# Syndrome-Based Hybrid ARQ with Reed-Solomon Codes and the Practice in Image Transmission over Rayleigh Fading Channel *

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## Abstract

*Two Reed-Solomon (RS) coded hybrid ARQ schemes based on syndrome calculation are proposed. For poor correction capacity RS code, the retransmission request is determined in the decoding procedure, which we call syndrome-ahead hybrid ARQ. As to powerful RS code, the request arises after decoding. The scheme thus gets the name of syndrome-behind hybrid ARQ. To guarantee the maximal throughput of the system, each retransmission is performed only once. We use analytical framework to evaluate the performance of the two schemes in Rayleigh fading channel. Simulation results are also provided to enhance the conclusion. The two hybrid ARQ scenarios are then applied to compressed image transmission, and the results further prove that the proposed ARQ schemes are powerful and effective to be used for a variety of fading channel applications.*

## 1. Introduction

In wireless communications, channels suffer from multipath effect, which causes fading and peaks of the signals and degrades the reliability drastically. Automatic-repeat-request (ARQ) systems and their hybrids have attracted huge attention as a powerful error-control technique to combat with the adverse channel condition, but most hybrid ARQ schemes decrease the throughput of the system or have a high complexity and additional lookup delay [1–6].

In this article, we investigate on a new Reed-Solomon (RS) coded hybrid ARQ protocol with low complexity and high throughput, which is suitable for time sensitive traffic, such as image data, in Rayleigh fading channel. The retransmission request is on the basis of syndrome calculation, and each retransmission is executed only once to guarantee the maximal throughput of the system. For convenience of analysis, the feedback channel is assumed to be error-free and delay-free.

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## 2. Architecture and Performance

The RS coded hybrid ARQ scenarios are divided into two types according to the error-correction capability: syndrome-ahead and syndrome-behind hybrid ARQs.

### 2.1. Syndrome-ahead hybrid ARQ

For poor RS code (e.g., RS (7, 5)), the error-correction capability is very low, and the error-detection capability is more important relatively. We set the code as an error-detection code and make the probability of error-detection maximal.

Assume that $d$ and $t$ are the minimum Hamming distance and error-correction capability of RS $(n,\ k)$ defined over $GF(q)$, respectively, then $d = n - k + 1$ and $t = \mathrm{INT}(n-k)/2$ [7]. Let $A_l$ denotes the number of code words whose weight are $l$ in the RS $(n,\ k)$ code. The weight distribution of the code can be got [8]

$$A_l = \begin{cases} 1 & l = 0 \\ 0 & 1 \le l < d \\ \binom{n}{l}\sum_{j=0}^{l-d}(-1)^j\binom{l}{j}(q^{l-d+1-j}-1) & d \le l \le n. \end{cases} \tag{1}$$

Now we consider the detail of the syndrome-ahead hybrid ARQ. In Berlekamp-Massey algorithm, the syndrome calculation of the received word is the first step of decoding [7]. The computing result determines whether to generate retransmission request or not. If the result is zero, no request arises, or else the request is sent to the transmitter for data retransmission. To get the probability of retransmission directly is very difficult, so we resort to an indirect method. According to (1), we can compute the probability of generating no retransmission request. This probability includes all the events that the received words belong to the set of code word, i.e.,

$$\begin{aligned}\mathrm{P_{nr}} &= \sum_{l=0}^{n} A_l \left(\frac{P_c}{q-1}\right)^l (1-P_c)^{n-l} \\ &= (1-P_c)^n + \sum_{l=d}^{n} A_l \left(\frac{P_c}{q-1}\right)^l (1-P_c)^{n-l}, \quad (2)\end{aligned}$$

where $P_c$ is the symbol error rate of the code. Thus, the probability of retransmission of course becomes

$$\begin{aligned}\mathrm{P_r} &= 1 - \mathrm{P_{nr}} \\ &= 1-(1-P_c)^n-\sum_{l=d}^{n} A_l \left(\frac{P_c}{q-1}\right)^l (1-P_c)^{n-l}. \quad (3)\end{aligned}$$

The lower bound of bit-error rate (BER) for RS $(n,\ k)$ has been proved to be [9]

$$P_b \geq \frac{1}{n} \sum_{i=t+1}^{n} \binom{n}{i} P_c^i (1-P_c)^{n-i} \left(i\frac{P_{bc}}{P_c}\right), \quad (4)$$

where $P_{bc}$ denotes average BER per code symbol. As mentioned above, the retransmission is performed only once, no matter whether the retransmission succeeds. Based on the protocol, the lower bound of BER for the syndrome-ahead hybrid ARQ can be written

$$P_b \geq \frac{\mathrm{P_r}}{n} \sum_{i=t+1}^{n} \binom{n}{i} P_c^i (1-P_c)^{n-i} \left(i\frac{P_{bc}}{P_c}\right). \quad (5)$$

Correspondingly, the throughput of the system is derived as

$$\eta = R_c/(1+\mathrm{P_r}) = k/[n(1+\mathrm{P_r})], \quad (6)$$

where $R_c$ represents the coding rate.

