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THE AUTOMATIC SOLUTION OF POWER-SYSTEM SWING-CURVE EQUATIONS

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SUMMARY

The paper details the problem of transient stability in electrical power systems and divides the methods used to date into four categories. These categories, with the relevant literature, are briefly reviewed.

The paper then describes the theory, method of construction and mode of operation of an automatic step-by-step computer for power-system swing curves; the machine has facilities for two generators only but may be extended in scope by the addition of equipment similar to that already employed. The limitations of this machine are discussed and a direct-analogue method for use with the generator units of a network analyser is then described.

The procedure for calibration of the direct analogue is given, and difficulties in the setting up of the analogues of damping power and inertia constant are mentioned. Comparison of the results of a simple swing-curve problem by integraph, step-by-step and analogue appears at the end of the paper.

Automatic generator-unit operation on a network analyser is possible with this type of direct-analogue equipment.

LIST OF PRINCIPAL SYMBOLS

A = Gain of motor field amplifier, mA/volt.
G_r = Gear ratio between velodyne and phase-shifter.
I = Output current of network analyser generator unit.
K_T = Tachometer constant, volts/deg/sec.
M_n = Inertia constant for the nth machine.
M = Inertia constant for the special case of a machine operating against an infinite busbar.
$(N-1)$, N, $(N+1)$, etc. = Subscripts denoted the steps of a calculation.
p = Feedback control setting, operating on feedback voltage v_p.
P_a = Net rotor accelerating power for a machine connected to an infinite busbar.
P_d = Damping coefficients for the special case of a machine operating against an infinite busbar.
P_{dn1}, P_{dn2}, etc. = Coefficients for the damping power between machines n and 1, n and 2, etc.
P_{en} = Electrical power output coefficient of the nth machine.
P_{mn} = Mechanical power input for the nth machine.
V = Output voltage of network analyser generator unit.
V_1 = Machine internal voltage.
V_2 = Voltage at infinite busbar.
v_p = Velodyne generator voltage, v_r = Reference voltage, v_w = Wattmeter output voltage } measured at the field amplifier input terminals.
X = Total reactance up to infinite busbar.
δ = Angular displacement of V_1 with respect to V_2, and also the angular displacement of the phase-shifter shaft of the analogue corresponding to synchronous-machine shaft displacement.
δ_n = Rotor angle in electrical radians, of the nth machine, with respect to a common reference axis.
θ = Velodyne shaft displacement, deg.

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(1) INTRODUCTION

The fundamental theory underlying power-system stability is well known, as also are the means most commonly adopted for ensuring the maintenance of stability. Systematic work on this type of problem dates from about 1924 and was at first mainly American.[1] Historically, studies of system stability commenced with the analytical determination of the power limits of two synchronous machines connected together through an impedance, and this is still the model used to-day for illustrating the general principles. It has been shown, for example, by Crary,[2] that the two-machine system may be represented by a single machine connected to an infinite busbar of inertia constant $M_1M_2/(M_1 + M_2)$. The equation of power balance for such a system is the basis for all methods of assessment of transient stability.

Early studies were mainly concerned with the determination of system layouts which were satisfactory from the standpoint of maximum steady-state power transfer, and only later was it realized that satisfactory operating characteristics under conditions of sudden disturbances were of greater importance. In order of increasing severity the major disturbances are as follows:

(*a*) Sudden large load increases.
(*b*) Switching operations.
(*c*) Faults with subsequent isolation of the faulty section and with, or without, subsequent operation of auto-reclosing circuit-breakers.

Several different types of stability have been considered from time to time, but of these it is *steady-state* and *transient* stability which are the generally accepted and major classifications. Steady-state stability is concerned with the operation of a power system under all normal conditions, this being taken to include the effects of governors, automatic voltage regulators, load fluctuations, etc. Transient stability is said to exist in a power system if, after an aperiodic disturbance, the individual parts of the system remain in synchronism. The well-known 'equal-area criterion' may be applied directly to a study of the transient stability of an equivalent two-machine system, but this suffers from the defect that the duration of disturbance is expressed in terms of changes of rotor angular displacement; this approach is impracticable, and it is necessary to determine the curves of generator load-angle against time. These latter are the so-called *swing curves.*

Transient stability only is considered here. Possible methods of solving the power-balance equations for the synchronous generators of a system are briefly reviewed, and two methods arising out of recent experimental work are described. One of these is an automatic equipment for performing step-by-step calculations when associated with the phase-angle control of an a.c. impedance-type network analyser; the other is an analogue method using essentially an electromechanical phase-modulator.

The first method uses a step-by-step process for solving the power-balance equation of the synchronous machine(s) and is fully automatic in its operation. Although it is satisfactory in operation, its use has now been discontinued in favour of the direct-analogue method, but it is thought to be of sufficient interest and educational value to merit inclusion. The analogue method, which is essentially a device for the solution of the second-order non-linear differential equation governing the power balance of a synchronous machine, has applications other than to the solution of stability problems; it may be applied, for example, to problems of hunting or to making the processes of network-analyser operation more automatic. The simulation of other phenomena associated with synchronous machines, such as governor and automatic-voltage-regulator operation, may subsequently be associated with this device.

