CHAPTER 1

Mathematical Modeling, Functions, and Change

Home ownership is a hallmark of the American dream. By looking at housing trends, homebuyers may forecast home values. Mathematical modeling is one primary tool used to forecast home values.

SECTION 1.1 Mathematical Modeling

LEARNING OBJECTIVES

- Identify real uses for symbolic, numeric, and graphical forms of mathematical models
- Analyze mathematical models and use them to create and answer real-world questions

GETTING STARTED

The word *model* has many everyday meanings. A model car is a replica of a real car. A role model is a person who represents admirable qualities such as honesty and integrity that others try to copy. A supermodel exhibits the physical qualities many people strive to reproduce. Thus, the word *model* is synonymous with *replica*, *representation*, *copy*, or *reproduction.*

In this section we give a broad overview of how *mathematical models* are used to represent real-world problems. In later chapters, we demonstrate the process for finding such a model for a given situation.

MATHEMATICAL MODEL

A **mathematical model** is a symbolic, numerical, graphical, or verbal representation of a problem situation.

Decision-Factor Equation

Mathematical models help us understand the nature of problem situations. They are often helpful in making predictions or solving problems in real-world situations.

One type of mathematical model is the *decision-factor equation*. We will use a decision-factor equation to model the decision process for purchasing a preowned vehicle.

When buying a vehicle, we carefully consider important features of a car such as price, manufacturer, engine type, fuel economy, color, model year, and body style, just to name a few. Suppose we are planning to purchase a preowned Toyota Corolla. We have decided that the three features most important to us are mileage, price, and color.

Table 1.1 shows Corollas meeting our criteria offered by Autotrader.com on July 6, 2007 in the vicinity of Indian Orchard, Massachusetts. We can use the data in this table to create a decision-factor equation that will produce a number called the *decision factor* that will help us decide which car to buy. The car that best fits our criteria will have the smallest decision factor.

Table 1.1

Car	Mileage	Price	Color
1	61,671	$11,495	Grey
2	23,258	$15,997	Red
3	14,865	$15,995	Silver
4	5,295	$16,495	Silver
5	35,671	$11,995	Red
6	3,446	$16,495	Grey

EXAMPLE 1 ■ A Decision-Factor Equation

Create a decision-factor equation with mileage, price, and color as the criteria. Assume that mileage is most important followed by price then color.

Solution *Mileage* and *price* have numeric values; *color* does not. We need to assign numeric values to the color options *red*, *grey*, and *silver* to use in our equation. We choose red = 1, grey = 2, and silver = 3, making *red* our first choice, *grey* our second choice, and *silver* our third choice.

One decision-factor equation for this situation is

$$\text{decision factor} = \text{mileage} + \text{price} + \text{color}$$

Using Car 1 from Table 1.1, we get

$$\begin{aligned}\text{decision factor} &= 61{,}671 + 11{,}495 + 2\\ &= 73{,}168\end{aligned}$$

Notice that the color number has a negligible effect on the decision factor. If we want the color to have a greater effect, we can modify the decision-factor equation. For example,

$$\text{decision factor} = \text{mileage} + \text{price} + (1000 \cdot \text{color})$$

We use this modified equation to calculate the decision factor for each of the six cars on the list, as shown in Table 1.2.

Table 1.2

Car	Mileage	Price	Color	Decision Factor	Rank
1	61,671	$11,495	Grey = 2	75,166	6
2	23,258	$15,997	Red = 1	40,255	4
3	14,865	$15,995	Silver = 3	33,860	3
4	5,295	$16,495	Silver = 3	24,790	2
5	35,671	$11,995	Red = 1	48,666	5
6	3,446	$16,495	Grey = 2	21,941	1

Using this model, we find that Car 6 has the lowest decision factor. Notice that it has the lowest mileage, the highest price, and the second-choice color.

To quadruple the effect of the price on the decision factor, we can modify the equation as shown.

$$\text{decision factor} = \text{mileage} + (4 \cdot \text{price}) + (1000 \cdot \text{color})$$

This gives the results shown in Table 1.3.

Table 1.3

Car	Mileage	Price	Color	Decision Factor	Rank
1	61,671	$11,495	Grey = 2	109,651	6
2	23,258	$15,997	Red = 1	88,246	5
3	14,865	$15,995	Silver = 3	81,845	3
4	5,295	$16,495	Silver = 3	74,275	2
5	35,671	$11,995	Red = 1	84,651	4
6	3,446	$16,495	Grey = 2	71,426	1

Although the numerical value of the decision factors changed, Car 6 still has the lowest decision factor. We decide to buy Car 6.

Mathematical Models Presented Numerically

Just as we may use equations to model a situation, we may also use a table of values. For example, one representation of a mathematical model for the increasing number of registered vehicles in the United States is a table of data such as Table 1.4.

Table 1.4

Year	Number of Registered Vehicles
1980	155,796,000
1990	188,798,000
1995	201,530,000
2000	221,475,000
2001	230,428,000
2002	229,620,000
2003	231,390,000

Source: Statistical Abstract of the United States, 2006, Table 1078

Whether in symbolic form (like a decision-factor equation) or numerical form (like a table of data), one of the purposes of a mathematical model is to make sense of the world around us. As we examine the data in Table 1.4, we may ask a variety of questions about the situation: *How is the number of registered vehicles in the United States changing? To whom is this information important?* or *Why is the number of registered vehicles in the United States changing the way it is? Is the number of registered vehicles keeping pace with the increase in population?* We explore these questions in the following examples.

EXAMPLE 2 ■ Analyzing a Mathematical Model in Numerical Form

Describe the change in the number of registered vehicles in the United States. Then identify to whom this analysis may be important and why.

Solution The number of registered vehicles tends to be increasing. We can examine the amount of increase by subtracting one value from the next. The results are shown in Table 1.5.

Table 1.5

Year	Number of Registered Vehicles	Difference
1980	155,796,000	33,002,000
1990	188,798,000	12,732,000
1995	201,530,000	19,945,000
2000	221,475,000	8,953,000
2001	230,428,000	−808,000
2002	229,620,000	1,770,000
2003	231,390,000	

By examining these differences, we can describe how the number of registered vehicles is changing. From 1980 to 1990, the number increased by more than 33 million vehicles (about 3.3 million vehicles per year). From 1990 to 1995, the number increased by nearly 13 million vehicles (about 2.6 million vehicles per year). In the next 5-year interval (1995 to 2000), there was an increase of about 20 million vehicles (4 million vehicles per year). From 2000 to 2001, the number increased by nearly 9 million vehicles. From 2001 to 2002, the number dropped by about 808,000 vehicles. Finally, from 2002 to 2003, the number of registered vehicles increased by nearly 2 million vehicles.

Although most drivers may not care about these statistics, many government agencies do. For example, state motor vehicle divisions that recognize these trends may be able to better forecast tax revenue from licensing fees. State and county officials may use these data to help shape plans for upgrades to transportation infrastructure such as roads, bridges, and highways.

Table 1.6

Year	Population of United States
1980	227,726,000
1990	250,132,000
1995	266,557,000
2000	282,402,000
2001	285,329,000
2002	288,172,000
2003	291,028,000

Source: Statistical Abstract of the United States, 2006, Table 2

In Example 3, we continue to explore the data about the number of registered vehicles in the United States by comparing it to the U.S. population data shown in Table 1.6.

EXAMPLE 3 ■ Thinking about Trends in Data

Offer a possible reason the number of registered vehicles in the United States is changing the way it is. Is the number of registered vehicles keeping pace with the increase in population?

Solution One reason for the increase in the number of registered vehicles in the United States is that the population is increasing. We expect that the number of registered vehicles will keep up with the population. In Table 1.7 we compare the number of registered vehicles data with the U.S. population data to check our assumptions. We see that, indeed, as the population increases, the number of registered vehicles tends to increase. This is true everywhere except for the change from 2001 to 2002, where the number of registered vehicles decreased even though the U.S. population increased.

Table 1.7

Year	Number of Registered Vehicles	Population of United States
1980	155,796,000	227,726,000
1990	188,798,000	250,132,000
1995	201,530,000	266,557,000
2000	221,475,000	282,402,000
2001	230,428,000	285,329,000
2002	229,620,000	288,172,000
2003	231,390,000	291,028,000

We can better understand these trends by computing the number of cars per capita (per person) for each of the given years. This value is found by dividing the number of registered vehicles for the given year by the population in that year. As shown in Table 1.8, the number of registered vehicles per capita initially increased but leveled off in the early 2000s.

Table 1.8

Year	Number of Registered Vehicles	Population of United States	Number of Vehicles Per Capita
1980	155,796,000	227,726,000	0.68
1990	188,798,000	250,132,000	0.75
1995	201,530,000	266,557,000	0.76
2000	221,475,000	282,402,000	0.78
2001	230,428,000	285,329,000	0.81
2002	229,620,000	288,172,000	0.80
2003	231,390,000	291,028,000	0.80

Mathematical Models Presented Graphically

Newspapers and magazines present information in graphical form every day. Let's use the context of median home prices in the metropolitan Phoenix area to investigate a mathematical model presented graphically.

The metropolitan Phoenix area was one of the fastest growing areas in the United States in the early 2000s. As a result, home prices skyrocketed in 2005. The graph in

Figure 1.1 shows that the median price of homes in the Phoenix area increased from $155,800 in the fourth quarter of 2003 to $260,190 in the first quarter of 2006.

In the graph, the fourth quarter of 2003 (October–December) is represented by $t = 0$. The first quarter of 2004 (January–March) is $t = 1$, the second quarter of 2004 (April–June) is $t = 2$, and so on. The vertical axis of the graph shows the median price in dollars.

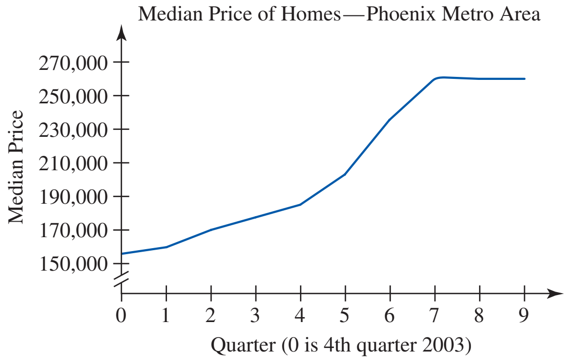


Figure 1.1

EXAMPLE 4 ■ **Analyzing a Mathematical Model Presented Graphically**

Describe the trend seen in the graph (Figure 1.1) of the median home price in the metropolitan Phoenix area.

Solution The median home prices increased from the fourth quarter of 2003 ($t = 0$) until the third quarter of 2005 ($t = 7$). Starting in quarter 7 (third quarter 2005), median home prices stabilized at approximately $260,000. Median home prices increased very quickly between quarter 4 (fourth quarter 2004) and quarter 7 (third quarter 2005).

EXAMPLE 5 ■ **Using a Mathematical Model**

Based on the graph (Figure 1.1) of the median home price in the metropolitan Phoenix area, determine the best time to have sold a home between the fourth quarter of 2003 and the first quarter of 2006.

Solution Assuming that circumstances were such that a homeowner could decide to either wait to sell or sell immediately, it would have been wise for the homeowner to wait to sell until quarter 7 (third quarter 2005). This is when the median price peaked.

SUMMARY

In this section you learned that a mathematical model is a representation of a real-world situation. You discovered that this representation may be symbolic (as in the decision-factor equation), numerical (as in the population data), or graphical (as in the median home price). We will use mathematical models throughout the text to solve problems and make predictions.

1.1 EXERCISES

SKILLS AND CONCEPTS

In Exercises 1–5, use the following decision-factor equation.

$$\text{decision factor} = 100(\text{gas mileage}) + 1000(\text{style}) - \frac{1}{10}(\text{miles})$$

where the style variable is as follows

$$\text{style} = \begin{cases} 2 & \text{if economy car} \\ 1 & \text{if midsize sedan} \\ 0 & \text{if other} \end{cases}$$

1. What kind of a car does this decision-factor equation suggest is desired? Why?

2. Decide which of the following cars is the best to buy. Explain why the results make sense.

Car A: 2002 Toyota Echo (economy car) with 18,298 miles that gets 35 miles per gallon

Car B: 2002 Chevrolet Suburban (sport utility vehicle) with 16,000 miles that gets 16 miles per gallon

Car C: 2002 Dodge Stratus (midsize sedan) with 19,845 miles that gets 28 miles per gallon

3. Suppose the decision factor is 2500 on a car that gets 25 miles per gallon and has 20,000 miles on the odometer. What style car is this?

4. If the decision factor is −1500 on a midsize sedan with 45,000 miles on the odometer, what is the car's gas mileage?

5. Based on the three cars in Exercise 2, the highest decision factor is 3670.2. Now consider another car that gets 30 miles per gallon and is a midsize sedan. What is the maximum mileage it can have to generate a decision factor greater than 3670.2?

For Exercises 6–10, write a decision-factor equation that will accurately score each type of car desired. Use the variables style, color, mileage, cost, year. *Be sure to define any scales used for any of these variables, as necessary.*

6. A relatively new red sports car. Money is not an object.

7. An economical, older-model vehicle for a family of five. Anything will do.

8. A vintage model car that could be restored and shown at car shows.

9. An off-road vehicle that hides dirt well and is very reliable.

10. A vehicle that could be used in the construction field. The company colors are blue and white. Price is not a concern.

SHOW YOU KNOW

11. Write an explanation of what a mathematical model is and how the concept is similar and dissimilar from the non-mathematical use of the word *model*.

12. What are some reasons that mathematical models are created and studied?

13. Describe the different ways that mathematical models may be represented.

MAKE IT REAL

For Exercises 14–18, analyze the mathematical model given in numerical form. Then answer the questions that follow each numerical model.

14. Number of Homes Sold The number of resale homes sold in the Metropolitan Phoenix area for selected quarters is given in the table.

Quarter	Number of Homes Sold
Fourth Quarter 2003	18,350
First Quarter 2004	19,460
Second Quarter 2004	28,760
Third Quarter 2004	27,580
Fourth Quarter 2004	26,315
First Quarter 2005	27,325
Second Quarter 2005	30,705
Third Quarter 2005	30,715
Fourth Quarter 2005	22,090

Source: **www.poly.asu.edu**

a. Describe when the number of homes sold is increasing and when the number of homes sold is decreasing.

b. Compare the difference in the number of homes sold from one quarter to the next. What patterns do you notice?

15. McDonald's Revenue per Year The total annual revenue for McDonald's Corporation for the 6-year period beginning in 2000 is given in the table.

Year	Total Revenue ($ millions)
2000	14,243
2001	14,870
2002	15,406
2003	17,140
2004	19,065
2005	20,460

Source: **www.mcdonalds.com**

a. Describe the trend seen in these data. Are revenues increasing or decreasing over time?

b. Compare the difference in revenue from one year to the next. What patterns do you notice?

16. McDonald's Revenue and Number of Locations The total annual revenue for McDonald's Corporation compared to the number of store locations is given in the table.

Number of Locations	Total Revenue ($ millions)
28,707 in 2000	14,243
30,093 in 2001	14,870
31,108 in 2002	15,406
31,129 in 2003	17,140
31,561 in 2004	19,065
31,886 in 2005	20,460

Source: **www.mcdonalds.com**

a. Describe the trend seen in these data. What connections are there between the increasing revenues and the number of McDonald's locations?

b. Predict the revenue when there are 32,000 locations. Justify your answer.

17. Value of a Car According to www.bankrate.com, vehicles depreciate by about 15% each year. The table projects the value of a 2006 Ford Mustang with a sticker price of $19,439.

Year	Value
2006	$19,439
2007	$16,523
2008	$14,045
2009	$11,938
2010	$10,147

***Source*:** **www.kbb.com**

a. What will be the value of a 2006 Ford Mustang in the year 2011?

b. Find the difference in the value of the 2006 Ford Mustang from one year to the next. What patterns do you notice?

c. Will the value of the 2006 Ford Mustang ever be $0? If so, when? If not, why not?

18. **Teacher Salary Comparison** Over 60% of men not in the teaching profession earn a higher salary than men who are teachers. The table shows how much more money the average college-educated male non-teacher made as compared to the average male teacher.

Year	Percent More Earned by Non-Teachers as Compared to Teachers
1940	−3.6%
1950	2.1%
1960	19.7%
1970	33.1%
1980	36.1%
1990	37.5%
2000	60.4%

Source: **www.nea.org**

For example, in 1990 male non-teachers made 37.5% more than male teachers on average.

a. Describe the trend observed in these data.

b. Why was there such a big jump in the percentage of non-teachers who earn a higher salary than teachers from 1990 to 2000?

c. What does the –3.6% in 1940 indicate about salaries of male teachers?

For Exercises 19–23, analyze the mathematical model given in numerical form. For each model,

a. Describe any trends you notice.

b. Write and answer at least two questions related to the situation.

19. **Teacher Salary Comparison** Over 16% of women not in the teaching profession earn a higher salary than women who are teachers. The table shows how much more money the average college-educated female non-teacher makes as compared to the average female teacher.

Year	Percent More Earned by Non-Teachers as Compared to Teachers
1940	−15.8%
1950	−11.2%
1960	−12.7%
1970	−3.1%
1980	−3.7%
1990	4.5%
2000	16.4%

Source: **www.nea.org**

For example, female non-teachers made 3.7% less than female teachers in 1980 but by 2000 female non-teachers made 16.4% more than female teachers.

20. **NBA Minimum Salary** The table gives the minimum salary paid in 2005–2006 to NBA players with the given years of service to the league.

Years of Service	Minimum Salary
0	$398,762
1	$641,748
2	$719,373
3	$745,248
4	$771,123
5	$835,810
6	$900,498
7	$965,185
8	$1,029,873
9	$1,035,000
10+	$1,138,500

Source: **www.insidehoops.com**

21. **Super Bowl Ticket Prices** The table shows the price of a Super Bowl ticket for selected Super Bowls.

Super Bowl	Ticket Face Value
I (1)	$10
V (5)	$15
X (10)	$20
XV (15)	$40
XX (20)	$75
XXV (25)	$150
XXX (30)	$300
XXXV (35)	$325
XL (40)	$600

Source: **www.superbowl.com**

22. **Price of Gasoline** The table shows the average price per gallon for unleaded, regular gasoline for selected years.

Year	Average Price
1990	$1.16
1995	$1.15
1997	$1.23
1998	$1.06
1999	$1.17
2000	$1.51
2001	$1.46
2002	$1.36
2003	$1.59
2004	$1.88

Source: Statistical Abstract of the United States, 2006**, Table 722**

23. **Comparing Price of Gasoline and Annual Fuel Consumption** The table shows the average price of unleaded, regular gasoline

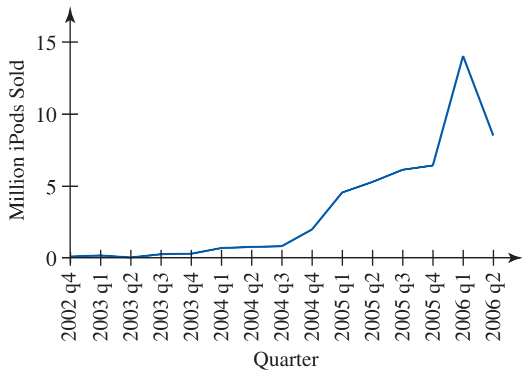
for selected years and the fuel consumption at that price. (Note: The years are not shown.)

Price of Gasoline	Fuel Consumption (billions of gallons)
$1.06	155.4
$1.15	143.8
$1.16	130.8
$1.17	161.4
$1.23	150.4
$1.36	168.7
$1.46	163.5
$1.51	162.5
$1.59	169.6

Source: Statistical Abstract of the United States, 2006, **Table 1085**

For Exercises 24–28, analyze the mathematical model given in graphical form. Then answer the questions that follow each graph.

24. Apple iPod Sales The graph shows iPod unit sales per quarter.



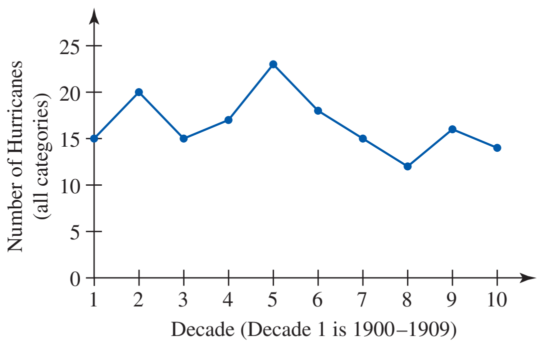
Source: **www.applematters.com**

a. Describe the amount of iPod sales from quarter to quarter. Are sales increasing or decreasing? Are they increasing or decreasing quickly or slowly?

b. Provide a possible explanation for the slow sales initially followed by a rapid increase in sales.

c. Provide a possible explanation for the drastic drop in sales in the beginning of 2006.

25. Number of Hurricanes by Decade The graph shows the number of hurricanes that have hit the mainland United States each decade, beginning in 1900–1909.



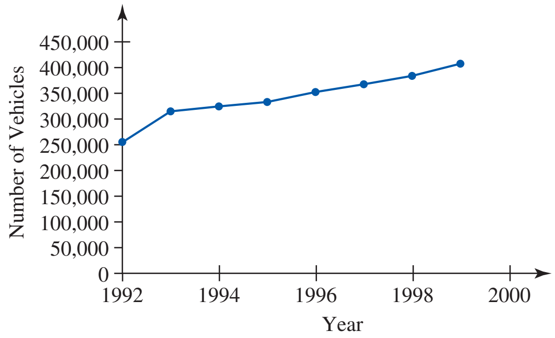
Source: **www.aoml.noaa.gov**

a. Over what time period did the number of hurricanes decrease the longest?

b. In which decade did the greatest number of hurricanes strike the mainland United States?

c. In which decade did the least number of hurricanes strike the mainland United States?

26. Alternative-Fuel Vehicles The graph shows the total number of alternative-fuel vehicles in use in the United States for selected years.



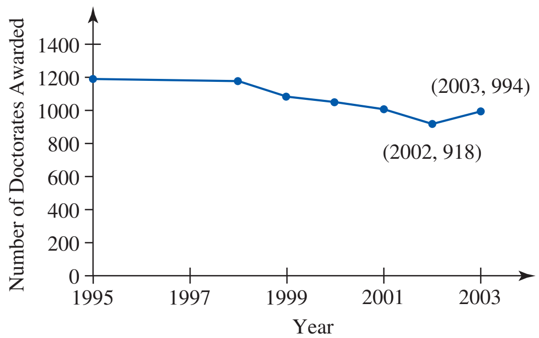
Source: **Quantitative Environmental Learning Project at www.seattlecentral.org**

a. What is the overall trend in the number of alternative-fuel vehicles in use in the United States?

b. Approximately how many more alternative-fuel vehicles are in use in 1999 compared to 1992?

c. Predict how many alternative-fuel vehicles will be in use in 2000.

27. Doctoral Degrees in Mathematics The graph shows the number of doctoral degrees in mathematics awarded in selected years.



Source: Statistical Abstract of the United States, 2006, **Table 784**

a. Describe the trend in the number of doctoral degrees in mathematics awarded from 1995–2002.

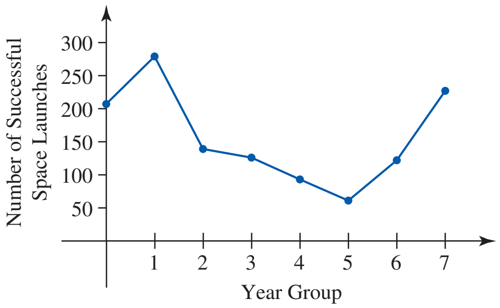
b. How many more doctoral degrees in mathematics were awarded in 2003 than in 2002?

c. If the trend from 2002 to 2003 continued, how many doctoral degrees in mathematics were awarded in 2006?

28. Successful Space Launches in the United States The following data and graph show the number of successful space launches in the United States for selected groups of years.

Year	Year Group	Number of Successful Launches
1957–1964	0	207
1965–1969	1	279
1970–1974	2	139
1975–1979	3	126
1980–1984	4	93
1985–1989	5	61
1990–1994	6	122
1995–2002	7	227

***Source: Statistical Abstract of the United States, 2006*, Table 784**



a. What factors may have caused the dramatic decrease in the number of successful space launches from the 1960s to the early 1970s?

b. What factors may have caused the dramatic increase in the number of successful space launches from the late 1980s and into the 1990s?

c. How many successful launches each year, on average, were there between 1957 and 1964?

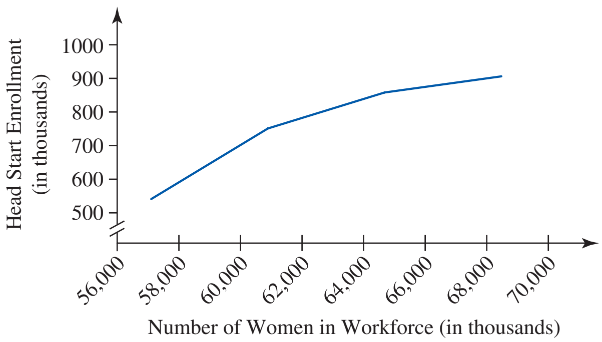
d. How many successful launches each year, on average, were there between 1985 and 1989?

For Exercises 29–33, analyze the mathematical model given in graphical form. For each model,

a. Describe any trends you notice.

