bayesian & frequentist definitions of probability

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- the probability of flipping a coin and getting heads is ½
- the probability of rolling snake eyes, that is, two 1s on two dice, is 1/36
- the probability of Apple's stock price going up today is 0.75

frequentist definition of probability

$$P(E) = \lim_{n \to \infty} \frac{n_E}{n}$$

Bayesian definition of probability

- indifferent between winning
  - $1 if event E occurs, or
  - winning $1 if you draw a blue chip from a box with 1,000 × p blue chips +1,000 × (1-p) white chips
- equating the probability of event E, P(E), to the probability of drawing a blue chip from this box, p

$$P(E) = p$$

confidence intervals

Example: Based on a 2015 Pew Research poll on 1,500 adults: "We are 95% confident that 60% to 64% of Americans think the federal government does not do enough for middle class people."

- 95% of random samples of 1,500 adults will produce confidence intervals that contain the true proportion of Americans who think the federal government does not do enough for middle class people
- common misconceptions:
  - there is a 95% chance that this confidence interval includes the true population proportion
  - the true population proportion is in this interval 95% of the time

Source: http://www.pewsocialtrends.org/2016/02/04/most-americans-say-government-doesnt-do-enough-to-help-middle-class/

credible intervals

- allow us to describe the unknown true parameter not as a fixed value but with a probability distribution
- this will let us construct something like a confidence interval, except we can make probabilistic statements about the parameter falling within that range
  - **Example**: "*The posterior distribution yields a 95% credible interval of 60% to 64% for the proportion of Americans who think the federal government does not do enough for middle class people.*"
- these are called credible intervals

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