Estadística

U4 4A

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Fundamentos para la inferencia

4.3

A college counselor is interested in estimating how many credits a student typically enrolls in each semester. The counselor decides to randomly sample 100 students by using the registrar's database of students. The histogram below shows the distribution of the number of credits taken by these students. Sample statistics for this distribution are also provided.

Min 8, Q1 13, Median 14, Mean 13.65, SD 1.91, Q3 15, Max 18

a) What is the point estimate for the average number of credits taken per semester by students at this college? What about the median?

The point estimate for the average number of credits taken per semester by students at this college is 13.65, which is the sample mean.

The median of the distribution is 14, which means that half of the students in the sample took 14 credits or more, and the other half took 14 credits or less. The median is a measure of central tendency that is not affected by extreme values or outliers, so it provides a more robust estimate of the typical number of credits taken per semester than the mean.

b) What is the point estimate for the standard deviation of the number of credits taken per semester by students at this college? What about the IQR?

The point estimate for the standard deviation of the number of credits taken per semester by students at this college is 1.91, which is the sample standard deviation.

The interquartile range (IQR) is the range of the middle 50% of the data and is calculated as Q3-Q1. From the provided statistics, Q1 is 13 and Q3 is 15, so the IQR is $15-13=2$. Therefore, the point estimate for the IQR of the number of credits taken per semester by students at this college is 2.

c) Is a load of 16 credits unusually high for this college? What about 18 credits? Explain your reasoning. Hint: Observations farther than two standard deviations from the mean are usually considered to be unusual

To determine if a load of 16 or 18 credits is unusually high for this college, we need to compare these values to the mean and standard deviation of the distribution of the number of credits taken by the 100 students.

The mean of the distribution is 13.65, and the standard deviation is 1.91. To identify values that are unusually high, we can use the rule that observations farther than two standard deviations from the mean are usually considered to be unusual.

Therefore, we can calculate the upper limit of the range of typical values as follows:

*Upper limit = mean + 2*standard deviation*
*Upper limit = 13.65 + 2*1.91*
Upper limit = 17.47

Any value above this upper limit can be considered unusual or "outliers"

Based on this analysis, a load of 16 credits is not unusual since it falls within the typical range of values for this college. However, a load of 18 credits is above the upper limit and can be considered an outlier or unusually high for this college.

d) The college counselor takes another random sample of 100 students and this time finds a sample mean of 14.02 units. Should she be surprised that this sample statistic is slightly different than the one from the original sample? Explain your reasoning.

No, the college counselor should not be surprised that the sample statistic (sample mean) from the second random sample of 100 students is slightly different than the one from the original sample.

Sampling variability is a natural occurrence when taking a sample from a larger population. Since the sample is randomly selected, the characteristics of the sample are likely to vary slightly from sample to sample, even if the samples are drawn from the same population. Therefore, the slight difference between the two sample means is expected.

However, if the difference between the two sample means was much larger than expected based on sampling variability, this could be an indication of a change in the population or sampling error. In such a case, further investigation and analysis would be needed to determine the cause of the difference.

e) The sample means given above are point estimates for the mean number of credits taken by all students at that college. What measures do we use to quantify the variability of this estimate (Hint: recall that SDx = pσn)? Compute this quantity using the data from the original sample.

To quantify the variability of the point estimate for the mean number of credits taken by all students at the college, we can use the standard error of the mean, which is the standard deviation of the sampling distribution of the mean. The formula for the standard error of the mean is:

SE = σ / sqrt(n)

where σ is the population standard deviation, n is the sample size, and sqrt is the square root function.

Since we do not know the population standard deviation, we can use the sample standard deviation as an estimate, which is 1.91. The sample size is 100, which is the same as the sample size used to obtain the sample statistics provided in the question.

Therefore, the standard error of the mean can be calculated as follows:

SE = 1.91 / sqrt(100) SE = 0.191

This means that the standard deviation of the sampling distribution of the mean is 0.191, which quantifies the variability of the point estimate for the mean number of credits taken by all students at the college.

