Based on the paper by Jesus, and the paper "a low order system frequency response", I need to calculate GSFR, Frequency response, Delta Frequency and then transform that to achieve the Power required to compensate for the Frequency Response.

Borrowing from the low order response paper, the following values will be used, where , , , , , and respectively represent the effects of the governor droop, frequency dependence of load, mechanical gain factor, reheat time constant, damping factor, and natural frequency of the system. This is the same as done by Jesus.

$R = 0.05$	$H = 4.0\ s$	$K_m = 0.95$
$F_H = 0.3$	$T_R = 8.0\ s$	$D = 1.0$

Then we compute

$\omega_n = 0.559$	$\varsigma\omega_n = 0.438$	$\phi_1 = 131.94°$
$\varsigma = 0.783$	$\omega_r = 0.348$	$\phi_2 = 141.54°$
$\alpha = 6.011$	$\sqrt{1-\varsigma^2} = 0.622$	$\phi = -9.60°$

```
R = 0.05;
H = 4.0;
K = 0.95;
Fh = 0.3;
Tr = 8.0;
D = 1.0;
```

We could use the values as computed in the paper, and shown above, or calculate it ourselves. Calculating it ourselves will be used, simply for completeness. A P-step of 0.3 will be used for this example. (For Jesus' paper, all that matters is the magnitude, hence why it is not negative such as the original SFR paper)

```
syms t

Ps = 0.3;
```

$$\omega_n^2 = \frac{DR + K_m}{2HRT_R}$$

$$\varsigma = \left(\frac{2HR + (DR + K_m F_H)T_R}{2(DR + K_m)}\right)\omega_n$$

```
wn2 = (D*R+K)/(2*H*R*Tr)
```

```
wn2 = 0.3125
```

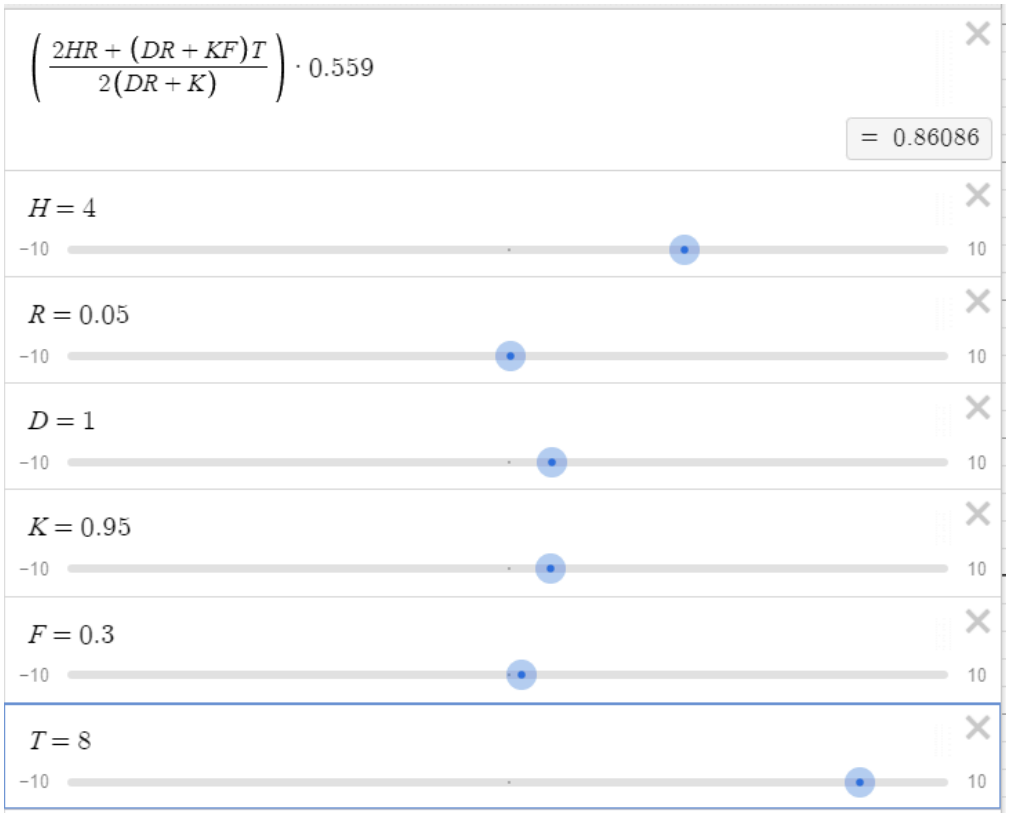
```
wn = sqrt(wn2)
```

```
wn = 0.5590
```

```
%c = 0.783
c = (((2*H*R)+(((D*R)+(K*Fh))*Tr))/(2*(D*R+K)))*wn %according to matlab and desmos, this result
```

```
c = 0.8609
```

