Sheet 3

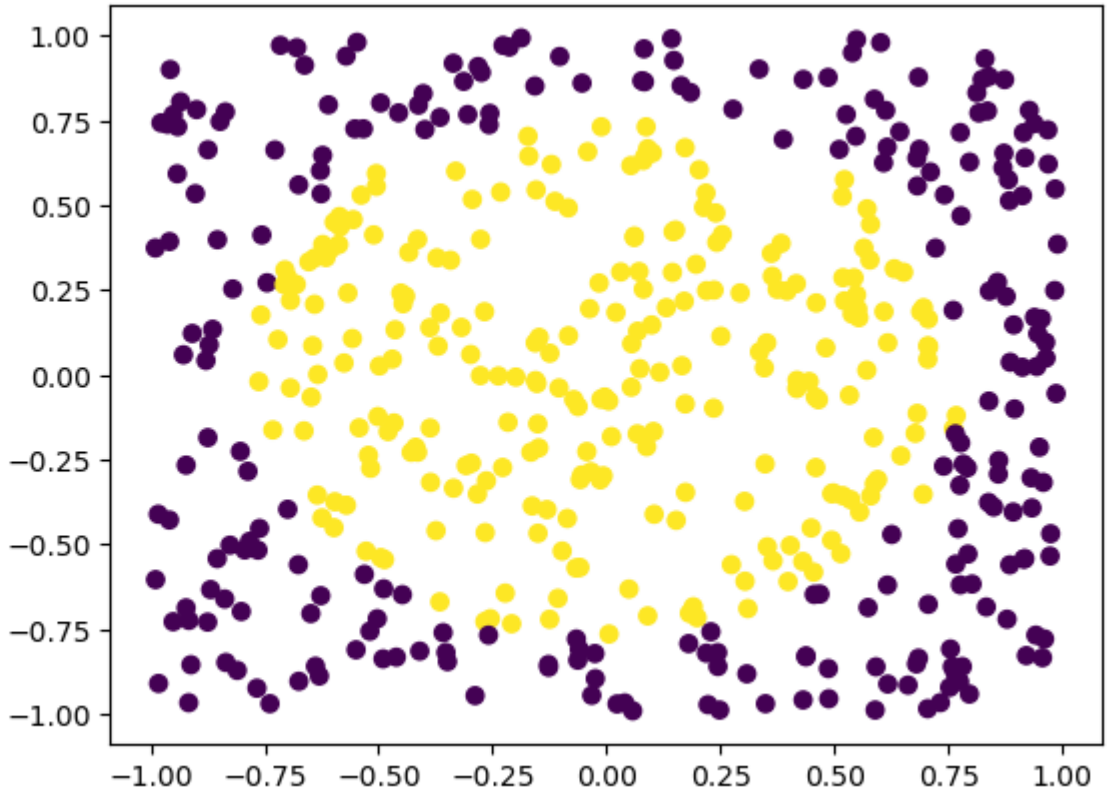
In [ ]:
```
import numpy as np
import matplotlib.pyplot as plt
import torch
```

2 Projection Trick

(a)

In [ ]:
```
# Load the data
xy = np.load('data03/data2d.npy')
labels = np.load('data03/labels.npy')

# TODO: Plot the data
```

In [ ]:
```
plt.scatter(xy[:,0], xy[:,1], c=labels);
```



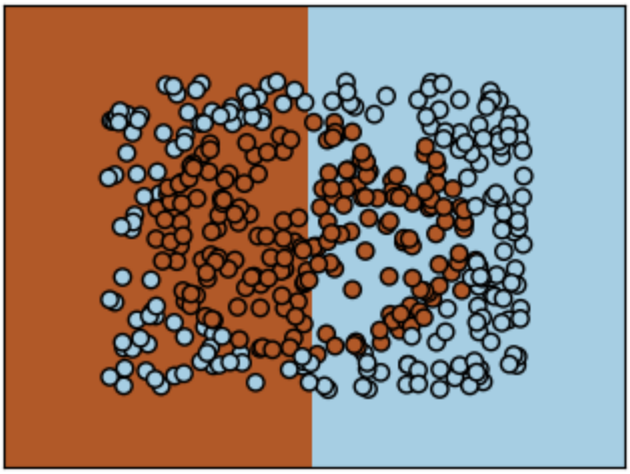
This cannot be separated by a linear decision boundary in 2D because one class is fully enclosed in the other.

