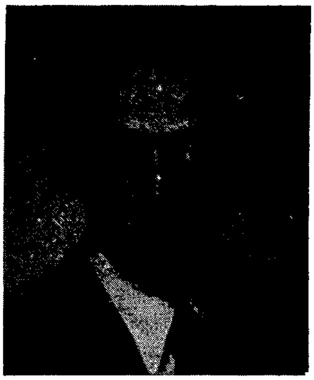
techniques, multiplexing, signaling and control technology, system design, and simulation. This work has contributed to the trunked mobile radio and cellular mobile radio-telephone systems being implemented today. He is now a Senior Member of the Technical Staff, and Manager of the Systems Research Laboratory, Corporate Research and Development Center, Motorola, Incorporated, with responsibilities for modulation techniques, signal processing, and communications systems.

Dr. Mikulski has been named a Dan Noble Fellow of Motorola and is a member of Tau Beta Pi and Sigma Xi.



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ARQ Schemes for Data Transmission in Mobile Radio Systems

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Abstract—**An important problem in land mobile radio communications is how to provide reliable data communications to the largest number of users. To explore this problem, several existing ARQ protocols are examined which have application to the land mobile radio channel, as well as some new protocol combinations. All protocols are analyzed for several key system performance measures which are verified by experimental means for static as well as fading channels. Finally, a conclusion is reached regarding a new protocol combination which is found to offer significant advantages over all other protocols explored.**

I. Introduction

THE problem of providing data communications over a land mobile radio channel has received considerable attention. Error detection combined with retransmission on request has been used for over two decades as a means of obtaining reliability in digital data transmission [1], [2]. Such systems are referred to as automatic RQ or ARQ, where RQ is the repeat-request symbol, attributed to a printing telegraph system patent dated 1943 [3]. When forward error correction (FEC) is used in conjunction with an ARQ protocol, it is called hybrid error control or hybrid ARQ. Over the years, a multitude of variations on the basic ARQ theme has been examined [4]–[21].

The distinguishing features of a land mobile radio (LMR) system, as far as the ARQ protocol is concerned, are a frequency or pair of frequencies shared by a large population of half-duplex users which cannot transmit and receive simultaneously, a wide range in signal conditions for users, and a signal condition which can fluctuate quickly in time due to multipath. The following sections will describe those variations of ARQ which are adaptable to an LMR system.

Section II will develop a new channel model for a fading land mobile radio channel. Section III will describe several ARQ protocols as well as some new protocol combinations. In Section IV the relationships will be developed to describe the various protocols in terms of channel failure probabilities, which will be examined in Section V. In

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Section VI the performance of the various protocols will be predicted and compared. Finally, in Section VII the undetected error probabilities will be developed and compared.

II. Multipath Fading Model

The characteristics of multipath are known in sufficient detail that accurate simulation is possible. For one well-known fading model programmed simulations exist which produce a random function resembling the strength of a multipath fading signal [22]. Hardware simulators also exist which can take a signal and vary its amplitude and phase to resemble multipath according to the same model [23]. The experimental results within this paper make extensive use of such a hardware fading simulator.

Another type of model, called a threshold model, will also be used. In this model a data system is assumed to have a sensitivity threshold. This assumption means that at signal strengths above this threshold communication is successful. Likewise, at signals below threshold, it is assumed to be unsuccessful. Thus, the probability of successful communication can be identified as the probability that the signal is above threshold [24]

$$P_{\text{success}} = 1 - F(r) = e^{-0.693r^2} \tag{1}$$

where r is the threshold to median signal ratio and $F(r)$ is the probability distribution of r. Since the signal amplitude pdf is Rayleigh, $F(r)$ is a Rayleigh distribution function. This can be interpreted as the success probability for a single transmitted bit, or a bound for short messages or very slow fading.

A more useful notion is the probability of success or failure for a data block of some measurable length. Define the block error probability P_f as the probability of the signal being below threshold somewhere during the message interval. By defining a gap as the interval between fades, a message lying entirely in a gap is assumed to be received error free with probability $1-P_f$.

To evaluate this probability the statistics of the gaps must be known; however, without knowing the gap length distribution the expected value can be calculated as the average time between fade occurrences $\overline{T}$ minus the average fade length $\bar{t}$ (see Fig. 1). For the threshold model, $\overline{T}$ has been found to be [24]

$$\overline{T} = 1/f_r \tag{2}$$

where

$$f_r = 2re^{-0.693r^2}f_e \tag{3}$$

is the fading rate across levels other than the median, and

$$f_e = V/\lambda \tag{4}$$

is the Doppler frequency for a vehicle speed V and carrier wavelength λ. Knowing only $\overline{T}$, the distribution function

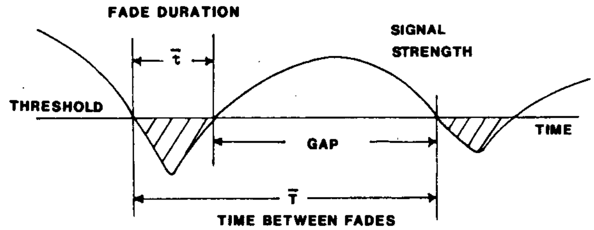

Fig. 1. Fades below a threshold.

