Performance Comparison of Type I, II and III Hybrid ARQ Schemes over AWGN Channels

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Abstract— **Basically, there are two techniques to control errors in data communications, ARQ and FEC. The combination of these two techniques yields type I, II and III Hybrid ARQ (HARQ) schemes. In this paper, we compare the performance of ARQ and different HARQ schemes based on theoretical results on the average number of transmissions and throughput on an AWGN channel. In type II HARQ, only a code combining strategy is considered which consists in alternatingly sending one of the outputs of a convolutional encoder.**

***Key Words*: ARQ, FEC, HARQ, Code Combining, AWGN.**

1 Introduction

A major concern in data communication is how to control transmission errors caused by the channel noise so that error free data can be delivered to the user. A solution to this problem is the use of Automatic Repeat reQuest ARQ) and Forward Error Correction FEC) schemes. Although ARQ systems are simple, easy to implement and provide high system reliability, they suffer from a severe decrease in throughput with an increase in channel error rate. FEC systems maintain constant throughput equal to the code rate) irrespective to the channel error rate. However, FEC systems have two major drawbacks. First, when a received sequence is detected in error, the sequence has to be decoded and the decoder output has to be delivered to the user regardless of whether it is correct or not. Second, in order to achieve high system reliability, a long powerful code must be used to correct a large number of error patterns. This makes the decoder hard to implement and expensive.

The advantage of obtaining high reliability in ARQ systems can be coupled with the advantage of FEC systems to provide a good throughput even in poor channel conditions. Such a system, which is a combination of the two basic error control schemes: FEC and ARQ, is called a Hybrid ARQ system. This scheme exploits the advantages of the conventional FEC and ARQ systems by combining them effectively. In fact, the FEC process reduces the frequency of retransmissions by correcting commonly occurring error patterns which increases the throughput. The ARQ protocol permits the retransmission of the erroneous packets instead of delivering them to the user.

Hybrid ARQ schemes are considered for the High-Speed Downlink Packets Access HSDPA) [1] which is currently being developed as the evolution of 3G systems to increase the data rate. This is why, we are interested in comparing the performance of different types of hybrid ARQ which are classified in the literature into three generic types: type I, II and III Hybrid ARQ. In type I HARQ, a fixed code rate is used and when a data packet is declared in error, the receiver rejects it and asks for its retransmission. This scheme is not adaptive to changing channel conditions and is the best suited for channels with a fairly constant noise or interference level. To adapt the channel encoder rate to the transmission reliability, type II HARQ has been introduced [3]. It is based on Incremental Redundancy IR) transmission by progressively sending parity bits to the receiver. Starting with a code rate that overcomes the nominal channel noise that is always present, incremental redundancy bits are provided by the transmitter as the channel degrades and all received packets are combined together to extract the information. Finally, type III HARQ is a particular case of the second scheme in which every transmission is self decodable. That is, the decoder has the possibility to extract the information either by combining all previously received sequences for the same data packet as it is the case in type II HARQ or by using only the last received sequence. This property allows type III HARQ to be the most adapted scheme for fading channels.

The paper is organized as follows. An overview of the basic elements of ARQ and HARQ schemes is presented in Section II. A theoretical analysis of their performance is also presented. Section III gives some numerical results and finally, section IV draws some conclusions.

2 Throughput Analysis

Performance evaluation of HARQ schemes is based on the evaluation of the average number of transmissions Tr and the throughput efficiency Thr, defined as the average number of accepted information bits per transmitted channel symbol. The following assumptions are made throughout the paper. An ideal Selective Repeat SR) strategy is used. A convolutional encoder of memory m is used for error correction and a perfect code detection is assumed. Furthermore, a noiseless feedback channel is available so that the receiver can reliably inform the transmitter of successfully decoded packets and finally a BPSK modulation is assumed.

2-1 ARQ Scheme

In this simplest ARQ scheme, we only use a code for error detection to generate packet retransmission requests. To each k information bits to be transmitted, n_p parity bits for error detection are appended. The sequence of $(k + n_p)$ bits is transmitted to the receiver. Each received packet declared in error is discarded and replaced by another copy of the same packet. The average number of transmissions is given by

$$Tr = \sum_{i=1}^{\infty} i \times P(tr = i), \qquad 1)$$

where tr is a random variable given a number of transmission,

$$P(tr = i) = (1-(1-p)^{(k+n_p)})^{(i-1)} \times (1-p)^{(k+n_p)}, \qquad 2)$$

$p = 1/2\ \mathrm{erfc}(\sqrt{E_s/N_0})$ for the Additive White Gaussian Noise AWGN) channel and E_s/N_0 is the Signal-to-Noise Ratio SNR) per received channel symbol. We deduce

$$Tr = \frac{1}{(1-p)^{k+n_p}}. \qquad 3)$$

The throughput Thr is therefore given by

$$Thr = \frac{1}{Tr} \frac{k}{(k+n_p)}, \qquad 4)$$

where the factor $k/(k+n_p)$ is the loss in throughput due to the added parity check bits for error detection.

