

the gamma-Poisson conjugate families

Poisson random variable

probability mass function for Poisson distribution

λ mean
variance

$$P[X = k] = \frac{\lambda^k}{k!} \exp(-\lambda) \text{ for } k = 0, 1, \dots$$

$$k! = k \times (k-1) \times \dots \times 1$$

$$\lambda > 0$$

A low-angle shot of the Golden Rider statue in Dresden, Germany. The statue is made of gilded copper and depicts a man in 17th-century military attire, including a plumed helmet and scale armor, riding a horse. The background is a clear blue sky with some light clouds.

1875 - 1894

15 cavalry units

200 casualties

λ average number
per year, per unit

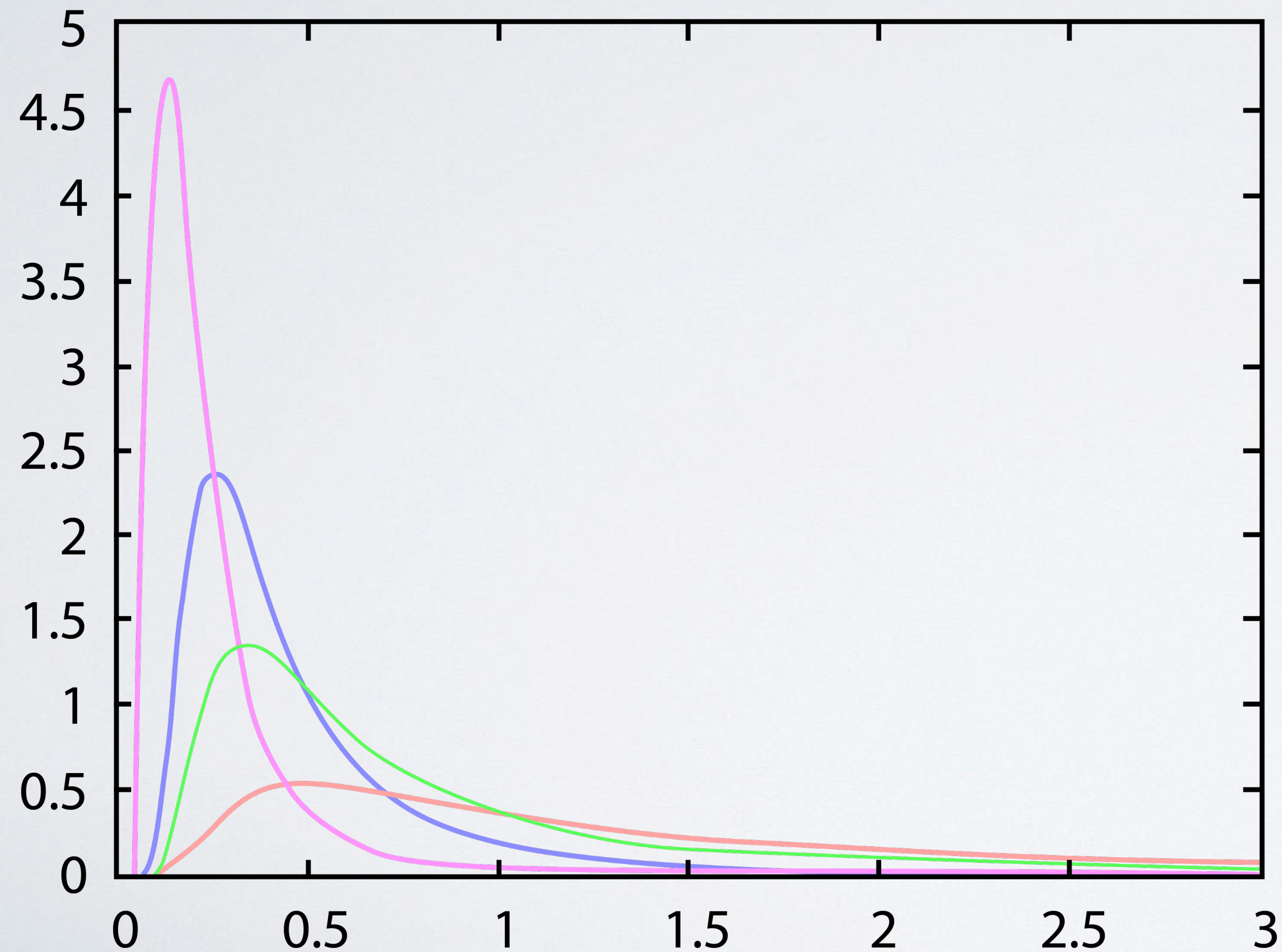
the Bayesian general

$$\lambda = 0.75$$

$$\text{std. dev.} = 1$$

gamma distribution

continuous non-negative random variable



parameters for the pdf for the gamma

$$k = 9/16 \quad \theta = 4/3$$

$$\text{mean} = k\theta = 0.75$$

$$\text{std. dev.} = \theta\sqrt{k} = 1$$

new parameters

$$x_1, x_2, \dots, x_n$$

$$k^* = k + \sum x_i$$

$$\theta^* = \frac{\theta}{(n\theta + 1)}$$

Prussian cavalry dataset

- ▶ $n = 300$ observations
- ▶ 200 casualties

$$\begin{aligned}k^* &= k + \sum_{i=1}^n x_i & \theta^* &= \frac{\theta}{n\theta + 1} \\&= (9/16) + 200 & &= \frac{4/3}{300 \times (4/3) + 1} \\&= 200.5625 & &= 0.0033\end{aligned}$$

has the general changed his mind?

	λ	uncertainty
before	0.75	1
after	0.67	0.047

summary

1. we learned about the **Poisson distribution**
2. we know the gamma-Poisson families are **conjugate**
3. we were given the formula for updating the **gamma parameters** after seeing data
4. we reanalyzed a **classic data set**