Unit 4 - Piecewise Functions, Function Composition, and More Function Transformations

General Notes

Module 14 - Function Transformations Part 2

Module 14 - Google Slides

> **Links:**
>
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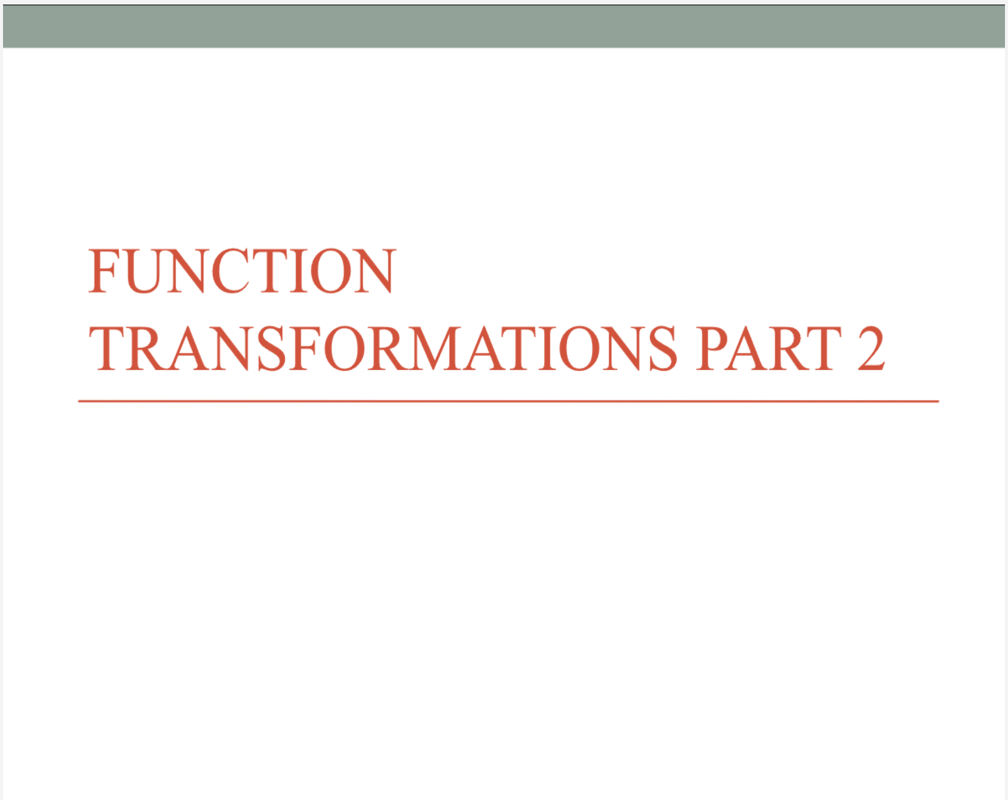
> 
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Function Transformations

The function $g(x) = af(x-c) + d$ is a transformed function in terms of the parent function $f(x)$.

The parameters **a**, **c**, and **d** transform the function in the following ways:

Parameter	Transformation
d	**Causes a vertical shift.** • If **d > 0**, then there is a shift up. • If **d < 0**, then there is a shift down.
a	**Causes a vertical stretch or compression.** • If **|a| > 1**, then there is a vertical stretch. • If **0 < |a| < 1**, then there is a vertical compression.
c	**Causes a horizontal shift** • If **c > 0**, then the shift is to the right. • If **c < 0**, then the shift is to the left.

Identify the Transformations

Identify Transformation One

Given the parent function is $f(x)$, the transformations used to create the transformed function $g(x)$ are:

> $g(x) = -2f(x) + 3$

- Vertical reflection
- Vertical stretch by a factor of 2
- Vertical shift up 3 units

Suppose that $f(x) = 2\sqrt{x} - 1$. The formula in terms of x for each of the functions is:

1. $g(x) = -2f(x) + 3$
2. $g(x) = -2(2\sqrt{x} - 1) + 3$
    - You could leave it like this, but it's better to follow the rest of the steps for readability.

3. $g(x) = -4\sqrt{x} + 2 + 3$
4. $g(x) = -4\sqrt{x} + 5$

Transformation One Image Format

Given the parent function is $f(x)$, list the transformations to create the following transformed function:

$$g(x) = -2f(x) + 3$$

- Vertical reflection
- Vertical stretch by a factor of 2
- Shift up 3 units

Suppose that $f(x) = 2\sqrt{x} - 1$. Write the formula in terms of x for each of the functions.

$$g(x) = -2f(x) + 3$$
$$g(x) = -2(2\sqrt{x} - 1) + 3$$
$$g(x) = -4\sqrt{x} + 2 + 3$$
$$g(x) = -4\sqrt{x} + 5$$

Identify Transformation Two

Given the parent function is ***f*(x)**, the transformations used to create the transformed function ***k*(x)** are:

> $k(x) = 0.5f(x - 4)$

- Vertical compression by a factor of 0.5
- Horizontal shift to the right 4 units

Suppose that $f(x) = 2\sqrt{x} - 1$. The formula in terms of x for each of the functions is:

1. $k(x) = 0.5f(x - 4)$
2. $k(x) = 0.5(2\sqrt{(x - 4)} - 1)$
   - The equation becomes **x - 4** inside the radical, because the input for ***f*** is **x - 4**, not just **x**.

- The radical sign goes above both the **x** and the **4**, hence the parentheses.

3. ***k*(x) = 1√(x - 4) - 0.5**
4. ***k*(x) = √(x - 4) - 0.5**

Transformation Two Image Format

Given the parent function is *f*(*x*), list the transformations to create the following transformed function:

$$k(x) = 0.5f(x-4)$$

- Vertical compression by a factor of 0.5
- Horizontal shift right 4 units

Suppose that $f(x) = 2\sqrt{x} - 1$. Write the formula in terms of x for each of the functions.

$$k(x) = 0.5f(x-4)$$
$$k(x) = 0.5\left(2\sqrt{x-4} - 1\right)$$
$$k(x) = 1\sqrt{x-4} - 0.5$$
$$k(x) = \sqrt{x-4} - 0.5$$

Identify Transformation Three

Given the parent function is ***f*(x)**, the transformations used to create the transformed function ***j*(x)** are:

> ***j*(x) = -4*f*(-x) - 0.5**

- Vertical reflection
- Vertical stretch by a factor of 4
- Horizontal reflection
- Vertical shift down 0.5 units

Suppose that ***f*(x) = 2√x - 1**. The formula in terms of ***x*** for each of the functions is:

1. ***j*(x) = -4(*ƒ*(-x) - 0.5)**
2. ***j*(x) = -4(2√-x - 1) - 0.5**
3. ***j*(x) = -8√-x + 4 - 0.5**
4. ***j*(x) = -8√-x + 3.5**

Transformation Three Image Format

Given the parent function is $f(x)$, list the transformations to create the following transformed function:

$$j(x) = -4f(-x) - 0.5$$

- Vertical reflection
- Vertical stretch by a factor of 4
- Vertical shift down 0.5 units
- Horizontal reflection

Suppose that $f(x) = 2\sqrt{x} - 1$. Write the formula in terms of x for each of the functions.

$$j(x) = -4f(-x) - 0.5$$
$$j(x) = -4(2\sqrt{-x} - 1) - 0.5$$
$$j(x) = -8\sqrt{-x} + 4 - 0.5$$
$$j(x) = -8\sqrt{-x} + 3.5$$