for the time between fade occurrences can be approximated as an exponential [25], where

$$P(T \leqslant t) = 1 - e^{-t/\overline{T}} = 1 - e^{-f_r t}. \tag{5}$$

The probability that the message lies within a gap can be expressed as the probability that the message starts in a gap times the probability that the next fade is further away than the message length L, and

$$P_f = 1 - P(\text{starts in gap}) \cdot P(T > L). \tag{6}$$

The probability of starting in a gap is just the probability that the starting moment is above threshold. Thus,

$$P_f = 1 - e^{-0.693r^2}e^{-f_r L} \tag{7}$$

which can be put in the convenient form

$$P_f = 1 - k_1(k_2)^{VL} \tag{8}$$

where

$$k_1 = e^{-0.693r^2} \tag{9}$$

and

$$k_2 = e^{-(2r/\lambda)e^{-0.693r^2}} \tag{10}$$

This relationship is illustrated in Fig. 2 for $r = 0.1$. When L or V goes to zero the fading process is stationary during the message. For this case P_f goes to $1-k_1$, which is the probability of the message being in a gap. When LV goes to infinity the message becomes very long compared to the fading process, and consequently P_f goes to unity. This is similar to, but slightly different in form from, a recent model by Hafez [8]. Both these models are improvements over previous models where the probability of error goes to zero as the length approaches zero [25], [26].

Another interesting characteristic of this model is that P_f remains constant for vehicle speeds inversely related to message duration. For example, a 1 s message at 1 mi/h would have the same P_f as a 1/2 s message at 2 mi/h. This is an intuitively satisfying result since the probability of error depends on the message duration relative to the fading process. This model is used in later sections, and is also compared to experimental results.

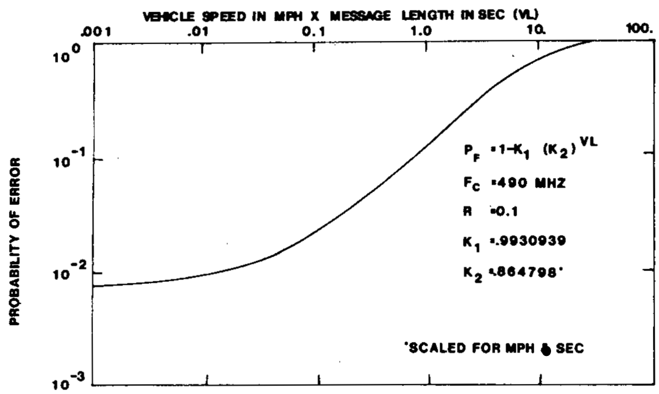

Fig. 2. Probability of error versus VL scaled in mi/h and s for 490 MHz carrier frequency.

III. ARQ Techniques

The ARQ techniques include the coding and retransmission request strategy used to deliver a message. The message itself is contained within a packet, with the source and destination addresses, as well as other useful routing information preceding the message in a header. The header is usually encoded separately from the message so that the header may be read before making a decision as to whether to read the following message section. This is desirable in a land mobile radio system where all users must read the header of every message in order to find messages with their destination address.

A. *SAW ARQ*

SAW is the notation for stop-and-wait ARQ. As the name implies, the message originator stops at the end of each transmission to await a reply from the receiver. The transmission may be followed by one of three possible events. The receiver may respond with an acknowledgment, or ACK, indicating that the message was correctly received; an NAK if it was not correctly received; or no response if the message header was not received. Anything other than an ACK will result in the message originator repeating the message. Theoretically, a message might be repeated forever while awaiting an ACK. Usually, a system implementation will contain a retransmission limit beyond which the message is returned as undeliverable.

When SAW ARQ is used on a single-frequency system, the time lost when waiting for an ACK is important because it represents wasted air time; however, the typical land mobile radio system uses two frequencies with duplex base stations. In this configuration the time is not wasted when awaiting an ACK. For example, after the base transmits a message to a mobile, it may immediately follow with a message to any other mobile without interfering with the first mobile's ACK on the return channel. The same is true of messages from mobile to base. Thus, SAW ARQ is well suited to land mobile radio channels. All the following ARQ variations will assume an SAW format.

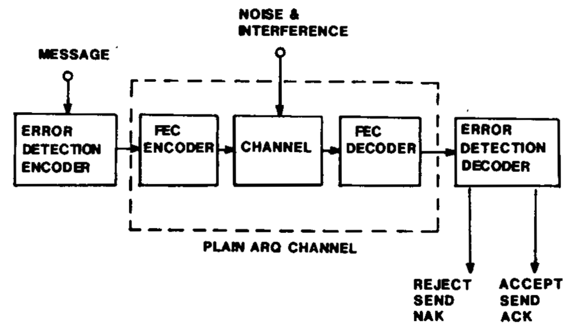

Fig. 3. Type I hybrid ARQ forward channel.

B. *Type I Hybrid ARQ*

Hybrid ARQ and hybrid error control are techniques that use forward error correction (FEC) AND error detection coding. In a Type I hybrid ARQ system the message and error detecting parity bits are further encoded with an FEC code. The bits added by the error correction code can be called error correction parity bits to distinguish them from the error detection parity bits. At the receiver, the error correction parity bits are used to correct channel errors. The FEC decoder then outputs an estimate of the message and the error detection parity bits. The FEC decoder output is then tested by the error detection decoder to determine if the message should be accepted as error free, or rejected as containing errors (see Fig. 3).