(2) A REVIEW OF THE PROBLEM AND METHODS OF SOLUTION

The general formulation of the swing-curve equations, after Dahl,[3] is as follows:

$$M_n \frac{d^2\delta_n}{dt^2} + P_{dn1}\frac{d(\delta_n - \delta_1)}{dt} + P_{dn2}\frac{d(\delta_n - \delta_2)}{dt} + \ldots + \ldots P_{en} f[(\delta_n - \delta_1), (\delta_n - \delta_2), \ldots] = P_{mn} \quad . \quad (1)$$

For the purpose of discussion of the various methods available for the solution of this problem, eqns. (1) may be simplified, and consideration given to the case of a single synchronous machine connected through a reactance to an infinite busbar. On making the usual assumptions that the machine and busbar voltages are constant in magnitude, and that the angular velocity of the voltage vector at the infinite busbar is unchanged throughout any disturbance, the power-balance equation for the generator may be written

$$M\frac{d^2\delta}{dt^2} + P_d\frac{d\delta}{dt} + \frac{V_1V_2}{X}\sin\delta = P_m \quad . \quad . \quad . \quad (2)$$

The equation has no explicit solution for δ but forms the basis of several methods of treating the transient stability problem, which, in effect, consist of performing a double integration of the power difference:

$$P_a = (P_m - \frac{V_1V_2}{X}\sin\delta) \quad . \quad . \quad . \quad . \quad (3)$$

$$= P_m - P_e \quad . \quad . \quad . \quad . \quad . \quad . \quad (3a)$$

in order to determine δ as a function of time.

Solution by this technique involves the assumption of constant inertia constant $M = H/180f$, where H is expressed in kilowatt-seconds per kilovolt-ampere rating. It is possible to allow for the change in M with instantaneous frequency of the rotor, although this effect is slight.

The methods available for solution may be classified into four main groups: (*a*) step-by-step methods, (*b*) digital-computer methods, (*c*) differential-analyser methods, (*d*) direct-analogue methods.

(2.1) Step-by-Step Methods

Step-by-step methods employ double numerical integration. The period of swing is divided into a number of discrete time-intervals of length Δt, whose value is of considerable significance to the accuracy of the result; in general, a time interval of 50 millisec may be expected to give an accuracy of 5%. Standard texts[2,3] give the procedure, making the following assumptions:

Machine Assumptions.
(i) The input power to each generator remains constant.
(ii) Damping power is negligible.
(iii) Each machine may be represented by a constant reactance in series with an e.m.f. of fixed magnitude and variable phase.

Convenient Analytical Assumptions.
(iv) The acceleration of the rotor is constant from the middle of one interval to the middle of the next.
(v) The rotor velocity is constant during each interval.

Assumptions (iv) and (v) involve the approximation that both the acceleration and the velocity diagrams are represented by stepped curves.

The simplified machine equation from eqns. (2), (3) and (3*a*) is

$$\frac{d^2\delta}{dt^2} = \frac{P_a}{M} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

The angular position at the end of the Nth interval may be shown to be

$$\delta_N = \delta_{(N-1)} + \Delta\delta_N \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where

$$\Delta\delta_N = \Delta\delta_{(N-1)} + \frac{P_a(\Delta t)^2}{M} \quad . \quad . \quad . \quad . \quad (6)$$

By means of eqns. (5) and (6), the swing curve of each of the machines in the system may be plotted, the accuracy of the method depending upon the number of points taken.

It will be seen that great accuracy is not obtainable if only on account of the sweeping assumptions which are made. However, the error is biased to the estimation of a conservative stability limit.

The above analysis has to be modified during the intervals in which the onset and clearance of a fault occur. If a discontinuity occurs at the beginning of an interval, the average value of P_a for the adjoining intervals may be used. If the discontinuity occurs in the middle of an interval, no modification is needed; and if it occurs at any other point in the interval, a weighted average P_a in both that and either the succeeding or preceding interval is used.

Similar methods of analysis, but with different assumptions, may be used,[3] but the routine given is the most convenient one for automatic working.

Stability problems may be solved with the aid of an a.c. network analyser using eqns. (4), (5) and (6). However, without additional computing aids the work is tedious. Mortlock[4] has described a semi-automatic computer which uses potentiometers for carrying out the double integration; a network analyser gives the appropriate values of power and the generator-unit phase-shifters are set by hand. A development of this apparatus has been described by Jones;[5] another form of semi-automatic computer for the same purpose has been described by Peterson.[6] It has been reported by Squires and Harder[7] that an automatic step-by-step calculator has been added to the Westinghouse network analyser.

(2.2) Digital-Computer Methods

Bennett, Dakin and Knight[8] have described the use of the high-speed digital computer at the University of Manchester for the solution of stability problems. Visual display of swing curves may be obtained with this method, whose utility for power-system engineers is equalled only by a continuous-recording direct-analogue computer. A disadvantage arises in the computing time which is necessary for the machine to obtain the equivalent network of the system. The existing machine at Manchester will derive the necessary matrix coefficients, i.e. the driving point and transfer admittances, for 50 independent node pairs and 10 synchronous sources in approximately half an hour. For example, if these figures are doubled, then for the same number of generators the time taken would be of the order of 7 hours. This method of solution of the swing-curve equations has been described by Gill.[10] Further information on the application of digital analysis to power-system problems has been given by Bills.[9]

The disadvantage mentioned above has been overcome by using a network analyser for the derivation of the necessary driving-point and transfer admittances, and has been mentioned by Rothe,[11] and Squires and Harder;[7] it is of course, self-evident that the network analyser must be sufficiently large for the purpose. It should be noted that the situation may alter in the near future when digital machines approximately ten times faster in operation than that at present installed in Manchester become available.

(2.3) Methods Involving the Use of Differential Analysers

The physical size of differential analysers limits their application to the problem; however, several attempts have been made to apply a combination of differential-analyser techniques, using either mechanical or electronic integrators and an a.c. network analyser. Although the earliest suggestions were made by Kimbark,[14] the first such application was due to Boast and Rector.[12] The use of a network analyser with a normal instrument system read by a human operator, who inserted information into a differential analyser used as an auxiliary computing device, has been described by Bekey and Schott.[15]

Van Ness[22] has described a method involving the use of double integration, in which the first integrator is electronic and the second, which drives the network-analyser phase-angle control, is electro-mechanical. There is no allowance for damping, and the inertia constant is inserted at the input of the first integrator. A modification of the method allows for the representation of saliency.

