MATPMD3 Financial Modeling Assignemnt

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2020.02.14

Assignment Introduction

The problems 1-4 involve calculating values of various instruments etc. You should present your workings but may use any computational method you choose to solve the equations you derive (e.g. excel, python etc). However, I would like to see how you do this so you should append your code or spreadsheet formulae to your assignment.
Problem 5 contains is more computational in nature. You should present your solution as a mini-project, giving a clear explanation of the background and theory. You should attach your code to your project and marks will be awarded for well written code (clear style, well commented and easy to follow), clear explanation of the background theory and presentation of your results.
Completed assignments may be uploaded to canvas or placed in my pigeon hole in the Computing Science and Mathematics division by the due date.

Date of Issue: January 27th 2020
Report Due by: February 14th 2020

https://financetrain.com/best-python-librariespackages-finance-financial-data-scientists/ contains a nice list of python libraries used in finance.

Question 1

Suppose that 12-month, 24-month, 36-month, 48-month and 60-month zero rates are 2.4%, 2.6%, 2.9%, 3% and 3.3% per annum with continuous compounding respectively. Calculate:
(a) the theoretical price of a bond with a face value of £200 and a maturity of 5 years that pays a coupon of 4% semiannually.

The theoretical price of a bond is its present value, P. Which is the sum of the present value of face value of the bond plus the present values of any coupon payments discounted at the appropriate interest rate.

$$\begin{aligned} P &= PV[F] + \sum_{k=1}^{n} PV_k \left[\frac{M}{m}\right] \\ &= FV[F]e^{RT} + \sum_{k=1}^{n} FV_k \left[\frac{M}{m}\right] e^{RT} \end{aligned}$$

Where:

- F is the face value of the bond $F = 200$
- M is the annual coupon payment $M = 0.04 * 200 = 8$
- m is the number of coupon payments per year $m = 2$
- c is the periodic coupon rate $c = \frac{M}{m} = 4$
- R is the relevant interest rate $R = \{0.024, 0.026, 0.029, 0.03, 0.033\}$
- t is the discount period, the time until payments are receieved $t = \{0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0\}$
- n is the total number of periods $n = m \times t = 10$

Filling in the numbers the theoretical price of the detailed bond is $P = £206.46$

(b) the bond's yield (please derive the expression for the yield, you may like to write some code to solve the expression)

The yield, y, is the single discount rate that makes present value of a bond equal to it's market price.

$$P = FV[F]e^{RT} + \sum_{k=1}^{n} FV_k \left[\frac{M}{m}\right] e^{RT} = FV[F]e^{yT} + \sum_{k=1}^{n} FV_k \left[\frac{M}{m}\right] e^{yT}$$

Solving for y gives a yield of of 3.268%

(c) the yield to maturity if the bond was purchased for £168 just after the second coupon payment.

The yield to maturity of a bond is the yield where the market price of the bond is price that it was purchased at. In this case the yield is calculated using a price, $p = £168$, removing the first two coupon payments from the calculation, and accounting for the fact that the bond now matures in 4 years (two coupon payments means one year has passed since the bond was issued.
The yield to maturity is calculated as 7.786%

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Question 2

Suppose interest rates fluctuates according to

$$r(t) = r + 0.03cos(0.2t)$$

(a) Derive an expression for the value of an investment made at t = 0 at some future time t with a risk-free interest rate r = 3%.

$$V(t) = V(0)e^{\int_0^t r(u)du}$$

$$= V(0)e^{\int_0^t (0.03+0.03cos(0.2t)dt}$$

where $g = 0.2t, dg = 0.2dt$

$$= V(0)e^{\int_0^t (\frac{0.03}{0.2} + \frac{0.03}{0.2}cos(g))dg}$$

$$= V(0)e^{\int_0^t (\frac{0.03}{0.2}g+0.15sin(g))}$$

$$= V(0)e^{\int_0^t (0.03t+0.15sin(0.2t))}$$

$$= V(0)e^{[0.03t+0.15sin(0.2t)]_0^t}$$

$$= V(0)e^{(0.03t+0.15sin(0.2t))}$$

(b) Calculate and plot the yield curve.

