# Unit 1 Notes

# Module 2 - Function Review, Function Notation, Inverse Functions

The Google Slides can be
found Here

## General Notes

- **Quantity:** A characteristic or attribute of some object you can imagine measuring.
    - When defining or identifying a quantity, we must be specific about what object, and what specific characteristic about that object, we're referring to.
- **Variable:** A character or symbol used to represent a quantity.
- **Evaluate:** To find the output of a function corresponding to a given input.
- **Solving:** To find the input of a function corresponding to a given output
- **Domain:** the set of all reasonable inputs values of a function.
- **Range:** The set of all corresponding output values of a function
- **Interval Notation:** [0, 100]
- **Inequality:** 0 <= x <= 100

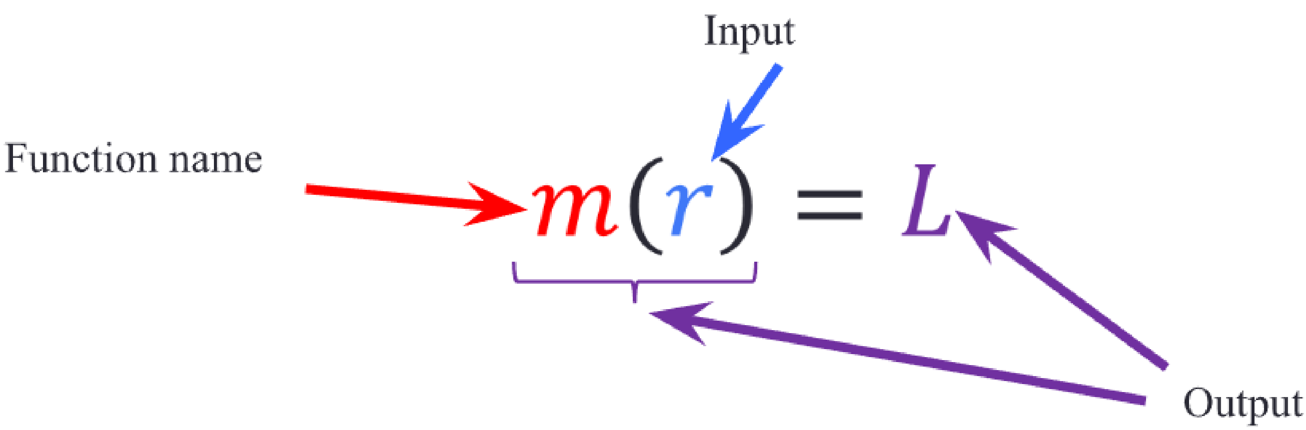
## Functions

### Function Language

We say "*output* as a function of *input*" or "*output* in terms of *input*"

- y as a function of x **or** y in terms of x
- *E.g. Length of the steel band with respect to the radius of the oil drums*

If an input points to more than one output, then it is **not** a function.



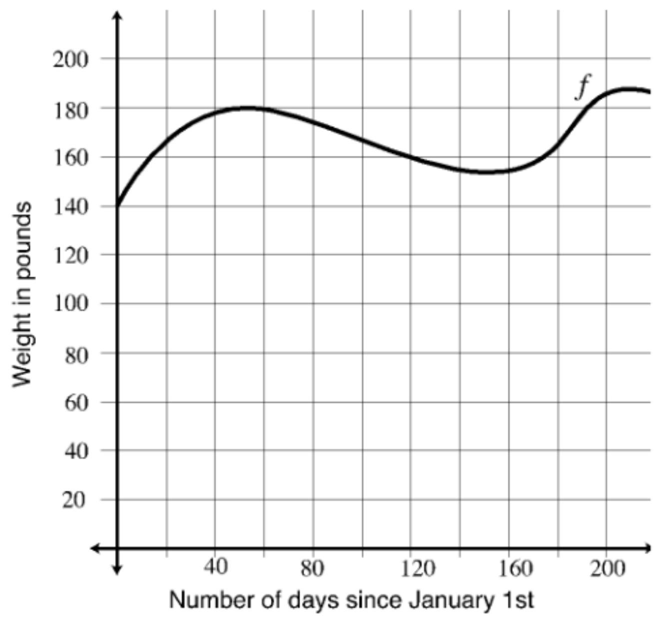

## Example 1

Suppose the following graph of the function $f$ represents John's weight (in pounds) as a function of time $t$, measured in days since January 1, 2008.

