Design a DPDA for the language $L = \{w \in \{a, b\}^* \mid w = a^i b^j a^k;\ i, j, k > 0;\ j = i + k\}$

$(s, a, Z_0) \to (s, aZ_0)$

$(s, a, a) \to (s, aa)$

$(s, b, a) \to (q, e)$

$(q, b, a) \to (q, e)$

$(q, b, Z_0) \to (q, bZ_0)$

$(q, b, b) \to (q, bb)$

$(q, a, b) \to (p, e)$

$(p, a, b) \to (p, e)$

$(p, e, Z_0) \to (f, Z_0)$

$F = \{f\}$

$i, j, k \geq 0$ case:

$(s, a, Z_0) \to (q, aZ_0)$

$(s, b, Z_0) \to (p, bZ_0)$

$(q, a, a) \to (q, aa)$

$(q, b, a) \to (p, e)$

$(p, b, a) \to (p, e)$

$(p, b, b) \to (p, bb)$

$(p, a, b) \to (r, e)$

$(p, e, Z_0) \to (f_0, Z_0)$

$(f_0, b, Z_0) \to (p, bZ_0)$

$(r, a, b) \to (r, e)$

$(r, e, Z_0) \to (f_1, Z_0)$

$F = \{s, f_0, f_1\}$

Design and write out in full a Turing machine that scans to the right until it finds two consecutive *a*'s and then halts. The alphabet of the Turing machine should be {a, b, #, ◇}.

$M = (Q, \Sigma, \delta, s, H)$

state	symbol under head	next state	action
q_0	◇	q_0	$\rightarrow$
q_0	#	q_0	$\rightarrow$
q_0	a	q_1	$\rightarrow$
q_0	b	q_0	$\rightarrow$
q_1	a	h	a
q_1	b	q_0	$\rightarrow$
q_1	#	q_0	$\rightarrow$

Design a Turing machine that adds 1 to the binary coded integer on its tape and then halts with initial and final IDs are as follows: (s, ◊<u>#</u><N>) |--* (h, ◊<u>#</u><N+1>) where <N> stands for the binary encoding of the integer N.

RS: $(s, \Diamond \underline{\#} w) \vdash^* (h, \Diamond \underline{\#} \# w)$

Label	Condition	Next TM
> A	–	$R_{\{\#\}} . L . B$
B	$\sigma = 1$	$0 . L . B$
	$\sigma = 0$	$1 . L_{\#} . h$
	$\sigma = \#$	$RS . R . 1 . L . h$

$(R_{\#} . 0 . L_{\#} . R . 1 . L . \#)$

Design a TM that decides the language which consists of strings where number of 0s is twice the number of 1s.

$(s, \underline{\sqcup}\#\, w)$ $\quad H = \{h_{yes}, h_{no}\}$ $\quad w \in \{0,1\}^*$

TM	Condition	Next TM
> A	–	$R_{\{0,1,\#\}} . B$
B	$\sigma = \#$	h_{yes}
	$\sigma = 0$	$x . L_{\#} . R_{\{0,1,\#\}} . C$
	$\sigma = 1$	$x . L_{\#} . R_{\{0,\#\}} . D$
C	$\sigma = \#$	h_{no}
	$\sigma = 1$	$x . L_{\#} . R_{\{0,\#\}} . E$
	$\sigma = 0$	$x . L_{\#} . R_{\{1,\#\}} . F$
D	$\sigma = \#$	h_{no}
	$\sigma = 0$	$x . L_{\#} . R_{\{0,\#\}} . E$
E	$\sigma = \#$	h_{no}
	$\sigma = 0$	$x . L_{\#} . A$
F	$\sigma = \#$	h_{no}
	$\sigma = 1$	$x . L_{\#} . A$