KBA-03 - Derivation of Parametric Equations for the Alpha Factor Common Cause Model

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1 Introduction

This KBA walks through the derivation of both the non-staggered and staggered alpha factor common cause equations. NUREG/CR-5485 provides detail on the derivation of the non-staggered alpha factor common cause equation but does not go into detail regarding the staggered equation.

2 Alpha Factor Common Cause Models

2.1 Non-Staggered

Page A-8 of NUREG/CR-5485 states: *In terms of the basic event probabilities, the alpha factor parameters for non-staggered testing are defined as:*

$$\alpha_k^{(m)} = \frac{\binom{m}{k} Q_k^{(m)}}{\sum_{k=1}^{m} \binom{m}{k} Q_k^{(m)}}$$

The total *frequency* of events involving a given component can be defined as:

$$Q_t = \sum_{k=1}^{m} \binom{m-1}{k-1} Q_k^{(m)}$$

For example, we can expand this expression out assuming we have a group of three components and obtain, for a given component (e.g., A):

$$Q_t = Q_1^{(3)} + 2Q_2^{(3)} + Q_3^{(3)} = I_A + C_{AB} + C_{AC} + C_{ABC}$$

Using these two equations we can derive an equation for the $Q_k^{(m)}$ terms as a function of the alpha factors.

$$\sum_{k=1}^{m} \binom{m}{k} Q_k^{(m)} \alpha_k^{(m)} = \binom{m}{k} Q_k^{(m)}$$

Using the fact that $\binom{m}{k} = \frac{m}{k}\binom{m-1}{k-1}$

$$\sum_{k=1}^{m} \binom{m}{k} Q_k^{(m)} \alpha_k^{(m)} = \frac{m}{k} \binom{m-1}{k-1} Q_k^{(m)}$$

$$k \sum_{k=1}^{m} \binom{m}{k} Q_k^{(m)} \alpha_k^{(m)} = m \binom{m-1}{k-1} Q_k^{(m)}$$

Summing both sides over k

$$\sum_{k=1}^{m} k\alpha_k^{(m)} \sum_{k=1}^{m} \binom{m}{k} Q_k^{(m)} = m \sum_{k=1}^{m} \binom{m-1}{k-1} Q_k^{(m)}$$

Recognizing the RHS has a term that is equal to Q_t

$$\sum_{k=1}^{m} k\alpha_k^{(m)} \sum_{k=1}^{m} \binom{m}{k} Q_k^{(m)} = mQ_t$$

By definition, $\sum_{k=1}^{m} k\alpha_k^{(m)} = \alpha_t$

$$\sum_{k=1}^{m} \binom{m}{k} Q_k^{(m)} = \frac{mQ_t}{\alpha_t}$$

Recognizing the LHS is the denominator of in the equation for $\alpha_k^{(m)}$*, and solving for* $Q_k^{(m)}$

$$Q_k^{(m)} = \frac{m}{\binom{m}{k}} \frac{\alpha_k^{(m)}}{\alpha_t} Q_t$$

$$Q_k^{(m)} = \frac{m}{\frac{m}{k}\binom{m-1}{k-1}} \frac{\alpha_k^{(m)}}{\alpha_t} Q_t$$

$$Q_k^{(m)} = \frac{k}{\binom{m-1}{k-1}} \frac{\alpha_k^{(m)}}{\alpha_t} Q_t$$

2.2 Staggered

A process very similar to that for the derivation of the Non-Staggered equation can be followed to derive the equation for a staggered test scheme. It must be recognized that the testing scheme can impact the estimator for the alpha factors, and thus:

$$\alpha_k^{(m)} = \frac{m\binom{m-1}{k-1} Q_k^{(m)}}{\sum_{k=1}^{m} m\binom{m-1}{k-1} Q_k^{(m)}}$$

Using this equation with the definition of Q_t defined in the non-staggered discussion:

$$m \sum_{k=1}^{m} \binom{m-1}{k-1} Q_k^{(m)} \alpha_k^{(m)} = m \binom{m-1}{k-1} Q_k^{(m)}$$

Summing both sides over k

$$m \sum_{k=1}^{m} \alpha_k^{(m)} \sum_{k=1}^{m} \binom{m-1}{k-1} Q_k^{(m)} = m \sum_{k=1}^{m} \binom{m-1}{k-1} Q_k^{(m)}$$

Recognizing that the RHS has a term that is equatl to Q_t

$$m \sum_{k=1}^{m} \alpha_k^{(m)} \sum_{k=1}^{m} \binom{m-1}{k-1} Q_k^{(m)} = mQ_t$$

Given that $\sum_{k=1}^{m} \alpha_k^{(m)} = 1$

$$m \sum_{k=1}^{m} \binom{m-1}{k-1} Q_k^{(m)} = mQ_t$$

Recognizing the LHS is the denominator of the equation for $\alpha_k^{(m)}$ *assuming a staggered test scheme, and solving for* $Q_k^{(m)}$

$$\alpha_k^{(m)} = \frac{m\binom{m-1}{k-1} Q_k^{(m)}}{mQ_t}$$

$$Q_k^{(m)} = \frac{m\alpha_k^{(m)} Q_t}{m\binom{m-1}{k-1}}$$

$$Q_k^{(m)} = \frac{1}{\binom{m-1}{k-1}}\alpha_k^{(m)} Q_t$$

3 References

1. NUREG/CR-5485, *Guidelines on Modeling Common-Cause Failures in Probabilistic Risk Assessment*, U.S. Nuclear Regulatory Commission, November 1998.