Common Cause Failure - The Basic Parameter Model and Alpha Factor Model

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1 Identification of Common Cause Basic Events - Boolean Approach

Before discussing the basic parameter model or the parametric Alpha Factor model, the idea of a common cause basic event (CCBE) is introduces. Take for example a system of three redundant components A, B, and C. The CCBEs for these components would be C_{AB}, C_{AC}, C_{BC} and C_{ABC}. The first event is the common cause event involving components A and B, and the fourth event is the common cause event involving three components. It is important to separate the idea of failure of a system and failures of a component. For that reason, we will first look at failures at the component level. Once again, given the example system of three redundant components, the complete set of basic events, including CCBEs, involving component A is:

$$I_A = Failure\,of\,component\,A\,due\,to\,independent\,causes$$

$$C_{AB} = Failure\,of\,components\,A\,and\,B\,from\,common\,causes$$

$$C_{AC} = Failure\,of\,components\,A\,and\,C\,from\,common\,causes$$

$$C_{ABC} = Failure\,of\,components\,A,\,B,\,and\,C\,from\,common\,causes$$

Thus,

$$A_t = I_A + C_{AB} + C_{AC} + C_{ABC}$$

Assuming now that our example system of three redundant components has a success criteria of 2-out-of-3. This can be phrased in two ways: (1) in terms of successes, or (2) in terms of failures. In terms of success, this means two components must remain operational for the system to be successful. In terms of failures, this means two components must fail to fail the system. As another example, if the success criteria was 3-out-of-3, in terms of success, this means that three components must remain operational for the system to be successful. In terms of failures, this would mean that only one component would need to fail to fail the system.
For our example system (success criteria of 2-out-of-3), the minimal cutsets at the cause level (i.e., at the common cause level) are as follows:

Combinations of Independent Failures:

$$(I_A, I_B) \quad (I_A, I_C) \quad (I_B, I_C)$$

CCBEs representing 2 of 3 failures:

$$(C_{AB}) \quad (C_{AC}) \quad (C_{BC})$$

CCBEs representing 3 of 3 failures:

$$(C_{ABC})$$

The reduced Boolean reprentation of the system failure at the cause leve can be written as:

$$S = (I_A)(I_B) + (I_A)(I_C) + (I_B)(I_C) + C_{AB} + C_{AC} + C_{BC} + C_{ABC}$$

At this point in the discussion, it is also beneficial to note that when splitting the total failure of a component into its independent and common causes, the events are assumed to be mutually exclusive. That is given the occurence of an independent or common cause basic event, the remaining independent or common cause basic events cannot occur. When we talk about probabilities of events this means that the probability of an event *given* another event has occurred, is equal to zero.

2 Parametric Representation of CCBE Probabilities

Quantification of fault trees requires transformation of system Boolean representation to an algebraic one involving probabilities of the basic events. The system failure probability can be calculated as follows:

$$P(S) = P(I_A \cap I_B) + P(I_A \cap I_C) + P(I_B \cap I_C) + P(C_{AB}) + P(C_{AC}) + P(C_{BC}) + P(C_{ABC})$$

PRA models typically assume that failure of redundant equipment are independent. That is, given a failure of one pieced of equipment, the probability of a reduant component failing remains the same. Or in mathematical terms:

$$P(I_A \cap I_B) = P(I_A)P(I_B|I_A) = P(I_A)P(I_B)$$

This simplifies the probability of the system failure to the following:

$$P(S) = P(I_A)P(I_B) + P(I_A)P(I_C) + P(I_B)P(I_C) + P(C_{AB}) + P(C_{AC}) + P(C_{BC}) + P(C_{ABC})$$

For simplicity, it is common to assume that probabilities of similiar events (redundant components) are the same. This is known as the *symmetry* assumption. This takes advantage of the symmetry associated with idnetically redundant components in reducing the number of parameters that need to be quantified. This can further simplifiy the system failure probability as:

$$P(I_A) = P(I_B) = P(I_C) = Q_1$$

$$P(C_{AB}) = P(C_{AC}) = P(C_{BC}) = Q_2$$

$$P(C_{ABC}) = Q_3$$

Therefore, the system failure probability can be written as follows:

$$P(S) = 3(Q_1)^2 + 3Q_2 + Q_3$$

The general form of the Q terms is defined as follows:

$$Q_k^{(m)} = Probability\, of\, a\, CCBE\, involving\, k\, specific\, components\, in\, a\, common\, cause\, component\, group\, of\, size\, m$$

