the normal-normal conjugate families



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normal-normal pair

normal distribution known σ unknown μ prior self-elicited self-elicited mean ν standard deviation τ

posterior density also normal

sometimes σ is known



normal-normal conjugate families

assume

- prior on unknown μ is normal
 - mean \boldsymbol{v}
 - \triangleright standard deviation τ
- \rightarrow data $x_1, x_2, ..., x_n$, are independent,
 - ightharpoonup come from normal with standard deviation σ

$$\nu^* = \frac{\nu\sigma^2 + n\bar{x}\tau^2}{\sigma^2 + n\tau^2} \quad \tau^* = \sqrt{\frac{\sigma^2\tau^2}{\sigma^2 + n\tau^2}}$$

mass of ammonium nitrate

known standard deviation = 0.2 milligrams

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mass = 10 milligrams?

standard deviation

of prior = 2
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prior is N(10, 2)

mass of ammonium nitrate

$$\nu^* = \frac{\nu\sigma^2 + n\bar{x}\tau^2}{\sigma^2 + n\tau^2}$$

$$= \frac{10 \times (0.2)^2 + 5 \times 10.5 \times 2^2}{(0.2)^2 + 5 \times 2^2}$$

$$=10.499$$

uncertainty of mass of ammonium nitrate

$$\tau^* = \sqrt{\frac{\sigma^2 \tau^2}{\sigma^2 + n\tau^2}}$$
$$= \sqrt{\frac{(0.2)^2 \times 2^2}{(0.2)^2 + 5 \times 2^2}}$$

uncertainty = 2.0

uncertainty of mass of ammonium nitrate

$$\tau^* = \sqrt{\frac{\sigma^2 \tau^2}{\sigma^2 + n\tau^2}}$$
$$= \sqrt{\frac{(0.2)^2 \times 2^2}{(0.2)^2 + 5 \times 2^2}}$$

uncertainty = 0.089

what has the chemist learned?

mass uncertainty
10mg 2

what has the chemist learned?

mass uncertainty 10.499mg 0.089

summary

- I. a new conjugate family, the normal-normal
- 2. the updating formulae for the posterior mean and standard deviation
- 3. worked example for a case in which one might reasonably know the **standard deviation** σ for the normal distribution