(2.4) Direct-Analogue Methods

The servo and electronic techniques, dating from the later years of the Second World War, made possible the use of direct-analogue methods associated with a.c. network analysers. Electronic phase-modulating systems have been described by Kaneff[17] and Shen and Packer.[18] Electro-mechanical methods have been proposed by Kusko,[13] Watkins,[16] Bauer,[20] Mortlock,[21] and very recently by Kaneff.[23]

All these methods, together with the one independently developed by the authors, determine automatically the net accelerating power, P_a, of the system, and use it in a servo system to control the angular position of the generator-unit phase-shifters and thereby determine the swing curves.

(3) AN AUTOMATIC STEP-BY-STEP COMPUTER FOR STABILITY STUDIES

(3.1) Essential Elements of the Computer

The components of a machine to solve eqns. (4), (5) and (6) are indicated in the block diagram of Fig. 1 and are four in number, namely

(*a*) A wattmeter to measure the quantity P_a.

(*b*) Two step-by-step integrators for the evaluation of the integral of P_a [eqn. (3)].

(*c*) A store into which is inserted initially the value of P_m, the electrical equivalent of the prime-mover input power.

(*d*) A mechanism for the sequential control of the operations of the machine.

For multi-machine problems items (*a*) and (*d*) may be centralized with consequent saving of equipment, but items (*b*) and (*c*) must be provided for each generator unit. If the wattmeter is centralized, it must be shared sequentially among the generators during each step of the calculation with consequent sacrifice of speed of operation. From Fig. 1 it will be seen that each generator has access to the wattmeter in turn through the banks of a uniselector. The integrators take the form of two storage locations between which information can be exchanged, with the facility of adding in a third quantity in one direction only.

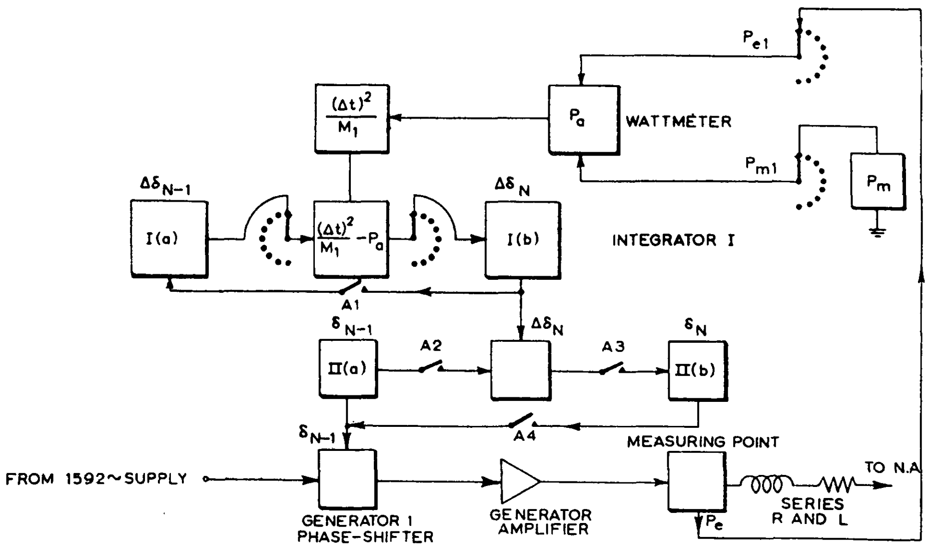

Fig. 1.—Schematic of computer.

Considering the Nth step of the calculation, with the uniselector wipers resting on the first bank contact corresponding to generator 1, the operation is as follows:

(i) The generator measuring points are connected to the wattmeter through one set of wipers and the P_m store through another. The wattmeter measures $P_m - P_e = P_a$.

(ii) The quantity P_a multiplied by the constant $(\Delta t)^2/M$ is inserted into integrator I, and added to $\Delta\delta_{(N-1)}$ to form $\Delta\delta_N$ in store I(*b*).

This solves eqn. (6) for this step.

(iii) $\Delta\delta_N$ is carried also into integrator II, where it is added to the quantity $\delta_{(N-1)}$ to form δ_N in store II(*b*). This solves eqn. (5) for this step.

(iv) The uniselector steps to the next contact and the above three steps are repeated for generator 2, *et seq.*

(v) When steps (i), (ii) and (iii) have been carried out for all the generators, and the quantities $\Delta\delta_N$ and δ_N stored in each I(b) and each II(b) respectively, step N is complete and relay A is operated. This has the effect of transferring the contents of each store I(b) and II(b) into the associated I(a) and II(a) stores, setting the integrators for the next step, and also setting the phase angles δ_N on to the phase shafts of the generators.

(vi) The uniselector is now reset to generator 1 and the above five steps are repeated with all the suffixes increased by one, i.e. the $(N + 1)$ step is now carried out.

To avoid the complex operation of taking average values for P_a for intervals which contain discontinuities, Δt is so arranged as to make all discontinuities fall in the middle of a period. The faults are applied and cleared by the machine through the banks of a second uniselector which counts the number of steps in the calculation and operates relays for this purpose at the appropriate times.

(3.2) Physical Arrangement of the Computer

(3.2.1) The Wattmeter.

The wattmeter was developed from an original idea by Bradshaw and Watkins; the first model was built by Watkins[23] during early work on a step-by-step computer at the Manchester College of Technology, but not using a velodyne as described below (Fig. 2).