## 2.2. Syndrome-behind hybrid ARQ

As to powerful RS code, we use the error-correction capability completely, thus causing the retransmission request after decoding. The syndrome of the decoded word (not a code word for decoding failure affair) determines the retransmission request. Zero result suggests the retransmission, otherwise no retransmission is performed. In this case, the probability of retransmission is just the probability of

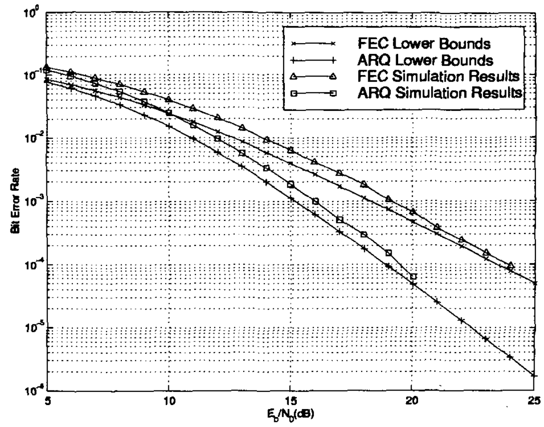


**Figure 1. BER of RS (7, 5) syndrome-ahead hybrid ARQ in Rayleigh fading channel.**

decoding failure, i.e.,

$$\mathrm{P_r} = \sum_{v=t+1}^{n} \left[\binom{n}{v}(q-1)^v - \sum_{s=0}^{t}\sum_{l=0}^{n} A_l N(l,v;s)\right] \left(\frac{P_c}{q-1}\right)^v (1-P_c)^{n-v}, \quad (7)$$

where

$$N(l,v;s) = \sum_{\substack{i,j,k \\ i+j+k=s \\ l+k-j=v}} \left[\binom{n-l}{k}(q-1)^k\right]\left[\binom{l}{i}(q-2)^i\right]\binom{l-i}{j}$$

denotes the number of error patterns of weight $v$ that are at distance $s$ from a particular code word of weight $l$.

The rest of performance is the same as syndrome-ahead scenario mentioned above (see (5) and (6)).

## 2.3. Simulation and Results

We choose two same-rate RS codes, RS (7, 5) and RS (63, 45), to verify our hybrid ARQ schemes. Both of the codes are combined with 8PSK modulation. The dimension of the code symbol alphabet for RS (7, 5) is the same as that of channel symbol alphabet of 8PSK signal. Obviously, each channel symbol represents one code symbol by Gray mapping rule. RS (63, 45) defined over $GF(64)$ cannot obey the one-to-one rule. Each code symbol of this code includes two concatenated channel symbols of 8PSK [11]. In Rayleigh fading channel, the symbol-error rate of MPSK