$$R(0,t) = \frac{1}{t-0}\int_0^t r(u)du$$

$$= \frac{1}{t}\int_0^t (0.03 + 0.03cos(0.2t)dt$$

where $g = 0.2t, dg = 0.2dt$

$$= \frac{1}{t}\int_0^t (\frac{0.03}{0.2} + \frac{0.03}{0.2}cos(g))dg$$

$$= \frac{1}{t}\int_0^t (\frac{0.03}{0.2}g + 0.15sin(g))$$

$$= \frac{1}{t}\int_0^t (0.03t + \frac{0.03}{0.2}sin(0.2t))$$

$$= \frac{1}{t}[0.03t + \frac{0.03}{0.2}sin(0.2t)]_0^t$$

$$= \frac{1}{t}(0.03t + \frac{0.03}{0.2}sin(0.2t))$$

$$= 0.03 + \frac{0.03}{0.02t}sin(0.2t)$$

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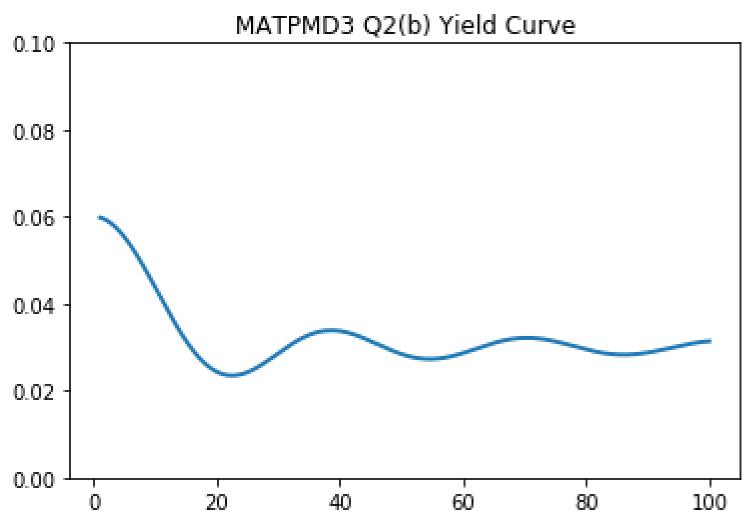
MATPMD3 Q2(b) Yield Curve
0.10
0.08
0.06
0.04
0.02
0.00
0
20
40
60
80
100

Question 3

Using a one step binomial tree, derive an expression for the value of a European call option for Δ units of stock, each with a value of S_0.

We are interested in valuing a European call option to buy a stock T years from now for a strike price, K, when it is worth S_0 now.
Suppose it is worth either S_{Tu} or S_{Td} in T years time, where $S_{Tu} > S_0 > S_{Td}$.
If the stock price is S_{Tu} then the value of the option is $S_{Tu} - K$ otherwise it is worthless as we would allow the option to expire rather than pay above the spot price by exercising it.

We set up a risk-free portfolio. Risk free because we would be as well invested at the risk free interest rate.
Our portfolio is made of a long position in Δ units of stock and a short position in one call option. Where Δ is the number of stocks that makes this portfolio risk-free.
If the stock price move from S_0 to S_{Tu} the value of the shares/stocks is $S_{Tu} \times \Delta$ and the value of the option is $(S_{Tu} - K)$ so the total value is $S_{Tu} \times \Delta - (S_{Tu} - K)$.
If the stock price move from S_0 to S_{Td} the value of the shares/stocks is $S_{Td} \times \Delta$ and the value of the option is £0 so the total value is $S_{Td} \times \Delta$.

The portfolio has been designed to be riskless by chosing a value of Δ such that the value of the portfolio is the same in both scenarios. That is:

$$S_{Tu} \times \Delta - (S_{Tu} - K) = S_{Td} \times \Delta$$
$$S_{Tu} \times \Delta - S_{Td} \times \Delta = (S_{Tu} - K)$$
$$(S_{Tu} - S_{Td}) \times \Delta = (S_{Tu} - K)$$
$$\Delta = \frac{(S_{Tu} - K)}{(S_{Tu} - S_{Td})}$$

To be riskless the portfolio must also (in the absence of arbitrage) earn the risk free rate of interest. For the Risk Free Interest Rate, R, the present value, PV, of the portfolio can be calculated by discounting the future value, FV.

$$FV = \Delta S_{Tu} - (S_{Tu} - K) = \Delta S_{Td}$$
$$PV = FVe^{RT}$$
$$= (\Delta S_{Tu} - (S_{Tu} - K))e^{RT} = (\Delta S_{Td})e^{RT}$$

And given that the present value of the stocks is ΔS_0 the present value of the call option, C_E can be calculated by subtracting from the present value of the portfolio:

$$C_E = (\Delta S_{Tu} - (S_{Tu} - K))e^{RT} - \Delta S_0 = (\Delta S_{Td})e^{RT} - \Delta S_0$$

