Unit 1 Notes

Module 2 - Function Review, Function Notation, Inverse Functions

The Google Slides can be found Here

General Notes

- **Quantity:** A characteristic or attribute of some object you can imagine measuring.
  - When defining or identifying a quantity, we must be specific about what object, and what specific characteristic about that object, we're referring to.
- **Variable:** A character or symbol used to represent a quantity.
- **Evaluate:** To find the output of a function corresponding to a given input.
- **Solving:** To find the input of a function corresponding to a given output
- **Domain:** the set of all reasonable inputs values of a function.
- **Range:** The set of all corresponding output values of a function
- **Interval Notation:** [0, 100]
- **Inequality:** 0 <= x <= 100

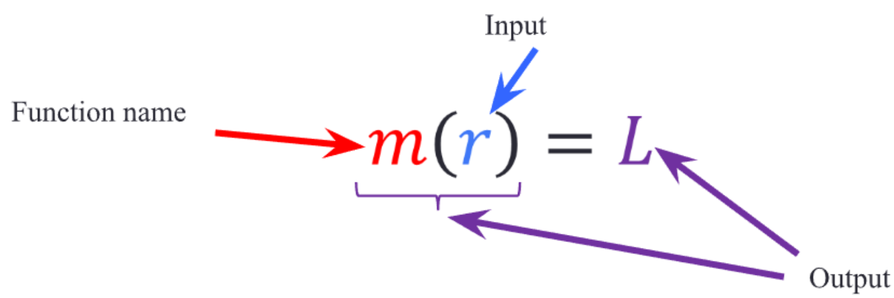
Functions

Function Language

We say "*output* as a function of *input*" or "*output* in terms of *input*"

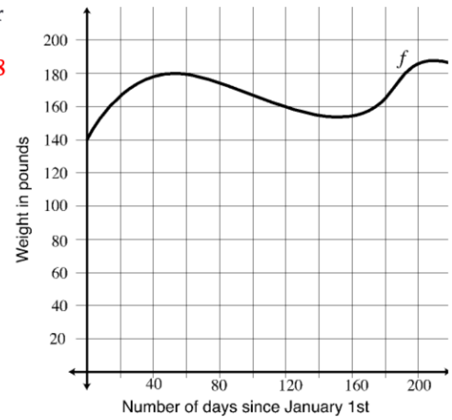
- y as a function of x **or** y in terms of x
- *E.g. Length of the steel band with respect to the radius of the oil drums*

If an input points to more than one output, then it is **not** a function.



Example 1

Suppose the following graph of the function f represents John's weight (in pounds) as a function of time t, measured in days since January 1, 2008.



a. Identify the input and output quantities for the function f.
input: t – number of days since Jan 1, 2008
output: $f(t)$ – John's weight in pounds

b. Evaluate $f(60)$. What does this value represent in the context of the problem?
$f(60) = 180$
John's weighs 180 pounds 60 days after Jan 1, 2008.

c. Solve $f(t) = 160$ for t using the graph. Describe what each solution represents.
$f(t) = 160$ when $t = 15, 119, \& 175$
Fifteen days after Jan 1, 2008, John weighed 160 pounds. John also weighed 160 pounds 119 days after Jan 1, 2008 and 175 days after Jan 1, 2008

Example 2

Given $m(x) = \frac{2x-3}{x+4}$,

a. Evaluate $m(4)$

$$m(4) = \frac{2(4) - 3}{4 + 4} = \frac{5}{8}$$

b. Evaluate $m(z)$

$$m(z) = \frac{2z - 3}{z + 4}$$

c. Evaluate $m(w + 2)$

$$m(w + 2) = \frac{2(w + 2) - 3}{(w + 2) + 4} = \frac{2w + 4 - 3}{w + 6} = \frac{2w + 1}{w + 6}$$

d. Solve $m(x) = 3$

$$3 = \frac{2x - 3}{x + 4}$$

$$3(x + 4) = 2x - 3$$

$$3x + 12 = 2x - 3$$

$$x = -15$$

Example 3 - Oil Drum Problem

For the Oil Drum Problem

It turns out that the function that determines the length of the steel band needed to tie three oil drums of radius r is