noticed something odd, which is that c is no 0.783 as indicated by the original paper. its.... 0.8609. wn is the same however, so I am unsure where the mistake has occured. Can confirm that Emico had the same results, because his Phi is also -0.14



moving on, the next set of equations are:

$$a=\sqrt{\frac{1-2T_R\varsigma\omega_n+T_R^2\omega_n^2}{1-\varsigma^2}}$$

$$\omega_r=\omega_n\sqrt{1-\varsigma^2}$$

$$\phi=\phi_1-\phi_2=\tan^{-1}\left(\frac{\omega_r T_R}{1-\varsigma\omega_n T_R}\right)-\tan^{-1}\left(\frac{\sqrt{1-\varsigma^2}}{-\varsigma}\right)$$

```
a = sqrt((1-2*Tr*c*wn+(Tr^2)*(wn^2))/(1-(c^2)))
```

```
a = 7.1677
```

```
wr = wn*(sqrt(1-(c^2)))
```

```
wr = 0.2844
```

```
phi = atan((wr*Tr)/(1-c*wn*Tr))-atan(sqrt(1-(c^2))/(-c))
```

```
phi = -0.1400
```

Next, we move to the actual function f(t), which will be denoted as f. This, according to the graph in the paper, is for per unit, so fo is not infact 50. This took me more than I would like to admit to notice. so for the mains power grid, the effects would be scaled by 50.

$$f(t) = f_0 - \frac{RP}{DR+K_m}[1 + \alpha e^{-\zeta\omega_n t} \sin(\omega_r t + \phi)]$$

```
hold on
for Ps = 0.1:0.05:0.3
    ft = 1 - ((R*Ps)/(D*R+K))*(1+a*exp(-c*wn*t)*sin(wr*t+phi))
    fplot(ft)
end
```

ft =

$$\frac{199}{200} - \frac{\sqrt{14}\ \sqrt{109}\ e^{-\frac{77\,t}{160}} \sin\left(\frac{\sqrt{2071}\ t}{160} - \frac{1260747285905809}{9007199254740992}\right)}{1090}$$

ft =

$$\frac{397}{400} - \frac{3\ \sqrt{14}\ \sqrt{109}\ e^{-\frac{77\,t}{160}} \sin\left(\frac{\sqrt{2071}\ t}{160} - \frac{1260747285905809}{9007199254740992}\right)}{2180}$$

ft =

$$\frac{99}{100} - \frac{\sqrt{14}\ \sqrt{109}\ e^{-\frac{77\,t}{160}} \sin\left(\frac{\sqrt{2071}\ t}{160} - \frac{1260747285905809}{9007199254740992}\right)}{545}$$