Identify Graph Transformations

Identify Graph Transformation One

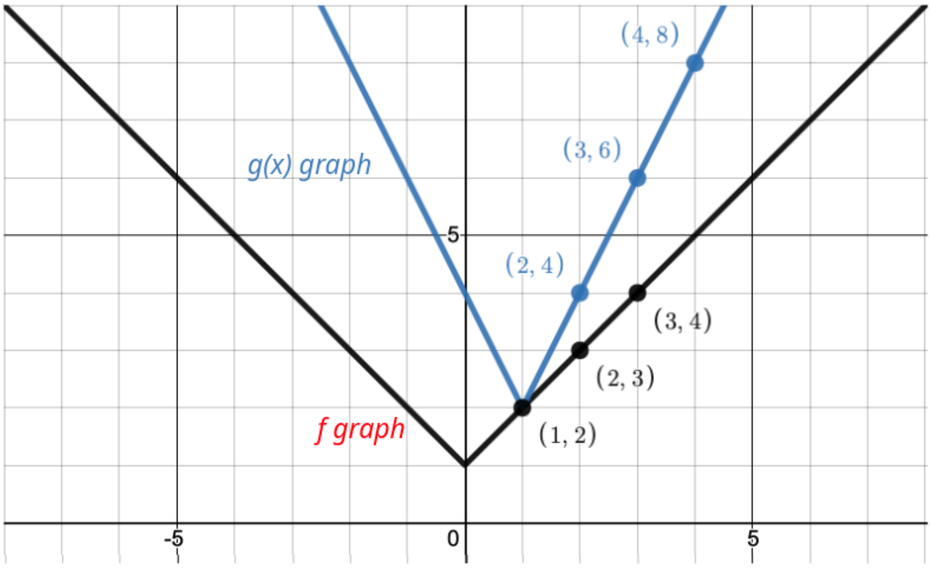
Suppose we were given the graph of the parent function ***ƒ*** and asked to graph the transformed function:

> **g(x) = 2*ƒ*(x — 1)**

The graph:

input	output
1	2
2	3
3	4

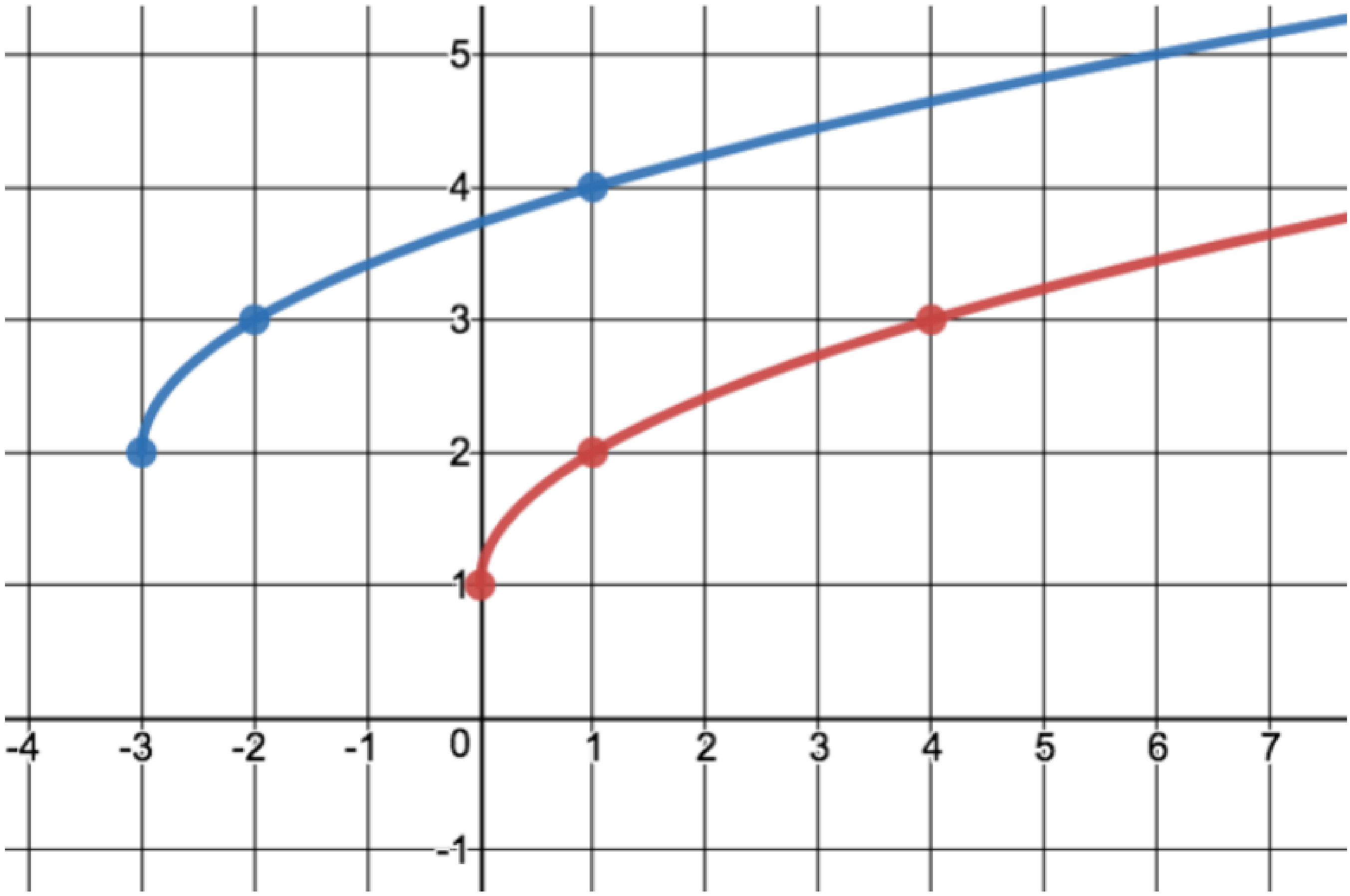
input	output
2	4
3	6
4	8



- The graph was shifted to the right 1 unit, and stretched vertically by a factor of 2.
    - The **x** inputs were shifted to the right 1 unit.
    - The **y** outputs were stretched vertically by a factor of 2.

Identify Graph Transformation Two

Suppose we were given the graph of the parent function ***r*** (in red) and the transformed function ***b*** (in blue) and asked to identify the transformations:



Based on the graph, the formula of ***b*** in terms of ***r*** is:

> ***b*(x) = r(x + 3) + 1**

To identify the transformations:

