Delft University of Technology

LITERATURE RESEARCH & PROJECT PROPOSAL

ME56010-22

Bi-threshold Gates for Mechanical Logic in Intelligent Metamaterials

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Abstract

This literature review and project proposal explores the emerging field of intelligent metamaterials, focusing on recent advancements in the design and fabrication of mechanical metamaterials with computational functionality. The review provides an overview of the state-of-the-art in this rapidly evolving field, including various design strategies and material systems. It also identifies gaps in the literature and proposes future research directions, such as the use of cellular automata as a basis for mechanical metamaterials.

The proposed project aims to develop a new class of intelligent metamaterials that can perform complex computations and information processing tasks in a parallel and distributed manner. The approach involves using networked multistable mechanisms interconnected logical responses to cyclic actuation, enabling the storage and manipulation of information. The proposed research will investigate the design, fabrication, and characterisation of these cellular automata-based metamaterials, by the use of bi-threshold elements to embody the logical functions. The project has the potential to significantly advance the field of intelligent metamaterials and open up new avenues for the development of advanced materials with unprecedented functionality.

Contents

1 Introduction

Metamaterials are architected periodic structures that exhibit unusual or unnatural material properties[1, 2], such as negative Poisson's ratio, negative refractive indices and more. The field of flexible mechanical metamaterials uses deformation and motion of mechanisms to create functional materials beyond the capabilities of a homogeneous continuous material[3].

Intelligent metamaterials have garnered renewed interest in recent times, with the blue-sky goal of developing materials that couple sensing, actuation, computation and communication[4]. When the material itself has information processing as a material property, truly distributed cognition and computation becomes possible. Such materials have numerous potential applications, such as morphing wing structures for aircraft that can change their shape mid-flight[5], robust robotic materials resilient to electromagnetic interference for aerospace, buildings with self-healing capabilities, or medical implants that can monitor and respond to changes in the body. Learning materials that can recognise patterns and adapt to novel situations[6] or materials congruent with the structure of the human brain are being pursued in the fields of neuromorphic metamaterials and morphological computing[7].

This literature review and project proposal explores the emerging field of intelligent metamaterials, focusing on recent advancements in the design and fabrication of mechanical metamaterials with computational functionality. A new approach of taking insights from the fields of cellular automata and threshold gates and a feasibility study in the embodiment of such a device as a research demonstrator is presented.

1.1 Problem Statement

In order to satisfy the description of an Intelligent Mechanical Metamaterial as we define them, such as system would need to fulfil the following requirements:

Tessellated Structure In order to be considered as a material and not a differentiated and integrated system of components, the metamaterial must tessellate and be complete in its tiled form. Boundary structures may be necessary but a material must be mostly homogeneous and consisting of a tiled unit cell structure.

State Information or Memory The metamaterial must be capable of encoding and storing information in a state element. This may be volatile or nonvolatile, but some aspect of the structure of the unit cell must function to remember the information it is processing.

Processing Capability To consider a metamaterial "intelligent" we require some sort of meaningful processing of the state information. Simple linear elastic deformation in response to stimulus from the environment would not suffice. Some meaningful, specifiable computation or logical 'decision making' must be granted by the structure of the unit cell.

Information Transmission Information that is processed by a single unit cell is very limited by the processing power of a single unit cell. Networking the cells to connect and process information collectively in parallel is necessary to transcend the power of a single unit cell. The processing capability of a single unit cell must be simple enough to scale down to make a macro scale material feasible. As such, the intelligence must be emergent from the complex connections of simple processing unit cells.

Mechanical While the overall field of metamaterials uses geometrical structure to define unusual, *mechanical* metamaterials specifically exploit motion, deformation, instabilities and elastic non-linearities to embody more exotic properties. As such, the scope of this project will be limited to this branch of metamaterials.

2 State of the Art

2.1 Search Approach & Strategy

Based on the requirements of a feasible intelligent metamaterial outlined in 1.1, a search strategy was developed. Each requirement has a area of literature that maps naturally to it. The literature search in each of these areas was seeded by widely cited review papers in each of those fields. From these seed papers, two strategies were taken to explore the literature further.

Keywords Keywords that were widely used and relevant to the goals of the project were documented and used to perform systematic searches on Google Scholar, ResearchGate and ScienceDirect.

Citation Network The online citation network visualiser CitationGecko[8] was used to explore and analyze citation networks based on the seed papers and subsequent bibliography. It works by importing a bibliography in BibTeX format and then retrieving citation data from Google Scholar. Once the citation data is retrieved, Citation Gecko constructs and visualises the citation network, where papers are represented as nodes and citations as directed edges between the nodes, as shown in

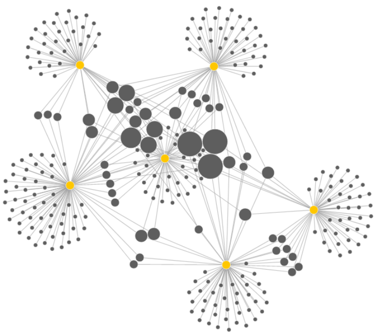


Figure 1: Citation Gecko example bibliography network. Yellow nodes are seed papers and grey nodes are citations. Larger grey nodes are cited by more seed papers.

2.2 Adaptable Mechanical Metamaterials

The tessellation requirement of course maps naturally to the field of flexible mechanical metamaterials. A recent comprehensive review of this area of the literature was conducted by Bertoldi et al[3]. A key insight from this paper that defines the fundamental reason that architecting materials leads to new properties is that *the heterogeneity of the structure breaks down the affine assumption.* In other words, designing geometric non-linearity into the structure of each component unit cell leads to non-linear elastic behaviour of the overall metamaterial. They also emphasise the potential of elastic instabilities (e.g. buckling or snapping) and large deformations for creating highly nonlinear behaviour. The outlook presented by this review concludes with several encouraging recommendations:

Complex Energy Landscapes Exploration of complex energy landscapes and frustrated materials and their rational design could allow for information storage and retrieval, and could be used to create truly useful robotic structures.

Actuation Strategies "The deep integration of actuation and the amplification of mechanical information are crucial to overcome the inevitable dissipative processes, and if combined with information processing (for example, using logic gates) would open the door to truly smart metamaterials"

Rational Design Challenges The review ends with a comment and challenge: "the rational design of metamaterials with a target property or functionality remains fiendishly difficult, and many designs so far have relied on luck and intuition." It lists many serendipitous discoveries that can be leveraged to create intelligent metamaterials, such as origami, but that rational first principles design is an unsolved problem and novel approaches are needed.

2.2.1 Rational Design

One example of such a "lucky" or intuitive framework for the rational creation of mechanical metamaterials that has recently been the focus of a section of the field is the ancient art of origami. Origami, and its less well known counterpart kirigami (which permits cutting of the sheet) are particularly interesting as they promise lamina-emergent structures[9]. Lamina emergent mechanisms are inherently easy to manufacture as they are 2D and non-subtractive. They are therefore easy to scale down in size (graphene kirigami[10]) and scale up in production volume.
Particularly the Miura-Ori pleat fold offers many avenues for exploration due to its multistable unit cells[11, 12]. However, I believe the lack of simple models of these structures and complexity of the lamina mechanics involved makes designing functional intelligent metamaterials with this approach intractable in the near future. The closest attempt has been in the recent work of Liu et al. with the exploitation of stacked Miura-ori sheets non-linear transition between stable states for the realisation of mechanical logic[13]. By designating a unique state to a permutation of states of three elements within each unit cell, and designing the transition map between those states by changing design parameters. This resulted in the ability to create simple logic gates, though through a fairly convoluted encoding-decoding procedure, as shown in Figure 2. Origami, while overall promising, does not seem to be a viable or pragmatic approach given the scope of this project.

