

(Homework VIII) Math 31AH Fall 2025

Assume  $\mathbb{N}, \mathbb{Q}, \mathbb{R}$  are used in the usual sense. We are using same notations as in class.  
 $\mathcal{L}(S)$  denotes the linear span of  $S$ , and  $\mathcal{E}_m$  denotes the standard basis of  $\mathbb{R}^m$ .

**Problem I.** Let  $S \subset \mathbb{R}^4$  be a linear subspace spanned by the vectors

$$\begin{bmatrix} 1 \\ 2 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 3 \\ 2 \\ 1 \end{bmatrix}.$$

Find an orthonormal set that spans  $S$ .