

Uppgift 1

Kolumner som är markerade med "COMP" är de som är jämförda.

A) $(a \rightarrow b) \rightarrow c = (a \vee c) \wedge (\neg b \vee c)$

a	b	c	$\neg b$	$(\neg b \vee c)$	$(a \vee c)$	COMP: $(a \vee c) \wedge (\neg b \vee c)$	$(a \rightarrow b)$	COMP: $(a \rightarrow b) \rightarrow c$
0	0	0	1	1	0	0	1	0
0	0	1	1	1	1	1	1	1
0	1	0	0	0	0	0	1	0
0	1	1	0	1	1	1	1	1
1	0	0	1	1	1	1	0	1
1	0	1	1	1	1	1	0	1
1	1	0	0	0	1	0	1	0
1	1	1	0	1	1	1	1	1

Bevisat enligt ovan.

B) $(a \vee b) \wedge (\neg a \vee b) = a \wedge b \vee \neg a \wedge b \Leftrightarrow (a \vee b) \wedge (\neg a \vee b) = (a \wedge b) \vee (\neg a \wedge b)$

a	b	c	$\neg a$	$(a \vee b)$	$(\neg a \vee b)$	COMP: $(a \vee b) \wedge (\neg a \vee b)$	$(a \vee b)$	$(\neg a \vee b)$	COMP: $(a \wedge b) \vee (\neg a \wedge b)$
0	0	0	1	0	1	0	0	0	0
0	0	1	1	0	1	0	0	0	0
0	1	0	1	1	1	1	0	1	1
0	1	1	1	1	1	1	0	1	1
1	0	0	0	1	0	0	0	0	0
1	0	1	0	1	0	0	0	0	0
1	1	0	0	1	1	1	1	0	1
1	1	1	0	1	1	1	1	0	1

Bevisat enligt ovan.

Uppgift 2

Identitet:

$$\begin{aligned}p \wedge S &= p \\ p \vee F &= p\end{aligned}$$

Tautologi:

$$p \vee \neg p = S$$

Motsägelse:

$$p \wedge \neg p = F$$

Kommutativitet:

$$\begin{aligned}p \vee q &= q \vee p \\ p \wedge q &= q \wedge p\end{aligned}$$

Distributivitet:

$$\begin{aligned}p \vee (q \wedge r) &= (p \vee q) \wedge (p \vee r) \\ p \wedge (q \vee r) &= (p \wedge q) \vee (p \wedge r)\end{aligned}$$

A) $a \wedge a = a$

$$\begin{aligned}a &= a \wedge S \text{ (Identitet)} \\ a \wedge S &= a \wedge (a \vee \neg a) \text{ (Tautologi)} \\ a \wedge (a \vee \neg a) &= (a \wedge a) \vee (a \wedge \neg a) \text{ (Distributivitet)} \\ (a \wedge a) \vee (a \wedge \neg a) &= (a \wedge a) \vee F \text{ (Motsägelse)} \\ (a \wedge a) \vee F &= a \wedge a \text{ (Identitet)}\end{aligned}$$

B) $a \wedge F = F$

$$\begin{aligned}a \wedge F &= (a \wedge F) \vee F \text{ (Identitet)} \\ (a \wedge F) \vee F &= F \vee (a \wedge F) \text{ (Kommutativitet)} \\ F \vee (a \wedge F) &= (a \wedge \neg a) \vee (a \wedge F) \text{ (Motsägelse)} \\ (a \wedge \neg a) \vee (a \wedge F) &= a \wedge (\neg a \vee F) \text{ (Distributivitet)} \\ a \wedge (\neg a \vee F) &= a \wedge \neg a \text{ (Identitet)} \\ a \wedge \neg a &= F \text{ (Motsägelse)}\end{aligned}$$

C) $a \wedge (a \vee b) = a$

$$\begin{aligned}a \wedge (a \vee b) &= (a \vee F) \wedge (a \vee b) \text{ (Identitet)} \\ (a \vee F) \wedge (a \vee b) &= a \vee (F \wedge b) \text{ (Distributivitet)} \\ a \vee (F \wedge b) &= a \vee (b \wedge F) \text{ (Kommutativitet)} \\ a \vee (b \wedge F) &= a \vee F \text{ (B)} \\ a \vee F &= a \text{ (Identitet)}\end{aligned}$$