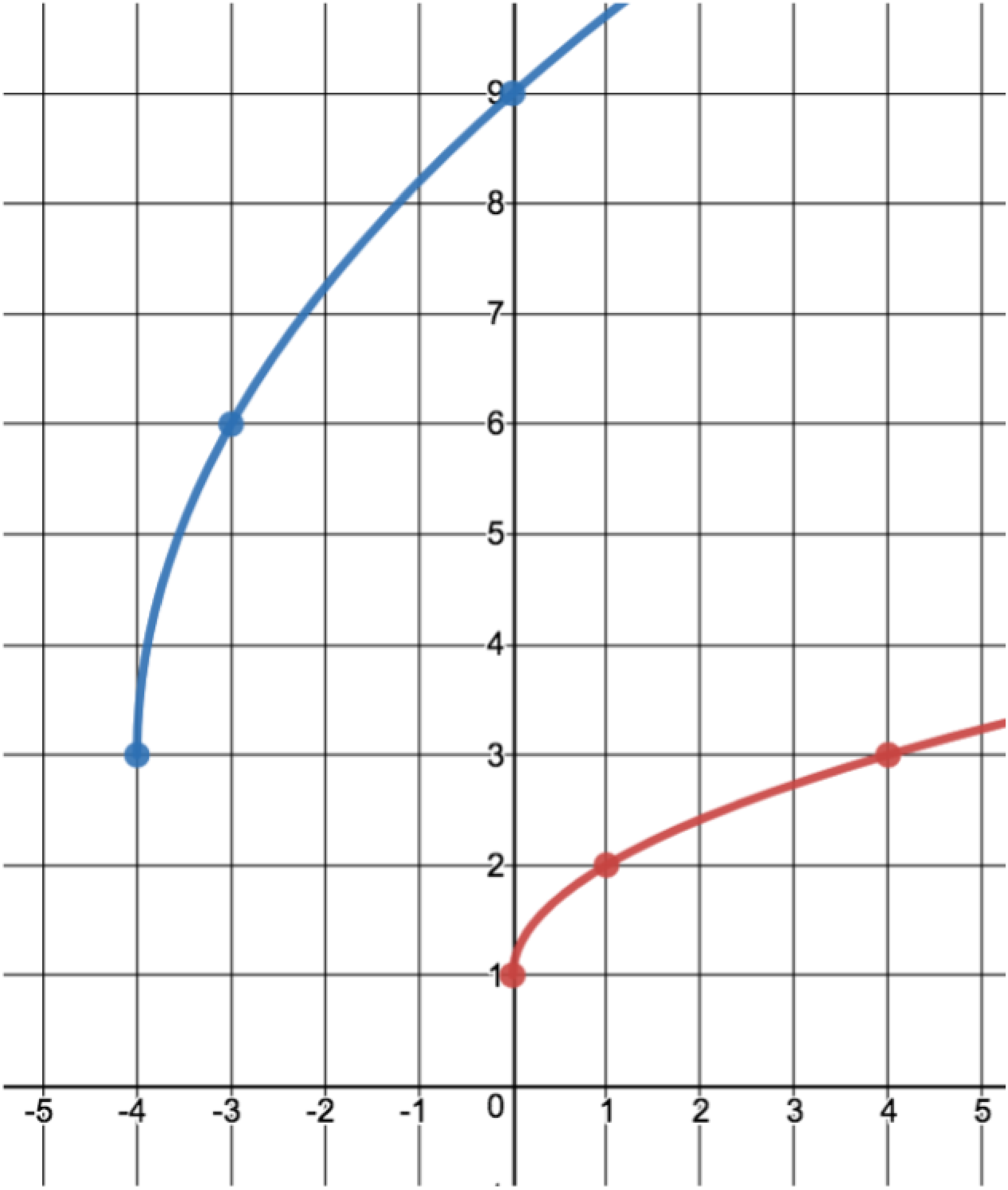
- **Vertical stretch or compression**
  1. Look for a change in outputs from on set of coordinates to the next on both graphs. If the parent function increases by 1 unit on both the *x* and *y* axes, but the transformed function increases by more than 1 unit, there is either a stretch or compression.
- **Horizontal shift**
  1. Look for how far the transformed function is shifted horizontally from the parent function.
- **Vertical shift**
  1. Look for how far the transformed function is shifted vertically from the parent function.
- **Vertical reflection**
  1. Look for a change in the vertical direction of the graph.

2. If the parent function is going one way vertically and the transformed function is vertically going the opposite way, there is a vertical reflection.

- **Horizontal reflection**
  1. Look for a change in the horizontal direction of the graph.
  2. If the parent function is going one way horizontally and the transformed function is horizontally going the opposite way, there is a horizontal reflection.

Identify Graph Transformation Three

Suppose we were given the graph of the parent function ***r*** (in red) and the transformed function ***b*** (in blue) and asked to identify the transformations:



Based on the graph, the formula of ***b*** in terms of ***r*** is:

> ***b***(x) = 3r(x + 4)

To identify the transformations:

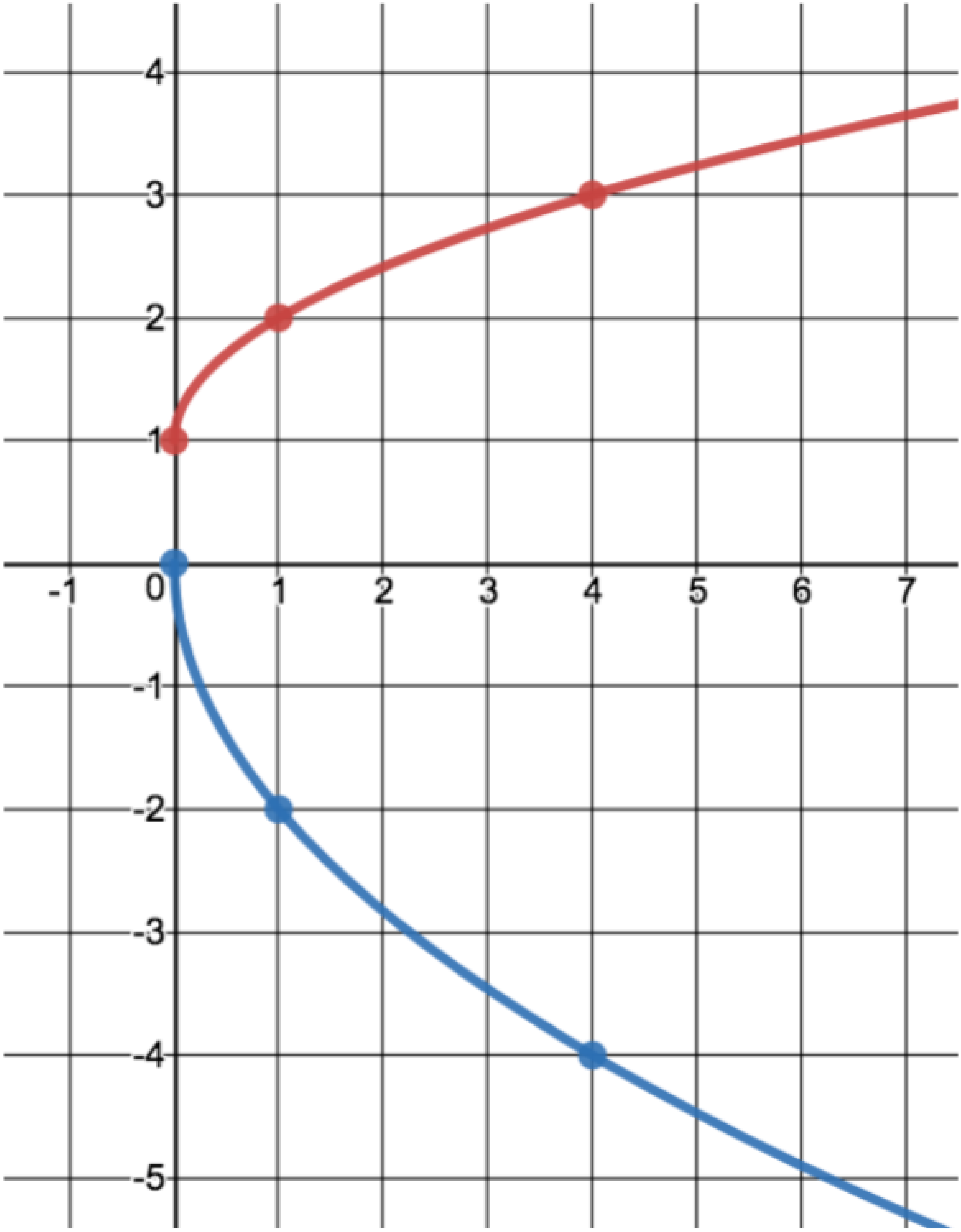
- **Vertical stretch**
  1. The parent function increases by 1 unit on both the *x* and *y* axes, but the transformed function increases by 3 units on the *y*-axis and 1 unit

   on the *x*-axis.
2. The increase in the transformed function is **3** times more than the increase in the parent function.
3. The parent function starts at **(0, 1)**, but if it were stretched by a factor of **3**, then the transformed function would start at **(0, 3)
   **.
   - You can make sure that the predicted stretch is correct by either drawing out the graph or by checking the other points and making sure that they all align with what the y-values should be. *Use a table if necessary*.
4. It's being stretched by a factor of **3**.
5. It's also helpful to draw out the stretched graph to make sure that it matches the transformed graph.

