Prove whether the language $L = \{a^{2^n} \mid n \geq 0\}$ is regular or not.

pumping length $n > 0$ , $w = a^{2^n}$ $|w| = 2^n \geq n$

$w = x.y.z$

(1) $|xy| \leq n$ (2) $|y| > 0$ (3) $x.y^i.z \in L$ for $i \in N$

$x = a^p$ $y = a^q$ $z = a^{2^n - p - q}$ $(p+q \leq n \,,\, q > 0 \Rightarrow 0 < q \leq n)$

$xy^2z = a^{2^n + q}$

min power of 2 that is greater than $2^n$ : $2^{n+1} = 2 \cdot 2^n = 2^n + 2^n$

$0 < q \leq n \Rightarrow 2^n < \underbrace{2^n + q \leq 2^n + n < 2^n + 2^n}_{\text{Contradiction. L. is not regular.}} \quad \underbrace{(n < 2^n)}_{\text{you can prove this by induction}}$

Prove whether the language L = $\{\cancel{\;} \mid m \neq n \,;\, m, n \geq 0\}$ is regular or not.

pumping length $n \geq 0$ $\qquad 0^m 1^n$

$w = 0^n 1^{(n+a)}$, $a > 0$ $\qquad |w| = 2n + a > n$

$w = xyz$ $\quad$ (1) $|xy| \leq n$ $\quad$ (2) $|y| > 0$ $\quad$ (3) $xy^iz \in L$ for $i \in \mathbb{N}$

$x = 0^p$, $y = 0^q$, $z = 0^{n-p-q}\, 1^{n+a}$ $\quad$ ( $p+q \leq n$ , $q > 0 \Rightarrow 0 < q \leq n$ )

$xy^iz = 0^p . 0^{qi} . 0^{n-p-q} . 1^{n+a} = 0^{n-q+qi} . 1^{n+a}$

$qi - q = a$

$q(i-1) = a$

$i = \frac{a}{q} + 1$

$\hookrightarrow 0^{n - \cancel{q} + \cancel{q} + \frac{n!}{\cancel{q}} \cdot \cancel{q}}$

$w = 0^n 1^{n+n!}$ : $i = \frac{n!}{q} + 1$

$x = 0^p$, $y = 0^q$, $z = 0^{n-p-q} . 1^{n+n!}$

$xy^{\left(\frac{n!}{q}+1\right)} . z = 0^{n+n!} . 1^{n+n!}$

Contradiction. L is not regular.

$L^c$ is regular $\Leftrightarrow$ $L$ is regular

$L_1 \cap L_2$

$D_1 \times D_2$

$$A = \{0^n 1^n \mid n \geq 0\} = \underbrace{L^c}_{} \cap \underbrace{0^* 1^*}_{\text{regular}}$$

If $L^c$ is regular, $A$ is regular

$A$ is not regular proven by pumping lemma, meaning $L^c$ is not regular

meaning $L$ is not regular.

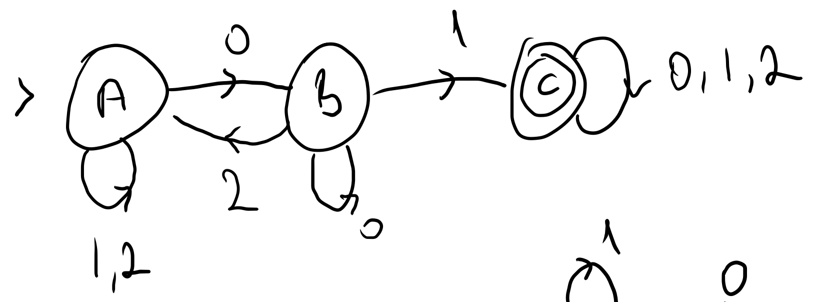
For alphabet $\Sigma = \{0, 1, 2\}$, design a DFA/NFA that accepts all strings that includes "01" but does not include "20", and minimize it.

$\delta(C1, 0) = C1$
$\delta(C2, 0) = C1$

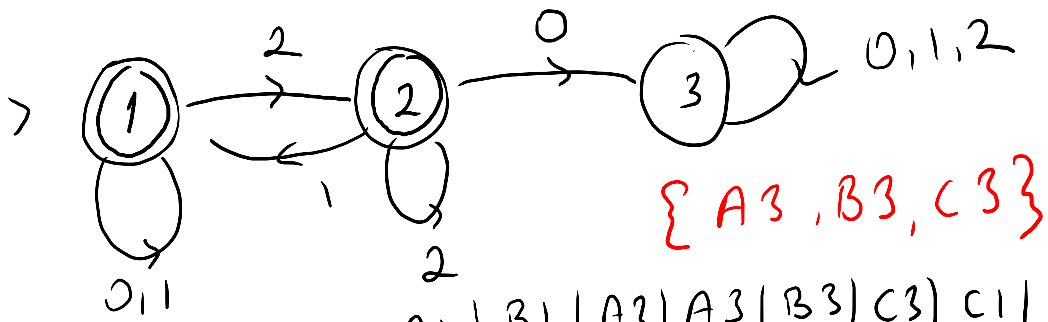
$L = L_1 \cap L_2$ , $L_1 = \{ w \in \{0,1,2\}^* \mid$ "01" is a substring of $w \}$ ,