2-2 Type I HARQ Scheme

The type I HARQ scheme [2] is based on a fixed code rate $R_c = l/v$ for error correction. In this strategy, when a received packet is detected in error, the receiver attempts to correct it. If the error is within the designed error-correcting capabilities of the code, it is corrected and the packet is delivered to the data sink. Otherwise, the receiver discards it and requests its retransmission via the return channel. When a convolutional encoder is used for error correction, the average number of transmissions is given by

$$Tr = \sum_{i=0}^{+\infty} P(D_d)^i = \frac{1}{1 - P(D_d)}, \qquad 5)$$

where D_d is the event "decoded sequence contains detected errors". Since the error events at different treillis levels are not independent, an exact evaluation of $P(D_d)$ is difficult. However, $P(D_d)$ is upper bounded by [4]

$$P(D_d) \leq 1 - (1 - P(E))^{\frac{k+n_p}{l}}, \qquad 6)$$

where $P(E)$ is the error event probability of the Viterbi decoding algorithm which is upper bounded by

$$P(E) \leq \sum_{d=d_{free}}^{+\infty} a_d \times P_d, \qquad 7)$$

where a_d is the number of incorrect paths at distance d and P_d is the probability that a wrong path at distance d is selected. For the AWGN channel, P_d is given by

$$P_d = \begin{cases} \displaystyle\sum_{j=\frac{d+1}{2}}^{d} \binom{d}{j} p^j (1-p) & d \text{ odd} \\ \displaystyle\sum_{j=\frac{d}{2}+1}^{d} \binom{d}{j} p^j (1-p)^{d-j} + \frac{1}{2}\binom{d}{d/2} [p(1-p)]^{d/2} & d \text{ even} \end{cases} \qquad 8)$$

for Hard decision decoding and by

$$P_d = Q\left(\sqrt{\frac{2dE_s}{N_0}}\right) \qquad 9)$$

for soft decision decoding, where $Q(x)$ is given by

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{+\infty} e^{-y^2/2} dy. \qquad 10)$$

From 5) and 6), we have

$$Tr \leq \frac{1}{(1-P(E))^{\frac{k+n_p}{l}}}, \qquad 11)$$

and

$$Thr = \frac{R_c}{Tr} \frac{k}{(k+n_p+lm)}, \qquad 12)$$

where the factor $k/(k+n_p+lm)$ is the loss in throughput due to the added parity bits for error detection and to the tail of lm known bits appended to each transmitted sequence.

The disadvantage of the type I HARQ scheme is that once the coding rate is fixed, all parity bits for error correction are transmitted even if they are not all needed, thus reducing the channel use efficiency. For this reason, type II HARQ strategy, developed in the next section, was proposed [4] to provide a partial remedy to this drawback.

2-3 Type II HARQ Scheme

Type II HARQ is based on an adaptive error correction technique known as Incremental Redundancy IR) procedure. The amount of redundancy bits for protecting data bits against channel impairments are gradually increased during the retransmission. Different strategies are considered in the literature: Code Combining [4], Generalized [5] and Selective Combining [6] type II HARQ. In this paper, we study type II with Code Combining which consists in alternately sending one of the outputs of a convolutional encoder. In this strategy, first transmission occurs without coding and successive repetition of the convolutional encoder outputs yields a family of repetition codes of decreasing rates equal to $1/i$ where i is the number of transmission tentative. This technique allows the system to adapt the channel encoder rate to the transmission quality. A correct reception occurs if the most recent received packet is declared error-free or if the combination of all received packets contains a correctable error pattern.

Let R_d and $D_d^{(i)}$ denote respectively the event "received packet is error free" and "decoded packet contains detected errors at ith transmission of the strategy". Due to the statistical dependency among the different events considered in this strategy, the exact evaluation of the average number of transmissions is difficult. However, we can lower and upper bound Tr as follows [4]

$$1 + P(R_d) + \sum_{i=1}^{\infty} P(R_d)^{i+1} \prod_{j=1}^{i} P(D_d^{(j)}) \leq Tr \leq 1 + P(R_d) + \sum_{i=1}^{\infty} P(D_d^{(i)}), \qquad 13)$$

where $P(R_d)$ and $P(D_d^{(i)})$ are given by

$$P(R_d) = 1 - (1-p)^{k+n_p+lm}, \qquad 14)$$

$$P(D_d^{(i)}) \leq 1 - (1 - P(E^{(i)}))^{\frac{k+n_p}{l}} \qquad 15)$$

where $P(E^{(i)})$ is the error event probability of the Viterbi decoding at the ith retransmission. The evaluation of 13) using the expression of $P(R_d)$ given by 14) and the approximation of $P(D_d^{(i)})$ by the lower bound given by 15), shows that the lower and upper bounds on Tr are nearly identical. This is why Tr can be approximated by the upper bound of 13) to have the worst performance.

