Local Frequency-Based Estimation of the Rate of Change of Frequency of the Center of Inertia

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***Abstract*—This letter proposes a novel technique for estimating the rate of change of frequency (RoCoF) of the center of inertia (CoI) in power systems. To offer a holistic picture of the system's frequency response, the proposed technique requires local frequency at the point of measurement, only. This enables the local estimation of active power deficit following a loss of generation (LoG) event, thus facilitating fast and effective remedial action. Extensive simulations conducted verify the effectiveness and applicability of the proposed technique.**

***Index Terms*—Center of inertia (CoI), frequency, inflection points, rate of change of frequency (RoCoF).**

I. Introduction

THE concept of center of inertia (CoI) enables the deployment of a single swing equation for describing power system dynamics as a whole [1]. It is applied in different areas such as transient stability studies, power system operational planning, frequency control, and under-frequency load shedding (UFLS) [1]–[4]. The CoI frequency is defined as a weighted average of frequencies of all synchronous generators. Knowing the aggregate system inertia and the rate of change of frequency (RoCoF) of the CoI makes it possible to estimate the size of active power deficit in the power system, caused by loss of generation (LoG) events [3]. To calculate the CoI RoCoF, frequencies of all synchronous generators must be known. This may be also achieved using other frequency measurements, provided that the bus impedance matrix is available with high accuracy [4]. Centralized approaches for calculating CoI frequency, however, introduce reliability and communication latency challenges, thereby limiting the utilization of the CoI concept in real-time applications.

This letter proposes an effective technique for estimating the CoI RoCoF without communication. Locally measured frequency is the only input to this technique. The resulting accuracy would be comparable to that by communication-based approaches. These features would facilitate the development and implementation of non-communication effective frequency control applications and UFLS schemes.

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II. Applications of the CoI Swing Equation

The swing equation is the fundamental equation governing generator dynamics. To obtain a single equation for describing dynamics of a power system with N synchronous generators, one can use frequency of the CoI, f_{CoI}, defined as follows

$$f_{CoI} = \left(\sum_{i=1}^{N} H_i S_i f_i\right)\left(\sum_{i=1}^{N} H_i S_i\right)^{-1} \tag{1}$$

where H_i, S_i and f_i are the inertia constant, rated apparent power and frequency of the i-th generator, respectively. Combining per-unit swing equations of all generators, one obtains [1]

$$2H_{CoI}\frac{df_{CoI}(t)}{dt} + D_{CoI}\Delta f_{CoI}(t) = \Delta P(t) \tag{2}$$

where H_{CoI} and D_{CoI} are the aggregate inertia constant and generator damping, respectively. The mismatch between active power generation and consumption are represented by $\Delta P(t)$.

The term $\Delta P(t)$ in (2) accounts for the dynamic variations of load and generation, in addition to the primary active power deficit caused by the LoG event. The huge number of loads in the system makes an accurate representation of their behavior impossible. A practical solution to overcome this problem is using (2) within a sufficiently short period of time following the event inception where the frequency dependency of load can be ignored. Within such a short period, the mismatch between active power generation and consumption may be assumed fairly constant and equal to the size of the tripped generator. Nevertheless, this applies only if voltage variations caused by the LoG event or voltage dependency of load are negligible.

The swing equation can be used to estimate the size of active power deficit following an LoG event. This would need the knowledge of the aggregate system inertia [5], [6], in addition to a reliable communication network for transmitting measured generator frequencies to the control center. If such a communication network was already available, it would be much straightforward and easier to directly monitor active power injected by every generator and/or the status of the corresponding circuit breakers in order to detect LoG events.

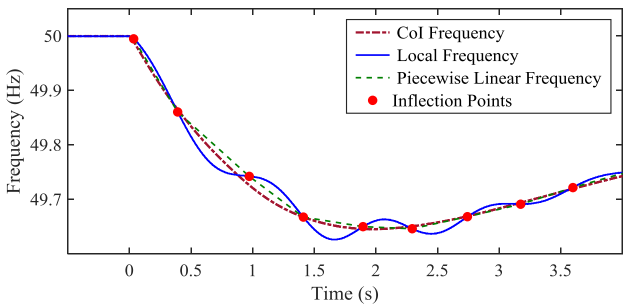

Fig. 1. Piecewise linear curve for estimating the CoI RoCoF.

