

Design a Turing Machine which would compute the following function: $(s, \diamond\#<x>, \diamond\#<y>) \mapsto (h, \diamond\#<x \sim y>, \diamond\#y)$ where $<x(y)>$ stands for the binary encoded integer $x(y)$, and $x \sim y = x - y$ if $x > y$; 0 otherwise.

TM	Condition	Next TM
>A	-	$R_{\#}^1 R_{\#}^2 . L^1 . L^2 . B$
B	$\sigma^1 \sigma^2 = \underline{00} \vee 11$	$0^1 . L^1 . L^2 . B$
	$\sigma^1 \sigma^2 = 10$	$1^1 . L^1 . L^2 . B$
	$\sigma^1 \sigma^2 = 01$	$x^1 . L^1 . L^2 . C$
	$\sigma^1 = \# \wedge \sigma^2 = \#$	h
	$\sigma^1 = \# \wedge \sigma^2 \neq \#$	D
	$\sigma^1 \neq \# \wedge \sigma^2 = \#$	$L_{\#}^1 . h$

$16 \rightarrow 2^4$
 $32 + 64 - 128$
 $+2^5 + 2^6 - 2^7 + 2^4$

01000110
 -00010010
 $\hline 01110100$

C	$\sigma^1 = 0$	$1^1 . L^1 . C$
	$\sigma^1 = 1$	$0^1 . R_{\#}^1 . 1^1 . L^1 . B$
	$\sigma^1 = \#$	$R_{\#}^1 . L^1 . E$
D	$\sigma^2 = 0$	$L^2 . D$
	$\sigma^2 = 1$	$L_{\#}^2 . R_{\#}^1 . L^1 . E$
	$\sigma^2 = \#$	h
E	$\sigma^1 \neq \#$	$\#^1 . L^1 . E$
	$\sigma^1 = \#$	$R^1 . 0^1 . L^1 . h$

Let M_1 and M_2 be single tape DTMs that decide the languages L_1 and L_2 respectively. Construct using a tabular format a 2-tape NDTM M that decides the language $L_1 \cdot L_2$.

(Assumptions: 1- M_1 and M_2 decide the languages with final configurations $(h_{yes}, \diamond \#)$ or $(h_{no}, \diamond \#)$)

2- If X and Y are DTMs that decide languages then $X.Y$ is defined as the sequential composition where control passes from X to Y *iff* X reaches the state h_{yes} .)

$(s, \diamond \# w, \diamond \#)$ $w = x.y$

TM	Condition	Next TM
$\triangleright A$	-	$R'.B$
B	$\sigma' = x \neq \#$	$R'.B$
	$\sigma' = x \neq \#$	$R^2.x^2.\#'.R'.R^2.C$
	$\sigma' = \#$	$L'_\# . D$
C	$\sigma' = x \neq \#$	$x^2.\#'.R'.R^2.C$
	$\sigma' = \#$	$L_\#^2 . L'_\diamond . R'.D$

D	-	$M_1^1 . M_2^2 . h_{yes}$

If \mathbf{M} is a Turing Machine that decides the language \mathbf{L} where the assumptions of question 2 hold, construct using a tabular format a 2-tape NDTM $\mathbf{M'}$ that decides the language $\mathbf{L^*}$.

($s, \diamond \# \omega, \diamond \#$)

TM	Condition	Next TM
$\triangleright A$	—	$R^1.B$
B	$\sigma^1 = \#$	hyes
	$\sigma^1 \neq \#$	C
C	$\sigma^1 = x \neq \#$	$R^1.C$
	$\sigma^1 = x \neq \#$	$R^2.x^2.R^1.R^2.D$
	$\sigma^1 = \#$	hno
D	$\sigma^1 = x \neq \#$	$x^2.R^1.R^2.D$
	$\sigma^1 = \#$	$L^1_{\#}.L^2_{\#}.R^1.R^2.E$
E	$\sigma^1 = \sigma^2 \neq \#$	$R^1.R^2.E$
	$\sigma^1 = \sigma^2 = \#$	$L^1_{\#}.L^2_{\#}.M^2.hyes$
	$\sigma^1 \neq \sigma^2 \wedge \sigma^2 \neq \#$	hno
	$\sigma^1 \neq \sigma^2 \wedge \sigma^2 = \#$	$L^2_{\#}.R^2.E$