Homework 1
AAE 512, Computational Aerodynamics
Purdue University
Due: 2022-01-24

1. Problem 2.12 in the textbook. Classify the following system of equations as elliptic, hyperbolic, or parabolic:
$$\beta^2 \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} = 0$$
$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0$$
2. Problem 2.13. Classify the following equation as elliptic, hyperbolic, or parabolic:
$$\frac{\partial^2 u}{\partial t^2} + \frac{\partial^2 u}{\partial x^2} + \frac{\partial u}{\partial x} = -e^{-kt}$$
Classify this, too:
$$\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial u}{\partial y} = 4$$

3. Problem 2.27. Solve the following partial differential equation:
$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, 0 \leq x \leq 1$$
$$u(0,t) = 0, u(1,t) = 0, u(x,0) = \sin(2\pi x)$$
4. Nonlinear pendulum problem. The equation of motion for a simple, nonlinear pendulum is given by:
$$\frac{d^2 \phi}{dt^2} + \frac{g}{R} \sin \phi = 0$$
$$\phi = \phi_0, \frac{d\phi}{dt} = 0, \text{for } t = 0$$
where R is the length of the pendulum, g is the acceleration of gravity, and ϕ is the angle from the vertical. To solve this equation numerically, recast it as two first-order equations:
$$\frac{du_1}{dt} = -\frac{g}{R} \sin u_2$$
$$\frac{du_2}{dt} = u_1$$
$$u_1(0) = 0, u_2(0) = \phi_0$$
For this problem, take $\phi_0 = 1, \frac{g}{R} = 1$, and integrate numerically out to $t = 10$. Use the following first order integration scheme: $u(t + \Delta t) \approx u(t) + \frac{du}{dt}\Delta t$. Do the integration out with $\Delta t = 10^0, 10^{-1}, 10^{-2}, \ldots, 10^{-8}$; that is, with ten steps up to one billion steps. Using your best solution as a reference, plot the error $|u_2(10) - u_{2,\text{ref}}(10)|$ versus Δt on a log-log plot. What is the slope of the error curve? **Write your program in C, C++, or Fortran!**