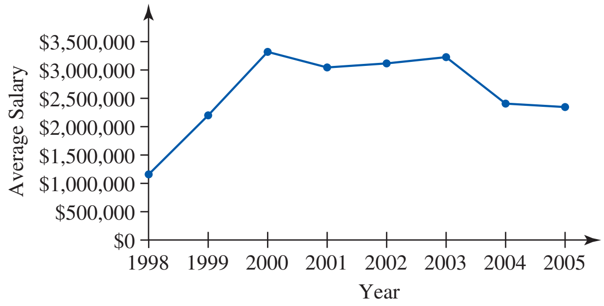
b. Write and answer at least two questions related to the situation.

29. The Labor Force and Head Start Enrollment The graph shows the number of children enrolled in the Head Start program as the number of women in the workforce increased.



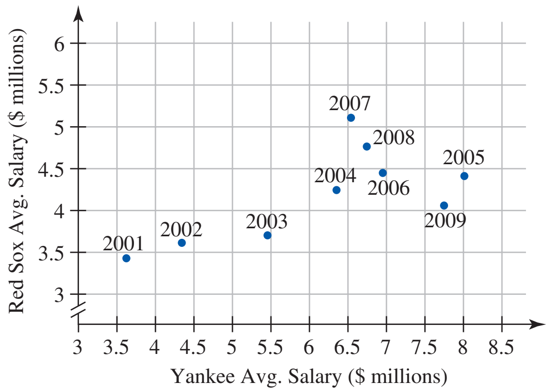
***Source:* Modeled from *Statistical Abstract of the United States, 2006*, Tables 564 and 579**

30. Arizona Diamondbacks Average Salary The graph shows the average salary for Arizona Diamondback players from 1998–2005.



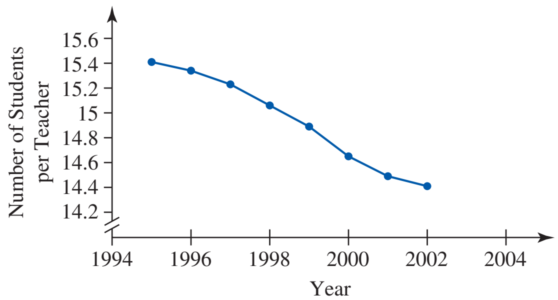
***Source:* sports.espn.go.com**

31. New York Yankees and Boston Red Sox Salaries The graph shows the average salary (in millions of dollars) for New York Yankee players compared to Boston Red Sox players from 2001–2009.



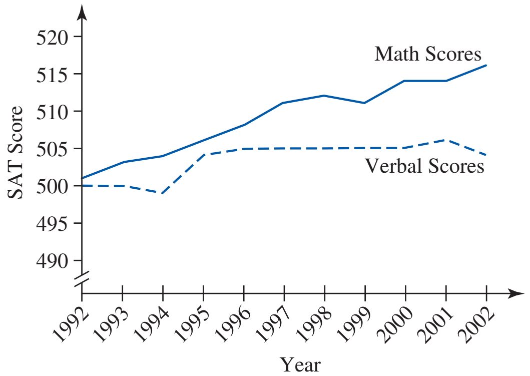
***Source:* Boston Red Sox, New York Yankees**

32. Student–Teacher Ratio The graph shows the number of students per teacher in Texas public schools for the specific years.



***Source:* www.sbec.state.tx.us**

33. Average SAT scores The graph shows the average SAT scores for students over the 10-year period from 1992–2002.



Source: www.collegeboard.com

■ STRETCH YOUR MIND

Exercises 34–35 are intended to challenge your understanding of mathematical models.

34. Explain the advantages and disadvantages of mathematical models.

35. A particular mathematical model in graphical form passes through all of the points in the given data set. A classmate claims that since the model fits the data perfectly, all predictions made using the model will be accurate. Explain why your classmate may be wrong.

SECTION 1.2 Functions and Function Notation

LEARNING OBJECTIVES

- Write and interpret functions using function notation
- Explain how a function is a process or a correspondence
- Solve function equations for a given variable using an equation, table, and graph

GETTING STARTED

One of the credit scores most widely used by financial institutions is the FICO® score formulated by the Fair Isaac Corporation. Engineer Bill Fair and mathematician Earl Isaac founded the Fair Isaac Corporation in 1958 to provide a way for lenders to quantify their investment risk. The loan interest rates that a consumer is offered depends largely upon the FICO score.

In this section we introduce the concept of function and show how to apply functions in real-world situations such as getting a loan to buy a car.

■ Functions

For a car loan, two primary factors come into play: the applicant's *credit score* and the *interest rate* a lender is willing to offer. Since both of these factors can change, we call them **variables** and denote them with letters such as c for *credit score* and r for *interest rate.* A numeric value that does not change is called a **constant**. For instance, a constant in this situation is the price of the car, assuming that the price will not change during the negotiation process.

> **VARIABLE**
>
> A **variable** is a quantity that changes value.

> **CONSTANT**
>
> A **constant** is a numeric value that remains the same.

Functions are typically expressed as a combination of variables and constants.

> **FUNCTION (SINGLE VARIABLE)**
>
> A **single-variable function** is a process or correspondence relating two quantities in which each input value generates exactly one output value.

For example, consider the household freezer. If we put water into a freezer, what will it become? Ice! Since the input (water) put into the freezer will become exactly one thing (ice), we can say that the freezer is a function. We represent this situation in Figure 1.2.

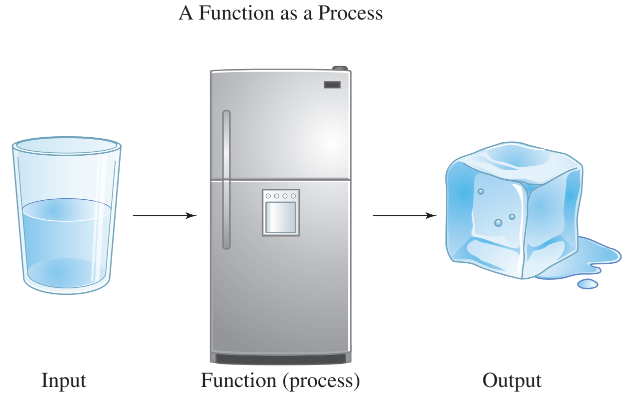


Figure 1.2

Alternatively, a function may be thought of as a *correspondence* between two sets of values. That is, each item from a set of inputs is matched with a single item from a set of outputs, as shown in Figure 1.3. Although a function process may be implied in the correspondence, the correspondence doesn't explicitly state what the process is.

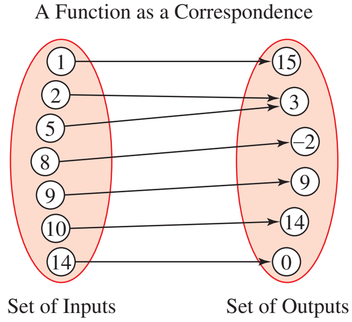


Figure 1.3

Function correspondences do not have to include numbers. Figure 1.4 shows a correspondence relating tennis tournaments with the locations of the tournaments.

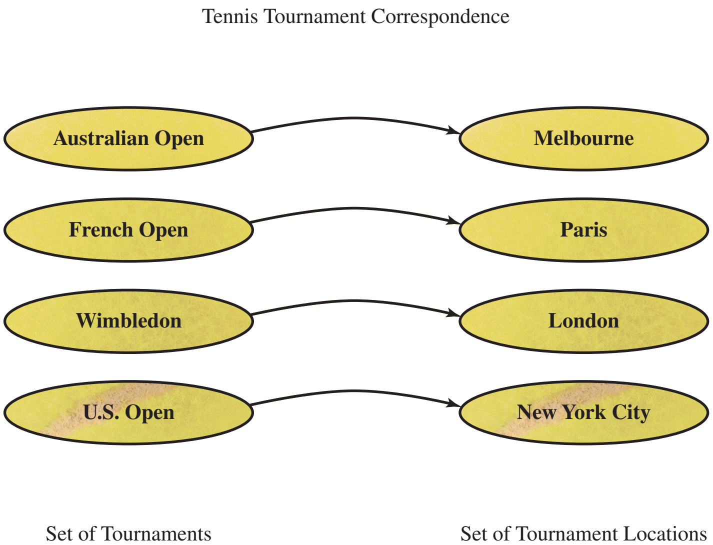


Figure 1.4

PEER INTO THE PAST

LEIBNIZ

Gottfried Wilhelm von Leibniz was born on July 1, 1646, in what is now Germany. Leibniz was a philosopher, theologian, and poet. He created much of the mathematical notation that we use today. He was the first to use the terms *coordinates* and *axes of coordinates*. The terms *function* and *variable* are also credited to Leibniz.

Source: www.mathforum.org

■ Determining If a Relationship Is a Function

Let's look at a function as a correspondence in a relationship between credit score and loan interest rate. A credit score can range from 300 to 850. According to Fair Isaac Corporation, most people have scores between 600 and 800. Table 1.9 shows how the FICO score impacts the interest rate a person is able to obtain on a loan.

Table 1.9

Credit Score c	Annual Interest Rate (percent) r
625	7.89
642	7.34
668	6.91
685	6.70
744	6.52
793	6.30

Source: www.fico.com
(Data is accurate as of 2007.)

The relationship between the person's credit score, c, and the annual interest rate, r, is a function because each credit score (input) produces only one interest rate (output). For example, a person who has a credit score (input) of 744 can secure an annual interest rate (output) of 6.52%.

In this example of a function as a correspondence, we examined two different sets (*credit score* and *annual interest rate*) and noted a link between them. Because the interest rate is affected by the credit score, we assume there is an implicit function process that relates the two sets. However, we do not know and thus cannot state explicitly what that process is.

We also need to be aware that there are relationships between variables that are not functions. For example, consider the relationship that exists between the US Airways' Italian destination cities from Tucson, AZ, and Reno, NV, shown in Figure 1.5.

U.S. Airways' Destination Cities in Italy

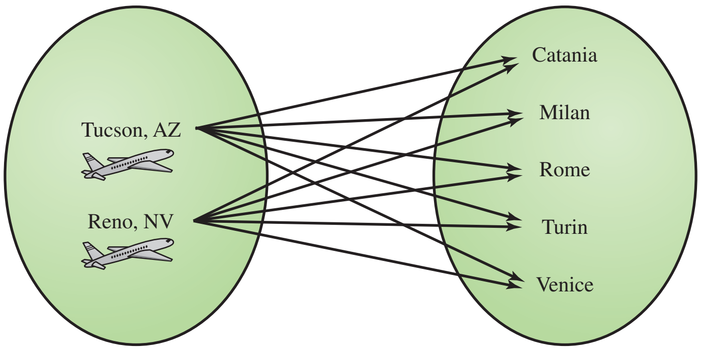


Figure 1.5

Source: Adapted from www.usairways.com

We can see that both Reno and Tucson have multiple destination cities in Italy. The fact that each input (origination city) has more than one output (destination city) makes this relationship not a function. To be a function, each input must only have one output—Tucson and Reno would need to have only one destination city each.

■ Vertical Line Test

If a relationship between two variables exists and is represented as a graph in the rectangular coordinate system (see Section 1.4), the **vertical line test** may be used to determine if the relationship is a function.

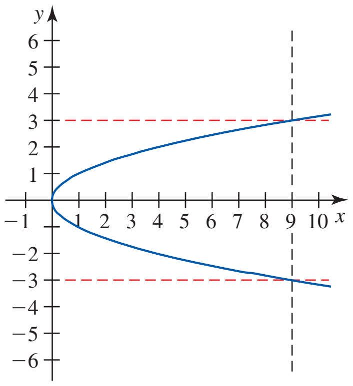


Figure 1.6

Vertical Line Test

If each vertical line drawn on a graph intersects the graph in at most one place, the graph is the graph of a function.

Using the vertical line test, we determine that the graph in Figure 1.6 is not a function because the vertical line crosses the graph more than once. For example, the vertical line shows that the input of 9 has two outputs, 3 and −3.

■ Function Notation

Functions may also be defined using **function notation**, a formal mathematical notation developed to communicate mathematical concepts on a universal scale. Consider the car loan scenario we discussed earlier. To express the relationship between the two variables in verbal terms we say that the interest rate, r, depends on the person's credit score, c. In mathematical terms we say that "r is a function of c." We are expressing the credit score c as an input variable whose value determines the interest rate r through a function correspondence. In function notation we denote this as $f(c) = r$. The input variable c is the **independent variable** and the output variable r is the **dependent variable** because the value of r depends on the value of c.

Function Notation

The component parts of function notation include an *input*, an *output*, and a *function name* as detailed in the following diagram.

function name / input / output

$$f(x) = y$$

The input x is called the **independent variable** and the output y is called the **dependent variable**. This equation may equivalently be written $y = f(x)$.

EXAMPLE 1 ■ Using Function Notation

Write each of the following sentences in function notation by choosing meaningful letters to represent each variable. Then identify the independent variable (input) and the dependent variable (output).

a. A person's weight is a function of the person's height.

b. The current gas price is a function of the amount of available crude oil.

Solution

a. Selecting w for the person's weight and h for the height, we represent this function as $f(h) = w$. The function f takes a value for the independent variable h and generates a value for the dependent variable w.

b. Selecting g for the gas price and c for the available crude oil, we represent this function as $f(c) = g$. The function f will take a value for the independent variable c and generates a value for the dependent variable g.

■ Evaluating Functions and Solving Function Equations

We can *evaluate* functions using function notation.

Evaluating a Function

The process of finding the *output* of a function that corresponds with a given *input* is called **evaluating a function**.

For example, if we say "evaluate $f(642)$," we mean "find the output value that corresponds with an input value of 642."

EXAMPLE 2 ■ Evaluating a Function from a Formula

Suppose that your current job pays \$11.50 per hour. Your salary, S, is calculated by multiplying \$11.50 by the number of hours you work, h. That is, *Salary* = \$11.50 · *hours* or, in function notation, $S = f(h) = 11.5h$. Evaluate $f(80)$ and explain what the numerical answer means.

PEER INTO THE PAST

EULER

Leonhard Euler (1707–1783) was the first to use the notation $f(x)$ as the symbol for a function. On page 268 of his work titled *Commentarii Academiae Scientiarum Petropolitanae* (Cajori, vol. 2), Euler introduces the notation as a way of writing "the function f of x."

Euler was from Basel, Switzerland, and was born to Margaret and Reverend Paul Euler. His father, a mathematician and a minister, groomed Leonhard to become a minister from birth. However, Leonhard chose not to be in the ministry and went on to study mathematics, physics, and engineering. He wrote over 866 books relating to these topics. It was once said that "*Euler calculated without apparent effort, as men breathe . . .*"

Euler had partial blindness when he was 20 and then lost almost all of his sight as he got older. Despite this difficult handicap, he persevered and discovered very important mathematical concepts.

Source: www.andrews.edu/~calkins/math/biograph/bioeuler.htm

Solution To evaluate $f(80)$ means to find the numerical value of S that results from "plugging in" 80 for the number of hours. We have

$$\begin{aligned} f(h) &= 11.5h \\ f(80) &= 11.5(80) \\ &= 920 \end{aligned}$$

In other words, when $h = 80$, $S = 920$. This means that if you work 80 hours your salary will be \$920.

Function notation is extremely versatile. Suppose we are given the function $f(x) = x^2 - 2x + 1$. We may evaluate the function at numerical values as well as nonnumerical values. For example,

$$\begin{aligned} f(2) &= (2)^2 - 2(2) + 1 \\ &= 4 - 4 + 1 \\ &= 1 \end{aligned} \qquad f(\Delta) = (\Delta)^2 - 2(\Delta) + 1$$

In each case, we replaced the value of x in the function $f(x) = x^2 - 2x + 1$ with the quantity in the parentheses. Whether the independent variable value was 2 or Δ, the process was the same.

EXAMPLE 3 ■ Evaluating a Function at Nonnumeric Values

Given $f(x) = 5x^2 - 9x + 4$, find $f(\square)$ and $f(\nabla + \diamond)$.

Solution Admittedly, it feels a bit strange to evaluate functions at nonnumeric values. Nevertheless, we use the exact same strategy as if we were using numeric values. We replace all x values on the right-hand side of the equation with the quantity in the parentheses.

$$f(\square) = 5(\square)^2 - 9(\square) + 4 \qquad f(\nabla + \diamond) = 5(\nabla + \diamond)^2 - 9(\nabla + \diamond) + 4$$

To find the input value that corresponds with a given output, we must solve a function equation.

Solving a Function Equation

The process of finding the *input* of a function that corresponds with a given *output* is called **solving a function equation**.

For example, if we say "solve $f(c) = 6.91$," we mean "find the input value that corresponds with an output value of 6.91."

EXAMPLE 4 ■ Solving an Equation from a Formula

As stated earlier, the salary from a job that pays \$11.50 per hour is given by the function $S = f(h) = 11.5h$. Solve $f(h) = 805$ for h. Explain what the numerical answer means in its real-world context.

Solution To solve $f(h) = 805$ means to find the number of hours h that must be put into function f to generate \$805 in salary. That is, we need to find the number of hours that must be worked to earn \$805. To find this, we set the salary, $f(h)$, equal to 805 and solve for h.

$$\begin{aligned} f(h) &= 11.5h \\ 805 &= 11.5h \\ \frac{805}{11.5} &= \frac{11.5h}{11.5} \\ h &= 70 \end{aligned}$$

This means that if you want to earn \$805 at a job that pays \$11.50 per hour, you must work 70 hours.

Condensed Function Notation

Mathematicians constantly look for ways to more efficiently communicate mathematical concepts. For example, we can represent a function that converts h hours into D dollars using the function notation $D = f(h)$. However, this notation becomes cumbersome when calculating particular values. If a person earns \$8 per hour, we have $D = f(h) = 8h$. Given that the person has worked 25 hours, we use the equation to calculate the worker's earnings as

$$\begin{aligned} D = f(25) &= 8(25) \\ &= 200 \end{aligned}$$

The multiple equal signs makes this difficult to read and understand quickly. To make things easier, we often rewrite the equation $D = f(h) = 8h$ as $D(h) = 8h$. This *condensed notation* is less cumbersome to work with and avoids the series of equal signs.

Condensed Function Notation

When calculating particular function values, it is customary to condense the function notation $D = f(h)$ to $D(h)$.

$$\overset{\text{output}}{D}(\overset{\text{input}}{h})$$

The equation $D(a) = b$ means that when $h = a$, $D = b$. This equation is written as $D = f(a) = b$ in standard function notation.

When we discuss inverse functions, we will see there are some mathematical concepts that are better understood using the standard function notation. So, although we frequently use condensed function notation, we will return to the standard function notation when discussing inverses.

EXAMPLE 5 ■ Evaluating a Function and Solving an Equation from a Table

The number of golf courses in the United States has increased over the years. As seen in Table 1.10, the number of golf facilities, G, is a function of the years since 1980, t. We write this in condensed function notation as $G(t)$. Use the table to do each of the following.

a. Solve $G(t) = 12{,}846$ for t and explain what the numerical value means in this context.

b. Evaluate $G(20)$ and explain what the numerical value means in this context.

Table 1.10

Golf Courses	
Years (Since 1980) t	**Golf Facilities** G
0	12,005
5	12,346
10	12,846
15	14,074
20	15,489

Source: Statistical Abstract of the United States, 2004–2005, Table 1240

Solution

a. To "solve $G(t) = 12{,}846$ for t" means to determine the year when the number of golf courses was 12,846. From the table we can see that there were 12,846 golf courses 10 years after 1980. That is, in 1990.

b. To evaluate $G(20)$ means to determine the number of golf courses in the year 20. When we locate 20 in the Years column of the table, we see that there were 15,489 golf courses in that year. In other words, in 2000 there were 15,489 golf courses in the United States.

EXAMPLE 6 ■ Evaluating and Solving a Function Equation from a Graph

The sales of blank audio cassettes in the United States have declined since 1990. As shown in Figure 1.7, the sales of blank audio cassettes, C, is a function of the years since 1990, t. We write this in function notation as $f(t) = C$. Use the graph to do each of the following.

a. Estimate $f(12)$ and explain what the solution means in this context. Then write the result in function notation.

b. Find a value of t such that $f(t) = 334$ and explain what the solution means in this context. Then write the result in function notation.

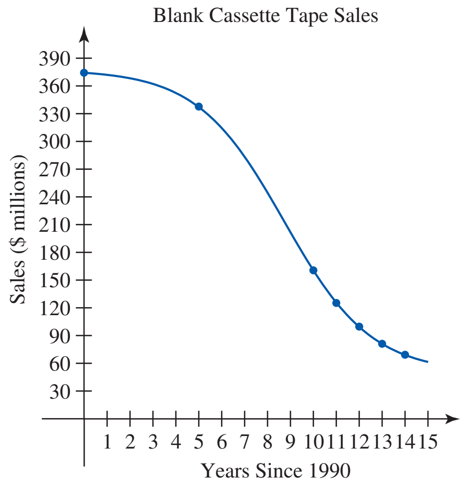


Figure 1.7

Solution

a. To estimate $f(12)$ means to find the value of the cassette tape sales for the year 12 years after 1990. That is, 2002. Using Figure 1.8, we find the t value of 12 on the horizontal axis, go "up" to the function graph, and then "over" to the vertical axis and find a C value of approximately 98.

Therefore, according to the mathematical model, in the year 2002 there were approximately \$98 million in blank cassette sales. In function notation, we write $f(12) = 98$.

b. To find a value of t such that $f(t) = 334$, we estimate the year in which there were \$334 million in audio cassette sales. Using Figure 1.9, we locate 334 on the vertical axis, go "over" to the function graph, and then "down" to the horizontal axis to find a t value of approximately 5.

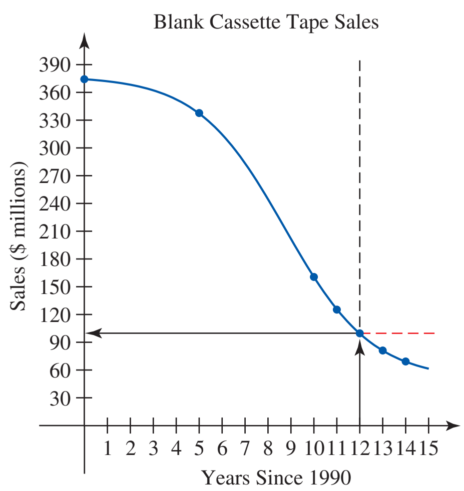


Figure 1.8

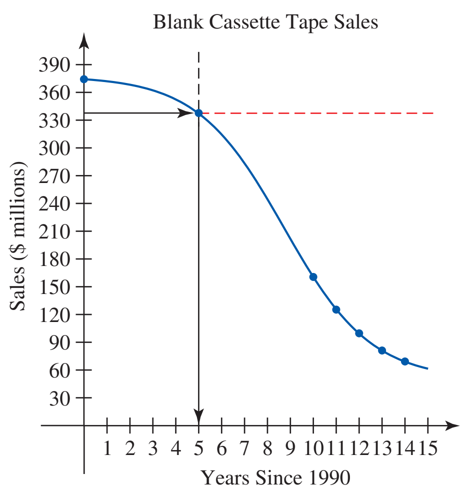


Figure 1.9

Therefore, audio cassette sales were \$334 million around 1995, according to the mathematical model. In function notation, we write $f(5) = 334$.

Multivariable Functions

Sometimes functions have more than one dependent variable. For example, a credit report contains information such as the number and type of accounts, bill-paying history, collection actions, outstanding debt, and the age of the accounts. The credit scoring system predicts who is most likely to repay a debt by awarding points for these creditworthiness factors. Thus, the credit scoring system is a mathematical model represented by a multivariable function.

Other common multivariable functions include the compound interest formula $A = P\left(1 + \frac{r}{n}\right)^{nt}$ and the formula for volume of a box $V = lwh$. We'll look into these in more detail in future sections.

SUMMARY

In this section you learned that mathematical models are represented by functions. You discovered that a function is a process or correspondence relating two quantities in which each input value generates exactly one output value. You learned that functions are expressed in function notation $f(x) = y$, where y is the output value, x is the input value, and f is the function name. You discovered that the output value is called the dependent variable and the input value is called the independent variable because the output value depends on the input value. You learned that to evaluate a function means to determine the output value given an input value and that to solve a function means to find the input value that yields a given output value. Finally, you saw that functions may be either single variable or multivariable.

1.2 EXERCISES

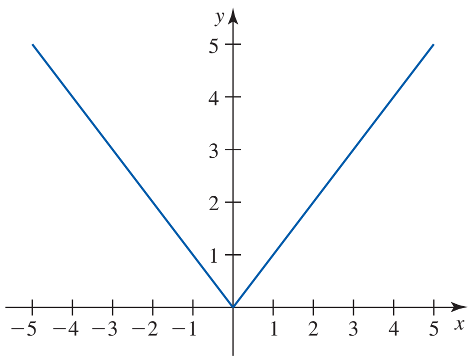
SKILLS AND CONCEPTS

For Exercises 1–5, write each of the following expressions in function notation by choosing meaningful letters to represent each variable. Also identify the independent variable (input) and the dependent variable (output).

