# Unit 1 Notes

# Module 2

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## General Notes

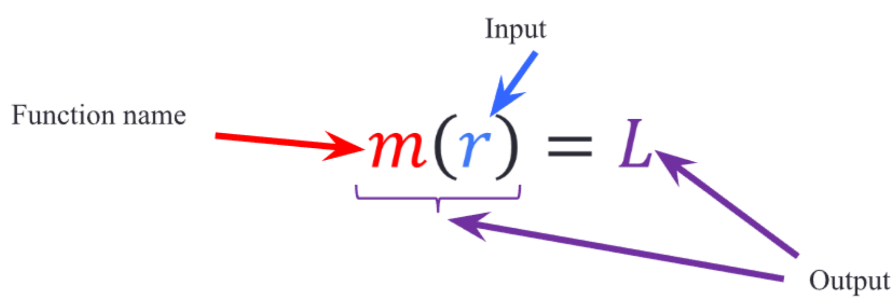
- **Quantity:** A characteristic or attribute of some object you can imagine measuring.
  - When defining or identifying a quantity, we must be specific about what object, and what specific characteristic about that object, we're referring to.
- **Variable:** A character or symbol used to represent a quantity.
- **Evaluate:** To find the output of a function corresponding to a given input.
- **Solving:** To find the input of a function corresponding to a given output
- **Domain:** the set of all reasonable inputs values of a function.
- **Range:** The set of all corresponding output values of a function
- **Interval Notation:** [0, 100]
- **Inequality:** 0 <= x <= 100

## Functions

### Function Language

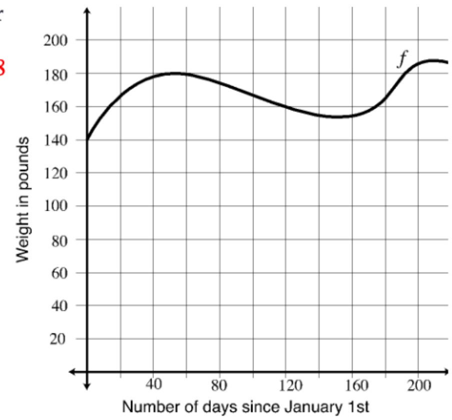
We say "*output* as a function of *input*" or "*output* in terms of *input*" - y as a function of x **or** y in terms of x - *E.g. Length of the steel band with respect to the radius of the oil drums*

If an input points to more than one output, then it is **not** a function.



**Example 1**

Suppose the following graph of the function *f* represents John's weight (in pounds) as a function of time *t*, measured in days since January 1, 2008.




a. Identify the input and output quantities for the function *f*.
input: $t$ – number of days since Jan 1, 2008
output: $f(t)$ – John's weight in pounds

b. Evaluate $f(60)$. What does this value represent in the context of the problem?
$f(60) = 180$
John's weighs 180 pounds 60 days after Jan 1, 2008.

c. Solve $f(t) = 160$ for $t$ using the graph. Describe what each solution represents.
$f(t) = 160$ when $t = 15, 119, \& 175$
Fifteen days after Jan 1, 2008, John weighed 160 pounds. John also weighed 160 pounds 119 days after Jan 1, 2008 and 175 days after Jan 1, 2008

**Example 2**

Given $m(x) = \frac{2x-3}{x+4}$,

a. Evaluate $m(4)$

$$m(4) = \frac{2(4) - 3}{4 + 4} = \frac{5}{8}$$

b. Evaluate $m(z)$

$$m(z) = \frac{2z - 3}{z + 4}$$

c. Evaluate $m(w + 2)$

$$m(w + 2) = \frac{2(w + 2) - 3}{(w + 2) + 4} = \frac{2w + 4 - 3}{w + 6} = \frac{2w + 1}{w + 6}$$

d. Solve $m(x) = 3$

$$3 = \frac{2x - 3}{x + 4}$$

$$3(x + 4) = 2x - 3$$

$$3x + 12 = 2x - 3$$

$$x = -15$$

**Example 3 - Oil Drum Problem**

## For the Oil Drum Problem

It turns out that the function that determines the length of the steel band needed to tie three oil drums of radius $r$ is

