comparing two independent means: hypothesis testing

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weight gain during pregnancy and mother's age

2004 survey of births on North Carolina

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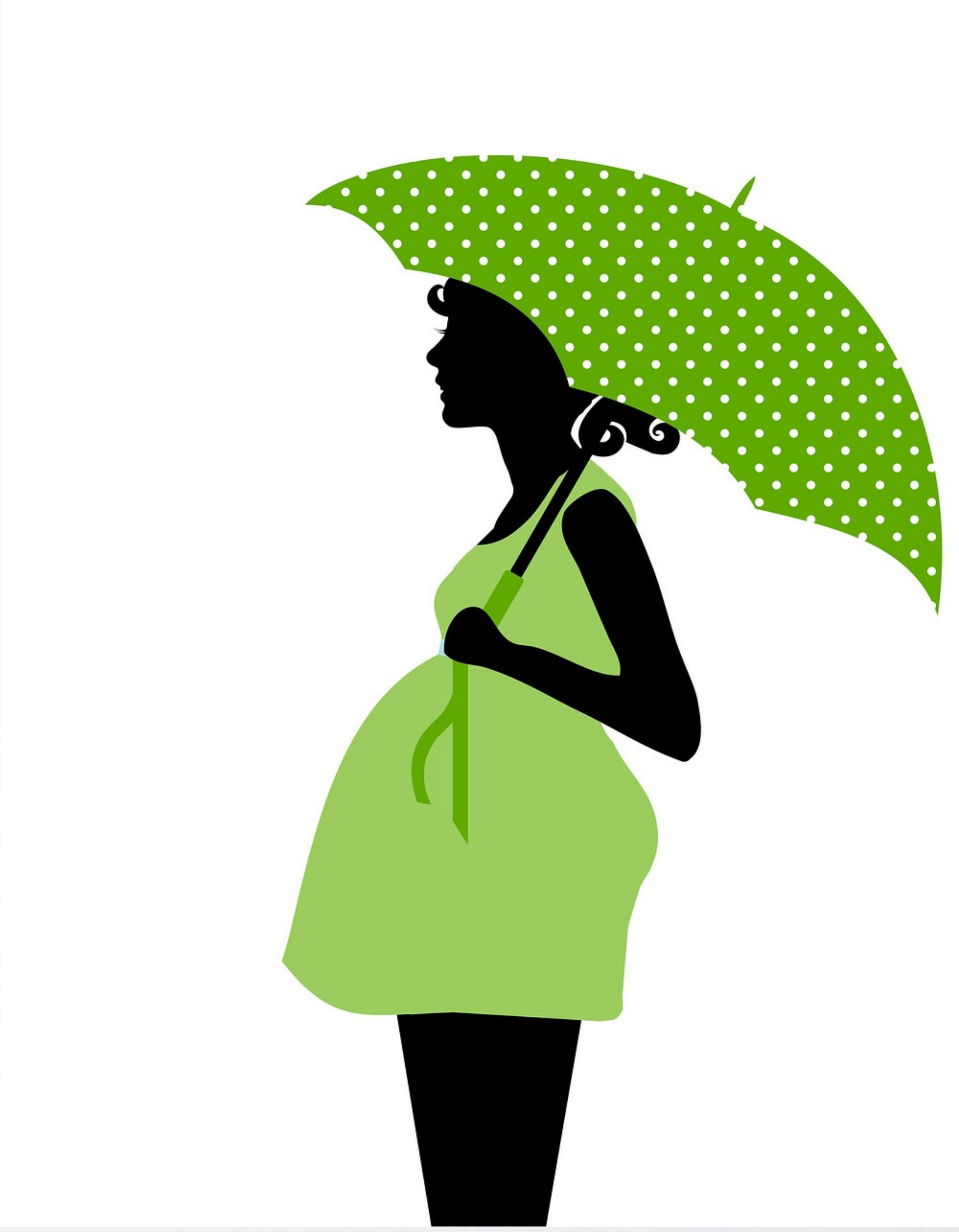
gained weight gain during pregnancy in pounds