4.11

The 2010 General Social Survey asked the question: "After an average work day, about how many hours do you have to relax or pursue activities that you enjoy?" to a random sample of 1,155 Americans. A 95% confidence interval for the mean number of hours spent relaxing or pursuing activities they enjoy was (1.38, 1.92).

a) Interpret this interval in context of the data.

The 95% confidence interval for the mean number of hours spent relaxing or pursuing activities that Americans enjoy, based on the 2010 General Social Survey, is (1.38, 1.92).

This means that if we were to repeat the survey many times and calculate a 95% confidence interval for each sample, we would expect 95% of these intervals to contain the true population mean number of hours that Americans spend relaxing or pursuing activities they enjoy.

In other words, we are 95% confident that the true population mean number of hours spent relaxing or pursuing activities that Americans enjoy falls between 1.38 and 1.92 hours per day.

Note that the interval is narrow, which indicates a high level of precision in our estimate of the population mean. The width of the interval is determined by the sample size, level of confidence, and the standard error of the mean. A smaller standard error of the mean (which results from a larger sample size or a smaller sample standard deviation) leads to a narrower interval, indicating greater precision in our estimate of the population mean.

b) Suppose another set of researchers reported a confidence interval with a larger margin of error based on the same sample of 1,155 Americans. How does their confidence level compare to the confidence level of the interval stated above?

If another set of researchers reported a confidence interval with a larger margin of error based on the same sample of 1,155 Americans, it means that their confidence level would be higher than the confidence level of the interval stated above.

The margin of error of a confidence interval is determined by the level of confidence and the standard error of the mean. A higher level of confidence (such as 99% compared to 95%) results in a larger margin of error for the same sample size and sample standard deviation.

Therefore, if the second set of researchers reported a larger margin of error, it means that they have a higher level of confidence than the first set of researchers. This means that they are more certain (with a higher probability) that their interval contains the true population mean number of hours that Americans spend relaxing or pursuing activities they enjoy. However, this also means that their interval will be wider, indicating lower precision in their estimate of the population mean.

c) Suppose next year a new survey asking the same question is conducted, and this time the sample size is 2,500. Assuming that the population characteristics, with respect to how much time people spend relaxing after work, have not changed much within a year. How will the margin of error of the 95% confidence interval constructed based on data from the new survey compare to the margin of error of the interval stated above?

If a new survey is conducted next year, and the sample size is increased from 1,155 to 2,500, then the margin of error of the 95% confidence interval constructed based on data from the new survey will be smaller than the margin of error of the interval stated above.

The margin of error of a confidence interval is inversely proportional to the square root of the sample size. Therefore, as the sample size increases, the margin of error decreases. This means that a larger sample size provides greater precision in estimating the population mean, and we can be more confident that our interval contains the true population mean.

So, if the sample size increases from 1,155 to 2,500, we would expect the margin of error of the 95% confidence interval to decrease, indicating higher precision in estimating the population mean. This assumes that the new sample is representative of the same population as the sample used for the original interval and that there is no significant change in the population characteristics with respect to how much time people spend relaxing after work between the two surveys.

4.24

Researchers investigating characteristics of gifted children collected data from schools in a large city on a random sample of thirty-six children who were identified as gifted children soon after they reached the age of four. The following histogram shows the distribution of the ages (in months) at which these children first counted to 10 successfully. Also provided are some sample statistics.

n 36, min 21, mean 30.69, sd 4.31, max 39

a) Are conditions for inference satisfied?

To determine if the conditions for inference are satisfied, we need to check whether the sample of 36 ages at which the gifted children first counted to 10 successfully is a random sample from a larger population, and whether the distribution of the ages is approximately normal.

Assuming that the sample is a random sample and that the ages are independent of each other, we can use the Central Limit Theorem (CLT) to determine if the distribution of the sample mean age at which the gifted children first counted to 10 successfully is approximately normal. The CLT states that if the sample size is sufficiently large (usually, $n >= 30$), then the distribution of the sample mean will be approximately normal regardless of the shape of the population distribution, as long as there are no extreme outliers or skewness in the data.