The wattmeter is servo-driven and is built round a single-dynamometer movement. The instrument is deflected by amplified voltage and current signals V_v and V_i, derived from a network analyser, but is restored to a null position by a torque produced in its coils at a second frequency which differs by about 50 c/s from

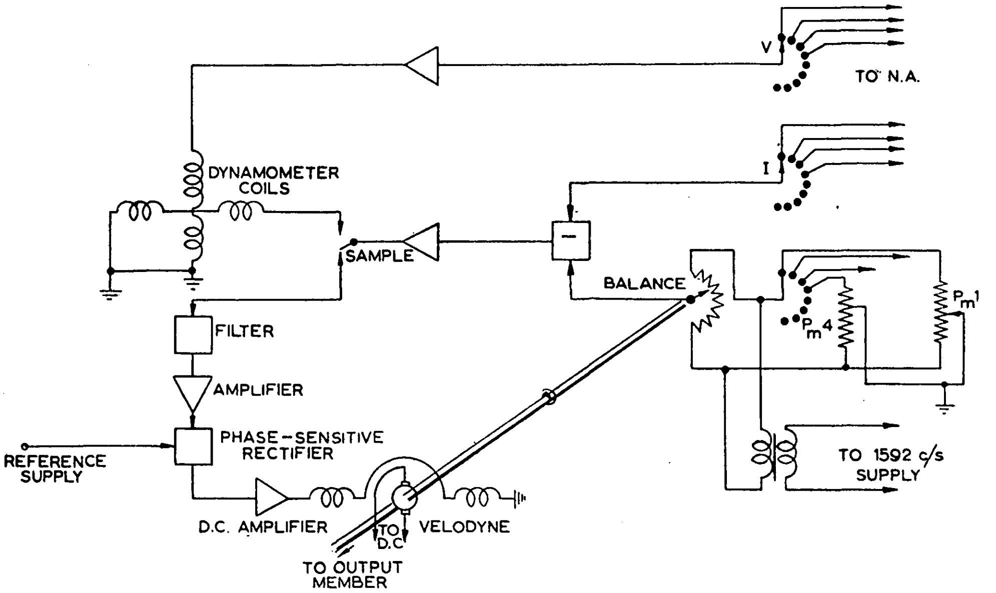

Fig. 2.—Schematic of wattmeter.

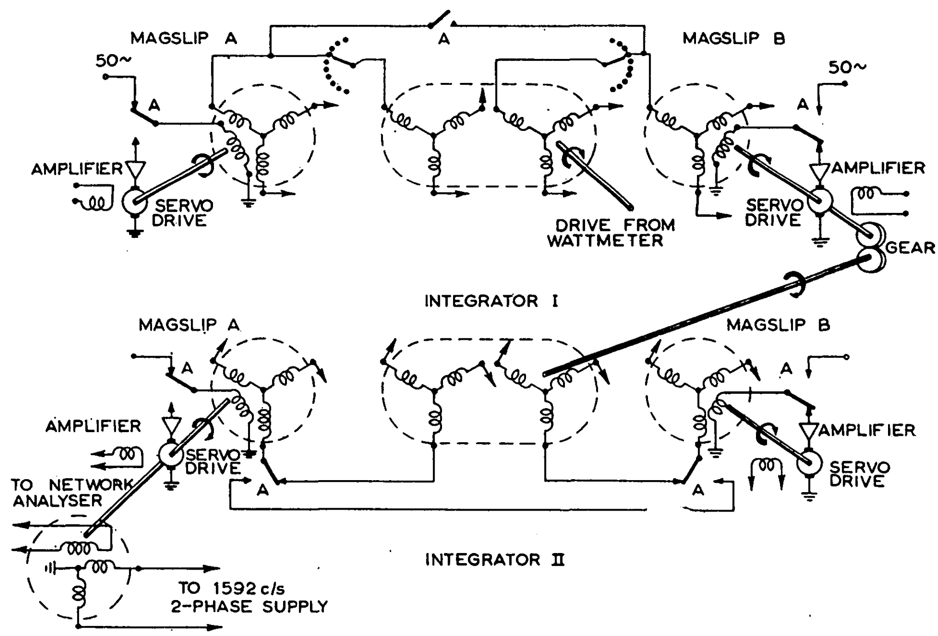

Fig. 3.—Integrator units.

the network-analyser frequency, which is 1 592 c/s. The null position is then taken to be the position of zero mutual inductance between fixed and moving coils. The angular deflection of the instrument can be measured at any instant by the magnitude and phase of the induced e.m.f., v_m, in the moving coil. The moving coil is periodically, i.e. 50 times a second, disconnected from its driving amplifier and connected instead to the input terminals of a servo system; v_m is first amplified, then detected by a phase-sensitive rectifier, and the resulting signal used to drive a velodyne. The velodyne shaft carries a potentiometer (called the 'balancing potentiometer') which controls the magnitude and direction of the second frequency-restoring torque. The servo system will thus set the wattmeter to the null position, where v_m is zero, when the power measured is proportional to the angular deflection of the balancing potentiometer.

(3.2.2) **The P_m Stores.**

The quantity P_m is subtracted from P_e by shifting the zero point of the balancing potentiometer. This is effected by putting a potentiometer in parallel with the balancing potentiometer and connecting its slider to earth. The P_m potentiometer for each appropriate generator is selected through one of the banks of the wattmeter uniselector.

(3.2.3) **The Integrators.**

The integrators use conventional magslip adding circuits (Fig. 3), but in this application information is transmitted in both directions so that magslip (*A*) and (*B*) each require to be servo-driven in turn; the direction of transmission of information is determined by relay A. Insertion of the quantity $(\Delta t)^2P_a/M$ into integrator I is effected by switching the follow-through transmitter associated with the wattmeter into the circuit by means of six banks on the wattmeter uniselector U. $\Delta\delta_N$ is carried into integrator II by connecting its follow-through transmitter mechanically to magslip I(*B*).

The flow of information in a typical adding circuit for the first three steps of a calculation is shown in Fig. 4.

(3.2.4) **The Controller.**

The controller is responsible for the sequence of operations during the solution of the problem. It consists of several relays and two uniselectors. One uniselector connects the wattmeter sequentially to the generators whilst the other counts the number of steps in the problem. This latter is responsible for applying faults and stopping the machine, etc., on the requisite step.