1. The amount of property tax you owe is a function of the assessed value of your home in dollars.
2. The length of your fingernails is a function of the amount of time that has passed since your last manicure.
3. The cost of mailing a letter is a function of the weight of the package in ounces.
4. The amount of water required for your lawn (in gallons) is a function of the temperature (in degrees).
5. A person's blood alcohol level is a function of the number of alcoholic drinks consumed in a 2-hour period.

In Exercises 6–15, use the vertical line test to determine whether the graph represents a function in the rectangular coordinate system.

6.



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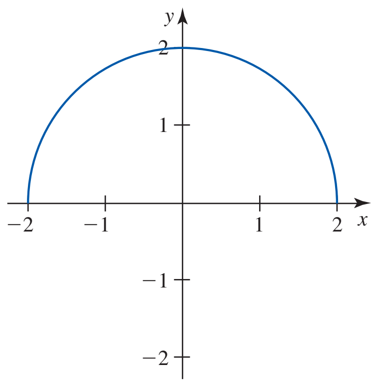

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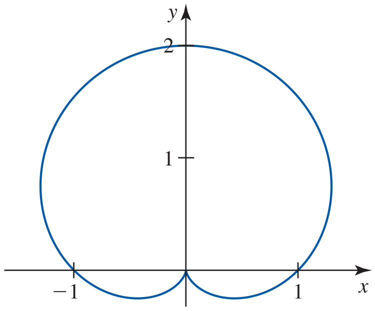

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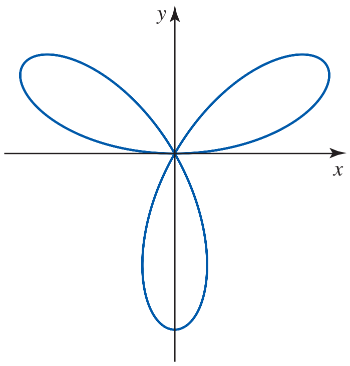

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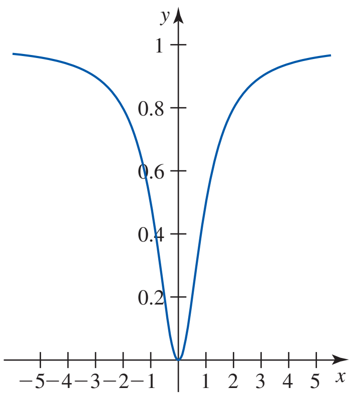

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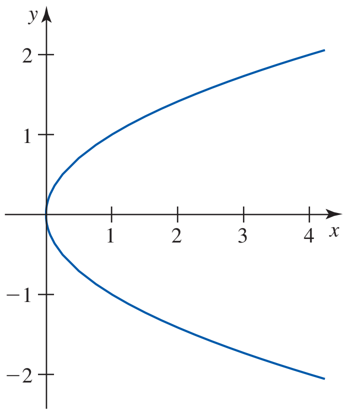
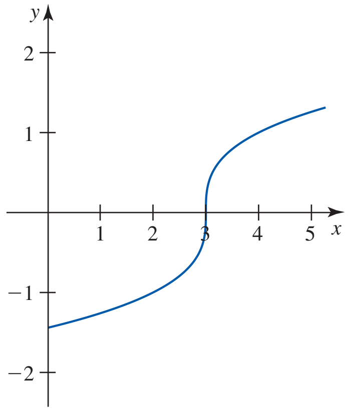


12.



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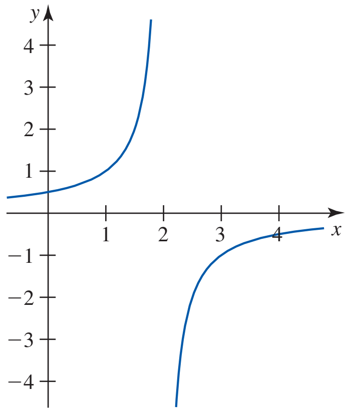

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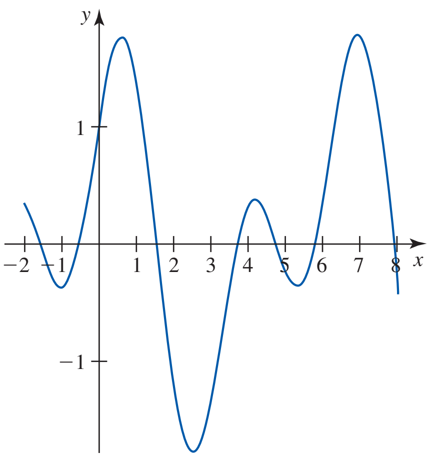

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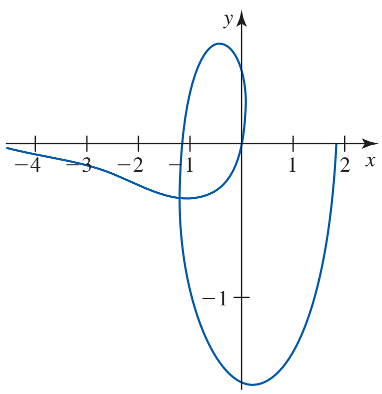


In Exercises 16–25, determine whether the table represents a function.

16.

x	y
9	7
10	7
11	7
12	7
15	7
18	7

17.

x	y
1	6
2	7
3	8
4	7
5	6
6	7

18.

x	y
9	10
10	9
11	8
10	7
9	6
8	5

19.

x	y
9	17
18	12
27	13
36	15
36	7
45	9

20.

x	y
−2	4
−1	1
0	0
1	1
2	4

21.

x	y
1.1	12
2.2	9
3.3	3
2.2	4
4.4	5

22.

x	y
1	7
1	7
1	7
1	7
1	7
1	7

23.

x	y
2	1
5	2
4	3
7	4
6	5
8	6

24.

x	y
19	17
19	7
19	−7
19	7
19	17
19	7

25.

x	y
3	17
4	13
5	9
3	17
4	13
5	19

In Exercises 26–35, evaluate the function for the specified value. Do not simplify. Note that the special symbols (such as #, Δ, and Θ) do not have any special mathematical meaning. They are just symbols.

26. $f(x) = 2x^2 - 3x$; $f(4)$

27. $v(t) = \dfrac{\sqrt{0.3t}}{10}$; $v(3)$

28. $r(s) = |9s^3 - 2s + 18|$; $r(-2)$

29. $t(v) = -v^2 + 3v - \dfrac{4}{v}$; $t(-4)$

30. $h(x) = 3^x - 17x + x^2$; $h(b)$

31. $m(x) = \sqrt{x^2 - 4x}$; $m(\# + 3)$

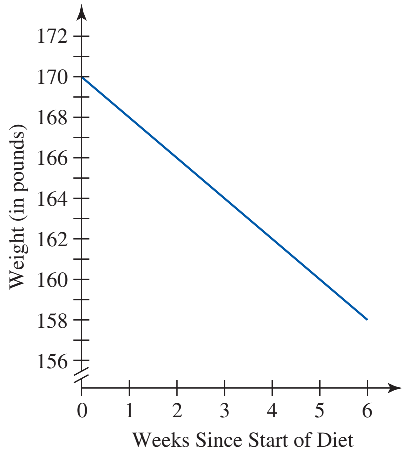
32. $n(d) = -2d + d^3$; $n(\square)$

33. $r(s) = |9s^3 - 2s + 18|$; $r(\Theta + \Delta)$

34. $t(v) = -v^2 + 3v - \dfrac{4}{v}$; $t(\Delta - \nabla)$

35. $h(x) = 3^x - 17x + x^2$; $h(b^2 + 7)$

36. Weight Loss Suppose that you go on a diet and the following graph displays your progress in losing weight. As seen in the graph, your weight (w) is a function of the time since your diet started ($t = 0$) because each week results in only one weight (in pounds).



a. Evaluate $w(3)$ and explain in a complete sentence what your solution means in its real-world context.

b. Find a value of t such that $w(t) = 160$ and explain in a complete sentence what your solution means in its real-world context.

c. Estimate $w(2.5)$ and discuss the accuracy of your prediction.

d. Estimate the solution to $w(t) = 100$ and discuss the accuracy of your approximation.

37. Per-Gallon Fuel Cost The fuel cost, C, of operating a car that goes 25 miles per gallon of gasoline is a function of the price of gasoline, g (in dollars per gallon), and the distance driven, D (in miles). The formula is

$$C(g, d) = \frac{gd}{25} \text{ dollars}$$

Use function notation to express the fuel cost of operation if gasoline costs \$2.76 per gallon and the car travels 310 miles. Then calculate the cost.

38. Grocery Cost A friend wants to purchase cereal, milk, and bananas on her next trip to the grocery store. The grocery bill in dollars, g, is a function of the number of boxes of cereal, c, gallons of milk, m, and bunches of bananas, b, she buys. Assume cereal is \$3.75 a box, milk is \$2.79 a gallon, and a bunch of bananas is \$1.15. The grocery bill is given by

$$g(c, m, b) = 3.75c + 2.79m + 1.15b$$

Use function notation to express the grocery bill if she buys two boxes of cereal, three gallons of milk, and one bunch of bananas.

SHOW YOU KNOW

39. Choose five exercises from this section. For each exercise, describe whether the function represented a process or a correspondence. Explain your reasoning.

40. Explain why the vertical line test is a valid way to check to see if a graph represents a function.

41. What is the practical advantage of requiring a function to have exactly one output for each input?

42. Give an example of a nonmathematical process that represents the function concept.

43. Give an example of a nonmathematical correspondence that represents the function concept.

MAKE IT REAL

44. Children in Preschool and Kindergarten Based on data provided by the Census Bureau, the number of American children age 3 to 5 years enrolled in preprimary school (preschool and kindergarten) can be modeled by the function

$$p(y) = 121.46y + 5231.31$$

where p represents the number of children enrolled (in thousands) and y is the number of years since 1980. (*Source:* www. census.gov)

Evaluate $p(27)$ and explain what the numerical answer represents in its real-world context. Then write the solution in function notation.

45. Consumer Expenditures The average annual expenditures of all U.S. consumers can be modeled by the formula

$$E(t) = 1019.65t + 27{,}861.97$$

where E represents the annual expenditures in dollars and t is the years since 1990. (*Source:* www.census.gov)

Evaluate $E(20)$ and explain what the numerical answer represents in its real-world context. Then write the solution in function notation.

46. College Enrollment The number of students enrolled for the Spring semester at Chandler-Gilbert Community College has been growing in recent years. The number of students enrolled can be modeled by the function

$$C(y) = 168.9y + 6741$$

where C is the number of students enrolled and y is the years since 2003. (*Source:* Modeled from data at www.azcentral.com)

Solve $C(y) = 9000$ for y and explain what the numerical answer represents in its real-world context. Then write the solution in function notation.

47. Homes Sales Price The median sales price of new homes since 1980 can be modeled by the formula

$$m(t) = 5844.95t + 56{,}589.91 \text{ dollars}$$

where m is the median sales price and t is the year since 1980. (*Source:* www.census.gov)

Solve $m(t) = 300{,}000$ for t and explain what the numerical answer represents in its real-world context. Then write the solution in function notation.

48. U.S. National Parks Visits The number of recreational visits to the National Parks of the United States is displayed in the table. The number of visits to the national parks, p, is a function of the year, t.

Year	Recreational Visits to U.S. National Parks (millions of people)
1990	258.7
1995	269.6
1999	287.1
2000	285.9
2001	279.9
2002	277.3
2003	266.1
2004	276.4

Source: www.census.gov

a. Solve $p(t) = 277.3$ for t and explain the meaning of the solution.

b. Evaluate $p(2000)$ and write a sentence explaining what the numerical value you find means in its real-world context.

c. Estimate $p(2010)$ and discuss the accuracy of your prediction.

d. Estimate the solution to $p(t) = 300$ and discuss the accuracy of your approximation.

49. Most Visited States by Foreigners The twelve most frequently visited American states by overseas travelers in 2004 are displayed in the following table. The number of visits, v, is a function of the U.S. state, s.

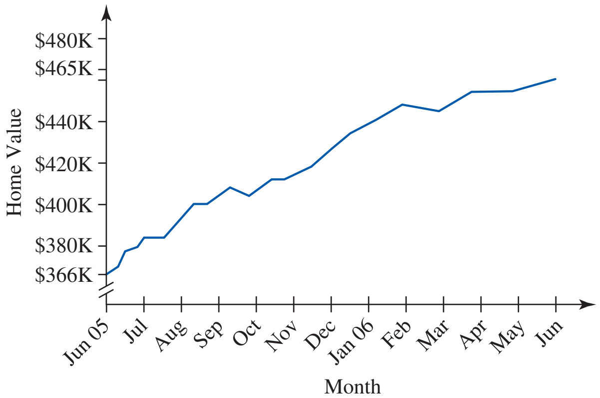
U.S. State	Number of Visits (in thousands)
New York	5426
Florida	4430
California	4207
Hawaii	2215
Nevada	1626
Illinois	975
Massachusetts	935
Texas	874
New Jersey	833
Pennsylvania	691
Arizona	630
Georgia	427

Source: www.census.gov

a. Solve $v(s) = 630$ for s and explain the meaning of the solution.

b. Evaluate $v(Hawaii)$ and explain the meaning of the result.

50. Home Value Zillow.com is a website that approximates the value of a home based on its address. Based on data from the website, we created the following graph showing the value of one Arizona home, h, as a function of the month, m.



a. Evaluate $h(\text{Aug05})$ and explain what the solution means in its real-world context.

b. Find a value of m such that $h(m) = 440{,}000$ and explain what the solution means in its real-world context.

c. Estimate the solution to $h(m) = 480{,}000$ and discuss factors that may affect the accuracy of your estimate.

STRETCH YOUR MIND

Exercises 51–52 are intended to challenge your understanding of functions and function notation.

51. A classmate solves the equation $f(x) = 4x - 3$ for x as follows.

$$f(x) = 4x - 3$$
$$f(x) - 4x = -3$$
$$x(f - 4) = -3$$
$$x = \frac{-3}{f - 4}$$

Explain what is wrong with this problem-solving process.

52. In the equation $x(f) = 9f + 6$, which variable is independent and which is dependent?

SECTION 1.3 Functions Represented by Tables and Formulas

LEARNING OBJECTIVES

- Determine if a data table represents a function
- Calculate and interpret the meaning of an average rate of change from a table
- Create and use basic function formulas to model real-world situations

GETTING STARTED

Both consumer debt and fuel prices for Americans have been increasing at an ever-increasing rate while total savings have decreased at an increasingly rapid rate. Although no one knows what the future will bring, we can forecast what might happen by analyzing tables of data related to these issues.

In this section we explain how to analyze functions in tables and formulas. We'll also demonstrate how to find average rates of change and use them to help forecast unknown results.

Before going on, let's step back and review what we know about mathematical models and functions. We've shown that a function is a process or correspondence relating a set of inputs with a set of outputs in such a way that each input is paired with a single output. We have also seen that many real-world situations and data sets can be effectively modeled using functions. In the remaining sections of this chapter, we look in greater detail at functions and function models of real-world data sets in four different ways: tables, formulas, graphs, and words (Figure 1.10).

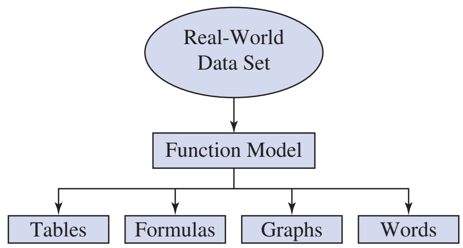


Figure 1.10

Determining If a Table Represents a Function

The Federal Reserve Board monitors total consumer debt (excluding loans secured by real estate). Table 1.11 shows several years of consumer debt with the year as the input and the total consumer debt as the output.

To determine if the table represents a function, we only need to verify that each input value corresponds with a single output value. That is, for each year, there can only be one value for the total consumer debt. Intuitively, we know that this must be the case, and a quick inspection of the table verifies this conclusion. Therefore, total consumer debt is a function of the calendar year.

Table 1.11

Year d	Total Consumer Debt ($ billions) C
1985	593.00
1990	789.10
1995	1095.80
2000	1556.25
2005	2175.25

Source: Federal Reserve Board

Average Rate of Change

One way to analyze a table of data is to calculate an *average rate of change.*

HOW TO: ■ CALCULATE AN AVERAGE RATE OF CHANGE

The average rate of change of a function f over an interval $[a, b]$ is calculated by dividing the difference of two outputs by the difference in the corresponding inputs. That is,

$$\text{average rate of change} = \frac{f(b) - f(a)}{b - a}$$

where $(a, f(a))$ and $(b, f(b))$ are any two data points in the table.

EXAMPLE 1 ■ Calculating and Interpreting an Average Rate of Change

Table 1.12 shows the age-adjusted death rate due to heart disease. (Age-adjusting is a statistical technique used by analysts to help avoid distortions in data interpretation when comparing disease rates over time.)

Table 1.12

Years since 1990 t	Age-Adjusted Death Rate Due to Heart Disease (deaths/100,000 people) r
0	321.8
1	313.8
2	306.1
3	309.9
4	299.7
5	296.3
6	288.3
7	280.4
8	272.4
9	267.8
10	257.6

Source: Statistical Abstract of the United States, 2006, Table 106

a. Calculate the average rate of change in the death rate between 1990 and 2000.

b. Interpret the real-world meaning of the average rate of change from part (a).

c. Use the average rate of change from part (a) and the death rate in 2000 to forecast the 2003 death rate. Explain why the forecasted 2003 death rate differs from the actual death rate reported by the Census Bureau (232.1 deaths per 100,000 people).

Solution

a. In 1990, the death rate was 321.8 deaths per 100,000 people. In 2000, the death rate was 257.6 deaths per 100,000 people. We calculate the average rate of change over the 10-year period.

$$\begin{aligned}\text{average rate of change} &= \frac{257.6 - 321.8}{10 - 0}\ \frac{\text{deaths per 100,000 people}}{\text{year}}\\ &= -6.42 \text{ deaths per 100,000 people per year}\end{aligned}$$

b. The average rate of change finds the rate at which the death rate changed each year *if we assume that the death rate changed by the same amount each year.* The rate of change is negative since the death rate is decreasing. If the death rate had decreased by the same amount each year over the 10-year period, it would have decreased by 6.42 deaths per 100,000 people each year.

c. To forecast the 2003 death rate, we assume that the average rate of change between 2000 and 2003 will be the same as that between 1990 and 2000. In part (a), we showed that the average rate of change was −6.42 deaths per 100,000 people each year. Because there are 3 years between 2000 and 2003, the 3-year change will be 3 times the average rate of change. We add this change to the 2000 death rate to forecast the 2003 death rate.

$$\begin{aligned}\text{2003 death rate} &= \text{2000 death rate} + 3(\text{average rate of change})\\ \text{2003 death rate} &= 257.6 + 3(-6.42)\\ &\approx 238.3\end{aligned}$$

We predict that the death rate due to heart disease in 2003 was 238.3 deaths per 100,000 people. Our projection differs from the death rate of 232.1 reported by the Census Bureau because we used the average rate of change between 1990 and 2000. The actual average rate of change between 2000 and 2003 was

$$\frac{232.1 - 257.6}{3 - 0} = -8.5 \text{ deaths per 100,000 people per year}$$

As Example 1 shows, we should exercise caution when using an average rate of change to forecast function values. The rate at which the function is changing could vary widely.

EXAMPLE 2 ■ Identifying Limitations in Using an Average Rate of Change

Refer to Table 1.12 and calculate the average rate of change in the death rate due to heart disease over the following time intervals: 1990 to 1991, 1992 to 1993, 1995 to 1996, and 1999 to 2000. Does the average rate of change between 1990 and 2000 (−6.42 deaths per 100,000 people per year) accurately represent those values?

Solution We calculate the average rate of change in the death rate due to heart disease over the given time intervals.

$$\text{1990 to 1991: } \frac{313.8 - 321.8}{1 - 0} = -8.0 \qquad \text{1992 to 1993: } \frac{309.9 - 306.1}{3 - 2} = 3.8$$

$$\text{1995 to 1996: } \frac{296.3 - 288.3}{5 - 4} = -8.0 \qquad \text{1999 to 2000: } \frac{257.6 - 267.8}{10 - 9} = -10.2$$

We see that the average rate of change varied widely, ranging from a 10.2 decrease to a 3.8 increase. The −6.42 average rate of change from 1990 to 2000 we calculated in Example 1 is a *constant* annual change that does not accurately predict wide variations in the actual annual rates of change for intermediate years.

EXAMPLE 3 ■ Calculating and Interpreting an Average Rate of Change

The movie *Ice Age: The Meltdown* was introduced on March 31, 2006. Table 1.13 shows the gross receipts for selected weekends within the first 9 weeks that the movie was shown in U.S. theaters.

Table 1.13

Weekend w	Gross Receipts (in millions) G
1	\$68.0
3	\$20.0
6	\$4.2
9	\$0.8

Source: www.the-numbers.com

a. Calculate the average rate of change for each pair of consecutive data points. Include units with each average rate of change.

b. Describe what is happening to the average rate of change for the first nine weekends.

Solution

a. The average rates of change are shown in Table 1.14.

Table 1.14

Weekend w	Gross Receipts (in millions) G	Average Rate of Change (in millions per weekend)
1	\$68.0	
		$\frac{20.0 - 68.0}{3 - 1} = -\$24.0$
3	\$20.0	
		$\frac{4.2 - 20.0}{6 - 3} \approx -\$5.3$
6	\$4.2	
		$\frac{0.8 - 4.2}{9 - 6} \approx -\$1.1$
9	\$0.8	

b. Between the first and third weekend, the average gross receipts were decreasing at an average rate of \$24.0 million per weekend. The theaters were bringing in an average of \$5.3 million less per weekend between the third and sixth weekends. Between the sixth and ninth weekend, gross receipts were decreasing by a lesser average amount of \$1.1 million per weekend. Notice that although the rates of change of the gross receipts are all negative, they are becoming less and less negative as time goes on. The *scatter plot* of the data shown in Figure 1.11 validates our conclusion. (A **scatter plot** is a graphical representation of a data table.)

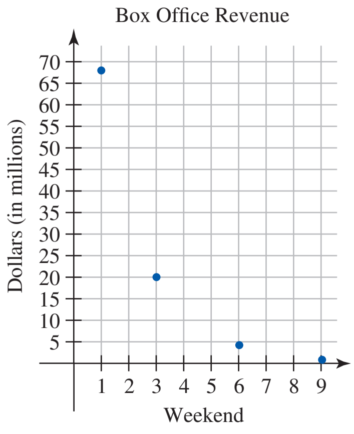


Figure 1.11

Predicting Unknown Data Values Using a Table

Despite the limitations of using the average rate of change to predict unknown data values, we often use it in forecasting when additional data are unavailable. Using the average rate of change is frequently more accurate than guessing.

EXAMPLE 4 ■ Estimating the Unknown Value of a Function at a Specified Input Value

Monthly life insurance premiums are based on the age and the gender of the insured. Table 1.15 records the monthly premium for a \$500,000 life insurance policy for a male. The table works well for the ages shown but if a man's age falls between those ages, we must estimate the monthly premium value. Estimate the monthly premium for a 53-year-old male.

Table 1.15

Life Insurance for Males	
Age a	Monthly Premiums for \$500,000 Worth of Coverage P
35	\$14
40	\$18
45	\$27
50	\$40
55	\$65
60	\$103
65	\$180
70	\$322

Source: www.Insure.com

Solution We first calculate the average rate of change for the data points that encompass the age 53: (50, \$40) and (55, \$65).

$$\begin{aligned} m &= \frac{P(55) - P(50)}{55 - 50} \frac{\text{dollars}}{\text{year of age}} \\ &= \frac{65 - 40}{55 - 50} \\ &= \$5.00 \text{ per year of age} \end{aligned}$$

For each year of age between 50 and 55, we estimate there is a \$5 increase in premium. Since 53 is 3 years more than 50, the premium for a 53-year-old male will be \$15 more than the premium for a 50-year-old male. The premium is given by

$$\begin{aligned} \text{age 53 premium} &= \text{age 50 premium} + (\text{average rate of change})(\text{difference in age}) \\ &= 40 + (5)(3) \\ &= 55 \text{ dollars} \end{aligned}$$

We estimate a 53-year-old male will pay a monthly premium of \$55 for \$500,000 worth of life insurance.

Functions Represented by Formulas

Although tables are useful, their value is limited because they can only show a finite number of values. In contrast, a function formula (equation) may be used to calculate any number of values. A **formula** is a succinct mathematical statement expressing a relationship between quantities.

For example, when a 154-pound woman exercises on a stair-step machine, the number of calories burned is related to the number of minutes she spends exercising on the machine. For every minute she exercises, she burns around 9 calories. (*Source:* www.fns.usda.gov) In other words, the total number of calories burned, C, is equal to nine times the number of minutes, m, spent operating the stair-step machine. Symbolically, we write

$$C(m) = 9m$$

To determine how many calories she burns in 5 minutes, we *evaluate* the function at $m = 5$.

$$C(m) = 9m$$
$$C(5) = 9(5)$$
$$= 45$$

When she exercises on the stair-step machine for 5 minutes, she burns 45 calories.

To determine how long it will take her to burn 150 calories, we set $C(m) = 150$ and solve the function for m. In other words, we substitute 150 for $C(m)$ and solve the resultant equation.

$$C(m) = 9m$$
$$150 = 9m$$
$$m \approx 16.67$$

If she wants to burn 150 calories, she must exercise on the stair-step machine for 16.67 minutes. Since most people don't end their timed exercise on a fraction of a minute, we'll round the number up to 17 minutes.

