Deep Neural Networks for Direction of Arrival Estimation of Multiple Targets with Sparse Prior for Line-of-Sight Scenarios

Signal Sparse Representation

$\Phi = [\phi_1, \phi_2, \dots, \phi_P]^T$: potential space of DOAs with $\Delta\phi = \phi_{p+1} - \phi_p \ \forall p$

$$y(t) = \sum_{p=1}^{P} a(\phi_p)\,\bar{s}_p(t) + n(t) \quad \text{where} \quad \bar{s}_p(t) = \begin{cases} s_k(t) & \text{if } \phi_p = \theta_k \\ 0 & \text{o.w.} \end{cases}$$

$$\Rightarrow R = \mathcal{E}\{y(t)y^H(t)\} = \sum_{p=1}^{P} \eta_p\, a(\phi_p)a^H(\phi_p) + \sigma_n^2 I_M, \quad \eta_p = \mathcal{E}\{\bar{s}_p(t)\bar{s}_p^H(t)\}$$

$$c_m = \sum_{p=1}^{P} \eta_p\, a(\phi_p)a^H(\phi_p)e_m + \sigma_n^2 I_M e_m, \quad e_m = [0\ 0\ \cdots\ 0\ \underset{m^{th}\text{ location}}{1}\ 0\ \cdots\ 0]^T \in R^{M\times 1}$$

(c_m: m^{th} column of R)

$$= [a(\phi_1)a^H(\phi_1)e_m \ \cdots \ a(\phi_P)a^H(\phi_P)e_m]\,\eta + \sigma_n^2 e_m \qquad (\eta = [\eta_1\ \eta_2\ \cdots\ \eta_P]^T)$$

$$= A_m \eta + \sigma_n^2 e_m$$

$$\rightarrow c = \text{vec}(R) = [c_1; c_2; \cdots; c_P]$$

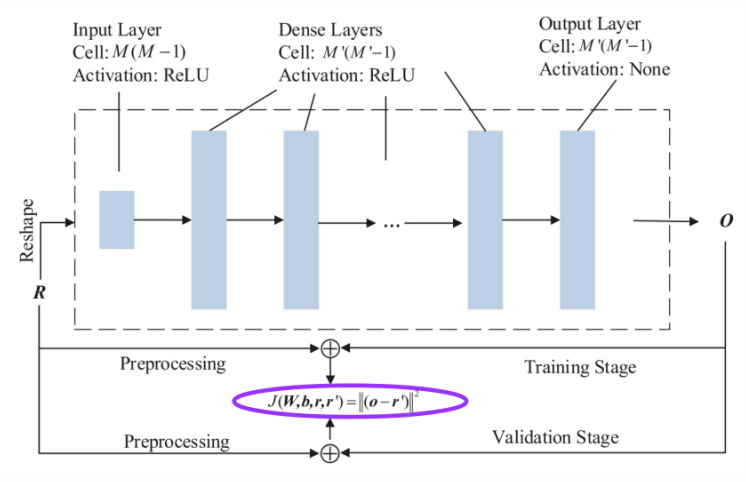
$$= [A_1; A_2; \cdots; A_M]\,\eta + \sigma_n^2 [e_1; e_2; \cdots; e_M]$$

$$= \tilde{A}\eta + \sigma_n^2 \vec{I}_n$$

($A^* \odot A = [a_1^* \otimes a_1 \ \ a_2^* \otimes a_2 \ \cdots \ a_P^* \otimes a_P]$)

$$\rightarrow \eta = \tilde{A}^H c \quad \text{(reconstructed sparse spatial spectrum)}$$

DNN Learning Framework



$$\tilde{r} = [R_{12}\ R_{13}\ \cdots\ R_{ij}\ \cdots\ R_{M-1,M}]$$

$$\hat{r} = [\text{Re}\{\tilde{r}\},\ \text{Im}\{\tilde{r}\}]$$

$$\rightarrow r = \frac{\hat{r} - \mu_r}{\sigma_r}$$

$$\rightarrow o^q = \begin{cases} \sigma(z^q) = \sigma(W^q o^{q-1} + b^q), & q = 1,2,\dots,Q-1 \quad (\sigma = \text{ReLU}) \\ W^q o^{q-1} + b^q, & q = Q \end{cases}$$

$$W^q = W^{q-1} - \mu\left[\frac{\partial J(W,b,r,r')}{\partial W^{q-1}}\right]$$

$$b^q = b^{q-1} - \mu\left[\frac{\partial J(W,b,r,r')}{\partial b^{q-1}}\right]$$