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# Prevalent Instability of Nonthermal Plasmas

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In thermally anisotropic plasmas or sheet pinches, the zero-order counterstreaming or directed streaming of like particles gives rise to instability modes that correspond essentially to the formation of local first-order pinches within the plasma. The marginal stability criteria for all such modes can be interpreted in terms of Bennett's pinch condition. The range of examples discussed includes Weibel's thermally anisotropic hot-electron plasma instability in null field, which is generalized for hot ions and finite magnetic field, and which is applied to interpenetrating neutral plasma streams, to a spherical electrostatic plasma-containment scheme, and to the Astron thermonuclear experiment. The familiar "mirror instability" is shown to be another example of a "pinch"-type mode. Sheet pinches with or without longitudinal magnetic field are generally unstable against tearing into subsidiary linear pinches.

## I. INTRODUCTION

THE self-focusing of a charged-particle stream in the presence of charge-neutralizing particles has been treated by Bennett.[1] For a cylindrical particle stream the self-focusing condition is

$$I^2 > 2A_s nmv_t^2, \tag{1}$$

where $I$ is the total current and $A_s$ the cross-sectional area of the stream; $n$ is the volume density, $m$ the mass, and $v_t$ the root mean square "thermal" velocity of the streaming particles. (The "thermal" pressure is assumed isotropic and very large compared with that of the neutralizing particles.) The condition expressed by Eq. (1) can be rewritten as

$$\tfrac{1}{2}A_s nr_c(v_s/v_t)^2 > 1, \tag{2}$$

where $r_c = e^2/mc^2$, $v_s$ is the mean directed velocity of the stream, $c$ the velocity of light, and $e$ the particle charge.

There exist two basic types of noncylindrical stream configuration that are not subject to self-focusing, even when the inequality of Eq. (2) is greatly exceeded:

A: Two identical interpenetrating particle streams (as in a thermally anisotropic plasma) give rise to no net current flow, and therefore to no self-focusing field.

B: A uniform infinite sheet stream of particles, while focused in the direction perpendicular to the sheet, remains unfocused in the direction transverse to the directed velocity within the sheet.

In configurations of these two types, the immunity against the simple Bennett condition derives from a symmetry property of the equilibrium state, and may therefore be expected to disappear in the presence of a suitable perturbation. The object of this paper is to point out that there does indeed exist a velocity-space instability corresponding to Bennett-type self-focusing and governed by marginal stability conditions very similar to Eq. (2). Many of the containment configurations currently envisaged for high-temperature plasmas happen to be precisely of types A and B as defined above, so that practical illustrations of the "self-focusing" instability are remarkably abundant.

The type-A configurations may be classified conveniently as follows:

A1: Thermally anisotropic plasmas (interpenetrating streams) in the absence of magnetic field. A configuration of this type, namely, a thermally anisotropic electron plasma with stationary neutralizing ions, has been shown unstable by Weibel[2] and Fried.[3] In the present analysis, the instability is shown to apply to an electrostatic plasma-confinement scheme[4,5] that has recently received considerable attention, and to interpenetrating plasma streams.[6]

A2: Thermally anisotropic plasmas in the presence of a weak magnetic field oriented in the direction of maximum thermal velocity. This type of configuration is again relevant to laboratory experiments with interpenetrating plasma streams.[6,7] A very interesting illustration is also afforded by a type of

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$E$ layer[8,9] that has been envisaged for the Astron thermonuclear experiment,[10] where the rms velocity transverse to the direction of current flow is so large that each of the two canceling transverse stream components exceeds the directed stream component in current density.

A3: Thermally anisotropic plasmas in the presence of an arbitrarily strong magnetic field oriented in the direction of minimum thermal velocity. This type of configuration is subject to the familiar "mirror instability."[11,12]

The type-B configurations may be classified as follows:

B1: Sheet-pinches with antiparallel magnetic fields on either side of the sheet. This type of configuration is illustrated by the $E$ layer,[8,9] as well as by certain nonrelativistic reverse-field configurations that are generated in "theta pinches"[13,14] and in linear pinch experiments.[15]

B2: Sheet pinches with differently oriented magnetic-field regions on either side, as in the surface of a "stabilized pinch"[16,17] or "inverse stabilized pinch."[18]

B3: Current sheets with parallel magnetic field on either side. A tubular current sheet of this type, with a large directed particle velocity, is subject to an instability mode essentially identical with the "mirror instability" that arises in the configurations of subtype A3. The point here is that the "mirror instability" applies not only to continuous thermally anisotropic plasmas with small particle gyroradii,[19] but also to the so-called plasmas that are produced

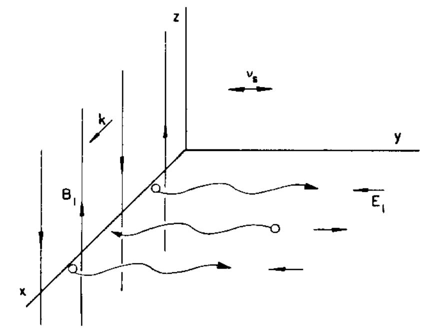


FIG. 1. Instability of a thermally anisotropic plasma with null zero-order field and maximum thermal velocity spread along the $y$ axis. (Unstable configuration of type A1.)

by trapping and neutralizing a high-energy ion beam in an axial magnetic field[20] so as to generate a single tubular layer of gyrating particles.

The instability modes that are considered here have certain common features.

(a) The coordinate in the direction of streaming (or counterstreaming) is ignorable (an exception occurs in Sec. II 2 c).

(b) The perturbation magnetic field is transverse to the direction of streaming, and tends to concentrate particles that are moving in the same direction (see Figs. 1 and 2).

(c) There is a resultant particle momentum transport (for subtypes A1 and A2), or a momentum transport plus a density transport (for subytpes A3 and B1, B2, B3), which in turn results in a current-density perturbation that accounts for the perturbation magnetic field.

