Convert the following grammar into CNF: G = ({S, A, B, C}, {0, 1}, R, S)

R:

S → 0A0 | 1B1 | BB

A → C

B → S | A

C → S | ε

$A \to BC \qquad B, C \in V$

$A \to a \qquad a \in T$

1) eliminate null productions

S → 0A0 | 00 | 1B1 | 11 | BB | B

A → C

B → S | A

C → S

2)

C → S → B → A
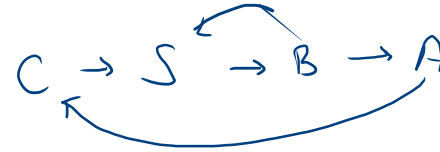
S → 0A0 | 00 | 1B1 | 11 | BB

A → 0A0 | 00 | 1B1 | 11 | BB

B → 0A0 | 00 | 1B1 | 11 | BB

C → 0A0 | 00 | 1B1 | 11 | BB

(ε, S)   (S, B)
(C, B)   (S, A)
(C, A)   (S, C)
(B, A)   (A, C)
(B, C)   (A, S)
(B, S)   (A, B)

3)

S → 0A0 | 00 | 1B1 | 11 | BB

A → 0A0 | 00 | 1B1 | 11 | BB

B → 0A0 | 00 | 1B1 | 11 | BB

zero → 0
one → 1

S → zero X | one Y | zero zero | one one | BB
A → zero X | one Y | zero zero | one one | BB
B → zero X | one Y | zero zero | one one | BB

X → A zero

Y → B one

CNF.

Convert the following grammar into CNF: G = ({S, A, B}, {a, b}, R, S)

R:

S → ASB | ϵ

A → aAS | a

B → SbS | A | bb

1) S → A SB | AB

A → aAS | a

B → SbS | A | bb

2)

B → A   (B, A)

S → A SB | AB

A → aAS | a

B → SbS | bb | aAS | a

X → b , Y → a ,

S → CB | AB

A → YC | a

B → SD | XX | YC | a

C → AS

D → XS

Determine whether the language L = {w ∈{a, b, c}* | w = $a^m b^m c^k$; m ≤ k} is context-free or not.

pumping length n : $|s| \geq n$, $s = u\,v\,w\,x\,y$

① $|vwx| \leq n$ ② $|vx| > 0$ ③ $u\,v^i\,w\,x^i\,y \in L$; $\forall i \in \mathbb{N}$

$S = a^n b^n c^n$ $|s| = 3n \geq n$

Case 1: $vx = a^q$, $q > 0$, $vx = b^q$, $vx = c^q$ $uwy = a^{n-q} b^n c^n \notin L$

Case 2: $vx = a^q b^r$, $vx = b^q c^r$ $uwy = a^n b^{n-q} c^n \notin L$

$uwy = a^{n-q} b^{n-r} c^n$ $uwy = a^n b^{n-q} c^{n-r} \notin L$ $uwy = a^n b^n c^{n-q} \notin L$

$uv^2 wx^2 y = a^{n+q} b^{n+r} c^n \notin L$ Not CFL

Determine whether the language L = {w ∈{a, b, c}* | w = $a^n b^m c^k$; k = m*n} is context-free or not.

pumping length n $|s| \geq n$, S = u v w x y

① $|vwx| \leq n$ ② $|vx| > 0$ ③ $u v^i w x^i y \in L$; $\forall i \in \mathbb{N}$

$S = a^n b^n c^{n^2}$

$vx = a^q$
$vx = b^q$ $\Rightarrow$ $uwy = a^{n-q} b^n c^{n^2} \notin L$ $(n-q) \cdot n = n^2 - qn \neq n^2$
$vx = c^q$

$\rightarrow$ $uwy = a^n b^{n-q} c^{n^2} \notin L$

$vx = a^q b^r$

$vx = b^q c^r$ $\rightarrow$ $uwy = a^n b^n c^{n^2-q} \notin L$

NOT CFL

$\downarrow$ $\rightarrow$ $uv^2 w x^2 y = a^{n+q} b^{n+r} c^{n^2} \notin L$ $(n+q)(n+r) = n^2 + qn + rn + qr \neq n^2$

$r \neq nq$
$nq \geq n$ $r < nq$

$uwy = a^n b^{n-q} c^{n^2-r} \notin L$ $n(n-q) = n^2 - nq \neq n^2 - r$

$0 < r \leq n-q < n$

Determine whether the language L = {w ∈ {a}* | w = a^p; p is prime} is context-free or not.

pumping length n $|s| \geqslant n$, $S = uvwxy$

① $|vwx| \leqslant n$ ② $|vx| > 0$ ③ $uv^iwx^iy \in L$; $\forall i \in \mathbb{N}$

$S = a^p$, $p \geqslant n$, p is a prime

$vx = a^q$, $0 < q \leqslant n$ $uv^iwx^iy = a^{\boxed{p + (i-1) \cdot q}}$

$i = p+1$ $\Rightarrow$ $p + pq = \boxed{p(q+1)}$ not prime   since $q > 0 : q \geqslant 1$

$q+1 \geqslant 2$

$uv^{p+1}wx^{p+1}y \notin L$: Not a CFL