CS 302
QUIZ 7

3 May 2023

***ANSWER** (10 pts)*

Assume L *a CFL and choose* $z = 0^n 1^{n-1} 0^n \in L$ *where* $|z| = 3n-1 > n$ *as desired . Then by the PL* $z = uvwxy$ *and by PL* $|vwx| \leq n$; $|vx| > 0$ *and* $uv^j wx^j y \in L$ *for all* $j=0,1,2,\ldots$

Hence by PL for $j=0$ *and* 2 , uwy *and* $uv^2wx^2y \in L$. *The following are possibilities for* vwx : *(i)* $vwx = 0^k$ *and* $k \leq n$; *(ii)* $vwx = 1^k$ *and* $k \leq n-1$;

(iii) $vwx = 1^i 0^j$ *or* $vwx = 0^j 1^i$ *and* $i+j \leq n$; $i \leq n-1$

If (i) holds then $uwy = 0^{n-q} 1^{n-1} 0^n$ *or* $uwy = 0^n 1^{n-1} 0^{n-q}$ *both are* **not** *in* L *since* $q > 0$!

If (ii) holds $uv^2wx^2y = 0^n 1^{n-1+q} 0^n$ *again not in* L *since* $q > 0$

If (iii) holds $uwy = 0^{n-q1} 1^{n-1-q2} 0^n$, $q1+q2 > 0$ *not in* L *unless* $q1=0$ *and* $q2 > 0$;

but then $uv^2wx^2y = 0^n 1^{n-1+q2} 0^n$ *not in* L *etc.*