Unit 4 - Piecewise Functions, Function Composition, and More Function Transformations

General Notes

Module 14 - Function Transformations Part 2

Module 14 - Google Slides

> **Links:**
>
> Slides | PDF
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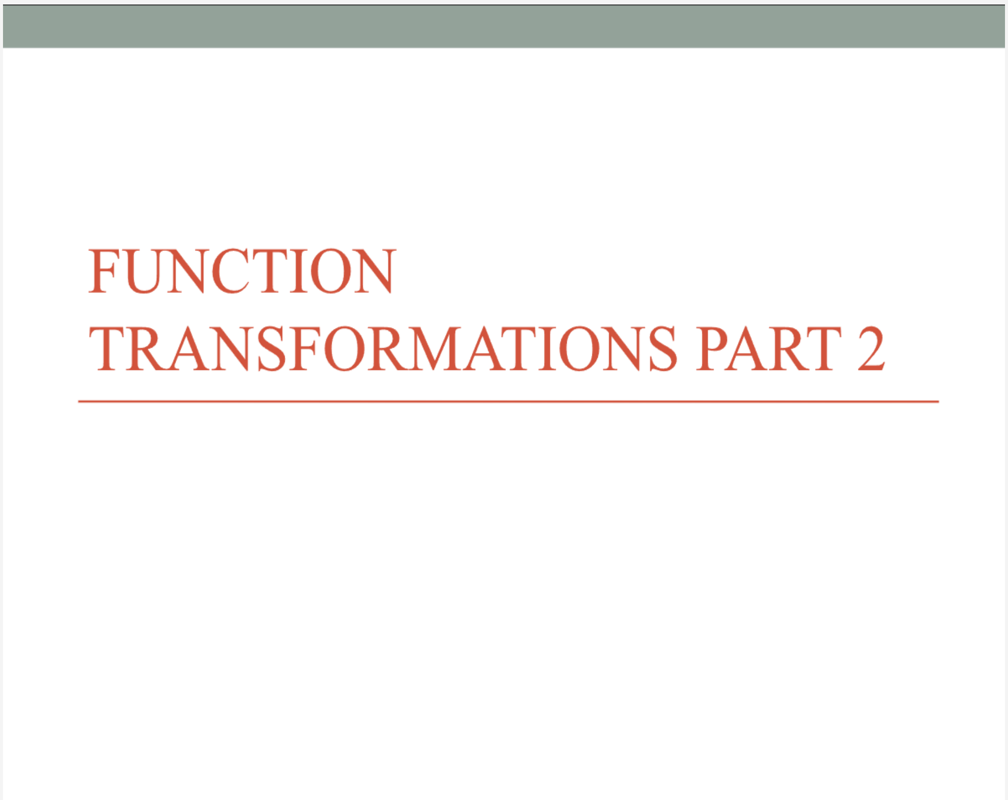
> 
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Function Transformations

The function $g(x) = af(x-c) + d$ is a transformed function in terms of the parent function $f(x)$.

The parameters **a**, **c**, and **d**
transform the function in the following ways:

Parameter	Transformation
d	**Causes a vertical shift.** • If **d > 0**, then there is a shift up. • If **d < 0**, then there is a shift down.
a	**Causes a vertical stretch or compression.** • If **|a| > 1**, then there is a vertical stretch. • If **0 < |a| < 1**, then there is a vertical compression.
c	**Causes a horizontal shift** • If **c > 0**, then the shift is to the right. • If **c < 0**, then the shift is to the left.

Identify the Transformations

Identify Transformation One

Given the parent function is $f(x)$, the transformations used to create the transformed function $g(x)$ are:

> $g(x) = -2f(x) + 3$

- Vertical reflection
- Vertical stretch by a factor of 2
- Vertical shift up 3 units

Suppose that $f(x) = 2\sqrt{x} - 1$. The formula in terms of x for each of the functions is:

1. $g(x) = -2f(x) + 3$
2. $g(x) = -2(2\sqrt{x} - 1) + 3$
   - You could leave it like this, but it's better to follow the rest of the steps for readability.

3. ***g*(x) = -4√x +2 + 3**
4. ***g*(x) = -4√x + 5**

Transformation One Image Format

Given the parent function is $f(x)$, list the transformations to create the following transformed function:

$$g(x) = -2f(x) + 3$$

- Vertical reflection
- Vertical stretch by a factor of 2
- Shift up 3 units

Suppose that $f(x) = 2\sqrt{x} - 1$. Write the formula in terms of x for each of the functions.

$$g(x) = -2f(x) + 3$$
$$g(x) = -2(2\sqrt{x} - 1) + 3$$
$$g(x) = -4\sqrt{x} + 2 + 3$$
$$g(x) = -4\sqrt{x} + 5$$

Identify Transformation Two

Given the parent function is ***f*(x)**, the transformations used to create the transformed function ***k*(x)** are:

> ***k*(x) = 0.5*f*(x - 4)**

- Vertical compression by a factor of 0.5
- Horizontal shift to the right 4 units

Suppose that ***f*(x) = 2√x - 1**. The formula in terms of ***x*** for each of the functions is:

1. ***k*(x) = 0.5*f*(x - 4)**
2. ***k*(x) = 0.5(2√(x - 4) - 1)**
   - The equation becomes **x - 4** inside the radical, because the input for ***f*** is **x - 4**, not just **x**.

   - The radical sign goes above both the **x** and the **4**, hence the parentheses.
3. ***k*(x) = 1√(x - 4) - 0.5**
4. ***k*(x) = √(x - 4) - 0.5**

Transformation Two Image Format

Given the parent function is *f*(*x*), list the transformations to create the following transformed function:

$$k(x) = 0.5f(x - 4)$$

- Vertical compression by a factor of 0.5
- Horizontal shift right 4 units

Suppose that $f(x) = 2\sqrt{x} - 1$. Write the formula in terms of x for each of the functions.

$$k(x) = 0.5f(x - 4)$$
$$k(x) = 0.5\left(2\sqrt{x - 4} - 1\right)$$
$$k(x) = 1\sqrt{x - 4} - 0.5$$
$$k(x) = \sqrt{x - 4} - 0.5$$

Identify Transformation Three

Given the parent function is ***f*(x)**, the transformations used to create the transformed function ***j*(x)** are:

> ***j*(x) = -4*f*(-x) - 0.5**

- Vertical reflection
- Vertical stretch by a factor of 4
- Horizontal reflection
- Vertical shift down 0.5 units

Suppose that ***f*(x) = 2√x - 1**. The formula in terms of ***x*** for each of the functions is:

1. $j(x) = -4(f(-x) - 0.5)$
2. $j(x) = -4(2\sqrt{-x} - 1) - 0.5$
3. $j(x) = -8\sqrt{-x} + 4 - 0.5$
4. $j(x) = -8\sqrt{-x} + 3.5$

Transformation Three Image Format

Given the parent function is $f(x)$, list the transformations to create the following transformed function:

$$j(x) = -4f(-x) - 0.5$$

- Vertical reflection
- Vertical stretch by a factor of 4
- Vertical shift down 0.5 units
- Horizontal reflection

Suppose that $f(x) = 2\sqrt{x} - 1$. Write the formula in terms of x for each of the functions.

$$j(x) = -4f(-x) - 0.5$$
$$j(x) = -4(2\sqrt{-x} - 1) - 0.5$$
$$j(x) = -8\sqrt{-x} + 4 - 0.5$$
$$j(x) = -8\sqrt{-x} + 3.5$$

Identify Graph Transformations

Identify Graph Transformation One

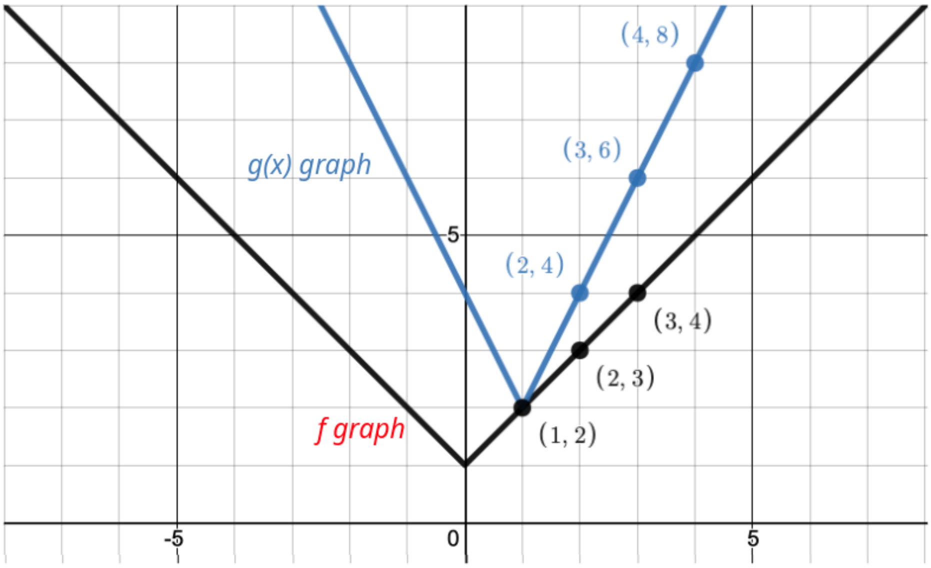
Suppose we were given the graph of the parent function f and asked to graph the transformed function:

> $g(x) = 2f(x — 1)$

The graph:

input	output
1	2
2	3
3	4

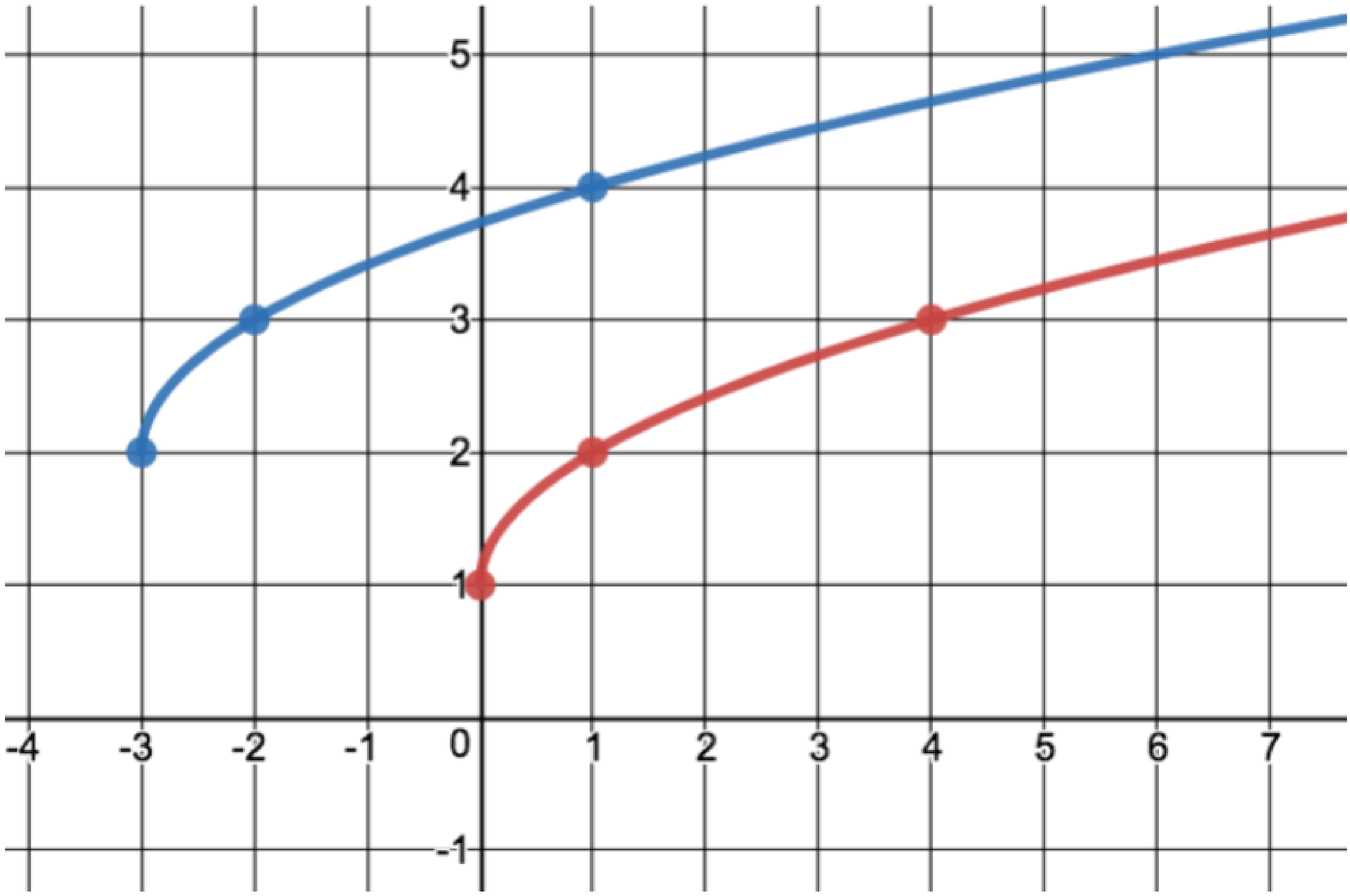
input	output
2	4
3	6
4	8



- The graph was shifted to the right 1 unit, and stretched vertically by a factor of 2.
    - The **x** inputs were shifted to the right 1 unit.
    - The **y** outputs were stretched vertically by a factor of 2.

