Sabancı University, CS 302 AUTOMATA THEORY

Answers to Final Examination

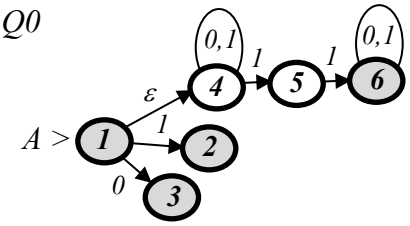
Answer 1 *(25 pts)*

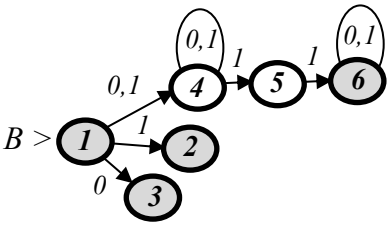
(a) *(5 pts) See the relevant slide.*

(b) *(8 pts)* ***E = e+0+1+ (0+1)*.1.1.(0+1)****

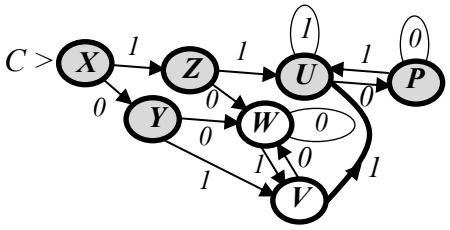
(c) *(12 pts)*

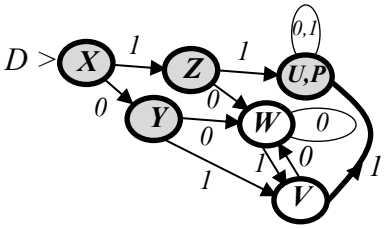
Q0





Q	***σ***	***Q'***
>X=1,4*	*0*	*Y*=4,3*
	1	*Z* =4,5,2*
*Y**	*0*	*W=4*
	1	*V=4,5*
*Z**	*0*	*W*
	1	*U*= 4,5,6*
W	*0*	*W*
	1	*V*
V	*0*	*W*
	1	*U**
*U**	*0*	*P* = 4,6*
	1	*U**
*P**	*0*	*P**
	1	*U**





	V	W	U	P	X	Y	Z
V		*1*	*0*	*0*	*0*	*0*	*0*
W			*0*	*0*	*0*	*0*	*0*
U					*2*	*1*	*1*
P					*2*	*1*	*1*
X						*1*	*1*
Y							*1*
Z							

Answer 2 *(25 pts)*

(a) *(9 pts) See the relevant slide.*

(b) *(8 pts)* ***n.m*** *= even number iff* ***n*** *OR* ***m*** *is an even number. Hence the regular expression* ***E = (0.0)* 1* + (0.0)* .0 . (1.1)*** *corresponds to* ***L*** *and hence* ***L*** *is regular.*

(c) *(8 pts) Given the constant* $N > 0$ *for the regular language we choose* $w = 0^N 1^{2N} 0^N \in L$.

Clearly $|w| = 4N > N$ *hence the hypothesis for the Pumping Lemma (PL) holds. Therefore by the PL* $w = xyz$ *and (i)* $|xy| \leq N$ *; (ii)* $|y| > 0$ *; (iii)* $xy^j z \in L$ *for* $j = 0, 1, \ldots$

Hence $xy = 0^k$, $y = 0^p$ *for* $p > 0$ *where* $k \leq N$ *and* $z = 0^{N-k} 1^{2N} 0^N$.

Therefore for $j = 0$, $x z = 0^{k-p} 0^{N-k} 1^{2N} 0^N = 0^{N-p} 1^{2N} 0^N \notin L$ *since* $p > 0$ *contradicting the PL and thus* ***L*** *is not regular.*

Answer 3 *(25 pts)*

(a) *(7 pts) See the relevant slide.*

(b) *(8 pts) Start with the grammar* $G' = (\{S, A\}, \{0, 1\}, R', S)$ *where* R' *is as below :*

$R' : S \rightarrow 0\,S\,00 \mid 0A00 \;;\; A \rightarrow 1A \mid 1$

Chomsky Normal Form steps :

1 - $S \rightarrow zero\ S\ zero\ zero \mid zero\ A\ zero\ zero \;;\; A \rightarrow one\ A \mid 1 \;;\; zero \rightarrow 0 \;;\; one \rightarrow 1$

2- CNF Grammar $G = (\{S, zero, one, A, B, C, D\}, \{0,1\}, R, S)$ *where* ***R*** *is as below :*

$R : S \rightarrow zero\ B \mid zero\ C \;;\; B \rightarrow S\ D \;;\; C \rightarrow A\ D \;;\; D \rightarrow zero\ zero$

$A \rightarrow one\ A \mid 1 \;;\; zero \rightarrow 0 \;;\; one \rightarrow 1$

(c) *(5 pts) DPDA is* $P = (\{q_0, q_1, q_2, f\}, \{0,1\}, \{0,1, Z_0\}, \delta, q_0, Z_0, \{f\})$ *where the transitions of* δ *are as below ;*

$(q_0, 0, Z_0) \rightarrow (q, 0Z_0)$

$(q, 0, 0) \rightarrow (q, 00)$

$(q, 1, 0) \rightarrow (q_1, 0)$

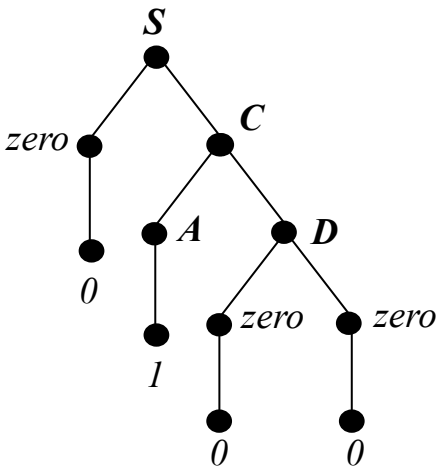
$(q_1, 1, 0) \rightarrow (q_1, 0)$

$(q_1, 0, 0) \rightarrow (q_2, 0)$

$(q_2, 0, 0) \rightarrow (q_1, e)$

$(q_1, e, Z_0) \rightarrow (f, Z_0)$

(d) *(5 pts)*



Answer 4 (25 pts)

(a) *(10 pts) See the relevant slides.*

(b) *(15 pts)*

Label	***Condition***	***TM***
M >	***-***	$R^1 A$
A	$\sigma = \#$	h_{YES}
	$\sigma \neq \# = x$	$\# R_{\#} LB$
B	$\sigma = x$	$\# L_{\#} RA$
	$\sigma = \#$	h_{YES}
	else	h_{NO}