ft =

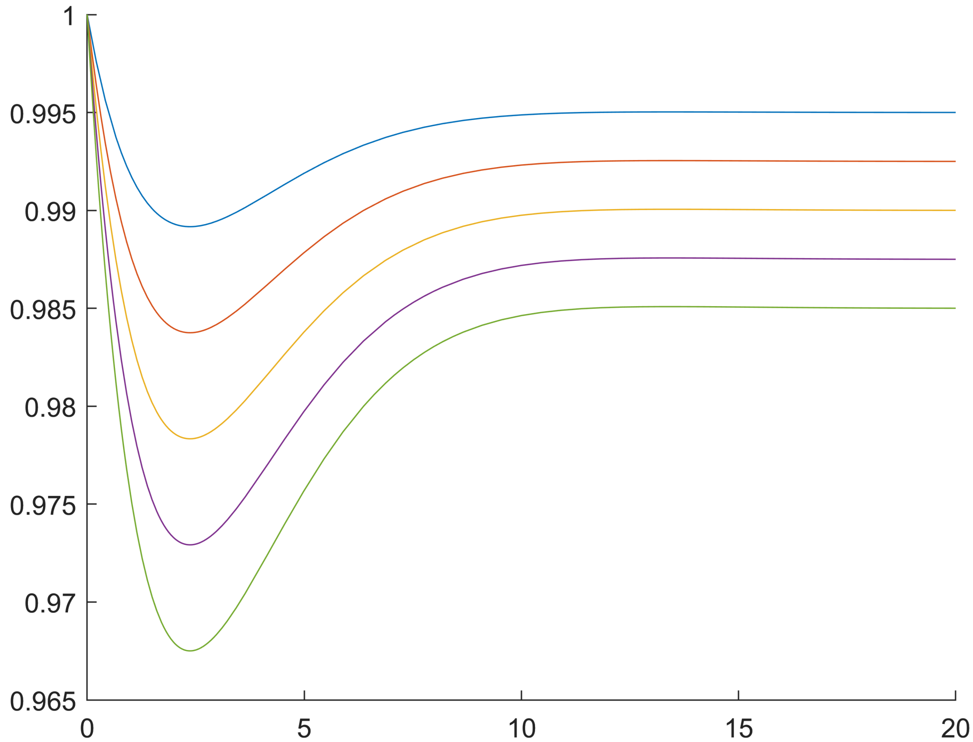
$$\frac{79}{80} - \frac{\sqrt{14}\ \sqrt{109}\ e^{-\frac{77\,t}{160}} \sin\left(\frac{\sqrt{2071}\ t}{160} - \frac{1260747285905809}{9007199254740992}\right)}{436}$$

ft =

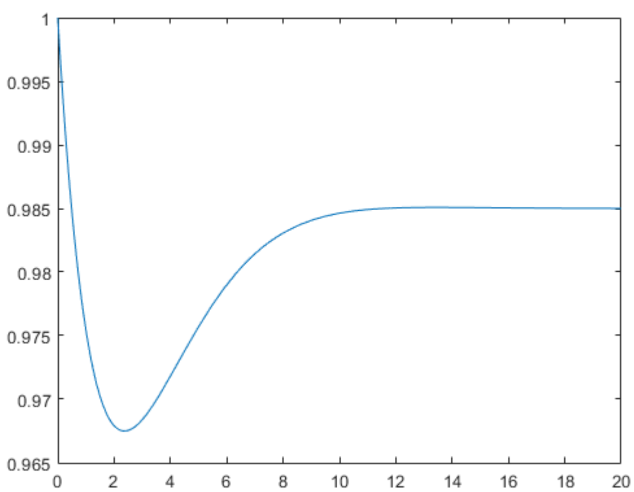
$$\frac{197}{200} - \frac{3\ \sqrt{14}\ \sqrt{109}\ e^{-\frac{77\,t}{160}} \sin\left(\frac{\sqrt{2071}\ t}{160} - \frac{1260747285905809}{9007199254740992}\right)}{1090}$$

```
hold off

xlim([0 20])
ylim([0.965 1])
```



This results in the following graph:



which looks right, given the assumptions from the SFR paper, which states that the oscillations beyond the first dip should be suitably dampened. Comparing to my Simulink model, the results are similar.