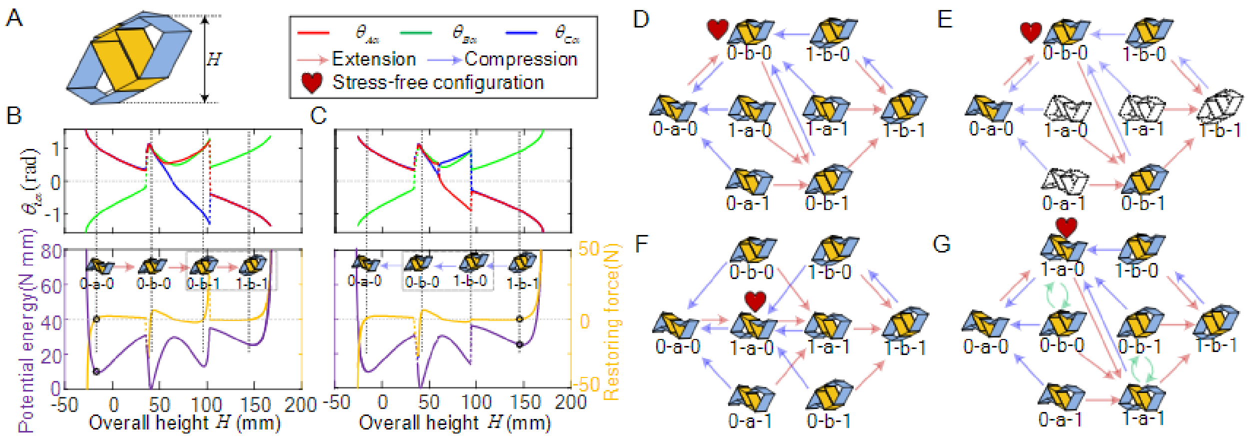
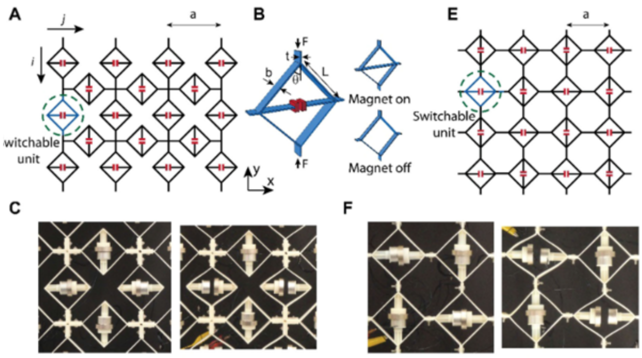


Figure 2: Architecture of logic gates (A) with and (B) without transition procedures. (C) Comparison to electronic logic circuits and (D) the completeness of the approach to embodying all of the 2^8 3-input Boolean functions.

2.2.2 Actuation Strategies

Different authors have taken different approaches to the external actuation of adaptable metamaterials. For the sake of cost and complexity, it is often desirable to have fewer and simpler actuators if possible. The actuators may also serve different roles in the functional control of the metamaterial. One approach was to embed piezoelectric actuators at the connection points of the unit cells[14]. In this work the metamaterial exhibits controllable shape morphing capabilities my the mechanical amplification of piezo-actuators. This allowed active tuning of the shape and properties of the material. One potential downside of this approach is the need for many individual actuators, and a bus and centralised controller, power supply and other auxiliary electronics.
Similarly, active locking and unlocking of electromagnetic latches was used as an actuation strategy[15]. This allowed granular control over the overall stiffness and Poisson's ratio of the metamaterial. While this strategy allowed more dramatic control over the materials properties, it also required dramatically more power, as well as an inability to scale down, as the power and efficiency of electromagnetic actuators diminishes rapidly as the dimensions decrease.
Other authors used active electromagnetic actuators at the links of the metamaterials to determine the stiffness of each joint such that it would have a programmable deformation shape[16]. They combined this with offline machine learning to find the stiffness network that would produce a desired shape morphing behaviour. However, all of these approaches of active actuators show promise for reprogrammable metamaterials, they do not do anything to imbue inherent intelligence or informational processing properties to metamaterials, as they all require external electronic computers to provide the intelligence.
A more global actuation approach was taken in the work of Van Hecke et al. on the effect of lateral confinement

on "holey sheet" metamaterials. In this approach, an elastomeric sheet with an architected periodic pattern of circular holes. They show that the lateral confinement of these sheets determines their nonlinear uniaxial compression response due to unstable collapse of the holes. They show that hysteretic behaviour can be generated in this manner. Hysteresis and multistability are necessary requirements of an inherently logical metamaterial. They develop so called hysterons that had transition pathways that could modify based on a skew in the loading[17, 18]. The "holey sheet" hysteron is shown in Figure 3b. This approach is similar to the work of Liu et al. in subsubsection 2.2.1, and aims to create logic systems in elastomeric metamaterials. Likewise, this approach is promising but intractable in the scope of this project.



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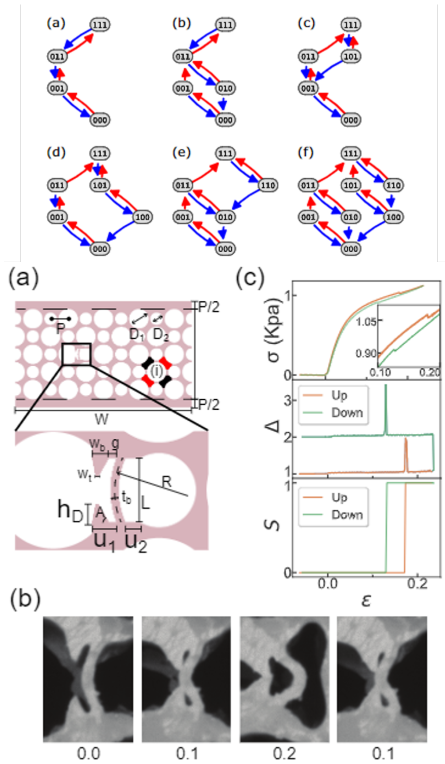
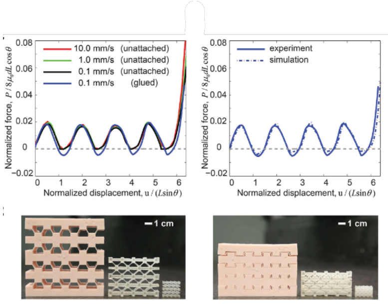


(b)

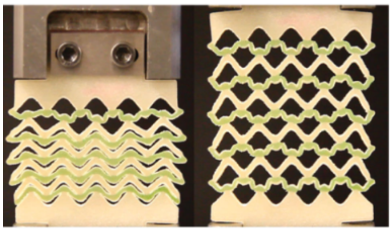
Figure 3: (a) electromagnetic locks that restructure the material by their configuratio (b) hysterons in holey elastomeric sheet metamaterial

2.2.3 Energy Landscaping

Intentional design of complex, highly nonlinear, and multistable energy landscapes could lead to useful, dense mechanical information storage and processing capabilities. Local minima in the landscape could represent states, and the directed transition flow between these states could be used to implement logical and computational processes. The majority of researchers have focused on the energy storage (or trapping) capabilities of multistable metamaterials[19, 20] without acknowledging the potential applicability of these arrays of bistable elements to intelligent metamaterials. Th They are primarily focused on energy disspation applications, observable by a sawtooth force-displacement response as in Figure 4a. However, Restrepo et al. introduce the concept of "Phase transforming cellular material" (Figure 4) and suggest that programmable structures where each phase of the material had a distinct property could be used for wave guiding, filtering and shape morphing[21]. There has also been work done to vary the geometric parameters of the bistable elements in the material to ensure the deformation sequence of the layers of the material is deterministic. Nevertheless, this approach taken by many in the field is, in my view, insufficiently developed to be fruitful within the scope of the project.



(a) Multistable force-displacement plot. Resting and compressed configurations



(b) Tension based multistability where the compressed configuration is the resting state.

(c) Multi phase structure

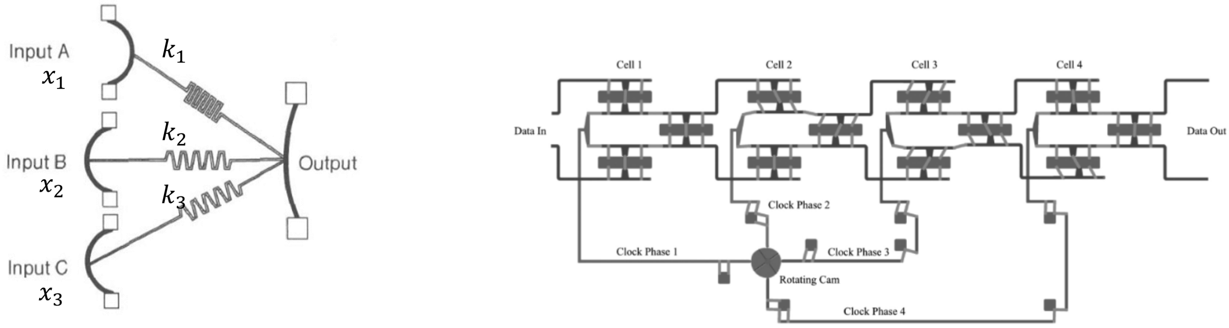
Figure 4

2.3 Mechanical Computing

The requirement of information processing for an intelligent metamaterial requires the exploration of another area of literature: mechanical computing. A 2021 perspective on the field[22] was the seed paper for the literature search. Mechanical computers are an ancient technology, from the Antikythera mechanism that made astronomical predictions in ancient Greece, to the designs of Charles Babbage's first automatic computer. According to the authors there has been a resurgence and renewed interest in mechanical computation since the domination of silicon semiconductor microelectronics and the Von Neumann architecture. They owe this emergence to advances in additive manufacturing, materials sciences, and the approaching limits of the current paradigm. They say that this problem poses challenges that require new theoretical and practical tools in various fields, including materials science, information theory, computer science, additive manufacturing, and robotics. They suggest that solutions to these challenges are likely to be found at the intersections of these fields.

