Unit 4 - Piecewise Functions, Function Composition, and More Function Transformations

General Notes

Module 14 - Function Transformations Part 2

Module 14 - Google Slides

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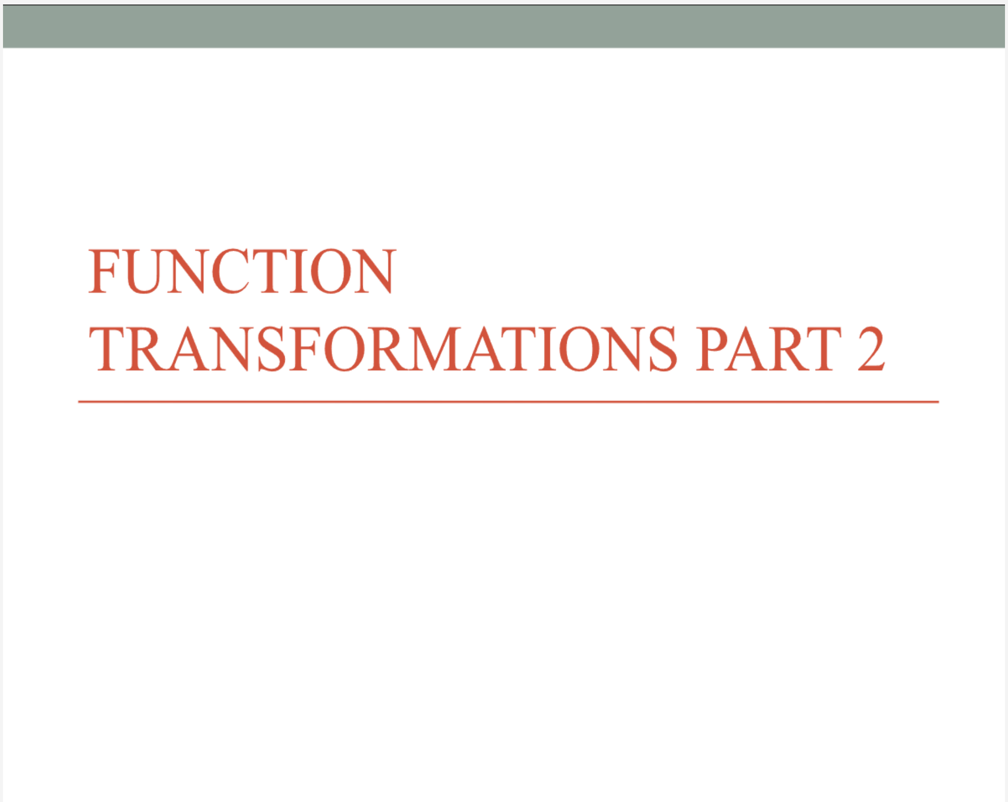
> 
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Function Transformations

The function ***g*(x) = a*ƒ*(x-c) + d** is a transformed function in terms of the parent function ***ƒ*(x)**.

The parameters **a**, **c**, and **d**
transform the function in the following ways:

Parameter	Transformation
d	**Causes a vertical shift.** • If **d > 0**, then there is a shift up. • If **d < 0**, then there is a shift down.
a	**Causes a vertical stretch or compression.** • If **|a| > 1**, then there is a vertical stretch. • If **0 < |a| < 1**, then there is a vertical compression.
c	**Causes a horizontal shift** • If **c > 0**, then the shift is to the right. • If **c < 0**, then the shift is to the left.

Identify the Transformations

Identify Transformation One

Given the parent function is ***f*(x)**, the transformations used to create the transformed function ***g*(x)** are:

> ***g*(x) = -2*f*(x) + 3**

- Vertical reflection
- Vertical stretch by a factor of 2
- Vertical shift up 3 units

Suppose that ***f*(x) = 2√x - 1**. The formula in terms of ***x*** for each of the functions is:

1. ***g*(x) = -2*f*(x) + 3**
2. ***g*(x) = -2(2√x - 1) + 3**
    - You could leave it like this, but it's better to follow the rest of the steps for readability.