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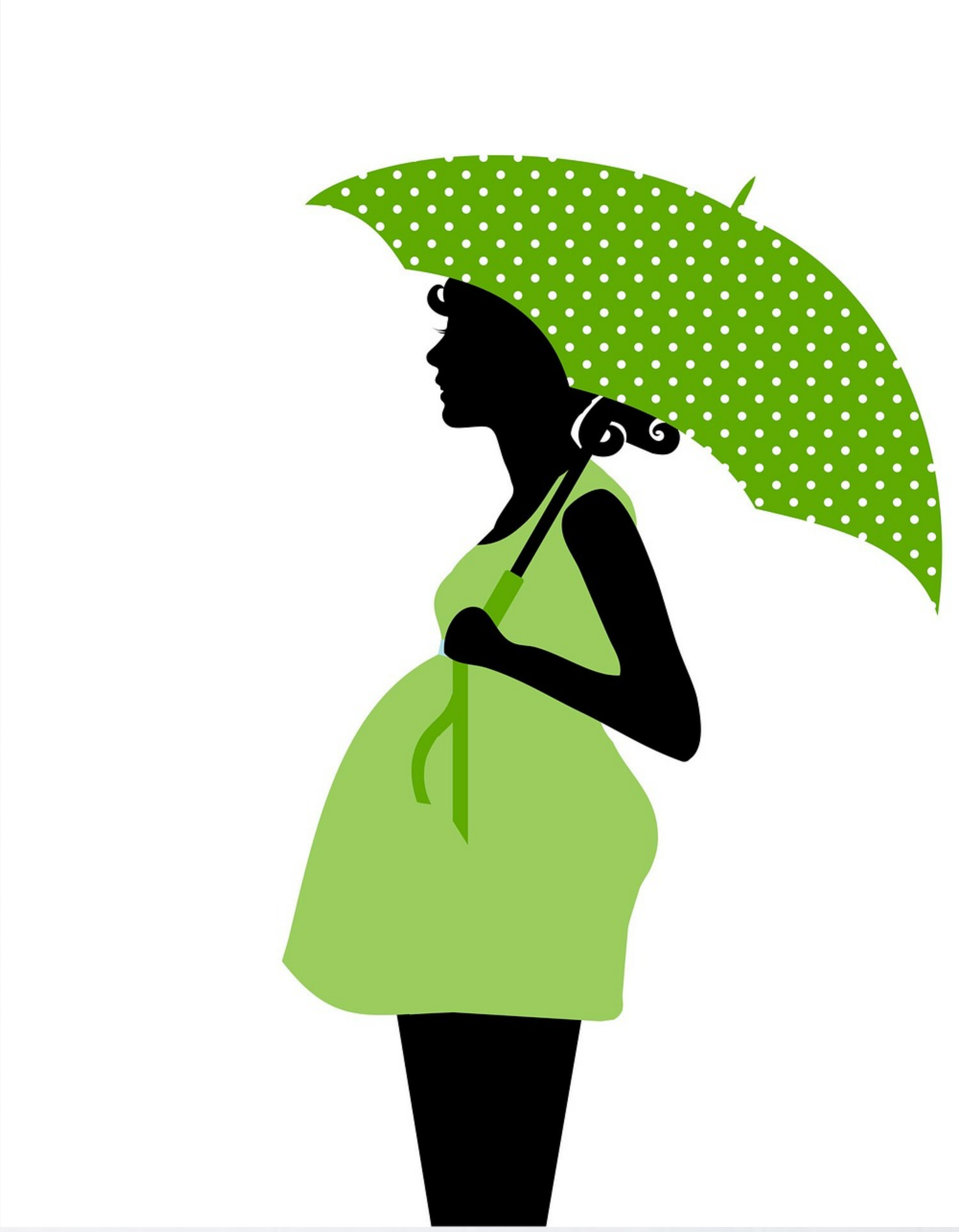

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weight gain during pregnancy and mother's age