a. Identify the input and output quantities for the function $f$.
input: $t$ – number of days since Jan 1, 2008
output: $f(t)$ – John's weight in pounds

b. Evaluate $f(60)$. What does this value represent in the context of the problem?
$f(60) = 180$
John's weighs 180 pounds 60 days after Jan 1, 2008.

c. Solve $f(t) = 160$ for $t$ using the graph. Describe what each solution represents.
$f(t) = 160$ when $t = 15, 119, \& 175$
Fifteen days after Jan 1, 2008, John weighed 160 pounds. John also weighed 160 pounds 119 days after Jan 1, 2008 and 175 days after Jan 1, 2008



## Example 2

Given $m(x) = \frac{2x-3}{x+4}$,

a. Evaluate $m(4)$

$$m(4) = \frac{2(4) - 3}{4 + 4} = \frac{5}{8}$$

b. Evaluate $m(z)$

$$m(z) = \frac{2z - 3}{z + 4}$$

c. Evaluate $m(w + 2)$

$$m(w + 2) = \frac{2(w + 2) - 3}{(w + 2) + 4} = \frac{2w + 4 - 3}{w + 6} = \frac{2w + 1}{w + 6}$$

d. Solve $m(x) = 3$

$$3 = \frac{2x - 3}{x + 4}$$

$$3(x + 4) = 2x - 3$$

$$3x + 12 = 2x - 3$$

$$x = -15$$

## Example 3 - Oil Drum Problem

# For the Oil Drum Problem

It turns out that the function that determines the length of the steel band needed to tie three oil drums of radius $r$ is

$$L = m(r) = (6 + 2\pi)r$$

- Evaluate $m(3.5)$

$m(3.5) = (6 + 2\pi)(3.5)$
$m(3.5) \approx 42.99$ feet
Solve $m(r) = 29$

- Solve $m(r) = 29$

$$29 = (6 + 2\pi)r$$

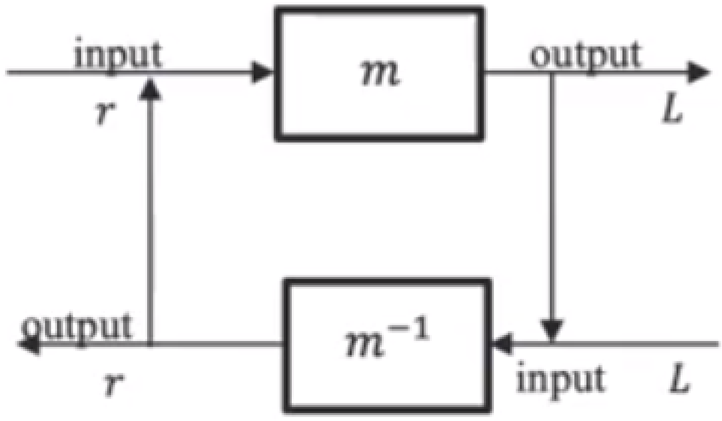
$$\frac{29}{6 + 2\pi} = r$$

$$2.36 \approx r$$

- `L` is the output
- `r` is the input

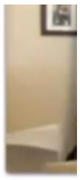
# Inverse Functions

An inverse function is a function that undoes the operations of an original function.