If the message is long, or if the channel signal strength is poor (high bit error rate), the probability of error-free transmission may approach zero, as shown in the previous section. Under these conditions the efficiency may be improved by using a Type I hybrid ARQ protocol rather than a simple ARQ protocol. This scheme lengthens each transmission with extra FEC parity bits to increase the success probability of each transmission. A coding gain can result if the reduction in the number of transmissions necessary to deliver a message compensates for the increased message length.

In strong signal, the Type I hybrid ARQ does not result in an efficiency improvement. When the signal is strong enough to deliver messages error free, then the extra FEC parity bits are wasted. Thus, a crossover point in signal strength exists between plain ARQ and Type I hybrid ARQ as far as efficiency is concerned.

C. *Type II Hybrid ARQ*

In a Type II hybrid ARQ scheme the FEC parity bits are not sent with the message and error detecting parity bits. Instead, the message originator alternates between message bits along with error detecting parity bits on one transmission, and only FEC parity bits on the next. The first point to observe is that when the first transmission is received error free, the FEC parity bits are never sent. Secondly, any error-free copy of the message and the error detecting parity bits delivers the message. Also note that if the code

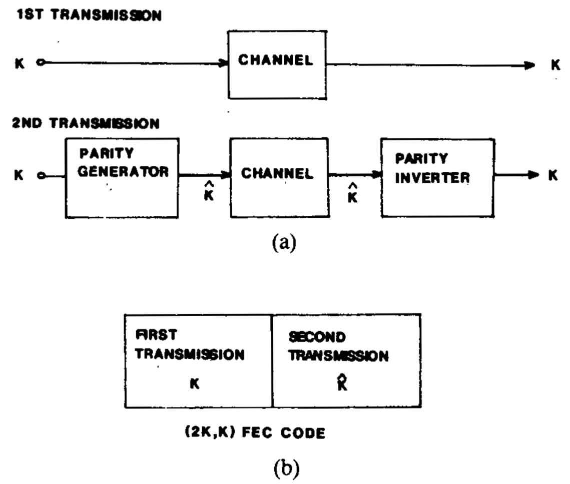

Fig. 4. Type II hybrid ARQ. (a) Data and parity transmissions. (b) Code formed by combining two consecutive transmissions.

is invertible [27] any error-free copy of the FEC parity bits delivers the message [see Fig. 4(a)]. Finally, any two consecutive transmissions, if neither is error free, can be combined for error correction, which may successfully deliver the message [see Fig. 4(b)]. The intent of Type II hybrid ARQ is to operate with the efficiency of plain ARQ in good quality signal and obtain the improvement of Type I hybrid ARQ in poor quality signal.

D. Selective Retransmission

When many blocks are transmitted at once, it can be useful to apply error detection to each block individually rather than to the message as a whole. The error detection on each block can then be used advantageously in an ARQ system by the receiver accepting individually any block received error free, and NAKing only those blocks found to contain detected errors. To do this the receiver must send back a request selecting the blocks to be retransmitted. This is called selective retransmission or SRT.

Either of the hybrid ARQ schemes may be combined with SRT ARQ. For a Type I hybrid SRT ARQ, each block in the SRT ARQ would be further encoded with an FEC code. This would lengthen each block while improving its success probability in poor signal conditions. A combination of SRT ARQ and Type II hybrid ARQ would imply that each block in the packet would be composed of message and error detecting parity bits, or FEC parity bits on alternate transmissions.

Fig. 5 illustrates typical channel transactions for these basic ARQ techniques. Both forward and reverse channel packets are shown on the same line for simplicity. The basic SAW ARQ transaction is illustrated in Fig. 5(a) showing all possible responses from the receiver (NAK, no response, ACK). In all ARQ variations the final ACK terminates the transaction. The SRT ARQ scheme is illustrated in Fig. 5(b) for a ten-block message. In this scheme the NAK returns the identity of the missed blocks

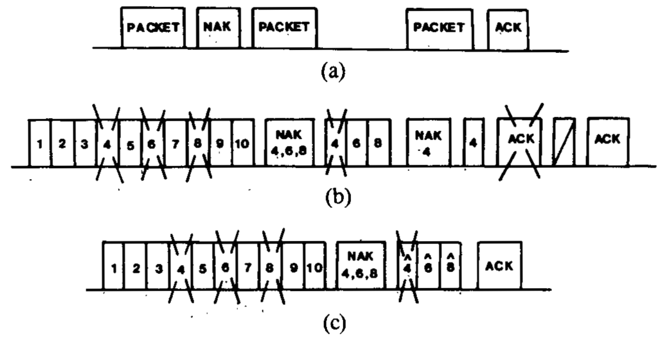

Fig. 5. ARQ transactions. (a) SAW ARQ. (b) SRT ARQ. (c) Type II hybrid SRT ARQ.

to the message originator. Also shown in this figure is an error on the reverse channel. The scheme illustrates a simple rule of repeating the last transmission for no-response or return channel errors. A Type II hybrid SRT ARQ combination protocol is illustrated in Fig. 5(c) for a ten-block message. The example shows three blocks recovered on the second transmission, which consisted of FEC parity bits on the blocks missed from the first transmission. The error-free FEC parity blocks can recover the missing message blocks by parity inversion. The remaining block containing detected errors is combined with the previously transmitted block for error correction to recover the missing message block (see Fig. 6).