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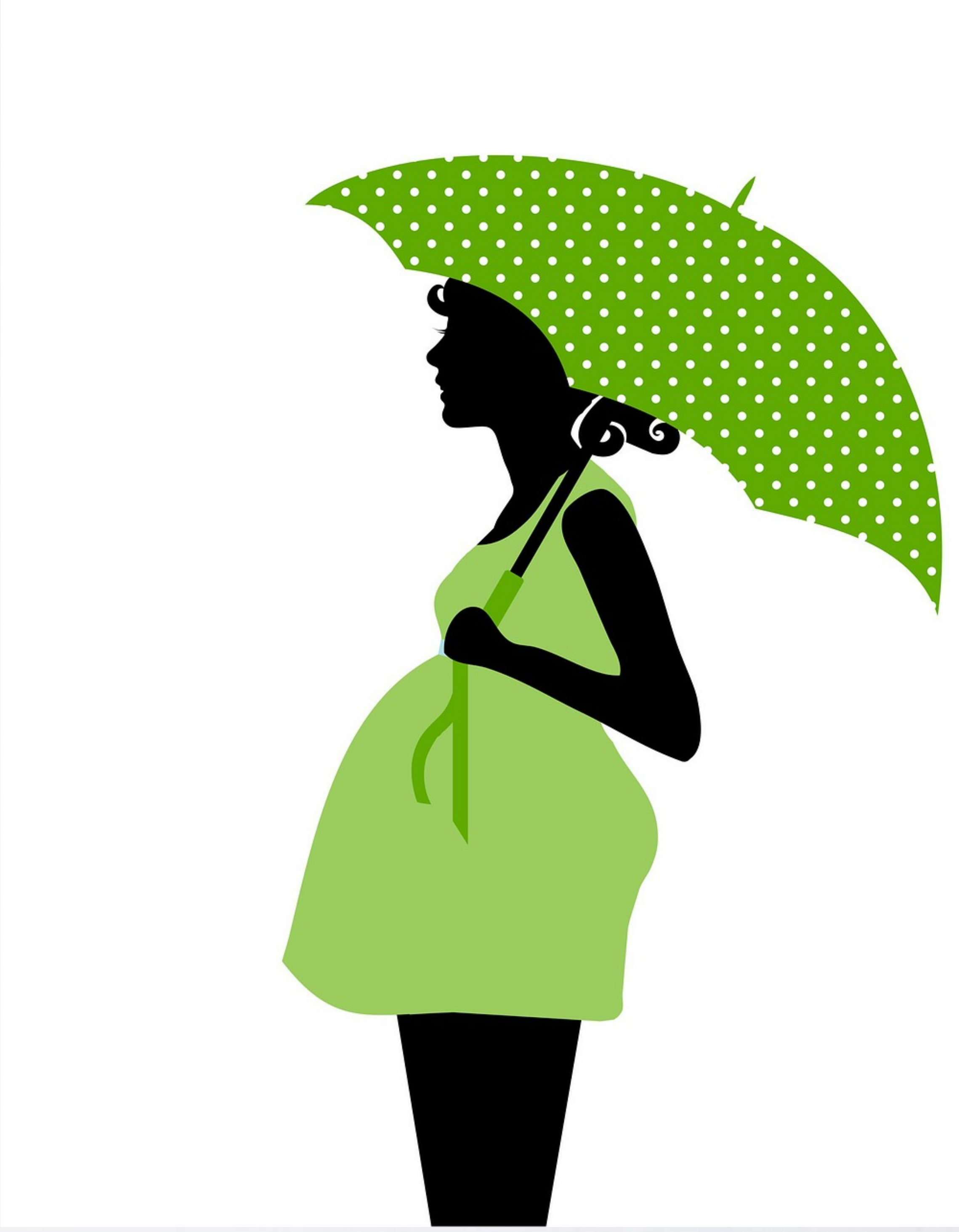
is the average weight gain of older mothers different from the average weight gain of younger mothers?



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- need prior distributions for α, μ, σ^2 under both hypotheses

Bayes factor

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$$
\begin{aligned}
BF[H_1 : H_2] &= \frac{p(\text{data}|H_1)}{p(\text{data}|H_2)} \\
&= \frac{\iint p(\text{data}|\alpha{=}0,\mu,\sigma^2)p(\mu,\sigma^2|H_1)\,d\mu\,d\sigma^2}{\int\iint p(\text{data}|\alpha,\mu,\sigma^2)p(\alpha|\sigma^2)p(\mu,\sigma^2|H_2)\,d\mu\,d\sigma^2\,d\alpha}
\end{aligned}
$$

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- Jeffreys-Zellner-Siow or 'JZS'

R code

```r
library(statsr)
data(nc)
bayes_inference(y=gained, x=mature, data=nc,type='ht',
                statistic='mean',  alternative='twosided', null=0,
                prior='JZS', r=1, method='theo', show_summ=FALSE)

## Hypotheses:
## H1: mu_mature mom  = mu_younger mom
## H2: mu_mature mom != mu_younger mom
##
## Priors: P(H1) = 0.5  P(H2) = 0.5
##
## Results:
## BF[H1:H2] = 5.7162
## P(H1|data) = 0.8511
## P(H2|data) = 0.1489
##
```

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next: putting it all together and summarizing results