Roukes[23] suggests that opportunity lies at the nanoscale, and that nanoelectromechanical systems (NEMS) may form a new paradigm once the limits of electronic devices is reached. He proposes that any realistic form of mechanical computing must use ultra-low dissipative connections between elements, i.e. compliant joints. Any losses to friction would be unacceptable at the nanoscale. To overcome the fan-out problem, the interconnection should ideally have the possibility of "gain" in the sense that external energy reservoirs, such as the elastic energy of springs, can be used to continually recharge the system by external sources. However, he does not provide concrete recommendations for implementing these systems, just that these are the requirements of a realistic, feasible system.

The inventor of public key encryption, Merkle, proposes two systems by which mechanical computing can not only be low power, but actually reversible[24]. A fundamental limitation of electronic computing is that logic gates creates entropy when information is destroyed by an irreversible logic operation. He proposes two systems: rod logic and buckled beam logic as shown in Figure 13a, for methods of computation that do not require inherent dissipation. These two proposed schemes show up in almost every subsequent paper on mechanical computation, regardless if their goal is to be reversible. Merkle himself published a study developing mechanical computing using only mechanical links and rotary joints[25]. In it they develop a theoretically Turing-complete computer using combinatorial logic, an example of the device is shown in Figure 13b. According to the authors, it is to date the simplest Turing complete mechanical computer. However, its architecture of combinatorial logic and sequential programming are incompatible with the requirements set out to embody an intelligent metamaterial, though its principles and base mechanisms could be used as constitutive elements. The following section goes into detail on current state of the art of development of mechanical Boolean elements



(a) (b)

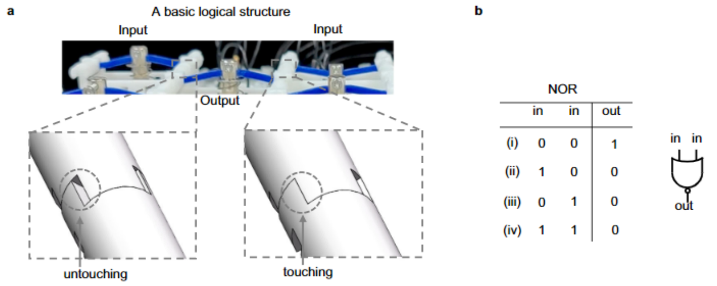
Figure 5: (a) buckled beam majority voter concept (b) bit-shift register embodied with links and rotary joints

2.3.1 Boolean Logic

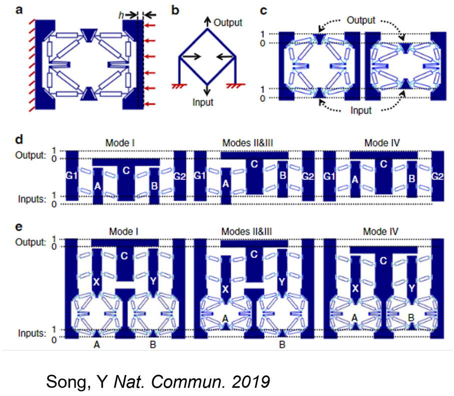
Many researchers have explored the implementation of Boolean logic in mechanical metamaterials. Boolean logic is a fundamental approach to computation, describing the operations of logical AND, OR, and NOT on binary values of 0 and 1. One concrete attempt to construct a mechanical metamaterial with the rod logic from Merkle[24] used 3-D printed bistable "bit cells" to propagate information by mechanical wave propagation. Rod logic elements implemented a simple AND gate that locked or unlocked a compliant door lock as a demonstration. Their approach focused on the mechanical signal transmission, and did not go very far into the logical function embodiment. Additionally, their approach also required resetting every time the signal fired, as energy is dissipated each time. And while their resultant device is cellular, it is specifically designed for a single purpose, so would more accurately be classified as a machine than a material, a classification that the authors themselves prefer, despite the title of the paper. Other works have done similar studies into signal propagation and simple linear logic functions[26, 27], addressing many issues with stable propagation and managing the energy landscape. However, the logical functionality seems like an afterthought in most papers and the low density of processing capability with this approach makes it unfeasible for intellegent metamaterials.
Mei et al. focused on creating a reprogrammable mechanical metamaterial (ReMM) capable of universal combinatorial and sequential logic, which has a multifunctional, programmable structure that can realise both NOR and NAND gates (necessary for more complex logical architectures. The ReMM can be reset and reprogrammed, and can also show signal transmission and bifurcation, and storage of information. The paper also demonstrates more complex logical functionality, such as a half-adder, crossover and S-R latch. According to the authors:

> the ReMM is expected to serve as a platform for constructing reusable, multi-functional, and reprogrammable robotic material with robust sensing-analyzing-response function, which can benefit the development of mechanical systems with embedded intelligence.

The authors accomplish this with simple bistable beams, and contact based logic. It is a very recent paper and further research is needed to show the scalability of this approach. As will be further discussed later, this architecture can be viewed as a type of cellular automata, where the state of each buckled beam is a function of the states of its predecessors, subject to sequential excitation, or "clocking". This idea is explored more fully in section 3.



(a)



(b)

Figure 6: (a)NOR gate implemented with bistable beams with revolute joint and contact based system. (b) Planar compliant mechanical logic gates

Several other authors have designed similarly complex logical functionality with different approaches. Song et al. designed a planar system that enables micro-mechanical logic gates to be manufactures with micro-additive technologies[28]. NOT, AND, OR, NOR and NAND gates were made using a system of planar buckled multi-stable mechanisms as shown in Figure 6b. The proposed benefits of this approach is that they can be integrated scaled down into micro-architected metamaterials. The compliant nature of the elements eliminates friction and allows for scalability. The paper only goes as far as constructing the gates and does not demonstrate integration into an interconnected system. I believe this approach has merit and should be expanded into a periodic system that can exhibit complex computation. Another work that takes another approach altogether is the doctoral thesis of M. Waheed[29], which gives a very comprehensive overview of the field of functional metamaterials. Logical building blocks are here build using 3-dimensional building blocks, as an effort to remove the limits of planar mechanisms, and create true 3-D materials. He also uses the rod logic system of Merkle, and creates some basic building blocks for logical systems using biased Von Mises trusses, basic bi-stable elements. However, this work leans on multi-material additive manufacturing technologies and does not seem scalable for microscale applications.

This review of mechanical computation strategies shows that multi-stability is one key aspect to generating both the nonlinear response necessary for embodying most logical functions, but also the storage and memory of state information. The next section reviews the state of the art and literature on compliant multi-stable mechanisms.

2.4 Multi-stable Compliant Mechanisms

This section aims to review the current literature on multi-stable compliant mechanisms, discussing the various design approaches, modelling techniques, and applications of these mechanisms.

Overview A 2008 doctoral thesis by Y. Oh gives a great overview of the field and foundational mathematics and mechanics necessary to synthesise multistable compliant mechanisms[30]. Two main approaches are covered, taking advantage of the instability of buckled configurations and of the bi-stability of a clamped-pinned beam. A particularly useful insight provided by this work is the canonical forms of the energy landscape and force-displacement graph of a tristable mechanism, as shown below in 7. The feasibility study in section 4 shows how this profile was achieved with a new tristable mechanism design.

Contact-based approaches Another approach to bi-stability is by use of contact-based approaches[31]. This design used a latch-lock mechanism to store the state information. Many MEMS devices use this approach but it is not practical for large periodic metamaterials. Contact based mechanisms exhibit fundamental energy loss and wear, as they rely on friction and sliding contact. Other works use magnetic systems instead of elastic materials to create multistable structures[32]. This approach suffers from the same scaling laws that make the electromagnetic active metamaterials did in the previous section. Contact also remains a problem, and the construction of such a unit cell would require assembly, impractical for large scale metamaterials.

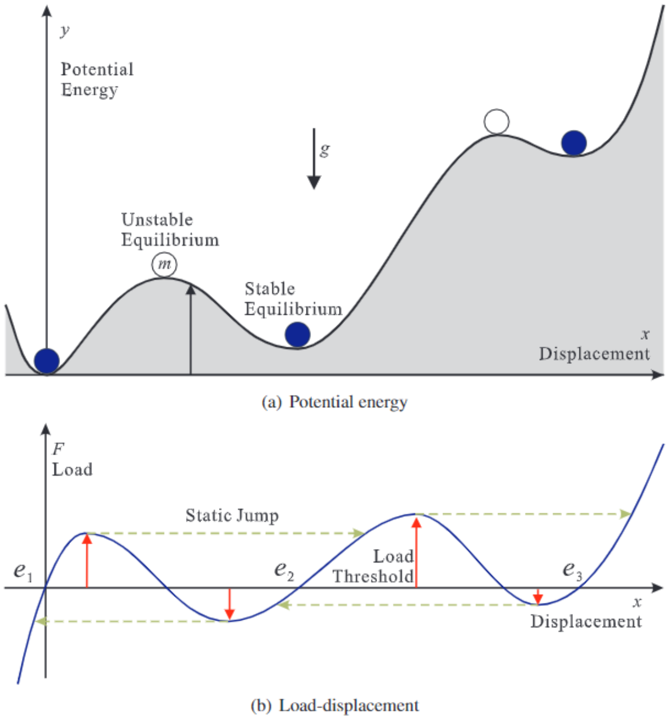


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Figure 7: Potential energy and load-displacement curves of a multistable system.