IV. Throughput Analysis

The development begins with calculations for the expected number of transmissions for the simpler schemes and then proceeds to the more complex ones. Real failure mechanisms such as message header or return channel failure probabilities are included in the analysis.

Let $F(i)$ be the probability of delivering a message in fewer than i transmissions. Then $1-F(i)$ is the probability that an ith transmission occurs. The expected number of transmissions necessary to deliver a message will be denoted $E(H)$, and is given by

$$E(H)=\sum_{i=1}^{\infty}[1-F(i)]. \tag{11}$$

The distribution function can be computed from

$$1-F(i)=\prod_{j=1}^{i-1}P(j) \tag{12}$$

where $P(j)$ is the probability of failure on the jth transmission. This assumes only that the failure probabilities are independent. An important special case is when the per transmission failure probabilities are identical, or $P(j)=P_f$ for all j. Then

$$E(H)=\sum_{i=1}^{\infty}[1-F(i)]=\sum_{i=1}^{\infty}P_f^{i-1}=\frac{1}{1-P_f}. \tag{13}$$

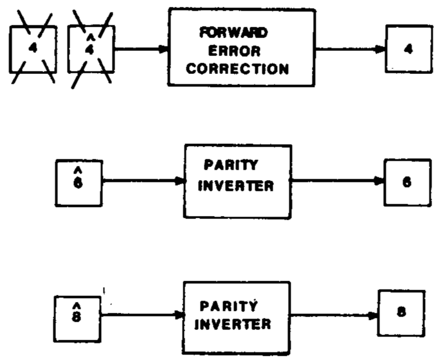

Fig. 6. Recovery of blocks from parity retransmission in a Type II hybrid SRT ARQ.

A. Non-SRT ARQ

$E(H)$ is a function of message length to the extent that P_f is a function of message length as was shown in Section II. To make meaningful comparisons between SRT and non-SRT techniques, P_f is defined as the block failure rate. Then a message is defined as a concatenation of N blocks. Now success on the ith transmission depends on all N blocks being successful, and this probability is

$$1 - P_N(i) = [1 - P(i)]^N \tag{14}$$

which assumes independence between blocks. For the equal probability case, the expected number of transmissions is given by

$$E_N(H) = \frac{1}{1 - P_N} = \frac{1}{(1 - P_f)^N}. \tag{15}$$

The function $E_N(H)$ is displayed in Fig. 7 versus block failure probability P_f for various values of message length. Note that the $E_N(H)$ increases more quickly with increasing message length.

B. SRT ARQ

In a selective retransmission scheme independence between blocks implies that the number of transmissions of each block is independent of the number of transmissions of any other block. Then the probability of requiring fewer than i transmissions for an N block message $F_N(i)$ is the same as N single-block messages *all* requiring fewer than i transmissions. Thus, it follows that

$$E_N(H) = \sum_{i=1}^{\infty} [1 - F_N(i)] = \sum_{i=1}^{\infty} \{1 - [F(i)]^N\}. \tag{16}$$

For the equal failure probability case

$$E_N(H) = \sum_{i=1}^{\infty} \left[1 - \left(1 - P_f^{i-1}\right)^N\right]. \tag{17}$$

This function is displayed in Fig. 8 for several message lengths versus block failure probability P_f. By comparison to the previous figure it can be seen that SRT requires fewer transmissions to deliver a message than a non-SRT scheme for messages of two or more blocks.

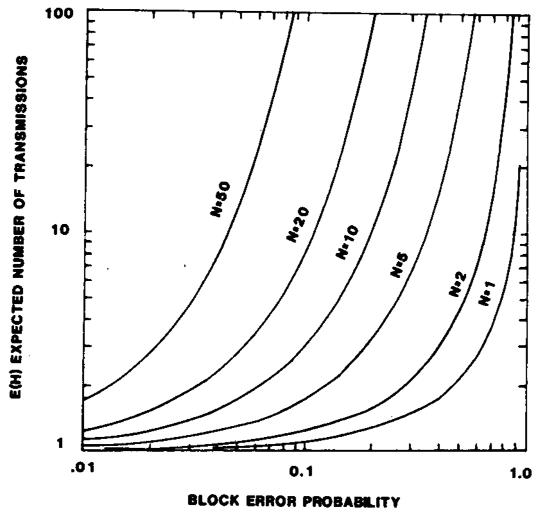

Fig. 7. $E(H)$, expected number of transmissions to deliver an N block message for ARQ.

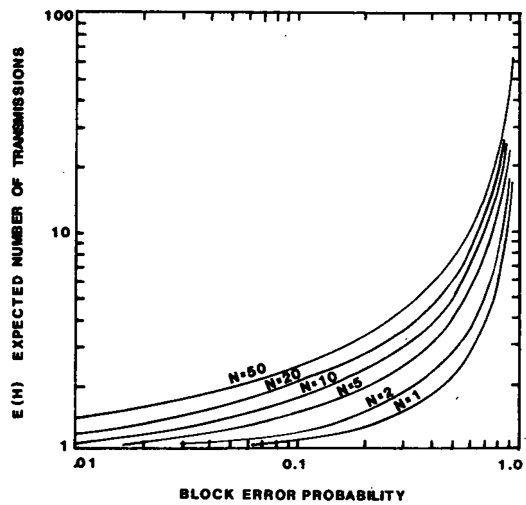

Fig. 8. $E(H)$, expected number of transmissions to deliver an N block message for SRT ARQ.