- This function inverses the oil drum problem's function.
- The `m` is the inverse, not a negative exponent
- This image shows that `L` is a function of `r` (the name of the function is `m`) and that `r` is a function of `L` (specifically, the function is the inverse of function `m` and is written **$m^{-1}$**).
- **y = f(x) means that x = $f^{-1}$(y) if the inverse of f is a function.**
- The inverse of a function is not always a function.
    - *A function has one input and one output*

The formula $F = p(c) = 1.8c + 32$ will input the temperature in degrees Celsius and output the temperature in degrees Fahrenheit.



a. Find the formula for the inverse function.

$$F = 1.8c + 32$$

$$F - 32 = 1.8c$$

$$\frac{F - 32}{1.8} = c$$

Since $F = p(c)$ means $c = p^{-1}(F)$,

$$c = p^{-1}(F) = \frac{F - 32}{1.8}$$

b. Evaluate $p^{-1}(50)$ and explain its meaning in the problem context.

$$p^{-1}(50) = \frac{50 - 32}{1.8} = 10$$

Meaning: When the temperature is 50° Fahrenheit, it is 10° Celsius.

- `F` is the output
- `c` is the input
- Always write the notation indicating that the input and outputs have switched.

## Finding The Inverse

1. Write the formula without the notation:
    1. **c = j(b) = 5b + 12**
    2. **c = 5b + 12**
2. Isolate the input until it's by itself

# Inverse of a Graph Example

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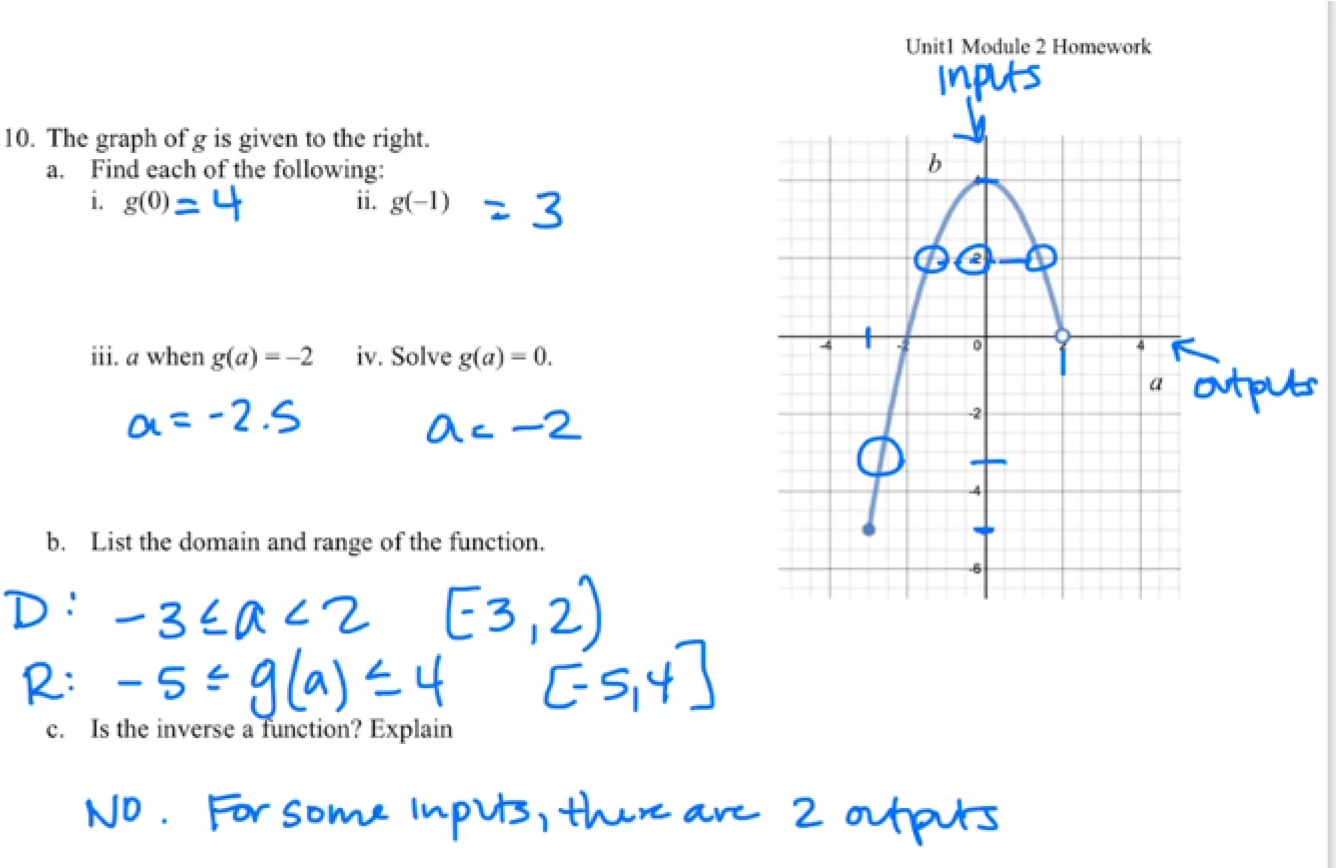