In [ ]:
```
from sklearn.model_selection import train_test_split

# Split the data into training and testing sets
xy_train, xy_test, labels_train, labels_test = train_test_split(xy, label
```

In [ ]:
```python
from sklearn.linear_model import LogisticRegression
# TODO: fit logistic regression]
clf = LogisticRegression().fit(xy_train, labels_train)

# TODO: compute the accuracy
score = clf.score(xy_test, labels_test)
print(f"The score is {score:.2f}.")

# TODO: visualize the decision boundary
x_min, x_max = xy[:, 0].min() - .5, xy[:, 0].max() + .5
y_min, y_max = xy[:, 1].min() - .5, xy[:, 1].max() + .5
h = .02  # step size in the mesh
xx, yy = np.meshgrid(np.arange(x_min, x_max, h), np.arange(y_min, y_max,

# Predict the class for each point in the meshgrid
Z = clf.predict(np.c_[xx.ravel(), yy.ravel()])
```

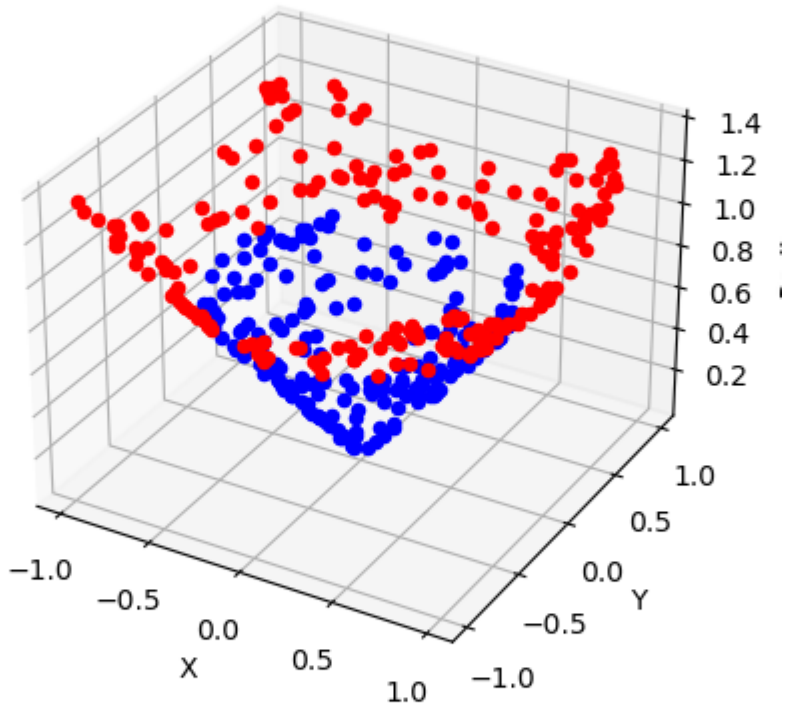
```
The score is 0.54.
```

In [ ]:
```python
# Put the result into a color plot
Z = Z.reshape(xx.shape)
plt.figure(1, figsize=(4, 3))
plt.pcolormesh(xx, yy, Z, cmap=plt.cm.Paired)

# Also plot the training points
plt.scatter(xy_train[:, 0], xy_train[:, 1], c=labels_train, edgecolors='k

plt.xlim(xx.min(), xx.max())
plt.ylim(yy.min(), yy.max())
plt.xticks(())
plt.yticks(())

plt.show()
```



b)

In [ ]:
```
# TODO: Come up with a nonlinear tranformation for the third feature. Com
def trafo3d(vec):
    # adds a third dimension to the data (radius)
    return np.column_stack([vec, np.sqrt(vec[:,0]**2 + vec[:,1]**2)])

xy_train_3d = trafo3d(xy_train)

# TODO: Show the enhanced data, e.g. with a 3D scatter plot
#       (https://matplotlib.org/stable/gallery/mplot3d/scatter3d.html).
fig = plt.figure()
ax = fig.add_subplot(projection='3d')
i = 0
while i < len(labels_train)-1:
    if labels_train[i] == 0:
        ax.scatter(xy_train_3d[i,0], xy_train_3d[i,1],xy_train_3d[i,2], c
        i +=1
    if labels_train[i] == 1:
        ax.scatter(xy_train_3d[i,0], xy_train_3d[i,1],xy_train_3d[i,2], c
        i +=1
ax.set_xlabel('X ')
ax.set_ylabel('Y ')
ax.set_zlabel('Radius')

plt.show()
```



