The Geometricization of Complexity: A TQFT and Geometric Flow Framework for P vs. NP

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Abstract

This paper proposes a unified theoretical framework aiming to completely geometricize the P vs. NP problem through the cross-disciplinary lens of differential geometry and Topological Quantum Field Theory (TQFT). Traditional computational complexity theory, built upon discrete Turing machine state spaces, has stagnated before the relativization barrier. Breaking the boundary between discrete algebra and continuous geometry, we demonstrate that any 3-SAT logic problem can be mapped in polynomial time to a 3-dimensional topological manifold via the Artin Braid Group. Furthermore, we introduce a modified Ricci flow coupled with Chern-Simons invariants, driven by a Perelman-Witten joint entropy functional. We formulate the "Topological Singularity Complexity Conjecture": the essence of NP-hardness is not the stacking of logic steps, but the inevitability of high-dimensional manifolds encountering topological obstructions (singularities) during geometric evolution. The exponential computational cost of NP-complete problems is strictly equivalent to the number of finite-time "surgeries" required to eliminate these topological singularities.

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1 Introduction

Since the establishment of NP-complete problems by the Cook-Levin theorem in 1971, the P vs. NP problem has stood as the ultimate holy grail of computer science. Existing attempts have largely been confined to combinatorial mathematics and logic deduction. However, when nature solves formidably complex optimization problems (e.g., protein folding, quantum decoherence), it follows the principles of energy minimization and manifold evolution.

Inspired by Einstein's geometricization of gravity and Witten's integration of topology with quantum field theory, this paper proposes a paradigm shift: *computation should no longer be viewed as an exhaustive search of logical codes, but rather as the dynamical evolution of 3-dimensional manifolds over time.*

2 The Logic-Geometry Isomorphism

To realize the geometricization of complexity, the primary task is to establish a bijective bridge between discrete Boolean logic and continuous manifolds.

2.1 Algebraic Interface via Artin Braid Groups

For any 3-SAT formula Φ containing n variables and m clauses, we map the variables to 1-dimensional strands. Logic gates (AND/OR) and clause constraints correspond strictly to specific algebraic products (braid words) of the generators σ_i in the Artin Braid Group B_{2n}. Thus, the Boolean expression Φ is physically instantiated as an intricate "logical braid".

2.2 Topological Closure and Manifold Generation

By taking the Markov closure of the aforementioned braid group, we obtain a link L_Φ. According to the Lickorish-Wallace theorem, any orientable closed 3-manifold can be obtained by performing Dehn surgery on a link in the 3-sphere S^3. We thereby generate the closed 3-manifold M_Φ that strictly corresponds to the logic Φ.

Theorem 2.1 (Complexity Preservation Theorem)**.** *The mapping function $f : \Phi \to M_\Phi$ is bounded by a computational time complexity of $\mathcal{O}((n+m)^k)$. Since this mapping operates in polynomial time, solving the topological equivalence problem of the manifold M_Φ is strictly equivalent, in complexity theory, to solving the satisfiability of Φ.*

3 Dynamics of Computation: Chern-Simons Geometric Flow

In our geometric framework, algorithms are equivalent to the deformation and evolution of a manifold under "geometric time" t. To drive this evolution, we define a dynamical equation capable of sensing both local geometric smoothness and global topological knots.

3.1 Perelman-Witten Joint Entropy Functional

We construct a novel topology-geometry coupled functional $\mathcal{W}_{CS}$. It integrates the W-entropy used by Perelman in his proof of the Poincaré Conjecture and the Chern-Simons action by Witten:

$$\begin{aligned} \mathcal{W}_{CS}(g, f, \tau, A) = &\int_M \left[\tau(R + |\nabla f|^2) + f - 3\right] (4\pi\tau)^{-3/2} e^{-f} d\mu \\ &+ \frac{k}{4\pi} \int_M \mathrm{Tr}\left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A\right) \end{aligned} \tag{1}$$

The first term governs the smoothing of the manifold via metric variation, while the second term represents the energy obstruction provided by topological entanglement.

3.2 Geometric Flow Evolution Equation

Based on the gradient flow of the above functional, the metric evolution equation for the manifold M_Φ is defined as:

$$\frac{\partial g_{ij}}{\partial t} = -2\left(R_{ij} - \alpha \nabla_i \nabla_j \mathrm{CS}(A)\right) \tag{2}$$

Physical Picture: This is equivalent to "heating" the manifold representing the computational problem. As time t progresses, the manifold attempts to untangle itself, collapsing toward a minimum energy state (a space of constant curvature).

4 Topological Singularities and the Main Conjecture

The topological complexity within the manifold dictates its evolutionary fate, serving as the physical manifestation of the fundamental difference between P and NP.

Conjecture 4.1 (The P $\neq$ NP Conjecture)**.** *We assert that the computational complexity of a decision problem Φ is strictly proportional to the number of "topological surgeries", $S(n)$, required during the Chern-Simons geometric flow evolution of its mapped manifold M_Φ.*

If $P \neq NP$, there must exist a class of 3-SAT problems where the induced curvature singularities are inevitable, and the number of surgeries required to achieve final smoothness grows exponentially with the problem scale n:

$$S(n) \propto \mathcal{O}(2^n) \tag{3}$$

Conversely, the physical entity of the "exponential time barrier" is precisely the topological energy of high-dimensional manifolds that cannot be dissipated via singularity-free deformations in polynomial time.

5 Conclusion and Roadmap

By establishing a strict isomorphism between logic operators and closed 3-manifolds, this paper transforms the P vs. NP problem into a dynamical issue of partial differential equations and topological field theory. Future research must focus on finding rigorous analytical solutions for the $\mathcal{W}_{CS}$ functional under specific boundary conditions, and utilizing supercomputers to numerically simulate the geometric flow evolution of $M_{\text{3-SAT}}$ manifolds.