Identify Graph Transformation Two

Suppose we were given the graph of the parent function ***r*** (in red) and the transformed function ***b*** (in blue) and asked to identify the transformations:



Based on the graph, the formula of ***b*** in terms of ***r*** is:

> ***b*(x) = r(x + 3) + 1**

To identify the transformations:

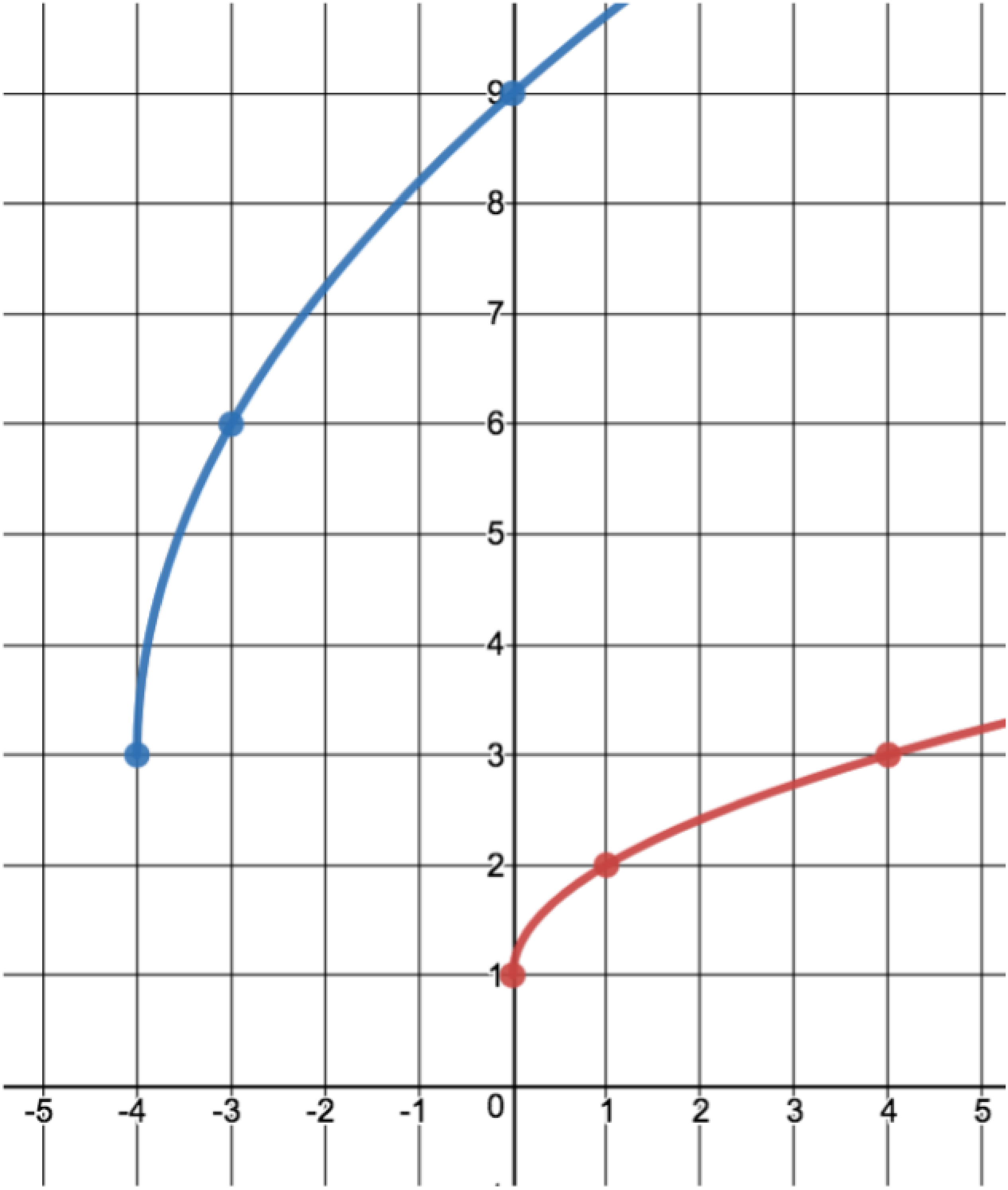
- **Vertical stretch or compression**
  1. Look for a change in outputs from on set of coordinates to the next on both graphs. If the parent function increases by 1 unit on both the *x* and *y* axes, but the transformed function increases by more than 1 unit, there is either a stretch or compression.
- **Horizontal shift**
  1. Look for how far the transformed function is shifted horizontally from the parent function.
- **Vertical shift**
  1. Look for how far the transformed function is shifted vertically from the parent function.
- **Vertical reflection**
  1. Look for a change in the vertical direction of the graph.

2. If the parent function is going one way vertically and the transformed function is vertically going the opposite way, there is a vertical reflection.

- **Horizontal reflection**
  1. Look for a change in the horizontal direction of the graph.
  2. If the parent function is going one way horizontally and the transformed function is horizontally going the opposite way, there is a horizontal reflection.

Identify Graph Transformation Three

Suppose we were given the graph of the parent function ***r*** (in red) and the transformed function ***b*** (in blue) and asked to identify the transformations:



Based on the graph, the formula of ***b*** in terms of ***r*** is:

> ***b*(x) = 3r(x + 4)**

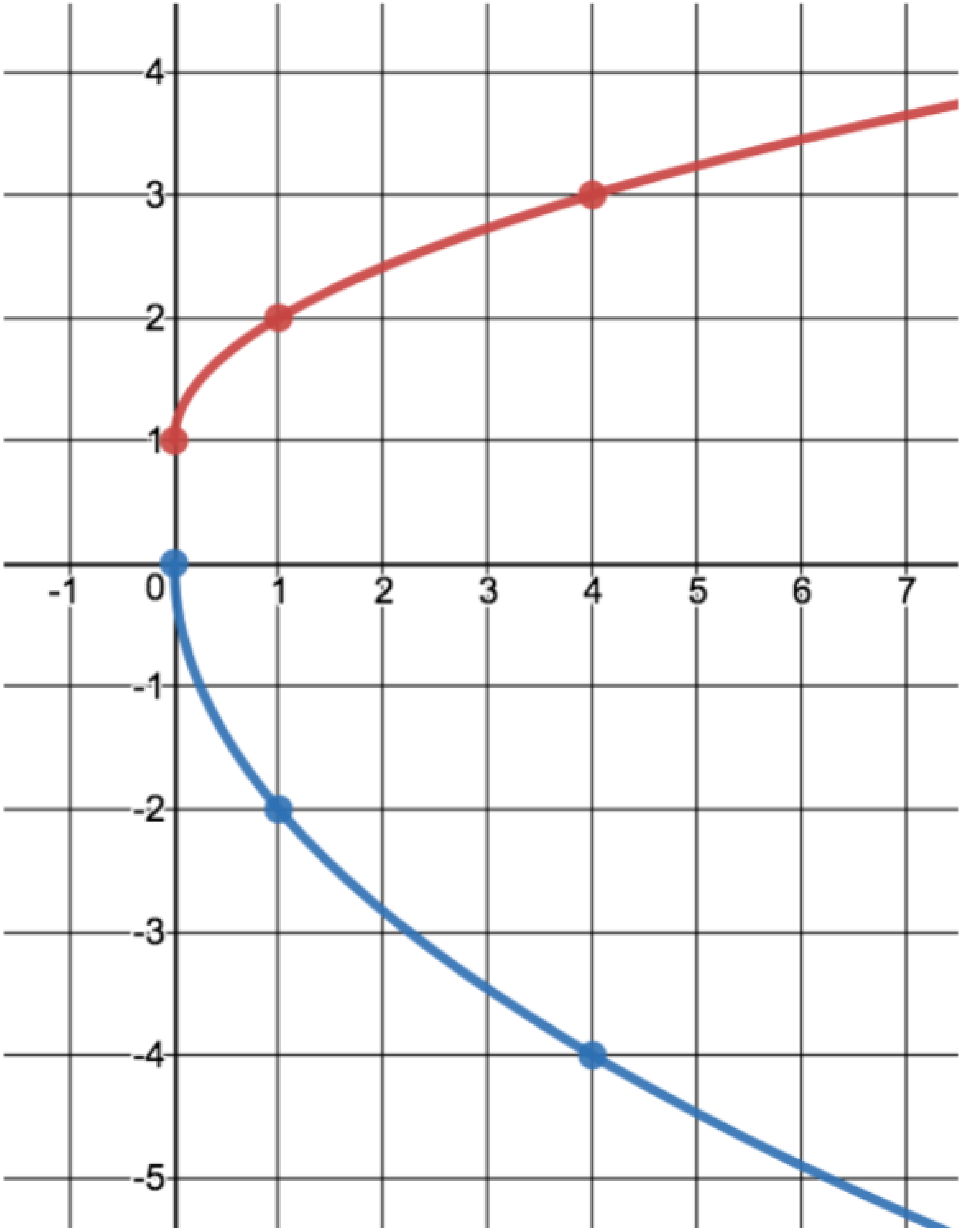
To identify the transformations:

- **Vertical stretch**
  1. The parent function increases by 1 unit on both the *x* and *y* axes, but the transformed function increases by 3 units on the *y*-axis and 1 unit

    on the *x*-axis.
    2. The increase in the transformed function is **3** times more than the increase in the parent function.
    3. The parent function starts at **(0, 1)**, but if it were stretched by a factor of **3**, then the transformed function would start at **(0, 3)
    **.
        - You can make sure that the predicted stretch is correct by either drawing out the graph or by checking the other points and making sure that they all align with what the y-values should be. *Use a table if necessary*.
    4. It's being stretched by a factor of **3**.
    5. It's also helpful to draw out the stretched graph to make sure that it matches the transformed graph.
- **Horizontal shift**
    1. The transformed function is shifted to the left by **4** units.

Identify Graph Transformation Four

Suppose we were given the graph of the parent function ***r*** (in red) and the transformed function ***b*** (in blue) and asked to identify the transformations:



Based on the graph, the formula of ***b*** in terms of ***r*** is:

> ***b*(x) = -2r(x) + 2**

To identify the transformations:

- **Vertical reflection**
    1. The parent function is going one way vertically and the transformed function is vertically going the opposite way, there is a vertical

reflection.

- **Vertical stretch**
  1. The transformed function decreases at twice the amount of the parent function.
- **Vertical shift**
  1. The transformed function is shifted up by **2** units.