A particular stock is currently priced at £40. Three months from now, it will be either £44 or £36. Find the value of a call option on one unit of this stock, when the strike price is £42 and the risk-free interest rate is 6%

$$S_{0u} = 44 \Rightarrow u = 1.1, S_{0d} = 36 \Rightarrow d = 0.9$$
$$f_u = max(S_{0u} - K, 0) = max(44 - 42, 0) = 2$$
$$f_d = max(S_{0d} - K, 0) = max(36 - 42, 0) = 0$$

The probability that the stock will increase in price in the next period is:

$$P = \frac{e^{-RT} - d}{u - d}$$

$$= \frac{e^{-0.06 \times 0.25} - 0.9}{1.1 - 0.9}$$
$$= 0.576$$

The expected value of the call option is then:

$$E(payoff) = p \times f_u + (1 - p) \times f_d$$
$$= 0.576 \times 2 + (0.424) \times 0$$
$$= £1.152$$

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Question 4

Suppose that a one-unit investment in a new portfolio that will track the FT 100 share index is priced at £1000. The shares in this index provide income equivalent to a continuous dividend yield at the nominal rate of 4% p.a. The risk-free interest rate in the economy is 5% p.a. nominal (to be compounded continuously).
What is the rational price for a forward contract to buy one unit in the portfolio two years from now?

Suppose that an arbitrager enters into contract to buy one share of this portfolio in T years. The current price of the share is S_0 and dividend yield of $q\%$ per year means the future value of the share will be discounted at q; $FV = S_0 e^{-qt}$. The arbitrager can borrow money at $r\%$ and borrows enough to buy one share in T years, the repayment due will then be $S_0 e^{-qt} e^{rt}$, or $S_0 e^{(r-q)t}$. With the share bought and the loan repaid the arbitrager will have profited to the value of $F_0 - S_0 e^{(r-q)t}$, as forward contracts are priced to eliminate the opportunity for arbitrage $F_0 = S_0 e^{(r-q)t}$.

In this example $S_0 = 1000$, $q = 0.04$, $r = 0.05$, and $T = 2$ so:

$$\begin{aligned} F_0 &= S_0 e^{(r-q)T}\} \\ &= 1000 \times e^{(0.05-0.04)\times 2} \\ &= £1020.20 \end{aligned}$$

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Question 5

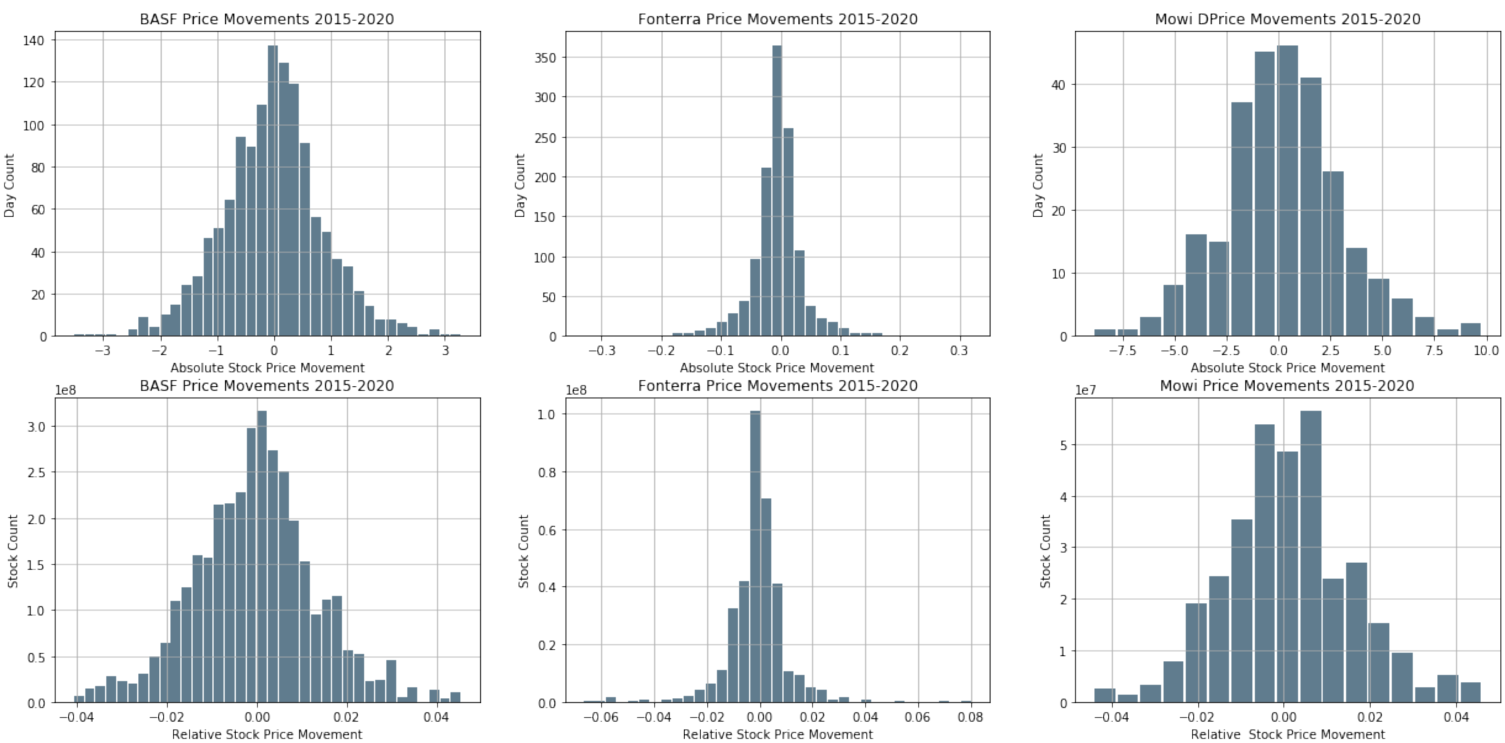
The underlying assumption in quantitative finance is that stock prices follow an Itô process of some kind and hence the [daily] movements are Normally distributed. In this problem you will investigate the validity of this assumption.
You will first need to get some data. Firstly, you will need to decide on a company and obtain the code with which it is listed; you can download the list of symbols for companies from http://investexcel.net/all-yahoo-finance-stock-tickers/. Once you have this code (called the ticker) you can download the historic prices from yahoo finance.
For example, if I wanted Google's (GOOG) prices, the relevant URL is https://finance.yahoo.com/quote/GOOG/his