Unit1 Module 2 Homework

10. The graph of $g$ is given to the right.

a. Find each of the following:

i. $g(0) = 4$ ii. $g(-1) = 3$

iii. $a$ when $g(a) = -2$ iv. Solve $g(a) = 0$.

$a = -2.5$ $a = -2$

b. List the domain and range of the function.

D: $-3 \le a < 2$ $[-3, 2)$

R: $-5 \le g(a) \le 4$ $[-5, 4]$

c. Is the inverse a function? Explain

NO. For some inputs, there are 2 outputs



## Example problem

- *Remember not to use linear regression when creating the linear formula, instead use the* ***point-slope form: y - b = m(x - a)***

A devastating freeze in California’s Central Valley in January 2007 wiped out approximately 75% of the state’s citrus crop. It turns out that the cost for a box of oranges is a function of the percentage of the citrus crop that was frozen, i.e. **c = g(P)**, where **c** is the price of a box of oranges and **P** is the percentage of the citrus crop that was frozen. When only 20% of the crop was frozen, the price for a box of oranges was $11.58. However, thepriceperboxwas$ 25.32 when 80% of the crop was frozen.

### Finding The Linear Function

1. Identify the two given points: **(20, 11.58)** and **(80, 25.32)**
2. Calculate the slope **(m)** using the formula: **m = (y2 - y1) / (x2 - x1)**, where **(x1, y1) = (20, 11.58)** and **(x2, y2) = (80, 25.32)**

3. Plug in the values into the formula to get: **m = (25.32 - 11.58) / (80 - 20) = (13.74) / (60) = 0.229**
4. Use the point-slope form of a linear equation to find the equation of the line: **y - y1 = m (x - x1)**, where **(x1, y1) = (20, 11.58)**
5. Plug in the values into the formula to get: **y - 11.58 = 0.229 (x - 20)**
6. Rearrange the equation to the standard form: **y = 0.229x + b**, where **b** is the y-intercept
7. Calculate the y-intercept using the formula: **b = y1 - m * x1**, where **(x1, y1) = (20, 11.58)**
8. Plug in the values into the formula to get: **b = 11.58 - (0.229 * 20) = 11.58 - 4.58 = 7**
9. The equation of the line is: **y = 0.229x + 7**, which represents the cost of a box of oranges as a function of the percentage of the citrus crop that was frozen.

## Finding The Inverse Function

1. Write the original function in the form **y = f(x): c = 0.229P + 7**
2. Replace **y** with **x** and **x** with **y**: **x = 0.229P + 7**
3. Solve for **P: x - 7 = 0.229P**, then **P = (x - 7) / 0.229**
4. The inverse function is: **P = (c - 7) / 0.229**

**Summary:**

1. To find the inverse of a function, switch the roles of **x** and **y**.
2. Write the original function in the form **y = f(x)**.
3. Replace **y** with **x** and **x** with **y**.
4. Solve for the original variable (in this case, P).
5. The inverse function is the result from step 4.