Environment Reactive While most of the literature in this area focuses on purely mechanically triggered bi-stability, one work developed a bistable unit element whose bistability was dependent on external chemical factors[33]. They used an absorbent polymer to create a buckled beam element that loses bistability once a sufficient quantity of liquid is absorbed from the environment. While the mechanics in this paper are fairly rudimentary, it does highlight the potential of metamaterials to be actuated and adapted to more than simple mechanical stimulus.

Binary Mechanical Properties Translating the displacement of a bi-stable element into tangible properties of the metamaterial has not been fully addressed in most works so far. Most papers are satisfied that the information can be "encoded" in the states of bi-stable elements, without regard for the "decoding" of utilisation of the state for realisation of novel properties. The work of Kuppens et al. develops an element wherein the stiffness is toggled between stiff and near-zero stiffness[34]. It does this by a balanced parallel arrangement of positive and negative stiffness elements. The negative stiffness element is toggled by the buckling of a beam. Recent work [35] used these elements to create a rudimentary metamaterial that learns. It learns a specific arrangement of lattice stiffnesses to attain a desired deformation under a given load. This work is still in early stages but is very promising outlook. Currently offline learning is used to determine the stiffnesses, but if onboard mechanical learning and information processing is built in, this would be a good first step towards realising functional intelligent metamaterials.

Pre-curved Beams The planar logic gates mentioned previously[28] and many other multistable elements that rely on buckling, require external actuation after manufacture. While this is unavoidable if a symmetric energy landscape is required, a relaxation of this requirement allows for the use of pre-curved beams[36] which are an inherently much more manufacturable and scalable solution. They do suffer the drawback of reduced bi-stability, and range of motion.

2.4.1 Tri-stable and multistable mechanisms

While this review has thus far focused on bistable elements, similar logical capabilities are also found in higher orders of multi-stability. A recent work by Zhang et al. that used kirigami techniques to make a highly non-linear spring created a tristable metamaterial structure capable of ternary logic operations, signal filtering and more[37]. The work, illustrated in Figure 8 is recent and shows promise, as the metamaterial is programmable and capable of relatively complex computation, with a large number of possible states. However, the manufacturing techniques used are not inherently scalable at they use multimaterial 3-D printing, and kirigami- contact based nonlinear springs. This approach adapted to overcome these shortcomings would show great promise in accomplishing an intelligent metamaterial.

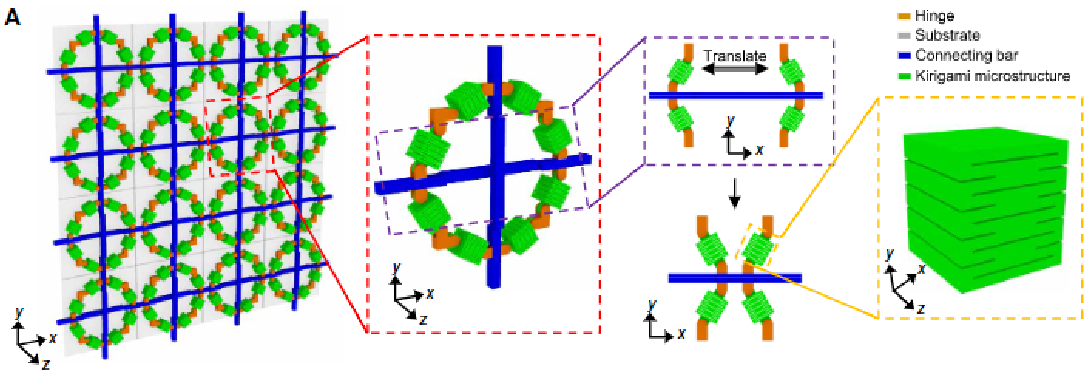


Figure 8: Schematic representaion of the kirigami based tri-stable mechanism and resultant metamaterial.

Orthogonal beam tri-stable mechanism The approach proposed by G. Chen et al. is a promising solution for achieving higher orders of multistability using a single bistable beam. By adding orthogonal bistable mechanisms in series with the original beam, a higher number of stable positions can be achieved, as shown in Figure 9. Importantly, this technique is both compact and does not require contact, making it highly desirable for many applications. Chen et al. demonstrated the feasibility of this approach for manufacturing at several length scales, suggesting that it has broad potential for use in a variety of settings. Notably, this approach also allows for the creation of multistable mechanisms with a desired number of stable positions, depending on the design requirements.

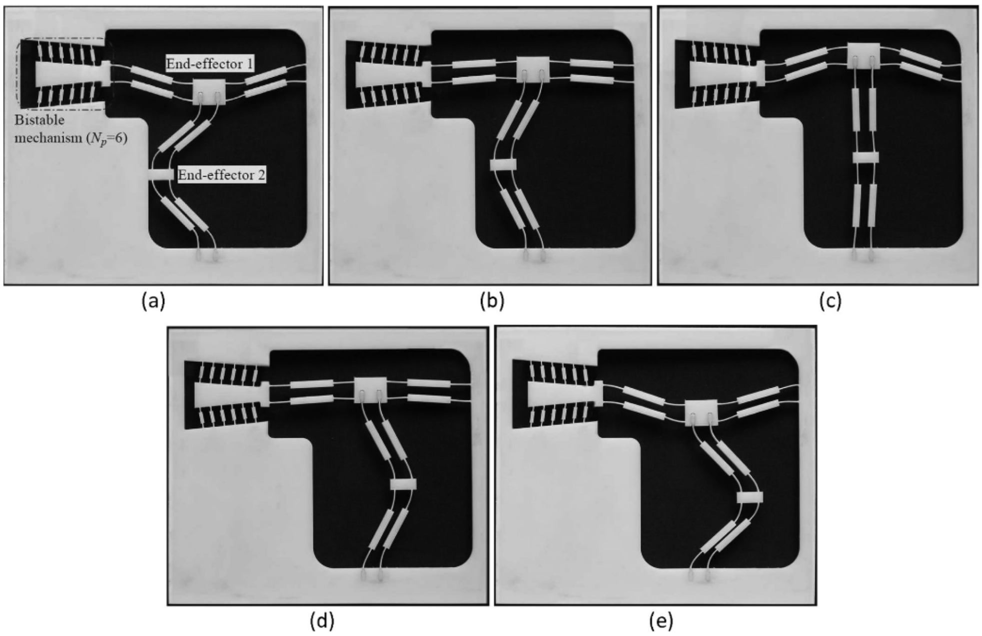


Figure 9: Multistability achieved by single bistable beam chained with successive orthogonal mechanisms.

Parallel Interconnected Bistable Mechanisms One of the most promising architectures for intelligent metamaterials takes the approach of simply arraying bistable beams in parallel[38]. This recent work by Kwakernaak et al. accomplishes counting and sequential information processing by architecting the structure and interconnections to create an irreversible response to cyclic actuation, as shown in Figure 10. They store information in the orientation of the buckled beams and create logical sequences by having beams transition states depending on its neghbours on each clocking cycle. They conclude the paper with the insight that this approach when expanded opens up

> "routes to create systems that are Turing-complete, such as 'rule 110'or Conway's game of life. Such 'cellular automata materials' would allow massively parallel computations *in materia*."

. This approach I believe is very promising, and addresses issues faced by many of the works presented in this review. In the following sections I will present an introduction and exploration into cellular automata as a basis for mechanical metamaterials, and the necessary elements to embody them.

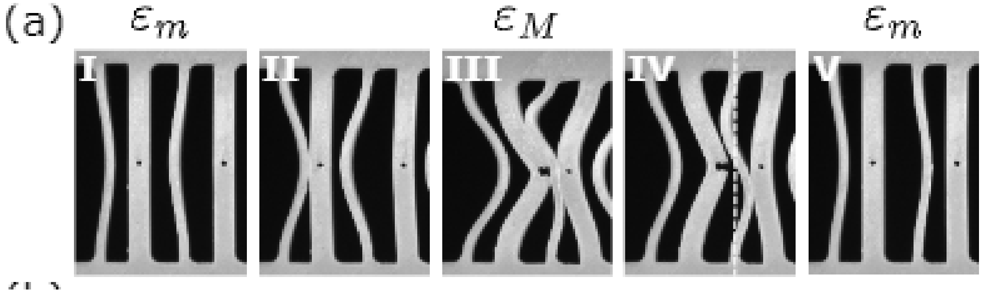


Figure 10: Evolution of the unit cell of counting metamaterial using simple cellular automata rules.

2.5 Gap of knowledge/ Research Question

Based on the literature review, a research question that arises is: How can the principles of cellular automata be used to design mechanical metamaterials with massively parallel computation capabilities?
This question aims to explore the potential of using cellular automata as a basis for mechanical metamaterials and how this approach could lead to the development of systems that are Turing-complete, such as rule 110 or Conway's Game of Life, allowing for massively parallel computations in materia. This approach could address the limitations of traditional logical and system architectures and open up new routes for the design of intelligent metamaterials and programmable matter[39]. These topics are further explored in the following sections.