3. ***g*(x) = -4√x +2 + 3**
4. ***g*(x) = -4√x + 5**

Transformation One Image Format

Given the parent function is $f(x)$, list the transformations to create the following transformed function:

$$g(x) = -2f(x) + 3$$

- Vertical reflection
- Vertical stretch by a factor of 2
- Shift up 3 units

Suppose that $f(x) = 2\sqrt{x} - 1$. Write the formula in terms of x for each of the functions.

$$g(x) = -2f(x) + 3$$
$$g(x) = -2(2\sqrt{x} - 1) + 3$$
$$g(x) = -4\sqrt{x} + 2 + 3$$
$$g(x) = -4\sqrt{x} + 5$$

Identify Transformation Two

Given the parent function is ***f*(x)**, the transformations used to create the transformed function ***k*(x)** are:

> ***k*(x) = 0.5*f*(x - 4)**

- Vertical compression by a factor of 0.5
- Horizontal shift to the right 4 units

Suppose that ***f*(x) = 2√x - 1**. The formula in terms of ***x*** for each of the functions is:

1. ***k*(x) = 0.5*f*(x - 4)**
2. ***k*(x) = 0.5(2√(x - 4) - 1)**
   - The equation becomes **x - 4** inside the radical, because the input for ***f*** is **x - 4**, not just **x**.

- The radical sign goes above both the **x** and the **4**, hence the parentheses.

3. ***k*(x) = 1√(x - 4) - 0.5**
4. ***k*(x) = √(x - 4) - 0.5**

Transformation Two Image Format

Given the parent function is $f(x)$, list the transformations to create the following transformed function:

$$k(x) = 0.5f(x-4)$$

- Vertical compression by a factor of 0.5
- Horizontal shift right 4 units

Suppose that $f(x) = 2\sqrt{x} - 1$. Write the formula in terms of x for each of the functions.

$$k(x) = 0.5f(x-4)$$
$$k(x) = 0.5\left(2\sqrt{x-4} - 1\right)$$
$$k(x) = 1\sqrt{x-4} - 0.5$$
$$k(x) = \sqrt{x-4} - 0.5$$

Identify Transformation Three

Given the parent function is ***f*(x)**, the transformations used to create the transformed function ***j*(x)** are:

> ***j*(x) = -4*f*(-x) - 0.5**

- Vertical reflection
- Vertical stretch by a factor of 4
- Horizontal reflection
- Vertical shift down 0.5 units

Suppose that ***f*(x) = 2√x - 1**. The formula in terms of ***x*** for each of the functions is:

1. ***j*(x) = -4(*f*(-x) - 0.5)**
2. ***j*(x) = -4(2√-x - 1) - 0.5**
3. ***j*(x) = -8√-x + 4 - 0.5**
4. ***j*(x) = -8√-x + 3.5**

Transformation Three Image Format

Given the parent function is $f(x)$, list the transformations to create the following transformed function:

$$j(x) = -4f(-x) - 0.5$$

- Vertical reflection
- Vertical stretch by a factor of 4
- Vertical shift down 0.5 units
- Horizontal reflection

Suppose that $f(x) = 2\sqrt{x} - 1$. Write the formula in terms of x for each of the functions.

$$j(x) = -4f(-x) - 0.5$$
$$j(x) = -4(2\sqrt{-x} - 1) - 0.5$$
$$j(x) = -8\sqrt{-x} + 4 - 0.5$$
$$j(x) = -8\sqrt{-x} + 3.5$$

