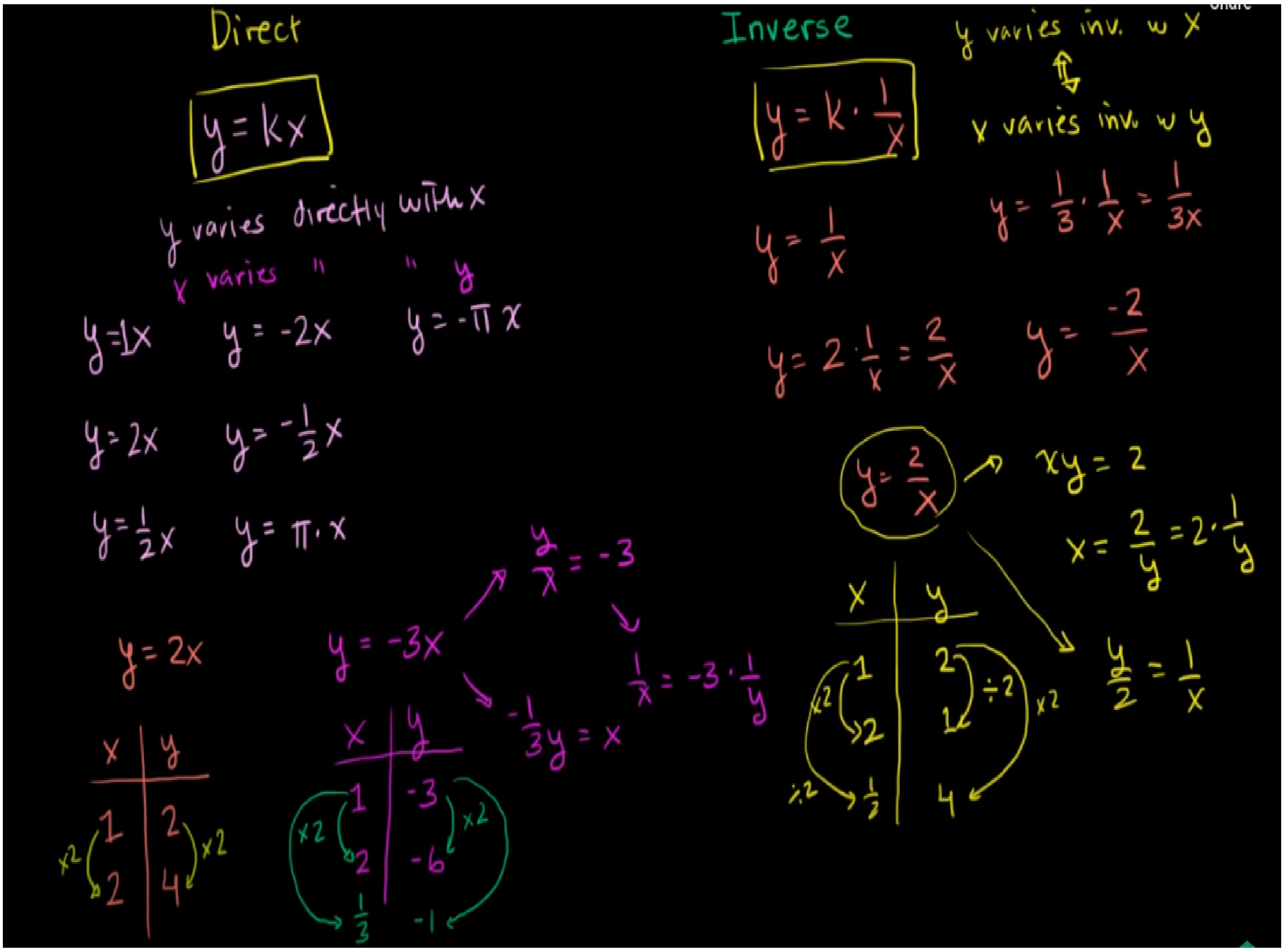
Module 1 - Modeling With Other Types of Functions

