Thermonuclear confinement system with twin mirror systems

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1. INTRODUCTION

The energy balance of a thermonuclear mirror confinement system with fast-atom injection is governed essentially completely by the figure of merit Q, which is the ratio of the thermonuclear power to the injection power. For a DT reactor with equal deuterium and tritium densities we have

$$Q=\frac{n^2\langle\sigma v\rangle}{4I}\frac{\varepsilon}{E_j}, \quad (1)$$

where n is the plasma density, $\langle\sigma v\rangle$ is the reaction rate, ε is the thermonuclear energy liberated per fusion event, including the energy liberated in the lithium blanket, and I and E_j are the current per unit volume and the energy of the injected atoms.

Experimental progress on the confinement of hot plasmas in open confinement systems with magnetic mirrors[1–3] holds out the hope that it will be possible to achieve the "classical" plasma confinement time in such systems, i.e., the confinement time set by the Coulomb scattering of ions. Calculations of the classical confinement time show that in ordinary open confinement systems[4] Q=1–1.5. The ratio of the useful electrical power to the thermonuclear power for a power plant with an open confinement system operating in the optimum mode[5] with direct conversion of the charged-particle energy into electrical energy, i.e., the efficiency of the power plant, is

$$\eta=\eta_t+\frac{\varepsilon_\alpha}{\varepsilon}\left(1-\frac{\Delta E_n}{\varepsilon_\alpha}\frac{P_r}{P_\alpha}\right)\eta_E(1-\eta_t)+\frac{\eta_E(1-\eta_t)}{Q}-\frac{1-\eta_t}{\eta_I Q}, \quad (2)$$

where η_t and η_E are the efficiencies of the thermal conversion and of the direct conversion, η_I is the efficiency of the injection system, ε_α is the energy of the α particles, P_r and P_α are the power loss due to radiation and the power transferred from α particles, and ΔE_n is the kinetic energy of the reacting nuclei which is transferred to the neutron. In the analysis of the system in ref. 5 the values $\eta_t=0.45$, $\eta_E=0.84$, and $\eta_I=0.87$ were adopted; corresponding to these values for Q=1–1.5 we have η=33–39%. If we adopt the least optimistic estimates, $\eta_t=0.4$, $\eta_E=0.8$, and $\eta_I=0.7$, and if we assume $\Delta E_n\sim100$ keV and $P_r/P_\alpha=0.1$, we conclude that in order to achieve a plant efficiency of η=30–40% we would need Q=2.3–5.85.

In the present paper we propose an open confinement system with twin magnetic mirrors, in which values $Q\gg1$ can in principle be attained. Figure 1 is a sketch of the magnetic field configuration in this system. Over the central part of the confinement system, which is the greater part of the system, the magnetic field B_0 is uniform. At each end of the confinement system there is a pair of magnetic mirrors with identical maximum magnetic field B_m. Each pair of mirrors forms an end mirror system of length L_k with a minimum field B_k between mirrors. For hydrodynamic stability of the plasma, the longitudinal barrel-shaped field B is supplemented by a transverse quadrupole field with gradient g in the end mirror systems. A transverse field in the opposite direction is also produced near the ends of the central mirror system, which is formed between the inner mirrors. This field is required to match the end minimum-B fields to the barrel-shaped central field. The magnetic lines of force of the confinement system in the plane in which the transverse field is precisely radial are shown at the top in Fig. 1.

Ions with energies on the order of 1 MeV are continuously introduced in the end mirror systems through atomic-beam injection. Ions with energies of a few tens of kiloelectron volts are introduced in the central part of the confinement system. The ratio of the injected currents is chosen such that the density of the hot plasma in the end mirror systems, n_k, is substantially higher than the plasma density at the center of the system, n_0. Since the electrons escape through the magnetic mirrors much faster than the ions do, because of Coulomb scattering, an ambipolar electric field is established in the plasma; this field automatically maintains equal electron and ion densities. Since the electron density is related to the ambipolar potentials by the Boltzmann law, an electrical potential well for ions forms at the center of the confine-

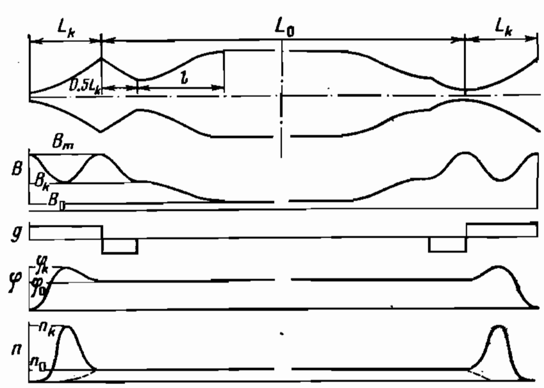

Fig. 1. Confinement system with twin mirror systems.

ment system. If the "central" ions are at a temperature corresponding to an energy considerably below the potential barriers, $\Delta\varphi=\varphi_k-\varphi_0$, the lifetime of the central ions becomes much longer than the confinement time corresponding to the confinement by the magnetic mirrors. As a result, a constant plasma density is established over the entire length of the central mirror system, L_0, including the regions near the mirrors. The plasma distribution in the end mirror systems is governed by the scattering of the fast ions confined in these systems. As shown by the dashed curves at the bottom in Fig. 1, the central ions can partially penetrate into the end mirror systems. The interaction of the relatively warm isotropic central plasma with the anisotropic fast ions can help arrange conditions for the loss-cone instabilities in the end mirror systems.