Next step is to create the GSFR function and Delta Frequency in the Laplace domain.

$$G_{SFR}(s) = \left(\frac{R{\omega_n}^2}{DR+K_m}\right)\left(\frac{1+T_R s}{s^2+2\zeta\omega_n s+{\omega_n}^2}\right)$$

```
%Make the transfer function
num = [Tr 1];
den = [1 2*c*wn wn^2];
transferFunc = tf(num,den);

GSFR = ((R*(wn^2))/(D*R+K))*transferFunc
```

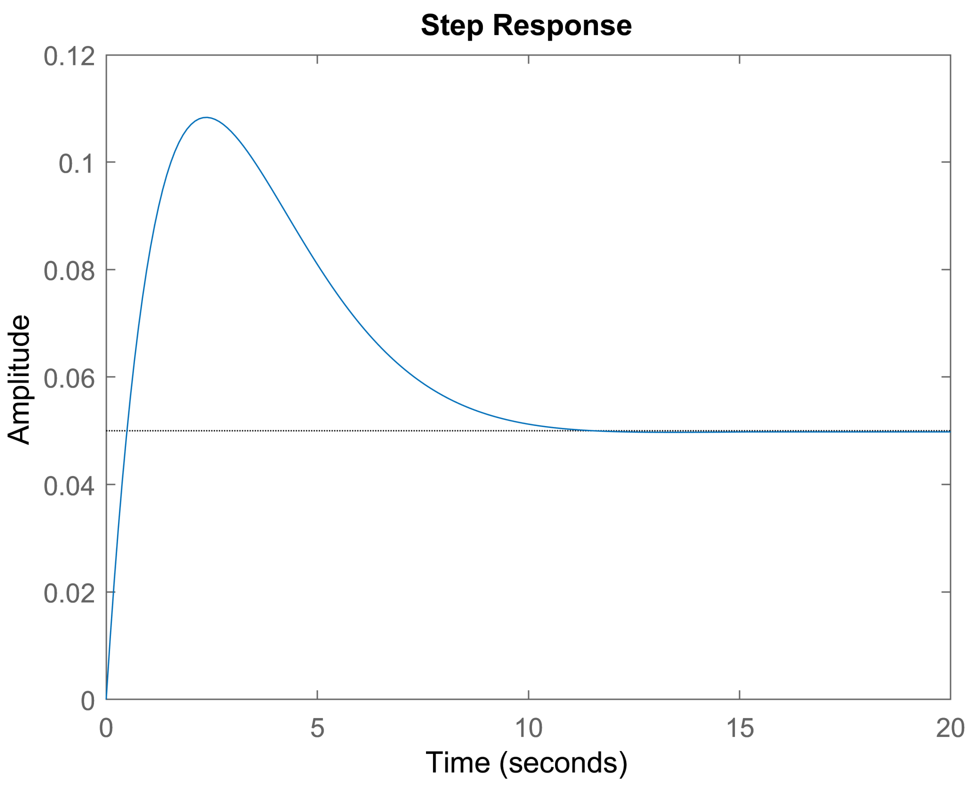
```
GSFR =

      0.125 s + 0.01563
  ------------------------
  s^2 + 0.9625 s + 0.3125

Continuous-time transfer function.
```

```
step(GSFR) % cannot use fplot because it doesnt support the transfer func

xlim([0 20])
ylim([0 0.12])
```



Now working to find the new frequency nadir in efforts to find the frequency change in t.

```
fs = 1 - ((R*Ps)/(D*R+K))
```

```
fs = 0.9850
```

```
tn = (1/wr)*atan((wr*Tr)/(c*wn*Tr-1))
```

```
tn = 2.3688
```

```
fn = 1 - ((R*Ps)/(D*R+K))*(1+a*exp(-c*wn*tn)*sin(wr*tn+phi))
```

```
fn = 0.9675
```

Next, we make:

$$f_n^{new} = \beta d_{tr} + f_n$$

$f_s - f_n$ = Dtr

```
B = 0.9; % Correction factor
Dtr = fs - fn
```

```
Dtr = 0.0175
```

```
fnew = B*Dtr + fn
```

```
fnew = 0.9833
```

Let $u(t)$ be the unit-step function occurring at $t = 0$. With the definitions just put forward and based on (9), the frequency correction needed is

$$\Delta f(t) = \left(f_n^{new} - f(t)\right)\left(u(t - t_1) - u(t - t_2)\right) \quad (11)$$