- **Horizontal shift**
  1. The transformed function is shifted to the left by **4** units.

Identify Graph Transformation Four

Suppose we were given the graph of the parent function ***r*** (in red) and the transformed function ***b*** (in blue) and asked to identify the transformations:



Based on the graph, the formula of ***b*** in terms of ***r*** is:

> ***b*****(x) = -2r(x) + 2**

To identify the transformations:

- **Vertical reflection**
    1. The parent function is going one way vertically and the transformed function is vertically going the opposite way, there is a vertical

reflection.

- **Vertical stretch**
    1. The transformed function decreases at twice the amount of the parent function.
- **Vertical shift**
    1. The transformed function is shifted up by **2** units.

Horizontal Reflections

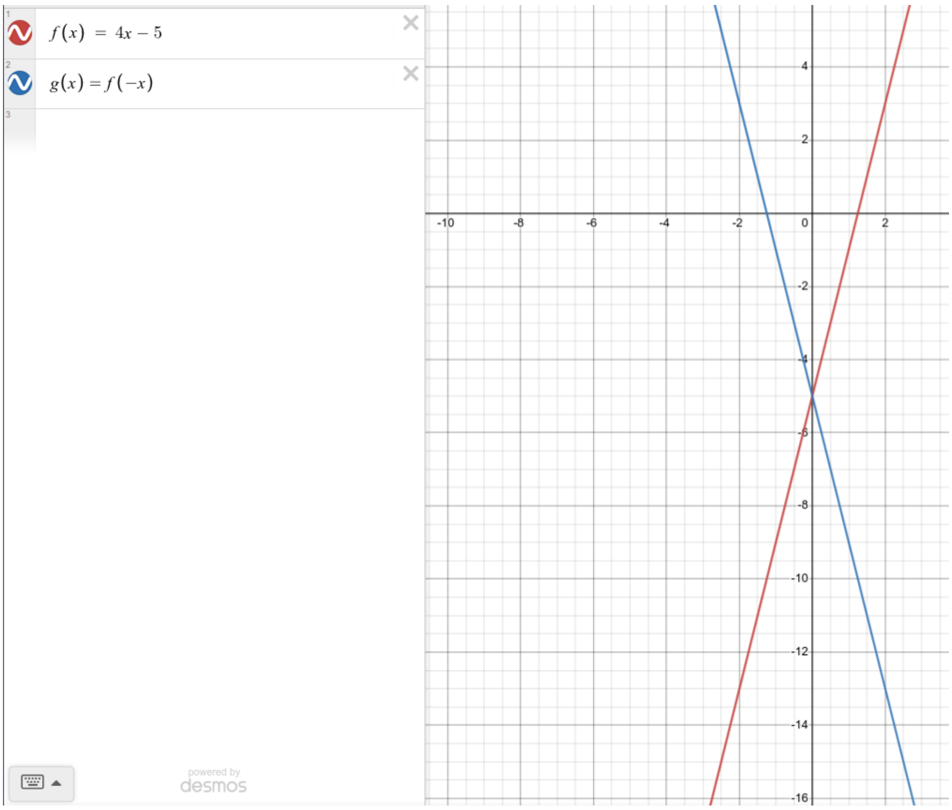
Suppose that the transformed function ***g*(x)** is:

> ***g*(x) = *f*(-x)**

Now suppose that the parent function ***f*** is:

> ***f*(x) = 4x - 5**

The Result:



In general, think of horizontal reflections as follows:

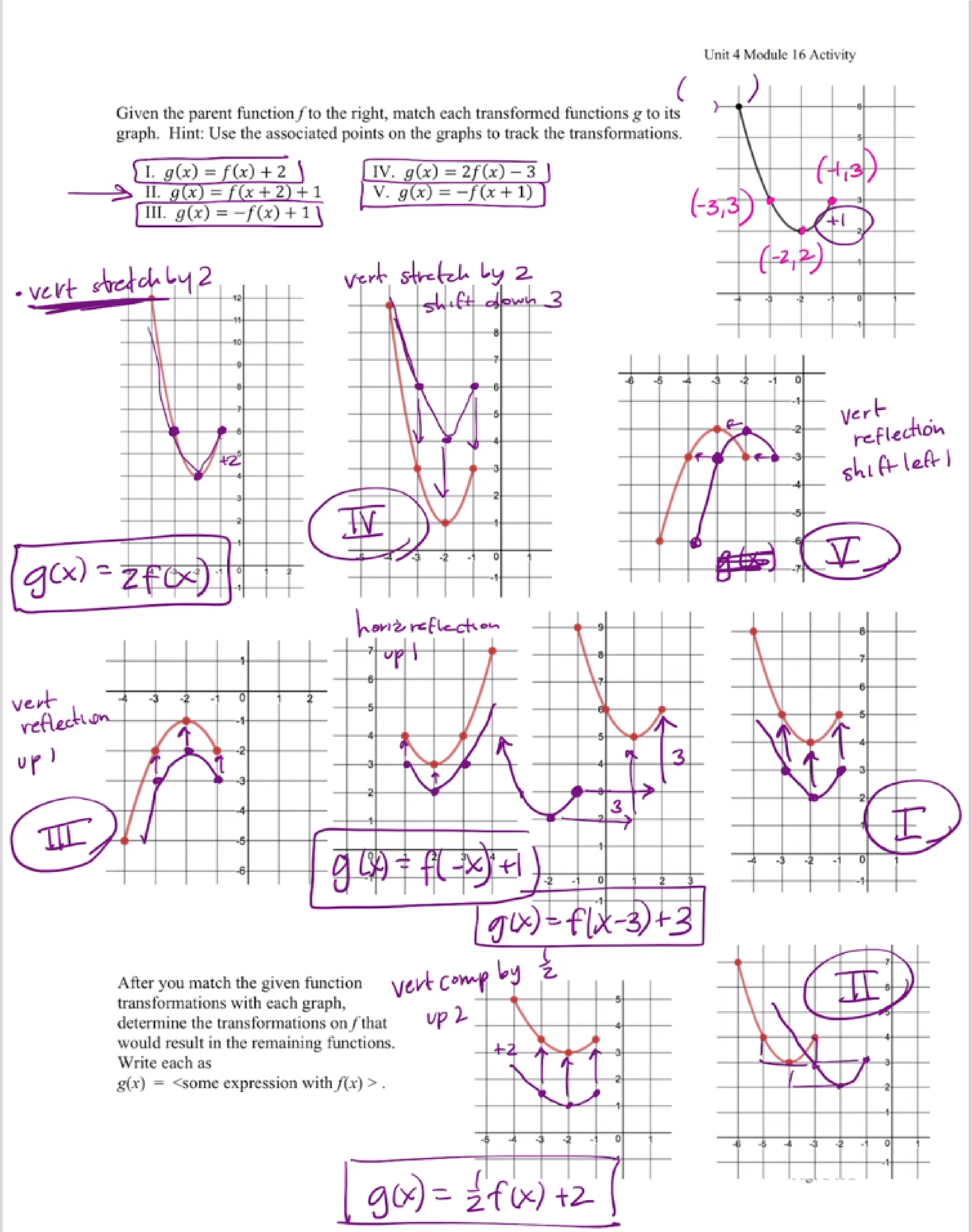
$$\underbrace{g(x)}_{\substack{\text{the output}\\ \text{of } g \text{ at } x}} \quad \underbrace{=}_{\text{is equal to}} \quad \underbrace{f(-x)}_{\substack{\text{the output of } f \text{ at}\\ \text{the opposite of } x}}$$