Horizontal Reflections

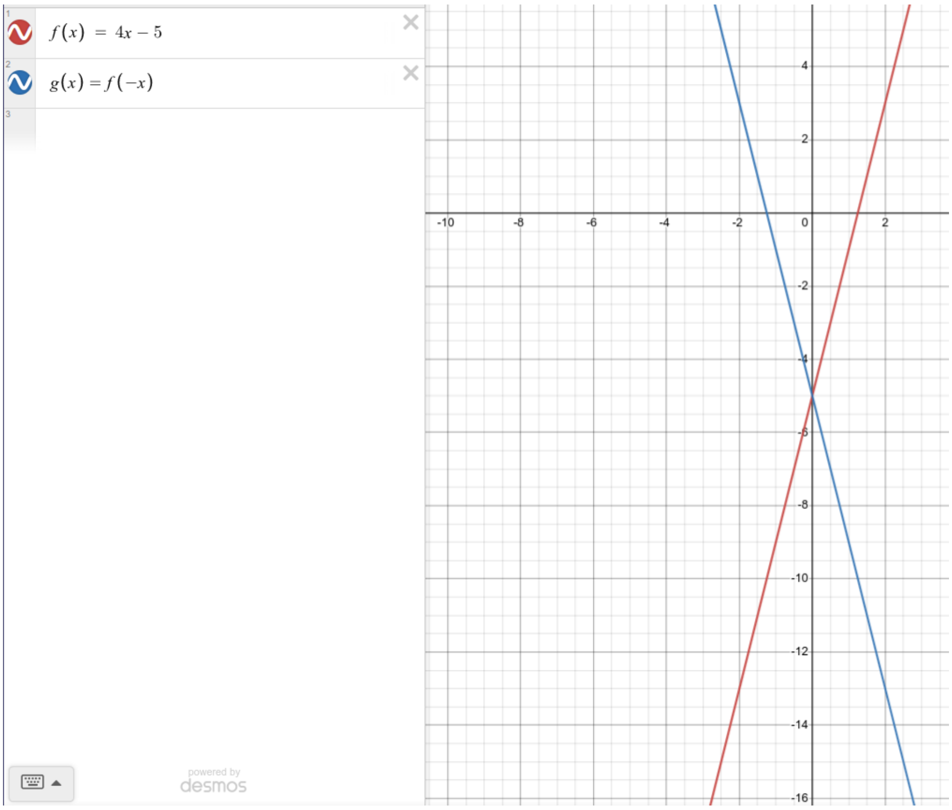
Suppose that the transformed function ***g*(x)** is:

> ***g*(x) = *f*(-x)**

Now suppose that the parent function ***f*** is:

> ***f*(x) = 4x - 5**

The Result:



In general, think of horizontal reflections as follows:

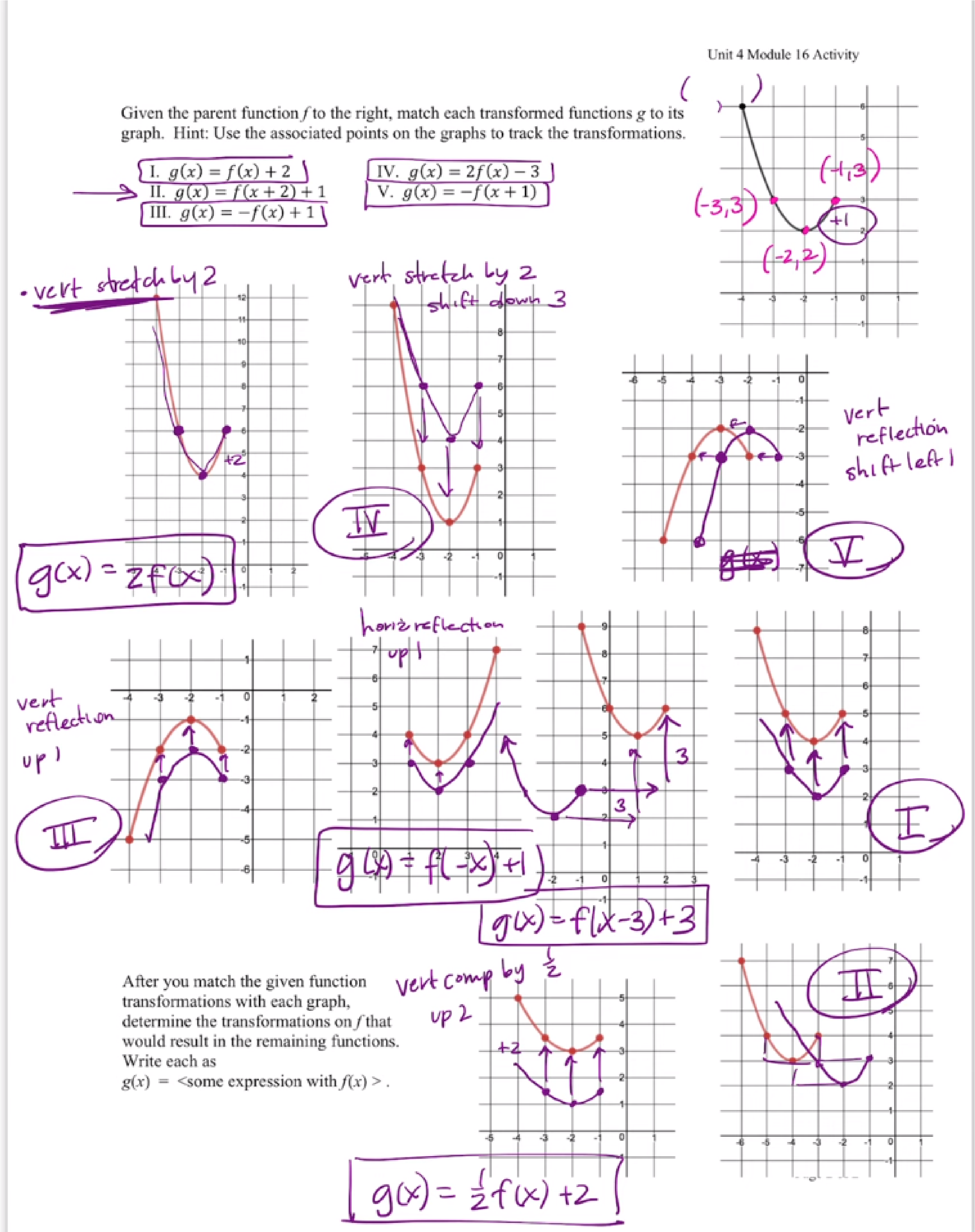
$$\underbrace{g(x)}_{\text{the output of } g \text{ at } x} \underbrace{=}_{\text{is equal to}} \underbrace{f(-x)}_{\text{the output of } f \text{ at the opposite of } x}$$

Transformations Sheet

Transformations Sheet One

Image (Transformed) Function (in terms of x)	Identify the Parent Function f	Transformations *(List how the graph would be affected)*	Write the transformed function in terms of the parent function f
1. $g(x)= 3x^2 + 2$	$f(x) = x^2$	• Vertical stretch by a factor of 3 • Vertically shifted up by 2	$g(x)= 3f(x)+2$
2. $h(x) = -x^3 + 3$	$f(x)= x^3$	• Vertical reflection • Shifted up 3	$g(x)= -f(x)+ 3$
3. $j(x)= \sqrt{x}+ 3$	$f(x)= \sqrt{x}$	• Horizontal reflection • Vertical shift up 3	$j(x)= f(-x)+3$
4. $k(x) = \frac{1}{3}(x-1)^2 + 2$	$f(x)= x^2$	• Vertical compression by a factor of $\frac{1}{3}$ • Horizontal shift right by 1 • Vertical shift up by 2	$k(x)= \frac{1}{3}f(x - 1)+2$
5. $m(x)= 2(\lvert x + 4\rvert+1)$	$f(x)=\lvert x\rvert+1$	• Vertical stretch by a factor of 2 • Horizontal shift left by 4	$m(x)= 2f(x + 4)$

Transformations Sheet Two



Unit 4 Module 16 Activity

Given the parent function f to the right, match each transformed functions g to its graph. Hint: Use the associated points on the graphs to track the transformations.

I. $g(x) = f(x) + 2$

II. $g(x) = f(x+2) + 1$

III. $g(x) = -f(x) + 1$

IV. $g(x) = 2f(x) - 3$

V. $g(x) = -f(x+1)$

After you match the given function transformations with each graph, determine the transformations on f that would result in the remaining functions. Write each as $g(x)$ = <some expression with $f(x)$> .

- *This sheet was created by Kacie Joyner*

Module 15 - Piecewise Functions

Module 15 - Google Slides

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Piecewise Function Definition

A **piecewise function** is a function that is defined in different ways for different intervals.

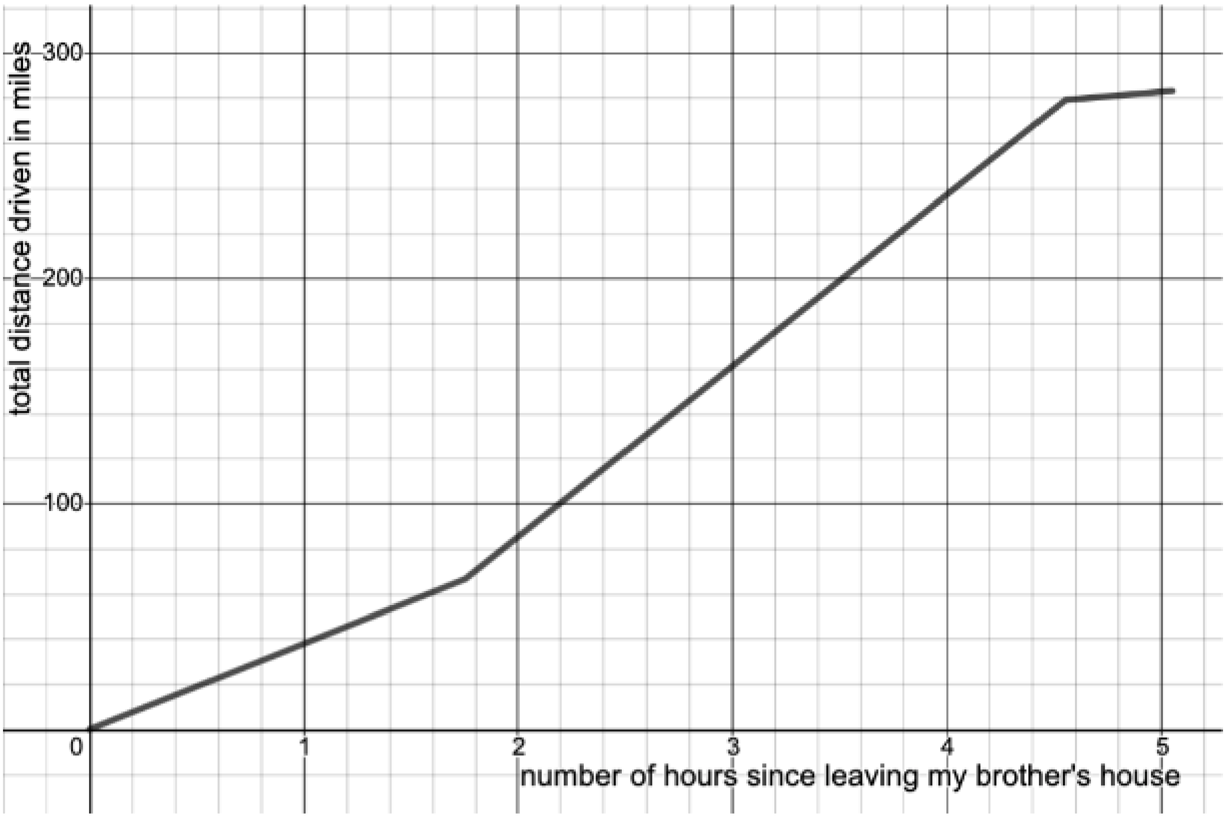
Roadtrip Problem

Initial Story

> On a recent road trip I traveled through various road conditions and weather patterns and so my average speed varied greatly throughout the day.
>
> I started the day at my brother's house in a rural area in northern Florida. Because he lives so far from the highway my average speed for the first 1.75 hours was only 38 mph. Once I hit the highway however I was able to drive faster and hence I averaged 76 mph for the next 2.8 hours. Unfortunately, as is often the case in Florida, a sudden torrential rainstorm came up and the visibility was so bad that for the next 30 minutes I only averaged 8 mph! Finally, the rain stopped, but by then I was so tired I needed to take a rest, so I pulled off at the next exit and found a McDonald's to sit at and surf the internet for a little while.

Roadtrip Graph

This graph represents the roadtrip problem:



Creating a Function for the Model

There are two options:

1. Create a separate function model per portion of the road trip and restrict the domain and range for each function.
    - The problem with this is that there are multiple models for the same road trip.
2. Use a piecewise function to model the road trip.

Goal: Create a function f to represent the total distance *(in miles)* traveled on my road trip, D, with respect to the amount of time *(in hours)* since I left my brother's house, t.

Creating Segments Per Portion of the Road Trip

We start by creating a function for each portion of the road trip:

1. **Segment 1:** Average speed of **38** mph for **1.75** hours
    1. **66.5** miles traveled in the first **1.75** hours.
        - *Gotten by multiplying **38** by **1.75**.*
    2. Practical domain starts at **[0, 0]** and ends at **[1.75, 0]**.
    3. Practical range starts at **[0, 0]** and ends at **[0, 66.5]**.

