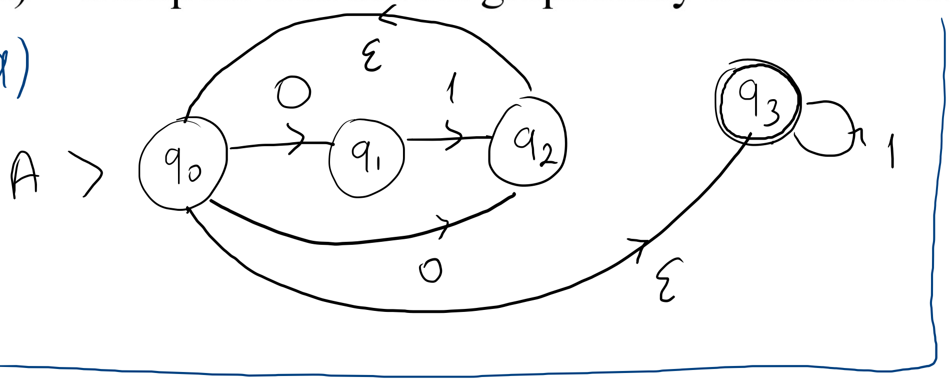
a) For the regular expression $\boldsymbol{E} = \boldsymbol{(0.1 + 0)^*1^*}$, sketch graphically an ε-NFA $\boldsymbol{A}$ with no more than 4 states that accepts the language corresponding to $\boldsymbol{E}$.

b) Compute and sketch graphically an NFA $\boldsymbol{B}$ (without ε-transitions) that is equivalent to $\boldsymbol{A}$.

c) Compute and sketch graphically a DFA $\boldsymbol{C}$ that is equivalent to $\boldsymbol{B}$.

d) Compute and sketch graphically a ***minimal state*** DFA $\boldsymbol{D}$ that is equivalent to $\boldsymbol{C}$.

a)



$ECLOSE(q_0) = \{q_0, q_3\}$

$ECLOSE(q_1) = \{q_1\}$ / $ECLOSE(q_3) = \{q_3\}$

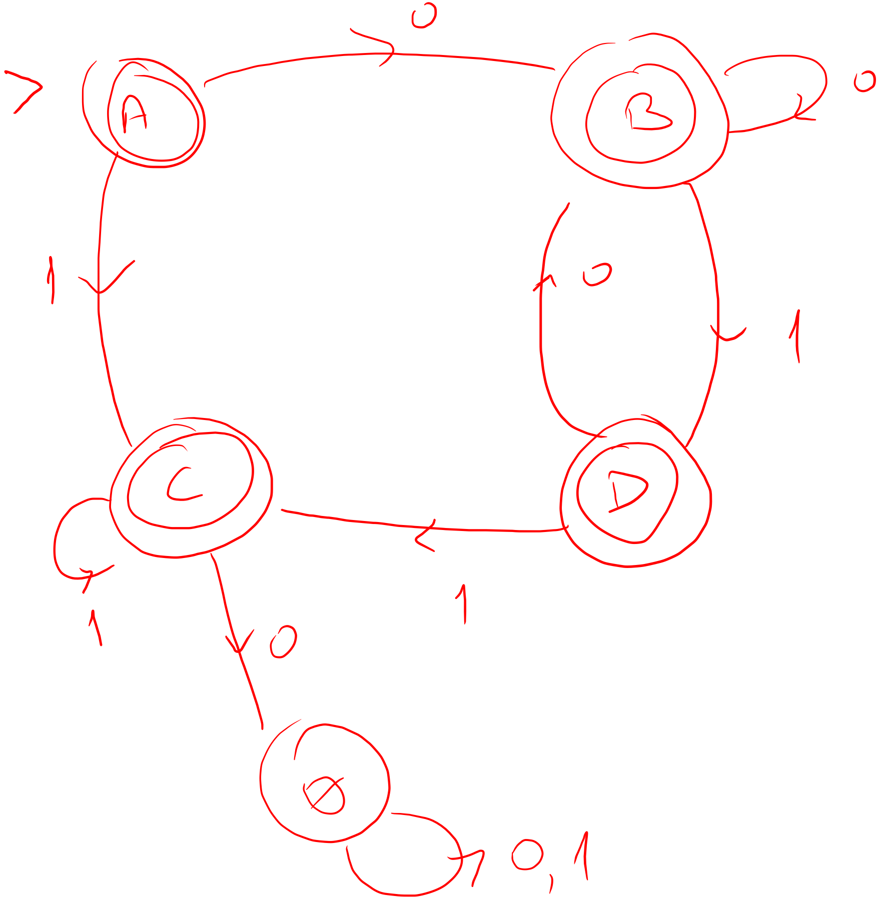
$ECLOSE(q_2) = \{q_2, q_0, q_3\}$

b)