The model that uses $Q_k^{(m)}$ to calculate the system failure probability is called the Basic Parameter (BP) model. In terms of the basic parameters, the total failure probability, Q_t of a specific component in a common cause group of m components is:

$$Q_t = \sum_{k=1}^{m} \binom{m-1}{k-1} Q_k^{(m)}$$

Other than just showing this equation, it's easier to expand this for an exmple common cause group size. So take again example a common cause group of three redundant components:

$$Q_t = \binom{2}{0} Q_1^{(3)} + \binom{2}{1} Q_2^{(3)} + \binom{2}{2} Q_3^{(3)}$$

$$Q_t = Q_1^{(3)} + 2Q_2^{(3)} + Q_3^{(3)}$$

What this means is that the total failure probability of a given component, let's say component A for example, is equal to the sum of an independent failure, two common cause basic event representing a failure of two components, and a common cause basic event representing a failure of three components. Looking back to Section 1.0 we can see that the this is exactly how we decomposed the failure of component A into its independent and common causes. Namely:

$$Q_t = Q_1^{(3)} + 2Q_2^{(3)} + Q_3^{(3)} = P(A_t) = P(I_A) + P(C_{AB}) + P(C_{AC}) + P(C_{ABC})$$

Taking it one step further we can also see that we can generate *all* events involving one or more component failures using the following equation. Note the subscript captial "T" is used to denote that the difference in the "total" being considered.

$$Q_T = \sum_{k=1}^{m} \binom{m}{k} Q_k^{(m)}$$

Once again, take for example a common cause component group of size 3, this would result in the following value for Q_T:

$$Q_T = \binom{3}{1} Q_1^{(3)} + \binom{3}{2} Q_2^{(3)} + \binom{3}{3} Q_3^{(3)}$$

$$Q_T = 3Q_1^{(3)} + 3Q_2^{(3)} + Q_3^{(3)}$$

What this means is that, for a common cause group of size 3, at the cause level, there will be 3 indepdent events, 3 common cause basic events representing failure of two components, and one common cause basic event representing failure of three components. Looking back to Seciton 1.0, we can see that this is the probability of component A OR B OR C occuring.

$$Q_T = 3Q_1^{(3)} + 3Q_2^{(3)} + Q_3^{(3)} = P(A_t \cup B_t \cup C_t)$$

It is beneficial to decompose this equation probabilistically. First we will compute the probability of A_t and B_t:

$$P(A_t \cup B_t) = P(A_t) + P(A_t) - P(A_t \cap B_t)$$

$$P(A_t \cap B_t) = P(C_{AB}) + P(C_{ABC})$$

$$P(A_t \cup B_t) = P(I_A) + P(C_{AB}) + P(C_{AC})_{+} P(C_{ABC}) + P(I_B) + P(C_{AB}) + P(C_{BC}) + P(C_{ABC}) - P(C_{AB}) - P(C_{ABC})$$

$$P(A_t \cup B_t) = P(I_A) + P(I_B) + P(C_{AB}) + P(C_{AC}) + P(C_{BC}) + P(C_{ABC})$$

The same process can be extended to include component C, which would result in the following probability:

$$P(A_t \cup B_t \cup C_t) = P(I_A) + P(I_B) + P(I_C) + P(C_{AB}) + P(C_{AC}) + P(C_{BC}) + P(C_{ABC})$$

And finally it can be seen more directly that:

$$Q_T = 3Q_1^{(3)} + 3Q_2^{(3)} + Q_3^{(3)} = P(I_A) + P(I_B) + P(I_C) + P(C_{AB}) + P(C_{AC}) + P(C_{BC}) + P(C_{ABC})$$

3 The Alpha Factor Model

For several practical reasons, it is often more convenient to rewrite $Q_k^{(m)}$ values in terms of other more easily quantifiable parameters. For this purpose a parametric model known as the Alpha Factor model is recommended. For more information on this reccommendatio, see NUREG/CR-5485.

It is noted that the terminology used in NUREG/CR-5485 and other various sources when discussing the Alph Factor mode gets, well, *weird*. When moving to discussion of the Alpha Factor Model in NUREG/CR-5485, it is stated that:

"The alpha-factor model develops CCF frequencies from a set of failure ratios and the total component failure rate. The parameters of the model are: Q_t = total failure frequency of each component due to all independent and common cause events, and α_k = the fraction fo the total frequency of failure events that occur in the system and involve the failure of k components due to a common cause."

The terminology shifts from talking about *probability*, and starts talking about *frequency*. This terminology should **not** be interepreted as being related to component failure rates. The intent of the term frequency in the Apha Factor model discussion is intended to reference the total frequency of events involving a specified number of component failures and not a failure rate. A better way to ook at this is that the BP model calculates the probability of system failure, an therefore the apha factor model also calculates a probability go a given number of components (i.e., k components) in a specified grup size (i.e., group size of m). To retain this idea we will constantly discuss units of the new values we are calculating to show that indeed, the result is a probability.

