mixtures of conjugate priors and MCMC

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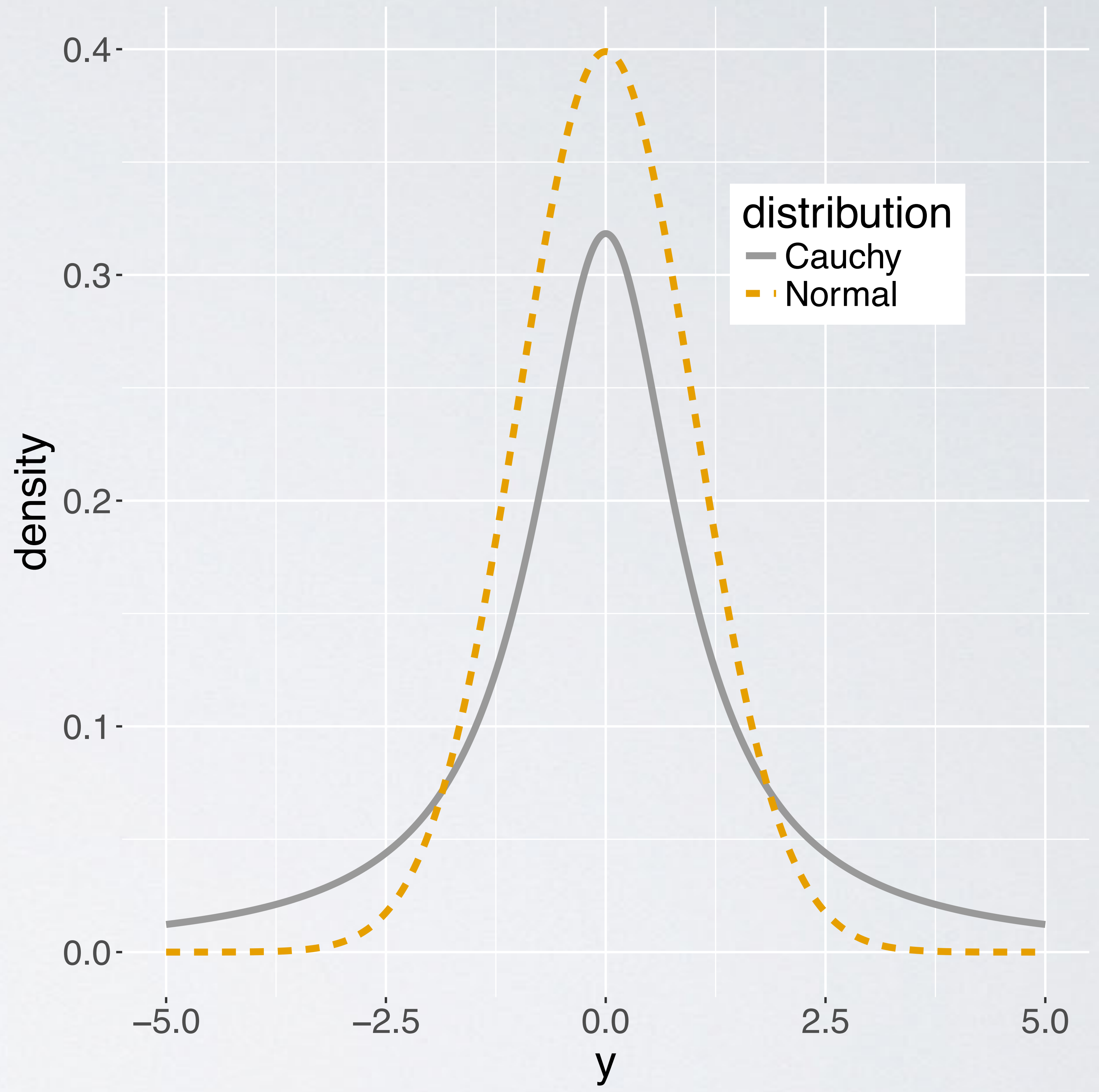
$$p(\mu \mid \sigma) = \frac{1}{\pi \sigma r}\left(1 + \frac{(\mu - m_0)^2}{\sigma^2 r^2}\right)^{-1}$$



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- Gibbs sampler or Markov chain Monte Carlo (MCMC)

MCMC algorithm

Pseudo Code

```
# initialize MCMC
sigma2[1] = 1; n_0[1]=1; mu[1]=m_0

#draw from full conditional distributions
for (i in 2:S) {
  mu[i]     = p_mu(sigma2[i-1], n_0[i-1],  m_0, r, data)
  sigma2[i] = p_sigma2(mu[i], n_0[i-1],    m_0, r, data)
  n_0[i]    = p_n_0(mu[i], sigma2[i],      m_0, r, data)
}
```