This enhanced frequency response is flattened from t_1 to t_2 and coincides with the original frequency response elsewhere. From (9), one can easily conclude that t_1 and t_2 are the first and second positive roots of the equation below:

$$f(t) - (\beta d_{tr} + f_n) = 0 \quad (10)$$

```
tau = vpasolve(ft-(B*Dtr+fn)==0,t)
```

tau = $0.56729274321346977795480309648809$

```
tau2 = vpasolve(ft-(B*Dtr+fn)==0,t,10)
```

tau2 = $8.1486716887171998250668189267115$

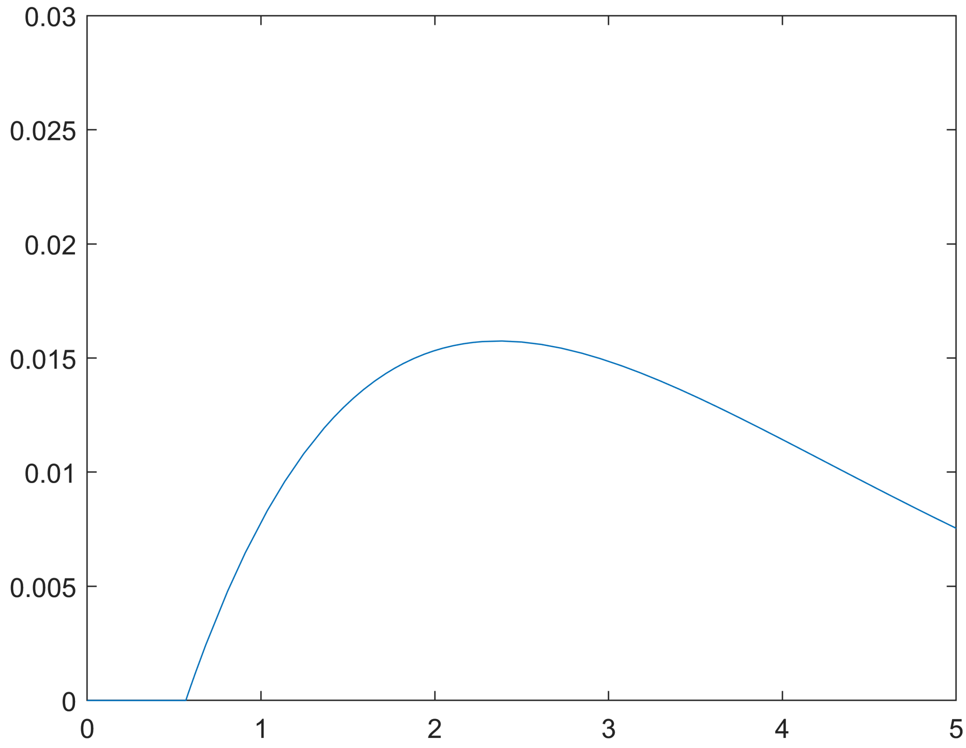
Dr. Azizi requested the equation without the second tau

```
dft = (fnew - ft)*(heaviside(t-tau))
```

```
dft =
```

$$\mathrm{heaviside}(1.0\,t - 0.56729274321346977795480309648809)\left(\frac{3\ \sqrt{14}\ \ \sqrt{109}\ e^{-\frac{77\,t}{160}}\sin\left(\frac{\sqrt{2071}\ t}{160} - \frac{12607}{90071}\right.}{1090}\right.$$

```
fplot(dft)
xlim([0 5])
ylim([0 0.03])
```



find delta P(s)

```
syms s;
dF = laplace(dft)
```

dF =

$$\frac{0.00039112444170280744176392161518062\,e^{-0.56729274321346977795480309648809\,s}\ (549777627765916324140}{}$$

```
G = ((R*(wn^2))/(D*R+K))*((1+Tr*s)/(s^2+2*c*wn*s+wn^2))
```