(3.3) Accuracy

The accuracy of the machine is limited to that of the network analyser, the integrators and the wattmeter. It is possible to make the last two accurate to better than 1%, but the network analyser will probably be no better than to 2·5%. The accuracy of wattmeter and integrators will be worst for small angles, and so provision is made for multiplying the quantities handled by the wattmeter and integrator I by 1, 5 or 10, the angle being reduced to its correct size by a gear connecting integrators I and II (Fig. 3).

(3.4) Setting up the Apparatus

The constant $(\Delta t)^2/M_{mn}$ and the magnification factor of 1, 5 or 10 mentioned above are inserted at each generator measuring-point on the network analyser by adjustment of a potential divider. P_m is inserted initially by backing off the steady-state power on the P_m potentiometer; this is done manually by stepping the uniselector round the various generators with only the wattmeter energized.

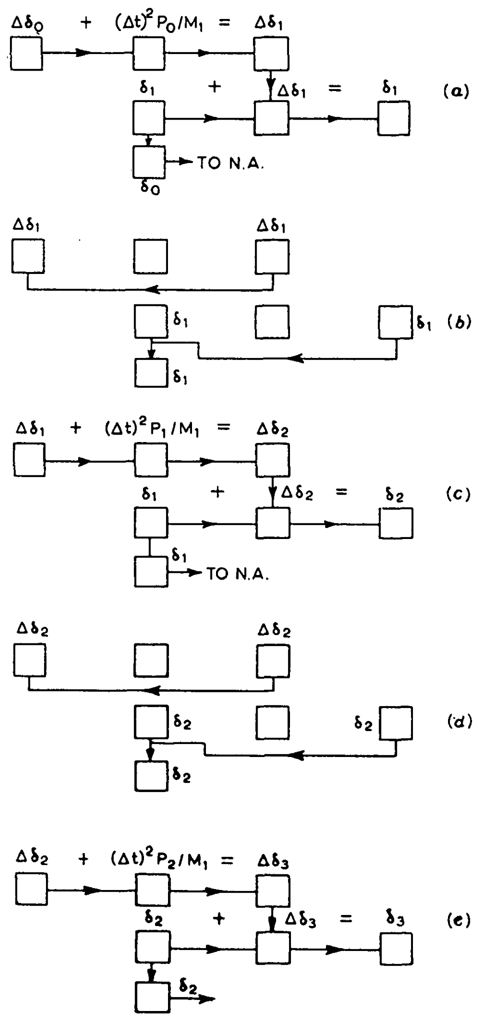

Fig. 4.—Flow of information for the first three steps of a calculation.

(*a*) Step 0.
(*b*) Transfer at end of step 0.
(*c*) Step 1.
(*d*) Transfer at end of step 1.
(*e*) Step 2.

(4) AN ANALOGUE METHOD FOR THE AUTOMATIC PHASE-ANGLE CONTROL OF NETWORK-ANALYSER GENERATOR UNITS

(4.1) Principle of the Analogue

Eqn. (2) may be written

$$M\frac{d^2\delta}{dt^2} + P_d\frac{d\delta}{dt} + (P_e - P_m) = 0 \quad . \quad . \quad . \quad (7)$$

This equation is similar in form to that of a second-order servo system with derivative feedback. The analogue described employs an electronic wattmeter and a velodyne[25] in a d.c. servo system driving a generator-unit phase-shifter. The arrangement of components is shown in the block diagram of Fig. 5. It will be seen that the wattmeter supplies a signal proportional to power output P_e and, after subtraction from a reference voltage proportional to P_m, the signal controls the velodyne motor; negative derivative feedback, proportional to P_d, the damping power, is applied from the velodyne generator. The loop is completed by coupling the velodyne mechanically to the phase-shifter. Under transient conditions, the system obeys an equation of the

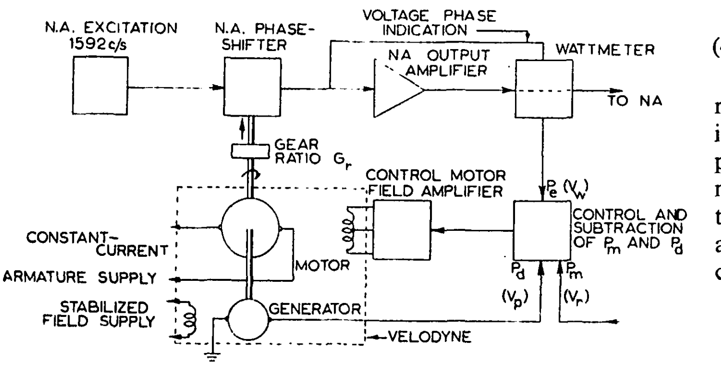

Fig. 5.—Schematic of simulator.

same form as the machine power relation, with the coefficients pre-set by adjustment of the controls.

Referring to Fig. 7, the motor field current is

$$I_f = A(v_r - v_w - pK_T\frac{d\theta}{dt}) \quad . \quad . \quad . \quad . \quad (8)$$

Since the inertia of the moving parts is constant, a parameter α may be introduced defining the acceleration (deg/sec²) for a step in the field current of 1 mA. For the machines used in the apparatus α has a value of 8. Hence the equation of the loop is

$$\frac{d^2\theta}{dt^2} = A(v_w - v_r - pK_T\frac{d\theta}{dt}) \quad . \quad . \quad . \quad . \quad (9)$$

This may be written as

$$\frac{G_r}{\alpha A}\frac{d^2\delta}{dt^2} + pG_rK_T\frac{d\delta}{dt} + (v_w - v_r) = 0 \quad . \quad . \quad (10)$$

It is seen that the analogue of power is simulator voltage.

