Logarithmic Functions

$y = \log_b(x)$
read
"y equals log base b of x,"
means
"y is the exponent we place on b to get x."

We can also think of it as answering the question "What exponent on b is necessary to get x ?"
That is, y is the number that makes the equation $b^y = x$ true.

Common and Natural Logarithms

Although any positive number other than 1 may be used as a base for a logarithm, there are two bases that are used so frequently that they have special names. A base-10 logarithm, $\log_{10}(x)$, is called the **common log**. When writing the common log, it is customary to omit the "10" and simply write $\log(x)$.

$y = \log(x)$
asks
"What exponent on 10 gives us x?"
"y is the exponent on 10 that gives us x"
$x = 10^y$

A base-e logarithm, $\log_e(x)$, is called the **natural log** (so named because $e \approx 2.71828$ is called the *natural number*). When writing the natural log we write $\ln(x)$ instead of $\log_e(x)$.

$y = \ln(x)$
asks
"What exponent on e gives us x?"
"y is the exponent on e that gives us x"
$x = e^y$

Example 1: Solve the equation $log_5(2x + 3) = 3$ for x.

Rewrite as an exponential equation. $5^3 = 2x + 3$
Solve for x like normal. $125 = 2x + 3$
$122 = 2x$
$61 = x$

Example 2: The total annual health-related costs in the United States in billions of dollars may be modeled by the function $H(t) = 30.917(1.1013)^t$ where t is the number of years since 1960. According to the model, when will health-related costs in the United States reach 250 billion dollars?

$250 = 30.917(1.1013)^t$
Simplify as much as possible. $8.086 = (1.1013)^t$
Write as a logarithmic equation $log_{1.1013}(8.086) = t$
Use your TI-84 calculator click "MATH", Select "logBASE(", and fill in the numbers.
Or use Desmos and enter as an expression.
$21.66 = t$
According to the model, 21.66 years after 1960 (8 months into 1982), the health-related costs in the United States reached 250 billion dollars.