(Homework VI) Math 31AH Fall 2025

Assume $\mathbb{N}, \mathbb{Q}, \mathbb{R}$ are used in the usual sense. We are using same notations as in class. $\mathcal{L}(S)$ denotes the linear span of S, and $\mathcal{E}_m$ denotes the standard basis of $\mathbb{R}^m$.

Problem I. Prove that the characteristic polynomial of the matrix $A \in M_n(\mathbb{R})$ defined as

$$\begin{bmatrix} \lambda Id_m & C \\ 0_{n-m \times m} & D \end{bmatrix}$$

is of the form $\chi(A) = (\lambda - x)^m \chi(D)$.