Simplify the following regular expression:

$(aa^*(aa^*b + b) + b + aa^*b + \varepsilon)(aa^*(aa^*b + b))^*$

Let's call $aa^* = P$: $((P(Pb+b)+\varepsilon)+(Pb+b)).(P(Pb+b))^*$

Use distribution law: $(P(Pb+b)+\varepsilon).(P(Pb+b))^* + (Pb+b).(P(Pb+b))^*$

$(L+\varepsilon).L^* = L^* \Rightarrow (P(Pb+b))^* + (Pb+b).(P(Pb+b))^*$

$Pb+b = aa^*b+b = (aa^*+\varepsilon).b = a^*b$ by distribution law.

$\Rightarrow (aa^*(a^*b))^* + a^*b.(aa^*(a^*b))^* = (aa^*b)^* + a^*b.(aa^*b)^*$ $(a^*a^* = a^*)$

By distribution law: $(a^*b+\varepsilon).(aa^*b)^*$

$a^* = (aa^*+\varepsilon)$: $a^*b = (aa^*+\varepsilon).b = aa^*b+b \Rightarrow (aa^*b+\varepsilon+b).(aa^*b)^*$

$= (aa^*b+\varepsilon).(aa^*b)^* + b.(aa^*b)^* = (aa^*b)^* + b.(aa^*b)^*$ (by $(L+\varepsilon).L^* = L^*$)

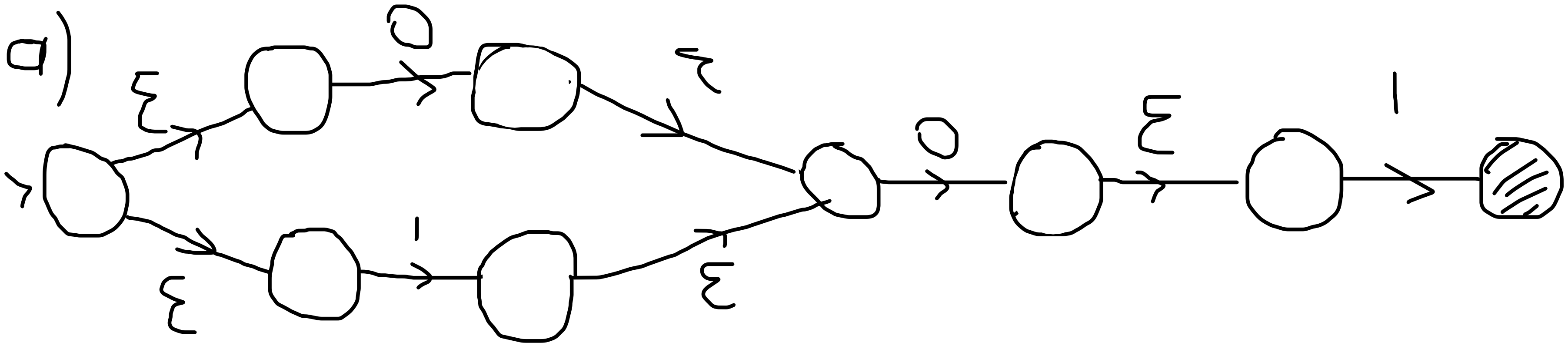
By distribution law: $\boxed{(\varepsilon+b).(aa^*b)^*}$

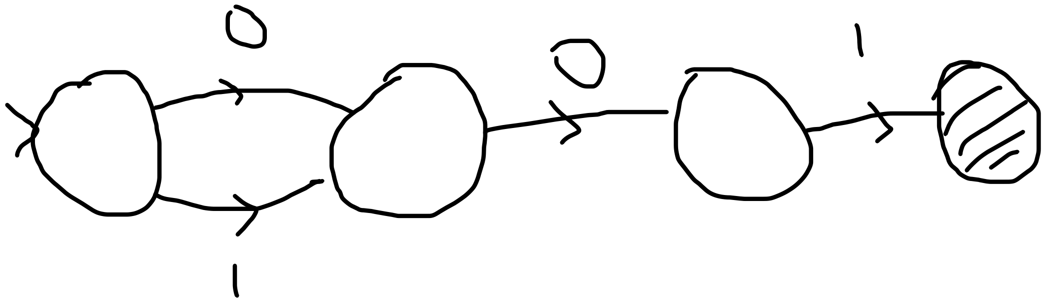
Convert the following regular expressions to ε-NFA's:

a) (0 + 1)01

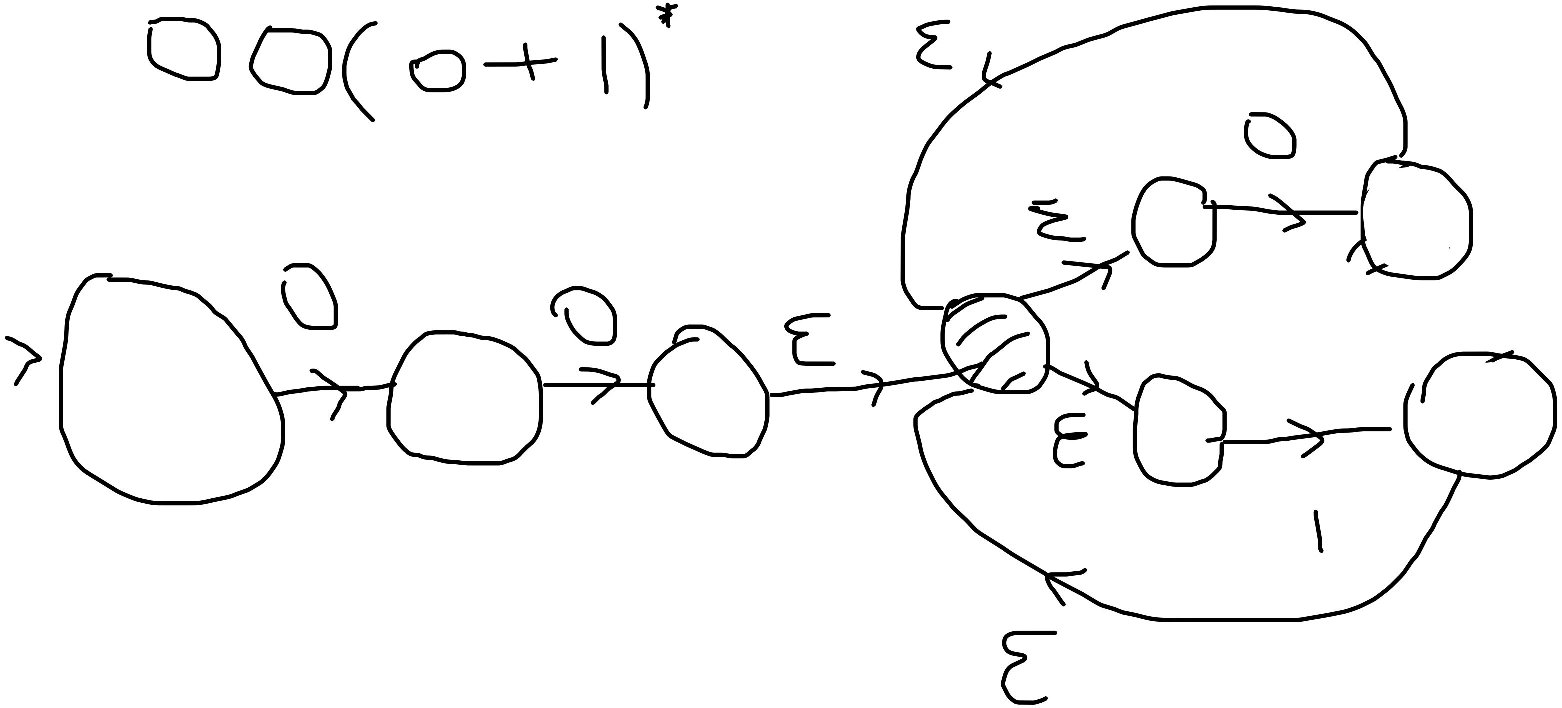
b) $00(0 + 1)^*$

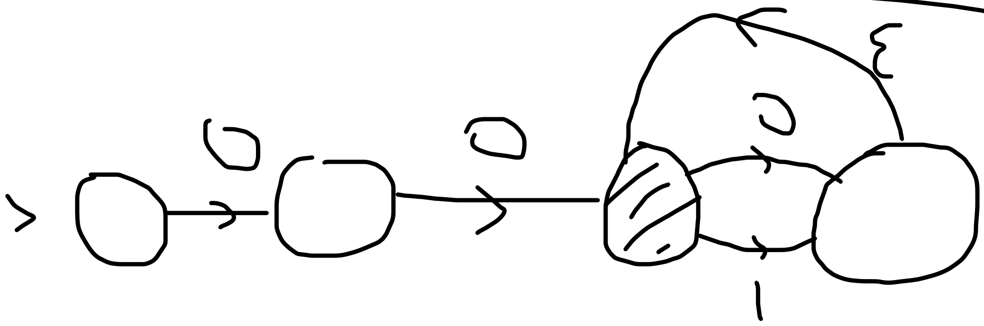
a)





