Subterranean Robot Locomotion

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1 Derivations and Equations

1.1 Horizontal Motion

$$\Delta x = a\Delta t^2$$

There are two important parts of the impact and resultant motion to consider. One is the part during which the impactor is still moving toward the chassis and approximate elastic deformation is occurring. The chassis reaches a maximum velocity due to the impact and then the second stage takes over. During the second stage, the motion of the entire chassis is governed by the initial maximum velocity and the pressure applied by the granular material.

$$\Delta x = (a_{impactor} - a_{granular})\Delta t^2_{impactor} + a_{granular}\Delta t^2_{stop}$$

From (paper on granular material motions citation), we see that the pressure from a granular material applied on the surface of an object within the granular material can be expressed as growing linearly with depth. $0.3\frac{N}{cm^3}$ was found to be the maximum pressure varying with depth in granular materials tested in Li's "A Terradynamics of Legged Locomotion on Granular Media". So this presents an upper bound for early experiments and calculations here and will be used in test calculations. However for the purpose of derivations and equations, I will notate it as σ.

$$\sigma_{x,z} = 0.3|z| \quad \frac{N}{cm^2}$$

This leads us to the acceleration of the chassis due to the pressure from the granular material.

$$a_{granular} = \frac{\sigma|z|A_{robot}}{m_{robot}}$$

The acceleration of the chassis due to the impactor is found by using the Young's modulus of the material and assuming that one material acts as a spring (the impactor is a likely choice as a softer material than the chassis).

$$a_{impact} = \frac{v_{max,impactor}A_{impactor}\sqrt{E\rho}}{m_{robot}}$$

The time from initial impact to the time the impactor reaches zero velocity is found through conservation of energy and momenta.

$$\Delta t_{impactor} = \sqrt{\frac{m_{impactor} L_{impactor}}{E A_{impactor}}}$$

The time from the impactor having zero velocity and when all kinetic energy has been transferred to the chassis (excluding losses to vibrations and heat) to the time when the chassis velocity reaches zero and all kinetic energy from it has been transferred to the granular material can be found through conservation of energy as well.

$$\Delta t_{stop} = \frac{m_{robot} v_{max,impactor}}{\sigma |z| A_{robot}}$$

Now that we have defined the pieces of the overall change in horizontal position, we can combine them.

$$\Delta x = (a_{impactor} - a_{granular}) \Delta t_{impactor}^2 + a_{granular} \Delta t_{stop}^2$$

$$\Delta x = (\frac{v_{max,impactor} A_{impactor} \sqrt{E\rho}}{m_{robot}} - \frac{\sigma |z| A_{robot}}{m_{robot}})(\frac{m_{impactor} L_{impactor}}{E A_{impactor}})$$

$$+ \quad (\frac{\sigma |z| A_{robot}}{m_{robot}})(\frac{m_{robot} v_{max,impactor}}{\sigma |z| A_{robot}})^2$$

Note: Scaling factors are required in use of the Young's Moduli and the

$$0.3 \frac{N}{cm^2}$$

experimentally determined value for yield stress of granular material at a certain depth.

1.2 Vertical Motion

For both updward and downward motion, these equations assume that the distance moved within the granular material does not affect the pressure exerted on the chassis by the granular material.

Upward Motion

$$\Delta x = (a_{impactor} - a_{gravitational} - a_{granular}) \Delta t_{impactor}^2 + (-a_{gravitational} + a_{granular}) \Delta t_{stop}^2$$

Downward Motion

$$\Delta x = (a_{impactor} + a_{gravitational} - a_{granular}) \Delta t_{impactor}^2 + (a_{gravitational} - a_{granular}) \Delta t_{stop}^2$$

1.3 Velocity

Velocity of the system in any direction requires a calculation or assumption of a specified number of impacts per unit time. This can be specified and then limited in the system or calculated as a maximum for an upper limit. It will be dependent on the maximum frequency at which the actuators can be activated in order to strike the chassis and then allow for enough time to allow the system to reach zero velocity.

1.4 Feasibility

General Motion Along Axis of Impact (Assuming acceleration due to impact is constant over short time)

$$\Delta x_{impact,direction} = (\Sigma a_{impact})\Delta t^2_{impactor} + v_{maxChassis}\Delta t_{stop} + \frac{1}{2}(\Sigma a_{stop})\Delta t^2_{stop}$$

$$\Sigma a_{impact} = a_{impactor} + a_{gravitational}\cos\theta - a_{granular,face} + a_{granular,back}$$

$$\Sigma a_{impact} = a_{impactor} + a_{gravitational}\cos\theta - \frac{\sigma A(z_{front} - z_{back})}{m_{robot}}$$

$$v_{maxChassis} = (\Sigma a_{impact})\Delta t_{impactor}$$

$$\Sigma a_{stop} = a_{gravitational}\cos\theta - a_{granular,face} + a_{granular,back}$$

$$\Delta t_{stop} = \frac{m_{robot}v_{max,impactor}}{\sigma|z|A_{robot}}$$

$$\Delta t_{impactor} = \sqrt{\frac{m_{impactor}L_{impactor}}{EA_{impactor}}}$$

1.5 Limitations

Requirements for Motion:

$$F_{impact} \geq F_\sigma$$

where F_σ is the granular material yield stress on the surface area of the robot in the direction in the direction of motion.

$$F_{impact} \geq \sigma * |z_{body} - \frac{1}{2} * h_{body} * \cos\theta| * \pi r^2_{body}$$

where θ is the angle formed between the body's z-axis and the world frame's z-axis, h_{body} is the height of the body, z_{body} is the z component of the location of the center of the robot system, and r_{body} is the radius of the circular face of the robot chassis in the direction of travel.