Identify Graph Transformations

Identify Graph Transformation One

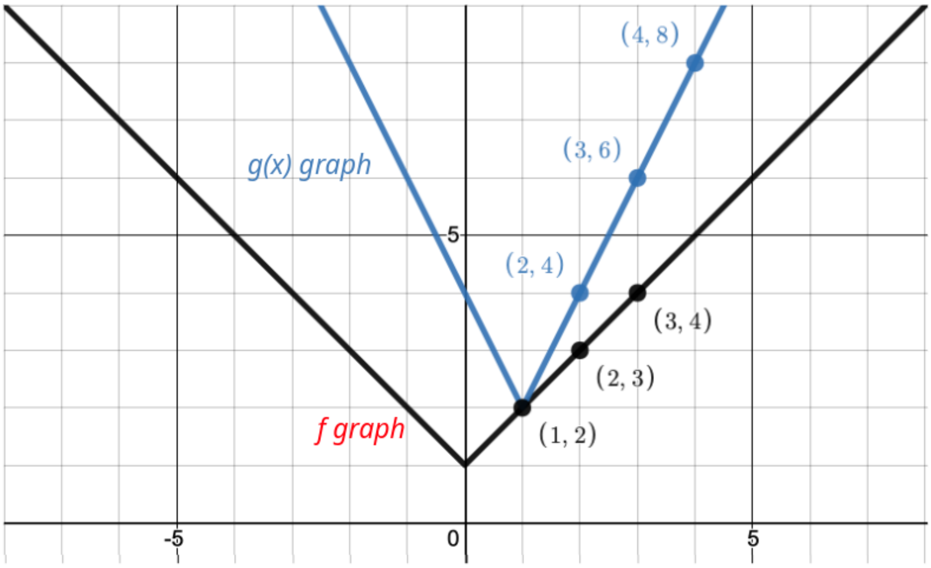
Suppose we were given the graph of the parent function ***f*** and asked to graph the transformed function:

> **g(x) = 2*f*(x — 1)**

The graph:

input	output
1	2
2	3
3	4

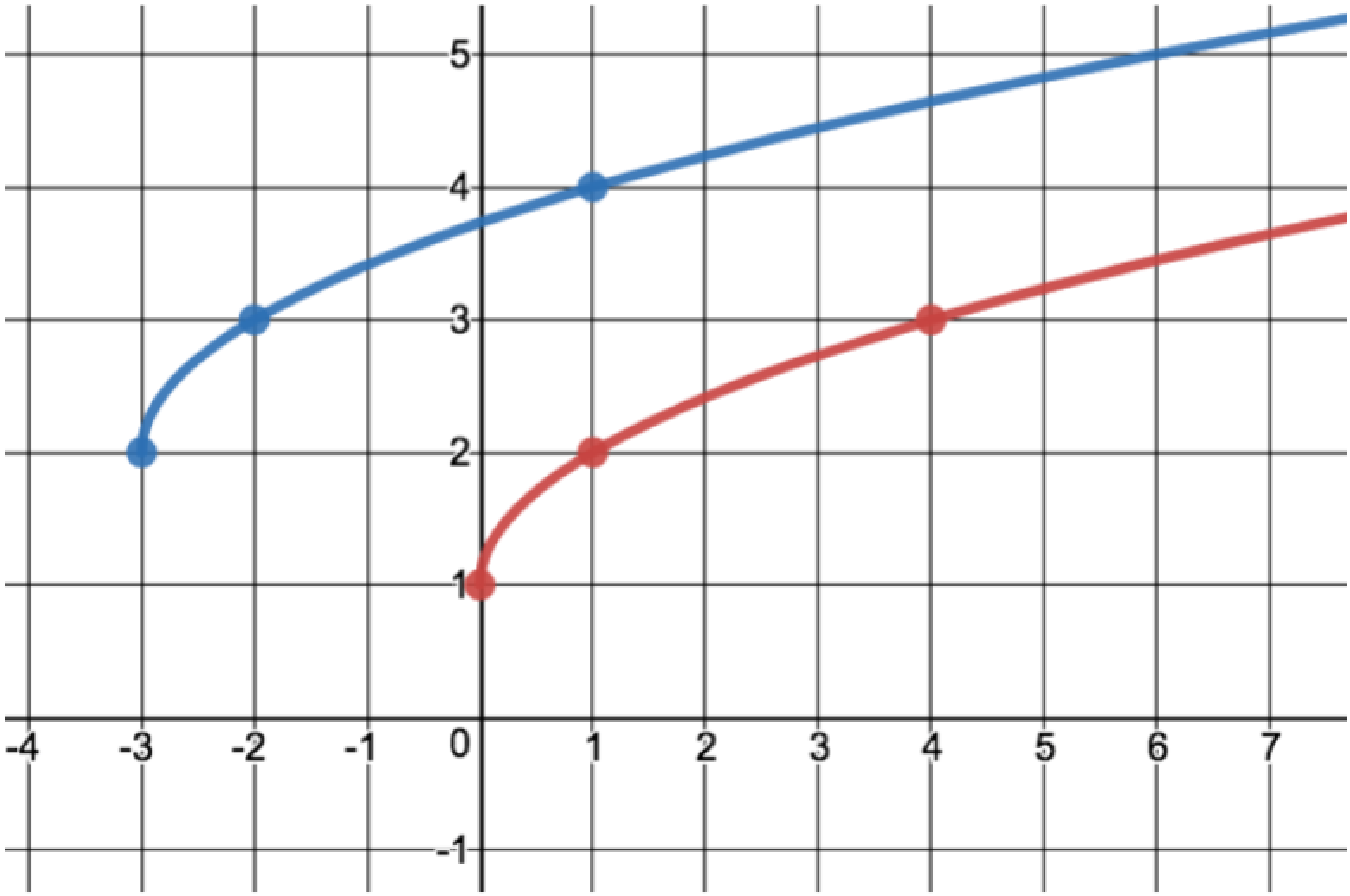
input	output
2	4
3	6
4	8



- The graph was shifted to the right 1 unit, and stretched vertically by a factor of 2.
    - The **x** inputs were shifted to the right 1 unit.
    - The **y** outputs were stretched vertically by a factor of 2.

Identify Graph Transformation Two

Suppose we were given the graph of the parent function ***r*** (in red) and the transformed function ***b*** (in blue) and asked to identify the transformations:



Based on the graph, the formula of ***b*** in terms of ***r*** is:

> ***b*(x) = r(x + 3) + 1**

To identify the transformations:

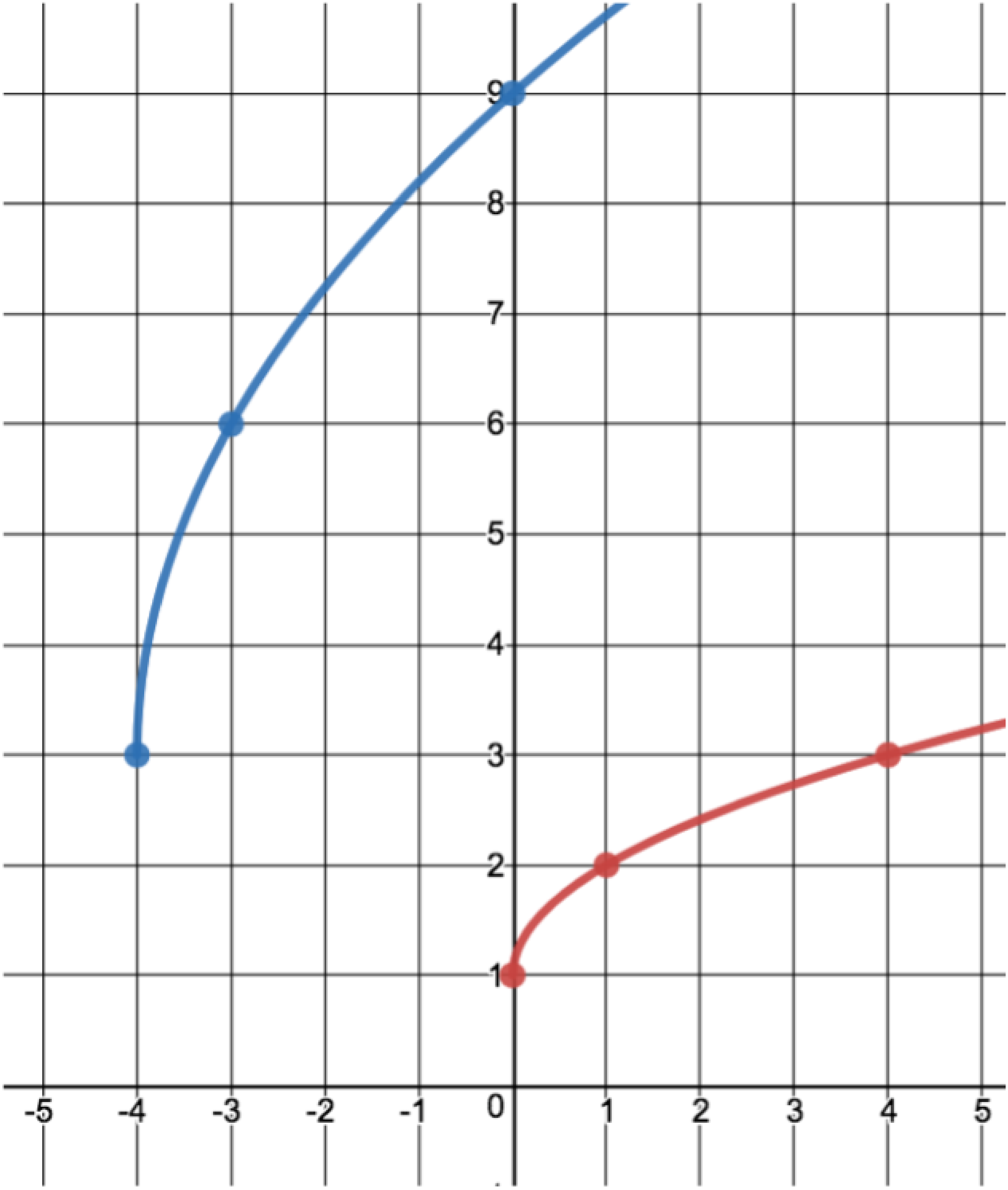
- **Vertical stretch or compression**
  1. Look for a change in outputs from on set of coordinates to the next on both graphs. If the parent function increases by 1 unit on both the *x* and *y* axes, but the transformed function increases by more than 1 unit, there is either a stretch or compression.
- **Horizontal shift**
  1. Look for how far the transformed function is shifted horizontally from the parent function.
- **Vertical shift**
  1. Look for how far the transformed function is shifted vertically from the parent function.
- **Vertical reflection**
  1. Look for a change in the vertical direction of the graph.