Is this calorie-burning formula, $C(m) = 9m$, a function? That is, does each value of the independent variable m result in exactly one dependent variable value C? Yes. A woman exercising for any number of minutes m would burn about the same number of calories C each time she exercised for m minutes in the same activity at the same level of intensity.

EXAMPLE 5 ■ Finding a Formula for a Cell Phone Plan

The Sprint Fair & Flexible cell phone plan costs one professor \$35.10 a month (including fees and taxes), if she does not go over her allotted 200 anytime minutes. However, she must pay \$0.28 a minute (including fees and taxes) for every minute over her limit. (*Source:* Sprint cell phone bill)

a. If she does not go over 200 minutes, the professor's monthly bill is \$35.10. Write a formula that she can use to calculate her monthly cell phone bill if she exceeds 200 minutes.

b. If she talks a half-hour over her monthly limit, what will be the total amount of her monthly cell phone bill?

Solution

a. The total cost, C, depends on the extra minutes, m, she uses. Since each extra minute costs \$0.28, the total cost will be the product of 0.28 and the number of extra minutes plus the cost of the basic plan. Symbolically, we write

$$C(m) = 0.28m + 35.10$$

b. A half-hour is 30 minutes, so $m = 30$. To calculate her bill, we evaluate $C(m)$ at $m = 30$.

$$C(m) = 0.28m + 35.10$$
$$C(30) = 0.28 \cdot 30 + 35.10$$
$$= 8.40 + 35.10$$
$$= 43.50$$

The professor's monthly phone bill is \$43.50.

For real-world situations that are complex and rely on multiple inputs, mathematicians have developed standardized formulas to help ordinary people make important life decisions. One such formula, the monthly loan payment formula, is illustrated in Example 6.

EXAMPLE 6 ■ Using a Multivariable Formula to Find the Amount of a Car Payment

The manufacturer's suggested retail price (MSRP) for a new 2006 Toyota Corolla was approximately \$15,000. (*Source:* www.toyota.com) As of May 30, 2006, Pentagon Federal Credit Union offered a 5-year new car loan with a 7.5% interest rate compounded monthly. The monthly payment formula is

$$M(p, i, n) = p\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right]$$

where

M = monthly payment in dollars
p = amount borrowed in dollars
n = number of monthly payments
i = interest rate per payment period as a decimal

What is the monthly payment on the loan if a person finances the full retail price of the car?

Solution To use the formula, we must first determine the value of each of the input variables: p, n, and i. Since \$15,000 is to be borrowed, $p = 15{,}000$. To determine the number of monthly payments, n, we must determine the number of months in 5 years.

$$\frac{5 \cancel{\text{ years}}}{1} \cdot \frac{12 \text{ months}}{1 \cancel{\text{ year}}} = 60 \text{ months}$$

So $n = 60$. To obtain the monthly periodic rate, i, we divide the decimal form of the annual rate, 7.5%, by the number of months in a year.

$$\begin{aligned} i &= \frac{0.075}{12} \\ &= 0.00625 \end{aligned}$$

The monthly periodic rate is 0.625%, but we use the decimal form in the formula.

Now that we know the values of the three input variables, we can calculate the payment amount.

$$\begin{aligned} M(p, i, n) &= p\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right] \\ M(15000, 0.00625, 60) &= 15{,}000\left[\frac{(0.00625)(1+0.00625)^{60}}{(1+0.00625)^{60} - 1}\right] \\ &= 15{,}000\left[\frac{(0.00625)(1.4533)}{1.4533 - 1}\right] \\ &= 15{,}000\left(\frac{0.009083}{0.4533}\right) \\ &\approx 300.57 \text{ rounded to the nearest cent} \end{aligned}$$

For a \$15,000 car financed with a 7.5% loan for a period of 5 years, the monthly payment is \$300.57.

EXAMPLE 7 ■ Using a Multivariable Formula to Determine a Loan Amount

On May 31, 2006, the national average rate on a 48-month car loan was 9.3%. (*Source:* www.bankrate.com) If we can afford a \$250 monthly car payment, what price car can we afford at this rate and loan length?

Solution We use the same formula as we did in Example 6. We know the monthly payment is \$250, so $M = 250$. Since the annual rate is 9.3%, the monthly periodic rate is

$$i = \frac{0.093}{12} = 0.00775$$

Finally, the loan period is 48 months, so $n = 48$. To find the affordable car price, which is the loan amount p, we substitute the values we know into the formula and solve for p.

$$\begin{aligned}
M(p, i, n) &= p\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right] \\
250 &= p\left[\frac{0.00775(1+0.00775)^{48}}{(1+0.00775)^{48} - 1}\right] \\
250 &\approx p\left[\frac{0.00775(1.4486)}{1.4486 - 1}\right] \\
250 &\approx p\left(\frac{0.01123}{0.4486}\right) \\
250 &\approx p(0.02503) \\
p &\approx 9988.92
\end{aligned}$$

Given a 9.3%, 48-month car loan with a monthly payment of \$250, we can afford to buy a vehicle that costs \$9,988.92.

You may have noticed in Example 7 that $\frac{250}{0.02503} \approx 9988.01$ instead of the 9988.92 shown. The difference in the two values occurs because of *round-off error.* To minimize the error introduced by rounding intermediate computations, we kept the calculated values in our calculator and used them to calculate our final answer instead of the rounded values we recorded in intermediate steps of the example. We deferred all rounding to the end of the problem. This approach helps reduce the error that occurs due to rounding and allows us to write the intermediate values with fewer significant digits.

SUMMARY

In this section you learned how to analyze functions in tables and formulas. Additionally, you discovered how to find average rates of change and use them to help forecast unknown results.

1.3 EXERCISES

SKILLS AND CONCEPTS

In Exercises 1–6 use the given formula to do each of the following.

a. Determine the indicated output value.

b. Interpret the meaning of the result.

1. The formula for calculating the perimeter of a rectangle, P, is $P(w, l) = 2w + 2l$, where w is the width in inches and l is the length in inches. Calculate the value of $P(5, 8)$.

2. Use the formula in Exercise 1 to calculate the value of $P(103, 808)$.

3. $V(r) = \frac{4}{3}\pi r^3$ is the formula for computing the volume of a sphere, V (in cubic centimeters), where r is the radius of the sphere (in centimeters). Compute the value of $V(100)$.

4. Use the formula in Exercise 3 to compute the value of $V(63)$.

5. The formula for calculating the area of a trapezoid, A, is $A(h, b_1, b_2) = \frac{h(b_1 + b_2)}{2}$, where h is the height, b_1 is one base, and b_2 is the second base. Each of the lengths is measured in meters. Calculate the value of $A(9, 14, 24)$.

6. Use the formula in Exercise 5 to calculate the value of $A(17, 25, 30)$.

SHOW YOU KNOW

7. Explain what is meant by "average rate of change." Particularly address the word *average*.
8. Find a situation where the average rate of change can be calculated. What does this calculation mean in the context of the situation?
9. Explain how someone's car insurance premium can be computed using a multivariable function. That is, describe which variables could be the independent variables and which could be the dependent variable.

MAKE IT REAL

For Exercises 10–15, do each of the following.

a. Determine if the table of data represents a function.
b. Calculate the average rates of change for consecutive pairs of data values.
c. Explain the meaning of the average rates of change in the context of the data.

10. Life Insurance Premiums

Life Insurance for Males		
Age a	Monthly Premiums for \$1,000,000 of Coverage P	Average Rate of Change
35	\$21	
40	\$30	
45	\$47	
50	\$75	
55	\$124	
60	\$198	
65	\$348	
70	\$628	

Source: www.Insure.com

11. Life Insurance Premiums

Life Insurance for Females		
Age a	Monthly Premiums for \$1,000,000 of Coverage P	Average Rate of Change
35	\$21	
40	\$25	
45	\$41	
50	\$57	
55	\$88	
60	\$130	
65	\$209	
70	\$361	

Source: www.Insure.com

12. Percent of 16-Year-Old Drivers

Year t	Percent of 16-Year-Olds with Driver's Licenses L	Average Rate of Change
1998	44.0%	
1999	37.0%	
2000	34.0%	
2001	32.5%	
2002	31.5%	
2003	30.5%	
2004	30.0%	

Source: Federal Highway Administration

13. Baby Girls' Average Weight

Age (in months) m	Weight (in pounds) W	Average Rate of Change
Birth	7.5	
6	16.0	
12	21.0	
18	24.0	
24	26.5	
30	28.5	
36	30.5	

Source: National Center for Health

14. Baby Boys' Average Weight

Age (in months) m	Weight (in pounds) W	Average Rate of Change
Birth	7.9	
6	17.4	
12	22.8	
18	25.9	
24	28.0	
30	29.8	
36	31.5	

Source: National Center for Health

15. Divorce Rates—Sixties and Seventies

Year t	Number of Divorces per 1000 Married Women, Age 15 and Older D	Average Rate of Change
1960	9.2	
1965	10.1	
1970	15.0	
1975	20.1	

Source: Rutgers National Marriage Project

In Exercises 16–29, use the data in the table to determine each solution.

16. Divorce Rates—From the Eighties

Year t	Number of Divorces per 1000 Married Women, Age 15 and Older D	Average Rate of Change
1980	22.7	
1985	21.7	
1990	20.7	
1995	19.7	
2000	18.7	
2005	17.7	

Source: **Rutgers National Marriage Project**

Estimate $D(1982)$ and interpret what it means in a real-world context.

17. China's Oil Demand

Year t	Demand (in millions of barrels per day) D	Average Rate of Change
2002	5.0	
2003	5.7	
2004	6.1	
2005	6.3	

Source: **International Energy Agency**

Estimate $D(2006)$ and interpret what it means in a real-world context.

18. Personal Bankruptcy in Japan

Year d	Number of Filings (in thousands) N	Average Rate of Change
1994	45.0	
1998	100.0	
2000	149.0	
2002	225.0	

Source: **Financial Services Agency**

Estimate the number of bankruptcy filings in 2001 and write your answer in function notation.

19. U.S. Lawmakers' Private Trips in 2006

Month m	Number of Trips N	Average Rate of Change
January	158	
February	61	
March	29	

Source: **Political Money Line**

Estimate the number of private trips that were given to U.S. Lawmakers in April 2006.

20. Amazon's Net Income

Year Y	Net Income ($ billions) R	Average Rate of Change
2002	3.9	
2003	5.3	
2004	6.9	

Source: **www.Amazon.com**

Estimate $R(2001)$ and interpret what it means in a real-world context.

21. Amazon's Net Income

Year Y	Net Income ($ billions) R	Average Rate of Change
2005	8.5	
2006	10.7	
2007	14.8	

Source: **www.Amazon.com**

Estimate the net income of Amazon for 2008. Write your answer in function notation.

22. Large U.S. Churches

Estimated Number of U.S. Churches with an Average Weekly Attendance of at Least 2000 People		
Year d	**Churches C**	**Average Rate of Change**
1960	0	
1970	10	
1980	115	
1990	300	
2000	600	
2003	800	
2005	1200	

Source: **Hartford Institute for Religion Research**

Estimate and explain the meaning of $C(1965)$.

23. Transportation Security Administration Fines

Year t	Fines F	Average Rate of Change
2002	279	
2003	3426	
2004	9741	

Source: **TSA**

a. What is the value of $F(2003)$ and what does it mean?

b. Estimate the number of fines issued by the TSA in 2005 and write your answer in function notation.

24. Daytime Emmy Award Viewers

Year t	Viewers (in millions) V	Average Rate of Change
2001	10.30	
2002	9.62	
2003	8.94	
2004	8.26	
2005	7.58	

Source: **Nielsen Media Research**

a. What is the value of $V(2003)$ and what does it mean?

b. How many viewers do you estimate watched the daytime Emmy awards in 2006? Explain your reasoning.

25. Exxon Mobil 2005 Net Income

Quarter q	Net Income (in billions) N	Average Rate of Change
2	6.9	
3	10.0	
4	10.5	

Source: **Exxon Mobil Corporation**

a. What is the value of $N(2)$ and what does it mean?

b. Estimate the net income for Exxon Mobil Corporation in the first quarter of 2006. Explain your reasoning.

26. Hispanic Marines

Year t	Percentage of Hispanic Marine Corps Recruits H	Average Rate of Change
2002	13.5%	
2003	14.5%	1.00
2004	16.0%	1.50
2005	16.5%	0.50

Source: **CNA Corporation**

a. What is the value of $H(2005)$ and what does it mean?

b. A newspaper stated, "The Marines are increasingly turning to Hispanics to fill their ranks." Do you agree with this conclusion?

27. Foreign Vehicle Demand in the U.S.

New Vehicles Sold in the U.S. (in thousands)			
Year	**Nissan**	**Honda**	**Toyota**
2000	700	1000	1400
2001	600	1050	1550
2002	620	1100	1550
2003	690	1190	1600
2004	830	1200	1800
2005	920	1250	1999

Source: **Autodata**

a. In each pair of consecutive years, which car had the largest average rate of change?

b. Which car would you predict had the largest average rate of change in 2006? Explain your reasoning.

28. Rates of Home Ownership

Year	United States	Arizona	Utah
1930	47.8%	44.8%	60.9%
1940	43.6%	47.9%	61.1%
1950	55.0%	56.4%	65.3%
1960	61.0%	63.9%	71.7%
1970	62.9%	65.3%	69.3%
1980	64.4%	68.3%	70.7%
1990	64.2%	64.2%	68.1%
2000	66.2%	68.0%	71.5%

Source: **U.S. Census Bureau, Housing and Household Economic Statistics Division Revised: 2004**

a. Which state has had the greatest increase over one decade? Include statistics to support your answer.

b. Which state has had the greatest decrease over one decade? Include statistics to support your answer.

29. U.S. Home Sales

2005 U.S. Home Sales ($ millions)		
Months m	**New Home Sales N**	**Existing Home Sales E**
February	1.24	6.87
March	1.30	6.88
April	1.27	7.08
May	1.28	7.07
June	1.29	7.22
July	1.36	7.06
August	1.27	7.20
September	1.24	7.19
October	1.34	7.02
November	1.23	7.01
December	1.27	6.68

Source: **Department of Commerce, National Association of Realtors**

a. Determine the months between which the function $N(m)$ is *increasing*. Support your decision with average rates of change.

b. Determine the months between which the function $N(m)$ is *decreasing*. Support your decision with average rates of change.

c. Determine the months between which the function $E(m)$ is decreasing. Support your decision with average rates of change.

30. Investment Account $B(p, r, n, t) = p\left(1 + \frac{r}{n}\right)^{nt}$ is the formula used to calculate the balance, B, in an investment account with a lump sum invested and interest compounded a fixed number of times a year for a certain number of years. The independent variables represent the following:

p is the amount invested.
r is the nominal interest rate as a decimal.
n is the number of times the compounding occurs in a year.
t is the years the money is invested.

Compute the value of $B(500, 0.05, 12, 2)$.

31. Investment Account Use the formula in Exercise 30 to compute the value of $B(350, 0.06, 12, 5)$.

32. Skid Distance The formula for calculating the minimum speed of a car in miles per hour at the beginning of a skid, S, is $S(d, f, n) = \sqrt{30\,dfn}$, where d is the skid distance in feet, f is the drag factor for the road surface, and n is the braking efficiency as a percent ($100\% = 1.00$). **(*Source:* www.harristechnical.com)** Calculate $S(60, 0.75, 1)$.

33. Skid Distance Use the formula in Exercise 32 to calculate $S(155, 0.2, 0.9)$.

In Exercises 34–35, use the formula to

a. Determine the value of the indicated input.

b. Interpret the meaning of the calculated value.

34. Commission-Based Income People who work in sales often earn a base salary plus a commission based on their sales. If the annual salary for a certain real estate agent is \$12,000 plus 6% of total sales, then

$$T(s) = 0.06s + 12{,}000$$

is the formula used to compute the total salary, T, as a function of the amount of sales, s, the person brings in. What is the value of s if $T = 30{,}000$?

35. Commission-Based Income Use the formula in Exercise 34 to find s given $T = 60{,}000$.

The formulas for Exercises 36–41 are standard measurement conversion formulas. **(*Source:* www.wikipedia.org)**

36. Temperature Conversion The formula for converting temperatures on the Kelvin scale, K, to temperatures on the Fahrenheit scale, F, is

$$F(K) = \frac{9}{5}K - 459.67$$

What is the value of K if $F = 98.33$?

37. Temperature Conversion Use the formula in Exercise 36 to find K given $F = 70$.

38. Temperature Conversion The formula

$$F(C) = \frac{9}{5}C + 32$$

converts the temperature on the Celsius, C, scale to an equivalent temperature on the Fahrenheit, F, scale. What is C if $F = 70$?

39. Temperature Conversion Use the formula in Exercise 38 to find F given $C = 70$.

40. Temperature Conversion The formula

$$C(F) = \frac{5}{9}(F - 32)$$

converts the temperature on the Fahrenheit scale, F, to the temperature on the Celsius scale, C. What is F if $C = 30$?

41. Temperature Conversion Use the formula in Exercise 40 to find F given $C = 10$.

42. Area of a Trapezoid The formula for the area, A, of a trapezoid is

$$A(h, b_1, b_2) = \frac{h(b_1 + b_2)}{2}$$

where h is the height, b_1 is one base, and b_2 is the second base. If $A = 50$, $b_1 = 7$, $b_2 = 13$, what is h?

43. Area of a Trapezoid Using the formula in Exercise 42, what is h if $A = 120$, $b_1 = 8$, $b_2 = 14$?

44. Investment Account Using the formula

$$B(p, r, n, t) = p\left(1 + \frac{r}{n}\right)^{nt}$$

what is p, if $B = 5395.40$, $r = 0.06$, $n = 12$, $t = 5$?

45. Investment Account Using the formula in Exercise 44, what is p, if $B = 5395.40$, $r = 0.03$, $n = 12$, $t = 5$?

46. Vehicle Cost A vehicle owner wants to calculate the total cost of his 2007 Jeep Compass with a MSRP of \$18,366. His monthly loan payment is \$317.54 for 5 years after he puts down a \$2000 down payment. **(Source: Car price at www.edmunds.com)**

a. Write a formula for the total amount he has paid toward the cost of the car (including down payment), T, as a function of the number of months he has made payments on the loan, m.

b. What is the total cost of the Jeep, after he has made all of the payments?

c. How much money has he paid in interest for his Jeep?

47. Vehicle Cost A student has a monthly car payment of \$172.55 a month for 6 years on a 2007 Blazing Blue two-door hatchback Toyota Yaris valued at \$13,210 in June 2006. **(*Source:* www.edmunds.com)** Her down payment was \$3000.

a. Write a formula for the total cost of the car, T, as a function of the number of months she will have to pay on the loan, m.

b. What is the total cost of the Yaris after she has paid all of the payments?

c. How much money has she paid in interest for her Toyota?

48. Car Payments The formula for monthly payments on a vehicle loan is

$$M(p, i, n) = p\left[\frac{i(1 + i)^n}{(1 + i)^n - 1}\right]$$

where

p = amount borrowed in dollars
n = number of monthly payments
i = interest rate per payment period as a decimal

a. Compute $M(2000, 0.005, 24)$.

b. Interpret the meaning of the value of the output, M.

49. Car Payments Use the formula in Exercise 48 to

a. Compute $M\left(600, \frac{0.098}{12}, 12\right)$.

b. Interpret the meaning of the value of the output.

50. Investment Future Value The formula for calculating the future value of an investment with constant periodic payments at a fixed rate for an established number of months is

$$FV(p, i, n) = p\left[\frac{(1 + i)^n - 1}{i}\right]$$

where

p = constant monthly payment
i = monthly interest rate
n = number of monthly payments

Compute $FV(10, 0.00425, 60)$ and explain the meaning of the value of the output.

51. Investment Future Value Use the formula in Exercise 50 to compute $FV(100, 0.00425, 60)$ and explain the meaning of the value of the output.

52. Investment Future Value An investor plans to contribute \$50 a month for 4 years in an ING Direct account with a 4.16% annual interest rate compounded monthly. (**Source: Rate quote at www.ingdirect.com**)

a. Write a formula for her investment account value using the formula found in Exercise 50.

b. Calculate the future value of the investment at 4 years.

53. Investment Future Value An investor plans to save \$75 a month for 5 years in a Citibank account earning 4.64% compounded monthly as advertised at www.direct.citibank.com in June 2006.

a. Write a formula for his investment account value using the formula found in Exercise 50.

b. Calculate the future value of the investment at 5 years.

■ STRETCH YOUR MIND

Exercises 54–55 are intended to challenge your understanding of functions.

54. Suppose that between 2000 and 2005, a person's salary increases at an average rate of \$2200 per year. Then, between 2005 and 2009, the person's salary increases at an average rate of \$1600 per year. What is the average rate of change in salary between 2000 and 2009?

55. The price of a particular item *increases* at an average rate of \$0.45 per month for the first 5 months of the year. For the last 7 months of the year, the price *decreases* by \$0.30 per month. What is the average rate of change in the price over the year?

SECTION 1.4 Functions Represented by Graphs

LEARNING OBJECTIVES

- Graph functions on the rectangular coordinate system
- Find a function's practical domain and practical range values from its graph
- Determine the vertical intercept (initial value) and horizontal intercepts of a function from a graph and interpret their real-world meanings

GETTING STARTED

Medical professionals often use growth charts like the one shown in Figure 1.12. By knowing how to read the graph, they are able to determine if the head circumference and weight of a child fall within an expected range. They may also predict the future growth of the child.

In this section we see how functions are represented graphically. We also discuss how to interpret a function graph.

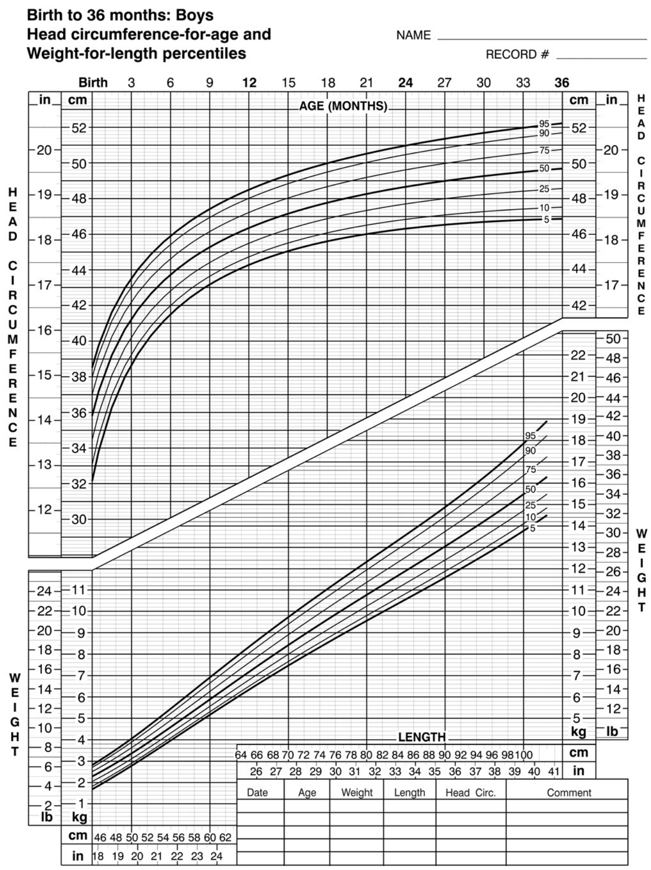


Figure 1.12

Source: www.cdc.gov

PEER INTO THE PAST

DESCARTES

René Descartes, born in France on March 31, 1596, was considered a "jack of all trades," contributing to mathematics, philosophy, anatomy, psychology, and optics. At the very young age of eight, he entered college. A sickly and feeble youth, Descartes was allowed by his parents to stay in bed until 11 o'clock in the morning—a habit he continued to do until the year of his death. While in bed, he tried to bridge the gap between the seeming certainty of mathematics and the controversial nature of philosophy.

In mathematics, Descartes unified algebra and geometry through Cartesian coordinates (rectangular or x-y coordinates). In his other work, Descartes explored the concepts of self, God, and mind. He asked "if our senses are all an illusion created by a malicious deceiver, what can we trust?" His answer was that we can doubt, and that the deceiver cannot cause us to doubt our own existence. Thus, the famous "cogito ergo sum"—"I think, therefore I am."

Source: http://www.math.psu.edu/tseng/class/descartes.html

■ Graphs of Functions

When given the equation of a function, we can generate a table of values and then plot the corresponding points on the **rectangular (Cartesian) coordinate system**. On the graph, y is frequently used in place of the function notation $f(x)$. That is, $y = f(x)$. Then the horizontal axis shows the value of the independent variable, x, and the vertical axis shows the value of the dependent variable, y.

Once we have drawn a sufficient number of points to determine the basic shape of the graph, we typically connect the points with a smooth curve. For example, the function $y = x^3 - 9x$ has the corresponding table of values (Table 1.16) and graph (Figure 1.13). (Note: Use caution when connecting the points with a smooth curve. Be sure that the graph transitions smoothly from one point to the next.)