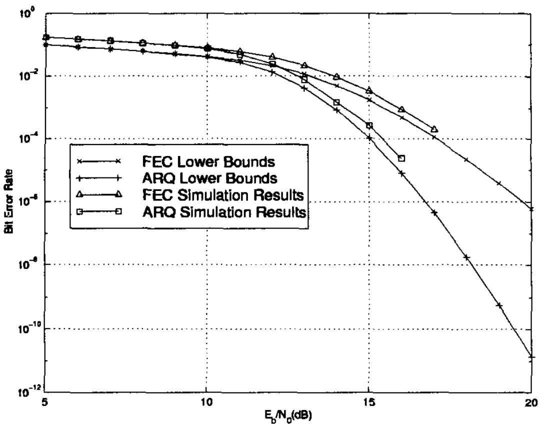


**Figure 2. BER of RS (63, 45) syndrome-ahead hybrid ARQ in Rayleigh fading channel.**

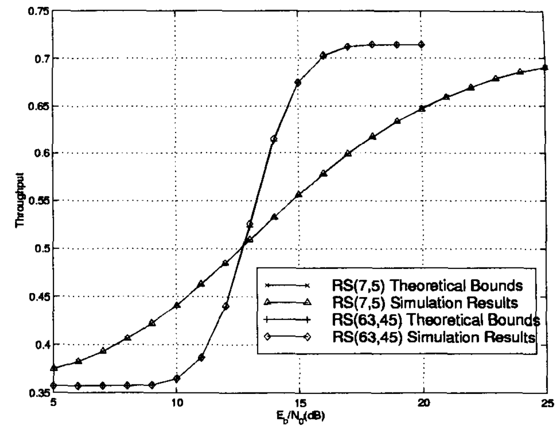


**Figure 3. Throughput of the two syndrome-based hybrid ARQs in Rayleigh fading channel.**

signal can be depicted approximately

$$P_s \approx 1 - \sqrt{\frac{mR_c \sin^2\left(\frac{\pi}{M}\right)\gamma}{1 + mR_c \sin^2\left(\frac{\pi}{M}\right)\gamma}}, \tag{8}$$

where $m$ and $M$ represent the bit number per channel symbol and the dimension of MPSK signal alphabet, respectively, and $\gamma = E_b/N_0$. In our experiment, $m = 3$ and $M = 8$. For RS (7, 5) coded 8PSK modulation, $P_c = P_s$ and $P_c = P_s(1 - P_s)$ in the case of RS (63, 45). If a Gray code is used for mapping code symbols to channel symbols, the average BER per ode symbol is approximated by

$$P_{bc} = P_s/m. \tag{9}$$

Submitting (9) into (5) yields the lower bound of BER for the hybrid ARQs

The theoretical and simulation results are shown in Fig. 1 through 3. The advantage of the hybrid ARQ can be observed clearly. BER curves of both ARQs descend sharply in contrast with the moderate drop of their counterparts, FEC schemes. In well-defined $E_b/N_0$ operating region, the syndrome-based ARQ curves indicate an increase in reliability of near and over two orders of magnitude over the FEC for RS (7, 5) syndrome-ahead and RS (63, 45) syndrome-behind ARQs, respectively. Furthermore, as shown in Fig. 3, the throughput of the system decreases very slightly, which guarantees the real-time transmission of the image traffic.

Both the hybrid ARQ schemes are applied to DCT-compressed image transmission in Rayleigh fading channel. Fig. 4 illustrates the results at $E_b/N_0 = 15$dB. As shown in Fig. 4A, there are a lot of distorted blocks in the distorted image. If FEC of RS (7, 5) is performed, we can get the improved image shown in Fig. 4B, from which, much noise has been removed. Once the syndrome-ahead hybrid ARQ is adopted, the quality of the image transmission is still higher, with only a few noisy blocks left. Comparison of Fig. 4D and 4E can serves as another powerful proof to the efficiency of the proposed ARQ protocol. With RS (63, 45) syndrome-behind ARQ, all the noisy blocks are cleaned up, and we get the completely recovered image shown in Fig. 4E, while Fig. 4D shows that with FEC alone, some ambiguity in the image still exists.

# 3. Conclusions

Two syndrome-based hybrid ARQ scenarios are presented. Syndrome-ahead hybrid ARQ adapts to poor RS code, which puts emphasis on error-detection, while syndrome-behind hybrid ARQ is suitable for powerful RS code, which emphasizes error-correction. The idea is also available for other hybrid ARQs.

Both the theoretical and simulation results show that the syndrome-based RS coded hybrid ARQs provide an extremely powerful error control system for a variety of fading channel applications, including real-time multimedia communication.

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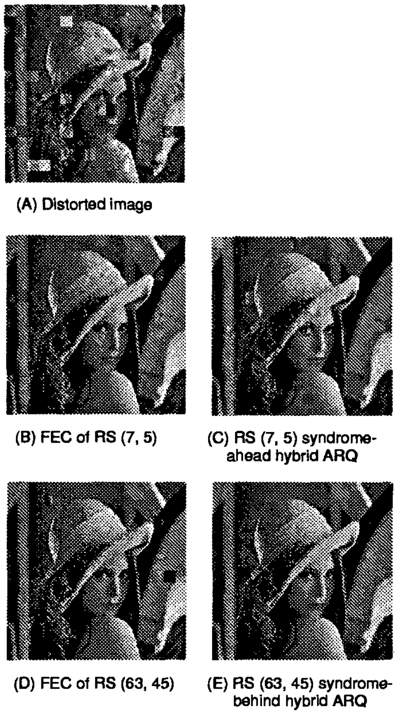


**Figure 4. Application of syndrome-based hybrid ARQs to image transmission over Rayleigh fading channel at $E_b/N_0 = 15\text{dB}$.**

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