3 Cellular Automata

This section introduces and explores the concept of cellular automata, for the purposes of exploring a new approach to the development of intelligent metamaterials. As mentioned in subsection 2.3,this approach has been suggested in literature before, though not explored in detail. Most of the information in this section has been adapted from the textbook Cellular Automata: A Discrete Universe[40].

3.1 Introduction to Cellular Automata

Cellular automata (CA) are mathematical models for simulating complex systems using a grid of discrete spatial cells that are updated in discrete time steps according to a set of rules that determine the state of each cell based on the states of its neighbouring cells.
The grid of cells is usually arranged in a regular lattice, such as a two-dimensional square or hexagonal grid, but can also be in just one or more dimensions. Each cell in the grid can be in one of a finite number of states, though most commonly a binary set such as "on" or "off", "alive" or "dead", or "black" or "white". The state of each cell is updated at each time step based on the states of its neighbouring cells, which can be defined using a neighborhood function that specifies which cells are considered neighbors of a given cell. The update rule for each cell is typically a deterministic function that maps the states of its neighboring cells to a new state for the cell itself. In summary, the key characteristics of a CA are as follows:

Discrete lattice of cells: The system substrate is composed of a one-, two- or three-dimensional lattice of cells.

Homogeneity: All cells within the lattice are equivalent.

Discrete states: Each cell is capable of taking on one of a finite number of possible discrete states.

Local interactions: Each cell is only able to interact with cells that are located within its local neighborhood.

Discrete dynamics: At each discrete unit time, each cell updates its current state according to a transition rule that takes into account the states of cells in its neighborhood.

It is clear that these characteristics map very clearly onto the requirements laid out in 1.1 and explored in Section 2.
Below, two examples of different types of CAs are detailed as examples. They are then explored as potential models for novel implementations of intelligent metamaterials.

3.2 Conway's Game of Life

John Conway's Game of Life is the most famous example of a cellular automaton. it was published in 1970 as an attempt to simplify Von Neumann's CA. It consists of a two-dimensional grid of cells, where each cell can be in one of two states, either alive or dead.

3.2.1 Rules of Life

The game begins with an initial configuration of living cells on the grid. At each time step of the game, the state of each cell is updated based on the states of its eight neighbouring cells. The update rules are based on a simple set of three conditions:

- Any live cell with fewer than two live neighbours dies, as if by underpopulation.
- Any live cell with two or three live neighbours lives on to the next generation.
- Any live cell with more than three live neighbours dies, as if by overpopulation.
- Any dead cell with exactly three live neighbours becomes a live cell, as if by reproduction.

The Game of Life, or just Life, is an example of an emergent complex and self-organising system. From an initial state of random noise, many patterns and structures emerge over time. Unchanging static patterns (also called "still lifes"), and periodically repeating structures are the simplest terminating configurations of game. However, even given the simplest of rules it is impossible to determine whether any given initial configuration will grow indefinitely or eventually terminate without directly simulating it.

3.3 Elementary Cellular Automata

3.3.1 Introduction to Elementary Cellular Automata

Elementary Cellular Automata (ECA) are a class of one-dimensional cellular automata where each cell can have only two possible states, usually labelled 0 and 1, and the updating state of each cell depends only on its own state and the state of its immediate neighbours. ECAs have been extensively studied since the 1980s, especially by Stephen Wolfram. They have proven to be a rich source of complex and interesting behavior, despite their simple rules. In his book "A New Kind of Science,"[41] Wolfram argues that ECAs are a fundamental model of computation and that they can be used to explain many natural phenomena.

3.3.2 Overview of 1D Cellular Automata

The cells are arranged in a line or lattice, and each cell has a finite number of possible states. At each time step, the state of each cell is updated based on a fixed rule, which is defined as a 3-input Boolean function. The rules are often given in the form of a lookup table or transition function, which specifies the next state of a cell based on its current state and the states of its neighbours. 1D CA have been used to model a wide variety of physical, biological, and social phenomena, and they have been studied extensively in the fields of mathematics, physics, and computer science. They are particularly useful for modelling systems that exhibit self-organisation, pattern formation, and emergent behaviour.

3.3.3 Explanation of the Wolfram Code

The Wolfram code is a method of encoding the rules for an ECA using a binary number. Each of the 256 possible rules for an ECA is assigned a unique code, which corresponds to a binary number between 0 and 255. The code is obtained by interpreting the eight possible configurations of a cell and its two neighbours as binary digits, and then converting the resulting binary number to decimal. The digits are ordered from left to right, with the first digit representing the neighbourhood where all three cells are alive, and the last digit representing the neighbourhood where all three cells are dead. The resulting binary number corresponds to one of 256 possible ECA rules, each with its own distinct behaviour and pattern formation. The Wolfram code has the advantage of being simple and compact, and it allows for easy comparison and analysis of different ECAs. It also highlights the fact that even though ECAs have simple rules, they can exhibit a wide variety of complex and interesting behaviours, as shown in Figure 11.

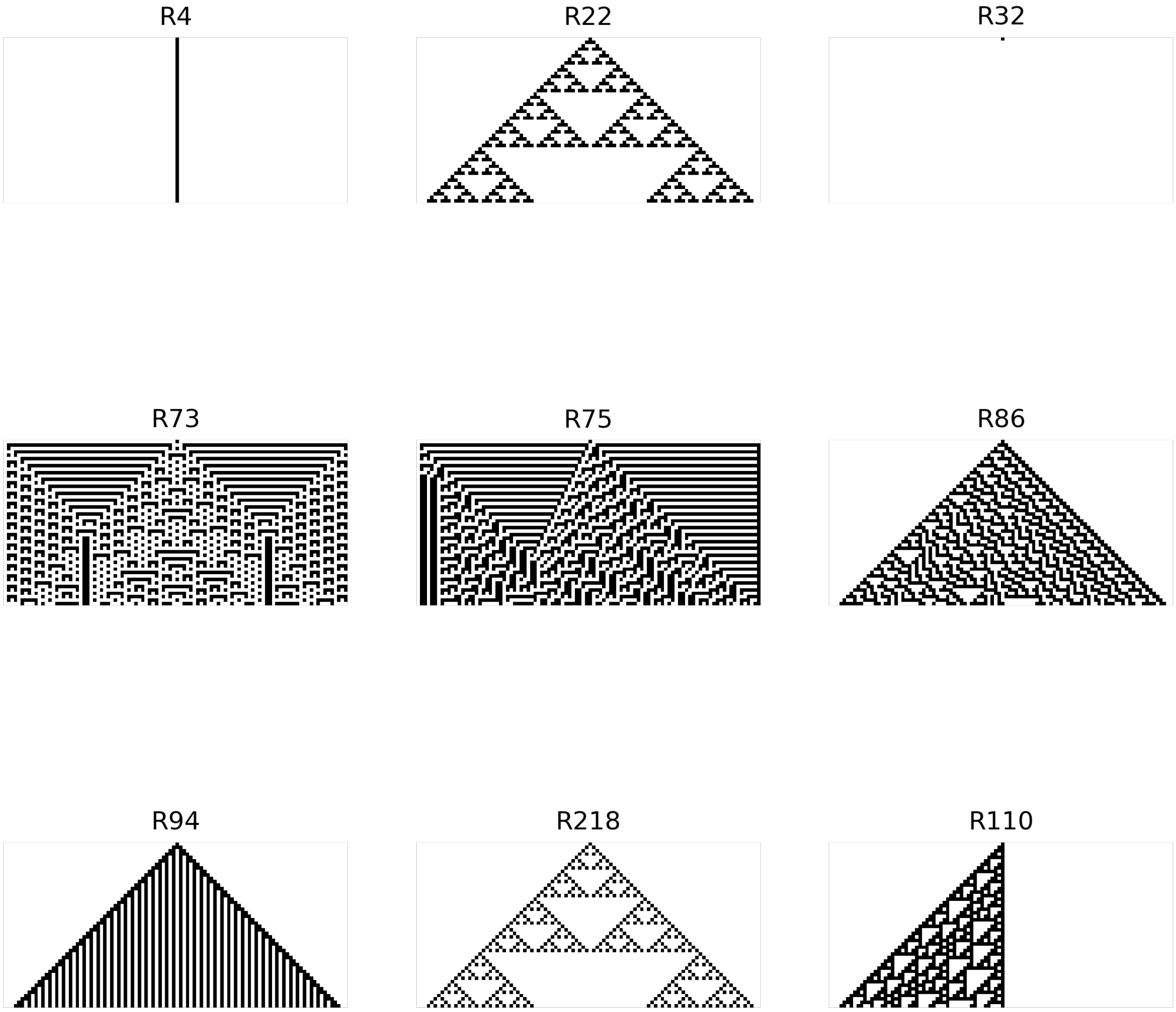


Figure 11: Various ECAs and their evolution after a single point is turned on. Complex emergent patterns and chaos can be observed from very simple rules

The properties and behaviour of ECAs depend on the specific rule that is being used. Some ECAs exhibit regular and repetitive behaviour, such as simple alternating patterns or periodic structures, while others exhibit complex and unpredictable behaviour, such as chaotic patterns or complex structures that evolve over time.