(a) Choose 3 tickers (you may pick any company you wish but they should be from different industries, e.g. tech, mining, food etc).

BASF - German Chemicals - https://www.basf.com/gb/en.html
Fonterra - New Zealand Diary - https://www.fonterra.com/nz/en/about.html
Mowi - Norwegian Fish Farming - https://mowi.com/

(b) Plot the daily movements (the amount gained or lost in a day) as a histogram.

I have plotted 6 graphs for this question: the first three were plotted considering absolute stock price movements and weighting each day equally' the other three were plotted considering relative stock price movements and weight the days according to the volume of sales of stock.



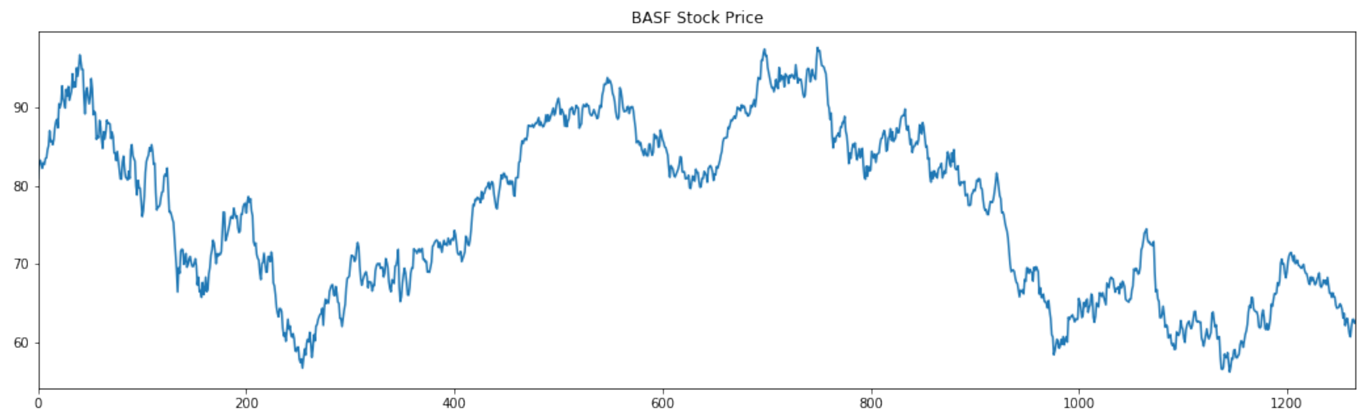
(c) Determine whether or not these daily fluctuations are Normally distributed? (A χ-squared test for a goodness of fit to a normal distribution will tell you whether or not a null hypothesis of normality is to be rejected).

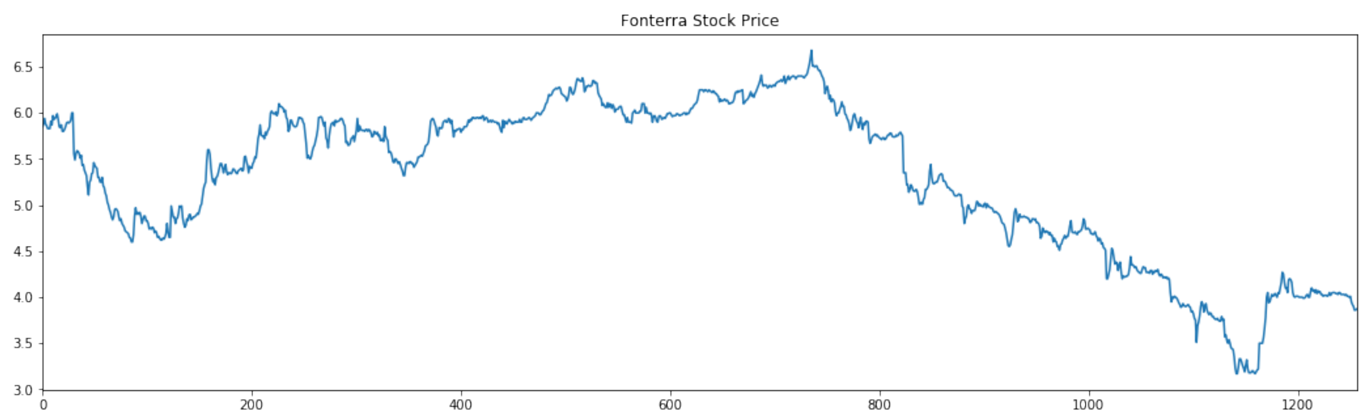
For the three companies considered the probability that the daily fluctuations are normally is given in the table below:

Company	BASF	Fonterra	Mowi
P	0.081	$8.07e^{-10}$	0.264

The null hypothesis that a company's daily stock price fluctuations are normally distributed cannot be rejected with any statistical significance for either BASF or Mowi.
However, there is strong evidence that daily stock price fluctuations are not normally distributed for Fonterra.

(d) Plot a time series plot of two of your stocks. Does the times series follow an Itô process? (I'm asking for your thoughts rather than a strict test of the condition).





I could believe that, generally, Fonterra's stock price does follow and Itô process, although it's very clear that occasionally the drift function changes. Specifically the Fonterra stock price looks to be drifting up until January 2018, then begins to drift down rather sharp until September 2019, before settling out.
This can be traced back to changes at the company: In early 2018 the then Chief Executive resigned following a massive writedown in the value of Fonterra assets; in September 2019 Fonterra announced record losses and a significant change in strategy.

BASF's stock price looks like a random walk with drift. I can well believe it follows an Itô process. As I understand it, the trend of BASF's stock price is illustrative of German slow donw in recent years. Beyond that broad movement, noise in the price is visible.

(e) Comment on your findings. Some things you may wish to comment on are:
• Are the prices Normally distributed over every timescale (can you identify a period where the movements are/are not Normally distributed)?
• Does the assumption of Normality only apply to some sectors?
• What happens in the case of a disruption (e.g. house builders share prices in the event of a market collapse, for example in 2006-2008, or food companies after a scare or fraud, for example coca-cola after the negative press for their Desani drink)

Fonterra was perhaps a bad company to pick. Fonterra is a cooperative, jointly owned by some 10500 dairy farmers and there has been a lot of talk in recent years about changing the ownership structure because the current structure makes their share price very sensitive to milk production and price. But I thought an edge case would be extra interesting. That's my bed; I;m lying in it.
Testing whether or not the daily fluctuations in share price for the three companies I chose might be normally distributed was interesting. The P values for Mowi and BASF were not statistically significant but were still small. However, the P value for Fonterra was minuscule. There is very strong evidence that, generally, the daily fluctuations in Fonterra's share price are not normally distributed suggesting that the share price does nto follow and Itô process. This changes significantly over specific time frames. Looking at the period between mid-January 2018, and mid-August 2019 testing if the daily share price fluctuations are distributed normally yields a P value of 0.056. This P value is bordering statistically significant, but on the insignificant side of the border and fully 8 orders of magnitude larger than the P value for the same test when the period 2015-2020 is considered.
The narrower period between mid-January 2018 and mid-August 2019 was considered because in early 2018 the then Chief Executive resigned following a massive writedown in the value of Fonterra assets, and in September 2019 Fonterra announced record losses and a significant change in strategy.
From this is seems that an Itô process is an entirely reasonable way to model share price moves outside of major upheavals.

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