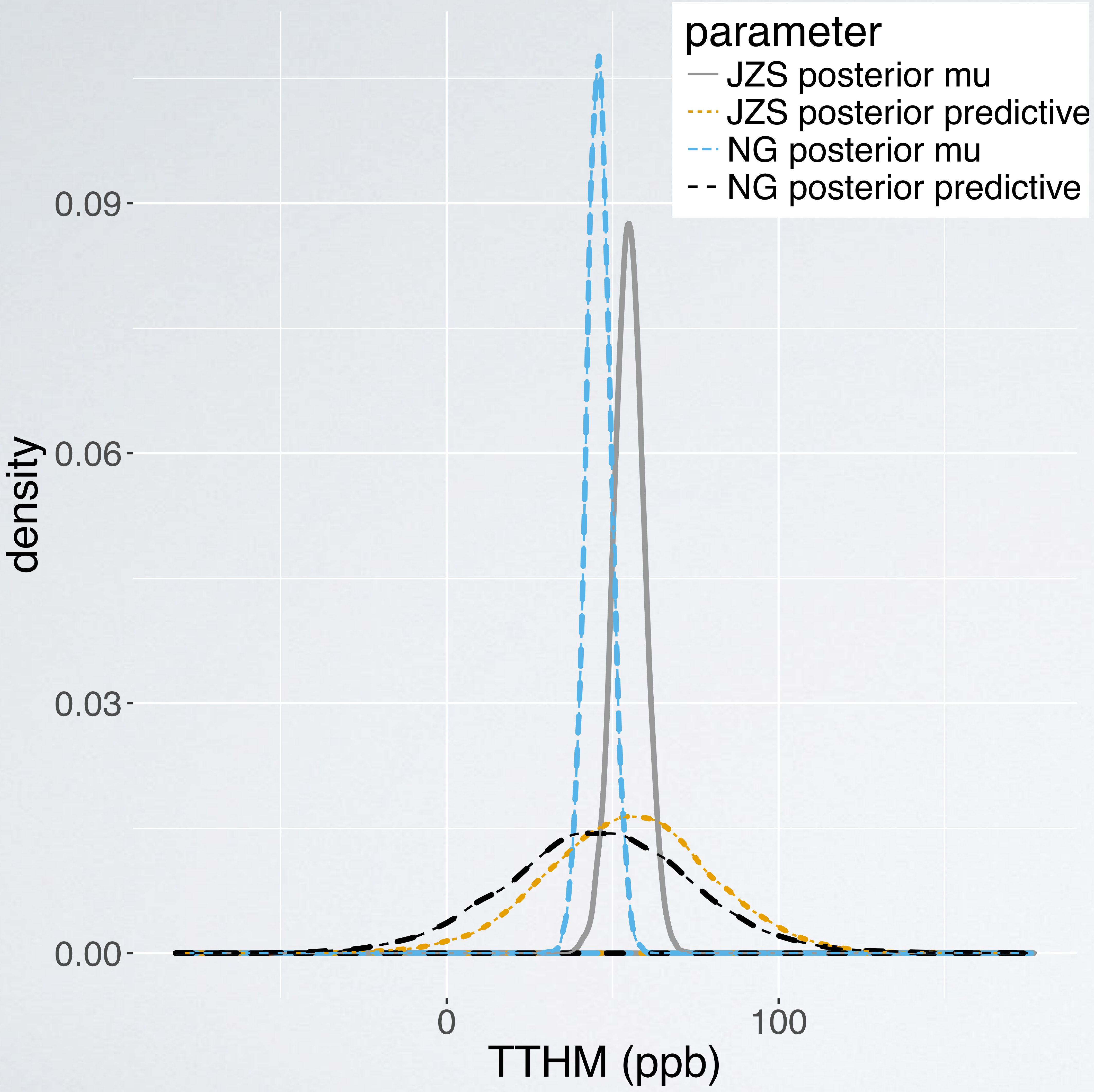
tap water example with Cauchy prior

R Code

```
bayes_inference(y=tthm, data=tapwater, statistic="mean",
                mu_0 = 35, rscale=1, prior="JZS",
                type="ci", method="sim")

## Single numerical variable
## n = 28, y-bar = 55.5239, s = 23.254
## (Assuming Zellner-Siow Cauchy prior:  mu | sigma^2 ~ C(35, 1*sigma)
## (Assuming improper Jeffreys prior: p(sigma^2) = 1/sigma^2
##
## Posterior Summaries
##              2.5%       25%      50%      75%    97.5%
## mu    45.5713714 51.820910 54.87345 57.87171 64.20477
## sigma 18.4996738 21.810376 23.84572 26.30359 32.11330
## n_0    0.2512834  2.512059  6.13636 12.66747 36.37425
##
```

comparison of posterior densities



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next: hypothesis testing and Bayes factors