Transformations Sheet

Transformations Sheet One

Image (Transformed) Function (in terms of x)	Identify the Parent Function f	Transformations *(List how the graph would be affected)*	Write the transformed function in terms of the parent function f
1. $g(x)= 3x^2 + 2$	$f(x) = x^2$	• Vertical stretch by a factor of 3 • Vertically shifted up by 2	$g(x)= 3f(x)+2$
2. $h(x) = -x^3 + 3$	$f(x)= x^3$	• Vertical reflection • Shifted up 3	$g(x)= -f(x)+ 3$
3. $j(x)= \sqrt{x}+ 3$	$f(x)= \sqrt{x}$	• Horizontal reflection • Vertical shift up 3	$j(x)= f(-x)+3$
4. $k(x) = \frac{1}{3}(x-1)^2 + 2$	$f(x)= x^2$	• Vertical compression by a factor of $\frac{1}{3}$ • Horizontal shift right by 1 • Vertical shift up by 2	$k(x)= \frac{1}{3}f(x-1)+2$
5. $m(x)= 2(\lvert x + 4\rvert+1)$	$f(x)=\lvert x\rvert+1$	• Vertical stretch by a factor of 2 • Horizontal shift left by 4	$m(x)= 2f(x+4)$

Transformations Sheet Two



Unit 4 Module 16 Activity

Given the parent function f to the right, match each transformed functions g to its graph. Hint: Use the associated points on the graphs to track the transformations.

I. $g(x) = f(x) + 2$
II. $g(x) = f(x + 2) + 1$
III. $g(x) = -f(x) + 1$
IV. $g(x) = 2f(x) - 3$
V. $g(x) = -f(x + 1)$

(-3,3) (-1,3) (-2,2) +1

• vert stretch by 2

$g(x) = 2f(x)$

vert stretch by 2
shift down 3

IV

vert reflection
shift left 1

V

vert reflection
up 1

III

horiz reflection
up 1

$g(x) = f(-x) + 1$

$g(x) = f(x-3) + 3$

I

After you match the given function transformations with each graph, determine the transformations on f that would result in the remaining functions. Write each as
$g(x)$ = <some expression with $f(x)$> .

vert comp by $\frac{1}{2}$
up 2

$g(x) = \frac{1}{2}f(x) + 2$

II

- *This sheet was created by Kacie Joyner*

Module 15 - Piecewise Functions

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Piecewise Function Definition

A **piecewise function** is a function that is defined in different ways for different intervals.

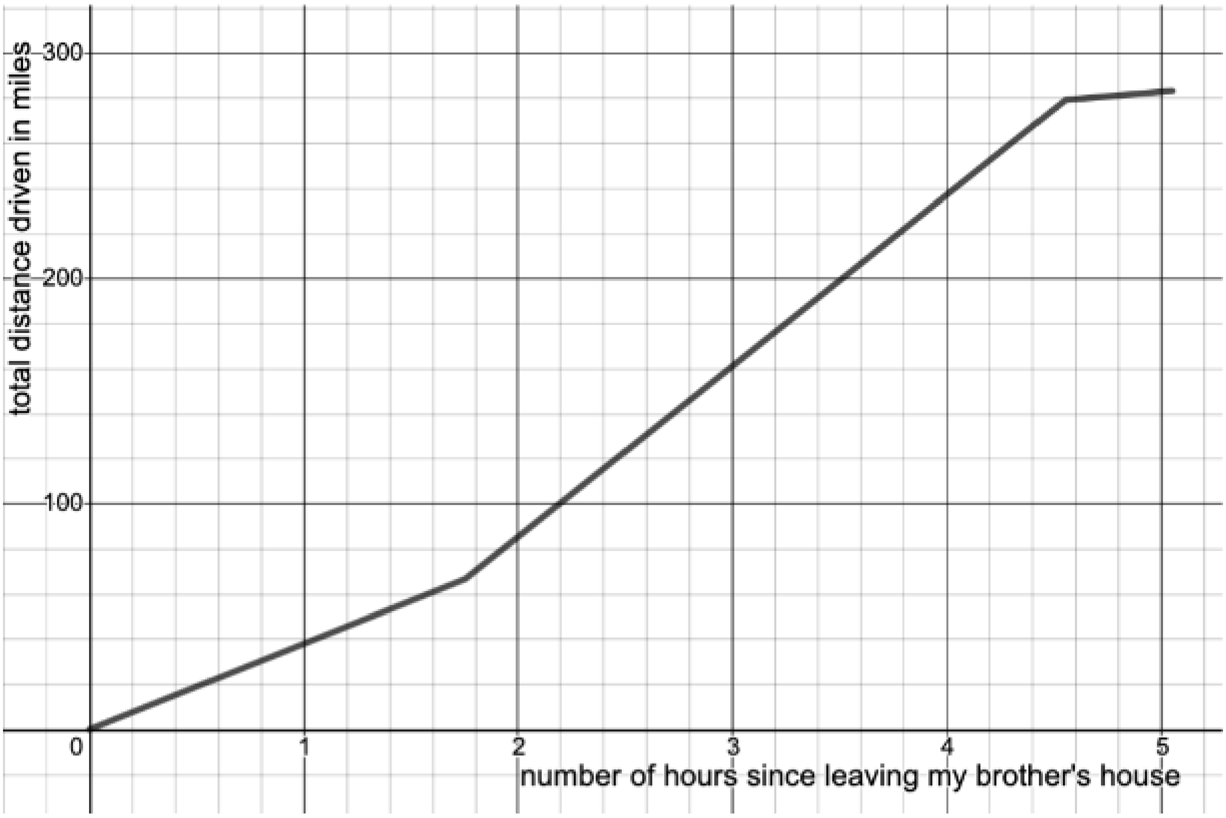
Roadtrip Problem

Initial Story

> On a recent road trip I traveled through various road conditions and weather patterns and so my average speed varied greatly throughout the day.
>
> I started the day at my brother's house in a rural area in northern Florida. Because he lives so far from the highway my average speed for the first 1.75 hours was only 38 mph. Once I hit the highway however I was able to drive faster and hence I averaged 76 mph for the next 2.8 hours. Unfortunately, as is often the case in Florida, a sudden torrential rainstorm came up and the visibility was so bad that for the next 30 minutes I only averaged 8 mph! Finally, the rain stopped, but by then I was so tired I needed to take a rest, so I pulled off at the next exit and found a McDonald's to sit at and surf the internet for a little while.

Roadtrip Graph

This graph represents the roadtrip problem:



Creating a Function for the Model

There are two options:

1. Create a separate function model per portion of the road trip and restrict the domain and range for each function.
    - The problem with this is that there are multiple models for the same road trip.
2. Use a piecewise function to model the road trip.

Goal: Create a function ***f*** to represent the total distance *(in miles)* traveled on my road trip, ***D***, with respect to the amount of time *(in hours)* since I left my brother's house, ***t***.

