

## \* Recursive code complexity analysis:-

→ Analysis will be based on the no. of instructions executed w.r.t the input.

## \* Problem 01:-

```
JS test.js > ...  
1 function foo(n) {  
2   if (n == 1) return 1;  
3   return n * foo(n - 1);  
4 }
```

Assume  $\text{foo}(5)$ , so line 2 will be constant.

Line 3 has 2 operations:-

(i) Product  $\rightarrow c$       (ii) Function call  $\rightarrow c$

→ But this does not mean that the entire operation is constant.

→ Whenever we call  $\text{foo}(5)$ , the function call goes to the call stack.

$\text{foo}(4)$
$\text{foo}(5)$

→ Whatever time  $\text{foo}(4)$  takes will be extra apart from the function call.

$\therefore$  Total instructions = no. of instructions in one function call  $\times$  Total no. of function calls

→ This might not work for certain relations like divide & conquer relations.

→ We can use this formula when the instructions are same in every function call.

$$\begin{array}{ccccccccc} \text{foo}(5) & + & \text{foo}(4) & + & \text{foo}(3) & + & \text{foo}(2) & + & \text{foo}(1) \\ \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow \\ c & & c & & c & & c & & c \end{array}$$

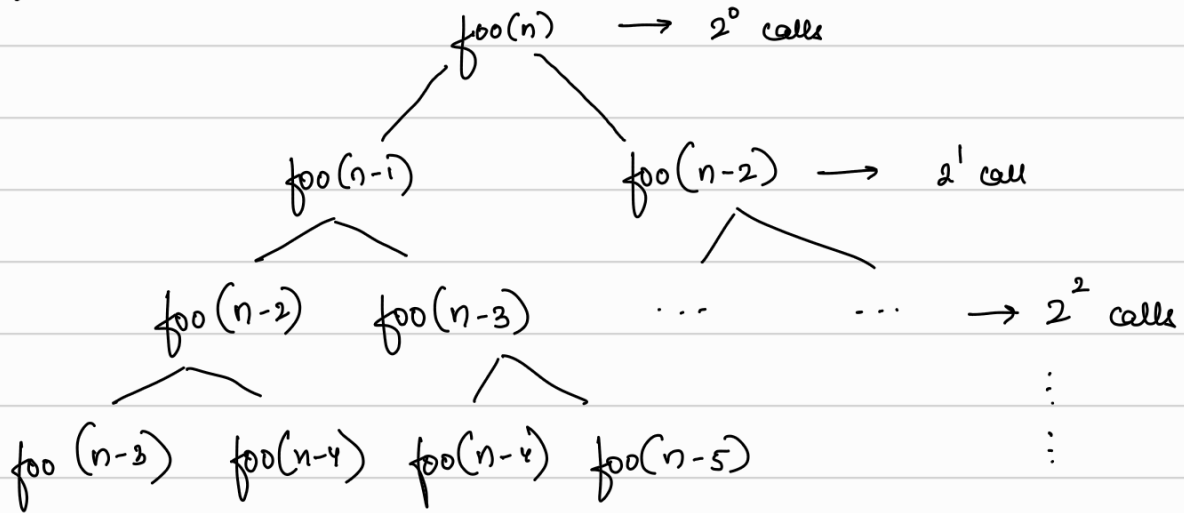
$$\therefore \text{Total instructions} = c \times n \\ \approx n$$

$\therefore \text{TC} = O(n)$

### \* Problem 2 :-

```
JS test.js > ...  
1 function foo(n) {  
2   if (n == 0 || n == 1) return n;  
3   return foo(n - 1) + foo(n - 2);  
4 }
```

→ In one function call, we have constant number of operations.



$$\begin{aligned}\therefore \text{Total calls} &\approx 2^0 + 2^1 + 2^2 + 2^3 + \dots + 2^{n-1} \\ &= \frac{1 \times (2^n - 1)}{2 - 1} \\ &= 2^n - 1 \\ &\approx 2^n \text{ function calls.}\end{aligned}$$

For every function call we are doing  $C$  operations.

$$\therefore \text{Total instructions} = 2^n \times C$$

$$\boxed{TC = 2^n}$$

### \* Problem 3 :-

```
JS test.js > ...  
1 function foo(n) {  
2   if (n == 0) return;  
3  
4   for (let i = 1; i <= n; i++) {  
5     // Some operations  
6   }  
7  
8   foo(n - 1);  
9 }
```

Here the function does not have constant number of operations because of the for loop, which is dependent on  $n$ . So we cannot use the previous formula.

$$\begin{array}{ccccccccc}
 \text{foo}(5) & + & \text{foo}(4) & + & \text{foo}(3) & + & \text{foo}(2) & + & \text{foo}(1) \\
 \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow \\
 C+5+C & & C+4+C & & C+3+C & & C+2+C & & C+1+C
 \end{array}$$

$$\therefore 5+2C + 4+2C + 3+2C + 2+2C + 1+2C$$

Replacing with n

$$\therefore n+2C + (n-1)+2C + (n-2)+2C + \dots + 1+2C$$

Ignoring 2C as its constant

$$\therefore n + (n-1) + (n-2) + \dots + 1 \rightarrow \text{Sum of natural numbers}$$

$$= \frac{n(n+1)}{2}$$

$$\therefore TC = O(n^2)$$

\* Problem 4 :-

```

test.js > ...
1 function foo(arr, n) {
2   // assume arr.length = k
3
4   if (n == 0) return;
5
6   for (let i = 1; i <= arr.length; i++) {
7     // Some operations
8   }
9
10  foo(arr, n - 1);
11 }
  
```

Here, the array length = k is constant everytime.  
In every function call, we execute a loop of  $O(k)$  & rest are constant operations. We have n function calls.

$$\therefore TC = O(n \times k)$$

\* Problem 5 :-

```

test.js > ...
1 function foo(arr, n) {
2   if (n <= 1) return 1;
3   return foo(n - 1) + foo(n - 1);
4 }
  
```

$$\therefore TC = O(2^n)$$