2. If the parent function is going one way vertically and the transformed function is vertically going the opposite way, there is a vertical reflection.

- **Horizontal reflection**
  1. Look for a change in the horizontal direction of the graph.
  2. If the parent function is going one way horizontally and the transformed function is horizontally going the opposite way, there is a horizontal reflection.

Identify Graph Transformation Three

Suppose we were given the graph of the parent function ***r*** (in red) and the transformed function ***b*** (in blue) and asked to identify the transformations:



Based on the graph, the formula of ***b*** in terms of ***r*** is:

> ***b*(x) = 3r(x + 4)**

To identify the transformations:

- **Vertical stretch**
    1. The parent function increases by 1 unit on both the *x* and *y* axes, but the transformed function increases by 3 units on the *y*-axis and 1 unit

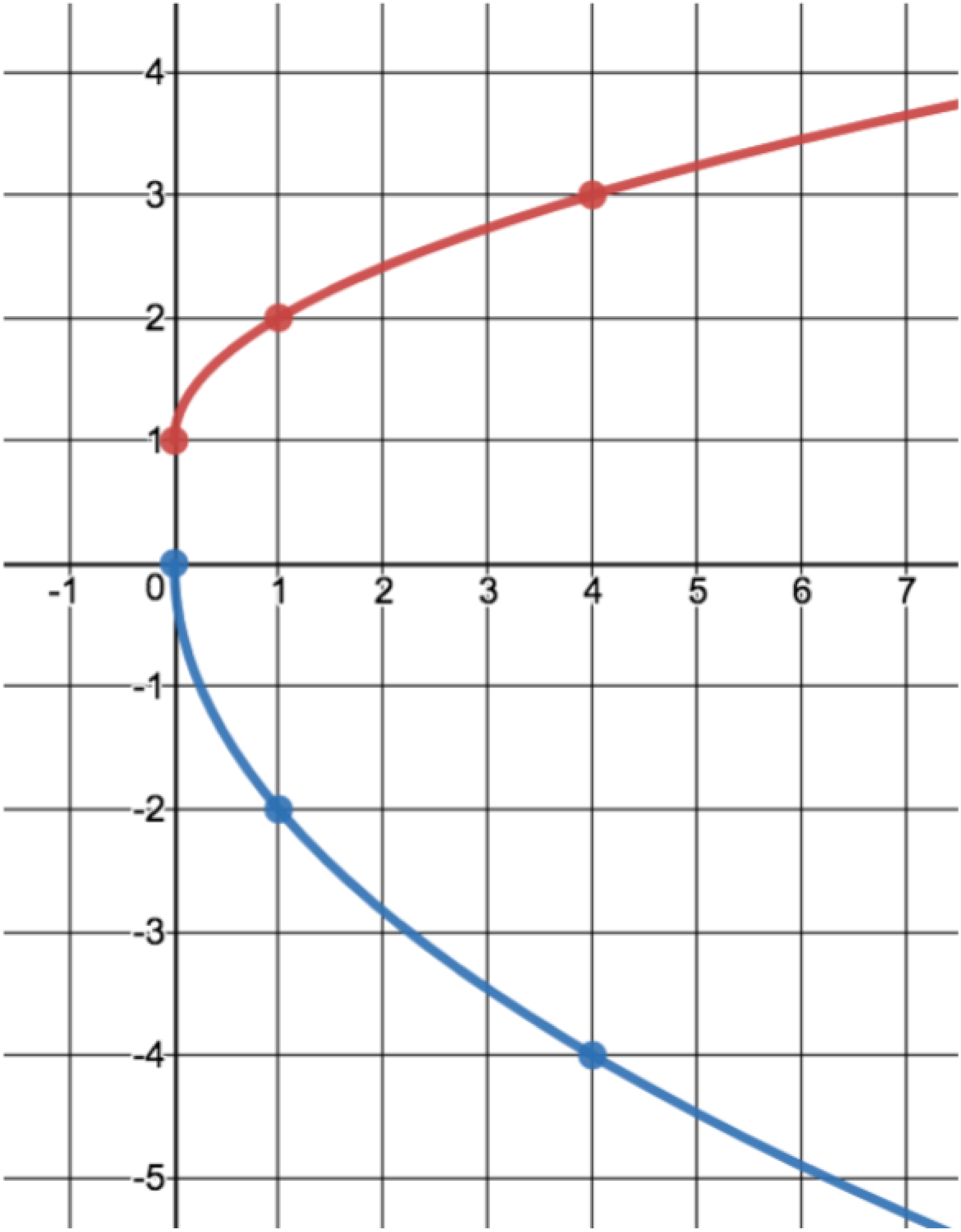
    on the *x*-axis.

    2. The increase in the transformed function is **3** times more than the increase in the parent function.
    3. The parent function starts at **(0, 1)**, but if it were stretched by a factor of **3**, then the transformed function would start at **(0, 3)
    **.
        - You can make sure that the predicted stretch is correct by either drawing out the graph or by checking the other points and making sure that they all align with what the y-values should be. *Use a table if necessary*.
    4. It's being stretched by a factor of **3**.
    5. It's also helpful to draw out the stretched graph to make sure that it matches the transformed graph.

- **Horizontal shift**
    1. The transformed function is shifted to the left by **4** units.

Identify Graph Transformation Four

Suppose we were given the graph of the parent function ***r*** (in red) and the transformed function ***b*** (in blue) and asked to identify the transformations:



Based on the graph, the formula of ***b*** in terms of ***r*** is:

> ***b*(x) = -2r(x) + 2**

To identify the transformations:

- **Vertical reflection**
    1. The parent function is going one way vertically and the transformed function is vertically going the opposite way, there is a vertical

reflection.

- **Vertical stretch**
    1. The transformed function decreases at twice the amount of the parent function.
- **Vertical shift**
    1. The transformed function is shifted up by **2** units.