state	input	next state
B > q_0	0	q_1, q_2, q_0, q_3
	1	q_3
q_1	0	$\emptyset$
	1	q_0, q_2, q_3
q_2	0	q_0, q_1, q_2, q_3
	1	q_3
B > $^*q_3$	0	$\emptyset$
	1	q_3

c)

state	input	next state
$\to \{q_0, q_3\}$ * A	0	q_0, q_1, q_2, q_3
	1	q_3
* B q_0, q_1, q_2, q_3	0	q_0, q_1, q_2, q_3
	1	q_0, q_2, q_3
* C q_3	0	$\emptyset$
	1	q_3
* D q_0, q_2, q_3	0	q_0, q_1, q_2, q_3
	1	q_3
$\emptyset$	0	$\emptyset$
	1	$\emptyset$

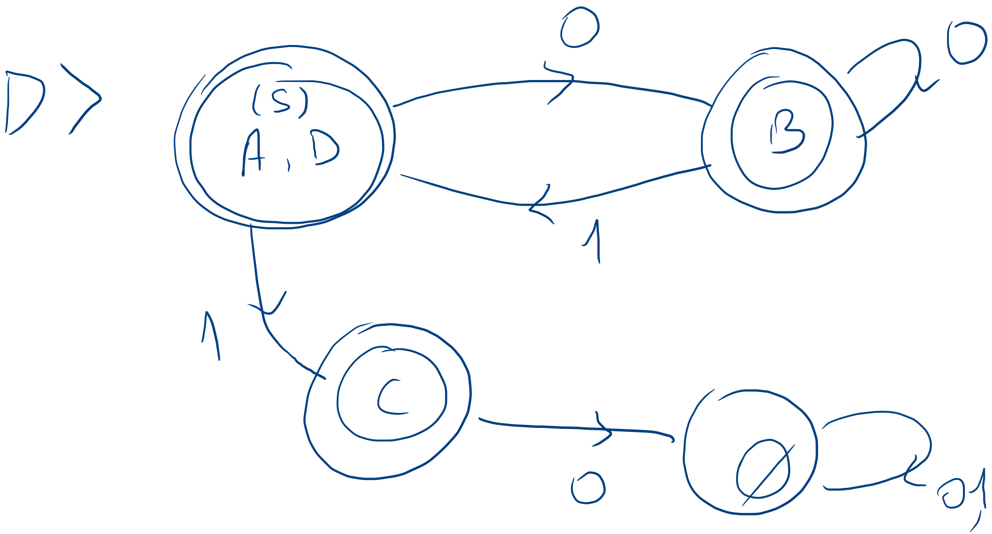


d)

	$\emptyset$	A^*	B^*	C^*	D^*
$\emptyset$		0	0	0	0
$^*A$			2	1	
$^*B$				1	2
$^*C$					1
$^*D$					

A and D are equivalent states

$S = \{A, D\}$ is the only equivalence class



Prove whether the language $L = \{w \in \{0, 1\}^* \mid w = 0^i1^j; \gcd(i, j) = 1\}$ is regular or not by either sketching an NFA that accepts it or by pumping lemma.

pumping length n;

$w = 0^n 1^{n+1}$; $|w| \geq n$, $w = x \cdot y \cdot z$.

① $|xy| \leq n$; ② $|y| > 0$; ③ $xy^iz \in L$; $\forall i \in \mathbb{N}$

$0^{n+1+q(i-1)} \cdot 1^n$

$xy = 0^p$ $y = 0^q$; $p \leq n$, $q > 0$; $z = 0^{n-p} \cdot 1^n$

$xy^iz = 0^{\cancel{p}-q} \cdot 0^{qi} \cdot 0^{n+1-\cancel{p}} \cdot 1^n = 0^{n+q(i-1)} \cdot 1^{n+1}$

$i = \frac{(n+1)!}{q} + 1$

$0^{n+(n+1)!} \cdot 1^{n+1}$

$L_2 = \{w \in \{0,1\}^* \mid \#0s > \#1s\}$

$w = 0^{n+1} \cdot 1^n$

$xy = 0^p$, $p \leq n$, $y = 0^q$, $q > 0$

$xz = 0^{n+1-q} \cdot 1^n$ $n+1-q < n+1$

$\notin L_2$ $n+1-q \leq n$ n