$$L = m(r) = (6 + 2\pi)r$$

- Evaluate $m(3.5)$

$m(3.5) = (6 + 2\pi)(3.5)$
$m(3.5) \approx 42.99$ feet
Solve $m(r) = 29$

- Solve $m(r) = 29$

$29 = (6 + 2\pi)r$

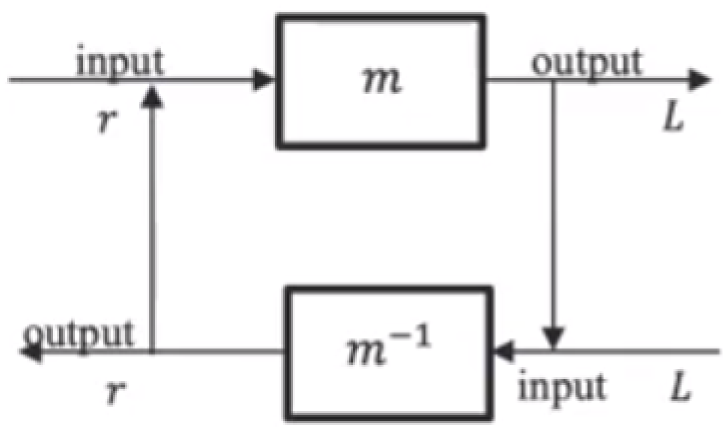
$$\frac{29}{6 + 2\pi} = r$$

$2.36 \approx r$

- `L` is the output
- `r` is the input

Inverse Functions

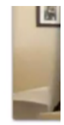
An inverse function is a function that undoes the operations of an original function.



- This function inverses the oil drum problem's function.
- The `m` is the inverse, not a negative exponent

- This image shows that `L` is a function of `r` (the name of the function is `m`) and that `r` is a function of `L` (specifically, the function is the inverse of function `m` and is written **m-1**).
- **y = f(x) means that x = f-1(y) if the inverse of f is a function.**
- The inverse of a function is not always a function.
  - *A function has one input and one output*

The formula $F = p(c) = 1.8c + 32$ will input the temperature in degrees Celsius and output the temperature in degrees Fahrenheit.



a. Find the formula for the inverse function.

$$F = 1.8c + 32$$

$$F - 32 = 1.8c$$

$$\frac{F - 32}{1.8} = c$$

Since $F = p(c)$ means $c = p^{-1}(F)$,

$$c = p^{-1}(F) = \frac{F - 32}{1.8}$$

b. Evaluate $p^{-1}(50)$ and explain its meaning in the problem context.

$$p^{-1}(50) = \frac{50 - 32}{1.8} = 10$$

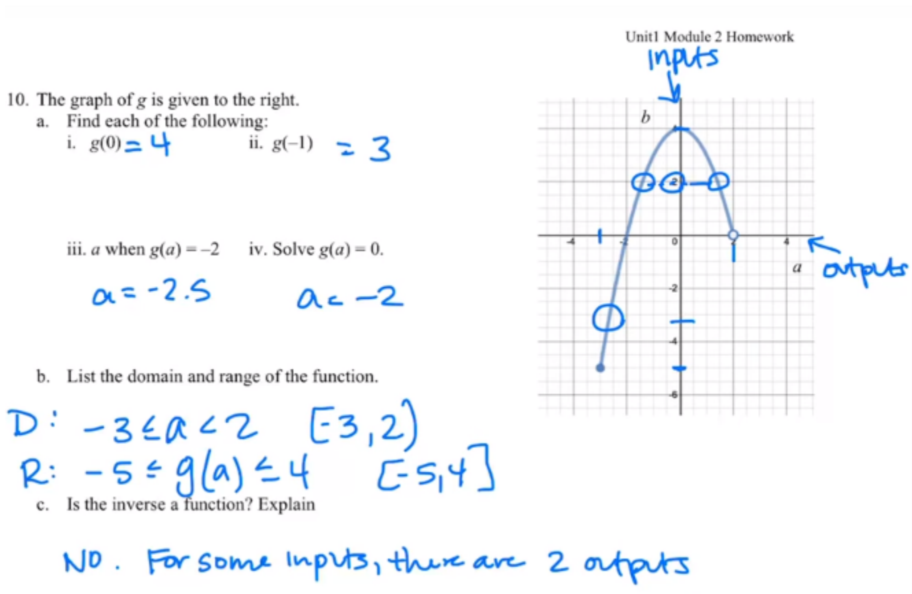
Meaning: When the temperature is 50° Fahrenheit, it is 10° Celsius.

- `F` is the output
- `c` is the input
- Always write the notation indicating that the input and outputs have switched.