(d) There exists a marginal stability condition similar to Eq. (2), (but opposite in sense) with the cross-sectional area $A_s$ to be interpreted either as proportional to the square of the instability wavelength (for subtype A1) or the square of the thermal

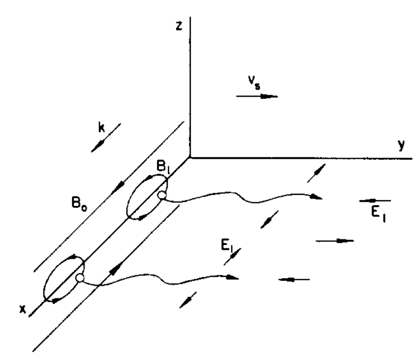


FIG. 2. Instability of a sheet pinch with zero-order $B_x$ field and directed particle velocity along the $y$ axis. (Unstable configuration of type B1.)

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gyroradius in the applied magnetic field (for subtypes A2 and A3) or the product of the instability wavelength and the thickness of the sheet stream (for subtypes B1 and B2), or the product of the thickness of the sheet stream and its radius of curvature (for subtype B3).

(e) When the marginal stability condition is violated, there exists an exponentially growing mode.

(f) The perturbation magnetic field is accompanied by an induced electric field such as to diminish the directed velocity of the streaming particles on the average. Thus, there is a transfer of energy from the particles to the perturbation magnetic field, and the particle distribution is modified so as to reduce its departure from the thermal distribution.

(g) The population of particles exhibiting the streaming (or counterstreaming) can couple with the neutralizing population via a charge-separation electric field transverse to the direction of streaming or via the induced electric field along the direction of streaming. For subtypes A1 and A2 there is no density modulation and, therefore, no transverse electric field. Both particle populations are free to move in the direction of streaming, and so the longitudinal induced electric field provides effective coupling. For subtypes A3 and B1, B2, and B3, there is a density modulation, with resultant coupling via the transverse electric field. The longitudinal electric field does not, in general, provide effective coupling, because there is a transverse zero-order magnetic field that inhibits acceleration of the neutralizing particles in the direction of streaming. If both particle populations exhibit streaming or thermal anisotropy (counterstreaming within each population) they act cooperatively to promote instability when the streaming of the two populations is oppositely directed (so as to increase the current flow) or when the thermal anisotropy is of the same kind for both populations.

(h) The marginal stability condition is most readily violated for long wavelengths (an exception occurs in Secs. II 2 a and b).

(i) The maximum growth rate attainable is limited by the transit time of the particle stream over a distance of the order of one instability wavelength.

In the present treatment a number of important idealizations are made that deserve comment:

Only first-order effects are considered. In view of the comments on energy transfer given in item (f), above, some of the instabilities found in the first-order treatment for configurations departing only slightly from thermal equilibrium are obviously trivial. In reality these "instabilities" correspond to bounded nonlinear oscillations.

Collisions are neglected. In some contexts, such as that of the electrostatic confinement scheme,[4] the $E$ layer,[8] or the high-temperature plasma in an axial field[19,20] this approximation is very good. On the other hand, the application of the present results for collisionless nonrelativistic sheet pinches to the typical pinch discharges obtained thus far in the laboratory[16,17,18] is quite unrealistic. Interestingly enough, the sheet-pinch instability modes that arise when there is a high collision rate (i.e., in the hydromagnetic approximation with finite conductivity[21]) bear a strong superficial resemblance to those obtained in the collisionless limit. In the hydromagnetic finite-conductivity case, however, there is a transfer of energy from the magnetic field to the particles, rather than conversely.

Unless otherwise specified, the neutralizing particle population is taken to have an isotropic thermal distribution. When possible, the mass and thermal velocity of the neutralizing particles are so chosen as to eliminate dependence on these parameters from the marginal stability condition and the instability growth rate. For subtypes A1 and A2, where the coupling is via the longitudinal electric field [see item (g), above], the dependence of the stability condition and the growth rate on the neutralizing particles can be eliminated by assuming their mass to be relatively very great. In that case, the current contribution due to their longitudinal acceleration becomes negligible. For subtype A1, the dependence on the neutralizing particles can also be eliminated from the marginal stability condition merely by assuming their thermal velocity to be finite. In that case, thermal intermixing cancels out the current contribution due to their longitudinal acceleration. (For subtype A2 the longitudinal magnetic field tends to prevent the intermixing.)

For subtypes A3 and B1, B2, and B3, where the coupling is via the transverse electric field, the assumption of very small (or null) thermal velocity eliminates reference to the neutralizing particles from the marginal stability condition. The added assumption of relatively small mass is needed to eliminate reference to the neutralizing particles from the instability growth rate (which depends on this mass in somewhat the manner that the contraction rate of a dynamic pinch[22] depends on the ion mass).

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A number of the equilibrium configurations considered are susceptible to velocity-space instabilities[23-25] of types other than the one that is analyzed here. The stability conditions and growth rates of some of these instabilities, particularly the electrostatic ones, may well be such as to prevent experimental realization of the equilibrium configuration in question, and thus make the present analysis of purely hypothetical interest.

## II. THERMALLY ANISOTROPIC PLASMAS

### 1. Null Magnetic Field

#### *a. Infinite Uniform Plasma*

An assembly of charged particles having the distribution function

$$f_0 = \frac{n_0}{(2\pi)^{\frac{3}{2}} a^2 v^3} \exp\left[-\frac{1}{2v^2}\left(\frac{v_x^2 + v_z^2}{a^2} + v_y^2\right)\right] \tag{3}$$

with charge neutralization provided by an assembly of infinitely massive particles, is subjected to the perturbation magnetic field

$$B_z = b_z e^{\omega t + ikx} \tag{4}$$

and the associated electric field

$$E_y = (i\omega/ck) b_z e^{\omega t + ikx}, \tag{5}$$

where $b_z$ is a constant.