3.3.4 Mapping of ECAs to Boolean functions

The mapping of the transition function of an ECA to a 3-input Boolean function can be done by considering the state of each cell and its two neighbouring cells as the three inputs to the Boolean function. The output of the function corresponds to the state of the cell in the next generation of the ECA.

3.4 Computation in CA

Cellular automata and related architectures have been extensively studied in natural sciences, mathematics, and computer science because they have the potential to perform complex computations efficiently and robustly, as well as model the behaviour of complex systems in nature. They have been used as models of physical and biological phenomena, such as fluid flow, galaxy formation, earthquakes, and biological pattern formation. They have also been studied as mathematical objects with formal properties that can be proven, and used as parallel computing devices for the high-speed simulation of scientific models and computational tasks such as image processing. Additionally, cellular automata have been used as abstract models to study emergent cooperative or collective behaviour in complex systems[42]. During the 1980s and 1990s, there was a concerted effort to implement large scale supercomputers built on massively parallel cellular automata hardware architectures[43]. The aim was to implement "programmable matter" that could be used to simulate an arbitrary 3-D material that could be modelled as simple interactions between cells, such as a lattice gas model[39]. The result was the CAM-8 computer, in the authors words the:

> "product of over a decade of Cellular Automata (CA) machine and modelling research by the Information Mechanics group at the MIT Lab for Computer Science. CAM8 is a parallel, uniform, scalable architecture offering unprecedented performance in the fine-grained modelling of spatially-extended systems. It provides a general-purpose instrument for the systematic exploration of a new band of the computational spectrum."

Unfortunately, the work on CAM-8 ended in 2001 due to setbacks, but showed great promise and the concept of programmable matter has since been picked up by researchers in many fields, from synthetic biology to metamaterials.
Both the Game of Life and ECAs have been shown to be capable of universal computation, meaning they both can theoretically compute anything that a Turing machine can compute. Rule 110 has long been shown to be capable of universal computation, by the use of an architecture called the cyclic tag system[44, 45]. John Conway proved in 1982 that the Game of Life can implement a universal computer and a universal constructor. In the meantime there has been a a lot of work done by enthusiasts to develop more and more advanced computers within life itself. Most recently in 2021, an improved, scalable version of an 8-bit programmable computer pattern was announced[46]. However, in some ways the goal of reproducing universal serial computation from a massively parallel architecture is a flawed approach from the start. It is a strong but flawed instinct to use a novel architecture to simply implement an already well known system. Others have tried to implement a unique method of computation using elementary cellular automata directly. A particle model was used to represent bits and processing instructions, which collide and output the result as more particles. This allowed for massively parallel arithmetic to be implemented directly in one-dimensional cellular automata.[47]. In this case, the single dimension was a constraint on the system that could be circumvented by higher dimensionality for greater functionality.Nevertheless, this work demonstrated the potential of very simple systems with clever use of their emergent properties.
More recently the focus has moved past the direct implementation of CAs to neuromorphic computing morphologies and cellular neural networks (CNNs). CNNs are similar to CAs, with the same spatial structures and connectivity, but operate as first-order dynamical systems, instead of discrete time "clocked" systems. One work demonstrates the capacity of these systems to implement arbitrary Boolean functions by the tuning of the weights of the transition function and connectivities[48].More modern trends in this field include memristor-based neuromorphic architectures for Cellular Neural Networks[49].However, this topic branches away from metamaterial implementations and is thus beyond the scope of this project, but it does demonstrate the future potential of this architecture in materials.

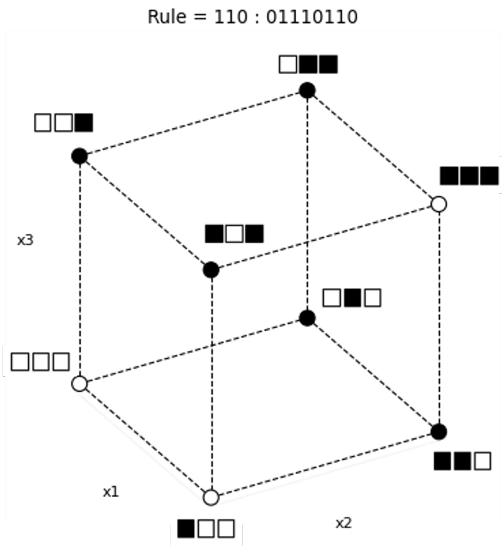
3.5 ECA as Boolean functions

This section has elucidated the potential of the cellular automata architechture for useful computational systems that also fulfil the requirements set out in Section 1.1. The question remains; what type of logical element could be implemented mechanically in order to embody a cellular automata into a metamaterial? To answer this question it is useful to make the observation that the transition rules for ECAs are simply 3-input, or "ternary" Boolean functions, one input for the state of each neighbour and the state of the cell itself. As we have shown in Section 2.4, tristable mechanisms have previously been used to implement ternary logic[37, 13] using origami and kirigami principles.

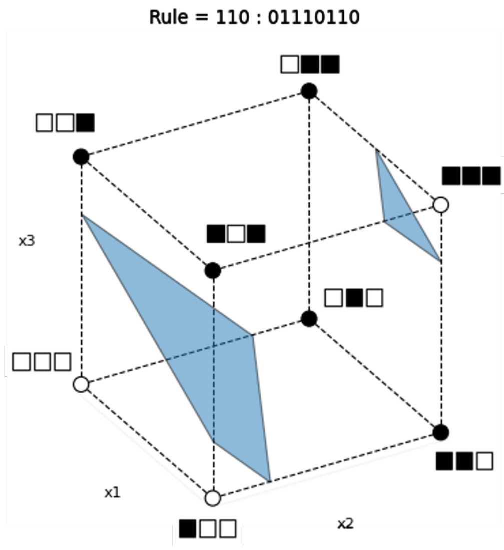
Figure 12: Caption

3.5.1 Ternary Boolean functions as unit cubes

While ternary Boolean functions can be represented as truth table, a more visual and useful representation is as a mapping from the unit cube in 3-dimensional space (where each axis represents an input variable) to the set 0, 1, which corresponds to the output of the function. The unit cube has eight corners, each corresponding to a unique combination of inputs, and each of these corners can be assigned a value of 0 or 1, depending on the output of the ternary Boolean function for that particular combination of inputs.
By assigning a colour (e.g. black for 1 and white for 0) to each corner of the cube, a visual representation of the Boolean function can be obtained. Each point within the cube corresponds to a unique combination of input variables, and the value of the Boolean function for that combination can be determined by looking at the colour of the corresponding corner.



(a)



(b)

Figure 13: (a) hypercube representation of Rule 110 ternary function (b)corresponding bi-threshold representation of the function.

3.5.2 Linear Separation

The benefit of this abstraction is that we can see visually that certain Boolean functions have the property known as *linear separability*. This means a single hyperplane defined by an orientation and offset can be used to separate the regions where the output is true or false. This plane actually represents a so called "threshold function", or "majority decision function", where the orientation of the plane represents the "weights" of the input variables, and the offset is a threshold on the weighted sum of the inputs beyond which the output is true. In fact, 104 of the 256 ternary Boolean functions are linearly separable by so called "threshold functions"[50]. It is possible to capture 230 out of the 256 functions if we allow *two* thresholds, corresponding to a second parallel plane. This fact is illustrated later in Figure 17. The next section will go into more detail on the theory of these majority decision/threshold functions, their applications and prospects for a valid approach to building logic and intelligence into mechanical metamaterials.

3.5.3 Game of Life Rules as Bi-threshold

Additionally, the rules of Game of Life can also be seen as a weighted sum with two thresholds. If all of the states of the neighbourhood of a central cell are weighted with a value of 1, and the central cell's state is weighted 0.5, then any sum between 2.5 and 3.5 will result in a cell being alive the next generation. This representation is shown in Figure 14.