C. Type II Hybrid ARQ

In the Type II hybrid scheme the failure probabilities are not identical on each transmission. Two probabilities are sufficient for a Type II scheme: a first-transmission probability P_1, and a retransmission probability P_2. Therefore,

$$\begin{aligned} E_N(H) &= [1 - F(1)] + \sum_{i=2}^{\infty} [1 - F(i)] \\ &= 1 + \sum_{i=2} \left[1 - \left(1 - P_1 P_2^{i-2}\right)^N\right]. \end{aligned} \tag{18}$$

In practice P_2 is at least as small as P_1; however, since the Type II system allows for more powerful forward error correction by combining the retransmissions with previously stored transmissions, P_2 would be expected to be smaller than P_1. To represent this concept, P_2 will be represented as a positive integer power of P_1. The higher the power, the greater the error correction capability of the FEC code. The resulting relationship for $E_N(H)$ is displayed in Fig. 9 for $N = 10$, and for various FEC code powers.

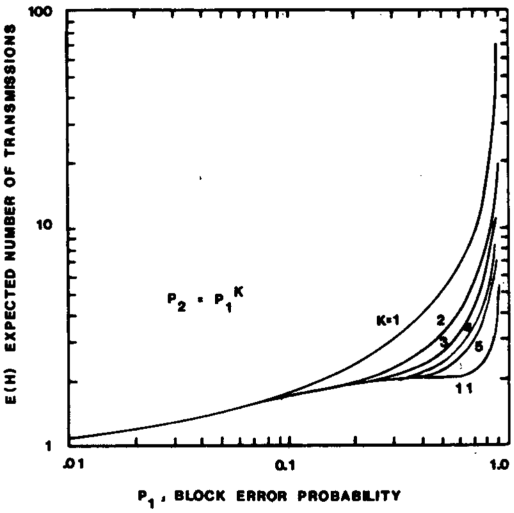

Fig. 9. $E(H)$, expected number of transmissions to deliver a 10 block message for Type II hybrid SRT ARQ for various retransmission failure probabilities.

D. Transmission Efficiency

In this section the transmission efficiency will be computed as the ratio of the number of information bits to the total number of transmitted bits. Since ACK's and NAK's are sent on a return channel, a separate efficiency is computed for each channel. The efficiency R of a non-SRT ARQ is given by

$$R = \frac{NL}{(H+NB)E_N(H)} \tag{19}$$

for an N block message where B is the number of bits per block, each containing L information bits, with an H bit long message header.

In an SRT scheme each block is transmitted $E_1(H)$, or $E(H)$ times, while $E_N(H)$ transmissions are necessary for the whole message. Then the efficiency is

$$R = \frac{L}{\frac{H}{N}E_N(H) + BE(H)}. \tag{20}$$

E. Header Sensitivity

When a header is missed, the transmission is wasted. If we let h represent the header failure probability, then h is the fraction of messages wasted. If $E_N(H)$ transmissions must be received to deliver a message, then $E_N(H)/(1-h)$ transmissions must be sent. The effect on efficiency is to decrease R by the same factor. In the non-SRT schemes

$$R = \frac{L(1-h)}{\left(\frac{H}{N}+B\right)E_N(H)} \tag{21}$$

and in the SRT case

$$R = \frac{L(1-h)}{\frac{H}{N}E_N(H) + BE(H)}. \tag{22}$$

F. ACK Sensitivity

When an ACK or NAK is missed the previous transmission is repeated. In a non-SRT scheme a missed NAK has no effect and may even be eliminated. The missed ACK, however, causes an extra transmission. Its effect is to increase $E_N(H)$ by the factor $a/(1-a)$ for an ACK failure probability of a. For the SRT scheme each NAK may be thought of as an ACK for any blocks accepted on that transmission. Therefore, both $E(H)$ and $E_N(H)$ are increased by the same factor. Then the resulting efficiencies, taking into account both header and ACK sensitivities, are, for the non-SRT schemes

$$R = \frac{L(1-h)}{\left(\frac{H}{N}+B\right)\left[E_N(H)+\frac{a}{1-a}\right]} \tag{23}$$

and for the SRT schemes

$$R = \frac{L(1-h)}{\frac{H}{N}\left[E_N(H)+\frac{a}{1-a}\right]+B\left[E(H)+\frac{a}{1-a}\right]}. \tag{24}$$

The feedback channel efficiency R_f is defined as the ratio of the total feedback channel bits to total forward channel bits. This is written in terms of the forward channel efficiency R and the ACK or NAK length in bits A. For non-SRT schemes where NAK's are not sent

$$R_f = \frac{R}{NL}\frac{A}{1-a} \tag{25}$$

and for SRT schemes

$$R_f = \frac{R}{NL}A\left[E_N(H)+\frac{a}{1-a}\right]. \tag{26}$$

V. Failure Probability Analysis

The expressions derived in the previous section all use failure probabilities. To compare particular schemes for land mobile radio channel, values will be needed for these failure probabilities. These will be derived in terms of the error weight distribution function $F_w(i)$. Then in turn, $F_w(i)$ will be obtained for a specific set of typical land mobile radio channel conditions.