$00(0+1)^*$

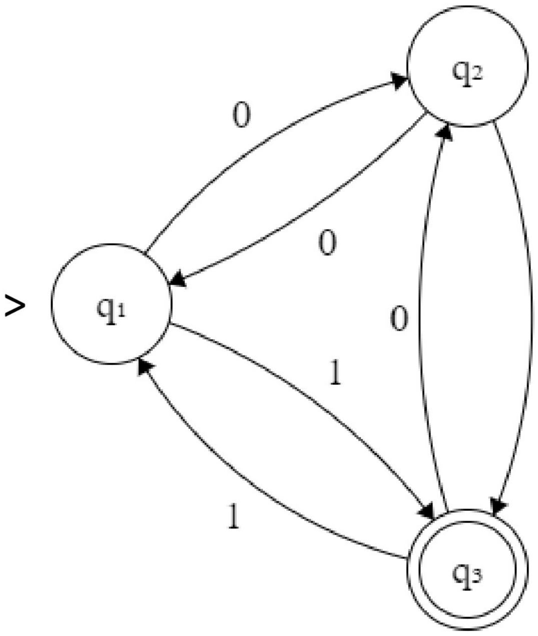




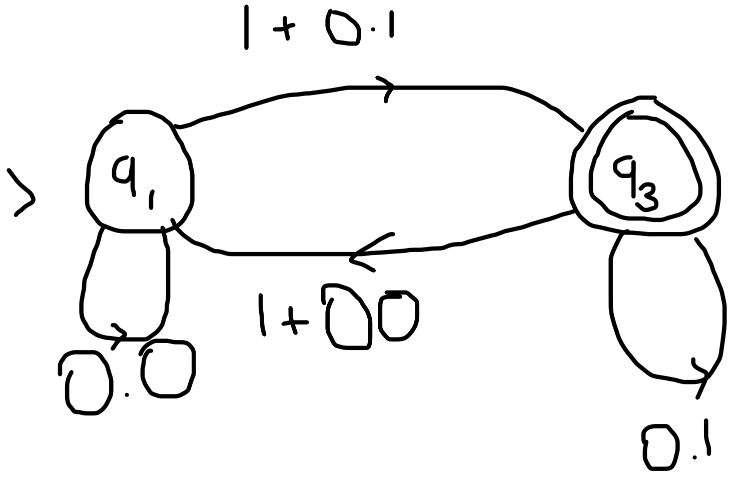
Give a regular expression for the following DFA using:

a) state elimination

b) inductive formula on R^k_{ij}



a)



$$RE: \left((00)^* \cdot (1+01) \cdot (01)^* \cdot (1+00)\right)^* \cdot \left((00)^* \cdot (1+01) \cdot (01)^*\right)$$

b) $$R^k_{ij} = R^{k-1}_{ij} + R^{k-1}_{ik} \cdot \left(R^{k-1}_{kk}\right)^* \cdot R^{k-1}_{kj}$$

$$R^3_{13} = R^2_{13} + R^2_{13} \cdot \left(R^2_{33}\right)^* \cdot R^2_{33}$$

The rest is exercise.

Prove whether the language L = {$a^m b^n c^k$ | k = m + n} is regular or not.

Pumping len: n

$w = a^n b^n c^{2n}$ $\quad |w| \geq n$

$w = x \cdot y \cdot z$

1. $|xy| \leq n$
2. $|y| > 0$
3. $x y^i z \in L$, $i = 0, 1, 2, \ldots$

$xy = a^p$, $p \leq n$

$y = a^q$, $p \geq q > 0$

$x = a^{p-q}$

$z = a^{n-p} \cdot b^n \cdot c^{2n}$

$x y^0 z \notin L$

$xz \in L$

$a^{p-q} a^{n-p} \cdot b^n \cdot c^{2n} = a^{n-q} b^n c^{2n}$

$2n - q \neq 2n$

NOT REGULAR!

Prove whether the language $L = \{w \in \{0, 1\}^* \mid w$ has an equal number of occurrences of "01"s and "10"s$\}$ is regular or not.