4. The function is ***D* = *ƒ*(*t*) = 38t; 0 ≤ *t* ≤ 1.75**
   ← *Practical domain*
2. **Segment 2:** Average speed of **76** mph for **2.8** hours
   1. **212.8 miles** in the next **2.8** hours.
   2. Practical domain: **1.75 ≤ *t* ≤ 4.55**
   3. Practical range: **66.5 ≤ *D* ≤ 279.3**
   4. The function: **D = *ƒ*(*t*) = 76(*t* - 1.75) + 66.5**
      - Created using either the **CROC** or the transformation values.
3. **Segment 3:** Average speed of **8** mph for **0.5** hours
   1. **4** miles in the next **0.5** hours.
   2. Practical domain: **4.55 ≤ *t* ≤ 5.05**
   3. Practical range: **279.3 ≤ *D* ≤ 283.3**
   4. The function: **D = *ƒ*(*t*) = 8(*t* - 4.55) + 279.3**

The final segment functions for the road trip are:

> 1. **D = *ƒ*(*t*) = 38t; 0 ≤ *t* ≤ 1.75**
> 2. **D = *ƒ*(*t*) = 76(*t* - 1.75) + 66.5; 1.75 ≤ *t* ≤ 4.55**
> 3. **D = *ƒ*(*t*) = 8(*t* - 4.55) + 279.3; 4.55 ≤ *t* ≤ 5.05**

Final Piecewise Function

The final piecewise function is:

$$D = f(t) = \begin{cases} 38t & \text{if} \quad 0 \le t \le 1.75 \\ 76(t - 1.75) + 66.5 & \text{if} \quad 1.75 < t \le 4.55 \\ 8(t - 4.55) + 279.3 & \text{if} \quad 4.55 < t \le 5.05 \end{cases}$$

How to Create a Piecewise Function

Piecewise Functions

A **piecewise function** is defined using two or more expressions over given intervals of the domain. Piecewise functions are written in the form

$$f(x) = \begin{cases} \text{Rule 1} & \text{if Condition 1} \\ \text{Rule 2} & \text{if Condition 2} \\ \text{Rule 3} & \text{if Condition 3} \\ \vdots & \vdots \end{cases}$$

The conditions define the input values for which each rule applies. The graphs of piecewise functions may be *continuous* or *discontinuous*. Intuitively, a **discontinuous** function is one with a "break," "hole," or "jump" in its graph and a **continuous** function is one whose graph can be drawn without lifting one's pencil.

Text Version:

> A **piecewise function** is defined using two or more expressions over given intervals of the domain.
>
> The conditions define the input values for which each rule applies. The graphs of piecewise functions may be *continuous* or *discontinuous*. Intuitively,
> a *
> *discontinuous function** is one with a "break", "hole", or "jump" in its graph and a **continuous function** is one whose graph can be drawn without lifting one's pencil.

Example Problems

1. ***f*(3) = 76(3 - 1.75) + 66.5 → D = 161.5**
2. **57 = *f*(*t*)**
    1. Plug in the value for each rule until you find the correct rule.
    2. For rule 1, it results in ***t*** = **1.5**.
        - This meets the condition for rule 1, so it is the correct rule.
    3. For rule 2, it results in ***t*** = **1.625**.
    4. For rule 3, it does not produce a valid result either.

Parking Garage Problem

Consider the given table, which shows the fees to park in the East Economy Garage at Sky Harbor International Airport in Phoenix, Arizona for a single day.

We see that for any time over **0** minutes through **60 minutes**, the fee is $4.00 * *; fortimeover * *60 * *through * *120minutes * *, thefeeis * *$**8.00**; and for any time over **120 minutes** (for one day), the fee is **$10.00**.

Table representing the parking garage problem:

Parking Time (minutes) ***m***	**Parking Fee (dollars)** ***F***
Over 0 through 60	**4.00**
Over 60 through 120	**8.00**
Over 120	**10.00**

First create functions for all three rules:

1. **F = *f*(*m*) = 4; 0 ≤ *m* ≤ 60**
2. **F = *f*(*m*) = 8; 60 ≤ *m* ≤ 120**
3. **F = *f*(*m*) = 10; 120 ≤ *m***

Then create a piecewise function:

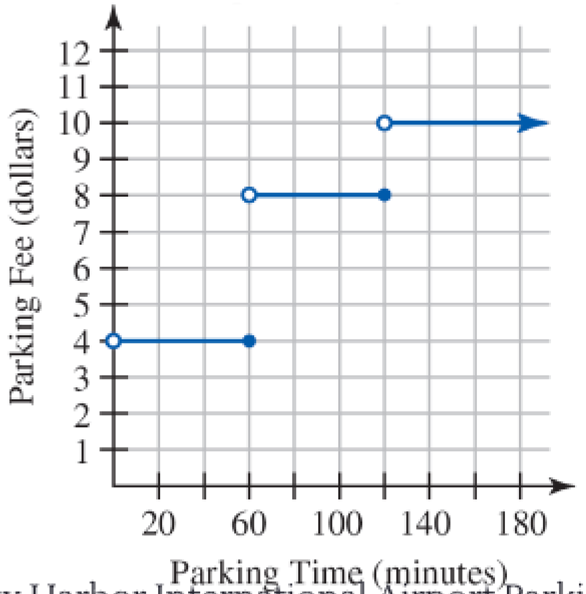
$$F(m) = \begin{cases} 4 & \text{if } \ 0 < m \leq 60 \\ 8 & \text{if } \ 60 < m \leq 120 \\ 10 & \text{if } 120 < m \leq 1440 \end{cases}$$

since there are 1440 minutes in 1 day

- **Note: F(*m*) is a single function defined in many *pieces*, not many functions.**

Parking Garage Graph

Graphing the above piecewise function shows that ***F*(*m*)** is a discontinuous combination of three linear functions.



Sky Harbor International Airport Parking Fees

- Use an open circle to denote that a value is not included in the function
- Use an arrow to denote that the function continues beyond the graph.

Additional Piecewise Function Problems

Piecewise Function Problem One

$$f(x) = \begin{cases} 2x+4 & \text{if} \quad x < -3 \\ x^2+1 & \text{if} \quad -3 \le x < 2 \\ 1.2^x & \text{if} \quad x \ge 2 \end{cases}$$

1. Evaluate ***f*(-5)**
    1. Use rule 1
    2. ***f*(-5) = 2(-5) + 4**

    3. **$f(-5) = -10 + 4$**
    4. **$f(-5) = -6$**
2. Evaluate **$f(2)$**
    1. Use rule 3
    2. **$f(2) = 1.2^2$**
    3. **$f(2) = 1.44$**
3. Evaluate **$f(5)$**
    1. Use rule 3
    2. **$f(2) = 1.2^5$**
    3. **$f(2) = 2.48832$**

Piecewise Function Problem Two

> An Airbnb host charges **$125 a night** for the first **3 nights** you stay at their location. The charge then drops to **$110 a night** for each additional night up to **7 nights**. After that, the rate drops to **$95 a night**.
>
> The host does not allow anyone to stay longer than **3 weeks**. There is also a one-time service fee of **$50**.
>
> Define a piecewise function, ***a*** to represent the cost of a stay for ***n*** nights at this Airbnb.

1. Function One
    1. Add **50** to the end to account for the service fee
    2. **$a(n) = 125(n) + 50;\ 1 \le n \le 3$**
2. Function Two
    1. Shift the function to the right by **3** to account for the first three nights
    2. Shift the function up by **125(3) + 50** to account for the price of the first three nights
        - Essentially plugging the max domain value of the previous function into the previous function as the input.
        - *Equals* ***425***
    3. **$a(n) = 110(n - 3) + 425;\ 4 \le n \le 7$**

3. Function Three
    1. Shift the function to the right by **7** to account for the first seven nights
    2. Shift the function up by **110(7 - 3) + 425** to account for the price of the first seven nights
        - Essentially plugging the max domain value of the previous function into the previous function as the
        - *Equals* ***865***
    3. The maximum for the domain is **21** nights
    4. ***a*(*n*) = 95(*n* - 7) + 865; 8 ≤ *n* ≤ 21**
4. Put the functions together

Problem Two Finished Piecewise Function

$$a(n) = \begin{cases} 125n + 50 & \text{if } 1 \leq n \leq 3 \\ 110(n-3) + 425 & \text{if } 4 \leq n \leq 7 \\ 95(n-7) + 865 & \text{if } 8 \leq n \leq 21 \end{cases}$$

Piecewise Function Problem Three

Federal income tax rates depend on the amount of taxable income received. The following tax rate schedule shows the tax rates for unmarried (single) filers for 2020 for the bottom 4 income levels.

NOTE: This means there is a tax rate of 10% on the first 9, 875oftaxableincome, arateof129,876 and 40, 125, arateof2240,126 and $85,525, etc

Rate	Income Tax Bracket
10%	0to9,875
12%	9, 876to40,125
22%	40, 126to85,525
24%	85, 526to163,300

1. Function One
    1. $T(I) = 0.10(I); 0 \le I \le 9875$
2. Function Two
    1. Shift the function to the right by **9875** to account for the first income level.
    2. Plug in the max value of the domain into *function one* to get the value to shift the function up by
        1. $T(I) = 0.10(9875)$
        2. **987.50**
    3. Final function: $T(I) = 0.12(I - 9875) + 987.50; 9876 \le I \le 40125$
3. Function Three
    1. Shift the function to the right by **40125** to account for the second income level.
    2. Plug in the max value of the domain into *function two* to get the value to shift the function up by
        1. $T(I) = 0.12(40125 - 9875) + 987.50$
        2. **4617.50**
    3. Final function: $T(I) = 0.22(I - 40125) + 4{,}617.50; 40126 \le I \le 85525$
4. Function Four
    1. Shift the function to the right by **85525** to account for the third income level.
    2. Plug in the max value of the domain into *function three* to get the value to shift the function up by
        1. $T(I) = 0.22(85525 - 40125) + 4617.50$
        2. **14605.5**
    3. Final function: $T(I) = 0.24(I - 85525) + 14605.50; 85526 \le I \le 163300$

Problem Three Finished Piecewise Function

$$T(I) = \begin{cases} 0.10\,I & \text{if } 0 \le I \le 9875 \\ 0.12(I - 9875) + 987.50 & \text{if } 9876 \le I \le 40125 \\ 0.22(I - 40125) + 4617.50 & \text{if } 40126 \le I \le 85525 \\ 0.24(I - 85525) + 14605.50 & \text{if } 85526 \le I \le 163{,}300 \end{cases}$$

Piecewise Function Problem Four

> A 2.4-mile swim, 112-mile bike ride, and a 26.2-mile run make up an Ironman Triathlon competition.
>
> A certain triathlete averages a swimming speed of 2.4 mph, a cycling speed of 18 mph, and a running speed of 10 mph.
>
> Assume there is no transitioning time from one segment of the race to another.

- *Source: <www.ironmanarizona.com>*
- Recall that **distance = rate * time**

Goal:

- Develop a piecewise function for the speed, ***S***, of the participant as a function of his time, ***t***, in hours.

To solve this, we need to understand the speed at any given time within the race. So we will alter the **distance** formula to get the **time** required to complete each portion of the race and put the results into a table to better understand the problem:

Segment	Distance (mi)	Rate (mph)	Time (hrs)
Swim	2.4 mi	2.4	1
Bike	112 mi	18	6.22
Run	26.2 mi	10	2.62

- The **Time** section was found by using **time = distance/rate**

1. Function One
    1. ***S*(*t*) = 2.4; 0 ≤ *t* ≤ 1**
2. Function Two
    1. ***S*(*t*) = 18; 1 < *t* ≤ 7.22**
        - It is **7.22** instead of **6.22**, because you need to add the time it took to swim to the time it took to bike.
3. Function Three
    1. ***S*(*t*) = 10; 7.22 < *t* ≤ 9.84**
        - It is **9.84** instead of **2.62**, because you need to add the time it took to swim and bike to the time it took to run.