*Note: The inverse of a function is not always a function. The inverse of a function is a function only if the original function is a one-to-one function.*

# Writing Out Functions

11. Given a function $p(s) = b$ where $s$ = the number of square mile of a forest and $b$ = the number of rabbits in the forest, explain the meaning of the following.

a. $p(100) = 85$

\# sq  # rab

When the size of a forest is 100 sq miles, there are 85 rabbits

b. $p(54)$

\# sq

\# rabb

The # of rabbits in a forest that is 54 sq miles

c. $p^{-1}(93)$

\# rabbits

\# sq mi

The # of square miles a forest is with 93 rabbits in it.

# Domain and Range

- **Domain:** the set of all reasonable inputs values of a function.
- **Range:** The set of all corresponding output values of a function

## Example 1

The function that determines the length of the steel band needed to tie three oil drums of radius $r$ is

$$L = m(r) = (6 + 2\pi)r$$

What would be a practical domain and range for this function? i.e., what values make sense for the radius and length of the steel band?

I would guess that the radius of an oil drum is not less than one half of a foot and probably not more than 5 feet. So the *practical domain* would be $0.5 \leq r \leq 5$ also written as $[0.5, 5]$ in interval notation.

Since a radius of 0.5 feet would require a steel band of 6.14 feet, and a since a radius of 5 feet would require a steel band of 61.42 feet, the *practical range* would be
$6.14 \leq L \leq 61.42$ also written as $[6.14, 61.42]$ in interval notation.

## Example 2

The letter grade earned on a test with respect to the percentage grade earned.

Practical Domain: $[0, 100]$

Practical Range: $\{A, B, C, D, F\}$

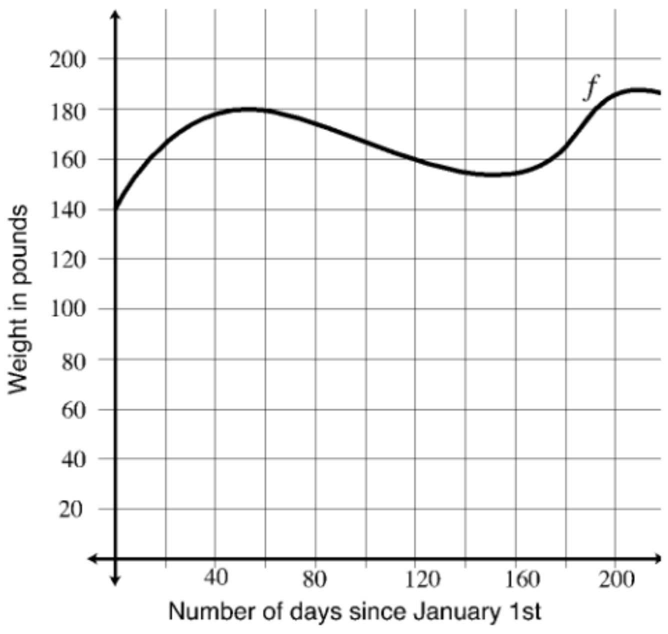
## Example 3

Suppose the following graph of the function $f$ represents John's weight (in pounds) as a function of time $t$, measured in days since January 1, 2008.

Determine the practical domain and range.

Practical Domain: $[0, 220]$

Practical Range: $[140, 190]$




# Intercepts

## Vertical Intercepts

The **Vertical Intercept** of a function is the coordinate point where the graph of the function crosses the vertical axis.

- This point will always be in the form **(0, b)**

- The vertical intercept can be found graphically by determining the point where teh graph crosses the vertical axis.
- The vertical intercept can be found in a table or algebraically by first determining the value of **b**.
    - To do this, find the output of the function for an input of 0 *(or **f(0)** = **b**)*. You then write the intercept in the form **(0, b)**.

## Horizontal Intercepts

The **Horizontal Intercept** of a function is the coordinate point where the graph of the function crosses the horizontal axis.

- The point will always be in the form **(a, 0)**.
- The horizontal intercept can be found graphically by determining the point where the graph crosses the horizontal axis.
- The horizontal intercept can be found in a table or algebraically by first determining the value of **a**.
    - To do this, find the input of the function for an output of 0 *(or solve for **a** when **f(a)** = **0**)*. You then write the intercept in the form **(a, 0)**.