The throughput efficiency is then given by

$$Thr = \frac{l}{Tr} \frac{k}{(k + n_p + lm)}. \qquad 16)$$

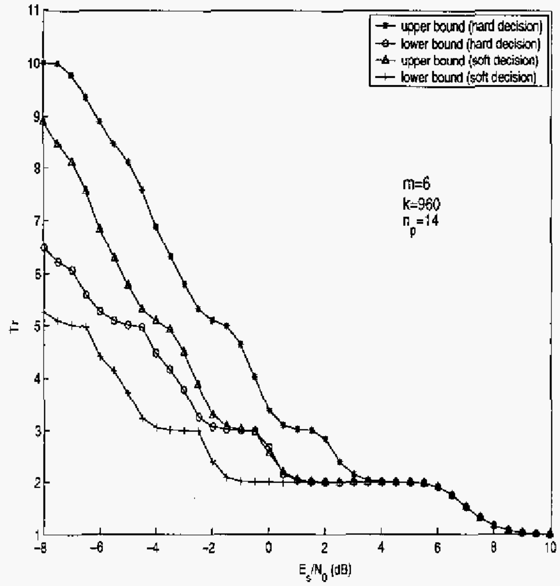

Fig. 1. Variation of the upper and lower bounds on the average number of transmissions Tr for type II HARQ with code combining.

2-4 Type III HARQ Scheme

The main drawback of IR HARQ schemes is that additional incremental coded bits are not in general self decodable. That is, the decoder must rely on both the initially transmitted packet as well as the additional incremental coded bits for decoding. In situation where a packet can be lost or severely damaged, it is desirable to have a scheme where incremental coded bits are self decodable. This idea has permitted to modify the second type of HARQ by exploiting the Complementary Punctured Convolutional CPC) codes [7] to create a new type of HARQ scheme called type III HARQ. Briefly, the CPC codes are composed of p equivalent codes [7] having the same rate $R_c = l/v$ and same distance spectra a_d. These codes are obtained from the same original rate $1/v_0$ code and when they are combining together the obtained code contains the original rate $1/v_0$ code. Due to the self decodability property, type III HARQ procedure has the choice to extract the information either from the last received packet or by combining all previous packets, as is generally the case with IR HARQ schemes [7]. We here emphasize the fact that type III HARQ packets are all coded with a correcting code which is not the case in type II HARQ with code Combining. Let $F^{(i)}$ denote the even "decoding failure at level i of the ARQ scheme" which is equivalent to the joint event $\{D_d(1), D_d(i)\}$ for $i \leq p$, and to the joint event $\{D_d(1), D_d(p)\}$ for $i \geq p$ where $D_d(i)$ is the event "decoded sequence in $C^{(i)}$, obtained by combining i equivalant codes is detected in error".

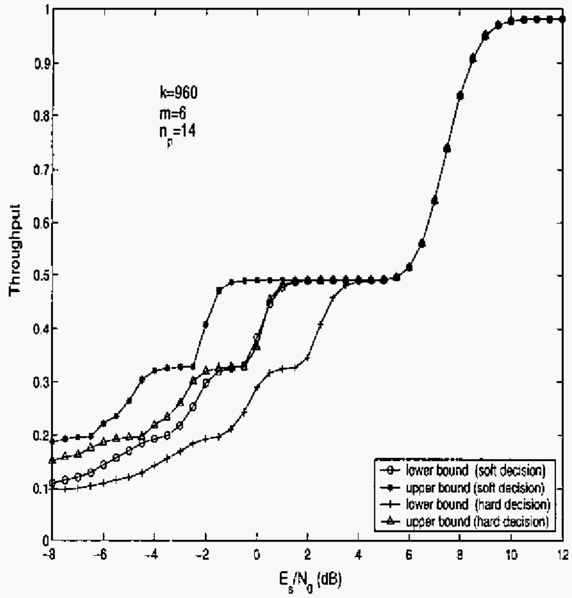

Fig. 2. Variation of the upper and lower bounds on the throughput for type II HARQ with code combining.