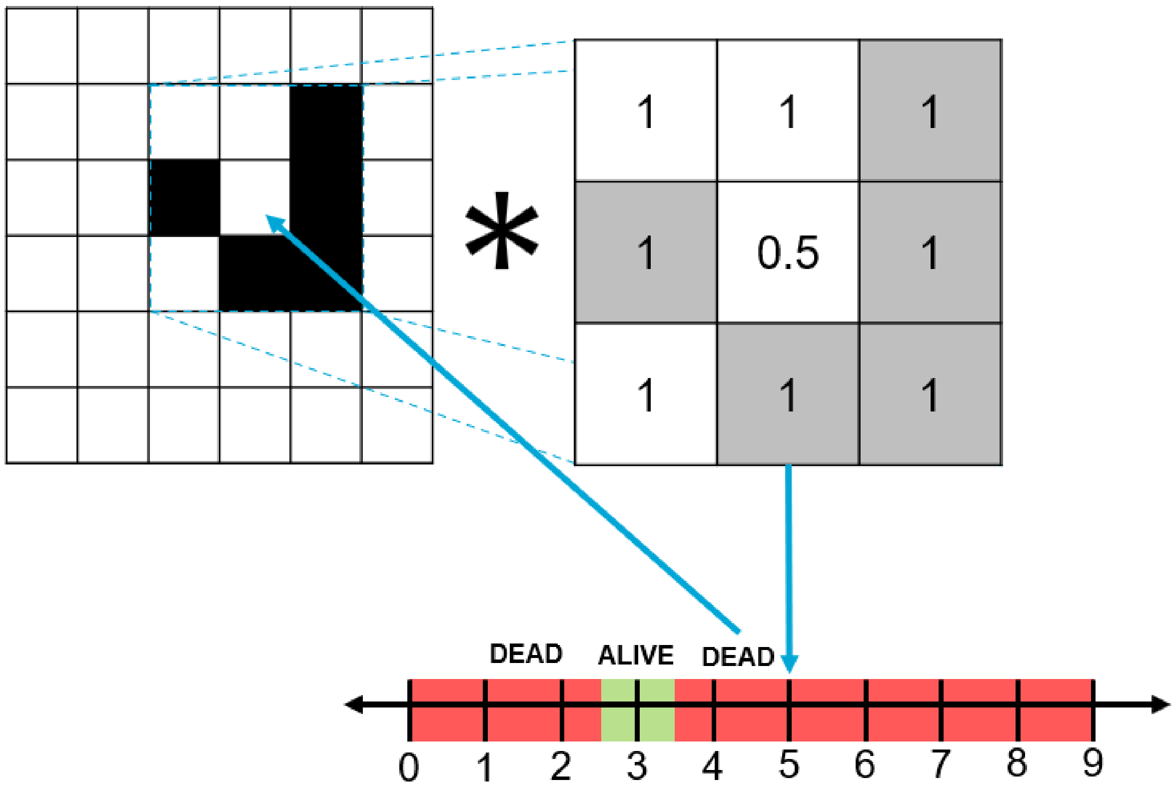


Figure 14: The game of Life can be represented as a bi-threshold system as shown

3.6 Threshold gates

3.6.1 Introduction

Threshold gates or elements are devices that can output either 0 or 1 depending on whether their weighted sum of inputs exceeds a certain threshold value. They are an alternative approach to conventional logic gates such as AND, OR and NOT gates. They are used in artificial neural networks, logic design and signal processing, though they remain a niche research area despite their utility. The theory of threshold elements was pioneered by Saburo Muroga, who published several papers on their properties, transformations and applications[51]. In this section, I will introduce the theory and discuss how it relates to the goal of developing intelligent metamaterials. I introduce the generalisation of threshold gates with multiple thresholds. As introduced in the previous section on One of the key components of this project is a bi-threshold gate that has two threshold values instead of one. A bi-threshold gate can cross two thresholds and has three regions in its input-output characteristic curve.

3.6.2 Threshold Logic

Threshold logic is a branch of logic that studies how to represent and manipulate Boolean functions using threshold elements. A Boolean function is any function that takes a finite number n of binary inputs (0 or 1) and produces a binary output. A threshold element or majority decision element is defined as an element whose output value is

$$\begin{aligned} &1, \quad \text{if } \sum_{i=1}^{n} w_i x_i \geq T \\ &0, \quad \text{if } \sum_{i=1}^{n} w_i x_i < T \end{aligned}$$

where w_i is the weighting factor for the input x_i and T is the threshold value for that element. For our purposes, we are most interested in ternary (3-input) Boolean functions, in order to implement ECAs in metamaterials, particularly the ones with the most interesting and powerful properties such as Rule 110.
It has been shown that only 104 out of the 256 possible ternary Boolean functions can be represented by a threshold gate[50], the linearly separable functions. The linearity of these functions limits their potential for computation.

3.6.3 Multi- and Bi-threshold gates

Multi-threshold threshold elements are a generalisation of the threshold element, in which k thresholds $k = (1, 2, 3, ...)$ are used instead of one. Each threshold crossed alternates the output between 0 and 1. The benefit is that any arbitrary Boolean function can be realised with a single threshold element with a sufficiently large k[52]. Of course, a higher k implies a higher complexity of the element itself, so it is desirable to minimise the required number of thresholds for a given application. The special case where $k = 2$ are known as bi-threshold elements and have particular relevance to this project. As shown previously, almost all (230/256) ECAs can be implemented with bi-threshold logic, as can Conway's Game of Life. The emergent computational properties of these systems implies in my view that a system of cellularly networked bi-threshold elements could have sufficient complexity to imbue a metamaterial with information processing capabilities. As shown previously, these systems behave vastly differently depending on their initial states[47], so could also be "programmed" to an arbitrary functionality and also respond to external stimulus after manufacture. Multi-threshold systems have also been shown to have significant space improvements over traditional logic gates. The area of a given Boolean function of n bits has an exponential (unbounded) size $O(exp^n)$ when made with AND, OR and NOT gates. It is shown that bi-threshold elements reduce this requirement to $O(n^2)$ and multi-threshold elements reduce it further to $O(n)$[53, 54].

3.6.4 Application and implementation

Physical implementations of these elements, called Multi-Threshold Threshold Gates, has been accomplished through the negative differential resistance devices, which operate in a monostable to bistable regime. They essentially act as an electronic bi-stable element, allowing for the implementation of these thresholds using voltages and resistances[55, 56].
Theoretical uses for bi-threshold elements specifically are currently being explored, though there are challenges to their general utility. Bi-threshold activation functions for neural networks has been shown to decrease the

number of layers, though the difficulty in optimising the element has been a barrier. This is because the optimisation of a bi-threshold activation has been shown to be an NP-complete problem, meing no efficient algorithm has been found to compute it. Despite this, the utility has still motivated researchers to find applications and other approaches to apply them to solve problems[57, 58, 59].

Despite this difficulty in applying the theory at large scale in computer science, mechanisms researchers have serendipitously designed a schema for making multi-threshold gates in compliant mechanisms[60]. This approach uses a single bistable unit as the output, and sequentially adds more orthogonal buckled beams to add more stable points.

4 Research Plan

4.1 Research Questions

This thesis project aims to propose a novel approach for intelligent metamaterials, departing from the current design paradigm of derived from electronic computing. A compliant bi-threshold element will be developed as a building block for an elementary cellular automata, to demonstrate the feasibility of this approach. The goal is to embody Rule 110 ECA and determine a general design schema to enable the embodiment of any ECA.

4.2 System Concept

This project proposes a mechanism concept based on Merkle buckled beam logic[24], which is extended with multi-stable compliant mechanisms[60] to embody bi-threshold logic. The system's constitutive elements are designed to include the decision element, which has a nonlinear response that encodes the threshold function, the storage or encoding of the output of the decision element, the interconnection and self-connection of the elements to tessellate the structure into an elementary cellular automaton (ECA) structure, the parametric design of the threshold element weights, such as the coupling springs between units, and the update clocking mechanism, which supplies energy to update the metamaterial state based on the transition rule.

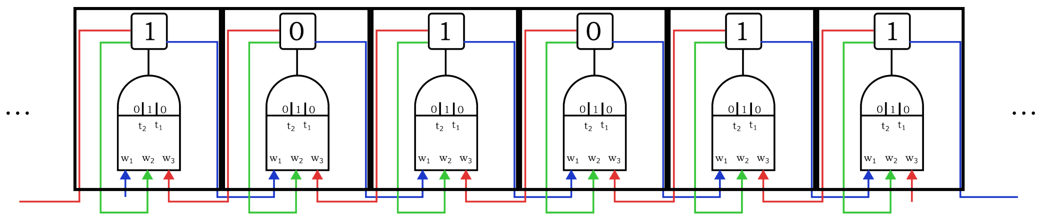
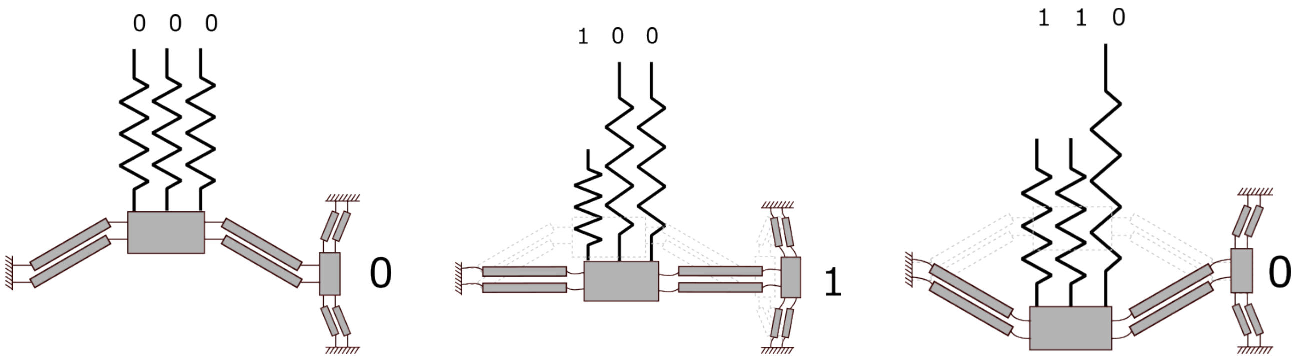


Figure 15: The proposed architecture of the metamaterial, where each unit cell has a bi-threshold decision element, a memory element and interconnections to its neighbours



(a) (b) (c)

Figure 16: The proposed decision element (a) in the ground state (b) when the first threshold has been crossed and (c) in the saturated state.