Table 1.16

x	y
–3	0
–2	10
–1	8
0	0
1	–8
2	–10
3	0

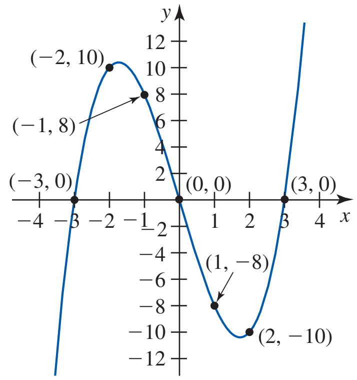


Figure 1.13

EXAMPLE 1 ■ **Estimating Function Values from a Graph**

Estimate $f(-3)$ and $f(2)$ using the graph of $f(x) = x^3 - 16x$ shown in Figure 1.14.

Solution

It appears from the graph in Figure 1.15 that $f(-3) \approx 20$ and $f(2) \approx -25$. Calculating these values with the algebraic equation, we see that our estimates are very close.

$$\begin{aligned} f(-3) &= (-3)^3 - 16(-3) \\ &= -27 + 48 \\ &= 21 \end{aligned} \qquad \text{and} \qquad \begin{aligned} f(2) &= (2)^3 - 16(2) \\ &= 8 - 32 \\ &= -24 \end{aligned}$$

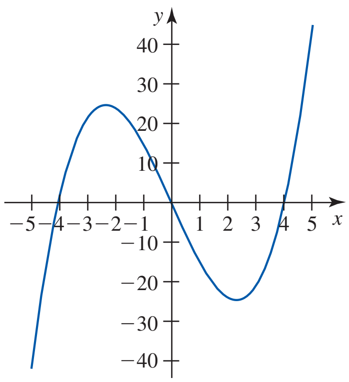


Figure 1.14

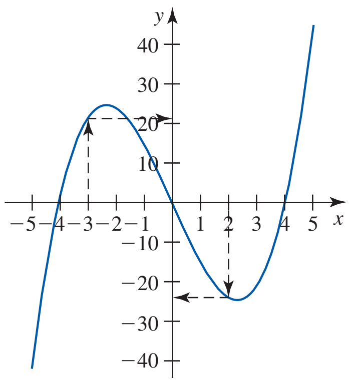


Figure 1.15

EXAMPLE 2 ■ Determining Input and Output Values from a Graph

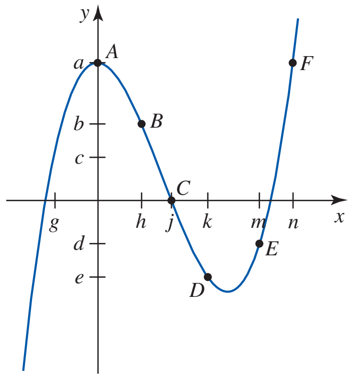


Figure 1.16

The graph in Figure 1.16 is a function $f(x)$ with labeled points A, B, C, D, E, and F; input values g, h, j, k, m, n, p, and q; and output values a, b, c, d, and e.

a. Evaluate $f(h)$.

b. Evaluate $f(n)$.

c. Evaluate $f(0)$.

d. Solve $f(x) = d$ for x.

Solution

a. To evaluate $f(h)$ means to find an output value for the input value h. In other words, we need to find a y-value for the x-value h. We locate h on the horizontal axis, go up to the function graph f, and then over to find the output value of b. Therefore, b is the answer, written in function notation as $f(h) = b$ or as the ordered pair (h, b). (Note that B is *not* the answer because B is simply the name of the point and not the output value of the point.)

b. Again, to evaluate $f(n)$ means to find an output value for the input value n. We locate n on the horizontal axis, go up to the function graph f, and then over to find the output value of a. Therefore, a is the answer, written in function notation as $f(n) = a$ or as the ordered pair (n, a).

c. To evaluate $f(0)$, we locate 0 on the horizontal axis, go up to the function graph f, and encounter the output value a. Therefore, a is the answer, written in function notation as $f(0) = a$ or as the ordered pair $(0, a)$.

d. This problem is different from parts (a)–(c). To "solve $f(x) = d$ for x" means that we know the output is d and we are to find the input value for x. To do this, we locate d on the *vertical axis*, go over to the function graph f to the point labeled E, and then up to the horizontal axis. When we do this we see that we are near the value m so our estimate is m. We can write the solution in function notation as $f(m) = d$ or as the ordered pair (m, d). We also observe that $f(x) = d$ for an unlabeled value of x between j and k and an unlabeled value of x to the left of g.

■ Domain and Range

When considering the graph of a function, we need to consider what values are reasonable for the input and output. The concepts of *domain* and *range* address this issue.

Domain and Range

- The set of all possible values of the *independent* variable (input) of a function is called its **domain.** When we limit the domain to values that make sense in the real-world context of a problem, we get the **practical domain.**
- The set of all possible values of the *dependent* variable (output) of a function is called its **range.** When we limit the range to values that make sense in the real-world context of a problem, we get the **practical range.**

(One way to keep the terms straight is to observe that the terms *in*put, *in*dependent variable, and doma*in* all contain the word *in*.) When a value from the domain is substituted for the independent variable and the corresponding output is evaluated, the result is a value of the dependent variable.

EXAMPLE 3 ■ Determining Practical Domain and Range

Determine (a) the practical domain and (b) the practical range for the head circumference of a boy who is in the 50th percentile as shown in Figure 1.17. Use the child's age as the independent variable, t, and the head circumference as the dependent variable, H.

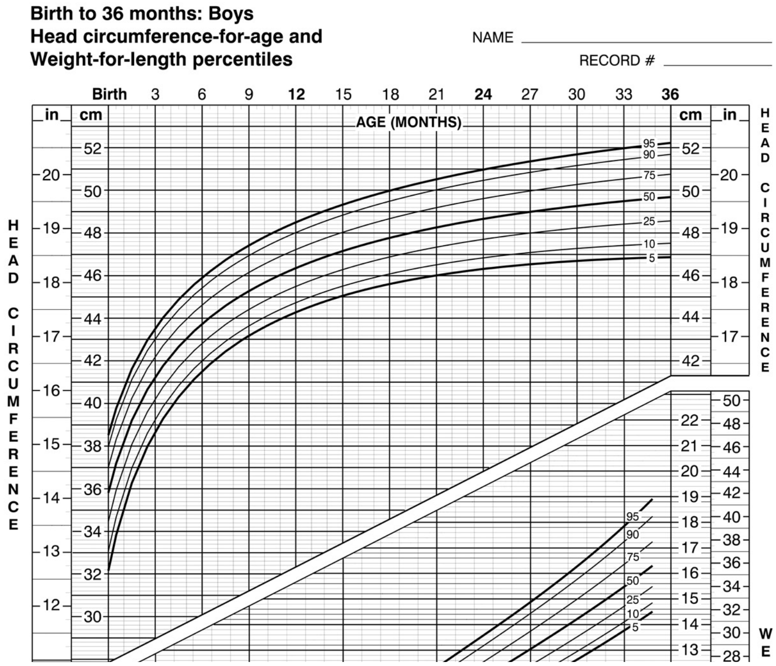


Figure 1.17

Source: www.cdc.gov

Solution

a. The practical domain for the head circumference function is $0 \leq t \leq 36$ because the graph goes from age 0 months to 36 months in the horizontal direction.

b. We find the curve on the graph marked with the 50. This is the graph representing the 50th percentile. The practical range for the function is $36 \leq H \leq 49.6$ because the head circumference ranges from 36 cm at birth to 49.6 cm at age 36 months.

■ Interpreting Graphs in Context

When we use a graph to model a real-world situation, it is important to keep track of the independent and dependent variables, and to understand the contextual meaning of the coordinates of a point. For example, the graphs of functions often intersect the vertical and horizontal axes at what we call the *vertical* and *horizontal intercepts.* A vertical intercept may also be referred to as the **initial value**. We investigate this in the following example.

EXAMPLE 4 ■ Interpreting the Real-World Meaning of a Graph

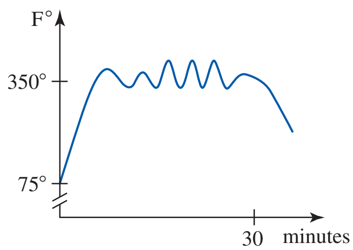


Figure 1.18

The graph in Figure 1.18 is a mathematical model of the temperature of an oven over time. The independent variable is time in minutes and the dependent variable is temperature in degrees Fahrenheit (°F). Describe what the graph indicates and include an explanation of any intercepts.

Solution The vertical intercept (where the graph intersects the vertical axis) shows that the oven started at approximately 75° (room temperature). We represent this by the ordered pair (0, 75). This vertical intercept may also be described as the initial value because at time 0 minutes (when we started keeping time) the initial oven temperature was 75°. The oven then heated up slightly past 350°. The oven then repeatedly cooled off a few degrees and heated up a few degrees above 350°, possibly trying to maintain a temperature of 350°. Near the 30-minute mark the oven may have been turned off, because it began to cool off. The graph does not cross the horizontal axis so there is no horizontal intercept. A horizontal intercept would show us the time in minutes when the oven temperature was 0°, and it is reasonable to assume that the oven temperature would never reach that low (unless it was in a landfill in the Arctic).

At first the relationship between data from a table and its associated graph may seem trivial. In reality, it is fundamental to developing a deep understanding of graphical representations of functions. Let's investigate this in Examples 5 and 6.

EXAMPLE 5 ■ Drawing the Graph of a Real-World Situation

Due to excessive air traffic, the air traffic control tower informs a pilot that she must go into a holding pattern and wait for her turn to land. Given that she has been told to circle the airport at a radius of 3 miles for 10 minutes, construct a graph to represent the distance (in miles) of the plane from the center (the airport) of its circular flight path as a function of the time (in minutes) during those 10 minutes.

Solution Initially many of us think that the graph of the distance of the plane from the airport is a circle because the plane is circling the airport. However, if we pay close attention to what the coordinates of the points mean we can create an accurate model. "A radius of 3 miles from the airport" means 3 miles from *the center* of the circular path. Thus the independent variable is time, t, (in minutes) and the dependent variable is distance, d, (in miles) *from the center of the circular flight path*. We begin by plotting points for 0 minutes, 1 minute, and 2 minutes, as shown in Figure 1.19: (0, 3), (1, 3), and (2, 3).

We can see that at each point the plane is 3 miles from the center. This pattern continues for the 10 minutes that the plane circles, forming the horizontal line at $d = 3$ shown in Figure 1.20.

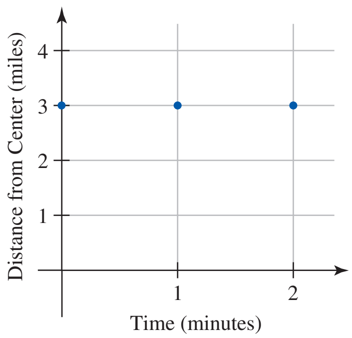


Figure 1.19

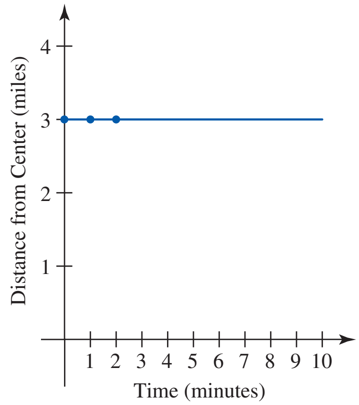


Figure 1.20

EXAMPLE 6 ■ Interpreting a Graph of a Real-World Situation

A child tosses a ball straight up and allows it to fall to the ground. He tosses the ball from 2 feet above the ground with an initial velocity of $3\,\frac{\text{feet}}{\text{sec}}$. We can model this by the formula $h(t) = -16t^2 + 3t + 2$. (Note: We demonstrate how to generate this formula in a later section.) Use the graph of the height function found in Figure 1.21 to answer the following questions.

a. Explain what the graph means in its real-world context.

b. Is the graph of the height function the actual path of the ball after the child tosses it? Explain.

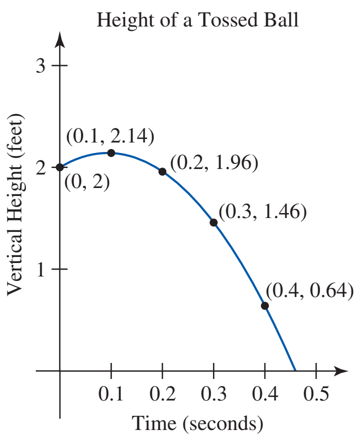


Figure 1.21

Solution

a. The graph shows an initial value (vertical intercept) at (0, 2), which means that at 0 seconds the ball was at 2 feet. The height of the ball increased to 2.14 feet and then changed direction and headed toward the ground.

b. No, it is the graph of the height as a function of the time. As noted in the problem description, the ball was tossed "straight up," so the actual path of the ball is a vertical path upward then downward.

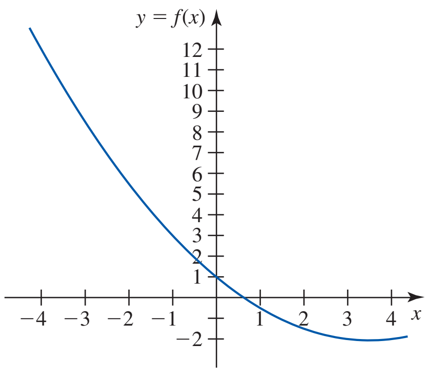
SUMMARY

In this section you learned that functions may be represented by graphs in addition to formulas and tables. You learned that graphs are made up of individual points, often connected with a smooth curve, that represent data. To understand what a graph means in context, you learned to focus on what the points represent in terms of the data. You discovered how to find the input and output of a function from its graph by locating values on the independent and dependent axes. Finally, using a graph and its real-world context you learned how to interpret the meaning of graphs, find the practical domain and range values, and determine and interpret the vertical (initial value) intercept.

1.4 EXERCISES

SKILLS AND CONCEPTS

In Exercises 1–5, refer to the graph of $f(x)$.



1. Evaluate $f(-3)$.

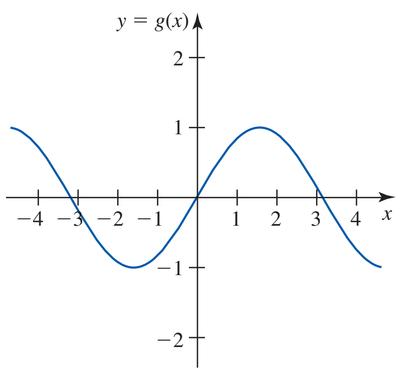
2. Solve $f(x) = 2$ for x.

3. Solve $f(x) = 0$ for x.

4. Evaluate $f(0)$.

5. Evaluate $f(4)$.

In Exercises 6–10, refer to the graph of $g(x)$.



6. Evaluate $g(-1)$.

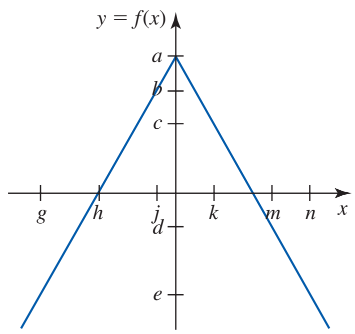
7. Solve $g(x) = 1.5$ for x.

8. Solve $g(x) = 0$ for x.

9. Evaluate $g(0)$.

10. Evaluate $g(1.5)$.

For Exercises 11–15, refer to the graph of $f(x)$ with x-values g, h, j, k, m, n and y-values a, b, c, d, e. Determine a possible solution to each equation and a function value for each expression.



11. Evaluate $f(g)$.

12. Solve $f(x) = a$ for x.

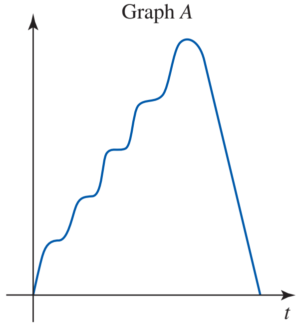
13. Solve $f(x) = e$ for x.

14. Evaluate $f(k)$.

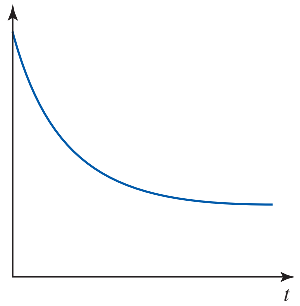
15. Evaluate $f(j)$.

For Exercises 16–20, match the five written scenarios with the most appropriate graph given in A–E. As you look at each graph, remember that time is advancing from left to right.

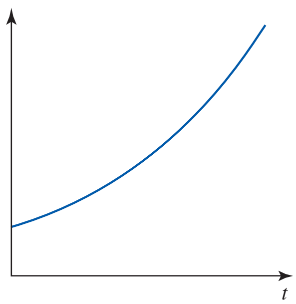
Graph A



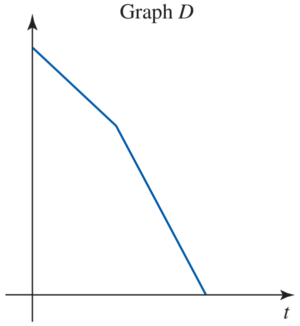
Graph B



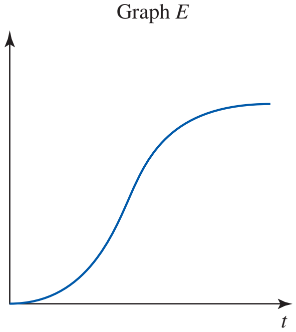
Graph C



Graph D



Graph E



16. It took me several breaths to inflate a balloon. As I started to tie it off, it slipped from my hand and flew around the room. The amount of air in the balloon is the dependent variable.

17. At the beginning of spring, the grass grew slowly and I seldom had to mow the lawn. By midsummer it was growing very fast, so I mowed twice a week. In fall, I only had to mow once in a while. When winter came I didn't mow at all. The cumulative number of times the lawn has been mowed to date is the dependent variable.

18. The amount of money in my savings account started out growing slowly because I didn't have much money in it. However, now that I have a larger amount in the account, interest is helping my balance grow much faster. The balance in my savings account is the dependent variable.

19. I put water in the ice-cube tray and placed it in the freezer to make ice. The temperature of the water in the ice-cube tray is the dependent variable.

20. I started walking to class at a constant pace but realized that I was going to be late so I started walking faster and maintained my fastest rate until I reached class. My distance from the classroom is the dependent variable.

SHOW YOU KNOW

For Exercises 21–25, use the graphs A–E (given in the instructions for Exercises 16–20) to answer each question.

21. Describe what the vertical and horizontal intercepts for graph A mean in its real-world context.

22. Describe what the vertical and horizontal intercepts for graph D mean in its real-world context.

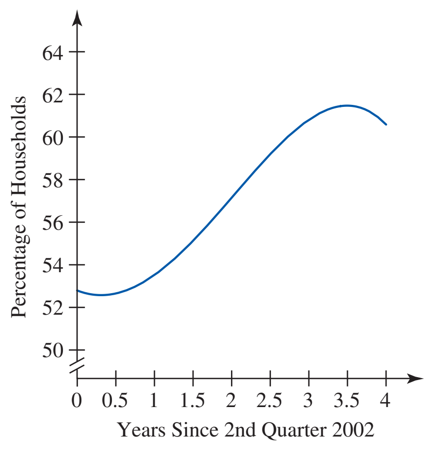
23. Describe what the vertical and horizontal intercepts for graph E mean in its real-world context.

24. Determine the practical domain and range for the function represented by graph C.

25. Determine the practical domain and range for the function represented by graph B.

MAKE IT REAL

26. Cell Phone Subscribers Industry statistics reveal that the number of cellular subscribers in the United States totaled nearly 195 million at the end of 2005. A recently completed study revealed that the majority of consumer households are now acquiring multiple cellular phones. The following graph shows the percentage of households with multiple cellular phones, m, as a function of the number of years since the second quarter of 2002, y.

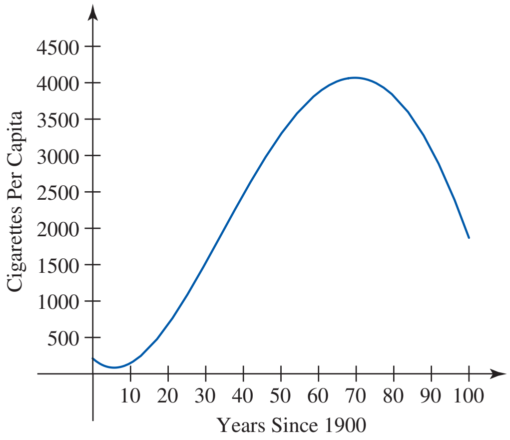


Source: **www.icrsurvey.com**

a. Determine the practical domain and range for $m = f(y)$.

b. Explain what the graph means in its real-world context.

27. Cigarette Consumption The number of cigarettes consumed by adults over the age of 18 in the United States, c, is a function of the years since 1900, t, and is given in the following graph.



Source: **www.infoplease.com**

a. Determine the practical domain and range for $c = f(t)$.

b. What conclusions about cigarette consumption can you draw from the graph?

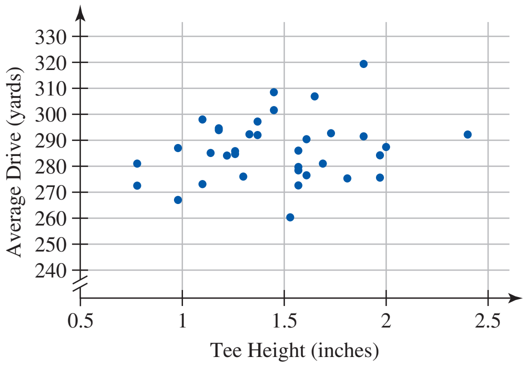
c. What does the vertical intercept mean in its real-world context?

28. Golf Pro Drives The following scatter plot shows data collected during a study at the driving range at Bay Hill Country Club. The data show the average drive in yards of 35 Professional Golf Association tour pros versus the height at which they tee the ball up (in inches). Use the plot of the data to answer the following questions.

a. Do these data represent a function? Explain.

b. Why do you think the data are spread out as they are? Provide a reasonable explanation.

c. If an additional professional golfer joins the study and chooses to tee up the ball 1.55 inches, do you feel confident in your ability to estimate how far of a drive he will hit?

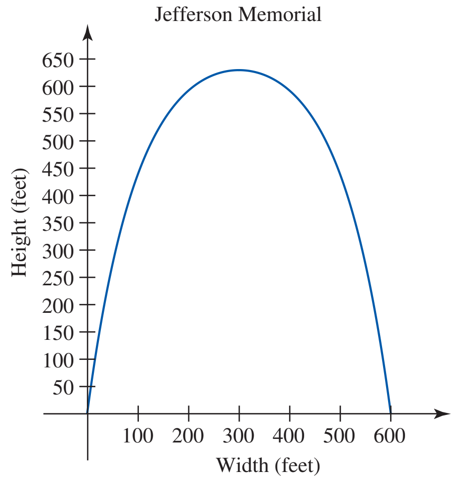


Source: Golf **Magazine, June 2006**

29. Jefferson Memorial The Jefferson Memorial (also commonly known as the St. Louis Gateway Arch) is an elegant monument to westward expansion in America. Located on the banks of the Mississippi River in St. Louis, Missouri, the 630-foot-tall stainless steel arch dominates the city skyline.

Philip Eppard/Shutterstock.com

A graphical representation of the arch based on the equation used by the architect is shown in the following figure. Use it to answer the following questions. (Note: The graphical representation does not take into account the thickness of the arch.)



a. Inside the arch is a tram that takes visitors to the top of the arch. Approximately how high off the ground will the tram be when it is 100 horizontal feet away from its original position (the origin)?

b. When the tram is 500 vertical feet above the ground, what is its horizontal distance from its original position (the origin)? (Note: There are two possible solutions.)

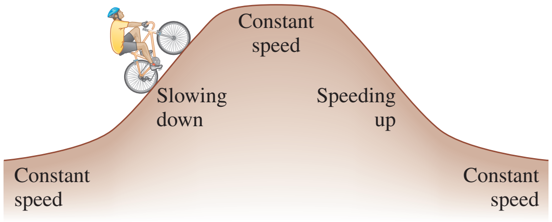
c. The *Titanic* was a famous cruise ship that ran into an iceberg and sank in 1912 even though it was touted as being virtually unsinkable. The ship had a length of 882 feet. If the *Titanic* had been placed lengthwise on the ground below the St. Louis Arch, would it fit between the two legs of the arch?

d. On July 2, 1982, the *St. Louis Post-Dispatch* posed the following hypothetical problem: If the *Spirit of St. Louis* airplane (wingspan of 46 feet) was flown directly through the middle of the arch, 200 feet above the base, how far would the wingtips be from the inside edge of the arch? Give an answer to this question and provide a justification for your result.

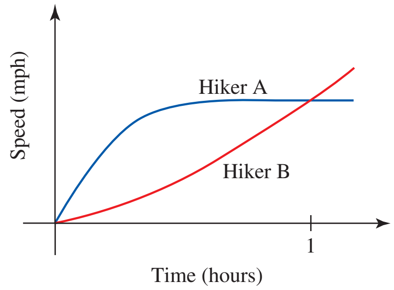
■ STRETCH YOUR MIND

Exercises 30–31 are intended to challenge your ability to work with graphs.

30. Cyclist Position The following diagram is a side view of an individual cycling up and over a hill. Considering the information provided on the diagram, draw a sketch of the speed of the cyclist as a function of the horizontal position of the cyclist from the start.