III. Local Estimation of the CoI RoCoF

Development of effective methods for calculating RoCoF has been a matter of cutting-edge research [7], [8]. RoCoF can be deployed to enhance the performance of many power system applications such as load shedding, islanding detection, and distributed generation control [8], [9]. The focus of this letter is on estimating the CoI RoCoF based only upon local frequency measurement with no need of wide-area frequency monitoring. The motivation behind proposing such an approach is that simplicity is the essence of many local protection/control practices considering the demand placed on them in terms of speed, robustness, and reliability.

The nature of power system electromechanical dynamics is such that following an LoG event, local frequencies begin to oscillate around the CoI frequency and ultimately converge to it when frequencies of all generators become the same [10]. For example, subsequent to the disconnection of Generator 5 in the IEEE 39-bus test system, the frequency response at Bus 10 would be as shown in Fig. 1. Extensive simulations suggest that once the local frequency curve is upward, it mainly lies below the CoI frequency curve. Conversely, when the frequency curve becomes downward, it essentially lies above the CoI frequency curve. The deviation between local and CoI frequencies can be well approximated by an exponentially decaying sinusoid. This leads to the idea that each local frequency intercepts the CoI frequency at around the inflection points of the former.[1] As shown in Appendix, for a simple two-source system with no damping, the inflection points of local frequencies lie exactly on the CoI frequency curve.

Let $Q_i(t_i,\ f(t_i))$ denote the i-th inflection point on the local frequency curve, observed from left to right. The event inception point is denoted by $Q_0(t_0, f(t_0))$. Connecting consecutive Q_i's by straight lines yields a piecewise linear curve. The so-created piecewise curve for the frequency measured at Bus 10 in the foregoing example is drawn in Fig. 1 in dashed black. The inflection points of any local frequency are expected to lie ideally on, and practically in a close proximity to the CoI frequency curve. The idea here is to consider the slope of each segment of the piecewise curve an estimate of the CoI RoCoF over the corresponding interval. In an ideal case where the inflection points of the local frequency lie exactly on the CoI frequency, the slope of each segment will be identical to the average RoCoF of the CoI on that interval.

[1] An inflection point is a point on the curve at which a change of curvature occurs, i.e., the curve changes from being upward to downward or vice versa.

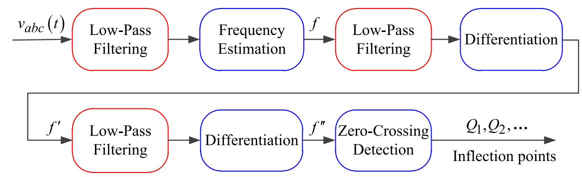

Fig. 2. Detection of inflection points of the local frequency curve.

In practice, inflection points of a frequency curve can be obtained by finding zero-crossing instants of its second derivative with respect to time. The low-frequency oscillations of frequency are of main interest to most applications, as they reflect electromechanical dynamics of synchronous generators. Thus, the effect of noise and high-frequency oscillations can be mitigated by low-pass filtering frequency signals. Fig. 2 shows a simple block diagram used in this letter for detecting the inflection points of frequency.

IV. Potential Application of the Proposed Technique

The CoI RoCoF estimation technique proposed in this letter can be readily deployed for responding to large LoG events in power systems in terms of UFLS with no need of communication. Let us assume there are n locations across the power system dedicated to UFLS. The maximum amount of load available for this at Bus i is denoted by $\delta P_{\max,i}$. To be able to compensate for the largest credible LoG event in the system, which is assumed to be of the size $\Delta P_{\text{Largest}}$, it is needed that

$$\sum_{i=1}^{n} \delta P_{\max,i} = \Delta P_{\text{Largest}} \tag{3}$$

Let us consider an LoG event of size ΔP. The estimated CoI RoCoF and the size of load to be shed at Bus i are denoted by m_i and δP_i, respectively. This generation deficit can be compensated for by shedding load at every Bus i as below

$$\delta P_i = \left(\frac{2m_i H_{CoI}}{\Delta P_{\text{Largest}}}\right) \delta P_{\max,\,i} \tag{4}$$