Problem Four Finished Piecewise Function

$$S(t) = \begin{cases} 2.4 & \text{if } 0 \le t \le 1 \\ 18 & \text{if } 1 < t \le 7.22 \\ 10 & \text{if } 7.22 < t \le 9.84 \end{cases}$$

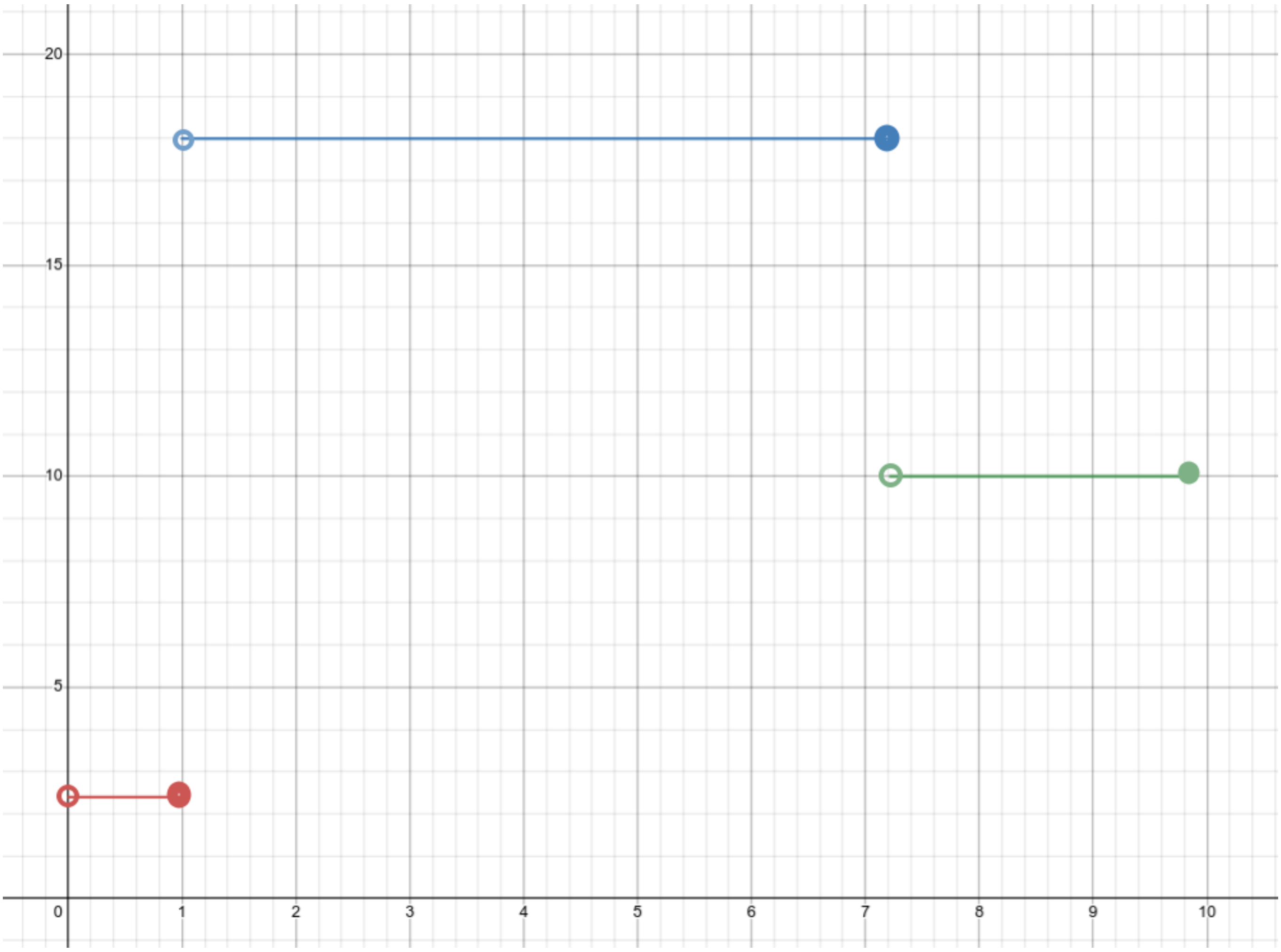
Problem Four Meaning

Meaning behind each piece of the piecewise function:

1. ***S*(*t*) = 2.4; 0 ≤ *t* ≤ 1**
    - While swimming the first hour, the participant had an average speed of 2.4 mph.

2. ***S*(*t*) = 18; 1 < *t* ≤ 7.22**
    - For the next 6.22 hours, the participant had an average speed of 18 mph while biking.
3. ***S*(*t*) = 10; 7.22 < *t* ≤ 9.84**
    - For the last 2.62 hours, the participant had an average speed of 10 mph while running.

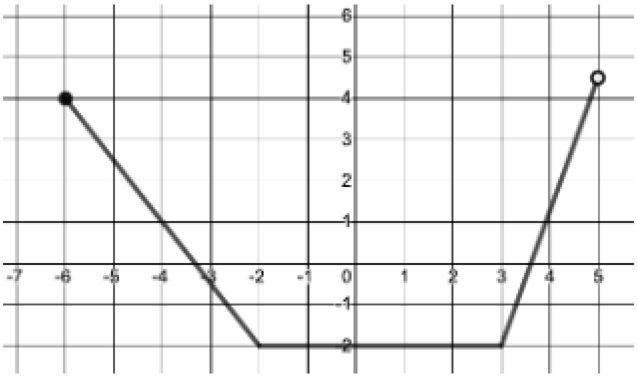
Problem Four Piecewise Graph



Piecewise Function Problem Five - Graphs

Piecewise Graph One

Determining the piecewise function of the graph:



First we need to figure out the function that is being used per interval:

> Formula 1: **m = (y2 - y1) / (x2 - x1)**
>
> Formula 2:

1. Function One
    1. Find the slope
        1. ***Point 1:* [-6, 4]** & ***Point 2:* [-2, -2]**
        2. $^{\mathbf{-2 - 4}}/_{\mathbf{-2 - (-6)}}$
        3. $^{\mathbf{-6}}/_{\mathbf{-4}}$
        4. **1.5**
    2. Find the transformations (using only the relevant section)
        1. The graph is shifted to the left **6**
        2. The graph is shifted up **4**
    3. Find the domain: **-6 ≤ *x* ≤ -2**
    4. Final function: **ƒ(*x*) = 1.5(*x* + 6) + 4; -6 ≤ *x* ≤ -2**
2. Function Two
    1. The output is just **-2**
    2. Find the domain: **-2 < *x* ≤ 2**
    3. Final function: **ƒ(*x*) = -2; -2 < *x* ≤ 2**
3. Function Three
    1. Find the slope
        1. ***Point 1:* [3, -2]** & ***Point 2:* [5, 4.5]**
        2. $^{\mathbf{4.5 - 3}}/_{\mathbf{5 - 3}}$
        3. $^{\mathbf{1.5}}/_{\mathbf{2}}$
        4. **0.75**
    2. Find the transformations (using only the relevant section)
        1. The graph is shifted to the right **3**
        2. The graph is shifted down **2**

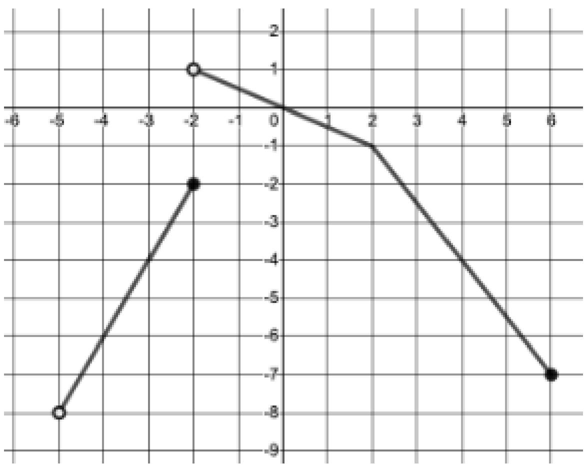
3. Find the domain: **3 < *x* < 5**
4. Final function: ***f*(*x*) = 0.75(*x* - 3) - 2; 3 < *x* < 5**

Piecewise Graph One Finished Function

$$f(x) = \begin{cases} -1.5(x+6)+4 & \text{if } -6 \le x \le -2 \\ -2 & \text{if } -2 < x \le 3 \\ 3.25(x-3)-2 & \text{if } 3 < x < 5 \end{cases}$$

Piecewise Graph Two

Determining the piecewise function of the graph:



1. Function One
    1. Find the slope
        1. ***Point 1:* [-5, -8]** & ***Point 2:* [-2, -2]**
        2. $^{-2 - (-8)} / _{-2 - (-5)}$
        3. $^{6}/_{3}$
        4. **2**
    2. Find the transformations (using only the relevant section)
        1. The graph is shifted to the left **5**
        2. The graph is shifted down **8**
    3. Find the domain: **-5 < x ≤ -2**
    4. Final function: ***f*(*x*) = 2(*x* + 5) - 8; -5 < x ≤ -2**
2. Function Two
    1. Find the slope
        1. ***Point 1:* [-2, 1] & *Point 2:* [2, -1]**

        2. $^{-1 - 1}/_{2 - (-2)}$
        3. $^{-2}/_{4}$
        4. **-0.5**
    2. Find the transformations (using only the relevant section)
        1. The graph is shifted to the left **2**
        2. The graph is shifted up **1**
    3. Find the domain: **-2 < *x* ≤ 2**
    4. Final function: ***f*(*x*) = -0.5(*x* + 2) + 1; -2 < *x* ≤ 2**
3. Function Three
    1. Find the slope
        1. ***Point 1:*** **[2, -1]** & ***Point 2:*** **[6, -7]**
        2. $^{-7 - (-1)}/_{6 - 2}$
        3. $^{-6}/_{4}$
        4. **-1.5**
    2. Find the transformations (using only the relevant section)
        1. The graph is shifted to the right **2**
        2. The graph is shifted down **1**
    3. Find the domain: **2 < *x* ≤ 6**
    4. Final function: ***f*(*x*) = -1.5(*x* - 2) - 1; 2 < *x* ≤ 6**

Piecewise Graph Two Finished Function

$$f(x) = \begin{cases} -2(x+5)-8 & \text{if } -5 < x \le -2 \\ -0.5(x+2)+1 & \text{if } -2 < x \le 2 \\ -1.5(x-2)-1 & \text{if } 2 < x \le 6 \end{cases}$$

Solving For Functions vs Graphs

How To Solve For Functions

1. Determine the domain
    - The maximum domain for the current function will serve as both the horizontal shift and the minimum domain for the next function.
        - **Example:** If the domain is **-2 ≤ *x* ≤ 2**, then the horizontal shift for the next function is **2 to the left** and the minimum domain for the next function is **-2**.

- This can usually be found with common sense or within the problem. Remember that it should pick up where the maximum domain of the previous function left off.