In [ ]:
```
# TODO: again, fit and evaluate logistic regression
clf2 = LogisticRegression(random_state = 0).fit(xy_train_3d, labels_train
pred2 = clf2.predict(trafo3d(xy_test))

print(clf2.score(trafo3d(xy_test), labels_test))
```

```
0.98
```

3) Linear regions of MLPs

(a)

In [ ]:
```
import torch
import torch.nn as nn
import torch.nn.functional as F
```

In [ ]:
```
# TODO: define NN architecture
class Net(nn.Module):
    def __init__(self):
        super(Net, self).__init__()
        self.hidden_layer = nn.Linear(2, 20)
        self.output_layer = nn.Linear(20, 1)

    # x represents our data
    def forward(self, x):
        x = self.hidden_layer(x)
        x = F.relu(x)
        x = self.output_layer(x)
        return x
```

Hidden layer: $W \in \mathbb{R}^{2\times 20}$ and $b \in \mathbb{R}^{20}$;

Output layer: $W \in \mathbb{R}^{20\times 1}$ and $b \in \mathbb{R}^{1}$

Sums up to 81 parameters (real numbers).

(b)

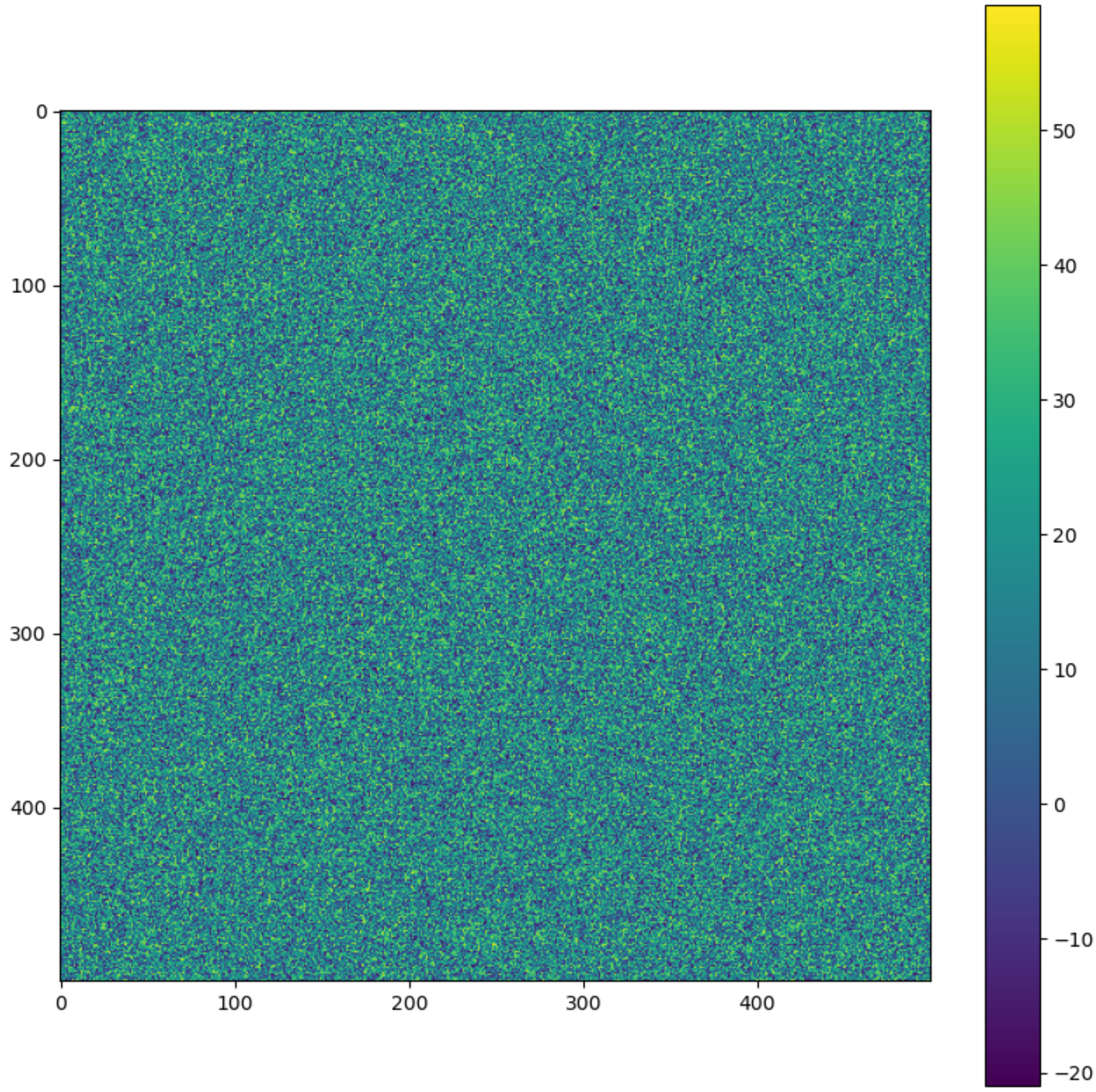
In [ ]:
```
indata = (torch.rand((500, 500, 2)) - 0.5) * 200
```