31. Hiker Position The given graph represents speed vs. time for two hikers. (Assume the hikers start from the same position and are traveling in the same direction.)



a. What is the relationship between the position of Hiker A and Hiker B 1 hour into the hike? In other words, is Hiker A or Hiker B ahead or are they at the same position? Justify your conclusion.

b. Which hiker is going faster 30 minutes into the hike? Explain how you know.

c. Between 45 and 60 minutes into the hike, is one hiker pulling away from the other? Justify your response.

SECTION 1.5 Functions Represented by Words

LEARNING OBJECTIVES

- Convert words representing function relationships into symbolic and graphical representations
- Translate functions given in equations, tables, and graphs into words

GETTING STARTED

Environmentalists and concerned citizens have actively promoted Earth-friendly policies over the past several decades. In 1970, Congress created the Environmental Protection Agency (EPA) "to protect human health and the environment." (*Source:* www.epa.gov) One of the roles of the EPA is to monitor air quality and crack down on polluters. By analyzing air quality trends, the EPA is able to measure its own effectiveness.

In this section we discuss how to convert words representing function relationships into symbolic and graphical representations. We also detail how to translate functions given in equations, tables, and graphs into words. These skills may be used by all of us to interpret public reports such as those issued by the EPA.

■ Recognizing a Function in Words

One of the EPA goals is to reduce smog-forming nitrogen oxides (NO_x). In a 2007 report, the EPA stated, "In 2007, . . . sources emitted approximately 506,000 tons of NO_x, an overall decrease of about 1,300 tons from 2006." (*Source: NO_x Budget Trading Program: Compliance and Environmental Results 2007*, p. 17).

We can represent the relationship between the 2006 and 2007 nitrogen oxide emission levels using functions. We let the variable x represent the 2006 emissions level and the variable y represent the 2007 emissions level. Since the 2007 level is 1300 tons less than the 2006 level, the relationship between x and y is

$$y(x) = x - 1300$$

What was the 2006 emissions level? Since we know $y = 506{,}000$ tons, we can find x.

$$506{,}000 = x - 1300$$
$$x = 507{,}300$$

The 2006 level was 507,300 tons.

The process for converting a function given in words to symbolic notation is given in the box below. With a little practice, you will become proficient in translating words into symbols for simple mathematical relationships. For more complex relationships, you will need to develop additional mathematical skills to accomplish Step 2 of the process.

HOW TO: ■ TRANSLATE A FUNCTION GIVEN IN WORDS INTO SYMBOLIC NOTATION

1. Identify the things that are being related to each other.
2. Express the mathematical relationship between the things using words.
3. Select variables to represent the things being related to each other.
4. Rewrite the mathematical relationship from Step 2 using the variables from Step 3 and appropriate mathematical notation.

EXAMPLE 1 ■ Translating a Function in Words into Symbolic Notation

In 2008, the Bureau for Economic Analysis (BEA) issued a press release detailing per capita income rankings for 2007. An excerpt from the report is provided here.

> Connecticut led the nation with a per capita income of $54,117. . . . Mississippi had the lowest per capita income of all states, $28,845. . . . [The national average was $38,611.] (*Source:* www.bea.gov)

a. How many dollars above (or below) the national average is the per capita income of Connecticut and Mississippi?

b. Let n represent the national average per capita income. Use function notation to represent the per capita income of each state as a function of the national average per capita income.

Solution

a. We calculate the difference between each state's average per capita income and the national average per capita income.

$$54{,}117 - 38{,}611 = 15{,}506 \qquad 28{,}845 - 38{,}611 = -9766$$

The per capita income of Connecticut was $15,506 above the national average in 2007. The per capita income of Mississippi was $9766 below the national average in 2007.

b. Let c represent the per capita income of Connecticut and m represent the per capita income of Mississippi. We have

$$c(n) = n + 15{,}506 \quad \text{and} \quad m(n) = n - 9766$$

We can often represent functions given in words graphically, as demonstrated in Example 2. Graphical representations are especially useful in depicting rates of change.

EXAMPLE 2 ■ Translating a Function in Words into a Graph

Apple Computer Corporation has captured the attention of millions of consumers with its innovative iPod digital music player. The company published the following in its 2005 Annual Report.

> Net sales of iPods rose $3.2 billion . . . during 2005 compared to 2004. Unit sales of iPods totaled 22.5 million in 2005 . . . [up] from the 4.4 million iPod units sold in 2004. (*Source:* Apple Computer Corporation 2005 Annual Report, p. 32)

The report further indicated that iPod sales revenue was $1.3 billion in 2004 and that 0.9 million iPod units were sold in 2003, generating $0.3 billion in revenue. Use a graph to model the revenue from iPod sales as a function of iPod units sold. Then use the graph to predict sales revenue when 15 million iPods are sold. (Note: The terms *sales*, *revenue*, and *sales revenue* are often used interchangeably.)

Solution We are asked to draw a graph representing iPod sales revenue as a function of iPod units sold. We first construct a data table to record what we know (Table 1.17). We make a scatter plot for the data and then draw a continuous, smooth curve through the data points, as shown in Figure 1.22.

From the graph, it appears that when 15 million iPod units are sold the sales revenue will be roughly 3.5 billion dollars.

Table 1.17

Year	Units Sold (in millions)	Revenue ($ billions)
2003	0.9	0.3
2004	4.4	1.3
2005	22.5	$1.3 + 3.2 = 4.5$

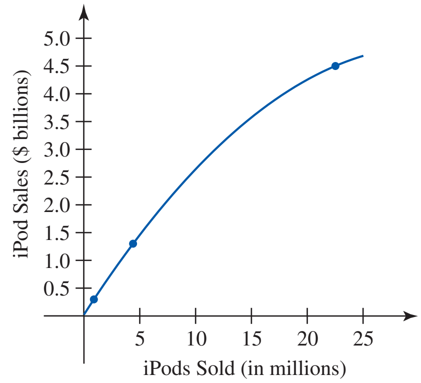


Figure 1.22

As stated earlier, graphs are especially useful in depicting rates of change. The graph in Example 2 was increasing, indicating that an increase in units sold corresponded with an increase in sales revenue. The graph is also curved downward, indicating that the rate at which sales revenue is increasing is decreasing. In other words, as the number of iPods sold increases, the revenue per unit sold decreases.

■ Interpreting the Meaning of a Function Using Words

Although it is important to be able to translate words into function notation and graphs, it is equally important to be able to reverse the process. A mathematical model is of no use if we are unable to interpret the meaning of the results in a real-world context. As we show in Example 3, keeping track of the units, or quantities represented by the variables, is one of the best ways to make sense out of function results.

EXAMPLE 3 ■ Interpreting in Words the Meaning of a Function Equation

Based on data from 1990 to 2003, the number of golf facilities in the United States can be modeled by

$$g(t) = \frac{4633}{1 + 59.97e^{-0.2567t}} + 12{,}000 \text{ facilities}$$

where t is the number of years since the end of 1990. (*Source:* Modeled from *Statistical Abstract of the United States, 2004–2005*, Table 1240) What does $g(23) = 15{,}982$ mean?

Solution The function g looks very complex and intimidating. In fact, we might initially think we will not be able to answer the question. However, let's apply a little mathematical reasoning.

What is the meaning of each of the variables? The independent variable t is the number of years since the end of 1990. Since $t = 23$, we are evaluating the function at the point in time 23 years after the end of 1990.

$$1990 + 23 = 2013$$

That is, we are evaluating the function in 2013.

What is the meaning of the dependent variable g? From the function equation we see that g is the number of golf facilities in year t. Therefore, $g(23)$ is the number of golf facilities in year 2013. Thus $g(23) = 15{,}982$ means that at the end of 2013 there will be 15,982 golf facilities in the United States (according to the model).

What about the variable e? Actually, e is not a variable. It is an irrational number so commonly used that it has its own special name: e. The number $e \approx 2.71828$. In later chapters, we will discuss the origin of e and its many uses.

Interpreting the real-world meaning of a table of values also requires that we keep track of the meaning of the variables. This is especially true when we are asked to use computations to explain the meaning of table results.

Table 1.18

Years Since 1990 t	Per Capita Spending on Prescription Drugs (dollars) P
0	158
5	224
8	311
9	368
10	423
11	485
12	552
13	605

Source: Statistical Abstract of the United States, 2006, Table 121

EXAMPLE 4 ■ Interpreting in Words the Meaning of a Function Table

Describe in words the meaning and real-world significance of the data in Table 1.18.

Solution There are multiple ways to interpret the data from a table in words. Let's calculate the average rate of change and then write statements to describe the significance of the data in the table.

$$\begin{aligned}\text{average rate of change} &= \frac{605 - 158}{13 - 0}\,\frac{\text{dollars}}{\text{year}}\\ &= 34.38 \text{ dollars per year}\end{aligned}$$

The table shows that per capita spending on prescription drugs has increased rapidly. In 2003, this spending reached a new high ($605 per person). Although per capita prescription drug spending increased an average of $34.38 per year between 1990 and 2003, it jumped by $53 between 2002 and 2003. The table may indicate that future increases in spending are likely to be even more extreme.

In Example 4, we gave one interpretation of the data given in the table and gave our opinion on what we expected for the future. A different person summarizing the data may write factual statements very different from those we presented. Because there could be several ways to interpret the same data set correctly, another person's written analysis should not be rejected simply because it is not identical to one's own conclusions.

As was the case with symbolic and tabular representations of functions, keeping track of the meaning of the independent and dependent variables is essential to understanding the graphical results.

EXAMPLE 5 ■ Interpreting in Words the Meaning of a Function Graph

Malaria is a major killer of children under 5 years old in Africa. The World Health Organization and a variety of humanitarian organizations are working together with the people of Africa to implement innovative strategies to save lives. One such strategy includes the distribution of insecticide-treated nets (ITNs). The nets purportedly can reduce malaria transmission by more than half. Families who choose to sleep under the nets are protected during the times of the day when mosquitoes, the transmitters of the malaria infection, are most active.

Based on data from 1999–2003, the number of insecticide-treated nets distributed in Africa may be modeled as a function of the year. The raw data and a function model of the data are shown in Figure 1.23.

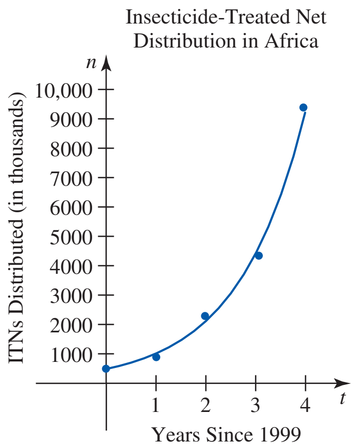


Figure 1.23

Source: Modeled from data at www.afro.who.net

Given that the independent variable t represents the years since 1999 and the dependent variable n represents the number of insecticide-treated nets distributed in Africa (in thousands), analyze and interpret in words the meaning of the graph.

Solution We first observe that the graph is increasing. That is, the number of insecticide-treated nets distributed is increasing as time goes on. In terms of malaria control and prevention, this is a good thing.

We next observe that the graph curves upward. This means that the rate at which the graph is increasing is also increasing. Notice that between roughly (0, 500) and (1, 1000), the average rate of increase in the model was about 500 thousand nets per year. However, between roughly (3, 4400) and (4, 9500), the average rate of increase in the model was about 5100 thousand nets per year. We summarize these results in words as follows.

> The distribution of insecticide-treated nets in Africa increased from about 500 thousand in 1999 to about 9500 thousand in 2003. The rate at which the nets were being distributed was also increasing. Between 1999 and 2000, the annual distribution rate increased by about 500 thousand nets per year. However, between 2002 and 2003, the annual distribution rate increased by about 5100 thousand nets per year. Since the model graph is curved upward, we anticipate that future rates of increase will be even more dramatic, at least in the short term.

Readers who want to become engaged in this humanitarian issue on a personal level may visit www.nothingbutnets.net to learn about easy ways to get involved.

SUMMARY

In this section you learned how to represent verbal descriptions of relationships using function concepts. You also discovered how to interpret in words the meaning of functions in equations, tables, and graphs. These skills will be used extensively throughout the rest of this book and the rest of your life.

1.5 EXERCISES

SKILLS AND CONCEPTS

In Exercises 1–7, be sure to identify the meaning of any variables you use in your function equation, table, or graph.

1. **Gasoline Prices** On July 10, 2008, the average price of gas in Utah was $4.109 per gallon. (*Source:* www.utahgasprices.com) Use function notation to represent the cost of buying g gallons of gas at this price.

2. **Disposable Personal Income** The following is an excerpt from a May 1, 2006, press release issued by the Bureau of Economic Analysis.

   > [monthly] disposable personal income (DPI) increased $78.4 billion . . . in March [2006] (*Source:* U.S. Bureau of Economic Analysis)

   Use function notation to represent the disposable personal income at the end of March 2006 as a function of disposable personal income at the end of February 2006.

3. **Wheat Production** The following is an excerpt from May 12, 2006, press release issued by the National Agricultural Statistics Service of the U.S. Department of Agriculture.

   > Winter wheat production is forecast at 1.32 billion bushels, down [0.18 billion] from 2005. Based on May 1 conditions, the U.S. yield is forecast at 42.4 bushels per acre, 2.0 bushels [per acre] less than last year. (*Source:* www.usda.gov)

   Create a table that gives total winter wheat production as a function of the number of bushels produced per acre.

4. **Orange Juice Concentrate** The following is an excerpt from a May 12, 2006, press release issued by the National Agricultural Statistics Service of the USDA.

   > Florida frozen concentrated orange juice (FCOJ) yield for the 2005–2006 season, at 1.62 gallons per box at 42.0 degrees Brix, is increased from the 1.58 gallons last season. (*Source:* www.usda.gov)

(Note: 42 degrees Brix means the juice has 42 grams of sucrose sugar per 100 grams of liquid.)

Draw a function graph that gives the Florida frozen concentrated orange juice yield as a function of the number of years since the 2004–2005 season. Then use the graph to forecast the yield for the 2007–2008 season.

5. **Arizona University Enrollment** The following is an excerpt from the Arizona University System FY 2003–2004 Annual Report published by the Arizona Board of Regents.

- In Fall 2003, the universities' headcount enrollment increased by 1,595 students, from 113,869 to 115,464.
- Arizona's universities are expecting tremendous growth in the next two decades. Student enrollments are projected to increase to more than 170,000 students by the year 2020. (***Source:*** **www.abor.asu.edu**)

Assuming that annual enrollment will continue to increase by 1595 students per year, draw a function graph that models the enrollment at Arizona universities between Fall 2002 and Fall 2020. Then use the graph to estimate the enrollment in Fall 2010.

6. **Apple Macintosh Sales** Apple Computer Corporation published the following in its 2005 Annual Report.

> Total Macintosh net sales increased \$1.4 billion . . . during 2005 compared to 2004. Unit sales of Macintosh systems increased 1.2 million units . . . during 2005 compared to 2004. (***Source:*** **Apple Computer Corporation 2005 Annual Report, p. 32**)

The report further indicated that 3.3 million Macintosh computers were sold in 2004, generating sales revenue of \$4.9 billion. In 2003, 3.0 million Macintosh units were sold, generating \$4.5 billion in revenue. Create a graph to model the revenue from Macintosh sales as a function of Macintosh units sold.

7. **Community College Enrollment** In her May 2006 speech to faculty and staff, the president of Chandler-Gilbert Community College indicated that although fall semester enrollments continued to increase, they were not increasing as rapidly as they had in previous years. (***Source:*** **author's notes**) Draw a graph to represent fall enrollments as a function of time.

SHOW YOU KNOW

8. A classmate claims that one of the most important things to focus on when translating a function given in words into symbolic notation is to determine what the variables and constants are that are involved in the problem. Do you agree or disagree? Explain your reasoning.

9. Explain how to determine which variables should be assigned to the horizontal and vertical axes when translating a function given in words to a graphical representation.

10. For the function $f(x) = y$, explain what each part of the symbolic notation means. In other words, what does the f, x, and y represent in terms of inputs and outputs of functions?

11. A classmate states, "My weekly income is a function of the number of hours I work at \$9.00 per hour." In creating the equation of the mathematical model, what are the advantages of choosing the variable w to represent *weekly income* and the variable h to represent *hours worked*?

12. When interpreting the meaning of a function graph using words, what information do the labels on the horizontal and vertical axes reveal?

MAKE IT REAL

In Exercises 13–19, explain in words the real-world meaning of the indicated function value.

13. **College Attendance** Based on Census Bureau data from 2000–2002 and projections for 2003–2013, *private* college enrollment can be modeled by

$$P(x) = 0.340x - 457 \text{ thousand students}$$

where x is the number of students (in thousands) enrolled in *public* colleges. (***Source:*** **Modeled from *Statistical Abstract of the United States, 2006*, Table 204**) What does $P(14{,}000) = 4303$ mean?

14. **Greenhouse Gas Emissions** Based on data from 1990–2002, carbon dioxide emissions can be modeled by

$$C(g) = 1.095g - 1743 \text{ million metric tons}$$

where g is the total amount of greenhouse gas emissions (in millions of metric tons). (***Source:*** **Modeled from *Statistical Abstract of the United States, 2006*, Table 362**) What does $C(6400) = 5265$ mean?

15. **Apple Computer Net Sales** Based on data from 2001–2005, the net sales of Apple Computer Corporation can be modeled by

$$S(c) = 1.45c - 408 \text{ million dollars}$$

where c is the cost of sales (in millions of dollars). (***Source:*** **www.apple.com**) For what value of c does $S(c) = 0$ and what does that mean for Apple Computer?

16. **Life Expectancy** Based on data from 1980–2003 and projections for 2005 and 2010, the average life expectancy of a woman can be modeled by

$$f(m) = 0.045645m^3 - 9.9292m^2 + 720.34m - 17350 \text{ years}$$

where m is the average life expectancy of a man (in years) born in the same year as the woman. (***Source:*** **Modeled from *Statistical Abstract of the United States, 2006*, Table 96**) What does $f(73) = 79$ mean?

17. **Per Capita Pineapple Consumption** Based on data from 1999–2003, the per capita consumption of pineapple can be modeled by

$$p(f) = 1.461f^3 - 35.40f^2 + 285.9f - 766.4 \text{ pounds}$$

where f is the combined per capita consumption of apricots, avocados, cherries, cranberries, kiwis, mangoes, papayas, and honeydew melons (in pounds) consumed in the same year. (*Source:* Modeled from *Statistical Abstract of the United States, 2006*, Table 203) What does $p(9) = 4.4$ mean?

18. Per Capita Pineapple Consumption According to the model in Exercise 17, $p(19) = 1907$. What does this mean? Does it make sense in the real-world context of the problem? Explain.

19. Per Capita Pineapple Consumption According to the model in Exercise 17, $p(3) = -188$. What does this mean? Does it make sense in the real-world context of the problem? Explain.

Exercises 20–29 focus on interpretations of real-world data sets.

20. Marketing Costs for Farm Foods

Years Since 1990 t	Marketing Labor Cost (\$ billion) L
0	154.0
4	186.1
5	196.6
6	204.6
7	216.9
8	229.9
9	241.5
10	252.9
11	263.8
12	273.1
13	285.9

Source: Statistical Abstract of the United States, 2006, Table 842

Analyze and interpret in words the meaning and real-world significance of the data in the table.

21. Spending on Medical Services

Years Since 1990 t	Per Capita Spending on Physician and Clinical Services (dollars) P
0	619
5	813
8	914
9	954
10	1010
11	1085
12	1162
13	1249

Source: Statistical Abstract of the United States, 2006, Table 121

Analyze and interpret in words the meaning and real-world significance of the data in the table.

22. Chicken Egg Production

Years Since 1990 t	Egg Production (billions) E
0	68.1
5	74.8
7	77.5
8	79.8
9	82.9
10	84.7
11	86.1
12	87.3
13	87.5
14	89.1

Source: Statistical Abstract of the United States, 2006, Table 842

Analyze and interpret in words the meaning and real-world significance of the data in the table.

23. Bowling The table shows the variable number of bowling establishments and bowling membership based on data from 1990, 1995, and 2000–2004.

Tenpin Bowling Establishments b	Bowling Membership (thousands) M
7611	6588
7049	4925
6247	3756
6022	3553
5973	3382
5811	3246
5761	3112

Source: Statistical Abstract of the United States, 2006, Table 1234

Analyze and interpret in words the meaning and real-world significance of the data in the table.

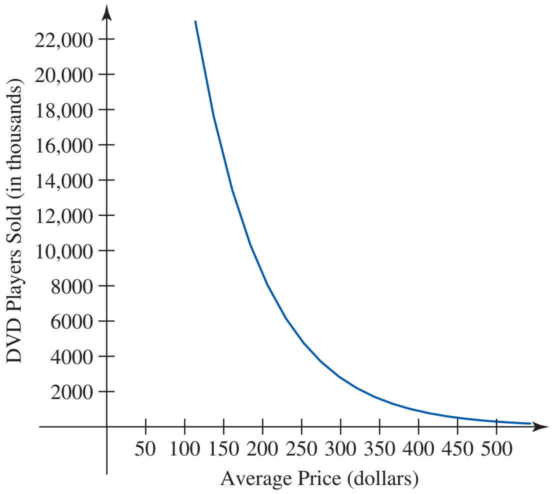
24. Movie Theaters The table shows the movie attendance and number of motion picture screens based on data from 2000–2004.

Movie Attendance (millions) m	Motion Picture Screens S
1421	38,000
1487	37,000
1639	36,000
1574	37,000
1536	37,000

Source: Statistical Abstract of the United States, 2006, Table 1234

Analyze and interpret in words the meaning and real-world significance of the data in the table.

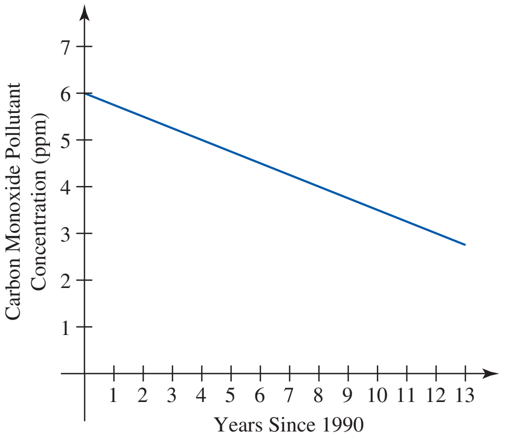
25. DVD Player Sales Based on data from 1997–2004, DVD player sales can be modeled by the function shown in the graph.



Source: **Modeled from Consumer Electronics Association data (www.ce.org)**

Describe in words the meaning and real-world significance of the function graph.

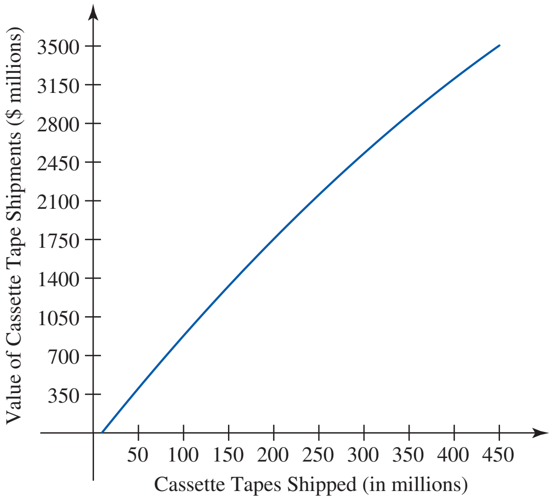
26. Carbon Monoxide Pollution Based on data from 1990–2003, carbon monoxide pollutant concentration can be modeled by the linear function shown in the graph.



Source: **Modeled from** ***Statistical Abstract of the United States, 2006*****, Table 359**

Describe in words the meaning and real-world significance of the function graph.

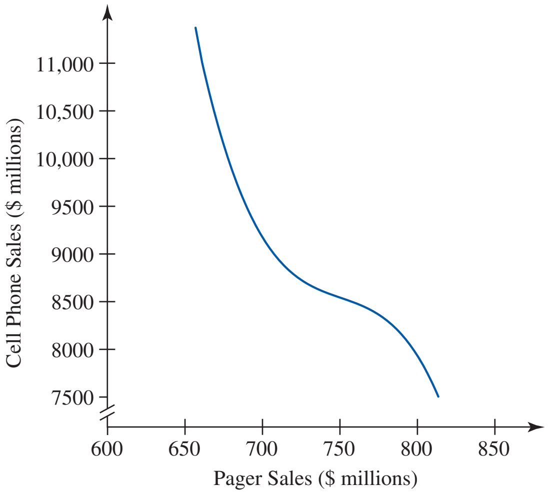
27. Cassette Tape Shipment Value Based on data from 1990–2004, the value of music cassette tapes shipped can be modeled by the function shown in the graph.



Source: **Modeled from** ***Statistical Abstract of the United States, 2006*****, Table 1131**

Describe in words the meaning and real-world significance of the function graph.