The Alpha Factor Model defines a factor (namely an alpha factor) as follows:

$$\alpha_k^{(m)} = \binom{m}{k} \frac{Q_k^{(m)}}{Q_T}$$

This factor is defined as *the fraction of events involving failure of k components due to a shared cause*. Noting that the subscript of the Q term in the denomiator is a capital T, we can rewrite this equation as follows.

$$\alpha_k^{(m)} = \frac{\binom{m}{k} Q_k^{(m)}}{\sum_{k=1}^{m} \binom{m}{k} Q_k^{(m)}}$$

Yet again, coming back to our example system of three redundant components, we can calculate three alpha factors: $\alpha_1^{(3)}$, $\alpha_2^{(3)}$, and $\alpha_3^{(3)}$. For demonstration purposes we will only focus on the calculation of $\alpha_1^{(3)}$.

$$\alpha_1^{(3)} = \frac{\binom{3}{1} Q_1^{(3)}}{\binom{3}{1} Q_1^{(3)} + \binom{3}{2} Q_2^{(3)} + \binom{3}{3} Q_3^{(3)}}$$

$$\alpha_1^{(3)} = \frac{3Q_1^{(3)}}{3Q_1^{(3)} + 3Q_2^{(3)} + Q_3^{(3)}}$$

From this expansion, it can be more easily seen that $\alpha_1^{(3)}$ represents the ratio of observing an independent event out of the entire set of independent and common cause events for the specified

common cause group. Once again, it is noted that the term in the numerator is a probability, and the summation in the denominator is a probability.
Given the mathematical expression for the alpha factor, and the mathematical expression for Q_T, we can define basic event probabilities as a function of Q_T and the alpha factors.

$$\binom{m}{k} Q_k^{(m)} = [\sum_{k=1}^{m} \binom{m}{k} Q_k^{(m)}] \alpha_k^{(m)}$$

One nice "trick" is that the binomial coefficient can be re-written using the following identity:

$$\binom{m}{k} = \frac{m}{k} \binom{m-1}{k-1}$$

Which helps the simplification:

$$\frac{m}{k} \binom{m-1}{k-1} Q_k^{(m)} = [\sum_{k=1}^{m} \binom{m}{k} Q_k^{(m)}] \alpha_k^{(m)}$$

$$\frac{k}{m} [\sum_{k=1}^{m} \binom{m}{k} Q_k^{(m)}] \alpha_k^{(m)} = \binom{m-1}{k-1} Q_k^{(m)}$$

Summing both sides over k results in:

$$\frac{1}{m} [\sum_{k=1}^{m} \binom{m}{k} Q_k^{(m)}] \sum_{k=1}^{m} k \alpha_k^{(m)} = \sum_{k=1}^{m} \binom{m-1}{k-1} Q_k^{(m)}$$

$$\sum_{k=1}^{m} \binom{m}{k} Q_k^{(m)} = \frac{m}{\sum_{k=1}^{m} k \alpha_k^{(m)}} \sum_{k=1}^{m} \binom{m-1}{k-1} Q_k^{(m)}$$

$$\sum_{k=1}^{m} \binom{m}{k} Q_k^{(m)} = \frac{m}{\sum_{k=1}^{m} k \alpha_k^{(m)}} Q_t$$

The right side of this equation can then be plugged back into the equation for the alpha factor:

$$\alpha_k^{(m)} = \frac{\binom{m}{k} Q_k^{(m)}}{\frac{m}{\sum_{k=1}^{m} k \alpha_k^{(m)}} Q_t}$$

Solving for $Q_k^{(m)}$

$$Q_k^{(m)} = \frac{\alpha_k^{(m)} m Q_t}{\binom{m}{k} \sum_{k=1}^{m} k \alpha_k^{(m)}}$$

$$Q_k^{(m)} = \frac{m}{\binom{m}{k}} \frac{\alpha_k^{(m)}}{\sum_{k=1}^{m} k \alpha_k^{(m)}} Q_t$$

Replacing $\binom{m}{k}$ with $\frac{m}{k}\binom{m-1}{k-1}$

$$Q_k^{(m)} = \frac{k}{\binom{m-1}{k-1}} \frac{\alpha_k^{(m)}}{\sum_{k=1}^{m} k \alpha_k^{(m)}} Q_t$$

This equation represents the probability of k components failing in a common cause group size of m. Furthermore this equation represents a non-staggered test scheme. Note that all variables are either factors, or probabilities. This relationship allows us to calculate the $Q_k^{(m)}$ terms given the specific alpha factors, and the total component failure probability.