In [ ]:
```
my_nn = Net()
```

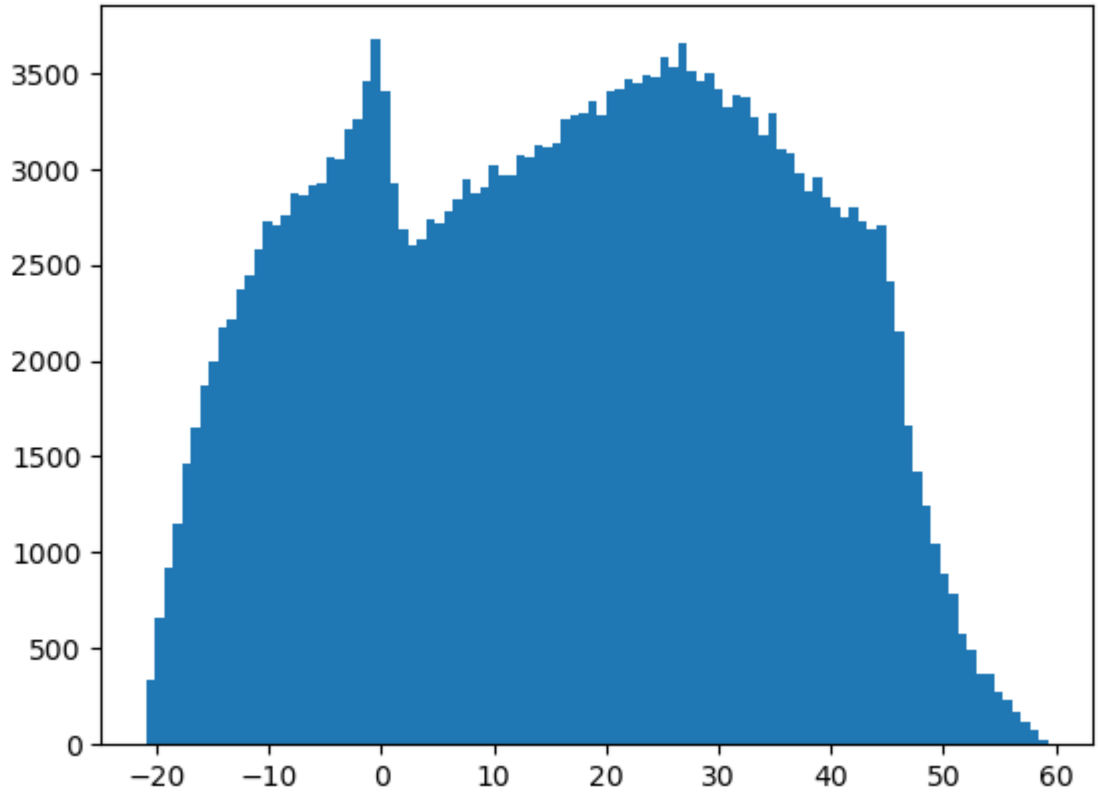
In [ ]:
```
output = my_nn(indata)[:,:,0].detach().numpy()
```

In [ ]:
```
plt.figure(figsize=(10,10))
plt.imshow(output, cmap='viridis')
plt.colorbar()
```