Finding The Inverse

1. Write the formula without the notation:
   1. **c = j(b) = 5b + 12**
   2. **c = 5b + 12**
2. Isolate the input until it's by itself

Inverse of a Graph Example


Unit1 Module 2 Homework

10. The graph of g is given to the right.
  a. Find each of the following:
    i. $g(0) = 4$ ii. $g(-1) = 3$
    iii. a when $g(a) = -2$ iv. Solve $g(a) = 0$.
    $a = -2.5$ $a = -2$
  b. List the domain and range of the function.
  D: $-3 \le a < 2$ $[-3, 2)$
  R: $-5 \le g(a) \le 4$ $[-5, 4]$
  c. Is the inverse a function? Explain
  NO. For some inputs, there are 2 outputs

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Example problem

- *Remember not to use linear regression when creating the linear formula, instead use the* ***point-slope form: y - b = m(x - a)***

A devastating freeze in California's Central Valley in January 2007 wiped out approximately 75% of the state's citrus crop. It turns out that the cost for a box of oranges is a function of the percentage of the citrus crop that was frozen, i.e. **c = g(P)**, where **c** is the price of a box of oranges and **P** is the percentage of the citrus crop that was frozen. When only 20% of the crop was frozen, the price for a box of oranges was $11.58. However, the price per box was $25.32 when 80% of the crop was frozen.

Finding The Linear Function

1. Identify the two given points: **(20, 11.58)** and **(80, 25.32)**
2. Calculate the slope **(m)** using the formula: **m = (y2 - y1) / (x2 - x1)**, where **(x1, y1) = (20, 11.58)** and **(x2, y2) = (80, 25.32)**
3. Plug in the values into the formula to get: **m = (25.32 - 11.58) / (80 - 20) = (13.74) / (60) = 0.229**
4. Use the point-slope form of a linear equation to find the equation of the line: **y - y1 = m (x - x1)**, where **(x1, y1) = (20, 11.58)**
5. Plug in the values into the formula to get: **y - 11.58 = 0.229 (x - 20)**
6. Rearrange the equation to the standard form: **y = 0.229x + b**, where **b** is the y-intercept

7. Calculate the y-intercept using the formula: **b = y1 - m * x1**, where **(x1, y1) = (20, 11.58)**
8. Plug in the values into the formula to get: **b = 11.58 - (0.229 * 20) = 11.58 - 4.58 = 7**
9. The equation of the line is: **y = 0.229x + 7**, which represents the cost of a box of oranges as a function of the percentage of the citrus crop that was frozen.

Finding The Inverse Function

1. Write the original function in the form **y = f(x): c = 0.229P + 7**
2. Replace **y** with **x** and **x** with **y**: **x = 0.229P + 7**
3. Solve for **P: x - 7 = 0.229P**, then **P = (x - 7) / 0.229**
4. The inverse function is: **P = (c - 7) / 0.229**

Summary:

1. To find the inverse of a function, switch the roles of **x** and **y**.
2. Write the original function in the form **y = f(x)**.
3. Replace **y** with **x** and **x** with **y**.
4. Solve for the original variable (in this case, P).
5. The inverse function is the result from step 4.

Note: The inverse of a function is not always a function. The inverse of a function is a function only if the original function is a one-to-one function.

Writing Out Functions

11. Given a function $p(s) = b$ where s = the number of square mile of a forest and b = the number of rabbits in the forest, explain the meaning of the following.

a. $p(100) = 85$

\# sq # rab

When thesize of a forest is 100 sq miles, there are 85 rabbits

b. $p(54)$

\# sq

\# rabb

The # of rabbits in a forest that is 54 sq miles

c. $p^{-1}(93)$

\# rabbits

\# sq mi

The # of square miles a forest is with 93 rabbits in it.

Domain and Range

- **Domain:** the set of all reasonable inputs values of a function.
- **Range:** The set of all corresponding output values of a function

Example 1

The function that determines the length of the steel band needed to tie three oil drums of radius r is

$$L = m(r) = (6 + 2\pi)r$$

What would be a practical domain and range for this function? i.e., what values make sense for the radius and length of the steel band?

I would guess that the radius of an oil drum is not less than one half of a foot and probably not more than 5 feet. So the *practical domain* would be $0.5 \leq r \leq 5$ also written as $[0.5, 5]$ in interval notation.

Since a radius of 0.5 feet would require a steel band of 6.14 feet, and a since a radius of 5 feet would require a steel band of 61.42 feet, the *practical range* would be
$6.14 \leq L \leq 61.42$ also written as $[6.14, 61.42]$ in interval notation.

Example 2

The letter grade earned on a test with respect to the percentage grade earned.

