

Unit 1 Notes

Module 2

The Google Slides can be found [Here](#)

General Notes

- **Quantity:** A characteristic or attribute of some object you can imagine measuring.
 - When defining or identifying a quantity, we must be specific about what object, and what specific characteristic about that object, we're referring to.
- **Variable:** A character or symbol used to represent a quantity.
- **Evaluate:** To find the output of a function corresponding to a given input.
- **Solving:** To find the input of a function corresponding to a given output
- **Domain:** the set of all reasonable inputs values of a function.
- **Range:** The set of all corresponding output values of a function
- **Interval Notation:** $[0, 100]$
- **Inequality:** $0 \leq x \leq 100$

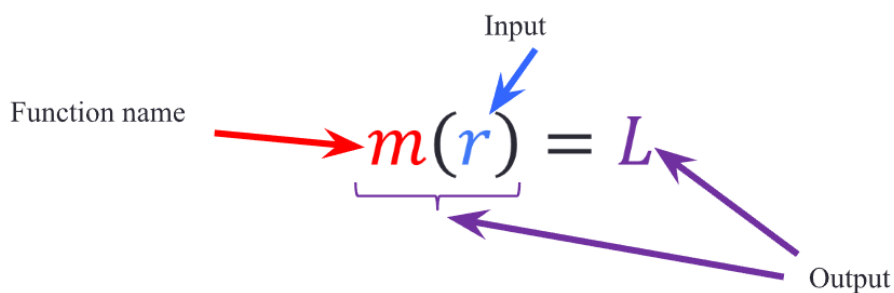
Functions

Function Language

We say “*output* as a function of *input*” or “*output* in terms of *input*”

- y as a function of x **or** y in terms of x
- *E.g. Length of the steel band with respect to the radius of the oil drums*

If an input points to more than one output, then it is **not** a function.



Example 1

Suppose the following graph of the function f represents John's weight (in pounds) as a function of time t , measured in days since January 1, 2008.

- a. Identify the input and output quantities for the function f .

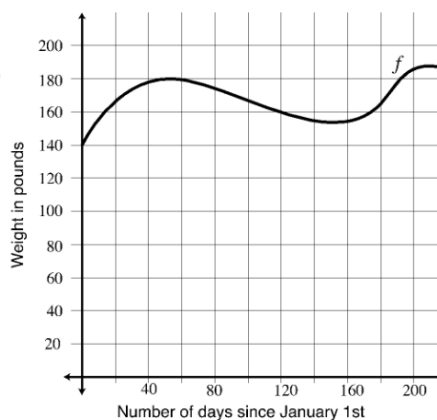
input: t – number of days since Jan 1, 2008
output: $f(t)$ – John's weight in pounds

- b. Evaluate $f(60)$. What does this value represent in the context of the problem?

$f(60) = 180$
John's weighs 180 pounds 60 days after Jan 1, 2008.

- c. Solve $f(t) = 160$ for t using the graph. Describe what each solution represents.

$f(t) = 160$ when $t = 15, 119, \& 175$
Fifteen days after Jan 1, 2008, John weighed 160 pounds. John also weighed 160 pounds 119 days after Jan 1, 2008 and 175 days after Jan 1, 2008



Example 2

Given $m(x) = \frac{2x-3}{x+4}$,

- a. Evaluate $m(4)$

$$m(4) = \frac{2(4) - 3}{4 + 4} = \frac{5}{8}$$

- b. Evaluate $m(z)$

$$m(z) = \frac{2z - 3}{z + 4}$$

- c. Evaluate $m(w + 2)$

$$m(w + 2) = \frac{2(w + 2) - 3}{(w + 2) + 4} = \frac{2w + 4 - 3}{w + 6} = \frac{2w + 1}{w + 6}$$

- d. Solve $m(x) = 3$

$$3 = \frac{2x - 3}{x + 4}$$

$$3(x + 4) = 2x - 3$$

$$3x + 12 = 2x - 3$$

$$x = -15$$

Example 3 - Oil Drum Problem

For the Oil Drum Problem

It turns out that the function that determines the length of the steel band needed to tie three oil drums of radius r is