General Notes

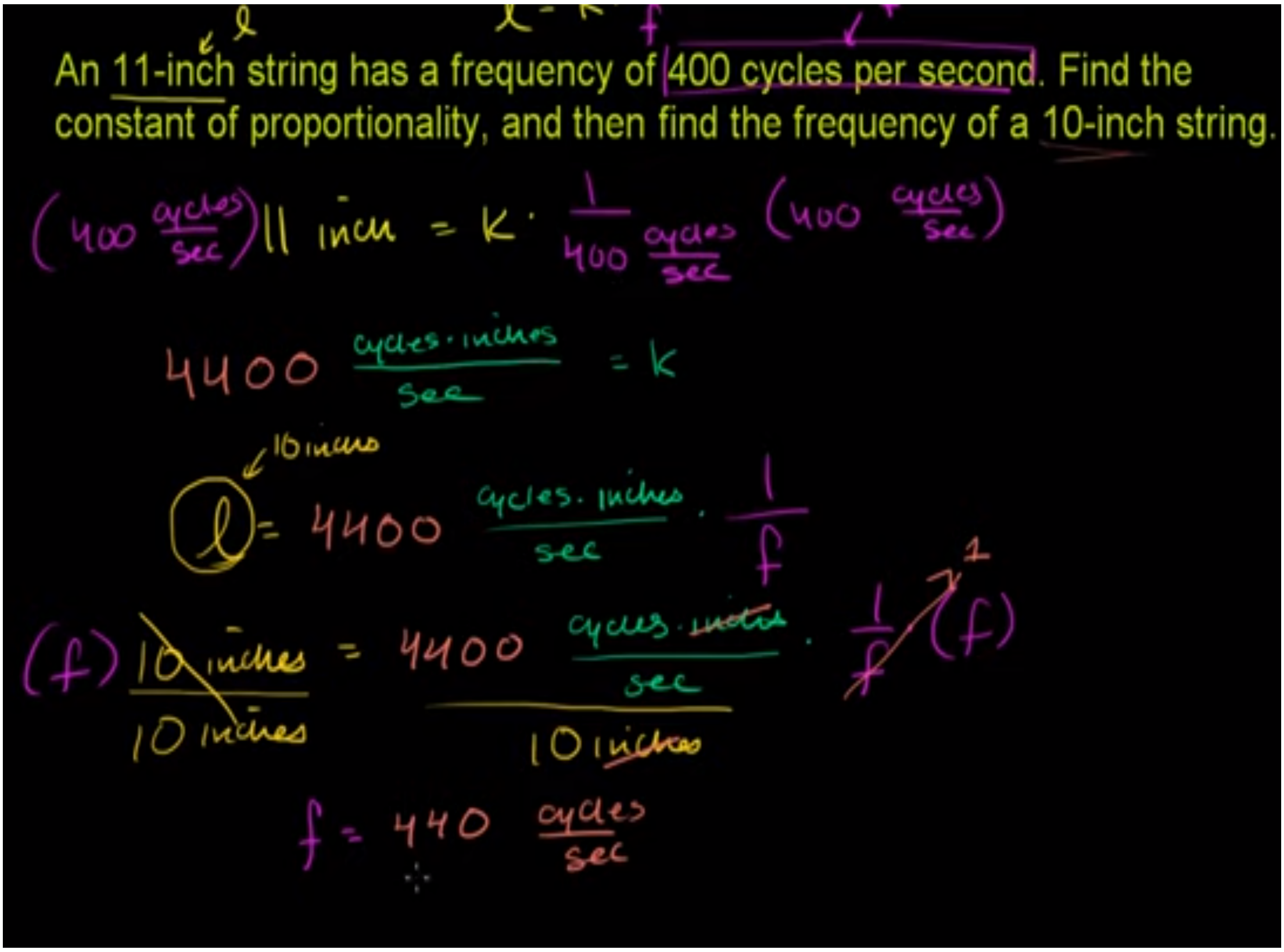
Video Resources

Direct and Inverse Variation



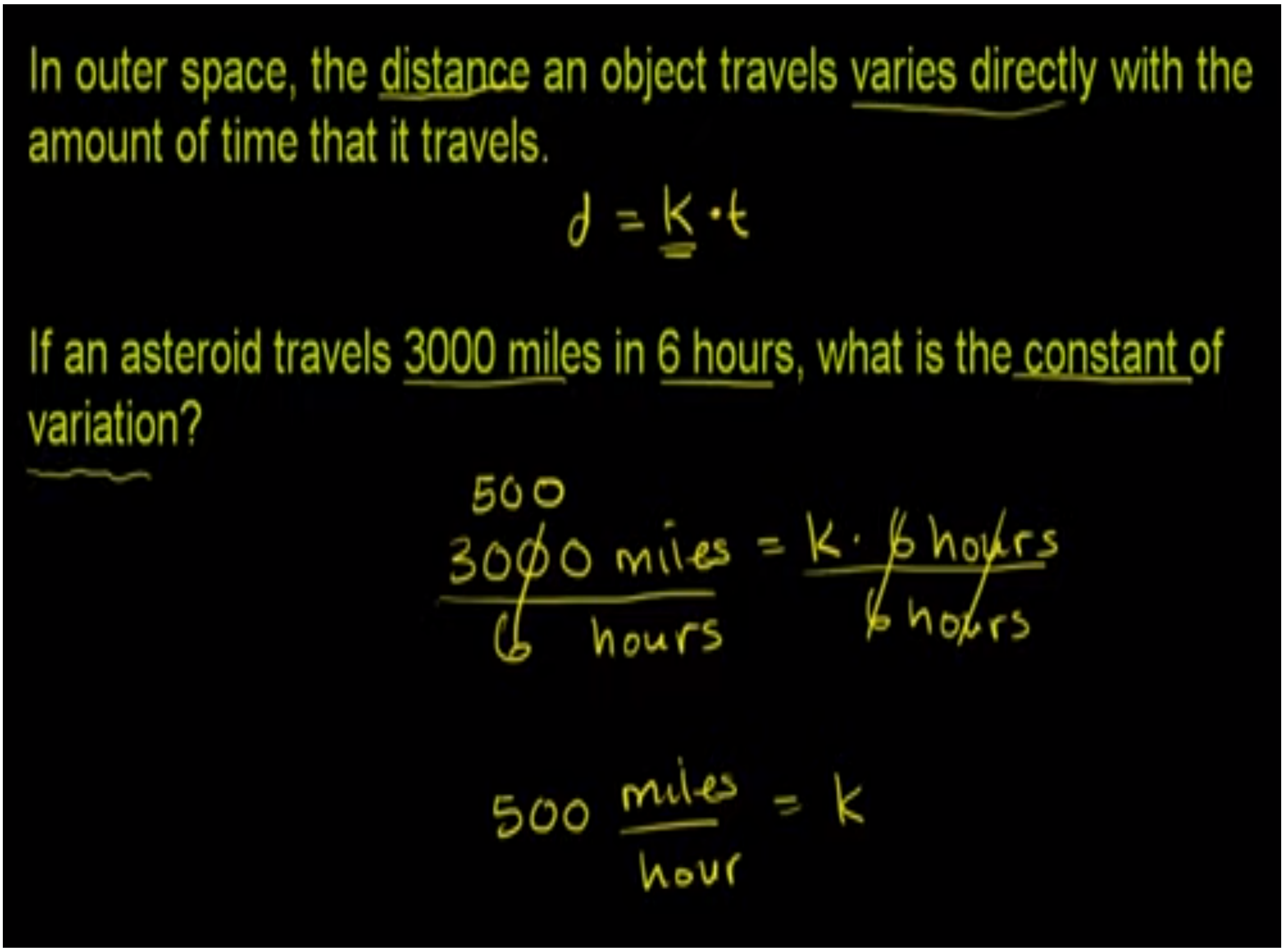
- **Direct Variation:** Scaling up **x** or **y** also scales the opposite by the same factor.
    - *I.e. If **x** is doubled, **y** is doubled.*
- **Inverse Variation:** Scaling up **x** or **y** inversely scales the opposite by the same factor.
    - *I.e. If **x** is doubled, **y** is halved.*
- As seen in the video and image, the equation can be rewritten in various ways to fool the reader. Because of this, always try to simplify the equation to either:
    - **Direct Variation: $y = kx$**
    - **Inverse Variation: $y = {}^{k}/_{x}$**

Inverse Variation Application



> **Full problem:**
>
> On a string instrument, the length of a string varies inversely as the frequency of its vibrations (the vibrations are what give string instruments their sound!).
>
> An 11-inch string has a frequency of 400 cycles per second. Find the constant of proportionality, and then find the frequency of a 10-inch string.

