Consider the language $L = \{w \in \{0, 1\}^* \mid w = 0^{2k}s;\ s \in \{0, 1\}^*;\ |s| = k\}$. State whether L is regular or not. Support your claim by either designing an NFA that accepts L or a regular expression corresponding to L; or by using pumping lemma.

pumping length n;

$w \in L$, $w = 0^{2n}1^{n}$; $|w| \geq n$ $\qquad w = x.y.z$

① $|xy| \leq n$; ② $|y| > 0$; ③ $xy^{i}z \in L \quad \forall i \in \mathbb{N}$

$x = 0^{p}$, $y = 0^{q}$; $p + q \leq n$, $q > 0$

$i = 0$: $xz = 0^{2n-q}.1^{n}$

L is not regular

$$\frac{3n-q}{3} \cdot 2 = 2n - \frac{2}{3}q \Rightarrow 2n - q < 2n - \frac{2}{3}q \ : \ xz \notin L \ \text{(Contradiction)}$$

Construct a CFG G that generates the language $L = \{w \in \{0, 1\}^* \mid w = 0^i 1^j 0^k;\ j > i + k;\ \underline{i,j,k > 0}\}$.
Then convert G into Chomsky Normal Form (CNF).

$G = (V, T, R, S)$, $V = \{S, A, B, C\}$, $T = \{0, 1\}$

$0^i 1^i \; 1^{j-i-k} \; 1^k 0^k$

R: $S \to ABC$

$A \to 0A1 \mid 01$

$B \to 1B \mid 1$

$C \to 1C0 \mid 10$

$A \to BC$ $\quad A, B, C \in V$

$A \to \sigma$ $\quad A \in V,\ \sigma \in T$

Incomplete

$(q_0, e, Z_0) \to (q_1, SZ_0)$

$(q_1, e, S) \to (q_1, ABC)$

$(q_1, e, A) \to (q_1, 0A1)$

$(q_1, e, A) \to (q_1, 01)$

$(q_1, 1, 1) \to (q_1, e)$ $\quad (q_1, e, Z_0) \to (f, Z_0)$

$(q_1, 0, 0) \to (q_1, e)$

one $\to 1$

zero $\to 0$

one $\to 1$

zero $\to 0$

$S \to ABC$

$A \to$ zero A one | zero one

$B \to$ one B | 1

$C \to$ one C zero | one zero

$\Rightarrow$

$S \to AX$, $X \to BC$

$A \to$ zero Y | zero one , $Y \to$ A one

$B \to$ one B | 1

$C \to$ one Z | one zero , $Z \to$ C zero

Construct a Deterministic Pushdown Automata (DPDA) for the language

$L = \{w \in \{0, 1\}^* \mid w = 0^n1^m;\ \underline{n \geq m} > 0\}$.

$P = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$, $Q = \{q_0, f, q_t\}$, $\Sigma = \{0, 1\}$, $\Gamma = \{0, Z_0\}$,

$F = \{f\}$

$(q_0, 0, Z_0) \rightarrow (q_0, 0Z_0)$

$(q_0, 0, 0) \rightarrow (q_0, 00)$

$(q_0, 1, 0) \rightarrow (f, e)$

$(f, 1, 0) \rightarrow (f, e)$

$(f, 1, Z_0) \rightarrow (q_t, Z_0)$ } Not a necessary transition

a) Design a TM that [redacted] decides the language $L = \{w \in \{0, 1\}^* \mid w = 0^n 1^m;\ n \geq m > 0\}$.

b) Design a TM that computes the function $(s, \underline{\#}w) \vdash^* (h, \underline{\#}w^c)$; where w^c is the compressed version of w, where all characters '\$' within w are removed. $w \in \{0,1,\$\}^*$, $w^c \in \{0,1\}^*$

$(s, \underline{\#}w)$

a)