4.3 Equivalence classes of 3 input Boolean functions

The primary aim of this project is to implement Rule 110 cellular automata, with the overarching goal of generalizing this approach to embody any of the 230 realizable ECAs. To achieve this, we aim to show that a subset of the ternary Boolean functions can be implemented, which all other functions are equivalent to. We define equivalent functions as those that can be obtained by permuting input variables, complementing one or more input variables, or complementing the output. Complementation refers to negation, where 1 becomes 0 and vice versa. It is shown in Figure 17 that all but two of these equivalence classes can be represented by single or bi-threshold gates. The

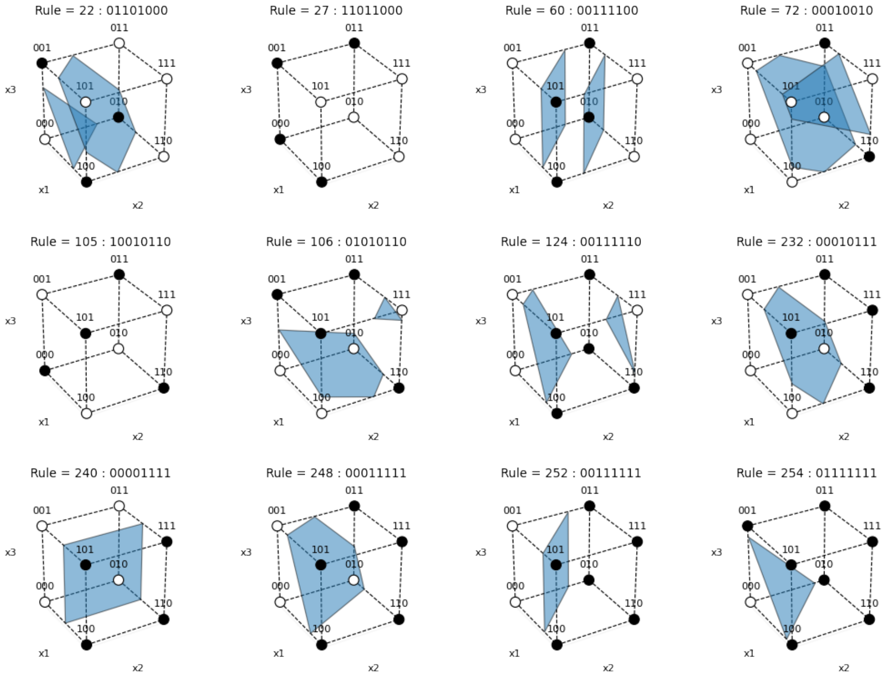


Figure 17: The 12 base functions of the equivalent classes, with the thresholds represented. Only the 26 functions associated rules 27 and 105 require three thresholds to represent.

4.4 Pseudo-Rigid body model of tristable element

As part of a feasibility study, a psuedo rigid body model was derived of the concept threshold gate in order to understand its mechanics. As can be seen in Figure 18, the PRBM consists of a crank slider mechanism with an additional compressive spring element to model the bistable beam element. The models geometry is parameterised by four lengths, H_1. L_1,H_2, L_2. The compliance of the mechanism is parameterised by three stiffnesses, two torsional k_θ, and k_ϕ, and one linear stiffness k_L. The mechanism takes the force F as input and the resultant state is the output displacements δ_1 and δ_2. The potential energy of the system is calculated by:

$$PE = k_\theta(\theta_0 - \theta)^2 + \frac{1}{2}k_\phi(\phi_0 - \phi)^2 + \frac{1}{2}k_L(L_0 - L)^2 \tag{1}$$

The input force F can be found by

$$F = \frac{dPE}{d\delta_1} \tag{2}$$

The nonlinear kinematics of the mechanism have been derived as follows:

$$\delta_2 = 2\left(\sqrt{L_1^2 + H_1^2 - (H_1 - \delta_1)^2} - L_1\right) \tag{3}$$

$$\theta = \tan^{-1}\left(\frac{H_1 - \delta_1}{L_1}\right) \tag{4}$$

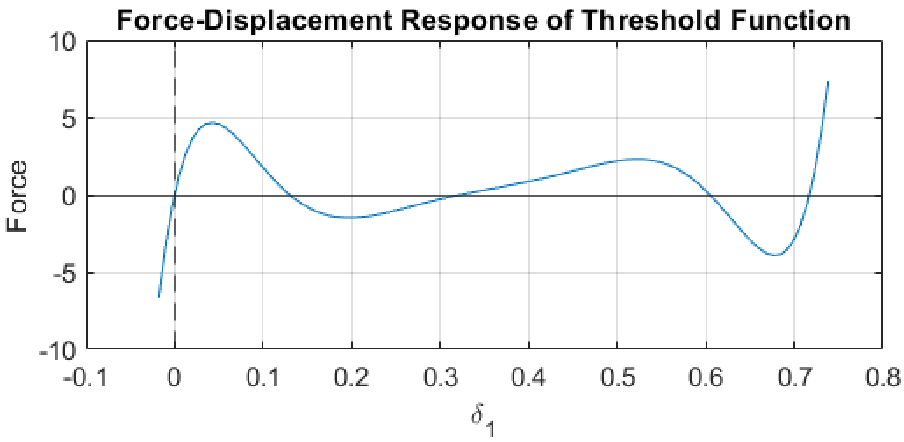
$$\phi = \tan^{-1}\left(\frac{H_2 - \delta_2}{L_2}\right) \tag{5}$$

$$L = \sqrt{L_2^2 + (H_2 - \delta_2)^2} \tag{6}$$

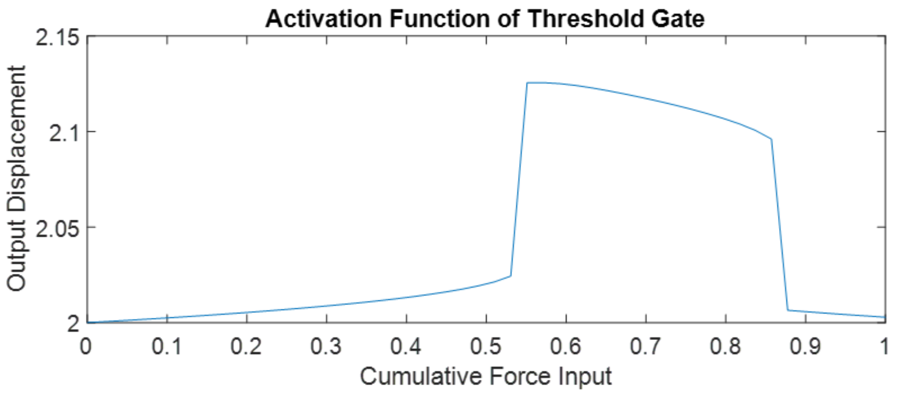
(a) (b)

Figure 18: The proposed decision element (a) psuedorigid body model parameterisation (b) Input displacement and stiffness parameterisation as constructed in MATLAB

The PRBM gives a useful analytical model to show the principle of operation of the mechanism. Preliminary results are shown in Figure 19. The tunable tri-stability of the mechanism agree with the results of Chen[60], partially validating the PRBM approach.



(a)



(b)

Figure 19: (a)Example force displacement plot generated by the PRBM. The three stable equilibria show disred multistability (b) Resultant output displacement over cumulative force input shows desired bi-threshold behaviour.

4.5 Research Plan

Based on the feasibility studies conducted so far, the plan for the next steps of the project is as follows:

1. Analysing the pseudo rigid body model to understand its properties and behaviour, and converting those insights to a detailed mechanical design.
2. Performing design and simulation steps to parameterize the properties of the unit cell in terms of its design parameters and the logical function we wish to embody.
3. Designing and developing the interconnection and tessellation systems needed to build a functional elementary cellular automaton for Rule 110.
4. Building and demonstrating the capabilities of the tessellated unit cell with Rule 110 ECA.
5. Quantifying approach performance and analysing the results.

4.6 Evaluation Criteria

The success of this project will be evaluated on several quantifiable metrics.

Threshold Design Freedom The degree of control we have over threshold values within the design space depends on the physical parameters of the design, which limits the values we can select for the thresholds. This limits the types of functions that can be created using this approach.

Interconnection Design Freedom The design freedom we have over the input weighting and interconnection system determines the complexity of the system that can be modelled. For example, in Figure 17, only weights of 1 and 2 were needed, but to account for negation, a NOT gate on some inputs will be required, and some other ratios of input weighting may be necessary. This is a design problem that must be solved to generalise this approach. The number of realisable systems (x/256) is a metric for the potential of this system.

Stability The robustness of the integrated system should be quantified by measuring the error rate during operation. The design should minimise errors, and ideally, the system should have a margin for manufacturing errors to ensure robustness.

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