$$L = m(r) = (6 + 2\pi)r$$

- Evaluate $m(3.5)$

$m(3.5) = (6 + 2\pi)(3.5)$
$m(3.5) \approx 42.99$ feet
Solve $m(r) = 29$

- Solve $m(r) = 29$

$$29 = (6 + 2\pi)r$$

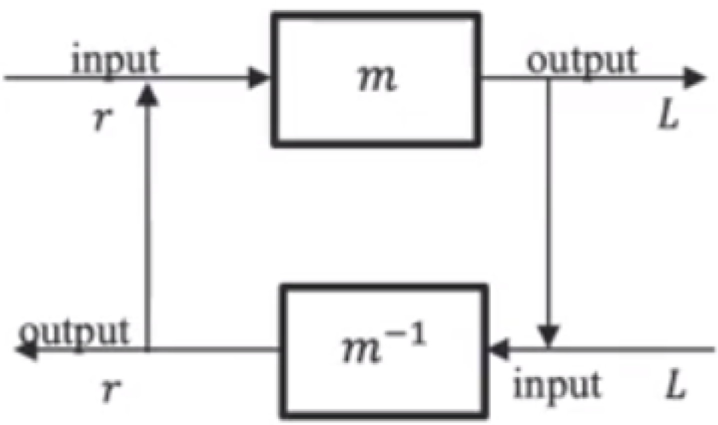
$$\frac{29}{6 + 2\pi} = r$$

$$2.36 \approx r$$

- `L` is the output
- `r` is the input

### Inverse Functions

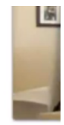
An inverse function is a function that undoes the operations of an original function.




- This function inverses the oil drum problem's function.
- The `m` is the inverse, not a negative exponent

- This image shows that `L` is a function of `r` (the name of the function is `m`) and that `r` is a function of `L` (specifically, the function is the inverse of function `m` and is written **m-1**).
- **y = f(x) means that x = f-1(y) if the inverse of f is a function.**
- The inverse of a function is not always a function.
  - *A function has one input and one output*

The formula $F = p(c) = 1.8c + 32$ will input the temperature in degrees Celsius and output the temperature in degrees Fahrenheit.



a. Find the formula for the inverse function.

$$F = 1.8c + 32$$

$$F - 32 = 1.8c$$

$$\frac{F - 32}{1.8} = c$$

Since $F = p(c)$ means $c = p^{-1}(F)$,

$$c = p^{-1}(F) = \frac{F - 32}{1.8}$$

b. Evaluate $p^{-1}(50)$ and explain its meaning in the problem context.

$$p^{-1}(50) = \frac{50 - 32}{1.8} = 10$$

Meaning: When the temperature is 50º Fahrenheit, it is 10º Celsius.

- `F` is the output
- `c` is the input
- Always write the notation indicating that the input and outputs have switched.

**Finding The Inverse**

1. Write the formula without the notation:
   1. **c = j(b) = 5b + 12**
   2. **c = 5b + 12**
2. Isolate the input until it's by itself

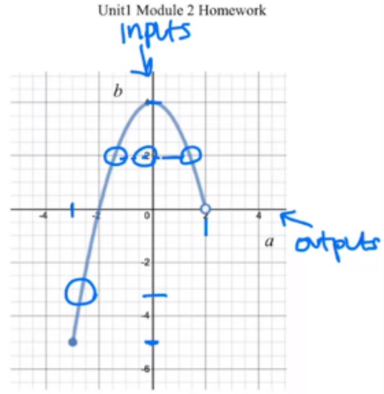
## Inverse of a Graph Example

Unit1 Module 2 Homework

10. The graph of $g$ is given to the right.
   a. Find each of the following:
      i. $g(0) = 4$   ii. $g(-1) = 3$

      iii. $a$ when $g(a) = -2$   iv. Solve $g(a) = 0$.

      $a = -2.5$   $a = -2$




   b. List the domain and range of the function.

   D: $-3 \le a < 2$   $[-3, 2)$

   R: $-5 \le g(a) \le 4$   $[-5, 4]$

   c. Is the inverse a function? Explain

   NO. For some inputs, there are 2 outputs

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## Writing Out Functions

11. Given a function $p(s) = b$ where $s$ = the number of square mile of a forest and $b$ = the number of rabbits in the forest, explain the meaning of the following.
   a. $p(100) = 85$

      # sq   # rab

      When the size of a forest is 100 sq miles, there are 85 rabbits

   b. $p(54)$

      # sq

      # rabb

      The # of rabbits in a forest that is 54 sq miles

   c. $p^{-1}(93)$

      # rabbits

      # sq mi

      The # of square miles a forest is with 93 rabbits in it.

## Domain and Range

- **Domain:** the set of all reasonable inputs values of a function.
- **Range:** The set of all corresponding output values of a function

**Example 1**

The function that determines the length of the steel band needed to tie three oil drums of radius $r$ is

$$L = m(r) = (6 + 2\pi)r$$

What would be a practical domain and range for this function? i.e., what values make sense for the radius and length of the steel band?

I would guess that the radius of an oil drum is not less than one half of a foot and probably not more than 5 feet. So the *practical domain* would be $0.5 \leq r \leq 5$ also written as $[0.5, 5]$ in interval notation.

Since a radius of 0.5 feet would require a steel band of 6.14 feet, and a since a radius of 5 feet would require a steel band of 61.42 feet, the *practical range* would be
$6.14 \leq L \leq 61.42$ also written as $[6.14, 61.42]$ in interval notation.

**Example 2**

The letter grade earned on a test with respect to the percentage grade earned.