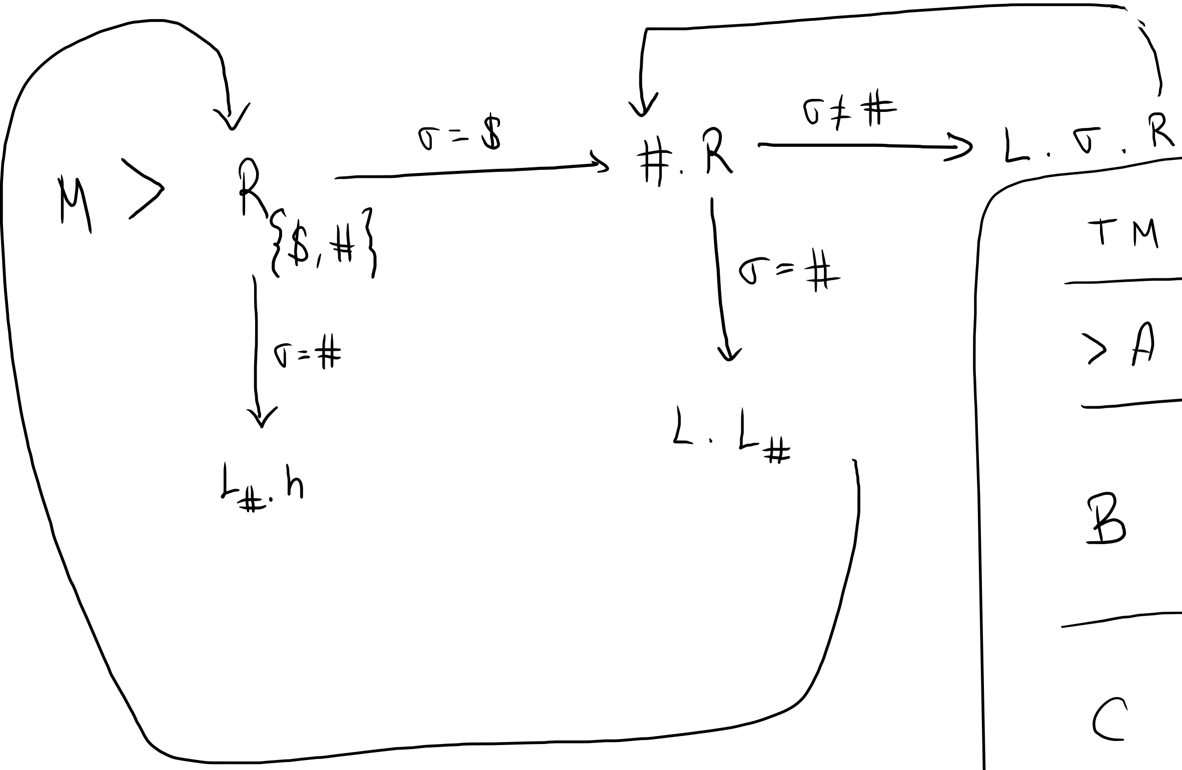
Label TM	Condition	Next TM
> A	—	R.B
B	$\sigma = 0$	$\#.R_\#.L.C$
	$\sigma = 1$	h_{no}
	$\sigma = \#$	h_{yes}
C	$\sigma = 1$	$\#.L_\#.R.B$
	$\sigma = 0$	$L_{\{1,\#\}}.D$ (B)
	$\sigma = \#$	h_{yes}

$\# \ \# \ \# \ \# \ 0 \ 1 \ \underline{0} \ \#\#\#$

D	$\sigma = 1$	h_{no}
	$\sigma = \#$	h_{yes}

b) $(s, \underline{\#} w) \vdash^* (h, \underline{\#} w^c)$

$\underline{\#} 0 1 0 \# \# \#$



TM	Condition	Next TM
> A	—	$R_{\{\$,\#\}} . B$
B	$\sigma = \$$	$\#.R.C$
	$\sigma = \#$	$L_\#.h$
C	$\sigma = \#$	$L.L_\#.A$
	$\sigma = x \neq \#$	$L.x.R.\#.R.C$