Direct Variation Application



Power Functions Definition

A function with the equation of the form:

> **$y = ax^b$**

where **a** and **b** are constants, is called a **power function**.

- The main difference between a power function and a polynomial function is that in a power function the exponent, **b**, can be any real number rather than just a positive integer.
- A power function is a single-term function, whereas a polynomial function may have multiple terms.

Solving Power Functions

Method 1 - Graphing

1. Input the table of values into Desmos and then use power regression to find the
   equation of the function.
   - The formula is: $\mathbf{y = ax^b}$
2. **a** is the slope of the line and **b** is the exponent.
3. Example: $\mathbf{y = 9.30693x^{0.12}}$

Method 2 - Algebraically

To solve a power function equation $c = ax^b$ for x, apply the following steps.

General Procedure for $c = ax^b$

1. Divide each side by a.

$$\frac{c}{a} = x^b$$

2. Raise each side to $\frac{1}{b}$.

$$\left(\frac{c}{a}\right)^{1/b} = (x^b)^{1/b}$$

3. Simplify.

$$x = \left(\frac{c}{a}\right)^{1/b}$$

Specific Procedure for $58 = 27x^{0.75}$

1. $\frac{58}{27} = x^{0.75}$

2. $\left(\frac{58}{27}\right)^{1/0.75} = (x^{0.75})^{1/0.75}$

3. $x = \left(\frac{58}{27}\right)^{1/0.75} \approx 2.77$

When solving power function equations using the streamlined method shown here, it is important to remember the meaning of the rational exponent. For example, since $0.75 = \frac{3}{4}$, we have

$$\begin{aligned} x &= \left(\frac{58}{27}\right)^{1/0.75} \\ &= \left(\frac{58}{27}\right)^{\frac{1}{3/4}} \\ &= \left(\frac{58}{27}\right)^{4/3} \\ &= \sqrt[3]{\left(\frac{58}{27}\right)^4} \end{aligned}$$

Rational Exponents and Radicals

Rational exponents can be expressed using radical notation. For example,

$$x^{1/2} = \sqrt{x}$$
$$x^{1/3} = \sqrt[3]{x}$$
$$x^{1/4} = \sqrt[4]{x}$$

If the exponent is not a unit fraction like these, we use properties of exponents to first rewrite the expression.

$$x^{2/3} = (x^2)^{1/3} = \sqrt[3]{x^2}$$
$$x^{0.35} = x^{35/100} = x^{7/20} = (x^7)^{1/20} = \sqrt[20]{x^7}$$