From the first-order Vlasov equation, one obtains for $f = f_0 + f_1 e^{\omega t}$,

$$f_1(\omega + ikv_x) + f_0 \frac{eb_z v_y}{mcv^2} e^{ikx}\left[v_x\left(1 - \frac{1}{a^2}\right) - \frac{i\omega}{k}\right] = 0. \tag{6}$$

The perturbed distribution gives rise to a current density

$$j_y = \frac{e}{c} e^{\omega t} \int_{-\infty}^{\infty} dv_x\, dv_y\, dv_z\, f_1 v_y. \tag{7}$$

$$j_y = -\frac{in_0 r_c b_z}{k} e^{\omega t + ikx}\left[\frac{1}{a^2} - 1 - \frac{1}{a^2} F(\Lambda)\right], \tag{8}$$

where $\Lambda = \omega/\sqrt{2}\, kva$, and

$$F(\Lambda) = \Lambda e^{\Lambda^2}\left[\pi^{\frac{1}{2}} - 2\int_0^{\Lambda} \exp(-\Lambda_1^2)\, d\Lambda_1\right] \tag{9}$$

$$= \pi^{\frac{1}{2}}\Lambda \quad \text{for} \quad \Lambda^2 \ll 1 \tag{10}$$

$$= 1 - \frac{1}{2\Lambda^2} \quad \text{for} \quad \Lambda^2 \gg 1. \tag{11}$$

Since $f_1$ is odd in $v_y$, there is no perturbation in density $n$, and therefore no charge-separation electric field $E_x$.

For simplicity, the displacement current will be neglected, which is appropriate for nonrelativistic plasmas.

In order that Eqs. (4) and (8) may be consistent, Maxwell's equation

$$j_y = (-1/4\pi)(\partial B_z/\partial x) \tag{12}$$

must be satisfied. Thus, one obtains the dispersion relation

$$\frac{4\pi n_0 r_c}{k^2}\left(\frac{1}{a^2} - 1 - \frac{1}{a^2} F\right) = 1. \tag{13}$$

The marginal stability condition ($\omega = 0$) is then

$$\frac{4\pi n_0 r_c}{k^2}\left(\frac{1}{a^2} - 1\right) < 1. \tag{14}$$

Note that for $4\pi n_0 r_c/k^2 \to \infty$ Eq. (13) yields $\omega \to kv$.

This type of instability was first reported by Weibel[2] for an anisotropic electron distribution in the presence of stationary ions, and has been interpreted by Fried[3] as a "mechanism for instability of transverse plasma waves." Alternatively, one may interpret the instability in terms of the generation of local pinches. The marginal stability condition of Eq. (14) is evidently equivalent to the Bennett condition [Eq. (2)], if one identifies $A_s = 8\pi/k^2$ and $v_s^2/v_t^2 = 1/a^2 - 1$.

If the mass $m_n$ of the neutralizing particles is taken to be finite, their contribution to the current density [Eq. (8)] alters the dispersion relation [Eq. (13)] so that one obtains

$$\frac{4\pi n_0 r_c}{k^2}\left[\frac{1}{a^2} - 1 - \frac{1}{a^2} F(\Lambda) - \frac{m}{m_n} F(\Lambda_n)\right] = 1 \tag{15}$$

where $\Lambda_n = \omega/\sqrt{2}\, kv_n$ refers to the neutralizing particle population. Note that if $v_n > 0$, the marginal stability criterion is unaltered, but for $v_n = 0$, one has $F(\Lambda_n) = 1$, and the condition $m/m_n > 1/a^2 - 1$ becomes sufficient for stability.

Applying Eq. (15) to a realistic situation, an anisotropic ion distribution in a medium of relatively "cold" electrons, one finds that the instability growth rate is limited by the mean thermal transit time of the electrons over a distance of the order of one instability wavelength.

#### *b. Electrostatically Confined Plasma in Spherically Symmetric Geometry*

A practical example of a highly anisotropic electron population in a medium of neutralizing ions is

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provided by an electrostatic plasma-confinement scheme that is being pursued by Farnsworth.[4]

Electrons are projected radially inward over the surface of a sphere, thus creating a potential well that traps the ions and permits establishment of a high-temperature plasma. The properties of the zero-order configuration have been analyzed in some detail by Elmore, Tuck, and Watson.[5]

For present purposes, the zero-order distribution function of the electrons need not be stated explicitly. A perturbation magnetic field $B_\phi$ is used. There are no forces to zero or first order in the $\phi$ direction, and so the $\phi$ and $v_\phi$ dependence can be eliminated from the distribution function. The zero-order distribution is then given by

$$f_0 = [f_r(r, v_r)/(2\pi)^{\frac{1}{2}}av] \exp(-v_\theta^2/2a^2v^2) \tag{16}$$

where $f_r$ is a function such that $f_0$ satisfies the zero-order Vlasov equation.

In order to simplify the analysis, a number of approximations are made, which are not necessarily realistic, but which do not alter the essential features of the problem. The quantity $(av)^2$ is assumed to be very small compared with the mean square of the radial velocity, $\overline{v_r^2}$. The product of the zero-order electron density per unit radius, times $\overline{v_r^2}$ is assumed to be a constant, which is written as $N_0v^2$. The boundary condition requiring spherical symmetry of electron injection at the surface of the sphere is neglected during the perturbation. The ions are assumed to have infinite mass.