(4.2) Time Scaling

It is not necessary for the time of duration of the swing curve on the actual power system to be reproduced on the simulator. If only on account of the inertia of the drive to the phase-angle control of the network-analyser generator unit, it is convenient to scale time and decrease the swing-curve frequency. If the system equation is expressed in the form of eqn. (7), a scaling factor s may be introduced such that $t' = st$, where t' is the scaled time of the simulator. The simulator equation may be written

$$M'\frac{d^2\delta}{d(st)^2} + P'_d\frac{d\delta}{d(st)} + P_e = P_m \quad . \quad . \quad . \quad (11)$$

i.e.
$$\frac{M'}{s^2}\frac{d^2\delta}{dt^2} + \frac{P'_d}{s}\frac{d\delta}{dt} + P_e = P_m \quad . \quad . \quad . \quad . \quad (12)$$

Thus, if the analogue inertia constant M' is made equal to s^2M, and the analogue damping constant P'_d is made equal to sP_d, the division of the simulator time-scale by s converts the curve derived from the simulator to that of the system equation.

If k_w is the wattmeter output voltage per unit power input, then eqn. (10) may conveniently be written with per-unit coefficients; this permits direct comparison with the coefficients of the machine eqn. (7), in order to satisfy the conditions above. If δ in eqn. (7) is expressed in electrical degrees, and the coefficients are in per-unit values, comparison of eqns. (7), (10) and (12) yield the following relationships:

$$\frac{G_r}{k_w\alpha A} = M = M'/s^2 \quad . \quad . \quad . \quad . \quad . \quad (13)$$

$$\frac{pG_rK_T}{k_w} = P_d = P'_d/s \quad . \quad . \quad . \quad . \quad . \quad (14)$$

in which all quantities except A and p are constants.

(4.3) Analogue Equipment

(4.3.1) The Measurement of P_e.

The requirements for the power-measuring device are that the response should be fast and that negligible burden should be imposed on the network analyser. The wattmeter measures power corresponding to the internal power of a synchronous machine which, for transient studies, is the power behind the transient reactance. Since the voltage at this point is usually assumed constant throughout the study, a phase-sensitive rectifier circuit may be employed: the arrangement is indicated in Fig. 6(*a*).

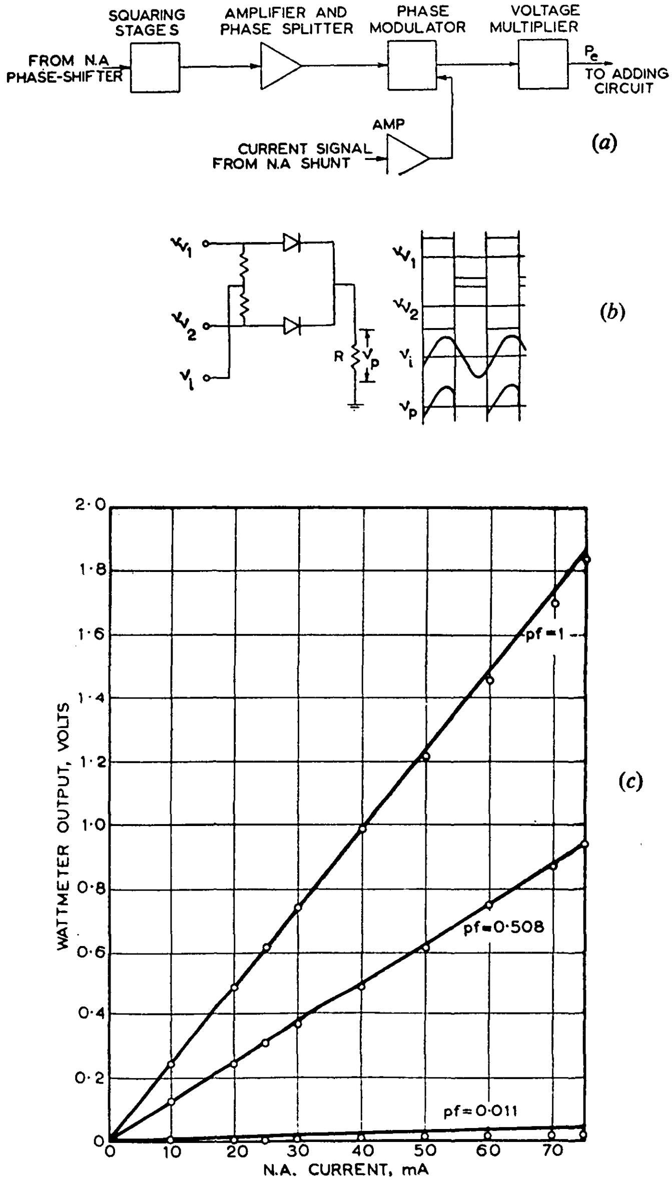

Fig. 6.—(*a*) Schematic of wattmeter.
(*b*) Operation of phase modulator.
(*c*) Wattmeter performance.

The straight lines indicate the expected performance calculated from the experimental point (25 mA, 0·620 volt). Other experimental points are encircled.

In Fig. 6(*b*) the square waves v_{v1} and v_{v2} are in phase and anti-phase, respectively, with the analyser voltage at the metering point. The passage of current due to the signal v_i, which is proportional to, and in phase with, the generator current, is governed by the sign of the voltage across the rectifiers, i.e. by the polarities of the square waves. While v_{v1} is positive and v_{v2} negative, current due to v_i flows and develops a voltage v_p in the load R. The d.c. output, being the average value of v_p, is a function of the magnitude of v_i and the phase difference between v_i and v_{v1}, i.e. the power-factor angle.

Let $v_i = V \sin \theta$ and let ϕ be the power-factor angle. Then, if the square waves v_v have sensibly vertical sides and high amplitude compared with v_i, the d.c. output of the wattmeter is given by

$$\frac{k}{\pi}\int_{\varphi}^{(180+\varphi)} V \sin \theta d\theta = \frac{2kV}{\pi} \cos \phi \quad . \quad . \quad . \quad . \quad (15)$$

where k depends on the circuit constants.