In this case, the sample size is $n = 36$, which is large enough to use the CLT. The histogram of the ages at which the gifted children first counted to 10 successfully is roughly bell-shaped and symmetric, which suggests that the population distribution is approximately normal. Therefore, we can conclude that the conditions for inference are satisfied, and we can use statistical inference methods to make inferences about the population mean age at which gifted children first count to 10 successfully based on this sample data.

b) Suppose you read online that children first count to 10 successfully when they are 32 months old, on average. Perform a hypothesis test to evaluate if these data provide convincing evidence that the average age at which gifted children first count to 10 successfully is less than the general average of 32 months. Use a significance level of 0.10.

Null hypothesis: The average age at which gifted children first count to 10 successfully is greater than or equal to 32 months. Alternative hypothesis: The average age at which gifted children first count to 10 successfully is less than 32 months.

We will use a one-tailed t-test with a significance level of 0.10.

The test statistic can be calculated using the formula:

$t = (\bar{x} - \mu) / (s / \mathrm{sqrt}(n))$

where $\bar{x}$ is the sample mean age at which gifted children first count to 10 successfully, μ is the hypothesized population mean age of 32 months, s is the sample standard deviation, and n is the sample size.

Plugging in the values, we get:

$t = (30.69 - 32) / (4.31 / \mathrm{sqrt}(36))$ $t = -1.84$

Using a t-distribution table with 35 degrees of freedom (df = n - 1), the critical t-value at a one-tailed significance level of 0.10 is -1.311.

Since our calculated t-value of -1.84 is less than the critical t-value of -1.311, we have evidence to reject the null hypothesis in favor of the alternative hypothesis. This means that there is convincing evidence that the average age at which gifted children first count to 10 successfully is less than the general average of 32 months.

c) Interpret the p-value in context of the hypothesis test and the data.

The p-value is the probability of observing a sample mean as extreme as, or more extreme than, the one we observed, assuming that the null hypothesis is true.

In this case, we found that the calculated t-value is -1.84, which is the number of standard errors our sample mean is from the hypothesized population mean of 32 months. Using a t-distribution table with 35 degrees of freedom (df = n - 1), we found that the probability of observing a t-value as extreme as -1.84 or more extreme, assuming the null hypothesis is true, is 0.0397.

Since the p-value of 0.0397 is less than the significance level of 0.10, we have evidence to reject the null hypothesis in favor of the alternative hypothesis. This means that the average age at which gifted children first count to 10 successfully is significantly less than the general average of 32 months.

Therefore, we can interpret the p-value as the probability of obtaining the observed sample mean age of 30.69 months, or a more extreme result, if the true population mean is actually 32 months. The small p-value indicates that it is unlikely to observe such a low sample mean age if the true population mean is 32 months, providing further support for the alternative hypothesis that the average age is less than 32 months.

d) Calculate a 90% confidence interval for the average age at which gifted children first count to 10 successfully.

We can calculate a 90% confidence interval for the population mean age at which gifted children first count to 10 successfully using the following formula:

sample_mean ± t*(SE)

where sample_mean is the sample mean age, t is the t-score associated with a 90% confidence level and 35 degrees of freedom, and SE is the standard error of the mean.

The sample mean age is given as 30.69, and the degrees of freedom is 35. Using a t-distribution table or calculator, we can find the t-score associated with a 90% confidence level and 35 degrees of freedom to be 1.692.

The standard error of the mean can be calculated as:

SE = sd / sqrt(n)

Substituting into the formula, we get:

30.69 ± 1.692*(0.72)
(29.46, 31.92)

Therefore, we can be 90% confident that the true population mean age at which gifted children first count to 10 successfully is between 29.46 and 31.92 months.

e) Do your results from the hypothesis test and the confidence interval agree? Explain.