The above formula is derived considering that the CoI swing equation estimates the size of LoG events as follows

$$\Delta P \approx 2m_i H_{CoI} \tag{5}$$

It can be concluded from (3)–(5) that $\sum_{i=1}^{n} \delta P_i \approx \Delta P$. This means the overall amount of load shed will be equal to the size of the active power deficit caused by the LoG event.

V. Performance Evaluation

In order to confirm the effectiveness of the proposed technique for estimating the average CoI RoCoF, extensive simulations are conducted on the IEEE 39-bus test system. In what follows, the RoCoF estimation error by the proposed technique is discussed first. Then, piecewise linear frequency curves are formed from

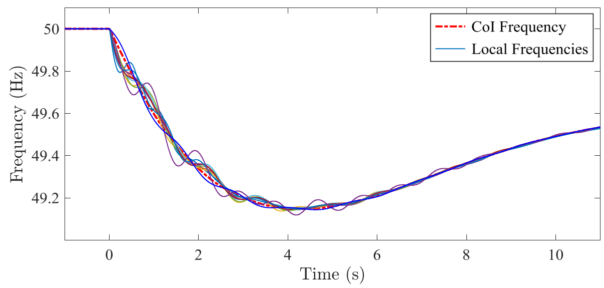

Fig. 3. Local frequencies following the outage of Generator 5.

local frequency measurements following an arbitrarily selected LoG event. In all simulations conducted, generated three-phase voltage waveforms are sampled with a sampling rate of 1.6 kHz and estimated frequencies are reported with a reporting rate of 50 Hz to feed the proposed technique with. The proposed estimation technique essentially relies on identifying low-frequency oscillations of local frequency signals. This is the reason that the LoG event size and/or high-frequency fluctuations of the local frequency signal would have no significant impact on the applicability of the proposed technique.

A total of more than 5,000 LoG events of different sizes and locations are simulated on the IEEE 39-bus test system. Then, the slope of the first segment of piecewise linear curves obtained is calculated. So-estimated RoCoFs are compared with the true CoI RoCoF following their corresponding LoG events. RoCoF estimation errors demonstrate a normal distribution with mean -0.7% and standard deviation 1.8%. It is observed that the average CoI RoCoF estimation error exceed 5% for none of LoG events simulated. Let μ and δ denote the mean and standard deviation of the estimation error by the proposed technique, respectively. Based on the 3δ criterion, the local CoI RoCoF estimation error will lie within the range $\mu\pm3\delta$ with a confidence level of 99.7%. Let us suppose there are k UFLS relays in the system, each of which is set to shed $1/k$ of its estimated LoG size. It follows that the total LoG size estimation error based on the slope of the first segment of piecewise linear frequency curves will be limited to the range $\mu \pm 3\delta/\sqrt{k}$. For a k of 10, this means the overall error of LoG size estimation will lie between -2.4% to $+1\%$ of the real LoG size. Considering other sources of inaccuracies and technical challenges of centrally calculating the CoI RoCoF, this would be quite acceptable for a good number of real-time applications.

As a walkthrough example, Generator 5 is tripped to create a 700 MW generation deficit in the system whilst the system inertia is reduced to 6.2 sec. Fig. 3 demonstrates local frequencies at a number of different buses in the system. As can be seen, all the local frequency curves follow a declining trajectory in the same way as the CoI frequency does, whilst demonstrating some exponentially decaying low-frequency oscillations around the CoI frequency. As described in Section III, inflection points on each frequency curve are found and connected to obtain the corresponding piecewise linear frequency curves shown in Fig. 4. It can be observed that removing low-frequency oscillations of frequency curves concentrates frequency curves around the CoI