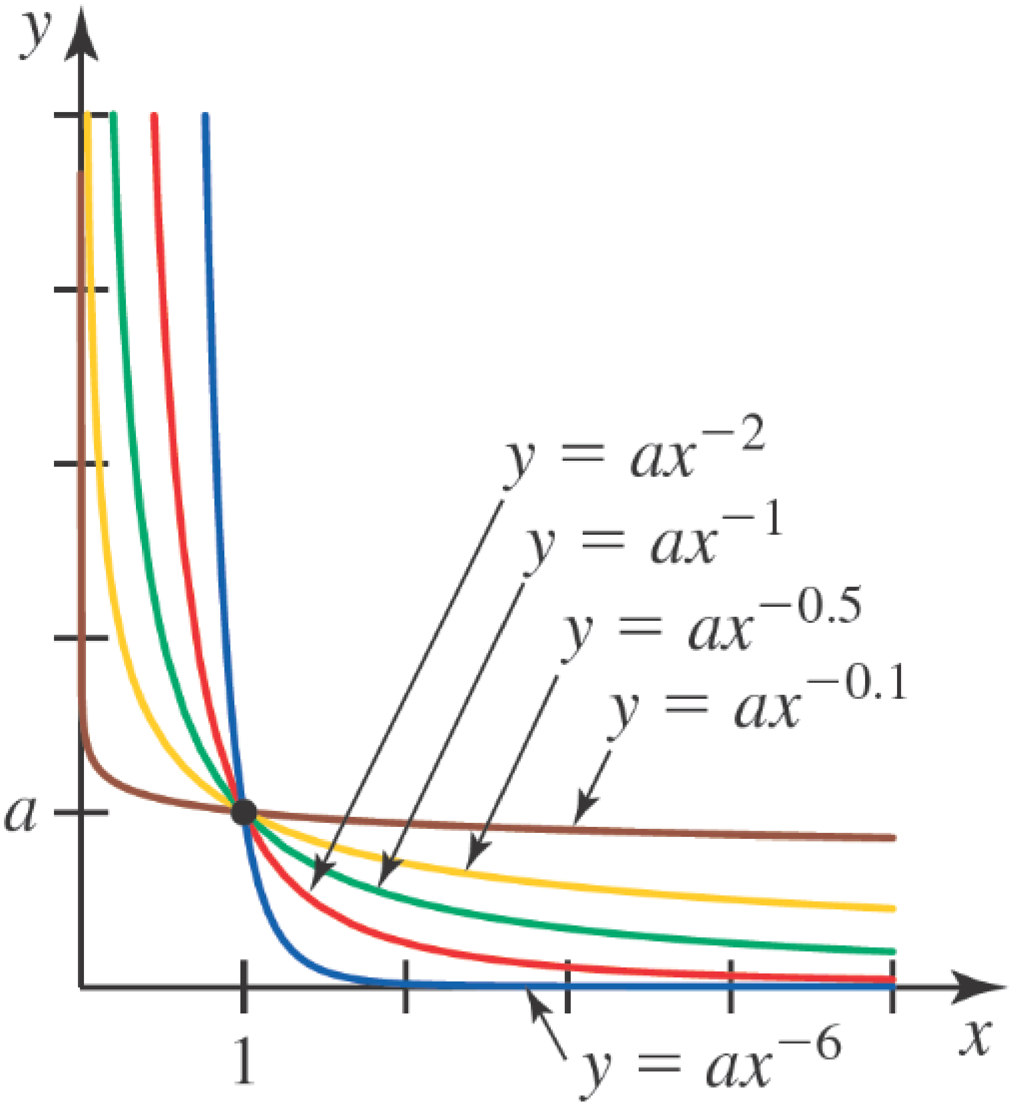
These radical expressions can also be written as

$$\sqrt[3]{x^2} = (\sqrt[3]{x})^2$$
$$\sqrt[20]{x^7} = (\sqrt[20]{x})^7$$

In general,

$$x^{m/n} = \sqrt[n]{x^m} = (\sqrt[n]{x})^m$$

Power Function Graph - `x > 0` and `b < 0`



Direct and Inverse Variation

Given a power function of the form **y = ax^b** with **a > 0**.

- If **b > 0**, then it is an **increasing** function and **x^b** and **y** vary **directly**.
- If **b < 0**, then it is a **decreasing** function and **x^b** and **y** vary **inversely**.

When determining if a power function should be used to model a set of data, look for signs of direct or inverse variation power functions.

Direct Variation

Given a power function of the form **$y = ax^b$** with **$a > 0$**.

It is Said as either:

- *"**y** varies directly with **x^b**"*
- "***y** is directly proportional to **x^b***"

a is called the **constant of proportionality**.

Inverse Variation

The power function with **b < 0** should be rewritten from **y = ax^b
** to **$y = ax^{-c}$**,
where **c** is a positive number equal to **$|b|$**.

- Alternate form useful for interpreting inverse variation.

Given a power function of the form (with **$c > 0$**):

$$y = ax^{-c} = \frac{a}{x^c}$$

It is Said as either:

- *"**y** varies inversely with **x^c**"*
- "***y** is inversely proportional to **x^c***"

a is called the **constant of proportionality**.

Real-world examples

Related Quantities	Formula	In Words
Cost of a fuel purchase and the amount of fuel bought	$C = kg$	The cost of a fuel purchase is directly proportional to the amount of fuel bought (purchased in **gallons**). **k** is the constant of proportionality and represents the fuel price per gallon.
Area of a circle and its radius	$A = \pi r^2$	The area of a circle is directly proportional to the square of its **radius**. π is the constant of proportionality.
Blood flow in an artery and the radius of the artery	$F = kr^4$	The rate at which blood flows in an artery (in **mL per minute**) is directly proportional to the fourth power of the **radius** of the artery. **k** is the constant of proportionality.
Average earnings per hour when paid a fixed amount of money to complete a task and hours worked	$A = k/x$	The average earnings per hour, **k**, is inversely proportional to the amount of money to complete a task and hours worked. **x** is the constant of proportionality and represents the fixed amount of money paid for the job.
Length of a 4-cubic-foot box with equal height and width and box width	$L = 4/w^2$	The length of a box, **L**, with equal height and width, **w**, is inversely proportional to the square of the width with a constant of proportionality, 4.