Out[ ]:
```
<matplotlib.colorbar.Colorbar at 0x7f6895aa2a70>
```



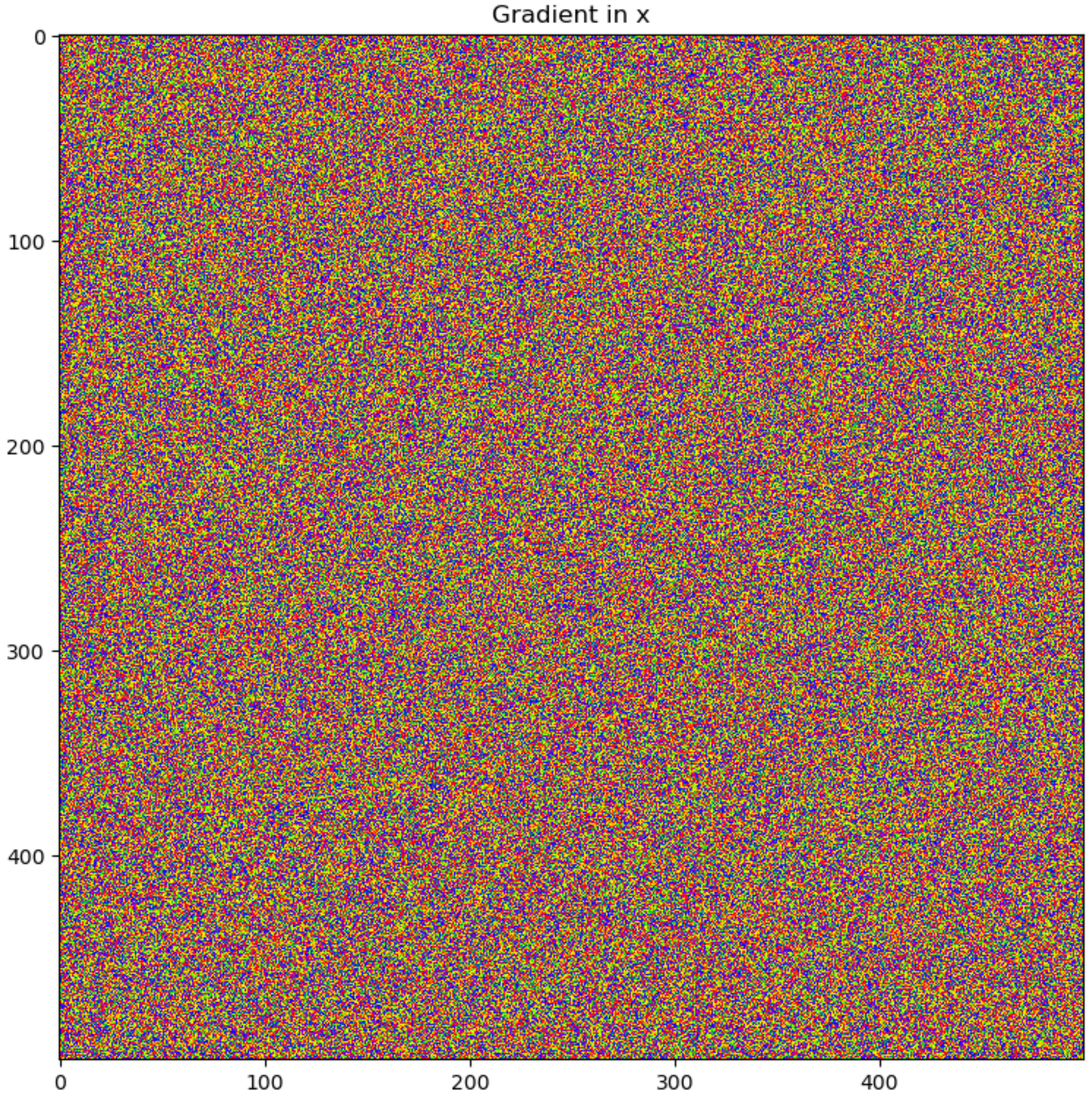
In [ ]:
```
plt.hist(output.flatten(), bins=100);
```