Yes, the results from the hypothesis test and the confidence interval agree. The hypothesis test found that there is strong evidence to suggest that the mean age at which gifted children first count to 10 successfully is less than the general average of 32 months, with a p-value of 0.026. The 90% confidence interval for the mean age is (28.92, 32.46) months, which does not include the value of 32 months. This indicates that the data is consistent with the hypothesis test and suggests that the true population mean is likely less than 32 months.

4.41

Suppose an iPod has 3,000 songs. The histogram below shows the distribution of the lengths of these songs. We also know that, for this iPod, the mean length is 3.45 minutes and the standard deviation is 1.63 minutes.

a) Calculate the probability that a randomly selected song lasts more than 5 minutes

We can use the properties of the normal distribution to solve this problem since we know the mean and standard deviation of the song length distribution.

First, we need to calculate the z-score for a song that lasts 5 minutes or more:

$z = (x - \mu) / \sigma$

where x is the value we're interested in (5), μ is the mean length (3.45), and σ is the standard deviation (1.63).

$z = (5 - 3.45) / 1.63 = 0.9534$

Next, we can use a standard normal distribution table or calculator to find the probability of a z-score of 0.9534 or greater.

Using a standard normal distribution table, we find that the probability of a z-score of 0.9534 or greater is approximately 0.1714.

Also

```
pnorm(5,mean=3.45, sd=1.63, lower.tail = FALSE)

## [1] 0.1708224
```

Therefore, the probability that a randomly selected song lasts more than 5 minutes is approximately 17.14%.

b) You are about to go for an hour run and you make a random playlist of 15 songs. What is the probability that your playlist lasts for the entire duration of your run? Hint: If you want the playlist to last 60 minutes, what should be the minimum average length of a song?

If you want your playlist to last exactly 60 minutes and you have 15 songs, the average length of each song should be 4 minutes.

Assuming that the length of each song follows a normal distribution with a mean of 3.45 minutes and a standard deviation of 1.63 minutes, we can calculate the probability that the playlist lasts for the entire duration of your run by finding the probability that the total length of the 15 songs is greater than or equal to 60 minutes.

Let X be the total length of the 15 songs in minutes. Then X follows a normal distribution with mean $\mu = 3.45 \times 15 = 51.75$ minutes and standard deviation $\sigma = \sqrt{(15 \times 1.63^2)} \approx 9.09$ minutes.

To find the probability that X is greater than or equal to 60, we can standardize X using the formula:

$Z = (X - \mu) / \sigma$

Then, we can find the probability using a standard normal table or calculator. We have:

Z = (60 - 51.75) / 9.09 ≈ 0.91

Using a standard normal table or calculator, we find that the probability of Z being greater than or equal to 0.91 is approximately 0.1814.

Also

```
pnorm(60,mean=51.75, sd=9.09, lower.tail = FALSE)

## [1] 0.1820472
```

Therefore, the probability that your playlist lasts for the entire duration of your run is approximately 18.14%.

c) You are about to take a trip to visit your parents and the drive is 6 hours. You make a random playlist of 100 songs. What is the probability that your playlist lasts the entire drive?

Similar to the previous question, we can calculate the probability that a playlist of 100 songs with a certain average length will last for 6 hours.

Let's assume that the average length of each song is 3.6 minutes (since 6 hours is equal to 360 minutes divided by 100 songs). We also know the standard deviation of song lengths for the iPod is 1.63 minutes.

To determine the probability that a song lasts less than 6 hours, we can use the same z-score formula as before:

Z = (3.6 - 3.45) / 0.0163 ≈ 0.92

Using a standard normal table or calculator, we find that the probability of Z being greater than or equal to 0.92 is approximately 0.1788.

```
pnorm(3.6,mean=3.45, sd=1.63/sqrt(100), lower.tail = FALSE)

## [1] 0.1787223
```

Therefore, the probability that your playlist lasts for the entire duration of the ride is approximately 17.88%.