Delft University of Technology

LITERATURE RESEARCH & PROJECT PROPOSAL
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Bi-threshold Gates for Mechanical Logic in Intelligent Metamaterials

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Abstract

Contents

0.1 Elementary Cellular Automata Formalism

1. State Space: $S = \{0, 1\}$

2. Neighborhood Configuration: N
   $N = (N_{-1}, N_0, N_1) \quad \text{where} \quad N_{-1}, N_0, N_1 \in S$

3. Rule Function: $f : S^3 \to S$

4. Rule Set: R

5. Cube Domain: $D \subset \mathbb{R}^3$
   Each vertex directly corresponds to a neighborhood configuration N, and its state is

6. Separating Planes: P
   Defined by a single normal vector $\mathbf{n}$ and different offsets $\{d_1, d_2, \ldots, d_n\}$.

7. Domain Classification Function: $\Delta : D \to \{0, 1, 2, 3\}$
   $\Delta(x) = \sum_{i=1}^{n} H(n_x \cdot x_x + n_y \cdot x_y + n_z \cdot x_z - d_i)$
   $H(z) = \begin{cases} 0 & \text{if } z < 0 \\ 1 & \text{if } z \geq 0 \end{cases}$

0.2 Wolfram Numbering Scheme for ECA

In the Wolfram numbering scheme for Elementary Cellular Automata (ECA), the rule set R can be uniquely identified by a single integer, which is the binary representation of the output states for all possible neighborhood configurations. For Rule 110, the binary representation is formed by considering all 8 possible 3-cell neighborhood configurations, starting from 111 down to 000.
For example, in Rule 110, the corresponding output states for these configurations are 01101110. Here's how it maps:

Neighborhood Configuration	Output State	Binary Position (b)
111	0	b_7
110	1	b_6
101	1	b_5
100	0	b_4
011	1	b_3
010	1	b_2
001	1	b_1
000	0	b_0

So, the Wolfram number for Rule 110 is obtained by reading the output states from b_7 to b_0 as a binary number: $01101110_2 = 110_{10}$.