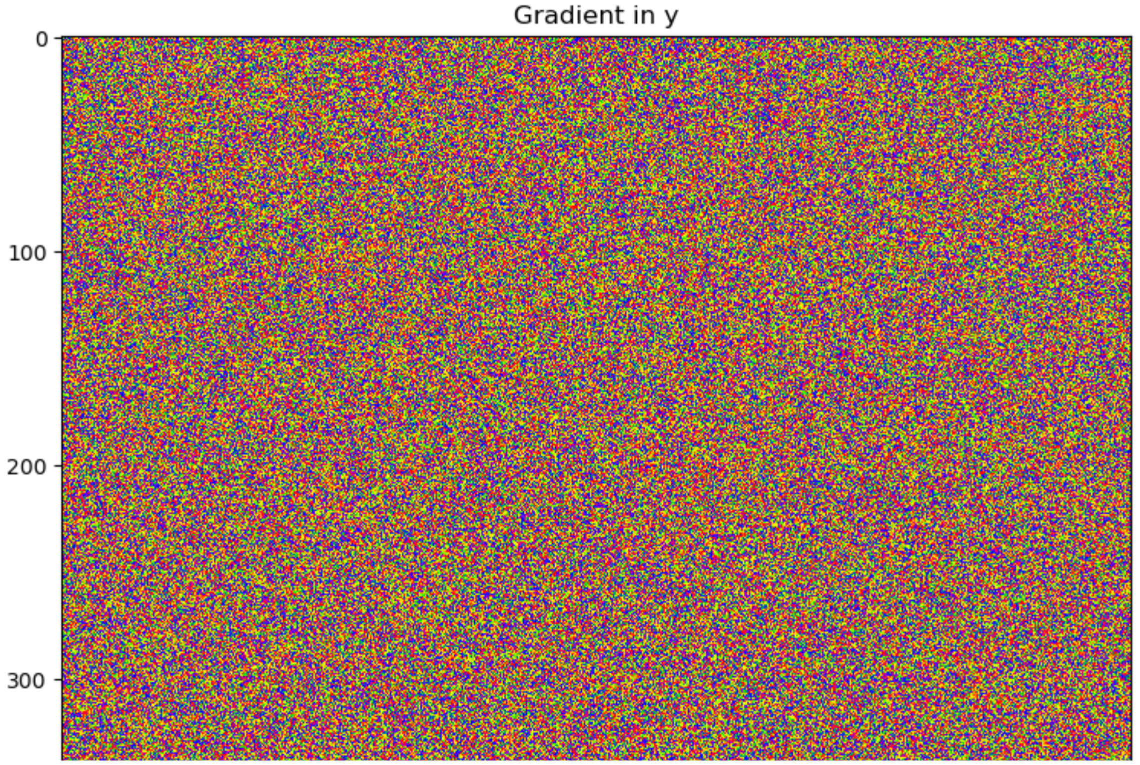


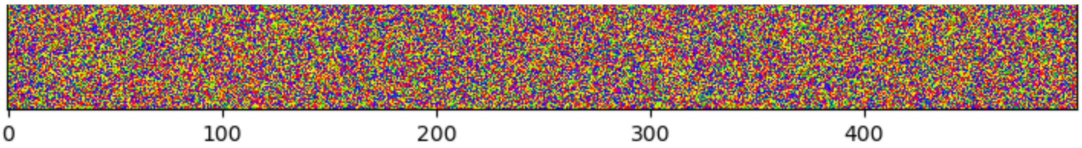
(c)

In [ ]:
```
[gradx, grady] = np.gradient(output)
```

In [ ]:
```
# Find the global min and max gradient values
f, axs = plt.subplots(2, 1, figsize=(10, 20))
# plot the gradients next to each other
axs[0].imshow(gradx, cmap='prism')
axs[0].set_title('Gradient in x')
axs[1].imshow(grady, cmap='prism')
axs[1].set_title('Gradient in y')

plt.show()
```