The perturbation magnetic field corresponding to these approximations is

$$B_\phi = \bar{b}_\phi \frac{1}{r}\frac{dP_n(\cos\theta)}{d\theta} e^{\omega t}, \tag{17}$$

where $P_n$ is the $n$th Legendre polynominal and $\bar{b}_\phi$ is a constant. The first-order Vlasov equation for $\omega = 0$ reduces to

$$\frac{v_\theta}{r}\frac{\partial f_1}{\partial\theta} - \frac{v_\theta v_r e B_\phi}{mca^2v^2} f_0 = 0, \tag{18}$$

from which one obtains

$$\frac{1}{r}\frac{\partial j_r}{\partial\theta} = \frac{r_cB_\phi}{a^2v^2}\int_{-\infty}^{\infty} dv_r\, dv_\theta\, v_r^2 f_0 \tag{19}$$

$$= \frac{N_0r_cB_\phi}{4\pi r^2a^2}. \tag{20}$$

The choice of $B_\phi$ made in Eq. (17) is seen to be consistent with Maxwell's equations

$$j_r = \frac{1}{4\pi r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta B_\phi), \tag{21}$$

$$j_\theta = -\frac{1}{4\pi r}\frac{\partial}{\partial r}(rB_\phi) = 0, \tag{22}$$

provided that

$$N_0r_c/a^2 = n(n+1). \tag{23}$$

Thus, the marginal condition against the most unstable mode ($n = 1$) is given by

$$N_0r_c/2a^2 < 1. \tag{24}$$

Since $r_c = 2.8 \times 10^{-13}$ for electrons, a sphere of $10^2$ cm radius would be limited to a *total* electron (or ion) population of $10^{15}$. Even allowing for the crudity of the present calculation, this result does not argue well for the usefulness of the electrostatic containment scheme in a thermonuclear reactor.

### c. *Interpenetrating Plasma Streams*

Two interpenetrating neutral plasma streams of infinite spatial extent can be described by ion and electron distribution functions of the form

$$f_0 = \frac{n_0}{2(2\pi)^{\frac{3}{2}}v^3}\exp\left[-\frac{1}{2v^2}(v_x^2+v_z^2)\right] \cdot\left\{\exp\left[-\frac{1}{2v^2}(v_y-v_s)^2\right] + \exp\left[-\frac{1}{2v^2}(v_y+v_s)^2\right]\right\}. \tag{25}$$

The stability analysis is essentially the same as for the thermally anisotropic plasma, the quantity $(1/a^2) - 1$ in Eq. (6) being replaced by $(v_s/v)^2 \equiv 1/\alpha^2$. The dispersion relation becomes

$$\frac{4\pi n_0 r_e}{k^2}\left\{\frac{1}{\alpha_e^2} - \left(1+\frac{1}{\alpha_e^2}\right)F(\Lambda_e) + \frac{m_e}{m_i}\left[\frac{1}{\alpha_i^2} - \left(1+\frac{1}{\alpha_i^2}\right)F(\Lambda_i)\right]\right\} = 1, \tag{26}$$

where the subscripts $e$ and $i$ refer to electron and ion parameters, respectively. If the electrons and ions have the same (finite) "temperature," then $m_e\alpha_e^2 = m_i\alpha_i^2$, and the marginal stability condition reduces to

$$8\pi n_0 r_e/k^2\alpha_e^2 < 1 \tag{27}$$

(where $n_0$ is the density of each species).

The range of practical applicability of these results is severely limited by the first-order treatment employed. As soon as the perturbation $B_z$ reaches sufficient amplitude to restrict the electron thermal motion in the $x$ direction, the situation begins to resemble that for null electron temperature [see discussion following Eq. (15)], and electrons are

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then accelerated locally by the $E_y$ field. The electron contribution to the current density $j_y$, which initially was equal to the ion contribution, then becomes opposite to the ion contribution, and the instability growth is arrested. Thus, the maximum energy density of the perturbation magnetic field is only of the order of the initial kinetic energy density of *electron* component of the interpenetrating streams.

Electron–electron and electron–ion collisions have an important modifying effect on the behavior of interpenetrating streams in laboratory experiments.[6,7] Moreover, such experiments are generally carried out in the presence of initial magnetic fields sufficiently strong to affect the electron thermal motion. Experimental results obtained to date do not indicate a significant interaction of interpenetrating plasma streams by the mode considered here, or for that matter by any other cooperative mode.

## 2. Magnetic Field Along Axis of Maximum Velocity

### *a. Infinite Uniform Plasma*

In the presence of a zero-order $B_y$ field, the zero-order distribution function of Sec. 1 a can still be used. The same $B_z$ perturbation is assumed as before [Eq. (4)]. The absence of a first-order $B_y$ perturbation will be verified below.

The first-order Vlasov equation now becomes

$$f_1(\omega + ikv_x) + \frac{eB_y}{mc}\frac{\partial f_1}{\partial \phi_y} + f_0\frac{eb_zv_y}{mcv^2}e^{ikx}\left[v_x\left(1 - \frac{1}{a^2}\right) - \frac{i\omega}{k}\right] = 0, \qquad (28)$$

where $\phi_y = \tan^{-1}(v_z/v_x)$.

In order to obtain the marginal stability condition, the limit $\omega \to 0$ is considered. For $\omega = 0$, the solution is

$$f_1 = f_0e^{ikx} \cdot \left[i\frac{eb_zv_y}{kmcv^2}\left(1 - \frac{1}{a^2}\right) + C\exp\left(-i\frac{v_zmck}{eB_y}\right)\right]. \qquad (29)$$

The constant $C$ is determined by considering terms of order $\omega/kv$ in Eq. (28). One obtains

$$C = i\frac{eb_zv_y}{kmcv^2a^2}J_0\left[\frac{(v_x^2 + v_z^2)^{\frac{1}{2}}mck}{eB_y}\right], \qquad (30)$$

where $J_0$ is the zero-order Bessel function.