$$L = m(r) = (6 + 2\pi)r$$

- Evaluate $m(3.5)$

$$m(3.5) = (6 + 2\pi)(3.5)$$

$$m(3.5) \approx 42.99 \text{ feet}$$

$$\text{Solve } m(r) = 29$$

- Solve $m(r) = 29$

$$29 = (6 + 2\pi)r$$

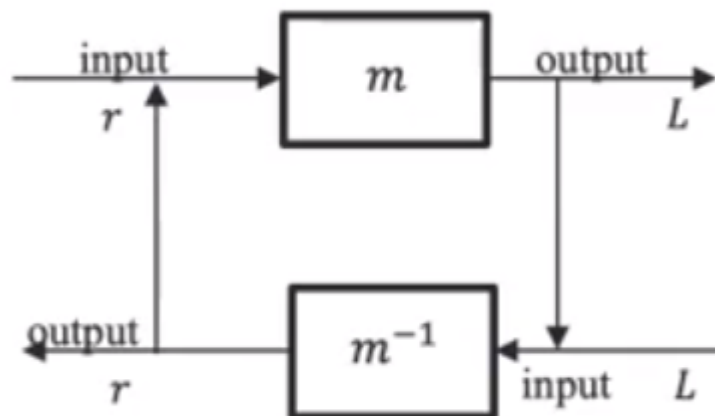
$$\frac{29}{6 + 2\pi} = r$$

$$2.36 \approx r$$

- L is the output
- r is the input

Inverse Functions

An inverse function is a function that undoes the operations of an original function.



- This function inverts the oil drum problem's function.
- The m is the inverse, not a negative exponent

- This image shows that L is a function of r (the name of the function is m) and that r is a function of L (specifically, the function is the inverse of function m and is written m^{-1}).
- $y = f(x)$ means that $x = f^{-1}(y)$ if the inverse of f is a function.
- The inverse of a function is not always a function.
 - A function has one input and one output

The formula $F = p(c) = 1.8c + 32$ will input the temperature in degrees Celsius and output the temperature in degrees Fahrenheit.



- a. Find the formula for the inverse function.

$$F = 1.8c + 32$$

$$F - 32 = 1.8c$$

$$\frac{F - 32}{1.8} = c$$

Since $F = p(c)$ means $c = p^{-1}(F)$,

$$c = p^{-1}(F) = \frac{F - 32}{1.8}$$

- b. Evaluate $p^{-1}(50)$ and explain its meaning in the problem context.

$$p^{-1}(50) = \frac{50 - 32}{1.8} = 10$$

Meaning: When the temperature is 50° Fahrenheit, it is 10° Celsius.

- F is the output
- c is the input
- Always write the notation indicating that the input and outputs have switched.

Finding The Inverse

1. Write the formula without the notation:
 1. $c = j(b) = 5b + 12$
 2. $c = 5b + 12$
2. Isolate the input until it's by itself

Inverse of a Graph Example

10. The graph of g is given to the right.

a. Find each of the following:

i. $g(0) = 4$ ii. $g(-1) = 3$

iii. a when $g(a) = -2$ iv. Solve $g(a) = 0$.

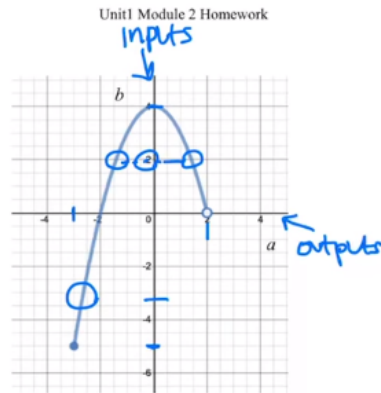
$a = -2.5$ $a = -2$

b. List the domain and range of the function.

$D: -3 \leq a < 2$ $[-3, 2)$
 $R: -5 \leq g(a) \leq 4$ $[-5, 4]$

c. Is the inverse a function? Explain

NO. For some inputs, there are 2 outputs



Writing Out Functions

11. Given a function $p(s) = b$ where s = the number of square mile of a forest and b = the number of rabbits in the forest, explain the meaning of the following.

a. $p(100) = 85$

#sq #rab

When the size of a forest is 100 sq miles, there are 85 rabbits

b. $p(54)$

#sq
#rabb

The # of rabbits in a forest that is 54 sq miles

c. $p^{-1}(93)$

#rabbits
#sq mi

The # of square miles a forest is with 93 rabbits in it.

Domain and Range

- **Domain:** the set of all reasonable inputs values of a function.
- **Range:** The set of all corresponding output values of a function

Example 1

The function that determines the length of the steel band needed to tie three oil drums of radius r is

$$L = m(r) = (6 + 2\pi)r$$

What would be a practical domain and range for this function? i.e., what values make sense for the radius and length of the steel band?

I would guess that the radius of an oil drum is not less than one half of a foot and probably not more than 5 feet. So the *practical domain* would be $0.5 \leq r \leq 5$ also written as $[0.5, 5]$ in interval notation.

Since a radius of 0.5 feet would require a steel band of 6.14 feet, and a since a radius of 5 feet would require a steel band of 61.42 feet, the *practical range* would be $6.14 \leq L \leq 61.42$ also written as $[6.14, 61.42]$ in interval notation.