Gradient in x
0
100
200
300
400
0
100
200
300
400

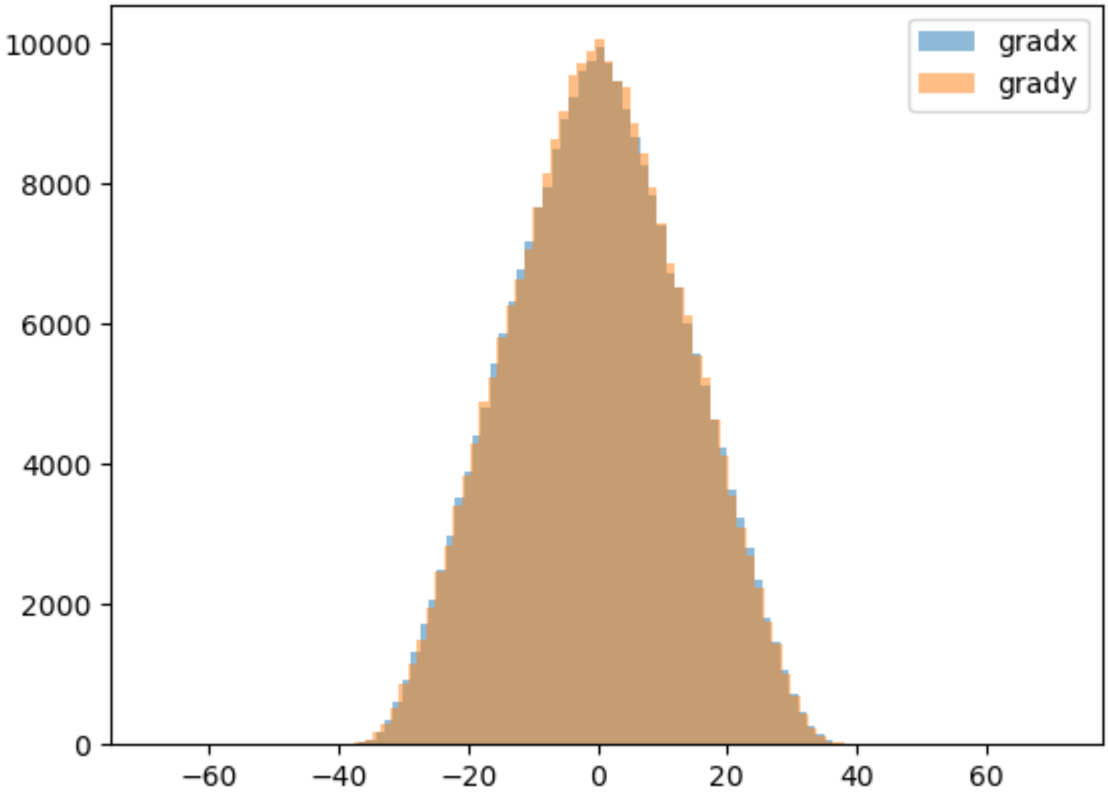

Gradient in y
0
100
200
300



We observe that there is a grid-like pattern of lighter areas. We do not know the values of the elements on this pattern because 'prism' is a repeating colourmap. It is also interesting to note that the probability distribution of the gradients peaks at 0 and tapers off in a weird shape that is *not* gaussian.

In [ ]:
```
plt.hist(gradx.flatten(), bins=100, alpha=0.5, label='gradx');
plt.hist(grady.flatten(), bins=100, alpha=0.5, label='grady');
plt.legend()
```

Out[ ]: <matplotlib.legend.Legend at 0x7f689595fb20>

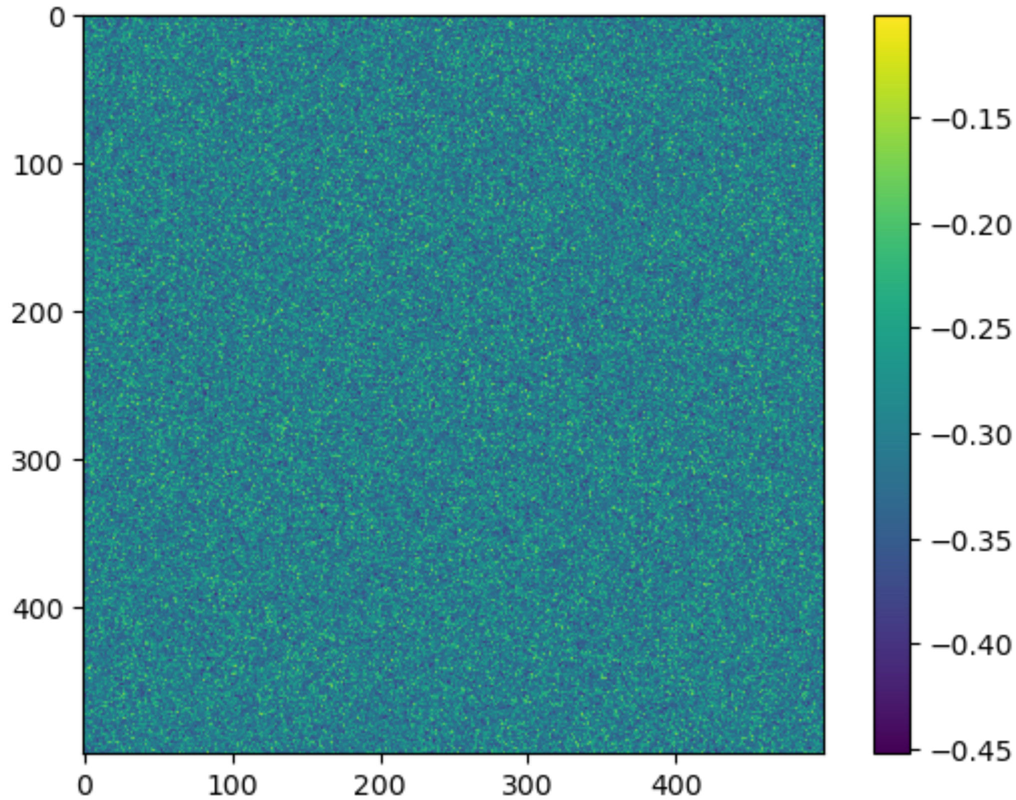


(d)