2. Determine the range
   - The maximum range for the current function will serve as both the vertical shift and the minimum range for the next function.
     - **Example:** If the range is **-2 ≤ *y* ≤ 2**, then the vertical shift for the next function is **2 up** and the minimum range for the next function is **2**.
   - The range can be determined by plugging in the domain values individually into the function.
     - If you already have the minimum range from the previous function, then you only need to plug in the maximum value for the next function's minimum.
3. Determine the slope
   - You can use the slope formula to find the slope between two points: $y_2 - y_1 / x_2 - x_1$
4. Determine the transformations
   - The horizontal shift is the minimum domain of the current function.
   - The vertical shift is the minimum range of the current function.
5. Determine the final piece of the piecewise function by separating the function and domain by a semicolon.
   - **Example: *f*(*x*) = 2(*x* - 2) + 2; -2 < *x* ≤ 2**
6. Repeat steps **1-5** until you have found the function for the entire graph.
7. Create the final piecewise function:

$$f(x) = \begin{cases} 38\,t & 0 \le x \le 1.75 \\ 76\ (t-1.75)+66.5 & 1.75 < x \le 4.55 \\ 8\ (t-4.55)+279.3 & 4.55 < x \le 5.05 \\ 0\ (t-5.05)+283.3 & 5.05 < x \le 6.05 \\ 78\ (t-6.05)+283.3 & 6.05 < x \le 7.55 \\ 35\ (t-7.55)+400.3 & 7.55 < x \le 8.3 \end{cases}$$

How To Solve For Graphs

1. Determine the domain

- You can find this by identifying the beginning and end of where the slope changes on the graph.
    - *I.e., The graph is going in a straight line, stops, and then goes in a different direction.*
- Make sure to check whether the point is inclusive or exclusive. You will usually write each functions minimum to be exclusive besides the very first one.
    - **Example: -2 < *x* ≤ 2**

2. Determine the range
    - You can find this by identifying the beginning and end of where the slope changes on the graph.
        - *I.e., The graph is going in a straight line, stops, and then goes in a different direction.*
3. Determine the slope
    - You can use the slope formula to find the slope between two points: $y_2 - y_1 / x_2 - x_1$
4. Determine the transformations
    - The horizontal shift is the minimum domain of the current function.
    - The vertical shift is the minimum range of the current function.
    - *Note: You can also visually look at the graph to determine the transformations. You will look at how horizontally and vertically shifted the beginning point of the line is.*
5. Determine the final piece of the piecewise function by separating the function and domain by a semicolon.
    - **Example: *f*(*x*) = 2(*x* - 2) + 2; -2 < *x* ≤ 2**
6. Repeat steps **1-5** until you have found the function for the entire graph.
7. Create the final piecewise function:

$$f(x) = \begin{cases} 38\,t & 0 \le x \le 1.75 \\ 76\ (t - 1.75) + 66.5 & 1.75 < x \le 4.55 \\ 8\ (t - 4.55) + 279.3 & 4.55 < x \le 5.05 \\ 0\ (t - 5.05) + 283.3 & 5.05 < x \le 6.05 \\ 78\ (t - 6.05) + 283.3 & 6.05 < x \le 7.55 \\ 35\ (t - 7.55) + 400.3 & 7.55 < x \le 8.3 \end{cases}$$

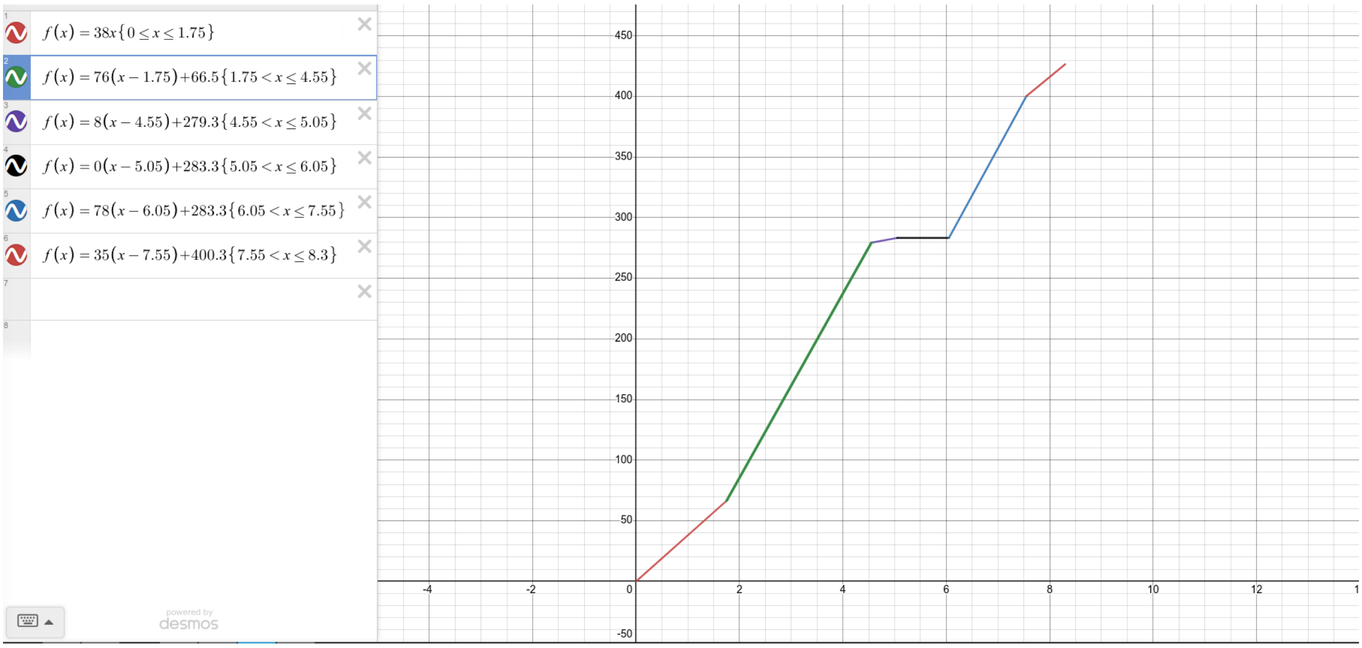
-

Note: You can create a piecewise function in Desmos by putting the rules/conditions after the function in bracketed parentheses and separating them with commas:

> $f(x) = 76(x - 1.75) + 66.5 \{1.75 < x \leq 4.55\}$

Example Graph

- **Graph Link:** https://www.desmos.com/calculator/jpyxmxnp48



Module 16 - Function Composition

Function Composition Activity

The Problem:

> A pebble is thrown into a lake and the radius of the ripple travels outward at **2 meters per second**.
>
> Your goal will be to determine the area inside the ripple based on the number of seconds elapsed since the pebble hit the water.
>
> - *Note: the formula for finding the area inside a circle is* $A = \pi r^2$

- r is the length of the radius of the circle.

Notes About The Problem:

1. The **radius** and **area** will increase as the time increases.
2. To create a function for the expanding area, you will need to figure out the radius at any given time.
   1. You can do this by multiplying the **time** by the **speed**.
   2. The new formulas:
      1. Radius: $\boldsymbol{r = f(t) = 2t}$
         - $\boldsymbol{t}$ is the time in seconds.
      2. Area with *Time* input: $\boldsymbol{A = \pi(2t)^2}$
      3. Area with *Radius* Input: $\boldsymbol{A = \pi r^2}$
3. Plugging in the previous function, to find the radius of the circle after three seconds:
   1. $\boldsymbol{A = \pi(3 \times 2)^2}$
   2. $\boldsymbol{A = \pi(6)^2}$
   3. $\boldsymbol{A = \pi(36)}$
   4. $\boldsymbol{A \approx 113.097}$ **square meters**

Function Composition Explanation

Function composition is when you take the output of one function and use it as the input for another

- The above problem is an example of function composition.
  - The output of the **radius function** is used as the input for the **area function**.

Writing Function Composition Functions

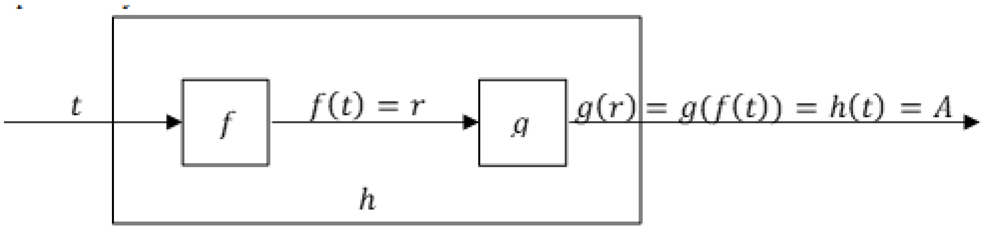
When an input to a function is itself a function, we write the function output in place of the variable it defines.

For example, given the following functions from the previous problem:

1. $\boldsymbol{r = f(t)}$
2. $\boldsymbol{A = g(r)}$

The final function would be written as:

> ***A* = *g*(*r*) = *g*(*f*(*t*))**



- The ***r*** in the ***g*(*r*)** function is replaced with the output of the ***f*(*t*)** function.
- The ***h*(*t*)** function is the same as the ***g*(*r*)** function.
- All the functions after ***g*** in the image are the same, but the variables are different.

Anytime you have a two-step process to find a result, you can model that with a **composite function**.

- **NOTE:** Function composition only makes sense when the output quantity of one function os the input to another.

Function Composition Examples

Function Composition Example 1

Example:

If I am shopping at a 15% off sale at Dillard's in Chandler and I want to know the final price I will have to pay for a particular item, I can use the following two functions.

Function 1: $s = f(r) = 0.85r$ where s represents the sale price of the item in dollars and r represents the original price of the item in dollars.

Function 2: $p = g(s) = 1.078s$ where p represents the final price of the item in dollars after the 7.8% sales tax has been added.

Let's say I want to know the final price I will pay for my jeans that normally sell for $54.95.

Step 1: find the sale price: $s = f(54.95) = 0.85(54.95) = 46.71$

Step 2: find the price with tax: $p = g(46.71) = 1.078(46.71) = 50.35$

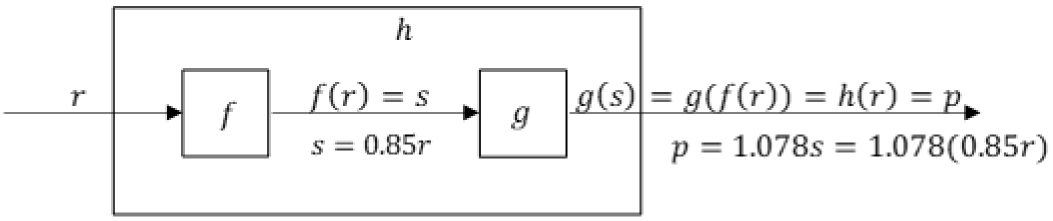
So, my final cost for the jeans is $50.35

However, the two functions I used can actually be combined, using function composition, as $p = g(s) = g(f(r))$, allowing me to find the cost in a single step.

To represent the final cost as a function of the original cost, we name a new function h and write $h(r) = g(f(r)) = 1.078(0.85r)$ where $h(r)$ represents the final cost in dollars.

Using this function, I find that the final price for my jeans is $p = h(54.95) = 1.078(0.85(54.95))$ which equates to $50.35.

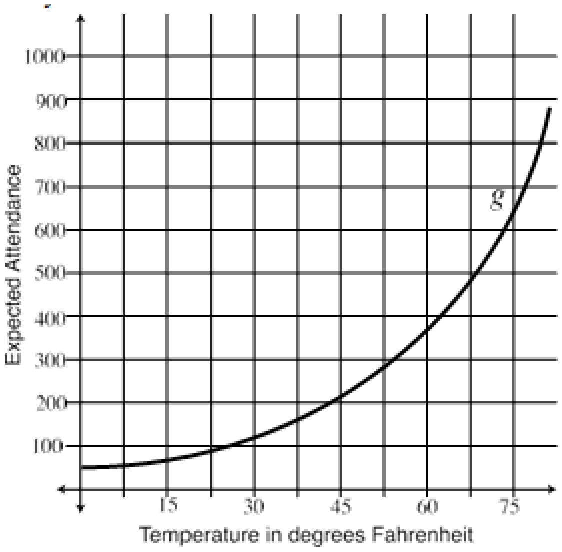
The following diagram illustrates the situation.

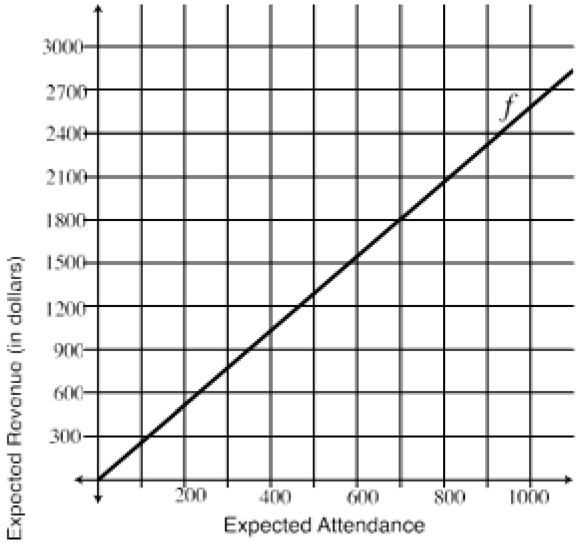


Function Composition Example 2

The graphs below show two different functions.

1. Function ***g*** takes as its input a temperature in degrees Fahrenheit and outputs the expected attendance at a neighborhood carnival.
2. Function ***f*** takes as its input the expected attendance at a neighborhood carnival and outputs the total expected revenue earned by the carnival.