G =

$$\frac{8\,s+1}{64\ \left(s^2+\frac{77\,s}{80}+\frac{5}{16}\right)}$$

```
dP = dF/G
```

dP =

$$\frac{0.025031964268979676272890983371 56\,e^{-0.56729274321346977795480309648809\,s}\,(54977762776591632.4140216}{}$$

where

$$\sigma_1 = 6.4e{+}65\,s^2 + 6.16e{+}65\,s + 2.0e{+}65$$

```
dpt = ilaplace(dP)
```

dpt =

$$\frac{6998117152103.0205440183635801077\,\sigma_5\,(\sigma_7 - 1)\,\sigma_1}{\sigma_2} - \frac{22393974886729.665740858763456345\,\Big(0.125}{}$$

where

$$\sigma_1 = 54977762776591632.414021674525815\,\sin(0.28442650632456884291693803355929\,t - 0.139971$$

$$\sigma_2 = 109955525553183264828043349051 63.0\,\sin(0.28442650632456884291693803355929\,t - 0.13997$$

$$\sigma_3 = e^{0.27300963267148233064074899018489 + t\,(-0.48125 + 0.28442650632456884291693803355929\,i) - 0.16135309301548797034025200769949\,i}$$

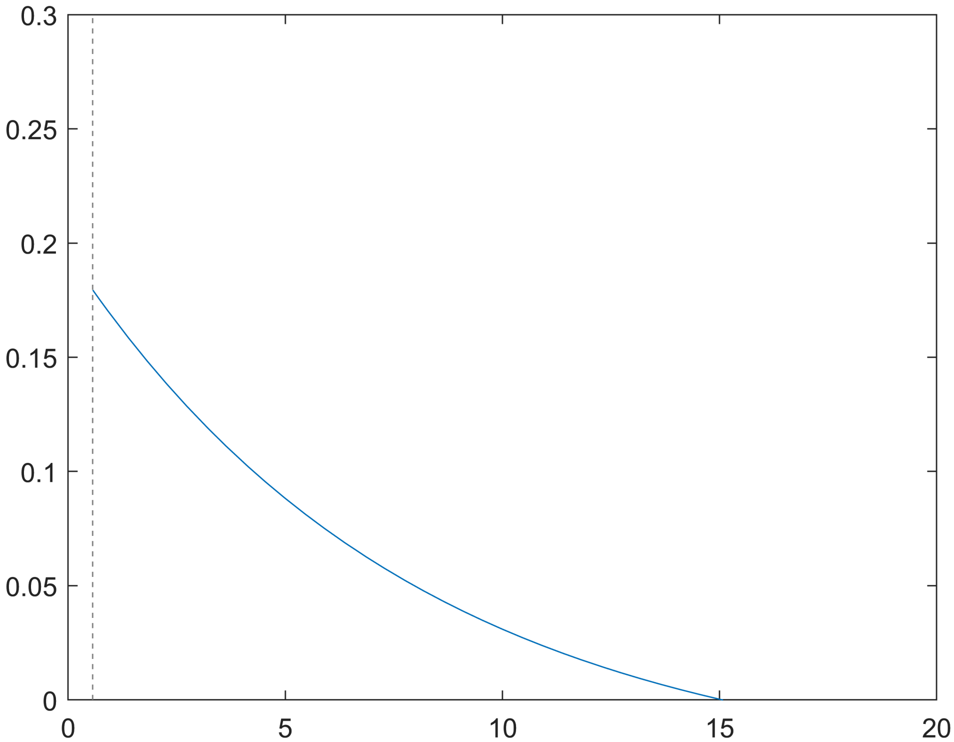
$$\sigma_4 = e^{0.27300963267148233064074899018489 + t\,(-0.48125 - 0.28442650632456884291693803355929\,i) + 0.16135309301548797034025200769949\,i}$$

$$\sigma_5 = \mathrm{heaviside}(1.0\,t - 0.56729274321346977795480309648809)$$

$$\sigma_6 = e^{0.070911592901683722244350387061012 - 0.125\,t}$$

$$\sigma_7 = e^{0.070911592901683722244350387061012 - \frac{t}{8}}$$

```
fplot(dpt)
xlim([0 20])
ylim([0 0.3])
```



0.3
0.25
0.2
0.15
0.1
0.05
0
0
5
10
15
20