Horizontal Reflections

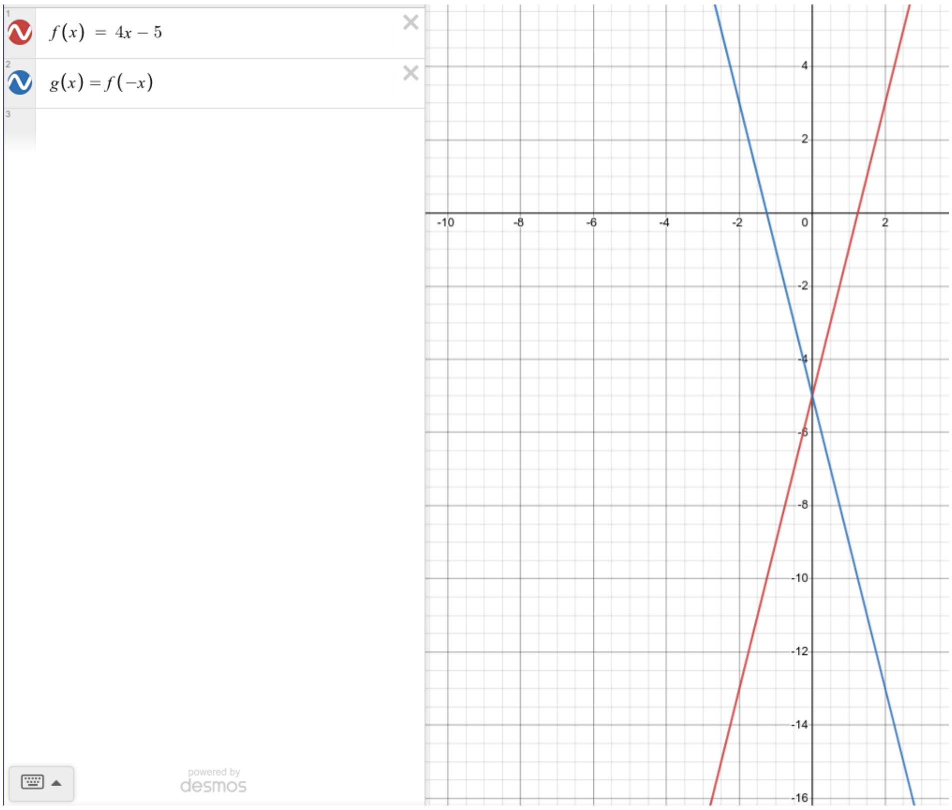
Suppose that the transformed function ***g*(x)** is:

> ***g*(x) = *f*(-x)**

Now suppose that the parent function ***f*** is:

> ***f*(x) = 4x - 5**

The Result:



In general, think of horizontal reflections as follows:

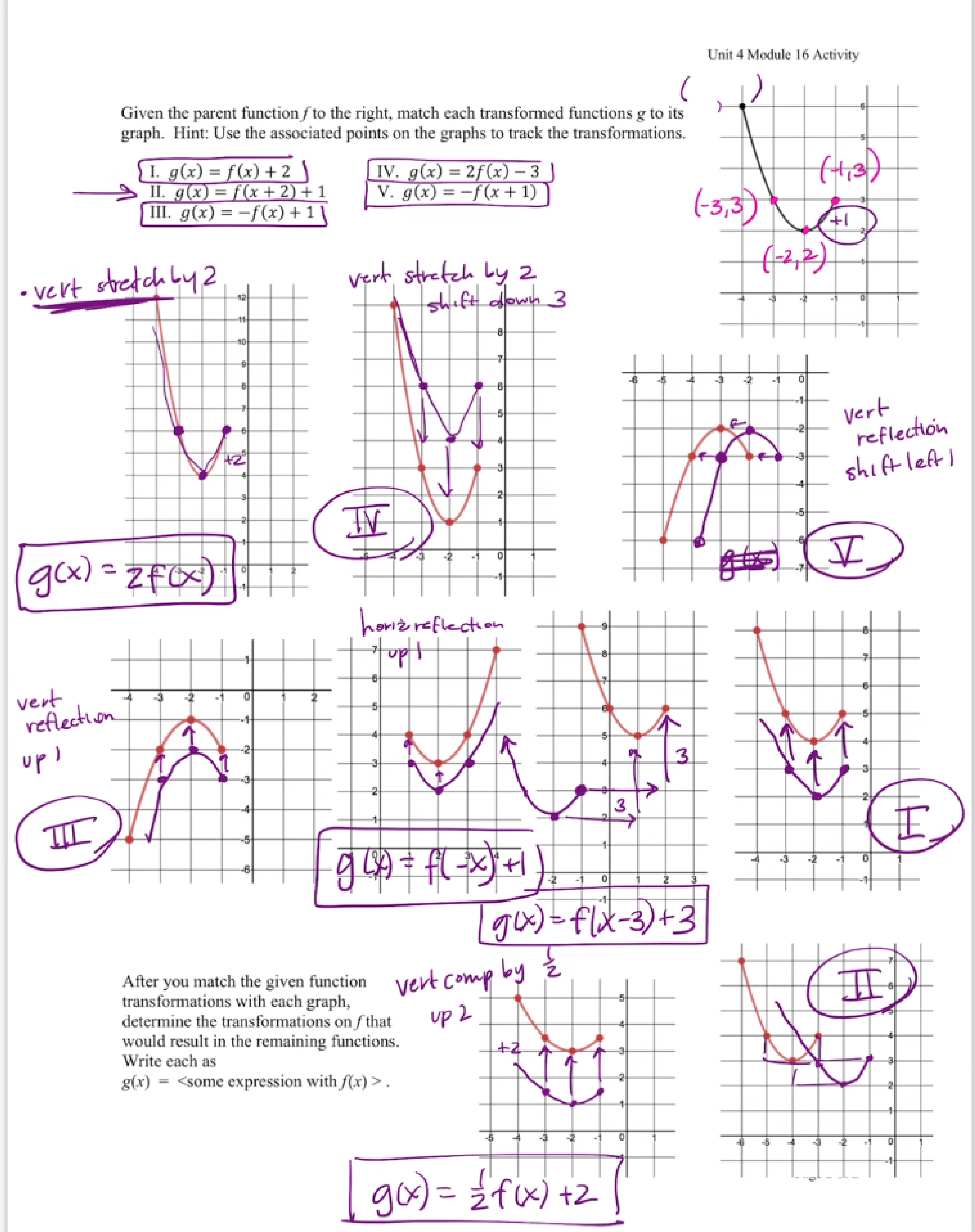
$$\underbrace{g(x)}_{\text{the output of } g \text{ at } x} \underbrace{=}_{\text{is equal to}} \underbrace{f(-x)}_{\text{the output of } f \text{ at the opposite of } x}$$

Transformations Sheet

Transformations Sheet One

Image (Transformed) Function (in terms of x)	Identify the Parent Function f	Transformations *(List how the graph would be affected)*	Write the transformed function in terms of the parent function f
1. $g(x) = 3x^2 + 2$	$f(x) = x^2$	• Vertical stretch by a factor of 3 • Vertically shifted up by 2	$g(x) = 3f(x) + 2$
2. $h(x) = -x^3 + 3$	$f(x) = x^3$	• Vertical reflection • Shifted up 3	$g(x) = -f(x) + 3$
3. $j(x) = \sqrt{x} + 3$	$f(x) = \sqrt{x}$	• Horizontal reflection • Vertical shift up 3	$j(x) = f(-x) + 3$
4. $k(x) = \frac{1}{3}(x-1)^2 + 2$	$f(x) = x^2$	• Vertical compression by a factor of $\frac{1}{3}$ • Horizontal shift right by 1 • Vertical shift up by 2	$k(x) = \frac{1}{3}f(x-1) + 2$
5. $m(x) = 2(\lvert x+4\rvert + 1)$	$f(x) = \lvert x\rvert + 1$	• Vertical stretch by a factor of 2 • Horizontal shift left by 4	$m(x) = 2f(x+4)$

Transformations Sheet Two



Unit 4 Module 16 Activity

Given the parent function f to the right, match each transformed functions g to its graph. Hint: Use the associated points on the graphs to track the transformations.

I. $g(x) = f(x) + 2$

II. $g(x) = f(x+2) + 1$

III. $g(x) = -f(x) + 1$

IV. $g(x) = 2f(x) - 3$

V. $g(x) = -f(x+1)$

After you match the given function transformations with each graph, determine the transformations on f that would result in the remaining functions. Write each as

$g(x)$ = <some expression with $f(x)$> .

- *This sheet was created by Kacie Joyner*

Module 15 - Piecewise Functions

Module 16 - Function Composition

Module 17 - Systems of Equations