Explaining The Meaning

1. Given ***f*(*g*(70))**
    1. **70** is the temperate in degrees Fahrenheit.
    2. The output of ***g*(70)** is the expected attendance at the carnival.
        - → ***525 expected attendance***
    3. The output of ***f*(525)** is the total expected revenue earned by the carnival.
        - → ***$1,350 total expected revenue***
    4. **Meaning:** When the temp is **70°F**, the carnival is
       expected to earn
       $1,350.
2. Given ***g*(*f*(70))**
    1. **70** is the expected attendance at the carnival.
    2. The output of ***f*(70)** is the total expected revenue earned by the carnival.
        - → ***$150 total expected revenue***
    3. The output of ***f*** ≠ input of ***g***.
    4. **Meaning:** There is no meaning, because the input of ***g*** is
       the temperature
       in degrees Fahrenheit and the input of ***f*** is the expected
       attendance at the carnival.
3. Given ***f*(*g*(*x*)) = 1800**, solve using the graphs for ***x***.
    1. Write it out: ***x*** → ***g*** → **___** → ***f*** → **1800**
       expected revenue
    2. Work your way backwards.

        1. Looking at the graph, when the expected revenue is **1800**, the expected attendance is **700**.
        2. ***x*** **→** ***g*** **→** **700** **→** ***f*** **→ 1800 expected revenue**
        3. Looking at the graph, when the expected attendance is **700**, the temperature is **77 degrees Fahrenheit**.
        4. **77 →** ***g*** **→ 700 →** ***f*** **→ 1800 expected revenue**
    3. **Meaning:** When the expected revenue is **$1,800**, the temperature is **77 degrees Fahrenheit**.

Function Composition Example 3

Given the following function, where ***ƒ*** is the temperate in degrees Fahrenheit and ***C*** is the temperature in degrees Celsius:

> $f = m(C) = {}^{9}/_{5}C + 32$

1. Determine the formula for the inverse $m^{-1}(f)$
    1. Rewrite the formula without the notation
        - $\rightarrow f = {}^{9}/_{5}C + 32$
    2. Subtract **32** from both sides
        - $\rightarrow f - 32 = {}^{9}/_{5}C$
    3. Multiply both sides by the reciprocal of ${}^{9}/_{5}$, which is ${}^{5}/_{9}$
        - $\rightarrow {}^{5}/_{9}(f - 32) = C$
    4. Rewrite the full notation
        - $\rightarrow {}^{5}/_{9}(f - 32) = C = m^{-1}(f)$
2. Evaluate $m(m^{-1}(115))$
    1. Calculate $m^{-1}(115)$
        1. $^{5}/_{9}(115 - 32) = C = m^{-1}(f)$
        2. **$^{5}/_{9}$(83) = C = $m^{-1}(f)$ **
        3. **46.11°C = C =** $m^{-1}(f)$

2. Calculate ***m*(46.11)**
    1. ***f* = *m*(*C*) = $\frac{9}{5}$(46.11) + 32**
    2. ***f* = *m*(*C*) = 83 + 32**
    3. ***f* = *m*(*C*) = 115°F**
3. ***m***$^{-1}$**(*m*(20))**
    1. Calculate ***m*(20)**
        1. ***f* = *m*(*C*) = $\frac{9}{5}$(20) + 32**
        2. ***f* = *m*(*C*) = 36 + 32**
        3. ***f* = *m*(*C*) = 68°F**
    2. Calculate ***m***$^{-1}$**(68)**
        1. **$\frac{5}{9}$(68 - 32) = C = *m***$^{-1}$**(
           f)**
        2. **$\frac{5}{9}$(36) = C = *m*$^{-1}$(*f*)
           **
        3. **20°C = C = *m***$^{-1}$**(*f*)**

Function Composition Example 4 - Reading a Table

Table 1 - *f*(x)

x	*f*(x)
-2	0
-1	3
0	4
1	-1
2	6
3	-2

Table 2 - *g*(x)

x	*g*(x)
-2	5
-1	3
0	2
1	1
2	-1
3	0

Image Version

x	$f(x)$
-2	0
-1	3
0	4
1	-1
2	6
3	-2

x	$g(x)$
-2	5
-1	3
0	2
1	1
2	-1
3	0

Solving For Various Function Compositions

1. ***f*(*f*(3))**
    1. Work your way backwards by finding the inner one first.
    2. Find ***f*(3)** in Table 1
        - **3** is the input in this scenario, so look for the corresponding output.
        - ***f*(3) = -2**
    3. Find ***f*(*f*(3))**
    4. ***f*(*f*(3)) = *f*(-2) = 0**
2. ***g*(*f*(-1))**
    1. ***f*(-1) = 3**

    2. $g(f(-1)) = g(3) = 1$
3. $g(g(0))$
    1. $g(0) = 2$
    2. $g(g(0)) = g(2) = -1$
4. $f(g(3))$
    1. $g(3) = 0$
    2. $f(g(3)) = f(0) = 4$
5. Given $f(g(x)) = 3$, find the value of x.
    1. For this one, we are looking for the matching input value that will produce an output of **3** for g.
        - *Note: We still work backwards from the inner one.*
    2. $g(-1) = 3$
    3. $x = 3$
6. $g^{-1}(3)$
    1. For inverse functions, it's the same as: $g(x) = 3$
    2. Find the output that makes $g(x) = 3$
    3. $g(-1) = 3$
    4. $g^{-1}(3) = -1$
7. $f^{-1}(0)$
    1. $f(x) = 0$
    2. $f(-2) = 0$
    3. $f^{-1}(0) = -2$
8. $g^{-1}(f^{-1}(3))$
    1. $f^{-1}(3) = -1$
    2. $g^{-1}(-1) = 2$
    3. $g^{-1}(f^{-1}(3)) = 2$
9. $g(f^{-1}(-2))$
    1. $f^{-1}(-2) = 3$
    2. $g(3) = 0$
    3. $g(f^{-1}(-2)) = 0$

Function Composition Example 5

Function Composition - Problem 5 Question

> A spherical snowball with a radius of **6.4 centimeters**, when brought inside, will melt in such a way that its radius decreases by a constant rate of
> **2 centimeters per minute**.
>
> - The formula for the volume of a sphere as determined by the radius of the sphere: $\mathbf{V = \frac{4}{3}\pi r^3}$

Formulas:

- Function f that outputs the radius r *(measured in centimeters)* with respect to time, t
  - $r = f(t) = 6.4 - 2t$
- Function g that outputs the volume V of the snowball *(measured in cubic centimeters)* with respect to the radius length of the snowball, r *(in centimeters)*
  - $V = \frac{4}{3}\pi r^3$
- Function h that outputs the volume V of the snowball *(measured in cubic centimeters)* with respect to time, t
  - $V = h(t) = g(f(t)) = \frac{4}{3}\pi(6.4 - 2t)^3$
- Evaluating $h(1.5)$ and its meaning
  - $h(1.5) = \frac{4}{3}\pi(6.4 - 2 \times 1.5)^3 = 164.64\text{cm}^3$
  - After **1.5 minutes** since the snowball was brought inside, the volume of the snowball is $\mathbf{164.64cm^3}$.

Function Composition Example 6 - Decomposing Functions

- *Note: There is more than one way to decompose a function.*

Below are example functions for $f(\text{m})$ and $\text{m} = g(x)$:

1. $f(g(x)) = 3(x - 1) + 5$

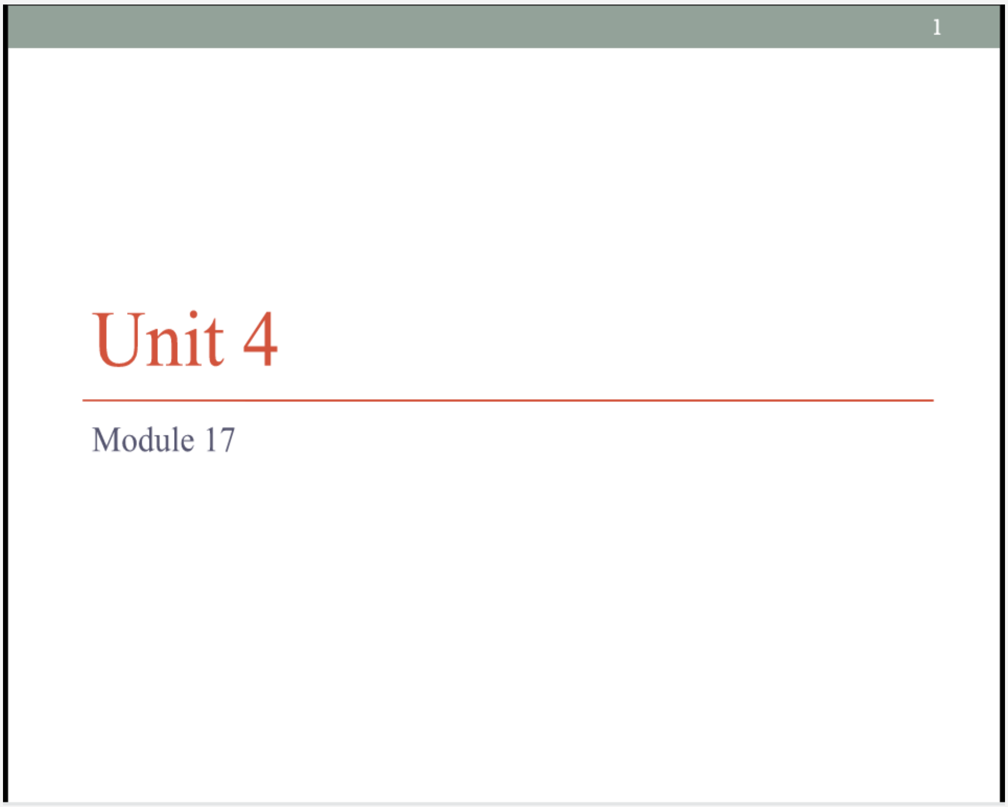
- If $m = g(x) = x - 1$, then $f(m) = 3m + 5$

2. $f(g(x)) = (x + 4)^2$
   - If $m = g(x) = x + 4$, then $f(m) = m^2$
3. $f(g(x)) = (x + 2)^2 + 3(x + 2) + 1$
   - If $m = g(x) = x + 2$, then $f(m) = m^2 + 3m + 1$
4. $f(g(x)) = \sqrt{x - 1}$
   - If $m = g(x) = x - 1$, then $f(m) = \sqrt{m}$
5. $f(g(x)) = {}^{500}/_{100 - x^2}$
   - If $m = g(x) = 100 - x^2$, then $f(m) = {}^{500}/_{m}$

Module 17 - Systems of Equations

Module 17 - Google Slides

> **Links:**
>
> Slides | PDF



Systems of Equations

- A systems of equations is a group of two or more equations.
- To solve for a system of equations, you must find the values of the variables that make all of the equations true.
- Some methods for solving systems of equations:
  - **The Substitution Method**
  - **The Elimination Method**
  - **The Graphical Method**

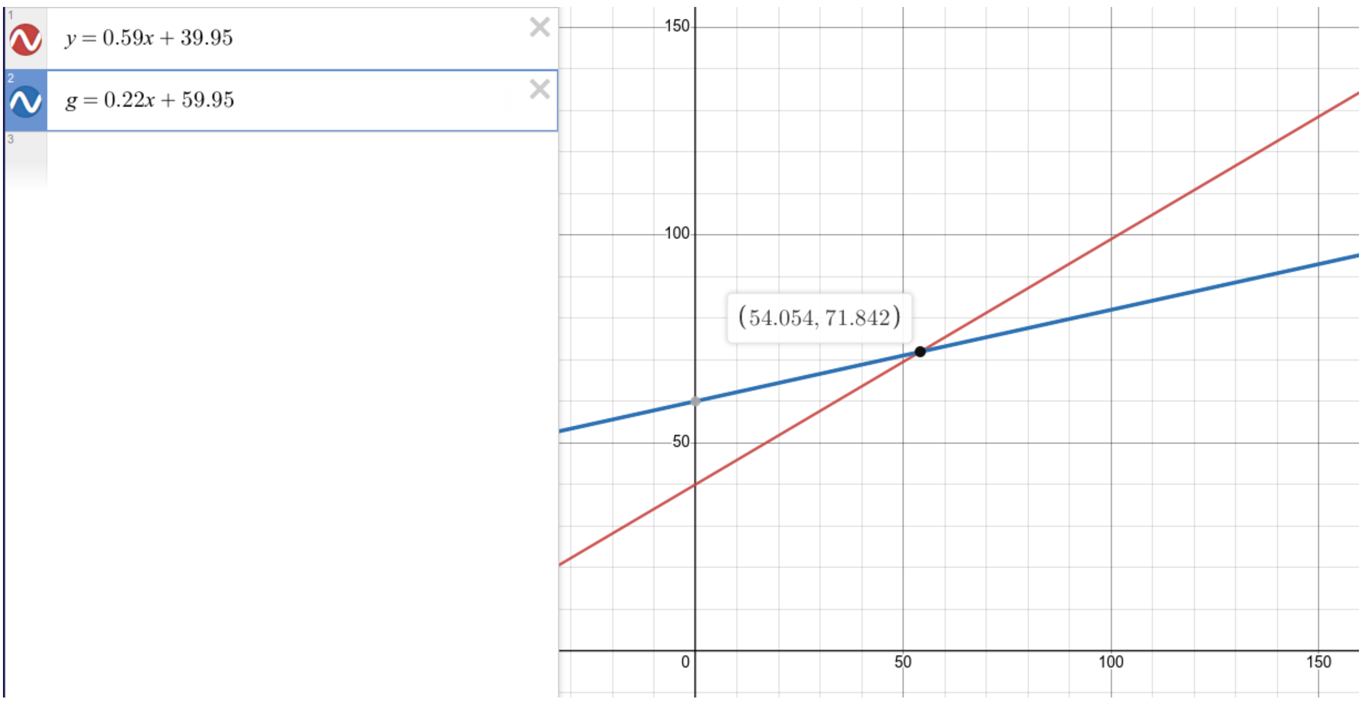
Systems of Equations - Example 1

> You just rented a new apartment and need to move your belongings from the old apartment to the new one. You contact two rental companies and receive the following information for the one-day cost of renting a moving van:
>
> - *Company A*: $39.95perday **, plus **$**0.59 per mile**
> - *Company B*: $59.95perday **, plus **$**0.22 per mile**

Formulas:

- *Company A*: ***y* = 39.95 + 0.59*x***
- *Company B*: ***y* = 59.95 + 0.22*x***

Graph:



Solution:

The point at which both equations will cost the same *(making both equations true)* is at **(x, y) = (54.054, 71.842)**.

