STABLE CONFINEMENT OF A HIGH-TEMPERATURE PLASMA*

R. F. Post, R. E. Ellis, and F. C. Ford
Lawrence Radiation Laboratory, University of California, Livermore, California

and

M. N. Rosenbluth
General Atomic, San Diego, California
(Received January 18, 1960)

Stable, long-time confinement of hot plasma in a "magnetic bottle" is the ultimate objective of high-temperature plasma research. Theoretically,[1-4] there are relatively few plasma-magnetic field configurations which should permit stable confinement. Even these, when experimentally tested, have until now been found to be unstable, forcing re-examination and extension of the theory. We wish in this Letter to report experiments which represent a contrary example, providing evidence for the existence of a stably confined plasma, observed under circumstances where the simplest hydromagnetic theory predicts instability.

Our two criteria for defining stable confinement are: (1) The observed confinement time shall be very much longer than the theoretically predicted exponentiation time of instabilities. (2) The rate of escape of the plasma out of the magnetic bottle shall agree reasonably with that expected from collisional diffusion (which ultimately limits the attainable time scale for any mode of plasma confinement).

To satisfy both criteria, the plasma must be hot and its particle density must not be too high. This is because rates of diffusion, being proportional to interparticle collision frequencies, become slower in a plasma the lower is its density and the higher is its temperature, whereas hydromagnetic instabilities are predicted to grow faster the higher the plasma temperature, at a rate which is essentially independent of plasma density.

In our experiments the hot plasma was confined in a Mirror Machine[5] and was produced by the adiabatic (i. e., slow) magnetic compression of a low-density burst of hydrogen plasma injected into the evacuated confinement chamber. Magnetic compression ratios ($B_{\text{final}}/B_{\text{initial}}$) of 1000 or more are used, resulting in a spindle of heated plasma typically 3 or 4 cm in diameter and about 20 cm in length. The most striking feature of this plasma is its very high electron temperature.

Following the 500-μsec compression period the magnetic field coils are short-circuited, so that the confining field decays slowly from its peak value, with a time constant of several milliseconds.

Preliminary evidence of plasma heating and long-time confinement in our experiments, first observed in 1953, was obtained by microwave methods. Subsequently the spatial and energy distribution of the confined plasma were measured and reported.[6] Typical values are:

1. Plasma particle density: initial, 10^{11} to 10^{12} cm^{-3}; compressed, 10^{13} to 10^{14} cm^{-3}.
2. Plasma electron temperature: initial, 10 ev; compressed, 10 to 25 kev.
3. Magnetic field: initial, 0 to 100 gauss; final, 10 000 to 40 000 gauss.
4. Calculated plasma β [pressure $\div (B^2/8\pi)$]: up to 0.08.
5. Observed total plasma confinement times: up to 30 milliseconds (typical half-life at peak compression, 2 milliseconds).

Confinement times and densities have been deduced by detection of x rays, by microwave radiometry and interferometry, by calorimetry, and by calculation from the initial densities, and most fruitfully by observing the time dependence of the flux of escaping fast electrons detected by scintillator plus photomultiplier placed near the magnetic axis outside one of the mirrors. (See trace of Fig. 1.) Electron temperatures were determined by absorber foils in front of

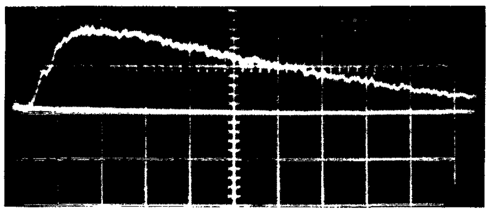

FIG. 1. Typical oscilloscope trace of a scintillator photometer signal—sweep time 5 milliseconds.

the scintillators. (See Fig. 2.) When the energy variation of the Coulomb cross section is folded into the data, a temperature for the confined electrons can be deduced. Corresponding information on ion energies is fragmentary, and largely inferential. From the necessarily low initial ion energies it can be calculated that the final mean ion energies could not be greater than 1 kev, i. e., substantially lower than the electron mean energy, a fact however, of secondary importance here. The observed confinement times are consistent within order of magnitude with those expected from collisional end losses, i. e., scattering into the escape cone.[7]

Hydromagnetic theory[1,2] predicts that a plasma confined in fields where the curvature of the field lines is for the most part concave toward the plasma (as in the Mirror Machine) should exhibit "flute" interchange instabilities essentially independent of the value of β, and of the division of plasma pressure between electrons and ions. Such instabilities would be expected to exponentiate in amplitude initially with a characteristic time constant, τ, determined by a characteristic length of the disturbance (here the length of the plasma spindle) divided by the plasma "sound" velocity $v=(p/\rho)^{1/2}$. Here

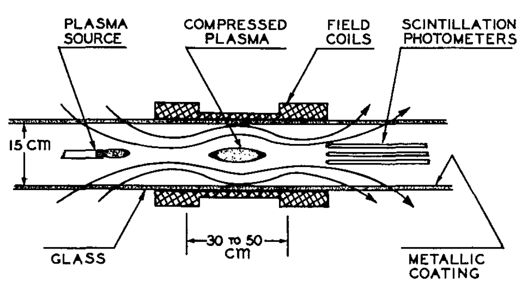

FIG. 2. Schematic arrangement of experimental apparatus.