$F_w(i)$, or the probability of fewer than i bit errors in a block, directly yields the failure probabilities for detection or correction codes. For error detection codes, P_f is $1-F_w(1)$. For an FEC code with error correction capability t, P_f is $1-F_w(t+1)$. Useful bounds exist for predicting the maximum error correcting capability t, for a given L and B [27].

In turn, $F_w(i)$ must be obtained for a given set of channel conditions. For static channels, independence between bits has been experimentally observed at data rates over 4800 bits/s on a standard land mobile radio set operating at 494 MHz. When independence holds, $F_w(i)$ can be computed by the binomial distribution function

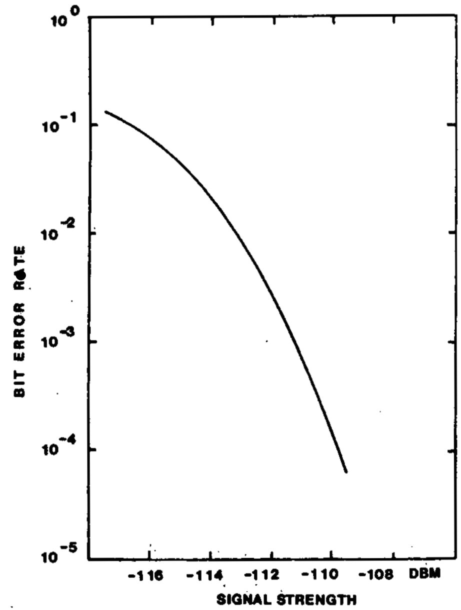

Fig. 10. Bit error rate versus static signal strength.

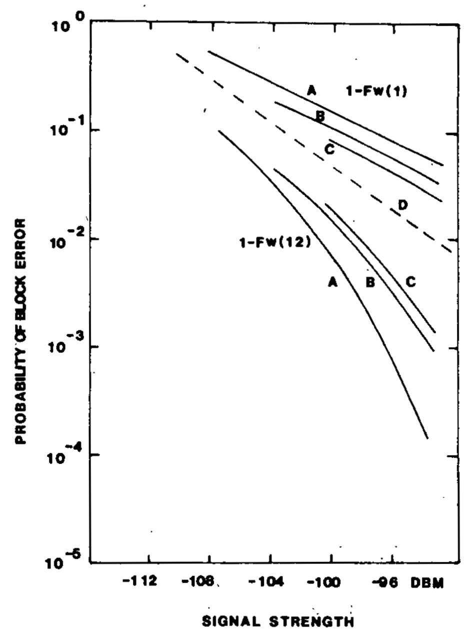

Fig. 11. Probability of block error versus signal strength in fading. A: 20 mi/h. B: 10 mi/h. C: 6 mi/h. D: Rayleigh distribution.

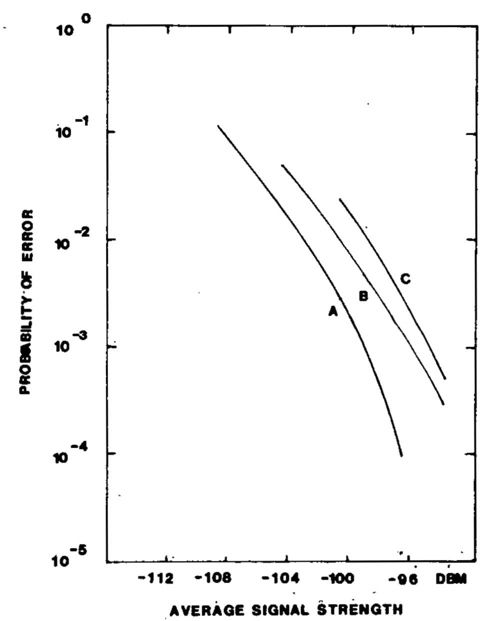

Fig. 12. Retransmission failure probability P_2 for a Type II hybrid SRT ARQ in fading. A: 20 mi/h. B: 10 mi/h. C: 6 mi/h.

with the bit error rate (BER) as the distribution parameter for any size block. The experimentally measured BER is shown in Fig. 10.

For fading channels the distribution cannot be obtained in this fashion. In Section II a model was presented which can predict $1 - F_w(1)$, or the error detection failure probability. Unfortunately, no model is known which can predict the rest of the distribution function. Experimental measurements were used to produce the distribution for 100 bit blocks at several fading rates. From these measurements the function $1 - F_w(i)$ was seen to follow a log-linear relationship. This suggested an approximation to this function of the form

$$1 - F_w(i) \approx e^{m(i-1)+b} \tag{27}$$

where the constants m and b are experimentally derived for each particular set of fading conditions.

From these measurements, models, and approximations, some conclusions can be reached as to the performance of error detection and correction coding in fading. For FEC coding, a block code was assumed with an error correction capability equal to the Hamming bound for 100 bit blocks with a rate of 1/2. The error detection and correction failure probabilities are shown for several different fading rates in Fig. 11. This shows that the FEC code yields a lower failure probability than the error detection code; however, as fading rates decrease, the failure probabilities move toward one another.