Creating Segments Per Portion of the Road Trip

We start by creating a function for each portion of the road trip:

1. **Segment 1:** Average speed of **38** mph for **1.75** hours
    1. **66.5** miles traveled in the first **1.75** hours.
        - *Gotten by multiplying **38** by **1.75**.*
    2. Practical domain starts at **[0, 0]** and ends at **[1.75, 0]**.
    3. Practical range starts at **[0, 0]** and ends at **[0, 66.5]**.

    4. The function is $D = f(t) = 38t$; $0 \le t \le 1.75$
       ← *Practical domain*
2. **Segment 2:** Average speed of **76** mph for **2.8** hours
    1. **212.8 miles** in the next **2.8** hours.
    2. Practical domain: $1.75 \le t \le 4.55$
    3. Practical range: $66.5 \le D \le 279.3$
    4. The function: $D = f(t) = 76(t - 1.75) + 66.5$
        - Created using either the **CROC** or the transformation values.
3. **Segment 3:** Average speed of **8** mph for **0.5** hours
    1. **4** miles in the next **0.5** hours.
    2. Practical domain: $4.55 \le t \le 5.05$
    3. Practical range: $279.3 \le D \le 283.3$
    4. The function: $D = f(t) = 8(t - 4.55) + 279.3$

The final segment functions for the road trip are:

> 1. $D = f(t) = 38t$; $0 \le t \le 1.75$
> 2. $D = f(t) = 76(t - 1.75) + 66.5$; $1.75 \le t \le 4.55$
> 3. $D = f(t) = 8(t - 4.55) + 279.3$; $4.55 \le t \le 5.05$

Creating a Piecewise Function

Piecewise Functions

A **piecewise function** is defined using two or more expressions over given intervals of the domain. Piecewise functions are written in the form

$$f(x) = \begin{cases} \text{Rule 1} & \text{if Condition 1} \\ \text{Rule 2} & \text{if Condition 2} \\ \text{Rule 3} & \text{if Condition 3} \\ \vdots & \vdots \end{cases}$$

The conditions define the input values for which each rule applies. The graphs of piecewise functions may be *continuous* or *discontinuous*. Intuitively, a **discontinuous** function is one with a "break," "hole," or "jump" in its graph and a **continuous** function is one whose graph can be drawn without lifting one's pencil.

Text Version:

> A **piecewise function** is defined using two or more expressions over given intervals of the domain.
>
> The conditions define the input values for which each rule applies. The graphs of piecewise functions may be *continuous* or *discontinuous*. Intuitively,
> a *
> *discontinuous function** is one with a "break", "hole", or "jump" in its graph and a **continuous function** is one whose graph can be drawn without lifting one's pencil.

For the road trip problem, the piecewise function is:

$$D = f(t) = \begin{cases} 38t & \text{if} \quad 0 \le t \le 1.75 \\ 76(t - 1.75) + 66.5 & \text{if} \quad 1.75 < t \le 4.55 \\ 8(t - 4.55) + 279.3 & \text{if} \quad 4.55 < t \le 5.05 \end{cases}$$

Example Problems

1. ***f*(3) = 76(3 - 1.75) + 66.5 → D = 161.5**
2. **57 = *f*(*t*)**
   1. Plug in the value for each rule until you find the correct rule.
   2. For rule 1, it results in ***t*** = **1.5**.
      - This meets the condition for rule 1, so it is the correct rule.
   3. For rule 2, it results in ***t*** = **1.625**.
   4. For rule 3, it does not produce a valid result either.

Parking Garage Problem

Consider the given table, which shows the fees to park in the East Economy Garage at Sky Harbor International Airport in Phoenix, Arizona for a single day.

We see that for any time over **0** minutes through **60 minutes**, the fee is $4.00 * *; fortimeover * *60 * *through * *120minutes * *, thefeeis * *$**8.00**; and for any time over **120 minutes** (for one day), the fee is **$10.00**.

Table representing the parking garage problem:

Parking Time (minutes) *m*	**Parking Fee (dollars) *F***
Over 0 through 60	**4.00**
Over 60 through 120	**8.00**
Over 120	**10.00**

First create functions for all three rules:

1. **F = *f*(*m*) = 4; 0 ≤ *m* ≤ 60**
2. **F = *f*(*m*) = 8; 60 ≤ *m* ≤ 120**
3. **F = *f*(*m*) = 10; 120 ≤ *m***

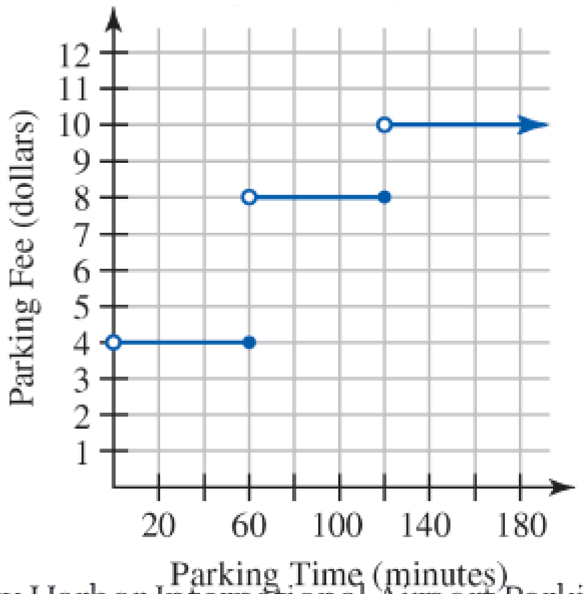
Then create a piecewise function:

$$F(m) = \begin{cases} 4 & \text{if} \quad 0 < m \leq 60 \\ 8 & \text{if} \quad 60 < m \leq 120 \\ 10 & \text{if } 120 < m \leq 1440 \end{cases}$$ since there are 1440 minutes in 1 day

- **Note: F(*m*) is a single function defined in many *pieces*, not many functions.**

Parking Garage Graph

Graphing the above piecewise function shows that ***F*(*m*)** is a discontinuous combination of three linear functions.



Sky Harbor International Airport Parking Fees

- Use an open circle to denote that a value is not included in the function
- Use an arrow to denote that the function continues beyond the graph.

Additional Piecewise Function Problems

Piecewise Function Problem One

$$f(x)=\begin{cases} 2x+4 & \text{if} \quad x<-3 \\ x^2+1 & \text{if} \quad -3\le x<2 \\ 1.2^x & \text{if} \quad x\ge 2 \end{cases}$$

1. Evaluate ***f*****(-5)**
   1. Use rule 1
   2. ***f*****(-5) = 2(-5) + 4**
   3. ***f*****(-5) = -10 + 4**
   4. ***f*****(-5) = -6**
2. Evaluate ***f*****(2)**
   1. Use rule 3
   2. ***f*****(2) =** $\mathbf{1.2^2}$

    3. ***ƒ*(2) = 1.44**
3. Evaluate ***ƒ*(5)**
    1. Use rule 3
    2. $f(2) = 1.2^5$
    3. ***ƒ*(2) = 2.48832**

Piecewise Function Problem Two

> An Airbnb host charges **$125 a night** for the first **3 nights** you stay at their location. The charge then drops to **$110 a night** for each additional night up to **7 nights**. After that, the rate drops to **$95 a night**.
>
> The host does not allow anyone to stay longer than **3 weeks**. There is also a one-time service fee of **$50**.
>
> Define a piecewise function, ***a*** to represent the cost of a stay for ***n*** nights at this Airbnb.