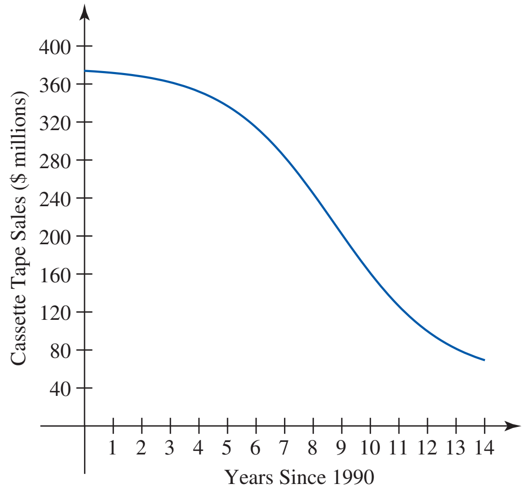
28. Cell Phones and Pagers Based on data from 2000–2004, cell phone sales can be modeled by the cubic function shown in the graph.



Source: **Modeled from** ***Statistical Abstract of the United States, 2006*****, Table 1003**

Describe in words the meaning and real-world significance of the function graph.

29. Blank Audio Cassette Tape Sales Based on data from 1990–2004, blank audio cassette tape sales can be modeled by the logistic function shown in the graph.



Source: **Modeled from *Statistical Abstract of the United States, 2006*, Table 1003**

Describe in words the meaning and real-world significance of the function graph.

■ STRETCH YOUR MIND

Exercises 30–34 are intended to challenge your ability to interpret the meaning of functions in words.

30. Income Ranking In March 2006, the Bureau for Economic Analysis (BEA) issued a press release detailing per capita income rankings for the states. An excerpt from the report is provided here.

> Connecticut led the nation with a per capita income . . . 38 percent above the national average. Louisiana's per capita income was . . . 28 percent below the national average . . . **(*Source:* www.bea.gov)**

Write the equation of a function that relates the per capita income of Connecticut to the per capita income of Louisiana.

31. Autism Study In June 2006, a son of one of the authors was diagnosed with autism, a condition commonly characterized by repetitive motions, limited communication skills, and impaired social skills. In the ensuing quest to better understand the condition, the author discovered that a popular theory was that the MMR vaccine was one of the causes of autism.

In March 2001, the *Journal of the American Medical Association* published the results of a study investigating this theory. The study indicated that the number of autism cases in the study population increased from 44 cases per 100,000 live births in 1980 to 208 cases per 100,000 live births in 1994. Over the same time period, the percentage of children receiving immunizations by the age of 24 months increased from 72% to 82%. **(*Source: JAMA*, Vol. 285, No. 9; March 2001)**

Based on these results, explain whether or not the study supports the hypothesis that the MMR vaccine is responsible for the increase in autism.

32. Darden Restaurants Financials Darden Restaurants, Inc., is the parent company of a number of popular restaurants including The Olive Garden, Red Lobster, Bahama Breeze, and Smokey Bones Barbeque & Grill. According to the company's 2005 annual report, the cost of sales increased from \$3.1 billion in 2001 to \$4.1 billion in 2005. During the same time period, revenue from sales increased from \$4.0 billion to \$5.3 billion. **(*Source:* Darden Restaurants, Inc., 2005 Annual Report, p. 60)** Answer the following questions using mathematical computations to substantiate your results.

a. Between 2001 and 2005, was the cost of sales increasing more rapidly than the revenue from sales? Explain.

b. What is an equation of a linear function that models revenue from sales as a function of cost from sales?

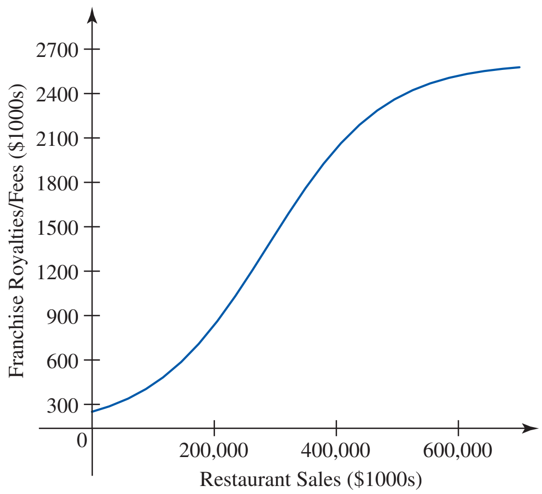
c. Given the model you found in part (b), what do you estimate the revenue from sales was when the cost of sales was \$3.9 billion?

d. According to the company's annual report, the *cost of sales* was \$3.9 billion in 2004 and the revenue from sales was \$5.0 billion. How well did your estimate in part (c) fit the actual data in the company's annual report?

33. Chipotle Mexican Grill Income In 2001, Chipotle Mexican Grill, Inc., earned \$131,598 thousand in revenue and had a net income of −\$24,000 thousand. Between 2001 and 2005, revenue and net income increased dramatically for the restaurant. In 2005, the company earned \$627,695 thousand in revenue and \$37,696 thousand in net income. **(*Source:* Chipotle Mexican Grill, Inc., 2005 Annual Report, p. 24)**

Write a function equation with a constant rate of change to model the net income of the company as a function of its revenue.

34. Chipotle Mexican Grill Revenue According to the company's 2005 annual report, Chipotle Mexican Grill, Inc., classifies its revenue as *restaurant sales* or *franchise royalties and fees.* A following graph relates franchise royalties and fees to restaurant sales, based on data from 2001–2005.



Source: **Chipotle Mexican Grill, Inc., 2005 Annual Report, p. 24**

Describe what the graph communicates regarding the relationship between sales and franchise revenue.

SECTION 1.6

Preview to Inverse Functions

LEARNING OBJECTIVES

- Explain the relationship between a function and its inverse
- Explain and use inverse function notation to solve real-world problems
- Find the inverse of a function from a table or graph and interpret its practical meaning

GETTING STARTED

Automobile engineers focused on improving a car's braking system need to be able to predict the car's braking distance (output) given the car's speed (input). Conversely, police officers need to be able to calculate a car's speed (output) based on its braking distance (input) as indicated by skid marks on the road. The mathematical concept of *inverse functions* allows the engineers and the police officers to get the information they need.

In this section we introduce the concept of inverse functions. Since we study inverse functions in depth in future sections, the purpose here is to develop a basic understanding of the inverse function concept.

The Inverse Function

Recall that a function is a process or correspondence relating two quantities in which each input value generates exactly one output value. Some functions have the additional characteristic that each output value has exactly one input value. Functions with this property are *reversible*. The inverse of a function reverses the process or correspondence of the original function. The function reversing the process or correspondence, if it exists, is called an **inverse function**.

Inverse Function

A function f^{-1} (read "f inverse") is the function whose inputs are the outputs of f and whose outputs are the inputs of f.

Figure 1.24 is a graphical representation of the concept that an inverse function reverses the correspondence between two data sets. The arrows in the figure point to the outputs of the function. Notice that in the inverse function correspondence, the set of inputs of the original function becomes the set of outputs for the inverse function. Similarly, the set of outputs of the original function becomes the set of inputs for the inverse function. Observe that the inverse is indeed a function since each input of the inverse function corresponds with a single output.

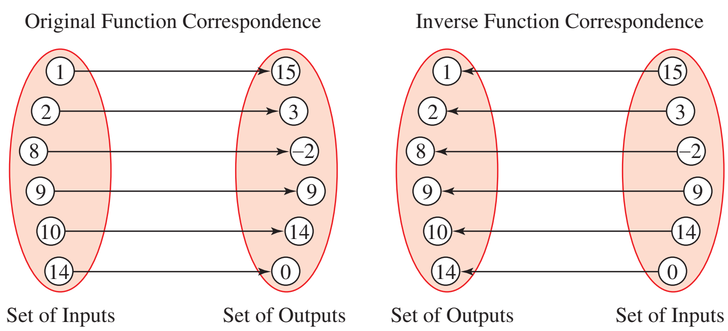


Figure 1.24

We can use a similar approach when looking at a function as a process, as shown in the following diagram.

$$\text{input } x \rightarrow \boxed{f} \rightarrow y \text{ output}$$
$$\text{output } x \leftarrow \boxed{f^{-1}} \leftarrow y \text{ input}$$

Notice that $y = f(x)$ and $x = f^{-1}(y)$. At this point in the text, the goal is not to compute inverse function models but rather to interpret their meaning in the context of different real-world situations. We revisit the inverse function concept repeatedly throughout the text as we introduce different types of functions. When we do, we will demonstrate how to find inverses algebraically. Let's now return to the braking distance scenario.

Modeling with Inverse Functions

Suppose you are in a car, traveling at a safe speed, and the traffic light ahead turns red. You press the brake pedal, and the car slows down and eventually stops. We call the distance that the car travels while the driver is braking the **braking distance**. We used the website www.phy.ntnu.edu.tw/ntnujava/viewtopic.php?t=224 to simulate this situation and record the data shown in Table 1.19.

Table 1.19

Braking Distance	
Speed (mph)	**Distance (feet)**
0	0.00
20	16.70
25	26.12
30	37.63
35	51.21
40	66.90
45	84.65
50	104.53

Using a technique called *quadratic regression*, we found the function in Example 1 to model the data set. We will explain this technique in detail in Chapter 4 as a part of our in-depth discussion of quadratic functions.

EXAMPLE 1 ■ Using a Braking Distance Model

The braking distance as a function of the speed of the car can be modeled by

$$T = f(S) = 0.042(S)^2 - 0.00077(S) - 0.0026$$

where T is the braking distance in feet and S is the speed of the car in miles per hour. Find $f(100)$ and explain what it means.

Solution The notation $f(100)$ means the braking distance when the car is traveling 100 mph. We compute the value of $f(100)$ by substituting 100 for the variable S in the function.

$$\begin{aligned} f(100) &= 0.042(100)^2 - 0.00077(100) - 0.0026 \\ &= 419.920 \text{ feet} \end{aligned}$$

According to the model, the braking distance for a car traveling 100 mph is 419.920 feet.

We have created a model to predict the braking distance (dependent variable) given the speed of the car (independent variable). But what if the braking distance is known and the speed is unknown? Can we determine the original speed of the car? Police department accident reconstruction officers often measure the skid marks left by a vehicle prior to an accident to answer this very question.

To simplify the mathematical model we will use in Example 2, we interpret the length of the skid marks left by the car to be exactly the braking distance. We also ignore the fact that variables other than braking distance and speed enter into this application. (Ignored variables include driving surface, driver alertness, and so on.)

Table 1.20

Braking Distance	
Speed (mph)	**Distance (feet)**
0	0.00
20	16.70
25	26.12
30	37.63
35	51.21
40	66.90
45	84.65
50	104.53

EXAMPLE 2 ■ Using Skid Marks to Determine the Speed of a Car

Table 1.20 shows the braking distance as a function of speed.

Assuming that the skid marks represent the braking distance of a car, describe how a police officer could use this data to determine the pre-accident speed of a vehicle.

Solution In Table 1.20, *speed* is the independent variable and *braking distance* is the dependent variable. However, for the police officer *braking distance* is the independent variable and *speed* is the dependent variable. To represent this, we switch the columns as shown in Table 1.21.

To estimate the speed, the police officer must find the braking distance (length of skid marks) and predict the corresponding speed. For example, if the braking distance was about 67 feet, he could predict that the car was traveling approximately 40 mph. The function used by the police officer is the *inverse* of the braking distance function.

Table 1.21

Braking Distance	
Distance (feet)	**Speed (mph)**
0.00	0
16.70	20
26.12	25
37.63	30
51.21	35
66.90	40
84.65	45
104.53	50

We say that two functions have an **inverse relationship** if the *independent* variable of the first function is the *dependent* variable of the second function and the *dependent* variable of the first function is the *independent* variable of the second function. That is, interchanging the inputs and outputs of one function yields the other function. For example, the functions $f(s) = t$ and $g(t) = s$ (shown in Tables 1.22 and 1.23) have an inverse relationship.

Table 1.22

Braking Distance as a Function of Speed $f(s) = t$	
Speed (mph) s	**Braking Distance (feet)** t
0	0.00
20	16.70
25	26.12
30	37.63
35	51.21
40	66.90
45	84.65
50	104.53

Table 1.23

Speed as a Function of Braking Distance $g(t) = s$	
Braking Distance (feet) t	**Speed (mph)** s
0.00	0
16.70	20
26.12	25
37.63	30
51.21	35
66.90	40
84.65	45
104.53	50

We say that f is the inverse function of g and that g is the inverse function of f. Symbolically, we write $f = g^{-1}$ and $g = f^{-1}$. Note that the independent variable of f (speed) is the dependent variable of g. Similarly, the dependent variable of f (braking distance) is the independent variable of g.

We may also represent the inverse relationship between speed and braking distance graphically. We demonstrate this in Example 3.

EXAMPLE 3 ■ Graphing the Braking Distance Function

Using the data in Table 1.22, draw a scatter plot of the braking distance data as a function of speed. Then graph the inverse of the braking distance function.

Solution For the braking distance function, the speed of the car is the independent variable and is measured along the horizontal axis. The braking distance is the dependent variable and is measured along the vertical axis. See Figure 1.25.

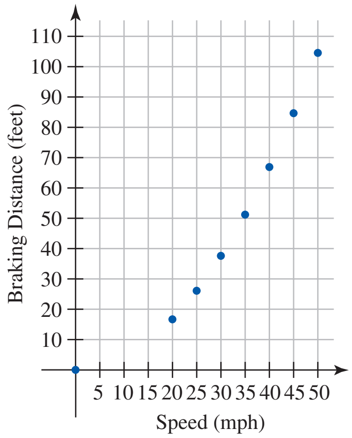


Figure 1.25

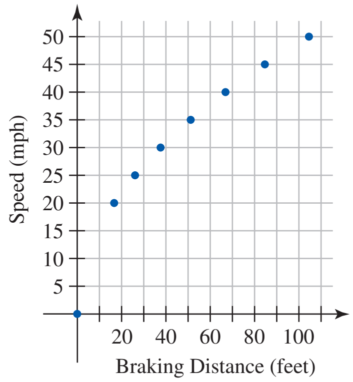


Figure 1.26

The inverse function is determined by interchanging the independent and dependent variables as shown in Figure 1.26. In this case, the braking distance (length of skid marks) becomes the independent variable and is measured along the horizontal axis. The speed of the car becomes the dependent variable and is measured along the vertical axis.

EXAMPLE 4 ■ Interpreting an Inverse Function Model

Super Bowl ticket prices have increased in the 40-year history of the game (see Table 1.24).

a. Interpret the meaning of $P = f(G)$ and $G = f^{-1}(P)$, where P represents the value of a ticket and G represents a particular Super Bowl game.

b. Calculate and explain the meaning of $P = f(20)$ and $G = f^{-1}(20)$.

Table 1.24

Super Bowl Game Number G	Ticket Face Value P
I (1)	\$10
V (5)	\$15
X (10)	\$20
XV (15)	\$40
XX (20)	\$75
XXV (25)	\$150
XXX (30)	\$300
XXXV (35)	\$325
XL (40)	\$600

Source: www.superbowl.com

Solution

a. The notation $P = f(G)$ indicates that the price of a Super Bowl ticket, P, depends on the Super Bowl game, G, for which it was used. That is, if we know the Super Bowl game number, we can determine the price of a ticket. The function $G = f^{-1}(P)$ is the inverse of $P = f(G)$. That is, if we know how much a ticket cost, we can tell which Super Bowl game that ticket represents.

b. $P = f(20)$ is the price of a ticket in Super Bowl 20 (XX), so $P = \$75$. $G = f^{-1}(20)$ is the game in which the price of the ticket was \$20, so $G = 10$: In Super Bowl X (10), the price of a ticket was \$20.

EXAMPLE 5 ■ Evaluating an Inverse Function Given a Graph

The graph in Figure 1.27 shows the function model $H = f(N)$ where H represents Head Start enrollment (in thousands) and N represents the number of women in the workforce (in thousands). Evaluate $f(66000)$ and $f^{-1}(600)$ and explain what each means in this context.

Solution The notation $f(66000)$ tells us to find the Head Start enrollment, H, when the number of women in the workforce, N, is 66,000,000 (remember that the numbers are in thousands). From the graph in Figure 1.28, we estimate that $H(66000) \approx 850$. That is, when there are 66,000,000 women in the workforce, Head Start enrollment is approximately 850,000.

The notation $f^{-1}(600)$ tells us to find the number of women in the workforce when Head Start enrollment is 600,000. Although we don't have a graph of f^{-1}, we can use the graph of f to find the desired result by reversing the process.

From the graph of f in Figure 1.29, we estimate that $f^{-1}(600) \approx 58{,}000$. That is, when Head Start enrollment is 600,000, the number of women in the workforce is about 58,000,000.

The relationship between Head Start enrollment and women in the workforce is a function correspondence. Although we related the variables to each other with a function, an increase in women in the workforce does not necessarily *cause* an increase in Head Start enrollment.

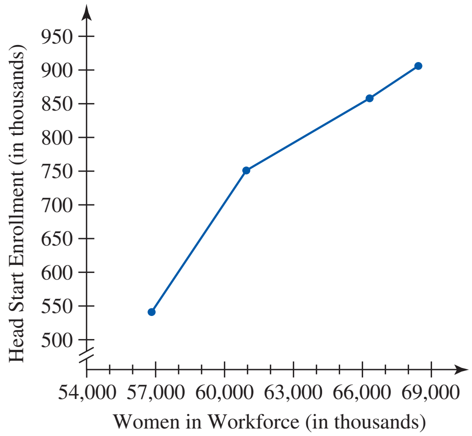


Figure 1.27

Source: Statistical Abstract of the United States, 2006

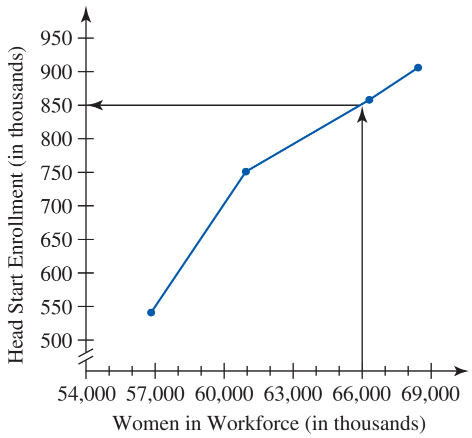


Figure 1.28

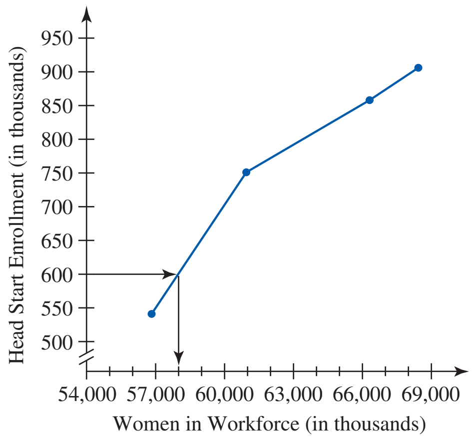


Figure 1.29

Inverse Relationships That Are Not Inverse Functions

Not all functions have an inverse function. Some processes cannot be reversed and some reversible processes do not yield exactly one output for each input to the inverse function. Example 6 shows one such function.

EXAMPLE 6 ■ Determining If an Inverse Relationship Is a Function

Projectile motion refers to the motion seen when an object, such as a baseball hit by a bat, is propelled into the air before returning to the ground due to the effects of gravity. Using the website www.exploratorium.edu/baseball/scientificslugger.html, we determined that if a hit ball has an initial velocity of 242 feet per second and is hit at an angle of 20°, the ball will travel 417 feet. The function $h = f(t)$ represents a model for the vertical height, h, of the baseball (in feet) as a function of the time, t seconds. This function is shown in the graph in Figure 1.30. Does the inverse relationship represent a function? Why or why not?

Solution: The inverse function, $f^{-1}(h) = t$, would allow us to determine at what time the ball was at a particular height. That is, if we wanted to know at what time the height of the ball was 90 feet, we could evaluate $f^{-1}(90)$. Using the graph of $f(t)$ shown in Figure 1.31, we find that there are two times where the height is 90 feet: at about 1.5 seconds into its flight and at about 3.6 seconds into its flight.

Although an inverse relationship exists, the inverse is not a function since there are two outputs for a single input.

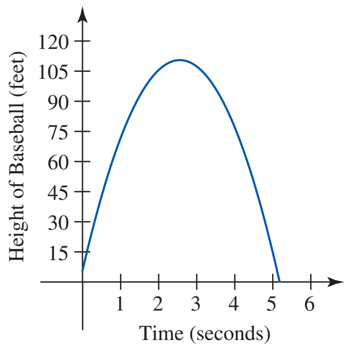


Figure 1.30

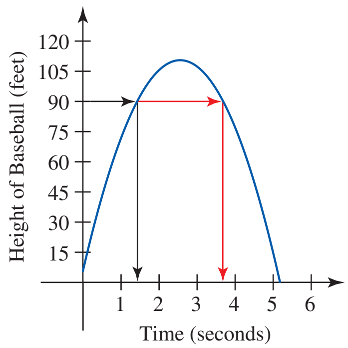


Figure 1.31

In Example 6, we saw that the baseball reaches a height of 90 feet two times. In cases such as this, it is not appropriate to use the function notation $f^{-1}(h) = t$ since the inverse does not represent a function. However, we can still discuss the idea of the inverse relationship by finding the two times as output values for a particular input value.

SUMMARY

In this section you discovered that if a situation may be explained as a function process or correspondence, the inverse function reverses the process or correspondence of the original function. You also learned how to evaluate inverse functions and interpret their meaning in the context of real-world situations. You also found that an inverse relationship can exist even though an inverse function does not.

1.6 EXERCISES

SKILLS AND CONCEPTS

For Exercises 1–10, do each of the following.

a. Identify the independent and dependent variables of the function.

b. Identify the independent and dependent variables for the inverse of the function.

c. Determine whether or not the inverse relationship represents a function. Justify your conclusion.

1. The height of water in a bathtub (in inches) that is being filled after the water has been running for t minutes.

2. Your distance from home t minutes after leaving home.

3. The length of your hair t days after getting a haircut.

4. The total amount of rain that falls on a person's lawn in Phoenix as a function of the days since June 1.

5. An electric bill total as a function of the temperature at which the thermostat is set.

6. The full name of each student in your class and the number of hours the student works during a given week.

7. The speed of a car as a function of the time since entering a freeway from the on-ramp.

8. The height of a child as a function of her age between birth and 10 years.

9. The weight of an adult as a function of his age between 18 and 40 years.

10. The number of points scored in each quarter of a basketball game.

Exercises 11–15 are based on the following data table, which shows the speed of a car in the car chase scene in the movie The Blues Brothers.

Time (seconds)	Speed (mph)
4	45
10	36
12	45
15	72
17	72
20	72
22	54
26	75
31	80
36	54
41	36

Source: **www.hypertextbook.com**

11. If S represents the speed of the car at a given time t, explain what the notation $S(t)$ represents.

12. Evaluate $S(22)$ and explain what it means.

13. Create a scatter plot of the function $S(t)$.

14. What does the inverse relationship represent in this context?

15. Create a scatter plot of the inverse relationship. Does the inverse relationship represent a function? Explain your reasoning.

SHOW YOU KNOW

16. Your friend missed class today and asks you what is meant by "inverse function." Write an explanation of the concept for your friend.

17. Construct two tables of values that would represent inverse functions of each other. Explain how you know that they are inverse functions.

18. What does the notation $x = f^{-1}(y)$ mean?

MAKE IT REAL

19. Percentage of Cremations The percentage of cremations as a function of years since 1980 can be modeled by $C = f(t) = 0.8071t + 9.5811$, where C is the percent of people cremated at death and t is the number of years since 1980. (***Source:*** **Modeled from data at www.cremationassociation.org**)

a. Describe what the notation $f^{-1}(C) = t$ means in this context.

b. Solve the equation $40 = 0.8071t + 9.5811$ and explain what the result means.

c. The inverse function for this situation is $f^{-1}(C) = 1.239C - 11.871$. Evaluate $f^{-1}(40)$ and explain the practical meaning of the solution.

d. Explain the relationship between the results in parts (b) and (c).

20. Female Participation in Athletics The number of females participating in high school athletics has been increasing steadily since 1990. (***Source: Statistical Abstract of the United States, 2001*****, Table 1241**) The number of females who participated in high school athletics can be modeled by

$$F = f(t) = 0.104t + 1.83$$

where F is the number of female participants (in millions) and t is the number of years since the 1990–1991 school year.

a. Describe what the notation $f^{-1}(F) = t$ means in this context.

b. Solve the equation $3 = 0.104t + 1.83$ and explain the practical meaning of the result.

c. The inverse function for this situation can be modeled by $f^{-1}(F) = 9.615F - 17.596$. Evaluate $f^{-1}(3)$ and explain the practical meaning of the result.

d. Explain the relationship between the results in parts (b) and (c).

21. Fiddler Crab Claws Male fiddler crabs have one large claw and one small claw. Scientists have found that there is a relationship between the weight of the large claw and the weight of the crab's body, as shown in the table.

Body Weight (grams)	Large Claw Weight (grams)
199.7	38.3
238.3	52.5
270.0	59.0
300.2	78.1
355.2	104.5
420.1	135.0
470.1	164.9
535.7	195.6
617.9	243.0

Source: Problems of Relative Growth**, J. S. Huxley, Dover, 1972, p. 12, Table 1**

a. Assume that the claw weight, C, is a function of the body weight, B. Describe what the notation $C = f(B)$ means in this context.

b. Create a scatter plot of the function $C = f(B)$. Then evaluate and interpret $C = f(300.2)$.

c. Create a scatter plot of the inverse function $f^{-1}(C) = B$. Explain why this is a function.

d. Evaluate $f^{-1}(52.5) = B$ using the table and interpret the practical meaning of the result.