Example 2

The letter grade earned on a test with respect to the percentage grade earned.

Practical Domain: $[0, 100]$

Practical Range: $\{A, B, C, D, F\}$

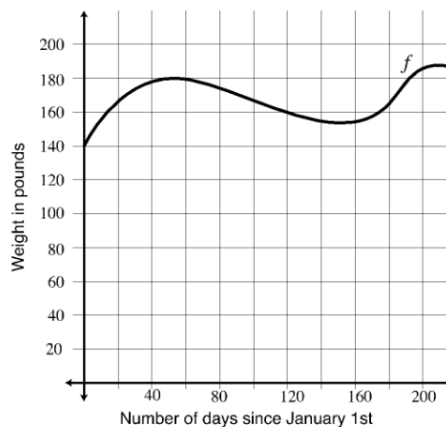
Example 3

Suppose the following graph of the function f represents John's weight (in pounds) as a function of time t , measured in days since January 1, 2008.

Determine the practical domain and range.

Practical Domain: $[0, 220]$

Practical Range: $[140, 190]$



Intercepts

Vertical Intercepts

The **Vertical Intercept** of a function is the coordinate point where the graph of the function crosses the vertical axis.

- This point will always be in the form $(0, b)$
- The vertical intercept can be found graphically by determining the point where the graph crosses the vertical axis.
- The vertical intercept can be found in a table or algebraically by first determining the value of b .
 - To do this, find the output of the function for an input of 0 (or $f(0)$ = b). You then write the intercept in the form $(0, b)$.

Horizontal Intercepts

The **Horizontal Intercept** of a function is the coordinate point where the graph of the function crosses the horizontal axis.

- The point will always be in the form $(a, 0)$.
- The horizontal intercept can be found graphically by determining the point where the graph crosses the horizontal axis.
- The horizontal intercept can be found in a table or algebraically by first determining the value of a .

- To do this, find the input of the function for an output of 0 (*or solve for a when $f(a) = 0$*). You then write the intercept in the form $(a, 0)$.

Constant Rate of Change

$$\frac{\text{change in output value}}{\text{change in input value}}$$

It is said that two quantities are related by a **constant rate of change (CROC)** if the *ratio of the changes in quantities* is always the same.

- Find the changes between each value in a table for all relevant columns, and use those in this formula.

Example 1

Change in time elapsed Δt	Time Elapsed in min t	Amount of water in a bath tub in gallons, a	Change in amount of water in tub Δa
	1	11.75	
1.5	2.5	14.375	2.625
2.5	5	18.75	4.375
3	8	24	5.25

- The triangle just means change

To figure out if the ratio is the same, the changes should all equal the same number when put into the formula:

$$\frac{\Delta a}{\Delta t} = \frac{2.625}{1.5} = \frac{4.375}{2.5} = \frac{5.25}{3} = 1.75$$

- Because all the numbers equal the same, it is **constant**.

This would be written as: ***For every additional minute that the water is left running, the amount of water in the bathtub increases by 1.75 gallons.***

Module 3

The Google Slides can be found [Here](#)

General Notes

Constant Rate of Change (Continued)

The value of the constant rate of change can always be determined by:

$$\text{Constant Rate of Change (CROC)} = \frac{\text{change in output value}}{\text{change in input value}}$$

Knowing this info, you can also get the other values.

Change in Output Value

$$\left(\frac{\text{change in}}{\text{output value}} \right) = (\text{CROC}) \cdot \left(\frac{\text{change in}}{\text{input value}} \right)$$

Change in Input Value

$$\frac{\text{change in output value}}{\text{CROC}} = \left(\frac{\text{change in}}{\text{input value}} \right)$$

Instead of always using the formula to find the change in input / output or the CROC, you can use repeated reasoning.

Imagine you have a pool with a hose in it filling it with water (it already has some in it). The CROC is **18.2**, and after **63 minutes**, there's **1382.6 gallons** inside. Instead of using formulas to find each value per different minute, you could create a formula:

1. Find Δt (*change in input value*)
 - $t - 63 = \Delta t$
 2. Find Δv (*change in output value*)
 1. $18.2(\Delta t) = \Delta v$
 2. $18.2(t - 63) = \Delta v$
 3. Find v (*total volume*)
 1. $\Delta v + 1382.6 = v$
 2. $18.2(t - 63) + 1382.6 = v$
- Δ means **Increment / Change**

The last one is the finished formula for a function that defines a relationship between the volume of water in the pool and the amount of time the pool has been filling. It can also be summarized as:

Change In Output

$$v = \text{CROC}(t - \text{reference input}) + \text{reference Output}$$

$$v = \text{CROC}(t - t_{\text{ref}}) + v_{\text{ref}}$$

General Form of a Linear Function

Whenever two quantities are related by a CROC, it's a line on a graph.