$L_2 = \{ w \in \{0,1,2\} \mid$ "20" is not a substring of $w \}$

$D_1$



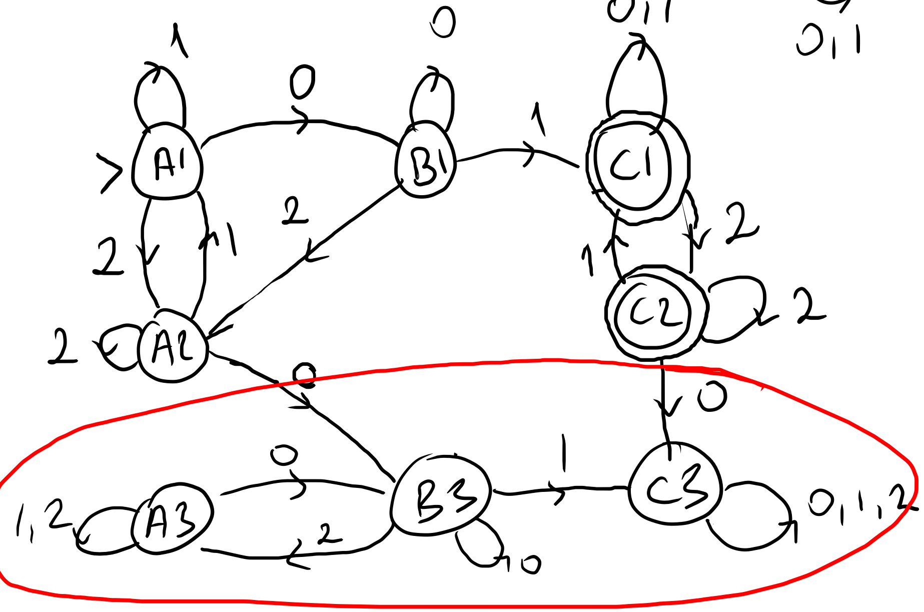

$D_2$




$\{A3, B3, C3\}$

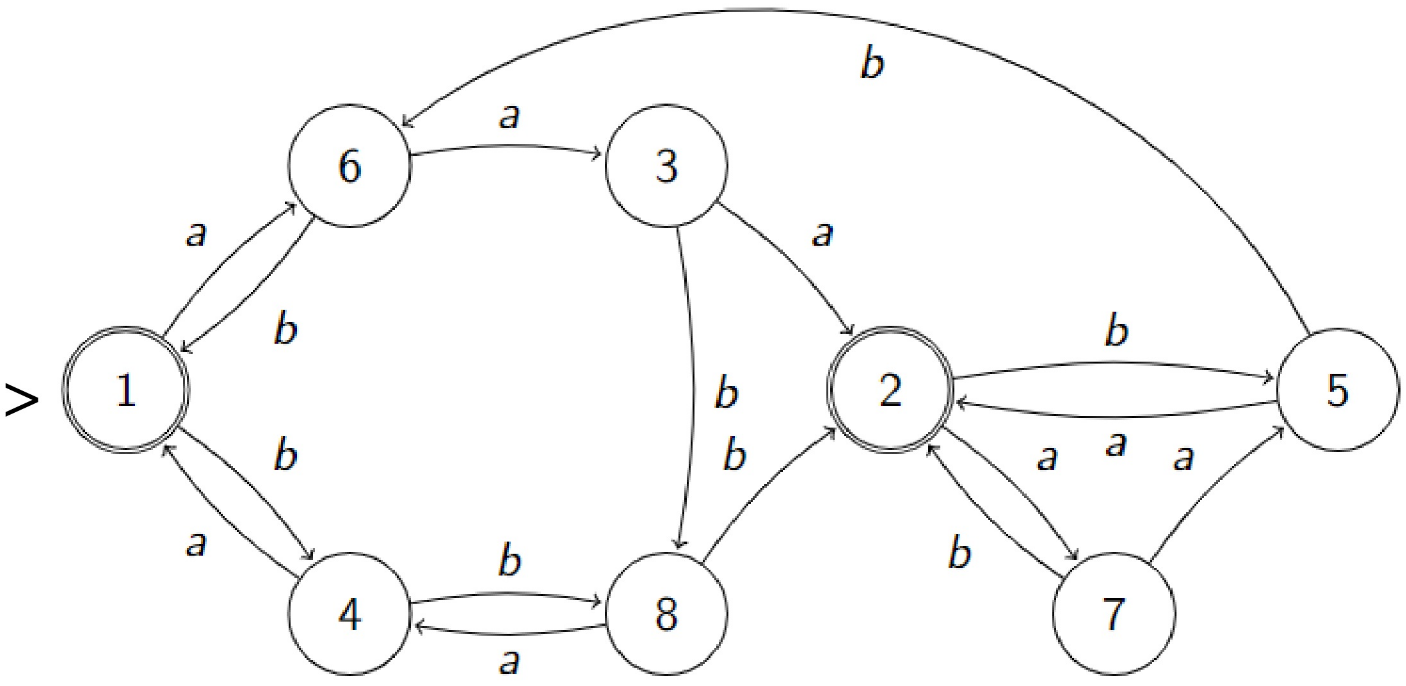
$\delta(A1, 0) = B1$
$\delta(B1, 0) = B1$
$\delta(A1, 1) = A1$
$\delta(B1, 1) = C1$