# Constant Rate of Change

$$\frac{\textit{change in output value}}{\textit{change in input value}}$$

It is said that two quantities are related by a **constant rate of change (CROC)
**
if the *ratio of the changes in quantities* is always the same.

- Find the changes between each value in a table for all relevant columns, and use those in this formula.

## Example 1

| Change in time elapsed $\Delta t$ | Time Elapsed in min $t$ | Amount of water in a bath tub in gallons, $a$ | Change in amount of water in tub $\Delta a$ |
|---|---|---|---|
| | 1 | 11.75 | |
| 1.5 | 2.5 | 14.375 | 2.625 |
| 2.5 | 5 | 18.75 | 4.375 |
| 3 | 8 | 24 | 5.25 |

- The triangle just means change

To figure out if the ratio is the same, the changes should all equal the same number when put into the formula:

$$\frac{\Delta a}{\Delta t} = \frac{2.625}{1.5} = \frac{4.375}{2.5} = \frac{5.25}{3} = 1.75$$

- Because all the numbers equal the same, it is **constant**.

This would be written as: ***For every additional minute* that the water is left running, the *amount of water* in the bathtub *increases* by 1.75 gallons.**

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# Module 3 - Constant Rate of Change and Linear Functions

The Google Slides can be found [Here]

# General Notes

# Constant Rate of Change (Continued)

The value of the constant rate of change can always be determined by:

$$\begin{array}{c}\textit{Constant Rate of Change}\\ (CROC)\end{array} = \frac{\textit{change in output value}}{\textit{change in input value}}$$

Knowing this info, you can also get the other values.

## Change in Output Value

$$\begin{pmatrix}\textit{change in}\\ \textit{output value}\end{pmatrix} = (CROC)\cdot\begin{pmatrix}\textit{change in}\\ \textit{input value}\end{pmatrix}$$

## Change in Input Value

$$\frac{\textit{change in output value}}{CROC} = \begin{pmatrix}\textit{change in}\\ \textit{input value}\end{pmatrix}$$

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Instead of always using the formula to find the change in input / output or the CROC, you can use repeated reasoning.

Imagine you have a pool with a hose in it filling it with water (it already has some in it). The CROC is **18.2**, and after **63 minutes**, there's **1382.6 gallons** inside. Instead of using formulas to find each value per different minute, you could create a formula:

1. Find **$\Delta$t** *(change in input value)*
    - **t - 63 = $\Delta$t**
2. Find **$\Delta$v** *(change in output value)*
    1. **18.2($\Delta$t) = $\Delta$v**

    2. **$18.2(t - 63) = \Delta v$**
3. Find ***v*** *(total volume)*
    1. **$\Delta v + 1382.6 = v$**
    2. **$18.2(t - 63) + 1382.6 = v$**

- $\Delta$ means **Increment / Change**

The last one is the finished formula for a function that defines a relationship between the volume of water in the pool and the amount of time the pool has been filling. It can also be summarized as:

### Change In Output

> **v = CROC(t - reference input) + reference Output**
>
> **$v = CROC(t - t_{ref}) + v_{ref}$**

## General Form of a Linear Function

Whenever two quantities are related by a CROC, it's a line on a graph.

- That's where the **Linear Function** comes from
- The process above can be used any time there's a CROC and a known reference point.

# Module 4 - Linear Functions, Average Rate of Change and Linear Regression

- [Google Slide Notes]

## General Notes

General form of a linear function:

$$v = \underbrace{\text{constant rate of change}\underbrace{\left(t - \text{reference input}\right)}_{\text{change in input}}}_{\text{change in output}} + \text{reference output}$$

$$v = \underbrace{\text{CROC}\underbrace{\left(t - t_{ref}\right)}_{\text{change in input}}}_{\text{change in output}} + v_{ref}$$

When the reference point is the vertical intercept, the formula simplifies:

1. **v = CROC(t - $t_{ref}$) + $v_{ref}$**
2. **v = CROC(t - 0) + output of VI**
3. **v = CROC(t) + Output of VI** or **y = m(x) + b**

- This formula is a *special case* of a Linear Function and can only be used if the reference point is the vertical intercept.

