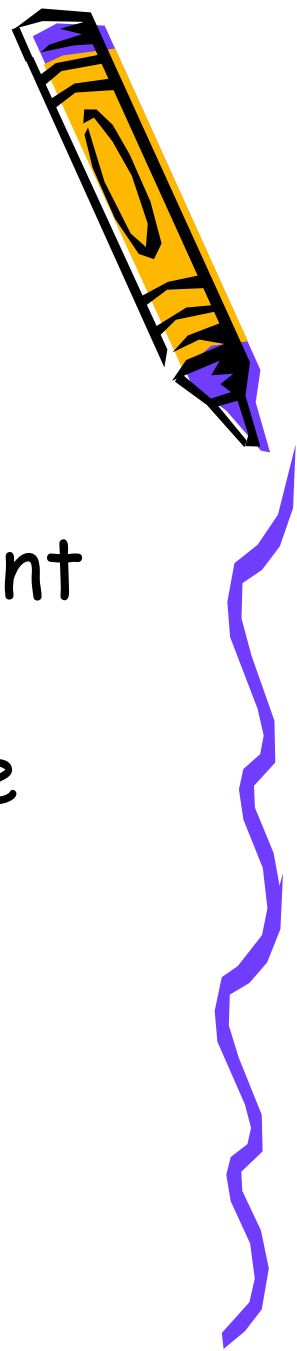


Communication

Can one easily understand the student's thinking, or is it necessary to make inferences and guesses about what the student was trying to do?



Computation and execution

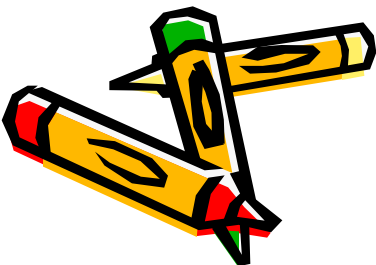
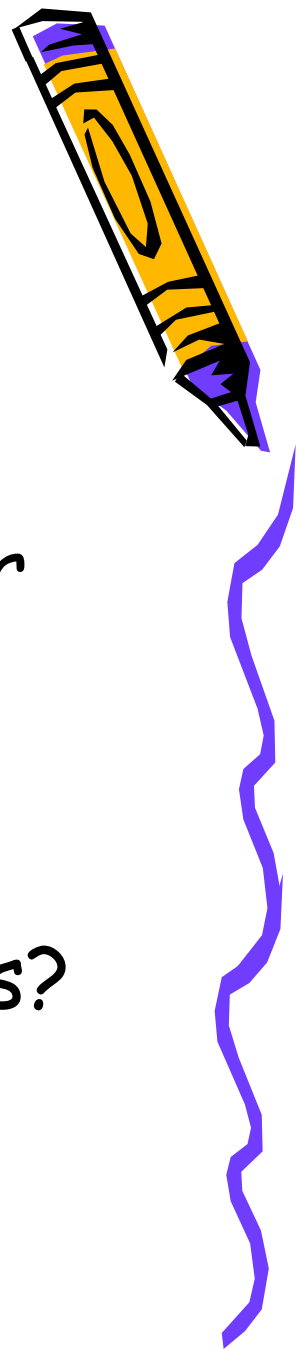


Given the approach that the student took to solve the problem, is the solution (including the steps of the process) performed in an accurate and complete?



Mathematical Insights

Does the student grasp the deeper structure of the problem and see how the process used to solve this problem connects it to other problems or real-world applications?



A problem is . . .



1) Something difficult to deal with or understand

2) An exercise in a textbook or examination

Oxford English Dictionary



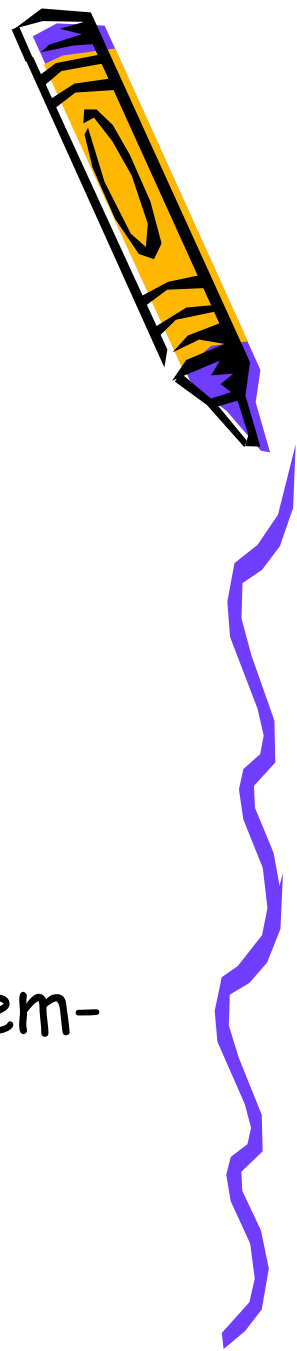
A problem is . . .



