Underdetermined DOA Estimation Under The Compressive Sensing Framework: A Review

M : # of sensors, L : # of snapshots, K : # of sources

$\bar{N}$: # of possible sources

$$\bar{\Theta} = \{\bar{\theta}_1, \bar{\theta}_2, \dots, \bar{\theta}_{\bar{N}}\}$$

$$y_{M\times L} = A_{M\times K}\, s_{K\times L} + n_{M\times L}$$

$$A = \left[a(\theta_1)\ a(\theta_2)\ \cdots\ a(\theta_K)\right] \qquad s = \begin{bmatrix} s_1^T \\ s_2^T \\ \vdots \\ s_K^T \end{bmatrix}$$

$$y_{M\times L} = \bar{A}_{M\times \bar{N}}\, \bar{s}_{\bar{N}\times L} + n_{M\times L}$$

$$\bar{A} = \left[a(\bar{\theta}_1)\ a(\bar{\theta}_2)\ \cdots\ a(\bar{\theta}_{\bar{N}})\right]$$

$$\bar{s}_n(t) = \begin{cases} s_k(t) & \text{if } \bar{\theta}_n = \theta_k \\ 0 & \text{otherwise} \end{cases}$$

n^{th} row of t^{th} snapshot

CS-Based DOA Estimation For A Single Snapshot

$$\min \|\bar{s}\|_1 \ \text{subject to} \ \|y - \bar{A}\bar{s}\|_2 < \varepsilon$$

$$\min \ \lambda_1 \cdot \|\bar{s}\|_1 + \lambda_2 \cdot \|y - \bar{A}\bar{s}\|_2$$

CS-Based DOA Estimation For Multiple Snapshots

$$\min \|\hat{s}\|_1 \text{ subject to } \|\mathrm{vec}\{y - \bar{A}\bar{s}\}\|_2 < \epsilon$$

$$\min \lambda_1 \cdot \|\hat{s}\|_1 + \lambda_2 \cdot \|\mathrm{vec}\{y - \bar{A}\bar{s}\}\|_2$$

where $\hat{s} = \left[\|\bar{s}_0\|_2 \ \|\bar{s}_1\|_2 \ \cdots \ \|\bar{s}_{\bar{N}-1}\|_2\right]^T$

CS-Based DOA Estimation Employing The Difference Coarray Concept

1. $z = \mathrm{vec}\{R_y\} = \tilde{A}\tilde{s} + \mathrm{vec}\{\sigma_n^2 I_{M\times M}\}$

where $\tilde{A} = A^* \odot A$ and $\tilde{s} = [\sigma_1^2 \ \sigma_2^2 \ \cdots \ \sigma_K^2]^T$

2. $z = \tilde{\bar{A}}\tilde{\bar{s}} + \sigma_n^2 \cdot \mathrm{vec}\{I_{M\times M}\}$

where $\tilde{\bar{A}} = \bar{A}^* \odot \bar{A}$ and $\tilde{\bar{s}}_n = \begin{cases} \sigma_k^2 & \text{if } \bar{\theta}_n = \theta_k \\ 0 & \text{otherwise} \end{cases}$

$\longrightarrow$ Let $\tilde{\bar{A}}^o = \left[\tilde{\bar{A}} \ \ \mathrm{vec}\{I_{M\times M}\}\right]$ and $\tilde{\bar{s}}^o = \left[\tilde{\bar{s}}^T \ \ \sigma_n^2\right]^T$

$$\Longrightarrow z = \tilde{\bar{A}}^o \tilde{\bar{s}}^o$$

$$\Longrightarrow \min \|\tilde{\bar{s}}_o\|_1 \text{ subject to } \|z - \tilde{\bar{A}}^o\tilde{\bar{s}}^o\|_2 \leq \epsilon$$

$$\min \lambda_1 \cdot \|\tilde{\bar{s}}_o\|_1 + \lambda_2 \cdot \|z - \tilde{\bar{A}}^o\tilde{\bar{s}}^o\|_2$$

Sparse Methods for DOA Estimation

Off - Grid Sparse Method

$$\bar{\Theta} = \{\bar{\theta}_1, \bar{\theta}_2, \dots, \bar{\theta}_{\bar{N}}\}$$

$$r = \bar{\theta}_{n+1} - \bar{\theta}_n \propto \frac{1}{\bar{N}}$$

For any DOA θ_k, suppose $\bar{\theta}_{n_k}$ is the nearest grid point with $|\theta_k - \bar{\theta}_{n_k}| \leqslant \frac{r}{2}$. Thus, the steering vector $a(\theta_k)$ can be approximated as:

Taylor expansion

$$a(\theta_k) \approx a(\bar{\theta}_{n_k}) + a'(\bar{\theta}_{n_k})(\theta_k - \bar{\theta}_{n_k})$$

$$\Longrightarrow \quad y = \Phi \bar{s} + n$$

$$\bar{s}_n(t) = \begin{cases} s_k(t) & \text{if } \bar{\theta}_n = \bar{\theta}_{n_k} \\ 0 & \text{otherwise} \end{cases}$$

$$\Phi = \bar{A} + \bar{B}\,\mathrm{diag}(\beta)$$

$$\bar{A} = [a(\bar{\theta}_1)\ a(\bar{\theta}_2) \cdots a(\bar{\theta}_{\bar{N}})], \quad \bar{B} = [a'(\bar{\theta}_1)\ a'(\bar{\theta}_2) \cdots a'(\bar{\theta}_{\bar{N}})]$$

$$\beta = [\beta_1\ \beta_2 \cdots \beta_{\bar{N}}] \text{ where } \beta_n = \begin{cases} \theta_k - \bar{\theta}_{n_k} & \text{if } \bar{\theta}_n = \bar{\theta}_{n_k} \\ 0 & \text{otherwise} \end{cases}$$

$$\boxed{\begin{aligned} &\Longrightarrow \min_{\bar{s}, \beta} \ \lambda_1 \|\hat{s}\|_1 + \frac{1}{2} \left\| \mathrm{vec}\{y - (\bar{A} + \bar{B}\beta)\bar{s}\} \right\|_2^2 + \lambda_2 \|\beta\|_2^2 \\ &\text{where } \hat{s} = \left[\|\bar{s}_0\|_2\ \|\bar{s}_1\|_2 \cdots \|\bar{s}_{\bar{N}-1}\|_2 \right]^T \end{aligned}}$$