# Average Rate of Change

When the rate is not constant, the rate is called the
**Average Rate of Change (AVROC)**.

- The average rate of change can be found between **any two points** by:
  1. finding the constant rate of change between those two points.
  2. Dividing the constant rate of change by the input.

## Example

| Number of weeks dieting | Brandon's weight in pounds |
|---|---|
| 0 | 196 |
| 2 | 187 |
| 7 | 190 |
| 12 | 184 |

- The average rate of change between week **0** and **7** is **-0.86**, but the rate does not go down constantly at this rate. Because of this, the average rate of change means in this scenario:

  > **IF** Brandon's weight had **changed by the same amount each week between week 0 and week 7**, he *would have* lost 0.86 pounds each week.

- If you needed to write this about a graph:

  > **IF** the function changed at a constant rate of change between **x = 0** and **x = 7**, we would have a line between those two points and that line *would have* a constant rate of **-0.86** (as shown in {color} on the graph).

- It's also important to use the data that is most closely related to the point in time that you are trying to find *(in this case, it would be between week **7** and **12**, not **0** and **7**)*.

The average rate of change is a *hypothetical* constant rate.

To find what the output would be at any specific non-given point, multiply the average rate of change by the change in input:

- **output = AROC(ΔInput)**

- The same as **output = (** $^{\text{output}}/_{\text{input}}$**)(Input)**

The average rate of change is calculated by dividing the changes of two outputs by the changes in the corresponding inputs. That is,

$$\text{Average Rate of Change} = \frac{\Delta\text{output}}{\Delta\text{input}} = \frac{f(b) - f(a)}{b - a}$$

where $(a, f(a))$ and $(b, f(b))$ are any two data points of the function $f$.

## Polynomial Example

**$g(x) = 2x^2 - 3x - 4$**

Determining the average rate of change between **x = 5** and **x = 1**:

1. **Average Rate of Change =** $^{g(5) - g(1)}/_{5 - 1}$
2. **$g(5) = 2(5^2) - 3(5) - 4 = 31$**
3. **$g(1) = 2(1^2) - 3(1) - 4 = -5$**
4. **Average Rate of Change =** $^{g(5) - g(1)}/_{5 - 1} = {}^{31 + 5}/_{4} = {}^{36}/_{4} = 9$

Image Reference:

$$\textit{Average Rate of Change} = \frac{g(5) - g(1)}{5 - 1}$$

$$g(5) = 2(5^2) - 3(5) - 4 = 31 \qquad g(1) = 2(1^2) - 3(1) - 4 = -5$$

$$\textit{Average Rate of Change} = \frac{g(5) - g(1)}{5 - 1} = \frac{31 - (-5)}{4} = \frac{36}{4} = 9$$

# Linear Regression

When data is not perfectly linear but is close, we create a **Linear Model**, a function that *models* the data. This is done using a process called **Linear Regression**.

How To Graph Linear Regression On Desmos

1. Click the plus button (Add an item)
2. Choose **Table**
3. Add some values (enter or copy/paste)
4. Change graph settings to better match the data.
5. Instead of $\mathbf{y_1 = mx_1 + b}$, use the **tilde (~)** symbol for the **equals (=)** sign: $\mathbf{y_1 \sim mx_1 + b}$ to use linear regression.
   - To use an exponential pattern, use: $\mathbf{y_1 \sim a * b^{x_1}}$
6. To predict a value, use either:
   1. **x = {desired x value}**
   2. **m({desired x value}) + b**

- It can be helpful to create a folder hitting the plus button, dragging the table into it, and then closing it.

## Coefficient of Determination

When computing a linear regression model, the $\mathbf{r^2}$ is known as the coefficient of determination, and it describes the strength of the fit of a linear regression model to a set of data.

**The closer the value of $r^2$ is to 1, the stronger the fit.**
$\mathbf{r^2}$ is always a value between **0** and **1**.