22. Buying Power of the Dollar Due to inflation, the buying power of the dollar decreases over time. Based on data from 1990–2004, the buying power of the current dollar can be modeled by

$$V(t) = -0.0107t + 0.836 \text{ of a 1980 dollar}$$

where t is the number of years after 1990. Assuming $V = f(t)$, what does $f^{-1}(0.44) = 37$ mean?

23. Apple Computer International Sales Based on data from 2001–2005, the international net sales of Apple Computer Corporation can be modeled by

$$I(d) = \frac{3217}{1 + 9446e^{-0.001704d}} + 2400 \text{ million dollars}$$

where d is the domestic net sales (in millions of dollars). (***Source:*** **www.apple.com**) Assuming $I = f(d)$, what does $f^{-1}(5597) = 8338$ mean?

24. Pager and Cell Phone Sales Based on data from 2000–2004, the amount of revenue generated by cell phone sales (in millions of dollars) can be modeled by

$$C = f(p) = -0.002745p^3 + 6.173p^2 - 4633\text{p} + 1{,}169{,}000$$

where p is the amount of revenue generated by pager sales (in millions of dollars). What does $f^{-1}(8077) = 811$ mean?

■ STRETCH YOUR MIND

Exercises 25–28 are intended to challenge your understanding of inverse functions. For these exercises, refer to the functions $g(y) = 2y + 4$ and $f(x) = \dfrac{x-4}{2}$.

25. Explain why it makes sense that the function $g(y) = 2y + 4$ is the inverse of the function $f(x) = \dfrac{x-4}{2}$.

26. Explain the meaning of the following diagram.

$$\begin{array}{rcl} \text{input } x \rightarrow & \boxed{f} & \rightarrow y \text{ output} \\ \text{output } x \leftarrow & \boxed{f^{-1}} & \leftarrow y \text{ input} \end{array}$$

27. Given $f(10) = 3$, find $f^{-1}(3)$. How can you use the function $g(y)$ to confirm your answer?

28. If $g(2) = 8$, find $g^{-1}(8)$. How can you use the function $f(x)$ to confirm your answer?

CHAPTER 1 Study Sheet

As a result of your work in this chapter, you should be able to answer the following questions, which are focused on the "big ideas" of this chapter.

SECTION 1.1

1. What is the purpose of a mathematical model?
2. What are the different representations of a mathematical model and what are the advantages of each?

SECTION 1.2

3. What is the relationship between the terms *independent variable*, *dependent variable*, *input*, *output*, *domain*, *and range?*
4. What is *function notation* and how does it work?
5. In mathematical terms, what do the words *solve* and *evaluate* mean?

SECTION 1.3

6. What is an average rate of change and what does it represent?

SECTION 1.4

7. What do we mean when we say *practical domain* and *practical range*?

SECTION 1.5

8. Why is it important to keep track of variables when working with functions in any real-world context?

SECTION 1.6

9. What does it mean for one function to be an inverse of another? Use tables and graphs as a part of your explanation.
10. What does the notation $x = f^{-1}(y)$ mean?

ENTIRE CHAPTER

11. How do you determine if a data set, graph, equation, or verbal expression represents a function? Your explanation should apply to each of the function representations listed.
12. How do you evaluate and solve a function in all of its representations?

REVIEW EXERCISES

SECTION 1.1

1. A high school student is trying to decide what college to attend after graduation. The student has identified cost, distance from home (closer is better), and reputation (on a scale of 0 to 10 with 0 being very bad and 10 being very good) as the most important characteristics in deciding which college to attend. Create an example of a decision-factor equation for this situation and explain why you created it in the way you did.

In Exercises 2–3, analyze the given mathematical model given in numerical form. Then, answer the questions that follow each numerical model.

2. **Attendance at NFL Football Games** The table shows the total attendance (in thousands) at all National Football League games for select years.

Year	Total Attendance (in thousands)
1985	14,058
1990	17,666
1995	19,203
2000	20,954
2001	20,590
2002	21,505
2003	21,639
2004	21,709

Source: ***Statistical Abstract of the United States, 2006*****, Table 1233**

a. Describe the trend observed in these data.

b. Based on the data, predict what the total attendance at NFL games will be in 2006. Explain how you arrived at your prediction.

3. **Tennis Participation by Age Group** The table shows the number of people who participate in the sport of tennis in the United States (in thousands) for the given age groups.

Age Group	Number of Participants (in thousands)
7–11	997
12–17	2054
18–24	1161
25–34	2312
35–44	1609
45–54	683
55–64	429
65 and over	327

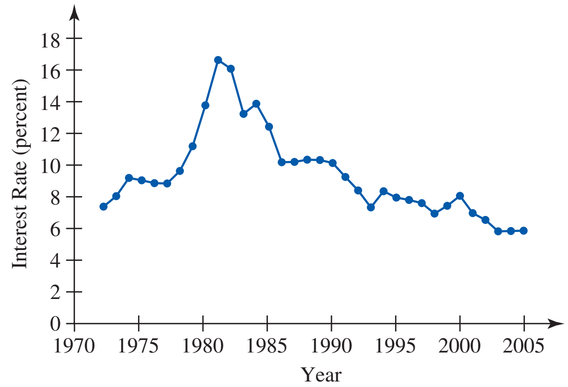
Source: ***Statistical Abstract of the United States, 2006*****, Table 1238**

a. Describe the trend observed in these data.

b. Why do you think the number of participants increases and decreases in the way that it does?

In Exercises 4–5, analyze the mathematical model given in graphical form. Then, answer the questions that follow each graph.

4. **U.S. Mortgage Interest Rates** The average interest rate for a home mortgage in the United States between 1970 and 2005 is shown in the graph.



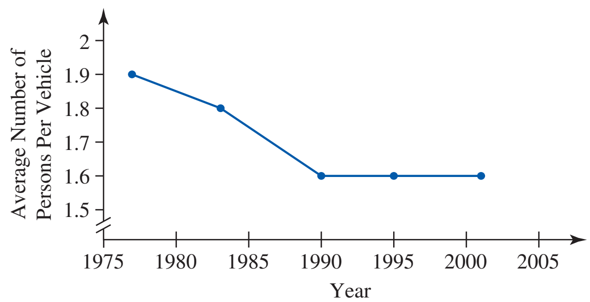
Source: **Modeled from www.federalreserve.gov data**

a. In what year was the interest rate the highest? What was the interest rate in this year?

b. Between what two consecutive years did the interest rate increase the most?

c. Between what two consecutive years did the interest rate decrease the most?

5. **Travel Trends** The average number of persons per vehicle on the road for any purpose between 1977 and 2002 is shown in the graph.



Source: **Modeled from** ***Statistical Abstract of the United States, 2006*****, Table 1254**

a. Describe the trend observed in this graph.

b. What does it mean to say the average number of persons per vehicle is 1.6?

c. From 1977 to 1990, the average vehicle occupancy declined. Give some possible reasons for this decline.

SECTION 1.2

6. Write the following expression in function notation by choosing meaningful letters to represent each variable. Also identify the independent variable (input) and the dependent variable (output).

The number of calories a person takes into his body is a function of the amount of food consumed by that person.

7. Evaluate the function $f(x) = -2x^2 + 3x$ at $x = -1$.

8. Income of Texans Based on data provided by the Bureau of Economic Analysis, the per capita income of Texans from 2000 to 2004 can be modeled by the formula

$$T = f(y) = 133.75y^3 - 666.71y^2 + 1131.61y + 28339.67$$

where T represents the per capita income of Texans (in dollars) and y is the years since 2000. (***Source:*** **www.bea.gov**) Evaluate $f(10)$ and explain what the numerical answer represents in its real-world context. Finally, write your solution in function notation.

9. Tuition and Fees of Four-Year Colleges The College Board, a nonprofit membership association of 4500 schools, colleges, and universities has consistently shown that the average tuition and fees for undergraduate students attending 4-year public universities are rising. The average cost of tuition and fees for 2004 in various regions of the country is shown in the table. The cost of tuition and fees, t, is a function of the U.S. region, r, and therefore, $f(r) = t$.

U.S. Region	2004 Average Cost of Tuition and Fees (in dollars)
New England	6839
Middle States	6300
Midwest	6085
Southwest	4569
South	4143
West	4130

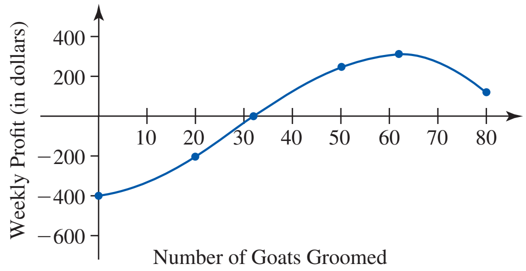
Source: **www.collegeboard.com**

a. Solve $f(r) = 4569$ for r and write a sentence explaining in words what your solution means in its real-world context.

b. Evaluate f(*New England*) and write a sentence explaining in words what the numerical value you find means in its real-world context.

10. New Business Profits Suppose that you have started a new company called Gus's Goat Grooming Service. The following graph of $p = f(g)$ displays your profit, p, in dollars for the number of goats you have groomed.

a. Estimate the maximum amount of profit made and how many goats are groomed to make that profit.



b. When are the profits negative? Why could this happen?

c. Evaluate $f(20)$ and explain what your solution means in its real-world context.

d. Find a value of g such that $f(g) = 300$ and explain what the solution means in its real-world context. Are there any other solutions for g such that $f(g) = 300$? If so, give them.

11. The Harris–Benedict Formula for Caloric Intake When designing a personal nutrition plan for yourself you should first calculate the number of calories your body burns in one day. This number is known as your total daily energy expenditure, E, or "maintenance level." The average value of E for women in the United States is between 2000 and 2100 calories per day and for men it is between 2700 and 2900 per day. (***Source:*** **www.weightlossforall.com**) These are only averages, however, as caloric expenditure can vary widely depending on one's level of fitness and activity level.

To calculate the total daily energy expenditure, E, we consider E a multivariable function dependent on variables including height in inches, h, weight in pounds, w, and age in years, a. E is also dependent on how active a person is so a level of activity factor is used to adjust the caloric intake. A factor of 1.55 is used for moderately active people.

Therefore, E is written in function notation as $E = f(h, w, a)$; this is known as the Harris–Benedict equation. The multivariable functions for both men and women who are moderately active are given below.

Men:

$$E(w, h, a) = 1.55(66 + 6.23w + 12.7h - 6.8a)$$

Women:

$$E(w, h, a) = 1.55(655 + 4.35w + 4.7h - 4.7a)$$

where w is in pounds, h is in inches, and a is in years.

a. Use function notation to express the total daily energy expenditure, E, for a moderately active male, who weighs 210 pounds, is 6 foot 2 inches tall (74 inches), and is 42 years of age. Calculate the total daily energy expenditure, E, for this person.

b. Write your total daily energy expenditure, E, in function notation and then calculate its value assuming that you are moderately active.

SECTION 1.3

12. Baby Girls' Average Height

Age (months) *m*	Height (inches) *H*	Average Rate of Change
Birth	19.4	
6	25.6	
12	29.0	
18	31.6	
24	33.8	
30	35.8	
36	37.4	

Source: National Center for Health

a. Calculate the average rates of change for consecutive pairs of data values.

b. Describe how the height changes as age increases.

13. Baby Boys' Average Height

Age (months) *m*	Height (inches) *H*	Average Rate of Change
Birth	19.6	
6	26.5	
12	29.8	
18	32.3	
24	34.5	
30	36.2	
36	37.8	

Source: National Center for Health

a. Calculate the average rates of change for consecutive pairs of data values.

b. Explain the meaning of the rates of change in the context of the data.

c. Estimate $H = f(27)$ and interpret what it means in a real-world context.

14. Perception Reaction Time Distance *Perception reaction time distance* is the distance a vehicle will travel during the time when a hazard first becomes visible to a driver and the driver takes action to avoid the hazard.

Perception Time of 1.4 Seconds for Nominal Hazard	
Miles Per Hour *m*	**Distance (in feet) *D***
30	61.60
40	82.13
50	102.67
60	123.20

Source: www.harristechnical.com/articles/skidmarks.pdf

Given that $D = f(m)$, estimate and explain the meaning of $f(35)$, $f(57)$, and $f(64)$.

15. Gross Movie Receipts The movie *Mission Impossible III* made its debut on May 5, 2006.

Weekend *w*	Receipts in Millions of Dollars *R*
1	47.743
2	25.009
3	11.350
4	7.002
5	4.685
6	3.021
7	1.343

Source: www.the-numbers.com

Estimate the receipts for the eighth weekend.

16. Skid Mark Distances on Snow

Speed (mph) *s*	Distance (feet) in Snow *D*
30	100
40	178
50	278
60	400

Source: www.harristechnical.com/articles/skidmarks.pdf

Given that $D = f(m)$, use the data in the table to estimate and explain the meaning of $f(47)$, $f(55)$, and $f(68)$.

17. Investment Account $B(p, r, n, t) = p\left(1 + \frac{r}{n}\right)^{nt}$ is the formula used to calculate the balance, B, in an investment account into which a lump sum is invested and interest is compounded a fixed number of times a year for a certain number of years. The independent variables represent the following:

p is the amount invested.
r is the nominal interest rate as a decimal.
n is the number of times the compounding occurs in a year.
t is the years the money is invested.

Compute the value of $B(5000, 0.067, 12, 3)$.

18. Vehicle Loan Payments The formula for monthly payments on a vehicle loan is

$$M(p, i, n) = p\left[\frac{i(1 + i)^n}{(1 + i)^n - 1}\right]$$

where

p = amount borrowed in dollars
n = number of monthly payments
i = interest rate per monthly payment period as a decimal

a. Calculate the monthly payment for a 2006 Chrysler Sebring Convertible with a MSRP of $26,115 in June 2006. (*Source:* price at www.edmunds.com) E-Loans has offered 5.89% APR for a 5-year loan.

b. How much interest will be paid over the life of the 5-year loan?

19. **Internet Plans** Nextel offered a BlackBerry plan that includes unlimited email and Internet with pay-as-you-go voice service for \$49.99 a month plus 20 cents per minute for voice service. (***Source:*** **nextelonline.nextel.com)**

a. Write an equation that models the monthly bill T as a function of the number of voice service minutes that have been used, m.

b. What is the total monthly bill (minus taxes and fees) when 100 voice service minutes are used?

20. The formula $C = f(F) = \frac{5}{9}(F - 32)$ converts the temperature on the Fahrenheit scale F to the temperature on the Celsius scale, C. What is F if $f(F) = 35$?

21. The formula for calculating the minimum speed of a car at the beginning of a skid S is $S(d, f, n) = \sqrt{30dfn}$

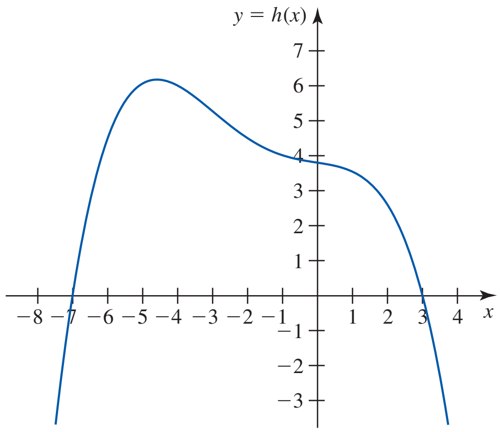
where

$d =$ skid distance in decimal feet
$f =$ drag factor for the road surface
$n =$ percent braking efficiency written as a decimal (i.e., $100\% = 1.00$)

How fast was a car traveling if it left a 64.3-foot skid on a road with a drag factor of 0.7 (asphalt) in a car with a braking efficiency of 100%?

SECTION 1.4

22. Estimate the function values using the following graph of $h(x)$ below.



a. Evaluate $h(-4)$.

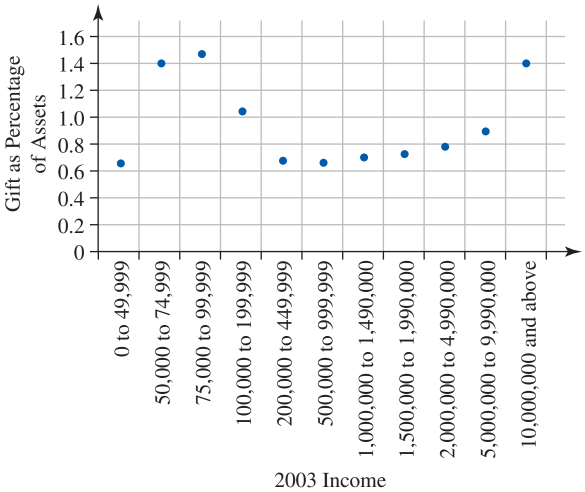
b. Solve $h(x) = 4$ for x.

c. Solve $h(x) = 0$ for x.

d. Evaluate $h(0)$.

e. Evaluate $h(3)$.

23. **Charitable Giving by Americans** In 2005, Americans gave an estimated \$260 billion to charity, marking a 5-year high in American giving. The rise came partly in response to a wave of natural disasters, including Hurricane Katrina, the South Asian tsunami, and the earthquake in Kashmir. The following graph shows the gift amount as a percentage of assets, g, as a function of the 2003 income, m.

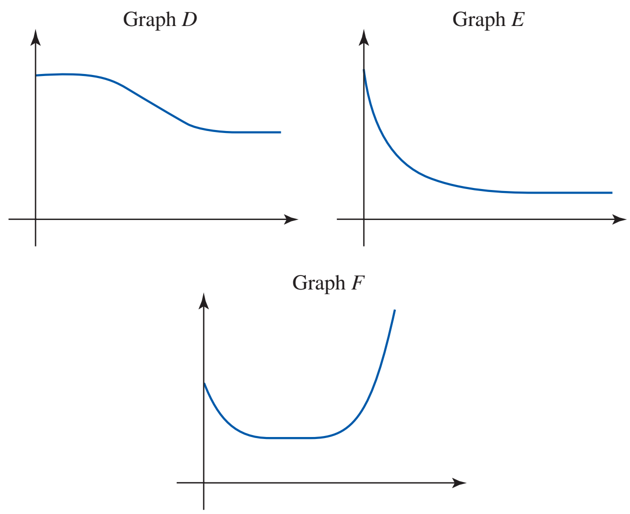


***Source: Newsweek*, July 10, 2006**

a. Determine the practical domain and range for $g = f(m)$.

b. Explain what the graph means in its real-world context.

24. Match the three written scenarios with the most appropriate graph given in D–F.



a. With the temperature more than 100° outside, I got in my car to cool off. I started the car and turned on the air conditioner. At first, the air conditioner did not blow cool air; however, after the car ran for a few minutes, the air conditioner began to cool down the car. Once the temperature inside the car was comfortable, I adjusted the air conditioner to maintain that temperature. The temperature inside the car is the dependent variable.

b. In 1964 I purchased a brand new car and as soon as I drove it off the lot the value of the car depreciated—dramatically for the first three years, then at a slower rate. I have kept the car and it is in great condition so the value has risen over the last 15 years. The value of the car in dollars is the dependent variable.

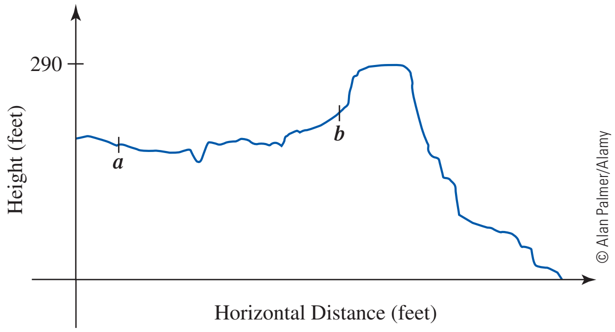
c. It has been a long time since I have worked out. I decided recently to begin walking on the treadmill to lower my heart rate. I walk three miles four times a week. When I started my workout regimen my heart rate was high, it stayed that way for six months and then slowly started to lower. Now it has reached a plateau. My heart rate in beats per minute is the dependent variable.

25. For graph D in Exercise 24, do each of the following.

a. Describe what the vertical intercept means in the real-world context.

b. Determine the practical domain and range for the function represented by the graph.

26. Gunsight Butte A picture of Gunsight Butte in Lake Powell near Page, Arizona, has been transposed onto a coordinate system that displays the elevation of the butte as a function of the horizontal distance. Assume that the horizontal and vertical scales of the graph are the same.



a. Explain what the graph means in its real-world context.

b. If the maximum height of Gunsight Butte is 290 feet, what can we say about the horizontal length of the butte as shown in the figure?

c. What do the vertical and horizontal intercepts mean in this context?

d. Suppose you were able to hike along the rim (top) of Gunsight Butte starting at the point labeled a and ending where the crown begins (labeled b). Estimate your change in horizontal position and in vertical position. Justify your numerical estimates.

SECTION 1.5

27. Gunsight Butte The graph of the vertical height of Gunsight Butte in Exercise 26 is given by a function $v = f(h)$, where v is the vertical distance (in feet) and h is the horizontal distance (in feet) measured from the origin of the coordinate system. Explain in words what $v(415) = 290$ means in its real-world context.

28. Chipotle Mexican Grill Sales Restaurant sales for Chipotle Mexican Grill increased every year between 2001 and 2005. In 2001, annual sales were \$131,331 thousand but they increased to \$625,077 thousand in 2005. Additionally, the *annual change* in restaurant sales was increasing between 2001 and 2005. (***Source:*** **Chipotle Mexican Grill, Inc, 2005 Annual Report, p. 24)** Draw a graph to model sales as a function of years since 2001.

SECTION 1.6

For situations in Exercises 29–30, do each of the following.

a. Use function notation to represent the scenario, if possible. (Be sure to identify the meaning of any variables used.)

b. Describe the advantages that knowing the inverse relationship to the situation would provide.

c. Determine whether or not the inverse relationship represents a function and justify your conclusion.

29. The price of a new electronics item (e.g., DVD player, MP3 player, plasma television) over time.

30. The temperature in your backyard from midnight one night to midnight the next.

31. Number of Children in Head Start The number of children enrolled in the Head Start program has increased as the number of women in the workforce has increased. (***Source: Statistical Abstract of the United States, 2006*, Tables 564 and 579)** The number of children enrolled in Head Start may be modeled by the function

$$H = f(N) = 0.0301N - 1134.9$$

where h is the number of children in the Head Start program (in thousands) and N is the number of women in the workforce (in thousands).

a. Describe what information the notation $f^{-1}(H) = N$ provides in this situation.

b. Solve the equation

$$700 = 0.0301N - 1134.9$$

and explain what the result means in the context of this situation.

c. The inverse function for this situation can be modeled by

$$f^{-1}(H) = N = 33.22H + 37{,}704.32$$

Evaluate $f^{-1}(700)$ and explain what it means in the context of this situation.

d. Explain the relationship between parts (b) and (c).

32. Daily Newspaper Circulation vs. Cable TV Subscribers Data reveal that as the number of cable television subscribers increases, the daily newspaper circulation decreases. (***Source:*** **www.census.gov)** Daily newspaper circulation as a function of cable TV subscribers can be modeled by the function

$$N = f(C) = -0.3824C + 81.574$$

where N is the daily newspaper circulation (in millions) and C is the number of cable television subscribers (in millions).

a. Describe what information the notation $f^{-1}(N) = C$ provides in this situation.

b. Solve the equation $60 = -0.3824C + 81.574$ and explain what the result means in the context of this situation.

c. The inverse function for this situation may be modeled by $f^{-1}(N) = C = -2.62N + 213.32$. Evaluate $f^{-1}(60)$ and explain what it means in the context of this situation.

d. Explain the relationship between parts (b) and (c).

33. Minimum Wage Salary In 2006, workers earning minimum wage earned \$5.15 per hour. (***Source:*** **www.dol.gov**) The weekly gross income of a worker earning minimum wage can be modeled by the function $D = f(h) = 5.15h$, where D is the gross income (in dollars) and h is the number of hours worked.

a. Describe what information the notation $f^{-1}(D) = h$ provides in this situation.

b. Solve the equation $206 = 5.15h$ and explain what the result means in the context of this situation.

c. The inverse function for this situation may be modeled by $f^{-1}(D) = h = \frac{D}{5.15}$. Evaluate $f^{-1}(206)$ and explain what it means in the context of this situation.

d. Explain the relationship between parts (b) and (c).

Make It Real Project

What to Do

1. Obtain a recent copy of a local, regional, or national newspaper or magazine.
2. Read through the paper looking for functions represented in words, tables, and graphs. Select a minimum of two samples for each function representation type (words, tables, and graphs).
3. For each sample, represent the data in all four function representations: formulas, tables, graphs, and words.
4. Select one of the function models and describe how you could use it to enhance your quality of life.

Where to Find Data

The following newspapers are typically accessible in public libraries:

- *The Wall Street Journal*
- *USA Today*
- *The New York Times*

The following magazines are typically accessible in public libraries:

- *Time*
- *Newsweek*
- *People Magazine*

Since many periodicals publish digital versions of their graphics and data online, you may find it helpful to access online versions for your report.