Practical Domain: $[0, 100]$

Practical Range: $\{A, B, C, D, F\}$

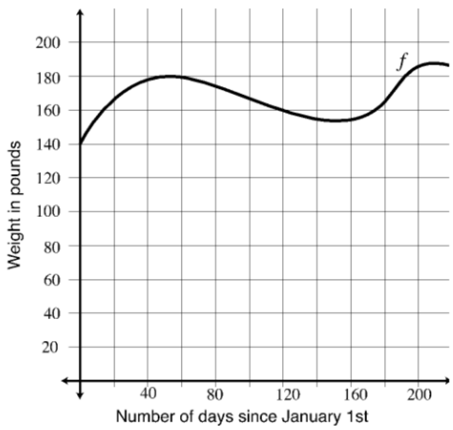
Example 3

Suppose the following graph of the function *f* represents John's weight (in pounds) as a function of time *t,* measured in days since January 1, 2008.

Determine the practical domain and range.

Practical Domain: $[0, 220]$

Practical Range: $[140, 190]$



Intercepts

Vertical Intercepts

The **Vertical Intercept** of a function is the coordinate point where the graph of the function crosses the vertical axis.

- This point will always be in the form **(0, b)**
- The vertical intercept can be found graphically by determining the point where teh graph crosses the vertical axis.
- The vertical intercept can be found in a table or algebraically by first determining the value of **b**.
  - To do this, find the output of the function for an input of 0 *(or* ***f(0)* = *b*)**. You then write the intercept in the form **(0, b)**.

Horizontal Intercepts

The **Horizontal Intercept** of a function is the coordinate point where the graph of the function crosses the horizontal axis.

- The point will always be in the form **(a, 0)**.
- The horizontal intercept can be found graphically by determining the point where the graph crosses the horizontal axis.
- The horizontal intercept can be found in a table or algebraically by first determining the value of **a**.

- To do this, find the input of the function for an output of 0 *(or solve for **a** when **f(a) = 0**)*. You then write the intercept in the form **(a, 0)**.

Constant Rate of Change

$$\frac{\text{change in output value}}{\text{change in input value}}$$

It is said that two quantities are related by a **constant rate of change (CROC)** if the *ratio of the changes in quantities* is always the same.

- Find the changes between each value in a table for all relevant columns, and use those in this formula.

Example 1

Change in time elapsed Δt	Time Elapsed in min t	Amount of water in a bath tub in gallons, a	Change in amount of water in tub Δa
	1	11.75	
1.5	2.5	14.375	2.625
2.5	5	18.75	4.375
3	8	24	5.25

- The triangle just means change

To figure out if the ratio is the same, the changes should all equal the same number when put into the formula:

$$\frac{\Delta a}{\Delta t} = \frac{2.625}{1.5} = \frac{4.375}{2.5} = \frac{5.25}{3} = 1.75$$

- Because all the numbers equal the same, it is **constant**.

This would be written as: ***For every additional minute* that the water is left running, the *amount of water* in the bathtub *increases* by 1.75 gallons.**

Module 3 - Constant Rate of Change and Linear Functions

The Google Slides can be found Here

General Notes

Constant Rate of Change (Continued)

The value of the constant rate of change can always be determined by:

$$\begin{matrix}\textit{Constant Rate of Change} \\ (\textit{CROC})\end{matrix} = \frac{\textit{change in output value}}{\textit{change in input value}}$$

Knowing this info, you can also get the other values.

Change in Output Value

$$\begin{pmatrix}\textit{change in} \\ \textit{output value}\end{pmatrix} = (\textit{CROC}) \cdot \begin{pmatrix}\textit{change in} \\ \textit{input value}\end{pmatrix}$$

Change in Input Value

$$\frac{\textit{change in output value}}{\textit{CROC}} = \begin{pmatrix}\textit{change in} \\ \textit{input value}\end{pmatrix}$$

Instead of always using the formula to find the change in input / output or the CROC, you can use repeated reasoning.

Imagine you have a pool with a hose in it filling it with water (it already has some in it). The CROC is **18.2**, and after **63 minutes**, there's **1382.6 gallons** inside. Instead of using formulas to find each value per different minute, you could create a formula:

1. Find **Δt** *(change in input value)*
   - **t - 63 = Δt**
2. Find **Δv** *(change in output value)*
   1. **18.2(Δt) = Δv**
   2. **18.2(t - 63) = Δv**
3. Find ***v*** *(total volume)*
   1. **Δv + 1382.6 = *v***
   2. **18.2(t - 63) + 1382.6 = *v***

- Δ means **Increment / Change**

The last one is the finished formula for a function that defines a relationship between the volume of water in the pool and the amount of time the pool has been filling. It can also be summarized as:

Change In Output

> **v = CROC(t - reference input) + reference Output**
> **v = CROC(t - tref) + vref**

General Form of a Linear Function

Whenever two quantities are related by a CROC, it's a line on a graph.