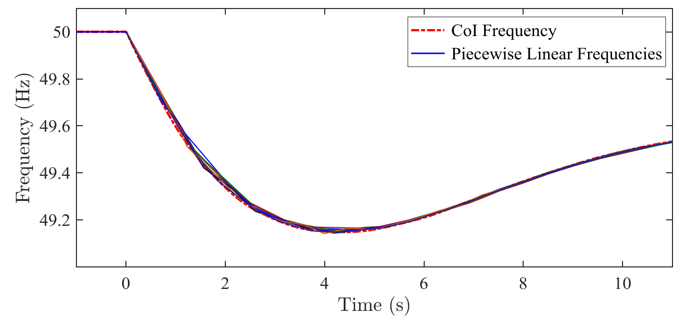

Fig. 4. Piecewise linear frequency curves following the outage of Generator 5.

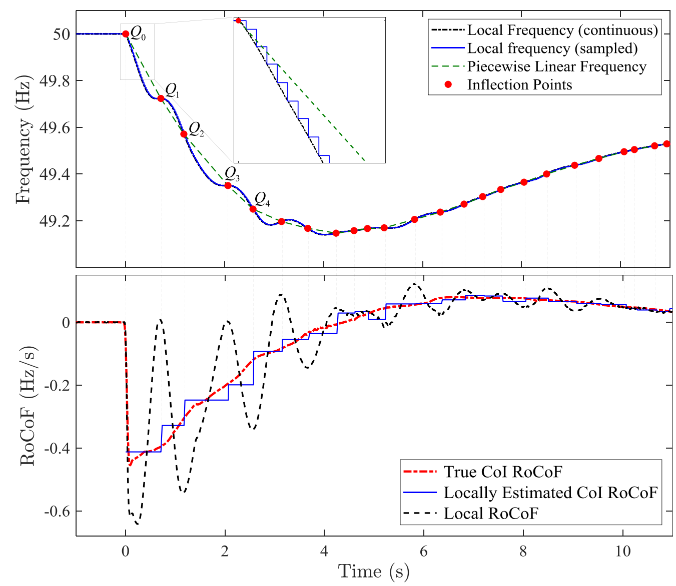

Fig. 5. Estimating the CoI RoCoF using frequency measurement at Bus 6.

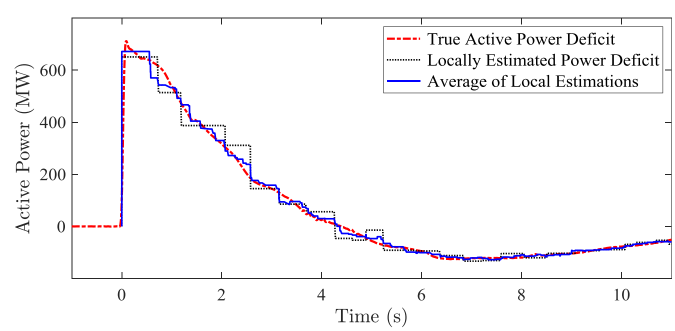

Fig. 6. Estimating the LoG size by the proposed technique.

frequency curve. Fig. 5 details how the CoI frequency and its RoCoF are estimated using the frequency signal at Bus 6, where inflection points, i.e., $Q_0, Q_1, \ldots$, are marked by red circles. The slope of each segment of the piecewise linear curve created is attributed to the average CoI RoCoF on the corresponding interval. It can be seen that the so-estimated RoCoF lies much closer to the true CoI RoCoF contrary to the local RoCoF at Bus 6. Fig. 6 shows the LoG size calculated from (5) based on the locally estimated CoI RoCoFs. As expected, the average of LoG size estimations at Buses 6, 11, 12, 19 and 20 lies much closer to the

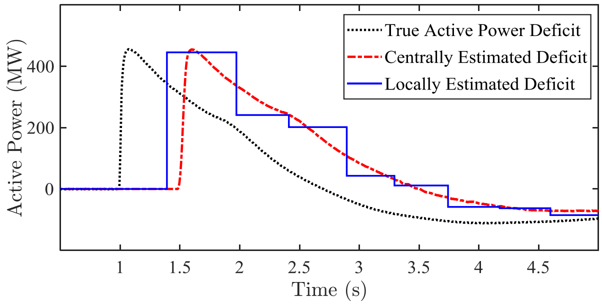

Fig. 7. Active power deficit calculated by the proposed technique based on the frequency measurement at Bus 10.

true active power deficit. This confirms that the overall performance is improved as the number of local estimations increases.