$$F_{impact}\Delta t_{impact} = \Delta p_{impactor} \implies F_{impact} = \frac{\Delta p_{impactor}}{\Delta t_{impact}}$$

$$\frac{m_{impactor}\Delta v_{impactor}}{\Delta t_{impact}} \geq \sigma * |z_{body} - \frac{1}{2} * h_{body} * \cos\theta| * \pi r_{body}^2$$

Given that the magnitude of $\Delta v_{impactor} = v_{max}$ and $\Delta t_{impact} = \sqrt{\frac{m_{impactor}L_{impactor}}{EA_{impactor}}}$ is known from earlier calculations above, we can substitute this and manipulate the previous inequality to get a bound on depth of travel.

$$d \leq \frac{1}{\sigma\pi r_{body}^2}\sqrt{\frac{EA_{impactor}k\Delta x_{compression}^2}{L_{impactor}}} + \frac{1}{2}h_{body}\cos(\theta)$$

where d represents depth below the surface of the granular material.

The limit on the angle of the rises on the helix has a theoretical maximum of 90 degrees from the horizontal. However, the minimum depends on the radius of the cylinder and the height of the extrusion. These form the base and height of a right triangle. Also factoring in are the width of the notches and flat portions.

$$\phi = \tan^{-1}\frac{nh_{helix}}{c - n(w+f)}$$

$$\phi \in [\phi_{min}, \phi_{max})$$

$$\phi_{max} = \frac{\pi}{4}$$

$$\phi_{min} = \arctan\frac{h_{helix}}{c - w - f}$$

During the release of the spring-loaded mechanism, the spring exerts forces on both the impactor and the chassis in opposite directions. The ideal circumstance for motion is one in which the spring force exerted on the chassis in the direction opposite the direction of motion is less than the sum of forces on the chassis in the direction of motion (including static friction, a gravitational component, and a depth-dependent yield stress pressure). This prevents the chassis from moving while the impactor is accelerated by the spring before impacting the impact surface.

$$F_{spring} < F$$

F of impact / area of impact = F other side / area other side

1.6 Definitions

F_{spring}: Force the spring applies to the bodies compressing it

d: Depth of the impactor

σ: pressure (Force per unit area)

r_{body}: radius of the cylindrical body of the robot

E: Young's modulus

$A_{impactor}$: Surface area of the impactor on the face which strikes the robot chassis

k: Spring constant

$\Delta x_{compression}$: Positive change in spring compression

$L_{impactor}$: Length of the impactor in the direction orthogonal to its impact face

h_{body}: Height of the robot body, that is the length of the chassis along the axis of motion from flat face to flat face of the cylinder

$m_{impactor}$: Mass of the impactor

v_{max}: Maximum velocity reached by the impactor during acceleration by the spring

$v_{impactor}$: Velocity of the impactor

v_{max}: Maximum velocity of the chassis, the initial velocity during the coasting phase

z_{body}, z: Depth of the body in the granular material

z_{front}, z_{back}: Depth of the face of the robot that gets struck by the impactor, and the opposite face, respectively

θ: Angle between the chassis of the robot (with the body frame z-axis aligned with the chassis opposite the direction of motion) and the world frame z-axis

$\Delta t_{impactor}$: Time between the impactor first contacting the chassis and the impactor reaching 0 velocity

Δt_{stop}: Time for the chassis to reach 0 velocity once it has started moving

a_{impact}: Accelerations acting on the robot chassis during the impact phase

$a_{impactor}$: Acceleration on the chassis due to the impactor

$a_{gravitational}$: Acceleration of gravity on the robot as a whole

$a_{granular,face}$: Acceleration from the granular material stress on the face of the robot in the direction of travel

$a_{granular,back}$: Acceleration from the granular material stress on the back face of the robot chassis

a_{stop}: Accelerations acting on the robot chassis during the coasting phase

$$\phi$$

: Angle of the helix rising notches with respect to the horizontal

$$\phi_{min}, \phi_{max}$$

: The minimum and maximum theoretical angles (respectively) for the helix rising notches with respect to the horizontal

n: The number of helix rising notches

h_{helix}: The height of the helix rising notches

w: The width of the helix rising notches

c: The circumference of the helix rotator

f: The width of the flat fillet portion at the top of the helix rising notches

$$d \leq \frac{1}{\sigma \pi r_{body}^2} \sqrt{\frac{EA_{impactor} k \Delta x_{compression}^2}{L_{impactor}}} + \frac{1}{2} h_{body} \cos(\theta)$$

$$F_{spring}$$

$$\Delta t_{impact} = \sqrt{\frac{m_{impactor} L_{impactor}}{EA_{impactor}}}$$

$$\Delta v_{impactor} = v_{max}$$

$$\frac{m_{impactor}\Delta v_{impactor}}{\Delta t_{impact}} \geq \sigma * |z_{body} - \frac{1}{2} * h_{body} * \cos\theta| * \pi r_{body}^2$$

$$\Delta x_{impact,direction} = (\Sigma a_{impact})\Delta t_{impactor}^2 + v_{maxChassis}\Delta t_{stop} + \frac{1}{2}(\Sigma a_{stop})\Delta t_{stop}^2$$

$$\Sigma a_{impact} = a_{impactor} + a_{gravitational}\cos\theta - a_{granular,face} + a_{granular,back}$$

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$$\Delta t_{stop} = \frac{m_{robot}v_{max,impactor}}{\sigma|z|A_{robot}}$$

$$\Delta t_{impactor} = \sqrt{\frac{m_{impactor}L_{impactor}}{EA_{impactor}}}$$