

## Assignment 2

November 15, 2025

1. Find the Fourier series expansion of the following periodic function of period 4,

$$f(x) = \begin{cases} 2+x, & \text{if } -2 \leq x \leq 0, \\ 2-x, & \text{if } 0 < x \leq 2. \end{cases}$$

$$f(x+4) = f(x).$$

2. Matrix  $M = \begin{bmatrix} A & B \\ C & 0 \end{bmatrix}$  is an orthogonal matrix. Find the value of  $|B|$ ?
3. Find the condition for which the given matrix  $A = \begin{bmatrix} \alpha + i\gamma & -\beta + i\delta \\ \beta + i\delta & \alpha - i\gamma \end{bmatrix}$  to be a unitary matrix.

4. Find the rank of the matrix  $\begin{bmatrix} 3 & 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 7 & 8 \\ 5 & 6 & 7 & 8 & 9 \\ 10 & 11 & 12 & 13 & 14 \end{bmatrix}$

5. Let  $\mathbf{A}$  and  $\mathbf{B}$  be two symmetric matrices of the same order. Show that the matrix  $\mathbf{AB}$  is symmetric if and only if  $\mathbf{AB} = \mathbf{BA}$ , that is, matrices  $\mathbf{A}$  and  $\mathbf{B}$  commute.

6. Find all the eigenvalues and corresponding eigenvectors of the given matrix  $\begin{bmatrix} 1 & 1 & i \\ 1 & 0 & i \\ -i & -i & 1 \end{bmatrix}$

7. Show that the given matrix is diagonalizable. Find the matrix  $\mathbf{P}$  such that  $\mathbf{P}^{-1}\mathbf{AP}$  is a diagonal matrix.  $\begin{bmatrix} 0 & 2 & 1 \\ 2 & 0 & 3 \\ 1 & -3 & 0 \end{bmatrix}$

8. Find the conditions that a matrix  $A = \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix}$  is orthogonal.

9. If  $\mathbf{x}, \mathbf{y}, \mathbf{z}$  are linearly independent vectors in  $\mathbb{R}^3$ . Then show that
  - (a)  $\mathbf{x}+\mathbf{y}, \mathbf{y}+\mathbf{z}, \mathbf{z}+\mathbf{x}$
  - (b)  $\mathbf{x}, \mathbf{x}+\mathbf{y}, \mathbf{x}+\mathbf{y}+\mathbf{z}$are also linearly independent in  $\mathbb{R}^3$ .

10. Prove that  $\langle \mathbf{u}, k\mathbf{v} \rangle = \bar{k} \langle \mathbf{u}, \mathbf{v} \rangle$ .

11. Define  $T: M_{23}(\mathbb{R}) \rightarrow M_2(\mathbb{R})$  by  $T\left(\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}\right) = \begin{bmatrix} c+3f & -b \\ -b & 4a-3d \end{bmatrix}$ . Verify that  $T$  is a linear transformation.
12. Let  $Q$  denote the set of all real skew-symmetric  $n \times n$  matrices. Verify that  $Q$  is a subspace of  $M_n(\mathbb{R})$ .  
 $S = \{A \in M_n(\mathbb{R}), A^T = -A\}$