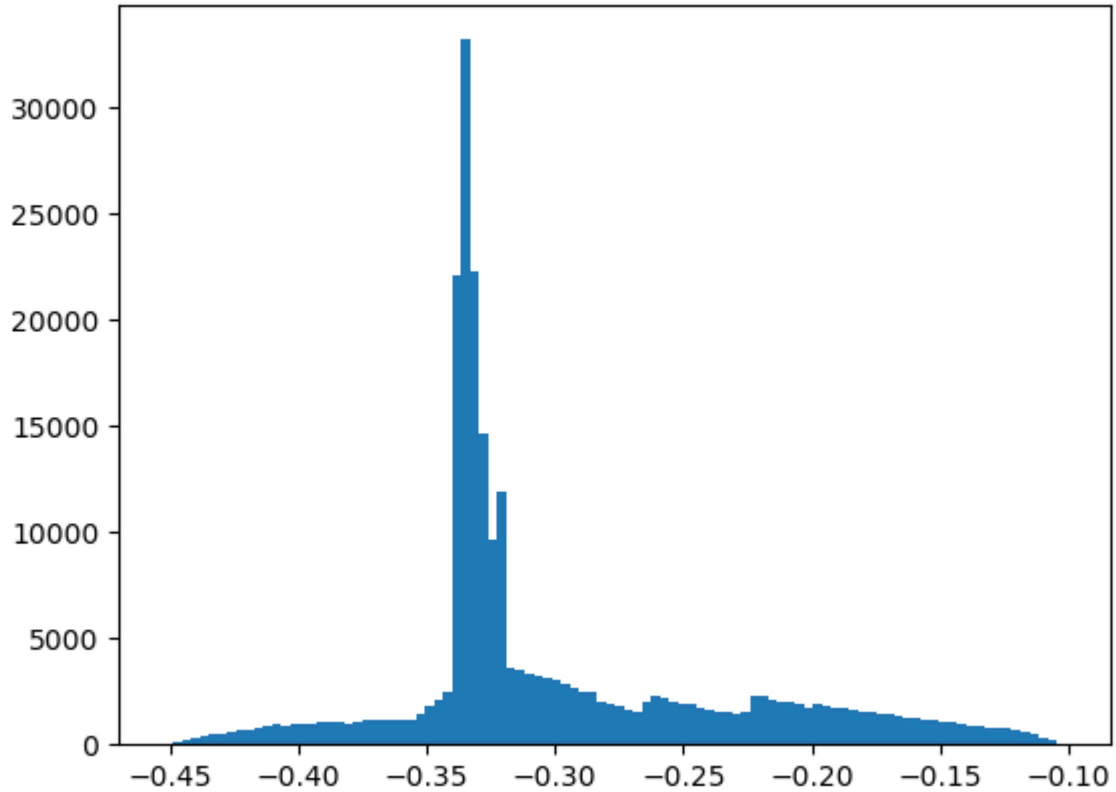
In [ ]:
```
# TODO: define NN architecture
class DeepNet(nn.Module):
    def __init__(self):
        super(DeepNet, self).__init__()
        self.hidden_layer_1 = nn.Linear(2, 5)
        self.hidden_layer_2 = nn.Linear(5, 5)
        self.hidden_layer_3 = nn.Linear(5, 5)
        self.hidden_layer_4 = nn.Linear(5, 5)
        self.output_layer = nn.Linear(5, 1)

    # x represents our data
    def forward(self, x):
        x = self.hidden_layer_1(x)
        x = F.relu(x)
        x = self.hidden_layer_2(x)
        x = F.relu(x)
        x = self.hidden_layer_3(x)
        x = F.relu(x)
        x = self.hidden_layer_4(x)
        x = F.relu(x)
        x = self.output_layer(x)
        return x
```

In [ ]:
```
# TODO: instantiate the model and make the visualizations as requested in
my_deepnn = DeepNet()
output = my_deepnn(indata)[:,:,0].detach().numpy()
plt.imshow(output, cmap='viridis')
plt.colorbar()
```

Out[ ]:
```
<matplotlib.colorbar.Colorbar at 0x7f68955be050>
```



In [ ]:
```
plt.hist(output.flatten(), bins=100);
```



Not much has changed compared to the shallow model. We have to note that the histogram of values is much more erratic for the deeper model than for the shallow one, the differences in the value distribution are quite pronounced here.

```
In [ ]:
```