It is seen that the wattmeter output is proportional to analyser current and power factor, but independent of analyser voltage. With constant voltage at the metering point, the multiplication $V \times I \cos \phi$ may be effected on a potentiometer calibrated in terms of per-unit analyser voltage and connected at the output of the wattmeter; this requires manual adjustment of the control prior to each swing-curve study. The output of the wattmeter is smoothed by an RC filter. The time-constant of this filter determines the speed of response of the wattmeter and is approximately 7 millisec, i.e. 10 cycles of base frequency. Performance of the wattmeter is illustrated in Fig. 6(c).

(4.3.2) **Generator-Unit Phase-Shifter Drive.**

The type 73 velodyne is suitable for the purpose of driving the 3 in magslip phase-shifter of the network analyser and the precision potentiometer necessary for presentation of angular information to either a cathode-ray or Duddell oscillograph.

The split field of the motor is supplied in push-pull from a three-stage negative-feedback amplifier, with the linear range extending up to rated field current in order to have maximum torque available. The circuit and performance of the amplifier are shown in Figs. 7 and 8 respectively. The necessary stabiliza-

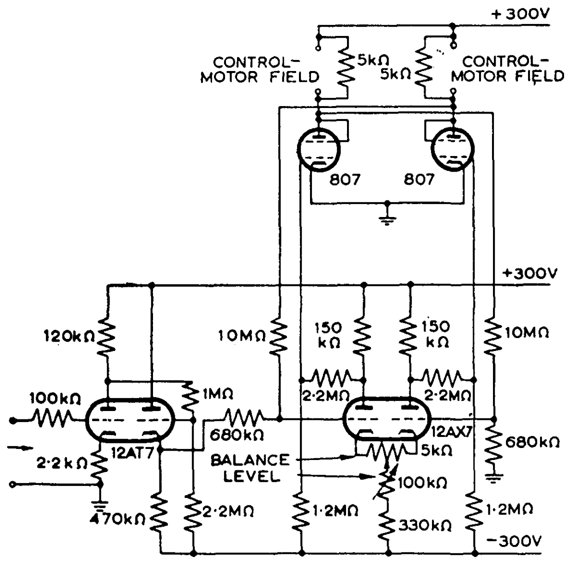

Fig. 7.—Control-motor field amplifier.

tion of motor armature current is effected with a simple degenerative circuit using one series valve and a shunt amplifier by which the current regulation at rated speed is reduced below 2%.

The velodyne tachometer-generator field is fed from a stabilized source which reduces the effect of supply-voltage variations by a factor of 10. The availability of derivative feedback also affords a means of neutralizing the effect of friction in the moving parts

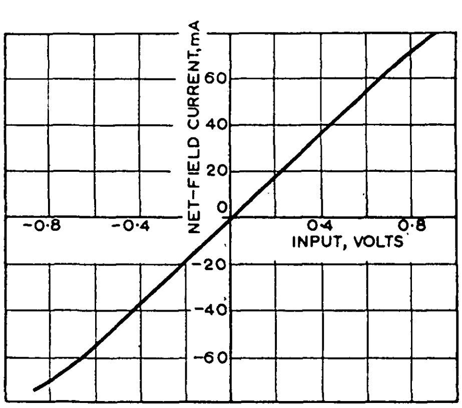

Fig. 8.—Gain characteristic of control-motor amplifier.

of the simulator. An approximate expression often employed for friction torque is

$$T = f' \frac{d\delta}{dt}$$

where T is friction torque, f' a friction coefficient and δ angular displacement. Thus, the effect of friction is to produce error in the first-order term of eqn. (10) and to impose a definite lower limit to the damping which occurs when the feedback control is at zero setting. The limitation can be removed by applying positive derivative feedback in a way similar to that employed for the simulation of damping power.

(4.3.3) **Reference Voltage and Adding Circuits.**

In transient-stability studies, the consideration of first-swing stability is of prime importance. For this reason, and owing to the delay in prime-mover governor operation subsequent to a sudden disturbance, the prime-mover input power may be assumed constant for transient-stability work with the simulator. Hence it is sufficient to represent P_m by an adjustable voltage which, according to eqn. (10), must be subtracted from the wattmeter output voltage corresponding to P_e.

Several methods of subtraction are available, but with a careful choice of circuit values, both the operation mentioned above and the addition of the voltage corresponding to asynchronous damping power may be carried out conveniently and with adequate accuracy by resistor networks.

(4.4) Simulator Operation

(4.4.1) **Simulator Controls.**

The adjustable analogue quantities are as follows:

Analogue quantity		*System quantity*
v_r	controls	P_m
A	controls	M
P	controls	P_d

The various controls of the simulator must be calibrated to facilitate rapid adjustment of system quantities and synchronous-machine parameters to the required values prior to swing-curve studies.

Most of the tests involved in the calibration have been carried out with the simulator in a static condition, the exception being the derivation of an acceleration constant for the control motor of the phase-shifter. The sequence of steps in the calibration is as follows:

(i) The maximum setting of the wattmeter voltage-multiplier is assigned a value equal to the highest relevant generator voltage likely to occur in normal problems. Intermediate points are immediately defined, since the setting is linear and proportional to the magnitude of the voltage to be represented. The control is adjusted to 1 per-unit voltage for the remaining steps in the calibration.

(ii) A static load is connected to the generator unit associated with the simulator. Adjustment of the P_m control to give zero difference power defines a point on the P_m calibration, and this is repeated for several values of the load.

(iii) Calibration of the M control is accomplished in two stages. First, the acceleration of the phase-shifter control motor is determined as a function of its field current. The relationship is linear and yields a constant giving the acceleration of the phase-shifter per milliampere step in the motor field current. Secondly, with a known and constant difference power at the input to the inertia-constant control, the setting of the control is varied and a graph of output against motor field current is obtained. Using the constant α, and the value of difference power, the scale of motor field current may be converted to per-unit power/deg/sec^2. By reference to the graph, the inertia-constant control may be calibrated in terms of these units. A more direct method would be to perform run-up tests with constant power differential and with variation of the inertia-constant setting: subsequent modifications, or other changes in any part of the apparatus, would entail a repetition of the acceleration-constant tests.