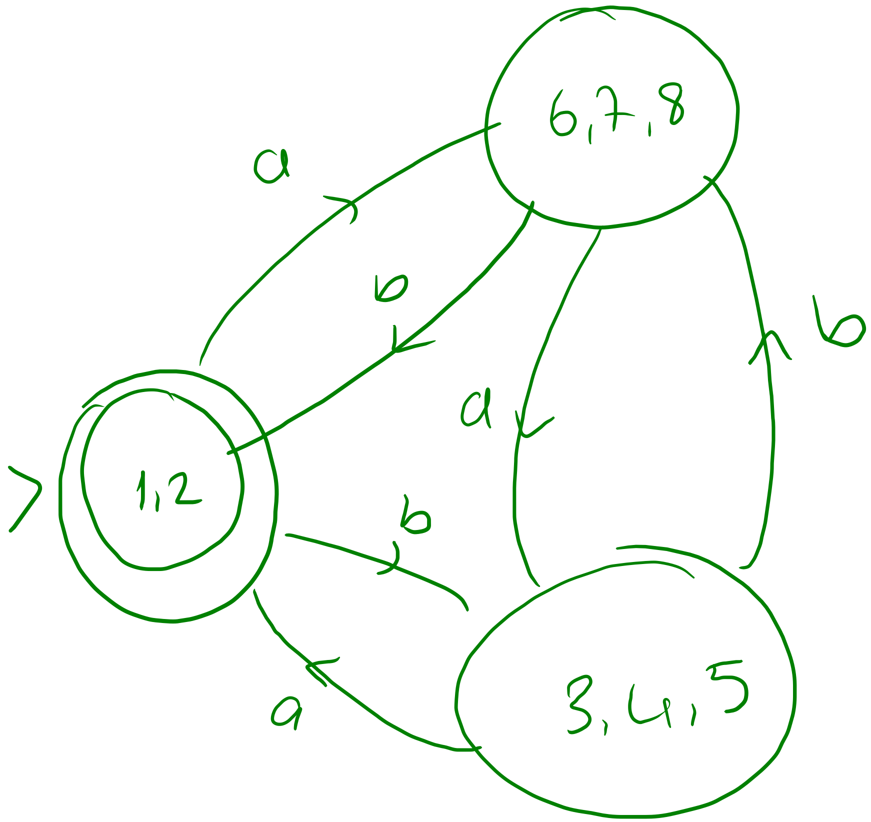
$\delta(A1, 0) = B1$
$\delta(A2, 0) = B3$
$\delta(A1, 1) = A1$
$\delta(A2, 1) = A1$
$\delta(A1, 2) = A2$
$\delta(A2, 2) = A2$




| | A1 | B1 | A2 | A3 | B3 | C3 | C1 | C2 |
|---|---|---|---|---|---|---|---|---|
| A1 | | 1 | 2 | 2 | 2 | 2 | 0 | 0 |
| B1 | | | 1 | 1 | 1 | 1 | 0 | 0 |
| A2 | | | | 3 | 3 | 3 | 0 | 0 |
| A3 | | | | | | | 0 | 0 |
| B3 | | | | | | | 0 | 0 |
| C3 | | | | | | | 0 | 0 |
| C1 | | | | | | | | 1 |
| C2 | | | | | | | | |

Minimize the given DFA.




{3,4,5}, {6,7,8}, {1,2}