Negative Exponents

Recall the following property of negative exponents:

$$x^{-p} = \frac{1}{x^p}$$

We can make sense of this rule by investigating patterns.

$$x^4 = x \cdot x \cdot x \cdot x$$

$$x^3 = \frac{x \cdot x \cdot x \cdot x}{x} = x \cdot x \cdot x$$

$$x^2 = \frac{x \cdot x \cdot x}{x} = x \cdot x$$

$$x^1 = \frac{x \cdot x}{x} = x$$

For each decrease in 1 of the exponent, we remove one factor of x.

$$x^0 = \frac{x}{x} = 1$$

Continuing to decrease the exponent by 1 and continuing to divide by x produces the following pattern.

$$x^{-1} = \frac{1}{x}$$

$$x^{-2} = \frac{\frac{1}{x}}{x} = \frac{1}{x^2}$$

$$x^{-3} = \frac{\frac{1}{x^2}}{x} = \frac{1}{x^3}$$

Inverses of Power Functions

Power functions that are strictly increasing or decreasing will have an inverse function.

Given: $\boldsymbol{f(x) = ax^n}$

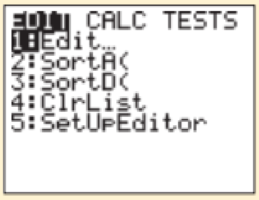
$$
\begin{aligned}
y &= ax^n \\
\frac{y}{a} &= x^n \\
\left(\frac{y}{a}\right)^{1/n} &= (x^n)^{1/n} \\
x &= \left(\frac{y}{a}\right)^{1/n} \\
f^{-1}(y) &= \left(\frac{y}{a}\right)^{1/n}
\end{aligned}
$$

- *inverse variation* and *inverse function* are not the same thing.

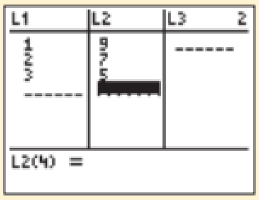
Power Function Regression on a Calculator

1. Press [2nd] then [0], scroll to `DiagnosticOn` and press [ENTER] twice. This will ensure that the correlation coefficient r and the coefficient of determination r^2 will appear.

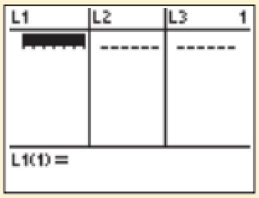
2. Bring up the Statistics Menu by pressing the [STAT] button.



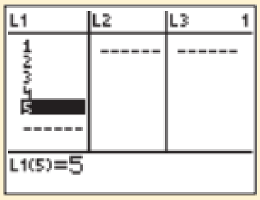
3. Bring up the List Editor by selecting `EDIT` and pressing [ENTER].



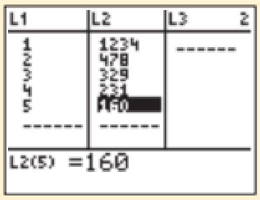
4. If there are data in the lists, clear the lists. Use the arrows to move the cursor to the list heading, `L1`, then press the [CLEAR] button and press [ENTER]. This clears all of the list data. Repeat for each list with data. (Warning: Be sure to use [CLEAR] instead of [DELETE]. [DELETE] removes the entire column.)



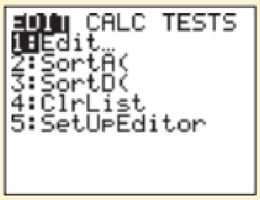
5. Enter the numeric values of the *inputs* in list `L1`, pressing [ENTER] after each entry.



6. Enter the numeric values of the *outputs* in list `L2`, pressing [ENTER] after each entry.



7. Return to the Statistics Menu by pressing the [STAT] button.



8. Bring up the Calculate Menu by using the arrows to select `CALC`. Use arrows to move down to `A:PwrReg`.

9. Calculate the power equation of the model by selecting `A:PwrReg` and pressing [ENTER] twice. The power regression model is $y = 1208.86x^{-1.22943}$ and has correlation coefficient $r = -0.9974$.