$p=(n_ikT_i+n_ekT_e)$ is the plasma pressure, and $\rho=(n_iM+n_em)$ is the plasma mass density. A lower limit to v is obtained by setting $T_i=0$, in which case $v\approx(kT_e/M)^{1/2}\approx 1.4\times10^8$ cm sec^{-1} in these experiments. Thus a typical exponentiation time of a hydromagnetic instability would be $\tau=L/v\approx0.14$ μsec. The growth times for other types of plasma instabilities, for example, unstable electron plasma oscillations, would be expected to be even shorter than this time. Since the observed total confinement times range up to as much as 10^5 times the longest of "instability times, they certainly satisfy our first criterion for stable confinement.

We may estimate the "classical" rate of radial collisional diffusion across the magnetic field (e. g., Spitzer[8]). The calculated initial diffusion rate at peak compression should be approximately 0.4 cm/sec, i. e., it should be barely detectable if at all during the confinement time. This fact then poses a severe test for our second stability criterion.

Using an array of scintillators one of us (Ellis) measured the flux of escaping energetic electrons as a function of time after peak compression. At any chosen time, the variation of this escaping flux with radius measures the instantaneous value of the radial distribution of particle density. Fig. 3 presents radial particle flux distribution curves obtained by averaging over many different individual compression cycles. When these data, normalized to the same height, are corrected for the slight expansion of the magnetic flux surfaces owing to the field decay, the curves of Fig. 4 result. These data show clearly that little,

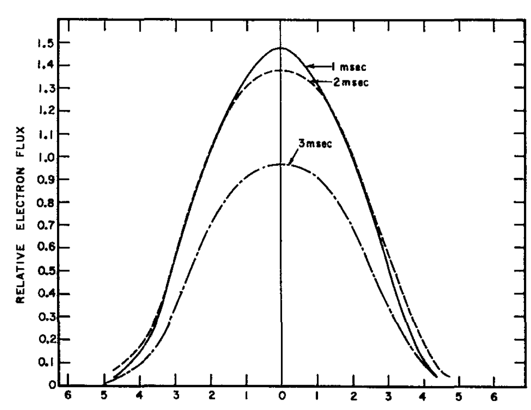

FIG. 3. Radial distribution of escaping electron flux as a function of time.

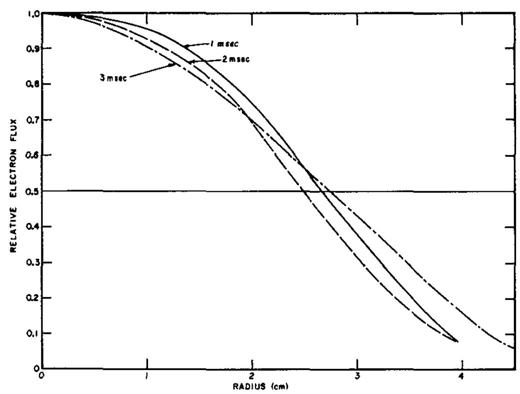

FIG. 4. Normalized radial flux distribution.

if any, radial diffusion of the bulk of the plasma electrons occurs during a period of several milliseconds after the peak compression. This conclusion is not altered qualitatively by examination of one set of traces. As analyzed by Perkins,[9] our data give a diffusion rate of 0 ± 40 cm/sec. Thus, the observations are not inconsistent with the theoretical value 0.4 cm/sec, but the statistical errors are too large to permit quantitative conclusions to be drawn. The velocity at one standard deviation is still some 10^5 times smaller than that predicted by the so-called "Bohm" diffusion formula, which might be expected to apply in the presence of intense plasma oscillations. Thus, these radial distribution measurements are not inconsistent with classical diffusion theory, and show no excessive diffusion rates of the kind which would be expected to accompany plasma turbulence.

Apparently, therefore, the plasmas obtained in these experiments satisfy the qualitative criteria defining stable confinement. Furthermore, the Van Allen radiation belt regions have many of the same features as our machine and evidently also represent a stably confined plasma.

Since the simple hydromagnetic theory predicts instability we must seek other explanations. For example:

1. Certain very special particle distributions in which most of the particles turn around near the peak of the mirrors may be theoretically stable.[2] This appears incompatible with our evidence on the plasma spatial distribution.

2. Owing to the ambipolar nature of diffusion losses from a Mirror Machine, axial and radial components of electric field can be expected to develop, which might cause differential rotation of successive flux shells of the plasma. This could be a scrambling mechanism which might inhibit flute instability growth. However, a calculation[10] of a related problem—stability of a pinch with differential rotation—indicates that this does not occur and that the effect of the rotation is to introduce another destabilizing effect—the centrifugal force.

3. The original instability calculations were done for infinite azimuthal wave number.[1,2] Due to the finite Larmor radius only perturbations with $m < \sim 10$ can be allowed in the machine. The hydromagnetic calculation can be generalized[1] to show that for low β any nonzero m would be unstable.[11] However, the more correct formulation in terms of the particle motion[2] has not yet been so generalized. It would be instructive to apply the new variational formulations[12,13] to the problem. Even these formulations are based on small Larmor radius and it remains remotely possible that in a plasma with radius about 5 times the ion Larmor radius strong additional stabilization comes about from finite Larmor radius effects.

4. Since all stability theory has been based on the linearized equations, the possibility remains that some stable configuration might be possible in which the oscillations are excited to a finite amplitude. However, since gross energetic considerations indicate that such oscillations would result in expulsion of the plasma, this explanation does not appear probable.

5. The most probable explanation for the stability is that the field lines are "tied" down outside the plasma by the metallic coatings on the chamber walls. During flute instability growth, charge separation occurs[2] in such a way that a potential difference exists between the crest and the trough of the instability. At low β only such "electrostatic" modes of growth are possible. Moreover, the fact that charges can move freely along the lines of force causes each line of force to be an equipotential. Even outside the mirror region there is likely to be enough low-density, low-temperature background plasma for this to be the case. However, the conducting end walls will not allow such potential differences to occur, but will drain charges off the flute, thus removing the electrostatic fields which drive the instability. Even an extremely thin metallic coating, such as we have, can be estimated to remove charges so efficiently that the effective growth rate is lowered from 10^7/sec to about 1/sec. Whether instabilities where growth rate is slower

than the precession around the machine can ever exist is problematical. They would certainly not be observed in these experiments.

If the purely electrostatic flute modes are thus eliminated, we must consider modes in which the magnetic field is altered in such a way that the field lines can carry the plasma out of the machine in the center, while remaining tied at the conducting end walls. In such a perturbation the plasma energy will be lowered, and the magnetic energy raised. Thus, we expect to find a critical value of β below which the plasma will be stable.[4] Calculations based on the hydromagnetic equations indicate that the critical β for our geometry is roughly 10%. More refined calculations[12, 13] would be useful, but our plasma pressure is probably sufficiently low that "tying" can produce stability.

Other possible instability mechanisms must also be considered. In addition to interchange instabilities, there are other classes of instabilities which may occur in a confined plasma. These are the various "velocity-space" instabilities, e. g.:

1. The so-called "Mirror Instability" which may occur when the equilibrium plasma pressure distribution is such that the component of the pressure tensor $\perp$ to the fields is greater than that along the field lines ($p_\perp > p_\parallel$), as in a Mirror Machine. A sufficient condition for stability against this can be obtained from the work of Newcomb.[14] At any point,

$$\beta < \{\tfrac{1}{2}[(\nabla p_\perp/p_\perp)/(\nabla B/B)]+1\}^{-1}, \qquad (1)$$

the gradients being taken along the field lines. For distributions with reasonable $p_\perp/p_\parallel$ ratios, (1) is relatively large,[15] being about 0.5 in typical cases; thus it is substantially larger than our observed values. However, for highly anisotropic pressure distributions this type of instability might be very serious.

2. Unstable electron plasma oscillations, arising from anisotropies in the electron angular distribution, have been considered by Harris[16] and others. A sufficient condition for stability against this type of disturbance is that $(\omega_p/\omega_{ce})^2 < 1$, i. e., plasma frequency less than electron cyclotron frequency. Stated in terms of a lower limit critical β for the electrons, this condition is[17]

$$\beta_e < 2kT_e/mc^2. \qquad (2)$$

Although representing a serious limitation for systems of high density and low electron temperature, the high electron temperature (25 kev) achieved in our experiments means that $\beta_e \approx 0.1$, probably larger than our experimental values.