(iv) The tachometer armature is disconnected and a constant voltage is applied to the damping-factor control. With the prime-mover power set to balance the electrical power, the output from the damping-factor control is varied and plotted against motor field current. The scale of the latter may be converted to power using the results of one of the tests in (iii) above. It may further be modified to read per-unit power/deg/sec^2 by dividing by a speed: this speed corresponds to the test voltage applied to the damping-power control and is obtained from the tachometer-voltage/speed curve.

(4.4.2) The Use of the Simulator in Steady-State and Transient-Stability Studies.

Normally, the setting-up on the network analyser of the equivalent sequence networks of a power system is followed by the tedious manual process of adjusting the analyser generator units until the required values of power and voltage, obtained from system information, are attained. The simulator described may be employed for this purpose as a preliminary to either load-flow or transient-stability studies, with the following procedure:

The supplies to the control motors are switched off, the equivalent inertia constants are set to minimum values and the damping factors to maximum values. The P_m controls and the generator-voltage magnitude controls are adjusted to the required values, and after the setting-up of the network analyser and interconnection of the generator units, the supplies to the control motors are switched on. Owing to their low equivalent inertia and high damping, they immediately position the phase-shifters to give the correct phase-angle and system conditions, provided an additional generator is available for supplying the system losses.

The accuracy of the automatic setting-up process is governed by the sensitivity of the motor to small field-current changes, as there is a dead band of field-amplifier inputs within which no change of phase angle results. Positioning accuracy may be increased quite simply by providing a gain switch which enables the amplifier to be saturated by small inputs, in which case the accuracy is limited only by the accuracy of the wattmeter [Fig. 6(*c*)].

When transient-stability studies are to follow the setting-up of the generator units, it is only necessary to adjust the inertia-constant and damping-factor controls to correspond to predetermined values before carrying out the switching operations on the analyser network.

(5) RESULTS

A suitable problem without damping, which had previously been analysed by means of an integraph, was chosen from Park and Bancker[26] and is shown in Fig. 9.

The problem was solved step by step and by the simulator and was checked against the published integraph solution. The step-by-step and integraph solutions were identical and are shown in curve A (full line). The analogue solution is shown as curve A (dotted line), and it will be observed that the simulator is pessimistic both as regards peak angle of swing and rate of change of angle. Thus the simulator shows an overswing of approximately 3° before recovery and attains peak angle of swing approximately 3 cycles (at system frequency) earlier in the case of the other solutions. The same problem, with allowance for damping power, $P_d = 0\cdot758 \times 10^{-4}$ per deg/sec, was then considered, the determination of P_d being an approximation due to Park.[27] The results are shown as curves B.

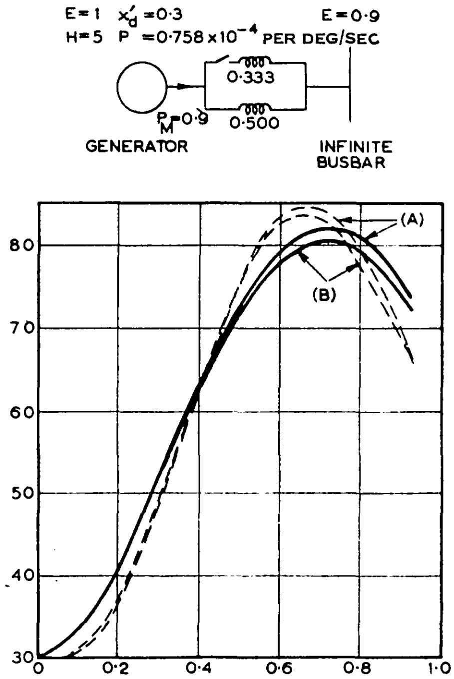

Fig. 9.—Calculated and simulator curves.

(A) Zero damping.
(B) Including damping.

(6) ECONOMY OF APPARATUS

Precise information from other workers in the field is difficult to obtain, but it appears that the direct-analogue equipment developed by the authors is competitive economically with other schemes of automatic generators for use with a.c. impedance-type network analysers. The equipment described in the second part of the paper, if manufactured commercially, would consist of an electronic chassis with wattmeter and velodyne controls and supplies, an electro-mechanical chassis containing voltage control setting, phase control, velodyne and gear-box, and a power-unit chassis. A large-scale commercially-produced network analyser could be thus equipped for the solution of swing curves, at a cost per generator not exceeding 1% of the cost of the complete installation. The accuracy obtainable would be within that normally required for power-system stability studies.

(7) CONCLUSIONS

The discrepancies in the results recorded may be attributed to the calibration of the damping and inertia controls; further work will be carried out on this aspect now that three automatic generators have been delivered for the large power-system simulator under construction at the College of Technology, Manchester. Provided that simulator operation is carried out on a slow time-scale (e.g. of the order of real time), voltage regulator action could be represented; circuits to this end involve simulation of the performance of the generator field circuit and regulator and the derivation of appropriate modulating signals. This work, as yet, is unpublished.

The automatic step-by-step method of solution of stability problems described in the paper is time-consuming, although it is faster than normal methods of desk computation with information derived from a network analyser. Two generators have been built using, in the main, suplus equipment, and would have been expensive if new equipment had been used. Furthermore, the machine is applicable only to transient-stability studies and, without further elaboration, cannot be used for saving time and setting up a network analyser for steady-state studies. The continuous-analogue method is rapid in operation and presents information in a convenient form; with heavy damping and increased external gain, the apparatus may be used as an auxiliary to a network analyser.

(8) ACKNOWLEDGMENTS

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[The discussion on the above paper will be found on page 161.]