- That's where the **Linear Function** comes from
- The process above can be used any time there's a CROC and a known reference point.

Module 4 - Linear Functions, Average Rate of Change and Linear Regression

- Google Slide Notes

General Notes

General form of a linear function:

$$v = \underbrace{\begin{matrix}\text{constant rate}\\ \text{of change}\end{matrix}\left(\underbrace{t - \begin{matrix}\text{reference}\\ \text{input}\end{matrix}}_{\text{change in input}}\right)}_{\text{change in output}} + \begin{matrix}\text{reference}\\ \text{output}\end{matrix}$$

$$v = \underbrace{\text{CROC}\left(\underbrace{t - t_{ref}}_{\text{change in input}}\right)}_{\text{change in output}} + v_{ref}$$

When the reference point is the vertical intercept, the formula simplifies:

1. **v = CROC(t - tref) + vref**
2. **v = CROC(t - 0) + output of VI**
3. **v = CROC(t) + Output of VI** or **y = m(x) + b**

- This formula is a *special case* of a Linear Function and can only be used if the reference point is the vertical intercept.

Average Rate of Change

When the rate is not constant, the rate is called the **Average Rate of Change (AVROC)**.

- The average rate of change can be found between **any two points** by:
  1. finding the constant rate of change between those two points.
  2. Dividing the constant rate of change by the input.

Example

Numberof weeksdieting	Brandon'sweight inpounds
0	196
2	187
7	190
12	184

- The average rate of change between week **0** and **7** is **-0.86**, but the rate does not go down constantly at this rate. Because of this, the average rate of change means in this scenario:

> **IF** Brandon's weight had **changed by the same amount each week between week 0 and week 7**, he *would have* lost 0.86 pounds each week.

- If you needed to write this about a graph: > **IF** the function changed at a constant rate of change between **x = 0** > and **x = 7**, we would have a line between those two points and that line > *would have* a constant rate of **-0.86** (as shown in {color} on > the graph).
- It's also important to use the data that is most closely related to the point in time that you are trying to find *(in this case, it would be between week **7** and **12**, not **0** and **7**)*.

The average rate of change is a *hypothetical* constant rate.

To find what the output would be at any specific non-given point, multiply the average rate of change by the change in input:

- **output = AROC(ΔInput)**
  - The same as **output = (Δ output/Δ input)(ΔInput)**

The average rate of change is calculated by dividing the changes of two outputs by the changes in the corresponding inputs. That is,

$$\text{Average Rate of Change} = \frac{\Delta\text{output}}{\Delta\text{input}} = \frac{f(b)-f(a)}{b-a}$$

where $(a, f(a))$ and $(b, f(b))$ are any two data points of the function f.

Polynomial Example

g(x) = 2x2 - 3x - 4

Determining the average rate of change between **x = 5** and **x = 1**:

1. **Average Rate of Change = g(5) - g(1)/5 - 1**
2. **g(5) = 2(52) - 3(5) - 4 = 31**
3. **g(1) = 2(12) - 3(1) - 4 = -5**
4. **Average Rate of Change = g(5) - g(1)/5 - 1 = 31 + 5/4 = 36/4 = 9**

Image Reference:

$$\mathit{Average\ Rate\ of\ Change} = \frac{g(5) - g(1)}{5 - 1}$$

$$g(5) = 2(5^2) - 3(5) - 4 = 31 \qquad g(1) = 2(1^2) - 3(1) - 4 = -5$$

$$\mathit{Average\ Rate\ of\ Change} = \frac{g(5) - g(1)}{5 - 1} = \frac{31 - (-5)}{4} = \frac{36}{4} = 9$$

Linear Regression

When data is not perfectly linear but is close, we create a **Linear Model**, a function that *models* the data. This is done using a process called **Linear Regression**.

How To Graph Linear Regression On Desmos

1. Click the plus button (Add an item)
2. Choose **Table**
3. Add some values (enter or copy/paste)
4. Change graph settings to better match the data.
5. Instead of **y1 = mx1 + b**, use the **tilde (~)** symbol for the **equals (=)** sign: **y1 ~ mx1 + b** to use linear regression.
   - To use an exponential pattern, use: **y1 ~ a * bx1**
6. To predict a value, use either:
   1. **x = {desired x value}**
   2. **m({desired x value}) + b**

- It can be helpful to create a folder hitting the plus button, dragging the table into it, and then closing it.

Coefficient of Determination

When computing a linear regression model, the **r2** is known as the coefficient of determination, and it describes the strength of the fit of a linear regression model to a set of data.

The closer the value of r2 is to 1, the stronger the fit. **r2** is always a value between **0** and **1**.