1. Function One
    1. Add **50** to the end to account for the service fee
    2. $a(n) = 125(n) + 50;\ 1 \leq n \leq 3$
2. Function Two
    1. Shift the function to the right by **3** to account for the first three nights
    2. Shift the function up by **125(3) + 50** to account for the price of the first three nights
        - Essentially plugging the max domain value of the previous function into the previous function as the input.
        - *Equals **425***
    3. $a(n) = 110(n - 3) + 425;\ 4 \leq n \leq 7$
3. Function Three
    1. Shift the function to the right by **7** to account for the first seven nights
    2. Shift the function up by **110(7 - 3) + 425** to account for the price of the first seven nights

- Essentially plugging the max domain value of the previous function into the previous function as the
- *Equals **865***

3. The maximum for the domain is **21** nights
4. ***a*(*n*) = 95(*n* - 7) + 865; 8 ≤ *n* ≤ 21**

4. Put the functions together

Problem Two Finished Piecewise Function

$$a(n) = \begin{cases} 125n + 50 & \text{if } 1 \le n \le 3 \\ 110(n-3) + 425 & \text{if } 4 \le n \le 7 \\ 95(n-7) + 865 & \text{if } 8 \le n \le 21 \end{cases}$$

Piecewise Function Problem Three

Federal income tax rates depend on the amount of taxable income received. The following tax rate schedule shows the tax rates for unmarried (single) filers for 2020 for the bottom 4 income levels.

NOTE: This means there is a tax rate of 10% on the first $9,875oftaxableincome,arateof12$9,876 and $40,125,arateof22$40,126 and $85,525, etc

Rate	Income Tax Bracket
10%	$0to$9,875
12%	$9,876to$40,125
22%	$40,126to$85,525
24%	$85,526to$163,300

1. Function One
   1. ***T*(*I*) = 0.10(*I*); 0 ≤ *I* ≤ 9875**
2. Function Two

    1. Shift the function to the right by **9875** to account for the first income level.
    2. Plug in the max value of the domain into *function one* to get the value to shift the function up by
        1. ***T*(*I*) = 0.10(9875)**
        2. **987.50**
    3. Final function: ***T*(*I*) = 0.12(*I* - 9875) + 987.50; 9876 ≤ *I* ≤ 40125**
3. Function Three
    1. Shift the function to the right by **40125** to account for the second income level.
    2. Plug in the max value of the domain into *function two* to get the value to shift the function up by
        1. ***T*(*I*) = 0.12(40125 - 9875) + 987.50**
        2. **4617.50**
    3. Final function: ***T*(*I*) = 0.22(*I* - 40125) + 4,617.50; 40126 ≤ *I* ≤ 85525**
4. Function Four
    1. Shift the function to the right by **85525** to account for the third income level.
    2. Plug in the max value of the domain into *function three* to get the value to shift the function up by
        1. ***T*(*I*) = 0.22(85525 - 40125) + 4617.50**
        2. **14605.5**
    3. Final function: ***T*(*I*) = 0.24(*I* - 85525) + 14605.50; 85526 ≤ *I* ≤ 163300**

Problem Three Finished Piecewise Function

$$T(I) = \begin{cases} 0.10\,I & \text{if } 0 \le I \le 9875 \\ 0.12(I-9875) + 987.50 & \text{if } 9876 \le I \le 40125 \\ 0.22(I-40125) + 4617.50 & \text{if } 40126 \le I \le 85525 \\ 0.24(I-85525) + 14605.50 & \text{if } 85526 \le I \le 163{,}300 \end{cases}$$

Piecewise Function Problem Four

> A 2.4-mile swim, 112-mile bike ride, and a 26.2-mile run make up an Ironman Triathlon competition.
>
> A certain triathlete averages a swimming speed of 2.4 mph, a cycling speed of 18 mph, and a running speed of 10 mph.
>
> Assume there is no transitioning time from one segment of the race to another.

- *Source: <www.ironmanarizona.com>*
- Recall that **distance = rate * time**

Goal:

- Develop a piecewise function for the speed, ***S***, of the participant as a function of his time, ***t***, in hours.

To solve this, we need to understand the speed at any given time within the race. So we will alter the **distance** formula to get the **time** required to complete each portion of the race and put the results into a table to better understand the problem:

Segment	Distance (mi)	Rate (mph)	Time (hrs)
Swim	2.4 mi	2.4	1
Bike	112 mi	18	6.22
Run	26.2 mi	10	2.62

- The **Time** section was found by using **time =** $^{\textbf{distance}}/_{\textbf{rate}}$

1. Function One
   1. ***S*(*t*) = 2.4; 0 ≤ *t* ≤ 1**
2. Function Two
   1. ***S*(*t*) = 18; 1 < *t* ≤ 7.22**

        - It is **7.22** instead of **6.22**, because you need to add the time it took to swim to the time it took to bike.
3. Function Three
    1. ***S*(*t*) = 10; 7.22 < *t* ≤ 9.84**
        - It is **9.84** instead of **2.62**, because you need to add the time it took to swim and bike to the time it took to run.

Problem Four Finished Piecewise Function

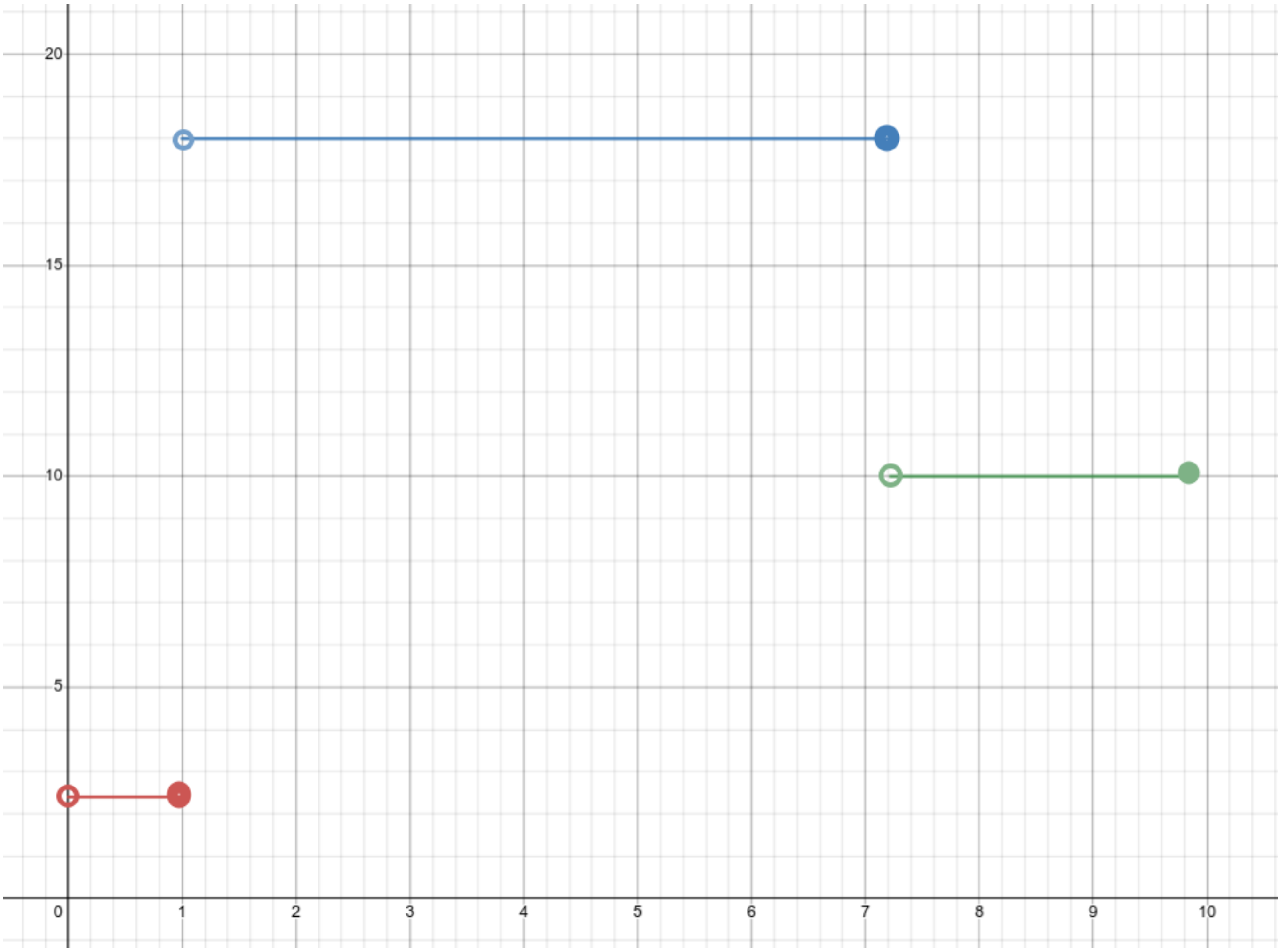
$$S(t) = \begin{cases} 2.4 & \text{if } 0 \le t \le 1 \\ 18 & \text{if } 1 < t \le 7.22 \\ 10 & \text{if } 7.22 < t \le 9.84 \end{cases}$$