For the Type II scheme the first transmission failure probability is that of the error detection code just described. The retransmission failure probability was calculated using moment generating functions which yield the probability that the sum of the error weights in two blocks exceeds the correction capability of a two-block, rate 1/2, FEC code. Again, the Hamming bound was used and the resulting retransmission failure probabilities are shown in Fig. 12.

VI. Predicted and Measured System Performance

For the channel conditions where the failure probabilities have been evaluated, the various ARQ protocols can be compared. The schemes will all share some basic choices to facilitate the comparison. The messages will be assumed to be 256 characters of 7 bits each. Each scheme will contain an additional message cyclic redundancy check (CRC) of 16 bits. Message headers will be 40 bits of preamble followed by two 100 bit blocks, each of a rate 1/2 FEC code. ACK's will be one block for non-SRT schemes and two blocks for SRT schemes to reflect the greater amount of information which must be returned in an SRT NAK.

The Type I hybrid coding will be assumed to be of a rate 1/2. Finally, SRT error detection will be assumed to be a 16 bit CRC per block.

For each protocol, a different packet length will result. For example, the 256 7 bit characters plus a 16 bit message CRC come to 1808 bits. At 100 bits per block, this is 18.08 blocks with a plain ARQ protocol. With a rate 1/2 FEC code per block, this is 36.16 blocks with a Type I hybrid ARQ scheme. If an additional 16 bits per block are used for error detection rather than for information, this becomes 53.17 blocks in a Type I hybrid SRT ARQ combination protocol. For error detection only per block, 21.52 blocks result in an SRT ARQ format. This is the same length for a Type II hybrid SRT ARQ combination format.

Fig. 13 illustrates the inverse of the efficiency (channel bits per data bit) in a fading environment, calculated with the expressions derived in the previous sections. Fig. 14 shows the same information for four of the five protocols, measured on a simulated system composed of a laboratory setup of an RF generator, fading simulator, radio, and special purpose microprocessor hardware.

Notice that at lower signal strengths the schemes require more channel bits per data bit due to increased retransmission activity. In strong signal conditions, each protocol achieves its asymptotic efficiency, based on its overall rate, when the channel becomes essentially error free. Other conclusions reached from this and from other data taken were that the SRT schemes continue to deliver messages at reasonable efficiency at lower faded signal strengths than the non-SRT schemes. Also, the plain ARQ, SRT ARQ, and Type II hybrid SRT ARQ perform the job of delivering messages with better efficiency in strong signal conditions than the Type I hybrid ARQ schemes. Finally, the best overall efficiency and sensitivity is offered by the Type II hybrid SRT ARQ.

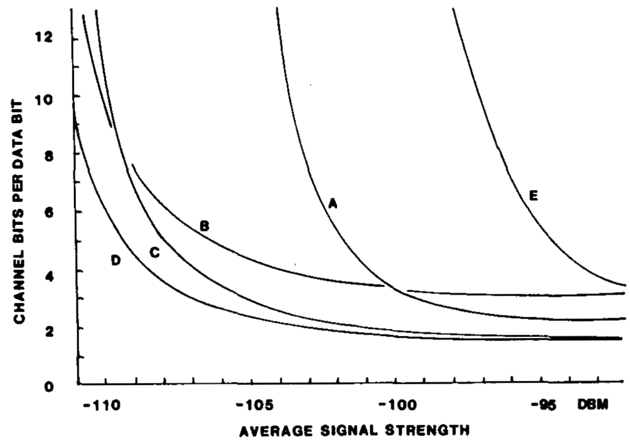

Fig. 13. Inverse of the transmission efficiency, 20 mi/h faded predictions. *A*: Type I hybrid ARQ. *B*: Type I hybrid SRT ARQ. *C*: SRT ARQ. *D*: Type II hybrid SRT ARQ. *E*: Plain ARQ.

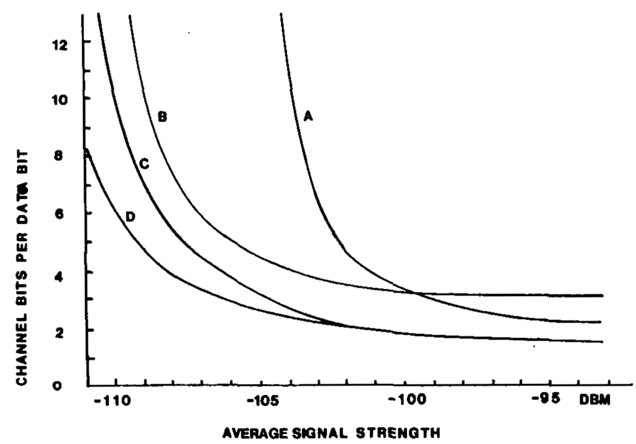

Fig. 14. Inverse of the transmission efficiency in 20 mi/h fading, experimentally measured. *A*: Type I hybrid ARQ. *B*: Type I hybrid SRT ARQ. *C*: SRT ARQ. *D*: Type II hybrid SRT ARQ.

