

Assignment 2

November 15, 2025

1. Find the Fourier series expansion of the following periodic function of period 4,

$$f(x) = \begin{cases} 2+x, & \text{if } -2 \leq x \leq 0, \\ 2-x, & \text{if } 0 < x \leq 2. \end{cases}$$

$$f(x+4) = f(x).$$

2. Matrix $M = \begin{bmatrix} A & B \\ C & 0 \end{bmatrix}$ is an orthogonal matrix. Find the value of $|B|$?

3. Find the condition for which the given matrix $A = \begin{bmatrix} \alpha + i\gamma & -\beta + i\delta \\ \beta + i\delta & \alpha - i\gamma \end{bmatrix}$ to be a unitary matrix.

4. Find the rank of the matrix $\begin{bmatrix} 3 & 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 7 & 8 \\ 5 & 6 & 7 & 8 & 9 \\ 10 & 11 & 12 & 13 & 14 \end{bmatrix}$

5. Let \mathbf{A} and \mathbf{B} be two symmetric matrices of the same order. Show that the matrix \mathbf{AB} is symmetric if and only if $\mathbf{AB} = \mathbf{BA}$, that is, matrices \mathbf{A} and \mathbf{B} commute.

6. Find all the eigenvalues and corresponding eigenvectors of the given matrix $\begin{bmatrix} 1 & 1 & i \\ 1 & 0 & i \\ -i & -i & 1 \end{bmatrix}$

7. Show that the given matrix is diagonalizable. Find the matrix \mathbf{P} such that $\mathbf{P}^{-1}\mathbf{AP}$ is a diagonal matrix. $\begin{bmatrix} 0 & 2 & 1 \\ 2 & 0 & 3 \\ 1 & -3 & 0 \end{bmatrix}$

8. Find the conditions that a matrix $A = \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix}$ is orthogonal.

9. If $\mathbf{x}, \mathbf{y}, \mathbf{z}$ are linearly independent vectors in \mathbb{R}^3 . Then show that
 (a) $\mathbf{x+y}, \mathbf{y+z}, \mathbf{z+x}$
 (b) $\mathbf{x}, \mathbf{x+y}, \mathbf{x+y+z}$
 are also linearly independent in \mathbb{R}^3 .

10. Prove that $\langle \mathbf{u}, k\mathbf{v} \rangle = \bar{k} \langle \mathbf{u}, \mathbf{v} \rangle$.

11. Define $T: M_{23}(\mathbb{R}) \rightarrow M_2(\mathbb{R})$ by $T\left(\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}\right) = \begin{bmatrix} c+3f & -b \\ -b & 4a-3d \end{bmatrix}$. Verify that T is a linear transformation.
12. Let Q denote the set of all real skew-symmetric $n \times n$ matrices. Verify that Q is a subspace of $M_n(\mathbb{R})$.
 $S = \{A \in M_n(\mathbb{R}), A^T = -A\}$