Practical Domain: $[0, 100]$

Practical Range: $\{A, B, C, D, F\}$

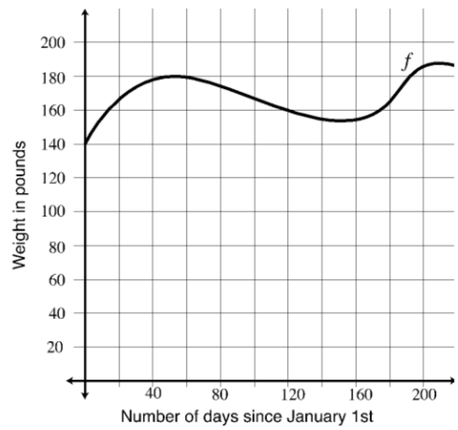
**Example 3**

Suppose the following graph of the function *f* represents John's weight (in pounds) as a function of time *t*, measured in days since January 1, 2008.

Determine the practical domain and range.

Practical Domain: $[0, 220]$

Practical Range: $[140, 190]$




## Intercepts

### Vertical Intercepts

The **Vertical Intercept** of a function is the coordinate point where the graph of the function crosses the vertical axis.

- This point will always be in the form **(0, b)**
- The vertical intercept can be found graphically by determining the point where teh graph crosses the vertical axis.
- The vertical intercept can be found in a table or algebraically by first determining the value of **b**.
  - To do this, find the output of the function for an input of 0 *(or* $\boldsymbol{f(0) = b}$*)*. You then write the intercept in the form **(0, b)**.

### Horizontal Intercepts

The **Horizontal Intercept** of a function is the coordinate point where the graph of the function crosses the horizontal axis.

- The point will always be in the form **(a, 0)**.
- The horizontal intercept can be found graphically by determining the point where the graph crosses the horizontal axis.
- The horizontal intercept can be found in a table or algebraically by first determining the value of **a**.

- To do this, find the input of the function for an output of 0 *(or solve for* $\boldsymbol{a}$ *when* $\boldsymbol{f(a) = 0}$*).* You then write the intercept in the form **(a, 0)**.

### Constant Rate of Change

$$\frac{\textit{change in output value}}{\textit{change in input value}}$$

It is said that two quantities are related by a **constant rate of change (CROC)** if the *ratio of the changes in quantities* is always the same.

- Find the changes between each value in a table for all relevant columns, and use those in this formula.

**Example 1**

| Change in time elapsed $\Delta t$ | Time Elapsed in min $t$ | Amount of water in a bath tub in gallons, $a$ | Change in amount of water in tub $\Delta a$ |
|---|---|---|---|
| | 1 | 11.75 | |
| 1.5 | 2.5 | 14.375 | 2.625 |
| 2.5 | 5 | 18.75 | 4.375 |
| 3 | 8 | 24 | 5.25 |

- The triangle_black just means change

To figure out if the ratio is the same, the changes should all equal the same number when put into the formula:

$$\frac{\Delta a}{\Delta t} = \frac{2.625}{1.5} = \frac{4.375}{2.5} = \frac{5.25}{3} = 1.75$$

- Because all the numbers equal the same, it is **constant**.

This would be written as: ***For every additional minute* that the water is left running, the *amount of water* in the bathtub *increases* by 1.75 gallons.**

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# Module 3

The Google Slides can be found Here

## General Notes

## Constant Rate of Change (Continued)

The value of the constant rate of change can always be determined by:

$$\begin{matrix}\textit{Constant Rate of Change} \\ (\textit{CROC})\end{matrix} = \frac{\textit{change in output value}}{\textit{change in input value}}$$

Knowing this info, you can also get the other values.

**Change in Output Value**

$$\begin{pmatrix}\textit{change in} \\ \textit{output value}\end{pmatrix} = (\textit{CROC}) \cdot \begin{pmatrix}\textit{change in} \\ \textit{input value}\end{pmatrix}$$

**Change in Input Value**

$$\frac{\textit{change in output value}}{\textit{CROC}} = \begin{pmatrix}\textit{change in} \\ \textit{input value}\end{pmatrix}$$

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Instead of always using the formula to find the change in input / output or the CROC, you can use repeated reasoning.

Imagine you have a pool with a hose in it filling it with water (it already has some in it). The CROC is **18.2**, and after **63 minutes**, there's **1382.6 gallons** inside. Instead of using formulas to find each value per different minute, you could create a formula:

1. Find **Δt** *(change in input value)*
   - **t - 63 = Δt**
2. Find **Δv** *(change in output value)*
   1. **18.2(Δt) = Δv**
   2. **18.2(t - 63) = Δv**
3. Find ***v*** *(total volume)*
   1. **Δv + 1382.6 = *v***
   2. **18.2(t - 63) + 1382.6 = *v***

- **Δ** means **Increment / Change**

The last one is the finished formula for a function that defines a relationship between the volume of water in the pool and the amount of time the pool has been filling. It can also be summarized as:

**Change In Output**

> **v = CROC(t - reference input) + reference Output**
> **v = CROC(t - tref) + vref**

## General Form of a Linear Function

Whenever two quantities are related by a CROC, it's a line on a graph.

- That's where the **Linear Function** comes from
- The process above can be used any time there's a CROC and a known reference point.