The resultant current density is given by

$$j_y = \frac{in_0r_cb_z}{k}e^{\omega t + ikx}\left\{\frac{1}{a^2}[1 - e^{-\epsilon}I_0(\epsilon)] - 1\right\}, \qquad (31)$$

where $\epsilon = (kr_g)^2$, $r_g = mavc/eB_y$, and $I_0$ is the zero-order imaginary Bessel function. Note that, since $f_1$ is odd in $v_y$, $j_x = j_z = 0$, so that the assumption of no perturbation in $B_y$ is self-consistent.

The marginal stability condition then becomes

$$\frac{4\pi n_0r_c}{k^2}\left\{\frac{1}{a^2}[1 - e^{-\epsilon}I_0(\epsilon)] - 1 - \frac{m}{m_n}e^{-\epsilon_n}I_0(\epsilon_n)\right\} < 1, \qquad (32)$$

where the subscript $n$ refers to parameters of the (thermally isotropic) neutralizing particle population. The quantity $e^{-\epsilon}I_0(\epsilon)$ goes as $1 - \epsilon$ for $\epsilon \to 0$ and as $(2\pi\epsilon)^{-\frac{1}{2}}$ for $\epsilon \to \infty$.

If the mass of the neutralizing particles is very small, so that $\epsilon_n \ll 1$, the instability is inhibited except for very large anisotropy $1/a^2 \approx m/m_n$. In what follows, the neutralizing particles will be assumed relatively massive, so that their effect can be neglected.

Equation (32) can be rewritten as

$$4\pi n_0r_cr_g^2(1/a^2 - 1)G(\epsilon) < 1, \qquad (33)$$

where

$$G(\epsilon) = \frac{1}{\epsilon}\left[1 - \frac{e^{-\epsilon}I_0(\epsilon)}{1 - a^2}\right].$$

Equation (33) is equivalent to the Bennett condition for $A_s = 8\pi r_g^2G$. Evidently, we have $G \leq 1$, and so, for any given $n_0$ and $a^2$, the instability can be prevented by taking $B_y$ sufficiently large.

For a low degree of anisotropy ($a^2 \approx 1$), the maximum value of $G$ is $8\pi(1 - a^2)^2/27$, corresponding to a wavenumber $k_m = 3/2\,(2\pi)^{\frac{1}{2}}\,r_g(1 - a^2)$.

For a high degree of anisotropy ($a^2 \ll 1$), the maximum value of $G$ is $1 - a\sqrt{3}$, corresponding to a wavenumber $k_m = 2a/\sqrt{3}\,r_g$.

It is of interest to note that the coefficient of $G$ in Eq. (33) is related to the "$\beta$" of the plasma. One finds

$$\beta \equiv 8\pi n_0mv^2a^2/B_y^2 = 8\pi n_0r_cr_g^2. \qquad (34)$$

The minimum "$\beta$" at which the instability can occur is found for $a^2 \to 0$, and equals $2a^2$. For a comparison with the stability criterion of the familiar "fire-hose instability" see Sec. II 3.

### *b. Interpenetrating Plasma Streams*

With a zero-order distribution function for counterstreaming particles, as in Sec. 1 c, one can use the same perturbation field and a similar mode of analysis as in Sec. 2 a, the quantity $(1/a^2) - 1$ in Eq. (28) being replaced by $(v_s/v)^2 \equiv 1/\alpha^2$. The gyroradius is now given by $r_g = mvc/eB_y$. With

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these substitutions, the marginal stability condition of Eq. (32) applies to interpenetrating streams.

If $B_y$ is such that $\beta \gg 1$ for electrons as well as ions, then the stability condition derived from Eq. (26) applies and there can be an instability for moderate values of $1/\alpha_e^2$. To achieve this regime, $B_y$ has to be exceedingly small, as may be judged from the consideration that the ratio of the magnetic field energy density to the ion kinetic energy density then has to be small compared with $\alpha_e^2 m_e/m_i$.

In the more realistic case where $\beta_i \gg 1$, $\beta_e \ll 1$, there can be no instability unless $\alpha_i^2 < m_e/m_i$.

### c. The E Layer

A plane infinite $E$ layer[8,9] is considered here, with a confining magnetic field

$$B_{x0} = 2v(2\pi n_0 m)^{\frac{1}{2}} \tanh(hz) \tag{35}$$

and a zero-order distribution function[26] for the electrons

$$f_0 = \frac{n_0}{2(2\pi)^{\frac{3}{2}} v^3} \frac{\exp\{-(1/2v^2)[(v_y - v_{sy})^2 + v_z^2]\}}{\cosh^2(hz)} \cdot \{\exp[-(1/2v^2)(v_x - v_{sx})^2] + \exp[-(1/2v^2)(v_x + v_{sx})^2]\}, \tag{36}$$

where

$$h^2 = 2\pi n_0 r_c v_{sy}^2/v^2 \tag{37}$$

and $n_0$ refers to the electron density at $z = 0$. The ions are assumed to be relatively massive and at low temperature, so that their effect on the growth of the present mode can be neglected.

The parameter $m$ is to be taken as referring to the relativistic mass of the electrons. The present analysis, which is aimed at obtaining a marginal stability criterion, is then equally applicable to relativistic and nonrelativistic electrons.

The magnetic-field perturbation is

$$B_y = b_y \psi(z) e^{\omega t + ikx}. \tag{38}$$

There is no associated perturbation in the $B_z$ field.

The first-order Vlasov equation for $\omega = 0$ is given by

$$ikv_x f_1 + v_z \frac{\partial f_1}{\partial z} + \frac{eB_{x0}}{mc} \frac{\partial f_1}{\partial \phi_x} + f_0 \frac{eb_y \psi v_{sx} v_z}{mcv^2} e^{ikx} = 0, \tag{39}$$

where $\phi_x = \tan^{-1}(v_y/v_z)$. For $v_{sy}^2 \gg v^2$, the $v_y$ dependence can be conveniently eliminated by integrating over $v_y$. To second order in $k/h$, one may write for the $B_y$-field dependence $\psi = \psi_0 + (k^2/h^2)\psi_2$, and the solution of Eq. (39) is given by

$$f_1 = f_0 \left\{ \frac{\exp[-(1/2v^2)(v_x - v_{sx})^2] - \exp[-(1/2v^2)(v_x + v_{sx})^2]}{\exp[-(1/2v^2)(v_x - v_{sx})^2] + \exp[-(1/2v^2)(v_x + v_{sx})^2]} \right\} e^{ikx} \frac{eb_y v_{sx}}{mchv^2} \left( \int_0^\eta \psi \, d\eta_1 - \frac{ik}{2h} \frac{v_x v_z}{v^2} + \frac{k^2 v_x^2 \eta}{2h^2 v^2} \right) \tag{40}$$

where $\eta = hz$, and where the subsequently verified result has been used that $\psi_0 = \mathrm{sech}^2\, \eta$.

The current density $j_x$ is given by

$$j_x = \frac{n_0 r_c v_{sx}^2}{v^2 h \cosh^2 \eta} e^{\omega t + ikx} \left( \int_0^\eta \psi \, d\eta_1 + \frac{k^2}{2h^2} \frac{v_{sx}^2}{v^2} \eta \right), \tag{41}$$

where $v_{sx}^2 \gg v^2$ has been assumed for convenience. The perturbation current density $j_y$ is null, and $j_z$ is such as to satisfy

$$ikj_x + \partial j_z/\partial z = 0.$$

The Maxwell's equations

$$j_x = (-1/4\pi)(\partial B_y/\partial z), \tag{42}$$

$$j_z = (1/4\pi)(\partial B_y/\partial x), \tag{43}$$

are then satisfied, provided only that

$$-\frac{d}{d\eta} \cosh^2 \eta \frac{d\psi}{d\eta} = \lambda \left( \psi + \frac{k^2 v_{sx}^2}{2h^2 v^2} \right), \tag{44}$$

where

$$\lambda = 4\pi n_0 r_c v_{sx}^2 / h^2 v^2. \tag{45}$$

The boundary conditions of this eigenvalue problem are $\partial\psi/\partial\eta = 0$ at $\eta = 0$, and $\psi = 0$ at $\eta = \infty$. The solution is $\lambda = 2 - \frac{3}{2}(kv_{sx}/hv)^2$, so that the marginal stability criterion becomes

$$(2\pi n_0 r_c v_{sx}^2 / v^2 h^2)[1 + \tfrac{3}{4}(kv_{sx}/hv)^2] < 1. \tag{46}$$

In terms of Eq. (37) (which expresses the condition for zero-order pressure balance), Eq. (46) can be rewritten as

$$v_{sy}^2 > v_{sx}^2[1 + \tfrac{3}{4}(kv_{sx}/hv)^2]. \tag{47}$$

The greater instability of modes with finite $k$ relates to the oscillation about the $z = 0$ plane which the $y$-directed particles execute in the $B_{x0}$ field at the characteristic frequency $\omega_s = eB_{x0}/mc$. When the wavenumber $k$ is such as to agree with the wavenumber $\omega_s/v_{sx} = 2hv^2/v_{sx}v_{sy}$ of the mean particle motion projected in the $x$ direction, a de-

[26] E. G. Harris, Naval Research Laboratory Report, NRL-4944 (1957).

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stabilizing effect results. In the present limit $v_{sx}^2$, $v_{sy}^2 \gg v^2$, this effect occurs for $(k/h)^2 \ll 1$, and is thus covered by the preceding analysis. The neglected higher-order terms in $k/h$ will presumably have a stabilizing effect.

The stability condition of Eqs. (46) and (47) apply rigorously to a cylindrical $E$ layer in the limit where the thickness is small compared with the radius of curvature ($Rh \gg 1$). The $x$ direction is then along the cylinder axis, and $z$ and $y$ correspond to the radial and azimuthal coordinates. An approximate analysis has been made of the effect of centrifugal force when $Rh$ is finite, and a maximum stabilizing factor $(1 + 2/Rh)$ is found. Initial plans for the Astron thermonuclear experiment[27] envisaged the achievement of field reversal for $v_{sx}^2 > 3v_{sy}^2$. In that case, Eq. (47) would indicate that regions of $B_\theta$ field would be generated spontaneously within the $E$ layer during the initial buildup. More recent studies[28] suggest that field reversal may be attainable for sufficiently small $(v_{sx}/v_{sy})^2$ so that the $B_\theta$ generation is avoided. The importance of this phenomenon to the feasibility of the $E$ layer remains to be evaluated in detail.

### 3. Magnetic Field Along Axis of Minimum Velocity

As in Sec. 2, we assume a zero-order $B_y$ field and make use of the distribution function of Eq. (3). We will now consider a field perturbation characterized by

$$B_z = b_z e^{\omega t + i(k_y y + k_z z)}, \tag{48}$$

$$B_y = B_{y0} - (k_z/k_y) b_z e^{\omega t + i(k_y y + k_z z)}, \tag{49}$$

$$E_x = -(i\omega/ck_y) b_z e^{\omega t + i(k_y y + k_z z)}. \tag{50}$$

This mode corresponds to the familiar "mirror instability" when $a^2 > 1$, and to the "fire-hose instability" when $a^2 < 1$. The marginal stability condition has been derived by a number of authors[11,12,29] and may be written as

$$(a^2 - 1)\beta \frac{k_z^2 - k_y^2/2a^2}{k_y^2 + k_z^2} < 1, \tag{51}$$

where $\beta$ has been defined in Eq. (34). The pressure due to the neutralizing particle population has been neglected. The value of $\beta$ for which the "mirror instability" can occur is lowest when $k_z^2 \gg k_y^2$. Equation (51) then reduces to

$$(a^2 - 1)\beta < 1. \tag{52}$$

Using Eq. (34), we may rewrite Eq. (52) in the form

$$\tfrac{1}{2} A_s n_0 r_c (a^2 - 1) < 1 \tag{53}$$

where $A_s = 16\pi r_g^2$. The similarity of Eqs. (2) and (53) suggests that the "mirror instability" consists basically of the formation of local "pinches" (see also Sec. III 3). The streaming velocity $v_s$ in this case is to be associated with the thermal velocity in the plane orthogonal to the magnetic field $B_y$.

The "fire-hose instability" does not belong to the family of pinch-type modes that are considered in this paper. For $a^2 < 1$, the counterstreaming along the $y$ axis does not give rise to self-focusing perturbation fields, and there is also no perturbation $i_y$. It is of interest to compare the marginal stability criterion for the "fire-hose instability" with that of the pinch-type mode of Sec. II 2 a, which occurs in the same type of zero-order configuration. From Eq. (51), we see that the "fire-hose instability" will occur most readily when $k_y^2 \gg k_z^2$. The marginal stability criterion then reduces to

$$\tfrac{1}{2}(1/a^2 - 1)\beta < 1. \tag{54}$$

(There is an error by a factor of 2 in the corresponding result given in reference 12.) For $a^2 \to 0$, Eq. (54) indicates that the minimum $\beta$ at which the instability can occur is given by $2a^2$. This is exactly the same criterion that was obtained for the pinch-type mode in Sec. II 2 a. For more general choice of $a^2$, one can show that the "fire-hose instability" always sets in at lower values of $\beta$ than the pinch-type mode. Which of the two instabilities is more important in practice depends in large measure on the parameters of the neutralizing particle population.

A theoretical point that is perhaps worth noting here is that the pinch-type mode is missed altogether in those instability analyses that begin with the assumption $kr_g \to 0$. In the case of finite-$\beta$ modes, a *complete* survey must evidently avoid the null-gyroradius approximation.

## III. SHEET-PINCH CONFIGURATIONS

### 1. Reverse-Field Pinches

The $E$-layer model introduced in Sec. II 2 c will serve here as the basic model for a collisionless sheet pinch with conducting walls that are remote from the current layer. We treat the case $v_{sx} = 0$ and consider perturbation fields of the form

$$B_z = b_z \psi(z) e^{\omega t + ikx}, \tag{55}$$

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[27] N. C. Christofilos (private communication, 1961).
[28] W. Heckrotte and V. K. Neil (private communication, 1962).
[29] H. P. Furth, Nuclear Fusion, Suppl., Part 1, 169 (1962).

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$$B_x = B_{x0} + i(b_z/k)(d\psi/dz)e^{\omega t + ikx}, \tag{56}$$

$$E_y = (i\omega/ck)b_z\psi e^{\omega t + ikx}. \tag{57}$$

The corresponding analysis has been carried out in references 29 and 30. The marginal stability criterion[29] (neglecting the pressure due to the neutralizing particle population) is

$$\tfrac{1}{2}A_s n_0 r_c (v_{sy}/v)^2 < \mu, \tag{58}$$

where $A_s = 8\pi/hk$ and the eigenvalue $\mu$ is obtained by solving the equation

$$\frac{1}{kh}\frac{d^2\psi}{dz^2} = \psi\left(\frac{k}{h} - \frac{\mu}{\cosh^2 hz}\right), \tag{59}$$

subject to the boundary condition $\psi = 0$ at the conducting walls. If these walls are at infinity (and for positive $k$), we have

$$\mu = 1 + k/h. \tag{60}$$

When $k/h \approx 1$, each half-wavelength region of the instability resembles a cylindrical Bennett pinch, and Eq. (58) then becomes appropriately similar to Eq. (2). A sheet pinch without nearby conducting walls is always unstable against "tearing" into cylindrical pinches, since we have from the zero-order conditions [Eq. (37)] that the left-hand side of Eq. (58) simply equals $2h/k$. Thus, the instability exists for all wavelengths such that $k < h$. The effect of nearby conducting walls is discussed in reference 29.

For purposes of obtaining the instability growth-rate and generalizing the zero-order field configuration (cf. Sec. III 3) it is sometimes useful to analyze the sheet pinch by an approximate method,[30] where all quantities are averaged over the $z$ and $v_z$ coordinates. This treatment becomes exact in the (thin-current-layer) limit $k/h \to 0$, $wh \to \infty$, for conducting walls located at $z = \pm w$. In this approximate analysis, the eigenvalue $\mu$ becomes

$$\mu = \coth kw \tag{61}$$

which is seen to agree with Eq. (60) when $k/h \to 0$ and to differ by a factor of 2 for $k/h \approx 1$. Equation (61) indicates that when $w$ is small enough, there is a strong stabilizing effect. In the Astron thermonuclear experiment, the presence of the outer conducting shell may therefore provide stability against the "tearing" mode.[29,30]

[30] V. K. Neil, Phys. Fluids **5**, 14 (1962).

### 2. General Field Rotation

When a $B_y$ field is superimposed on the sheet-pinch model of the last section, the angle over which the magnetic-field vector rotates as one passes from $z = -\infty$ to $z = \infty$ can be generalized. A particularly simple case arises if the $B_y$ field is constant. The zero-order distribution function is then unaltered. If we make the perturbation described by Eqs. (55–57), there now result particle drifts in the $x$ and $z$ directions, just as in the example of Sec. II 2 a. Due to the presence of only a *single* stream (rather than a counterstream) of particles in the present example, the particle drifts give rise to charge separation and to associated $E_x$ and $E_z$ fields. These fields are just such as to eliminate the particle drifts in the $x$ and $z$ directions to zero order in $\omega$. Thus, the marginal stability criterion of the last section applies unaltered in the presence of a constant $B_y$ field.