- If you're driving less than **54.054 miles**, then **Company A** is cheaper.
- If you're driving more than **54.054 miles**, then **Company B** is cheaper.

Systems of Equations - Example 2

Given two functions:

- **y = 3*x* - 4**
- **y = -*x* + 28**

Solution:

1. Solve for **x**.
    1. Use substitution method since both equations are already solved for **y **.
    2. **3*x* - 4 = -*x* + 28**
    3. **3*x* = -*x* + 32**
    4. **4*x* = 32**
    5. **x = 8**
2. Solve for **y**
    1. Plug in **x = 8** into either equation.
    2. **y = 3*x* - 4**
    3. **y = 3(8) - 4**
    4. **y = 24 - 4**
    5. **y = 20**
3. Check your solution of **(8, 20)** by plugging both values into both equations to ensure they are both accurate.
    1. **y = 3*x* - 4**
        1. **20 = 3(8) - 4**
        2. **20 = 24 - 4**
        3. **20 = 20**
    2. **y = -*x* + 28**
        1. **20 = -8 + 28**
        2. **20 = 20**

Systems of Equations - Example 3

Given two linear equations:

- $y = 3x + 1$
- $x + 4y = 30$

Solution:

1. Solve for **x** using substitution
    1. *Equation 1* $y = 3x + 1$ → *Equation 2* $x + 4y = 30$
    2. $x + 4(3x + 1) = 30$
    3. $x + 12x + 4 = 30$
    4. $13x + 4 = 30$
    5. $13x = 26$
    6. $x = 2$
2. Solve for **y** using substitution
    1. $y = 3x + 1$
    2. $y = 3(2) + 1$
    3. $y = 6 + 1$
    4. $y = 7$
3. Check your solution of **(2, 7)** by plugging both values into both equations to ensure they are both accurate.
    1. $y = 3x + 1$
        1. $7 = 3(2) + 1$
        2. $7 = 6 + 1$
        3. $7 = 7$
    2. $x + 4y = 30$
        1. $2 + 4(7) = 30$
        2. $2 + 28 = 30$
        3. $30 = 30$

Systems of Equations - Example 4

> Rashid works as a driver for both Lyft and Uber. After expenses such as insurance, taxes, and gas are taken out of his pay, he makes about **$20 every hour for Lyft** and about **$18 every hour for Uber**.
>
> - **L = number of hours worked for Lyft**
> - **U = number of hours worked for Uber**
>
> The following equations are true:
>
> - **3L = U**
> - **20L + 18U = 370**

Solution:

- **Meanings**
  - **3L = U**
    - For every **3 hours worked for Lyft**, Rashid works **1 hour for Uber**.
  - **20L + 18U = 370**
    - After working for both Lyft and Uber, Rashid makes **$370**.
    - **20L**
      - **20 dollars per hour pay × The amount of hours worked = Total money made working at Lyft**
    - **18U**
      - **18 dollars per hour pay × The amount of hours worked = Total money made working at Uber**
- **Solve for L**
  1. Because **3L = U**, we can substitute **U** for **3L** in the second equation.
  2. **20L + 18(3L) = 370**
  3. **20L + 54L = 370**
  4. **74L = 370**
  5. **L = 5**
- **Solve for U**
  1. **3L = U**
  2. **3(5) = U**

3. **15 = U**

- **Verify Solutions**
    1. **3L = U**
        1. **3(5) = 15**
        2. **15 = 15**
    2. **20L + 18U = 370**
        1. **20(5) + 18(15) = 370**
        2. **100 + 270 = 370**
        3. **370 = 370**
- Final Solution:
    - **L = 5 hours**
    - **U = 15 hours**

Systems of Equations - Example 5

> Chip and Dale both went to the store to buy snacks (chips and soda) for a party.
>
> - **Chip paid $18** (before tax) and bought **four 2-liters of soda** and **three bags of chips**.
> - **Dale paid $20** (before tax) and bought **four 2-liters** and **4 bags of chips**.
>
> *Note: Assume that they each paid the same price per item.*

Variables:

- **C = price of bag of chips**
- **S = price of 2-liter of soda**

Equations:

- **4S + 3C = 18**
- **4S + 4C = 20**

Solution:

1. Use the elimination method to solve for **C**, coefficients are the same for enough of the variables to leave only one left.
    1. Flip one of the equations to negative.

- *Equation 1:* **-4S - 3C = -18**
- *Equation 2:* **4S + 4C = 20**

2. Add the equations together by combining like terms.
3. **(-4S + 4S) + (-3C + 4C) = (-18 + 20)**
4. **C = 2**

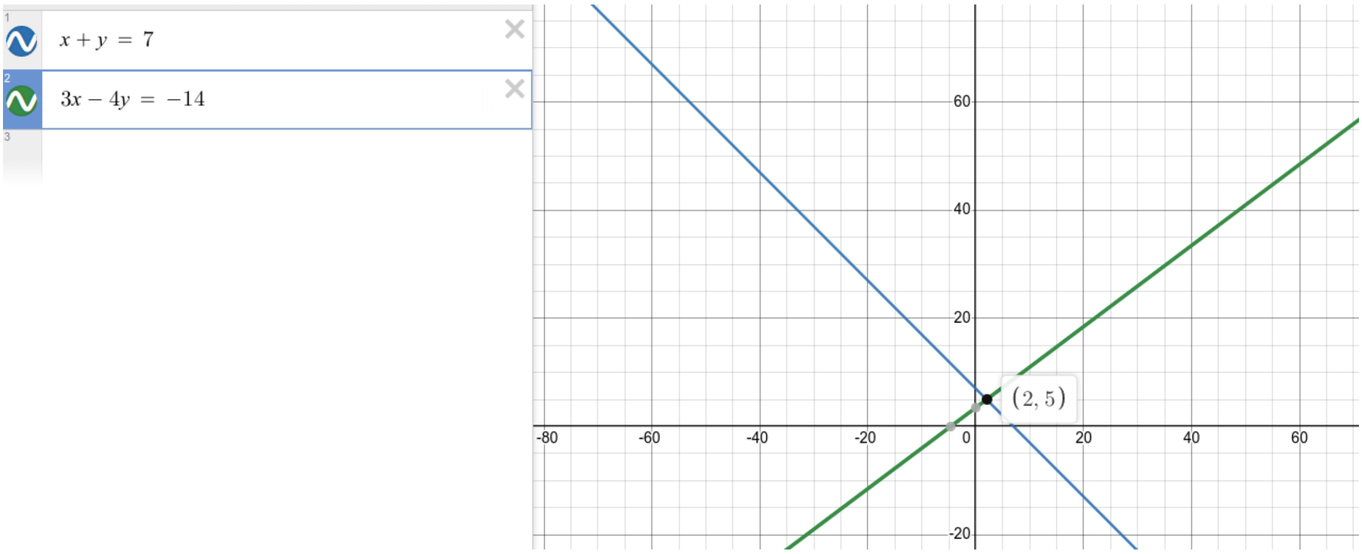
2. Solve for **S** by plugging in **C** into either equation.
    1. **4S + 3C = 18**
    2. **4S + 3(2) = 18**
    3. **4S + 6 = 18**
    4. **4S = 12**
    5. **S = 3**
3. Check your solution of **(2, 3)** by plugging both values into both equations to ensure they are both accurate.
    1. **4S + 3C = 18**
        1. **4(3) + 3(2) = 18**
        2. **12 + 6 = 18**
        3. **18 = 18**
    2. **4S + 4C = 20**
        1. **4(3) + 4(2) = 20**
        2. **12 + 8 = 20**
        3. **20 = 20**
4. Final Solution:
    - **C = 2**
    - **S = 3**

Systems of Equations - Example 6 - Graphing

Given the equations:

- **a + b = 7**
- **3a - 4b = -14**

Graph:



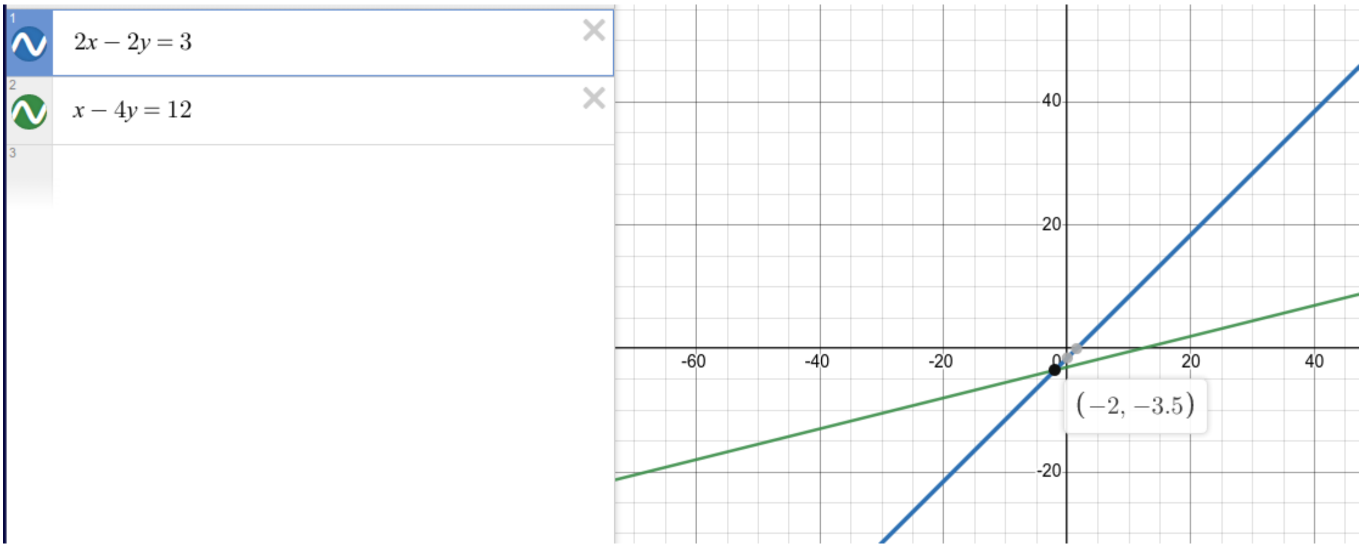
- Use **x** and **y** values instead of the random letters given.
- You can graph the equations exactly as they are given.
- Where the lines intersect is the solution to the systems of equations.

Systems of Equations - Example 7 - Graphing

Given the equations:

- **2m - 2n = 3**
- **m - 4n = 12**

Graph:



Systems of Equations - Example 8 - Graphing

Given the equations:

- **d = 2x - 12**
- **d = -4x + 18**

Graph: