Solution Method

The ODEs for mass and heat transfer and coupled and non-linear making them challenging to solve analytically. To get the simulation running initially the ODEs for heat and mass transfer were solved separately. The simplest numerical way of iterating an ODE is to use the Euler method.

To simplify the temperature ODE, the Reynold's number was set to zero, which means the drop is stationary and there is only diffusive heat transfer. This fixes the value of the Nusselt number as constant. Furthermore, the classical rapid mixing model was used meaning $f_2 = 1$ and $H_{\Delta T} = 0$. In addition to this to decouple the temperature ODE from the mass ODE, the term $\left(\frac{L_V}{C_L}\right)\frac{\dot{m}_d}{m_d}$ was ignored. Therefore the temperature ODE becomes:

$$\frac{dT_d}{dt} = \frac{f_2 Nu}{3Pr_G}\left(\frac{\theta_1}{\tau_d}\right)(T_G - T_d) \tag{1}$$

Which can be solved analytically to provide a reference point for the numerical solution. To make the solution steps clearer, define the constants:

$$\begin{aligned} A &= \frac{f_2 Nu}{3Pr_G}\left(\frac{\theta_1}{\tau_d}\right) \\ B &= T_G \end{aligned} \tag{2}$$

The ODE can then be solved for $T_{d_{n+1}}$ for a given increment in time Δt from a state where $T_d = T_{d_n}$:

$$\begin{aligned} \frac{dT_d}{dt} &= A(B - T_d) \\ \int_{T_{d_n}}^{T_{d_{n+1}}} \frac{dT_d}{(B - T_d)} &= \int_0^{\Delta t} A\ dt \\ \left[\ln(B - T_d)\right]_{T_{d_n}}^{T_{d_{n+1}}} &= \left[A\ dt\right]_0^{\Delta t} \\ -\ln(B - T_{d_{n+1}}) + \ln(B - T_{d_n}) &= A\Delta t \\ \ln\left(\frac{B - T_{d_{n+1}}}{B - T_{d_n}}\right) &= -A\Delta t \\ \frac{B - T_{d_{n+1}}}{B - T_{d_n}} &= e^{-A\Delta t} \\ B - T_{d_{n+1}} &= (B - T_{d_n})e^{-A\Delta t} \\ T_{d_{n+1}} &= B - (B - T_{d_n})e^{-A\Delta t} \end{aligned} \tag{3}$$

Hence the final analytical solution is:

$$T_{d_{n+1}} = T_G - (T_G - T_{d_n})e^{-\left(\frac{f_2 Nu}{3Pr_G}\left(\frac{\theta_1}{\tau_d}\right)\right)\Delta t} \tag{4}$$

The numerical solution uses the Euler forward method, which can be used to solve an ODE of the form:

$$\frac{dy}{dx} = f(x,\ y) \tag{5}$$

in increments of Δt, by assuming for a small timestep the output of the differential is constant. The formula is:

$$y_{n+1} = y_n + \Delta t f(x_n,\ y_n) \tag{6}$$

With $n+1$ being the output at the end of the timestep and n being the current value. Applying this result to equation 1:

$$T_{d_{n+1}} = T_{d_n} + \Delta t \left(\frac{f_2 Nu}{3Pr_G} \left(\frac{\theta_1}{\tau_d} \right) (T_G - T_d) \right) \tag{7}$$

The implementation of equation 7 requires the usage of a for loop to iterate through the time steps. Importantly, this loop must check to see if T_{d_n} has exceeded T_G, if this condition is met the loop must stop as the following results will be unphyiscal. The numerical solution process is shown in the flow chart in Figure .

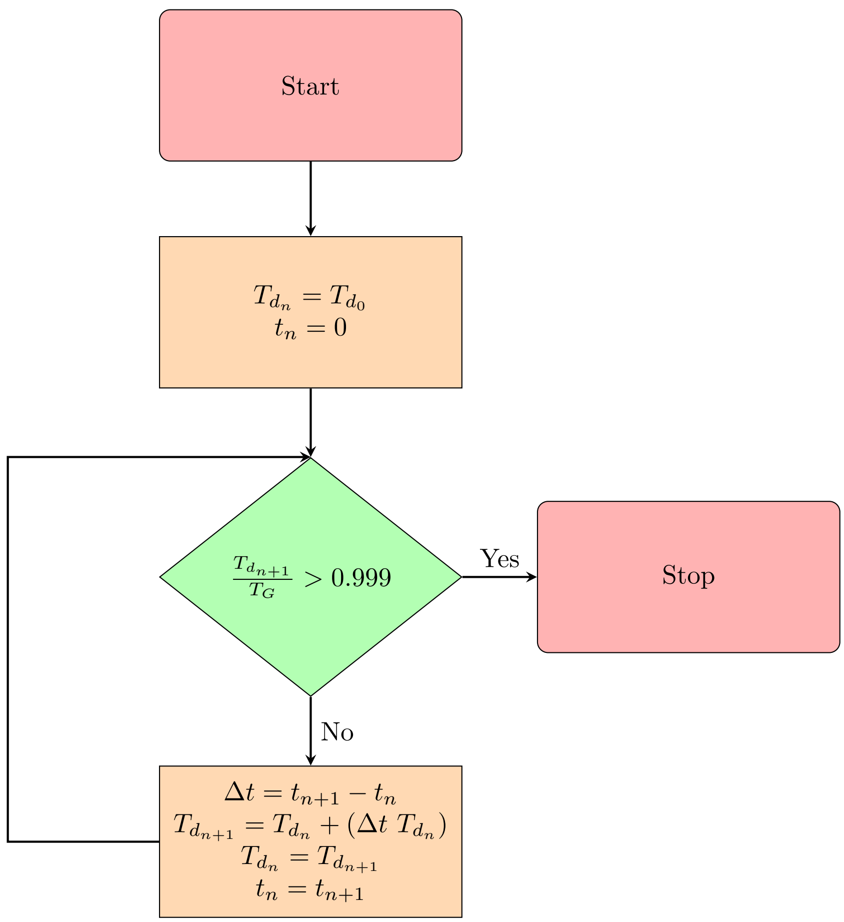

Figure 1: Flowchart for the temperature transfer iteration process.

The results from a simulation run for one droplet of water evaporating in air for a time step of $0.1s$ are shown in Figure . The error between the numerical and analytical solution are shown in Figure .

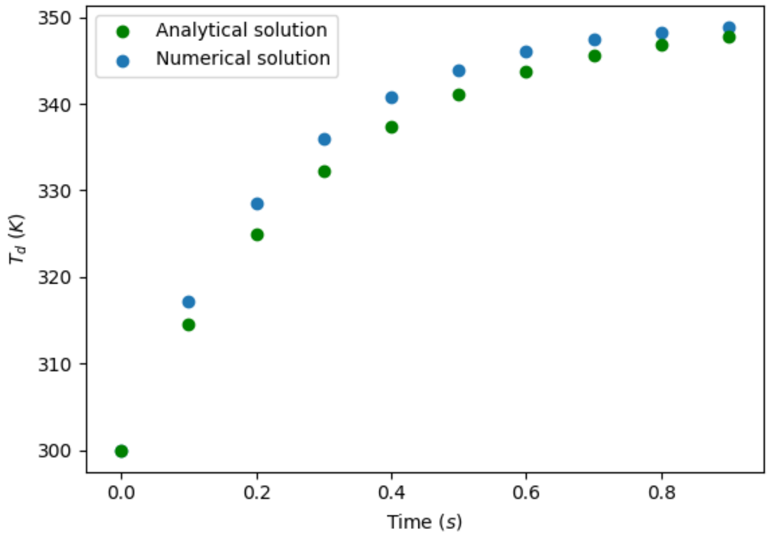

Figure 2: T_d for a droplet sized $D = 5mm$ with $Re_d = 0$, $T_{d_0} = 300K$, $T_G = 350K$ and $\Delta t = 0.1s$.

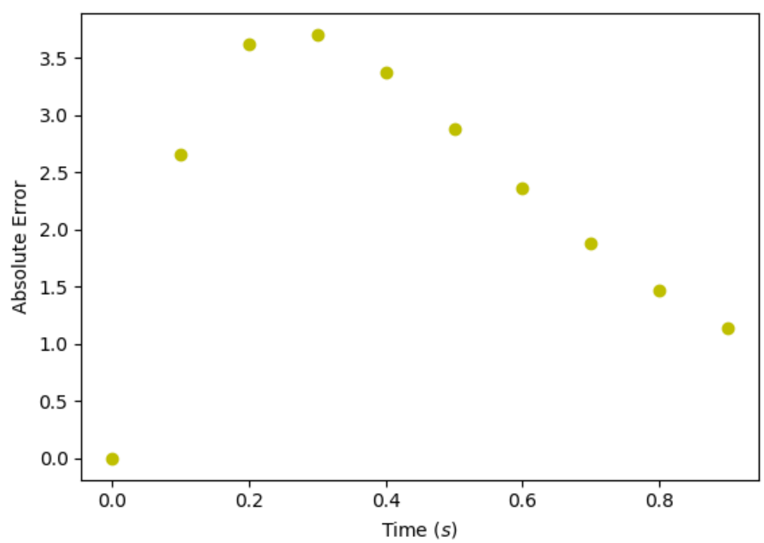

Figure 3: Absolute error between the analytical and numerical solution for the simplified temperature ODE.

For a smaller timestep of 0.01

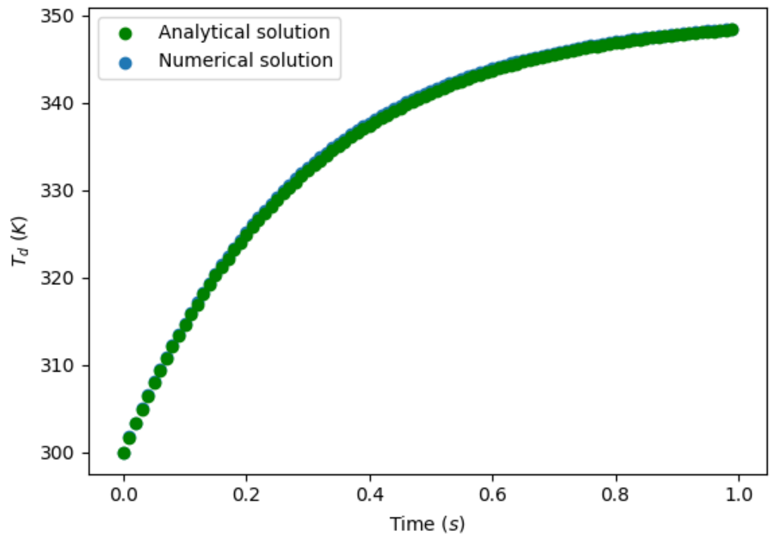

Figure 4: T_d for a droplet sized $D = 5mm$ with $Re_d = 0$, $T_{d_0} = 300K$, $T_G = 350K$ and $\Delta t = 0.01s$.

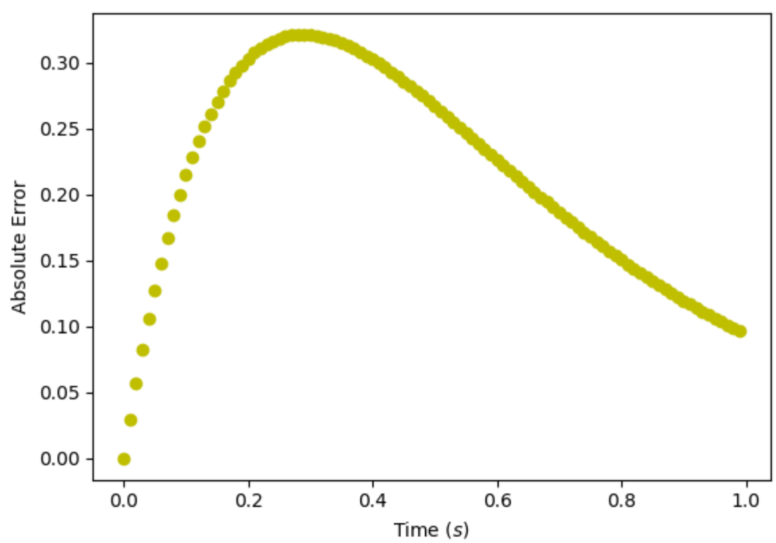

Figure 5: Absolute error between the analytical and numerical solution for the simplified temperature ODE.