### 3. Null Field Rotation

The thin-current-layer approximation that was introduced in Sec. III 1 has the advantage that it eliminates reference to the $z$ dependence of $B_{x0}$ from the first-order Vlasov equation. Accordingly, the approximate stability criterion of Eqs. (58) and (61) holds for arbitrary $B_{x0}$ values on either side of a plane current layer, including the case where $B_{x0}$ is of the same sign on either side. If the current layer is curved, the Maxwell's equations in the region outside the current layer are of course changed, and a different eigenvalue $\mu$ is obtained.

For the interesting special case of a thin tubular layer of gyrating particles in a nearly homogeneous axial field $B$, Neil[30] obtains the eigenvalue

$$\mu = (kr_{g1})^{-1}(1 - r_{g1}^2/r_w^2)^{-1} \tag{62}$$

in the limit $k \to 0$. The radius of the current layer is $r_{g1} = mv_{s\theta}c/eB$, where $v_{s\theta}$ corresponds to $v_{sy}$ in Eq. (58). The radius of an outer conducting shell is given by $r_w$.

In Sec. II 3 a, the suggestion was made that the "mirror instability" consists basically of the formation of local "pinches" within the plasma continuum. The result given in the present section allows this identification to be made more precise. If we consider the single tubular layer of gyrating particles to have a "$\beta$ value"

$$\beta_c = 16\pi m n_0 v_{s\theta}^2/r_{g1}B^2 h = 16\pi r_c n_0 r_{g1}/h \tag{63}$$

arising from the centrifugal pressure, then Eqs. (58) and (62) give the stability criterion

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$$\tfrac{1}{4}\beta_c(v_{s\theta}/v)^2(1 - r_{g1}^2/r_w^2) < 1 \qquad (64)$$

for "tearing" of the current layer into toroidal pinches. We note the close similarity between Eqs. (64) and (52). The main differences are the wall-stabilization factor $(1 - r_{g1}^2/r_w^2)$, which disappears for the small gyroradii of the continuous medium, and the factor of $\frac{1}{4}$, which reflects the greater stability of the single tubular layer relative to the cooperating adjacent layers of gyrating particles in the continuous medium.[29]

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# Plasma Electromagnetic Instabilities

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A gyrotropic plasma with $T_\parallel/T_\perp \equiv \gamma < 1$, where $T_\perp$ and $T_\parallel$ are the transverse and longitudinal temperatures, is shown to be unstable to a perturbation in the form of a transverse wave with a wave vector $\mathbf{k}$ such that $k^2 < k_0^2 = \{\omega_c^2(1-\gamma)^2 + \omega_p^2(1-\gamma)/\gamma\}/c^2$, $\omega_c$ and $\omega_p$ are the gyro and plasma frequencies. For $T_\parallel > T_\perp$ such an instability arises only if $\omega_c^2/\omega_p^2 > 1/\gamma(\gamma-1)$ and then only for $k^2 < k_0^2$. However, in the latter case, the phase velocity of the instability exceeds $c$ and the growth rate must be made to vanish because of relativistic considerations. No such instability arises for complete isotropy $T_\perp = T_\parallel$.

## I. INTRODUCTION

It is shown that a gyrotropic plasma can suffer from instabilities in the form of transverse electron waves that propagate along the direction of the uniform static magnetic field $\mathbf{B}_0$, provided the electron temperature[1] is anisotropic. In particular, all situations which have $T_\perp > T_\parallel$ are unstable, where $T_\parallel$ and $T_\perp$ are the temperatures along and perpendicular to $\mathbf{B}_0$, respectively. For $T_\perp < T_\parallel$ there is a region of stability determined by the magnitude of $B_0$. As expected these instabilities disappear for $T_\perp = T_\parallel$. The origin of these instabilities may be traced to the fact that in a gyrotropic plasma the transverse wave phase velocity $\nu$ can be much less than $c$ in which case there exist particles which travel with or faster than the wave.[2] The interaction of these particles with the magnetic vector of a transverse wave is considerable and leads to instabilities in the circumstances mentioned above.

## II. ANALYSIS

Consider a collision-free plasma of infinite extent immersed in a uniform magnetostatic field $\mathbf{B}_0$ whose electron distribution function $F(\mathbf{r}, \mathbf{v}, t)$ is governed by the Boltzmann equation

$$\frac{\partial F}{\partial t} + \mathbf{v}\cdot\nabla F - \frac{e}{m}\left(\mathbf{E} + \frac{1}{c}\mathbf{v}\times\mathbf{B}\right)\cdot\nabla_v F = 0. \qquad (1)$$

The electrons are neutralized by a positive ion distribution and in the following we will deal with oscillations that are much too rapid for any active role to be played by the ions. The electric and magnetic fields are determined by Maxwell's equations

$$\text{curl curl } \mathbf{E} = -\frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2} - \frac{4\pi}{c^2}\frac{\partial\mathbf{j}}{\partial t}, \qquad (2a)$$

$$\text{curl } \mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t}, \qquad (2b)$$

$$\text{div } \mathbf{E} = 4\pi\rho, \qquad (2c)$$

$$\rho = n_0 e - e\int F\, d^3v, \qquad (2d)$$

$$\mathbf{j} = -e\int \mathbf{v}F\, d^3v. \qquad (2e)$$

In the linear approximation we take as usual

$$\mathbf{B} = B_0\mathbf{z} + \mathbf{b}(\mathbf{r}, t), \qquad (3a)$$

[1] The word "temperature" is used here in the sense of the mean kinetic energy of the particle.
[2] F. L. Scarf, Phys. Fluids 5, 6 (1962).

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