As another example, the active power deficit over time is calculated from (5) for an LoG of size 450 MW based on locally and centrally estimated RoCoFs as shown in Fig. 7. The frequency response at Bus 10 following this event is shown in Fig. 1. The overall communication latency of the centralized approach is considered to be 500 ms. This is to account for communication between generators and the control center, and between the control center and local remedial action units. The true active power deficit, i.e., the mismatch between mechanical input and electrical output power of all generators, is also calculated. As can be seen, the accuracy of LoG size estimation by the proposed technique is quite comparable with that of the centralized one. With regard to real-time applications, the time delay of the proposed technique results from the time it takes to observe the first inflection point on the local frequency curve. Suitable curve fitting approaches can be employed to predict upcoming inflection points of the frequency curve in advance in order to reduce the foregoing time delay, e.g., to below 100 ms [10].

VI. Conclusion

This letter proposes an effective technique for estimating the rate of change of frequency (RoCoF) of the center of inertia (CoI) in power systems. Local frequency at the point of measurement is the only input needed to this end. The accuracy achieved is comparable to that of the centralized practice, whilst the former does not require a system-wide communication for collecting frequencies of all synchronous generators. The simplicity of the proposed technique in terms of input requirements and computational burden will pave the way for designing effective non-communication UFLS schemes and enhanced frequency control applications for power systems with high penetration of renewable generation. These can be used stand-alone or in parallel with their communication-based counterparts to improve reliability.

Appendix

The validity of the CoI RoCoF estimation technique is demonstrated for the case of a simplified two-source system shown in Fig. 8. Let the subscripts s and r denote sending- and receiving-end quantities in this system, respectively. The damping term

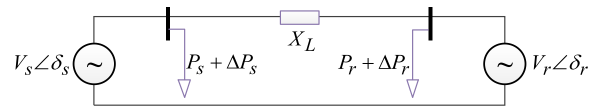

Fig. 8. A simplified two-source system following an LoG event.

and changes of the generation deficit over time are ignored. The generator rotor angle and line reactance are denoted by $\delta(t)$ and X_L, respectively. Assuming that the line reactance dominates the transient reactance of the generators,

$$\begin{cases} 2H_s\frac{df_s(t)}{dt} = \Delta P_s + V_sV_r\sin\left(\delta_s\left(t\right)-\delta_r\left(t\right)\right)/X_L \\ 2H_r\frac{df_r(t)}{dt} = \Delta P_r - V_sV_r\sin\left(\delta_s\left(t\right)-\delta_r\left(t\right)\right)/X_L \end{cases} \tag{A-1}$$

where an LoG of $\Delta P = \Delta P_s + \Delta P_r$ is supposed to have occurred at $t = 0$ sec in the system. The mechanical power inputs of the generators are assumed constant within the timeframe of interest. Differentiating the above equations with respect to t,

$$\begin{aligned} \frac{d^2f_s\left(t\right)}{dt^2} &= -\frac{H_r}{H_s}\frac{d^2f_r\left(t\right)}{dt^2} \\ &= \left[f_s\left(t\right)-f_r\left(t\right)\right]\frac{V_sV_r\cos\left(\delta_s\left(t\right)-\delta_r\left(t\right)\right)}{2H_sX_L} \end{aligned} \tag{A-2}$$

Equation (A-2) implies that the second derivative of generator frequencies with respect to time becomes zero once $f_s(t) = f_r(t)$. This means sending- and receiving-end frequencies would intercept each other on the CoI frequency curve. It follows that the frequency of each generator intercepts the CoI frequency at its inflection points.

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