3. Unstable ion plasma oscillations. Harris[18] has also pointed out the possibility, under special circumstances (vanishing ion velocity components parallel to the field lines) of unstable plasma ion oscillations which might occur if the condition $(\omega_{pe}/\omega_{ci})^2 < 1$ is violated. This is a very restrictive condition, occurring at densities $(M/m)^2$ lower than (2) above, i. e., at densities of about 10^7 ions/cm^3. It is not clear whether the fact that we do not see these instabilities represents a contradiction with Harris' prediction or whether a more realistic ion distribution would alter the theoretical conclusions.

Since experimental stability has apparently been attained, it is worth enumerating the ways in which we differ from most other plasma heating and confinement experiments: At no time are electric fields applied parallel to the magnetic field, thus avoiding beam instabilities; adiabatic (i. e., slow) magnetic compression is used as the sole heating mechanism; and the plasma is confined by an axially symmetric externally generated magnetic field, the lines of force of which are "tied" at the ends of the machine. However, other features of our plasma differ substantially from those desired for practical application: The electrons are much hotter than the ions and the plasma dimensions are not very large compared to the ion orbit size. Moreover, at our densities the confinement times, even though in apparent agreement with theory, are still shorter by two or more orders of magnitude than those required for self-sustaining fusion reactions. This deficiency can presumably only be remedied by achieving higher ion temperatures and relying on ambipolar effects to hold the electrons. While there is no apparent theoretical reason to expect that a change in these parameters will destroy stability, nevertheless we feel that only further experiments can solve such complicated questions.

In conclusion, although we cannot as yet provide a documented theoretical explanation, we believe we have adequately demonstrated, experimentally, a case of a stably confined hot plasma. Whether the results are of general interest, or are only a special case, must be determined by additional experimentation and theoretical work. Hopefully, the experiments provide an opening wedge into solving the problem of long-time magnetic confinement of a hot plasma.

We wish to acknowledge many helpful discussions with F. Coensgen, S. Colgate, A. Kaufman, and W. Newcomb, in addition to the past contributions of many others of the Mirror Machine group in the experimental work.

---

*Work done under the auspices of the U. S. Atomic Energy Commission.

[1]I. Bernstein, E. Frieman, M. Kruskal, and R. Kulsrud, Proc. Roy. Soc. (London) A244, 17 (1958).

[2]M. Rosenbluth and C. L. Longmire, Ann. Phys. 1, 120 (1957).

[3]J. Johnson, C. Oberman, R. Kulsrud, and E. Frieman, Phys. Fluids 1, 281 (1958).

[4]J. Berkowitz, H. Grad, and H. Rubin, Proceedings of the Second United Nations International Conference on the Peaceful Uses of Atomic Energy, Geneva, 1958 (United Nations, Geneva, 1959), Vol. 31, p. 177.

[5]R. F. Post, reference 4, Vol. 32, p. 245.

[6]F. Coensgen, F. C. Ford, and R. E. Ellis, reference 4, Vol. 32, p. 266; and reference 5.

[7]A. Garren, R. J. Riddell, L. Smith, G. Bing, L. R. Henrich, T. G. Northrop, and J. E. Roberts, reference 4, Vol. 31, p. 65.

[8]L. Spitzer, Physics of Fully Ionized Gases (Interscience Publishers, New York, 1956).

[9]W. A. Perkins, R. E. Ellis, and R. F. Post, American Physical Society Bulletin for Division of Plasma Physics Meeting, Monterey, Paper B9, p. 8, December, 1959 (unpublished).

[10]E. Gerjuoy and M. Rosenbluth (to be published).

[11]E. Frieman and R. Kulsrud, Advances in Applied Mechanics (Academic Press, New York, 1958), Vol. 5.

[12]M. Kruskal and C. Oberman, reference 4, Vol. 31, p. 137.

[13]M. Rosenbluth and N. Rostoker, reference 4, Vol. 31, p. 144.

[14]W. Newcomb, Ann. Phys. (to be published).

[15]R. F. Post, University of California Radiation Laboratory Report UCRL-5524, July, 1959 (unpublished); Proceedings of the Conference on the Theoretical Aspects of Controlled Fusion Research, Gatlinburg, Tennessee, April 28-29, 1959 [Atomic Energy Commission Report TID-7582 (unpublished)].

[16]E. Harris, Phys. Rev. Letters 2, 34 (1959).

[17]R. F. Post, Annual Review of Nuclear Science (Annual Reviews, Inc., Palo Alto, 1959), Vol. 9, p. 416.

[18]E. Harris, Proceedings of the Conference on the Theoretical Aspects of Controlled Fusion Research, Gatlinburg, Tennessee, April 28-29, 1959 [Atomic Energy Commission Report TID-7582 (unpublished)].