- That's where the **Linear Function** comes from
- The process above can be used any time there's a CROC and a known reference point.

Module 4

- Google Slide Notes

General Notes

General form of a linear function:

$$v = \underbrace{\text{constant rate of change} \left(\underbrace{t - \text{reference input}}_{\text{change in input}} \right)}_{\text{change in output}} + \text{reference output}$$

$$v = \underbrace{\text{CROC} \left(\underbrace{t - t_{ref}}_{\text{change in input}} \right)}_{\text{change in output}} + v_{ref}$$

When the reference point is the vertical intercept, the formula simplifies:

1. $v = \text{CROC}(t - t_{ref}) + v_{ref}$
 2. $v = \text{CROC}(t - 0) + \text{output of VI}$
 3. $v = \text{CROC}(t) + \text{Output of VI}$ or $y = m(x) + b$
- This formula is a *special case* of a Linear Function and can only be used if the reference point is the vertical intercept.

Average Rate of Change

When the rate is not constant, the rate is called the **Average Rate of Change (AVROC)**.

- The average rate of change can be found between **any two points** by:
 1. finding the constant rate of change between those two points.
 2. Dividing the constant rate of change by the input.

Example

Number of weeks dieting	Brandon's weight in pounds
0	196
2	187
7	190
12	184

- The average rate of change between week **0** and **7** is **-0.86**, but the rate does not go down constantly at this rate. Because of this, the average rate of change means in this scenario:

IF Brandon's weight had **changed by the same amount each week between week 0 and week 7**, he *would have* lost 0.86 pounds each week.

- If you needed to write this about a graph: **IF** the function changed at a constant rate of change between $x = 0$ and $x = 7$, we would have a line between those two points and that line *would have* a constant rate of **-0.86** (as shown in {color} on the graph).
- It's also important to use the data that is most closely related to the point in time that you are trying to find (*in this case, it would be between week 7 and 12, not 0 and 7*).

The average rate of change is a *hypothetical* constant rate.

To find what the output would be at any specific non-given point, multiply the average rate of change by the change in input:

- **output = AROC(Δ Input)**
 – The same as **output = (Δ output/ Δ input)(Δ Input)**

The average rate of change is calculated by dividing the changes of two outputs by the changes in the corresponding inputs. That is,

$$\begin{array}{c} \text{Average Rate} \\ \text{of Change} \end{array} = \frac{\Delta \text{output}}{\Delta \text{input}} = \frac{f(b) - f(a)}{b - a}$$

where $(a, f(a))$ and $(b, f(b))$ are any two data points of the function f .

Polynomial Example

$$g(x) = 2x^2 - 3x - 4$$

Determining the average rate of change between $x = 5$ and $x = 1$:

1. **Average Rate of Change** = $g(5) - g(1)/5 - 1$
2. $g(5) = 2(5^2) - 3(5) - 4 = 31$
3. $g(1) = 2(1^2) - 3(1) - 4 = -5$
4. **Average Rate of Change** = $g(5) - g(1)/5 - 1 = 31 + 5/4 = 36/4 = 9$

Image Reference:

$$\text{Average Rate of Change} = \frac{g(5) - g(1)}{5 - 1}$$

$$g(5) = 2(5^2) - 3(5) - 4 = 31 \quad g(1) = 2(1^2) - 3(1) - 4 = -5$$

$$\text{Average Rate of Change} = \frac{g(5) - g(1)}{5 - 1} = \frac{31 - (-5)}{4} = \frac{36}{4} = 9$$

Linear Regression

When data is not perfectly linear but is close, we create a **Linear Model**, a function that *models* the data. This is done using a process called **Linear Regression**.

How To Graph Linear Regression On Desmos

1. Click the plus button (Add an item)
2. Choose **Table**
3. Add some values (enter or copy/paste)
4. Change graph settings to better match the data.
5. Instead of **y1 = mx1 + b**, use the **tilde (~)** symbol for the **equals (=)** sign: **y1 ~ mx1 + b** to use linear regression.
 - To use an exponential pattern, use: **y1 ~ a * bx1**
6. To predict a value, use either:
 1. **x = {desired x value}**
 2. **m({desired x value}) + b**
 - It can be helpful to create a folder hitting the plus button, dragging the table into it, and then closing it.

Coefficient of Determination

When computing a linear regression model, the **r2** is known as the coefficient of determination, and it describes the strength of the fit of a linear regression model to a set of data.

The closer the value of r2 is to 1, the stronger the fit. r2 is always a value between 0 and 1.