

ball A and ball B both moving initially, 1-Dimension :

“1” stands for ‘before’ collision (initial) and “2” stands for ‘after’ collision (final)

$$\frac{(m_A - m_B)v_{A1} + 2m_B v_{B1}}{m_B + m_A} = v_{A2}$$

$$\frac{(m_B - m_A)v_{B1} + 2m_A v_{A1}}{m_B + m_A} = v_{B2}$$

It's dramatically more complicated in 2-D because only momentum is conserved separately in x and y dimensions. Kinetic energy is conserved when looking at the total final  $v$  of the balls (not the  $v_x$  final and  $v_y$  final separately)



## 2-D both moving

Final V in x direction for ball A

$$V_{\text{final } X A} =$$

$$= \frac{(m_A - m_B)(v_{AX} \cos \theta + v_{AY} \sin \theta) + 2m_B(v_{BX} \cos \theta + v_{BY} \sin \theta)}{m_B + m_A} \cos \theta - (v_{AX} \sin \theta + v_{AY} \cos \theta) \sin \theta$$

Final V in y direction for ball A

$$V_{\text{final } Y A} =$$

$$= \frac{(m_A - m_B)(v_{AX} \cos \theta + v_{AY} \sin \theta) + 2m_B(v_{BX} \cos \theta + v_{BY} \sin \theta)}{m_B + m_A} \sin \theta + (v_{AX} \sin \theta + v_{AY} \cos \theta) \cos \theta$$

theta is the angle of approach to the horizontal, that is, between the line connecting the centers of mass and the horizontal (for example see next slide --->)

Final V in x direction for ball B

$$V_{\text{final } X B} =$$

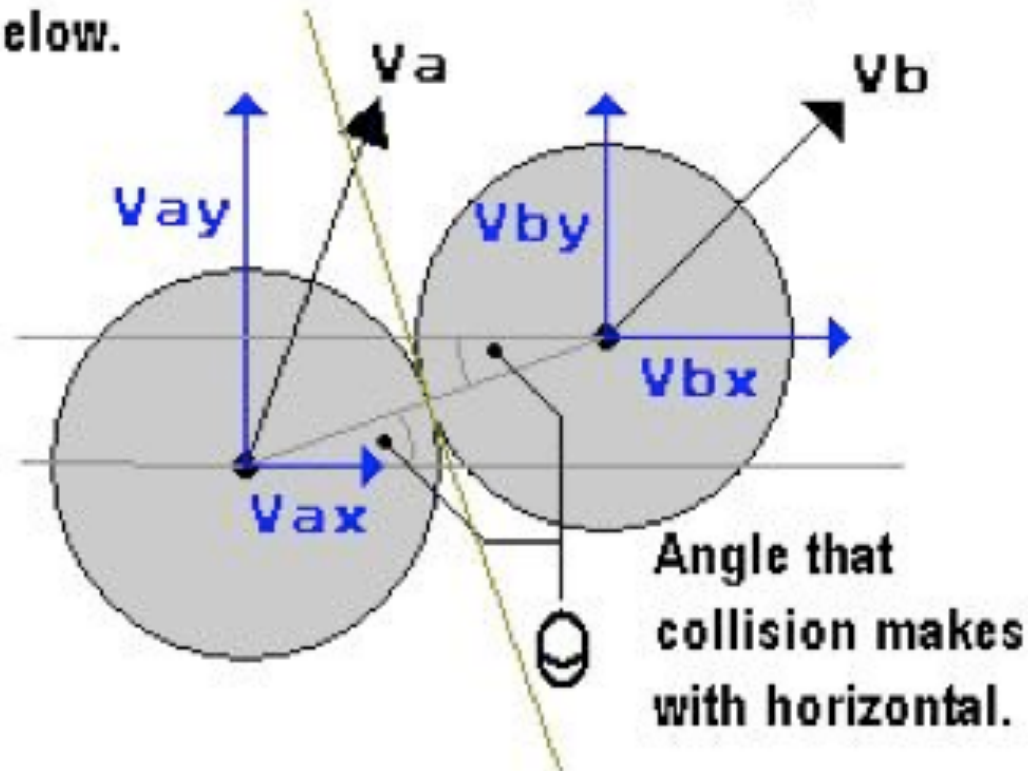
$$= \frac{(m_B - m_A)(v_{BX} \cos \theta + v_{BY} \sin \theta) + 2m_A(v_{AX} \cos \theta + v_{AY} \sin \theta)}{m_B + m_A} \cos \theta - (v_{BX} \sin \theta + v_{BY} \cos \theta) \sin \theta$$

Final V in y direction for ball B

$$V_{\text{final } Y B} =$$

$$= \frac{(m_B - m_A)(v_{BX} \cos \theta + v_{BY} \sin \theta) + 2m_A(v_{AX} \cos \theta + v_{AY} \sin \theta)}{m_B + m_A} \sin \theta + (v_{BX} \sin \theta + v_{BY} \cos \theta) \cos \theta$$

Two spheres that collide. The direction of collision is angled at the value of  $\theta$  shown below.



To get the final Total velocities for each ball, use the values obtained for the components on previous slide in pythagoras equation ( $v^2 = v_x^2 + v_y^2$ )