- A problem for one student, but an exercise for another
- A relationship between an individual and the task
- Based on an intellectual, rather than a computational, challenge



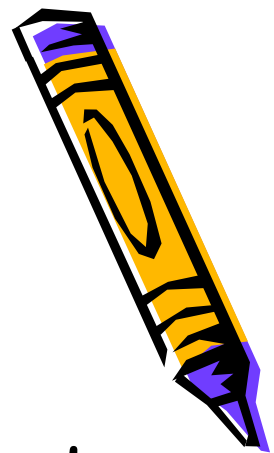
Teachers must distinguish between



- Using problem solving as a context for teaching concepts
- Teaching problem-solving strategies
- Teaching and assessing students' problem-solving abilities



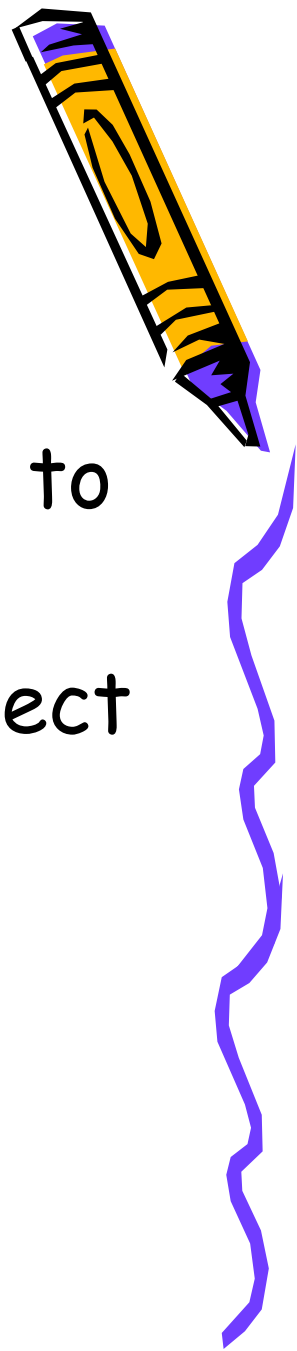
Teachers must distinguish between



- Mathematics as a static, unified body of knowledge
- Mathematics as an accumulation of facts, rules and skills
- Mathematics as a dynamic, continually expanding field of human creation and invention



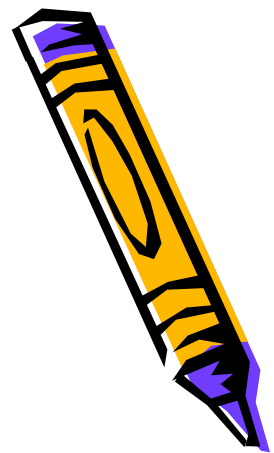
Problem-solving environment



- Ideas have potential to contribute to learning and should be respected
- Students have autonomy with respect to methods of solving problems
- Mistakes afford opportunities for examination and learning



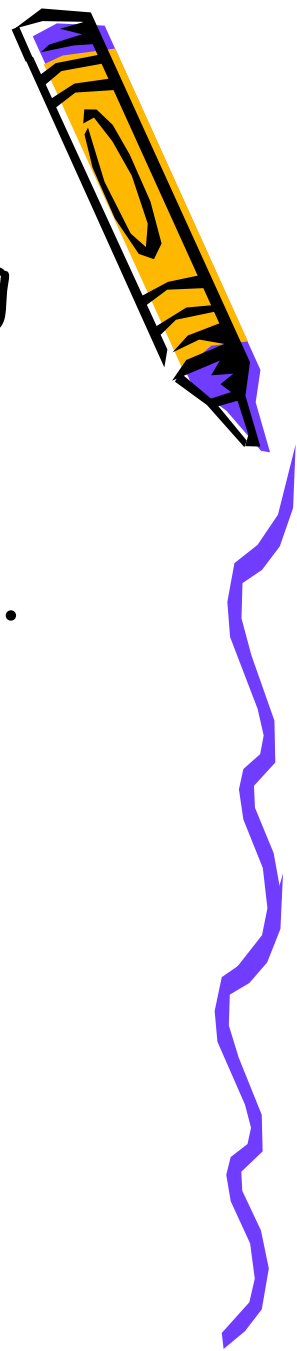
Problem-solving as a Vacation



- Good planning is necessary.
- The journey is more important than the destination.
- You will make side trips and have delays or detours.
- It is an experience that enriches the routine.



"At 8 am, a train left Town A traveling toward Town B at 50 miles per hour. Two hours later a train left Town B traveling toward Town A at 200 mph. Towns A and B are 1000 miles apart. Will the crash wake you up?"



... Not a problem!