VII. Undetected Error Rates

In the previous sections it has been assumed that each transmission results in one of two outcomes, i.e., success or failure. Actually, three outcomes are possible: success, detected error, or undetected error. These three probabilities per transmission are denoted P_c, P_d, and P_e. When these probabilities are identical for each transmission, the non-SRT undetected error probability per message has been found to be [27]

$$P(E)=\frac{P_e}{P_c+P_e}. \tag{28}$$

P_c can be expressed in terms of the failure probabilities previously defined, while P_e can be bounded based on the number of error detection parity bits m [28]

$$P_c=1-P_f \tag{29}$$

$$P_e \leqslant P_f 2^{-m}. \tag{30}$$

For an N block ARQ define P_{Nc}, P_{Nd}, and P_{Ne} as the N block counterparts of P_c, P_d, and P_e. For the non-SRT scheme, the undetected error probability is given by

$$P_N(E)=\frac{P_{Ne}}{P_{Nc}+P_{Ne}}=\frac{1-(1-P_f)^N}{1+(1-P_f)^N(2^m-1)}. \tag{31}$$

For the SRT schemes, first consider b parity bits per block instead of m parity bits for the whole message. The probability of undetected error for each block as a single message would be

$$P(E)=\frac{P_f}{P_f(1-2^b)+2^b}. \tag{32}$$

For all N blocks to be received correctly, none can contain detected errors, and therefore

$$P_N(E)=1-\left[1-\frac{P_f}{P_f(1-2^b)+2^b}\right]^N \tag{33}$$

If b bits per block and m bits per message are assumed for error detection, then

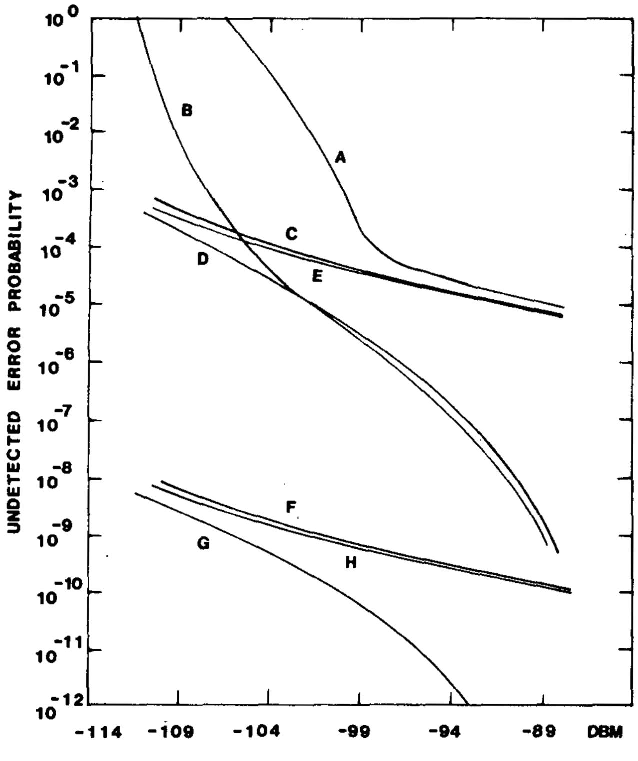

Fig. 15. Undetected error rates for 20 mi/h fading. *A*: Plain ARQ. *B*: Type I hybrid ARQ with whole message CRC. *C*: SRT ARQ. *D*: Type I hybrid SRT ARQ. *E*: Type II hybrid SRT ARQ with per block CRC. *F*: SRT ARQ. *G*: Type I hybrid SRT ARQ. *H*: Type II hybrid SRT ARQ with per block and whole message CRC.

$$P_N(E) = \frac{1-[1-P(E)]^N}{1+[1-P(E)]^N(2^m-1)} \tag{34}$$

which can be evaluated by substituting in (32) for $P(E)$ as the failure probability per block.

For the Type II hybrid scheme two sets of probabilities are needed: one set for the first transmission P_{c1}, P_{d1}, and P_{e1}, and another for the retransmissions P_{c2}, P_{d2}, and P_{e2}. Then $P(E)$ is equal to

$$P(E) = P_{e1} + P_{d1}\left(\frac{P_{e2}}{P_{e2}+P_{c2}}\right) \tag{35}$$

which can be calculated in terms of the per transmission failure probabilities P_1 and P_2, and the number of error detection parity bits. Fig. 15 illustrates an overall comparison of the resulting undetected error probabilities for all the analyzed schemes for 256 character message lengths. The SRT schemes are shown two ways: with per block only, and with whole message in addition to per block CRC's.

VIII. Conclusions

The land mobile radio channel exhibits a wide range of signal quality. This results in variations in time and space from low error rates to very high error rates. Because of this, the coding applied to the message and the retransmission request strategy has been found to have a dramatic influence on the overall system performance. It has also been found that the longer the message, the more important the coding and protocol become. Regarding the fading rate, an important characteristic was noted: that at slower fading rates, the success probability advantage of FEC coding over error detection coding is diminished.

Regarding the protocols examined, it was found that the best efficiency is exhibited by plain ARQ in strong signal; however, it also degrades the most in marginal signal conditions. Type I hybrid ARQ improves the poor signal sensitivity at the expense of the strong signal efficiency. SRT ARQ gives a much more dramatic improvement in sensitivity in fading signals with much less impact on efficiency than the Type I hybrid ARQ. A Type I hybrid SRT ARQ combination gains a little more sensitivity; however, it has the worst efficiency in strong signals. Finally, a new protocol combination, Type II hybrid SRT ARQ, was found to offer the best sensitivity and efficiency overall, and when both per block and whole message CRC's are used, also exhibits as low an undetected error rate as any other protocol tested.

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