Problem Four Meaning

Meaning behind each piece of the piecewise function:

1. ***S*(*t*) = 2.4; 0 ≤ *t* ≤ 1**
    - While swimming the first hour, the participant had an average speed of 2.4 mph.
2. ***S*(*t*) = 18; 1 < *t* ≤ 7.22**
    - For the next 6.22 hours, the participant had an average speed of 18 mph while biking.
3. ***S*(*t*) = 10; 7.22 < *t* ≤ 9.84**
    - For the last 2.62 hours, the participant had an average speed of 10 mph while running.

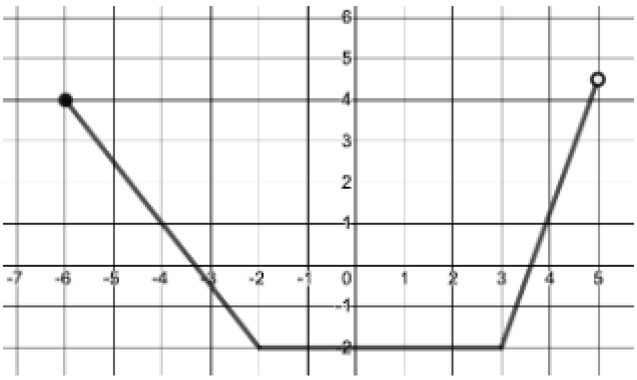
Problem Four Piecewise Graph



Piecewise Function Problem Five - Graphs

Piecewise Graph One

Determining the piecewise function of the graph:



First we need to figure out the function that is being used per interval:

> Formula 1: **m = (y2 - y1) / (x2 - x1)**
>
> Formula 2:

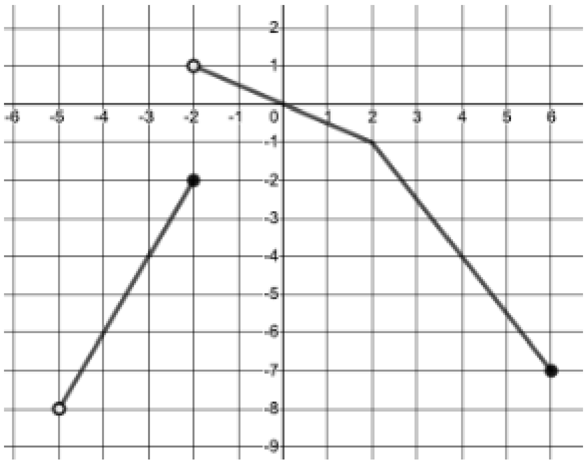
1. Function One
    1. Find the slope
        1. ***Point 1:*** **[-6, 4]** & ***Point 2:*** **[-2, -2]**
        2. $\frac{-2 - 4}{-2 - (-6)}$
        3. $\frac{-6}{-4}$
        4. **1.5**
    2. Find the transformations (using only the relevant section)
        1. The graph is shifted to the left **6**
        2. The graph is shifted up **4**
    3. Find the domain: $-6 \leq x \leq -2$
    4. Final function: $f(x) = 1.5(x + 6) + 4;\ -6 \leq x \leq -2$
2. Function Two
    1. The output is just **-2**
    2. Find the domain: $-2 < x \leq 2$
    3. Final function: $f(x) = -2;\ -2 < x \leq 2$
3. Function Three
    1. Find the slope
        1. ***Point 1:*** **[3, -2]** & ***Point 2:*** **[5, 4.5]**
        2. $\frac{4.5 - 3}{5 - 3}$
        3. $\frac{1.5}{2}$
        4. **0.75**
    2. Find the transformations (using only the relevant section)
        1. The graph is shifted to the right **3**
        2. The graph is shifted down **2**
    3. Find the domain: $3 < x < 5$
    4. Final function: $f(x) = 0.75(x - 3) - 2;\ 3 < x < 5$

Piecewise Graph One Finished Function

$$f(x) = \begin{cases} -1.5(x+6)+4 & \text{if } -6 \leq x \leq -2 \\ -2 & \text{if } -2 < x \leq 3 \\ 3.25(x-3)-2 & \text{if } 3 < x < 5 \end{cases}$$

Piecewise Graph Two

Determining the piecewise function of the graph:



1. Function One
    1. Find the slope
        1. ***Point 1:* [-5, -8]** & ***Point 2:* [-2, -2]**
        2. $^{-2 - (-8)}/_{-2 - (-5)}$
        3. $^{6}/_{3}$
        4. **2**
    2. Find the transformations (using only the relevant section)
        1. The graph is shifted to the left **5**
        2. The graph is shifted down **8**
    3. Find the domain: **-5 < x ≤ -2**
    4. Final function: ***ƒ*(*x*) = 2(*x* + 5) - 8; -5 < x ≤ -2**
2. Function Two
    1. Find the slope
        1. ***Point 1:* [-2, 1] & *Point 2:* [2, -1]**
        2. $^{-1 - 1}/_{2 - (-2)}$
        3. $^{-2}/_{4}$
        4. **-0.5**
    2. Find the transformations (using only the relevant section)
        1. The graph is shifted to the left **2**
        2. The graph is shifted up **1**
    3. Find the domain: **-2 < *x* ≤ 2**
    4. Final function: ***ƒ*(*x*) = -0.5(*x* + 2) + 1; -2 < *x* ≤ 2**
3. Function Three
    1. Find the slope
        1. ***Point 1:* [2, -1]** & ***Point 2:* [6, -7]**

    2. ${}^{-7 - (-1)}/_{6 - 2}$
    3. ${}^{-6}/_{4}$
    4. **-1.5**
2. Find the transformations (using only the relevant section)
    1. The graph is shifted to the right **2**
    2. The graph is shifted down **1**
3. Find the domain: **2 < *x* ≤ 6**
4. Final function: ***ƒ*(*x*) = -1.5(*x* - 2) - 1; 2 < *x* ≤ 6**

Piecewise Graph Two Finished Function

$$f(x) = \begin{cases} -2(x+5)-8 & \text{if } -5 < x \le -2 \\ -0.5(x+2)+1 & \text{if } -2 < x \le 2 \\ -1.5(x-2)-1 & \text{if } 2 < x \le 6 \end{cases}$$

Module 16 - Function Composition

Module 17 - Systems of Equations