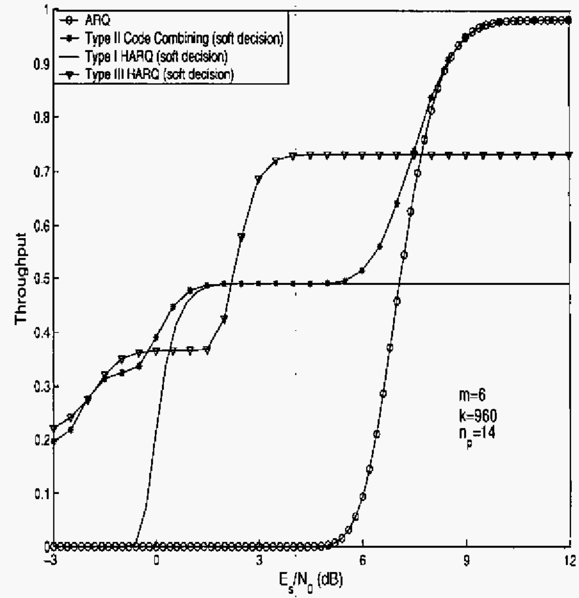

Fig. 3. Throughput comparison of ARQ and HARQ schemes.

The average number of packets transmitted per correctly decoded packet is given by [7]

$$Tr = 1 + P(F^{(1)}) + ... + P(F^{(1)}, .., F^{(i)}) + ... \quad (17)$$

Due to the statistical dependency among the joint events $\{D_d(1), D_d(i)\}$, the exact evaluation of Tr is difficult. However, we can bound each term in (17) as [7]

$$P(F^{(1)}, ..., F^{(i)}) \leq \begin{cases} P(D_d(i)), for \;\; i \leq p. \\ P(D_d(p))^j, for \;\; i = jp. \\ P(D_d(p))^j P(D_d(i-jp)), \\ \quad for \;\; jp < i < (j+1)p, j \geq 1 \end{cases} \quad (18)$$

Substituting (18) into (17) and rearranging terms, we obtain

$$Tr \leq \frac{1 + \sum_{i=1}^{p-1} P(D_d(i))}{1 - P(D_d(p))}. \quad (19)$$

The throughput is then lower bounded by

$$Thr = \frac{R_c}{Tr} \geq R_c \frac{1 - P\{D_d(p)\}}{1 + \sum_{i=1}^{p-1} P\{D_d(i)\}}. \quad (20)$$

3 Numerical Results

This section presents numerical results of the average number of transmissions and the throughput of ARQ and different HARQ schemes. The data stream is organized in packets of length $k = 960$ information bits, to which $n_p = 14$ bits are appended for error detection. Type I and II HARQ use a convolutional code (2,1,6) with generator polynomials (133,171). The free distance d_{free} and the distance spectra $a_{d_{free+j}}, j = 0, 1, 2, ...$ of repetition codes for Type II HARQ issued from code (2,1,6) are listed in table VI of [4]. In type III HARQ, we use a convolutional code (3,1,6) with generator polynomials (133,145,175). The free distance d_{free} and the distance spectra $a_{d_{free+j}}, j = 0, 1, 2, ...$ for this code and CPC codes are given in table II of [7].

Figure 1 and 2 show respectively the average number of transmission Tr and throughput Thr of type II HARQ with Code Combining. We can see that the use of soft decision decoding permits to reduce the average number of transmissions and to increases the throughput for low SNRs. Also, we can see that the two bounds on Tr agree at high and moderate SNRs, and differ at low SNRs. This is why, we can approximate Tr by its upper bound to obtain a lower bound on the throughput which is presented in figure 2 for soft and hard decisions.

Figure 3 shows that the throughput of type II HARQ with Code Combining is better than ARQ and type I HARQ for all SNRs and better than type III HARQ only at high SNRs. We note that even for low SNRs a useful throughput is maintained for both type III HARQ and type II HARQ with Code Combining due to respectively the transmission of complementary codes sequences and successive repetition of the convolutional encoder output. That is, during good channel conditions, a high code rate can be used, and as the channel degrades, the code rate can be lowered accordingly. Also, we can see that only the throughput of type II HARQ is equal to that of the ARQ at high SNRs since the first transmission occurs without coding. That is not the case for the others HARQ schemes since all transmitted packets are coded.

4 Conclusion

In this paper, we have given an overview of ARQ and different Hybrid ARQ schemes. We have compared their performances in terms of the average number of transmissions and throughput on an AWGN channel. We have shown that type II with Code Combining can be considered as the best scheme. Type III HARQ offers better performances than type II with Code Combining only at moderate SNRs.

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