The confinement of the central plasma along the field in this confinement system is electrostatic, but, in contrast with the Lavrent'ev electromagnetic confinement systems,[6–8] the intensity of the confining electric field is absolutely unrelated to the pressure of the central plasma. The length and profile of this field can be adjusted by adjusting the profile of the longitudinal magnetic field in the end mirror systems, which basically determines the axial profile of the fast-ion density. Ultimately, the longitudinal pressure of the central plasma is restrained by the magnetic field of the confinement system.

Ambipolar electric fields are unavoidable features of open confinement systems; the corresponding electric potentials in the plasma can reach[4] 100 kV. These fields are usually detrimental to plasma confinement, so that to make constructive use of them can hardly disrupt the operation of the confinement system seriously.

The volume of the end mirror system is an insignificant fraction of the overall volume of the confinement system. These end mirror systems do not play an important role in the production of thermonuclear energy; their basic purpose is longitudinal confinement of the central plasma. They act as composite mirrors for the straight magnetic confinement system. Since these end systems have a minimum field and a high plasma density, they also impart hydrodynamic stability to the plasma throughout the confinement system.

Kelley[9] has proposed a device consisting of three mirror systems, designed to reduce the loss due to the ambipolar potential through the cancellation of the ambipolar field in the central mirror system. As is shown below, this approach leads to a negligible improvement in the overall amplification factor Q.

To carry out calculations for this confinement system it is necessary, generally speaking, to solve the Fokker–Planck equation for the central and end mirror systems for each particle species. Since all the electrons and the central ions are confined primarily by potential barriers, their distribution functions will be approximately Maxwellian if the barriers are sufficiently high. Accordingly, to calculate the confinement time and the average energy with which a particle leaves the confinement system, we use the equations of Pastukhov,[10] altering them for the case of ions. The electrons are common to the entire system; they can be described by a common temperature T_e and a Boltzmann distribution $n_e \propto \exp(\varphi/T_e)$ over the entire length of the system under the condition $\exp(-\varphi_0/T_e) \ll 1$. The Fokker–Planck equation is solved in the end mirror systems for the ions only.

In carrying out calculations for the plasma behavior we ignore the finite bounce time of the particles between mirrors, the interactions of the electrons and ions with α particles, and the radiation from the electrons. We replace the deuterons and tritons by ions with an intermediate mass M_i defined by

$$\frac{1}{\sqrt{M_i}}=\frac{1}{2}\left(\frac{1}{\sqrt{M_D}}+\frac{1}{\sqrt{M_T}}\right). \tag{3}$$

The cross sections and reaction rate are determined from the interpolation equations and experimental data given in systematic form by Kozlov.[11] The Coulomb logarithms are calculated by the procedure of ref. 12.

2. PROPERTIES OF THE PLASMA IN THE CENTRAL CONFINEMENT SYSTEM

The substantial improvement in Q comes from the increase in the confinement time τ_0 of the central ions. According to Pastukhov,[10] this time is given by

$$\frac{n_0}{\tau_0}=I_0(1-\alpha_0)=\frac{\sqrt{2\pi}\,e^4\lambda_0^{ii}}{\sqrt{M_i}F(R_0/2R_k)}$$
$$\times\frac{n_0^2\exp-\Delta\varphi/T_{i0}}{\Delta\varphi\sqrt{T_{i0}}}\left[1+\frac{T_{i0}}{2\Delta\varphi}-\left(\frac{T_{i0}}{2\Delta\varphi}\right)^2\right];$$
$$\Delta\varphi=T_e\ln\frac{n_k}{n_0};\qquad \alpha_0=\frac{1}{2}\langle\sigma v\rangle_0\frac{n_0^2}{I_0};$$
$$F(x)=\sqrt{1+\frac{1}{x}}\ln\sqrt{1+\frac{1}{x}}+1\Big/\sqrt{1+\frac{1}{x}}-1. \tag{4}$$

Here α_0 is the degree of burnup of the central plasma, T_{i0} is the temperature of the central ions, R_0 and R_k are the mirror ratios for the central and end mirror systems, and λ is the Coulomb logarithm. From (4) we find the rough estimate $\tau_0 \sim \tau_i(n_k/n_0)^{T_e/T_{i0}}$, where τ_i is the ion relaxation time. We see that in addition to the condition $n_k > n_0$ we must have $T_e > T_{i0}$; the latter condition is satisfied since the injection energy of the central ions is $E_{j0} < \Delta\varphi + T_{i0}$. The central ions are held at a steady temperature by energy transfer from electrons; in turn, the electrons acquire energy from the fast ions in the end confinement systems or from auxiliary rf heating by an external rf field.

The energy balance for the central ions is described by the equations

$$E_{j0}+\langle E_0^{ei}\rangle=\left[\Delta\varphi+T_{i0}\left(1+\frac{T_{i0}}{2\Delta\varphi}\right)\right](1-\alpha_0)+\langle E_{ir}\rangle\alpha_0;$$
$$\langle E_0^{ei}\rangle=4\sqrt{2\pi}\,e^4\lambda_0^{ei}\frac{\sqrt{m}}{M_i}\frac{T_e-T_i}{T_e^{3/2}}\frac{n_0^2}{I_0};$$
$$\langle E_{ir}\rangle=\frac{3}{4}T_{i0}+\frac{2}{\sqrt{\pi}}\frac{1}{\sqrt{M_i}\langle\sigma v\rangle_0 T_{i0}^{3/2}}\int W^2\sigma(W)\,dW. \tag{5}$$

Here E_0^{ei} is the energy acquired by a central ion from electrons, E_{ir} is the energy of a reactant ion, W is the energy of the reactant particles in their center-of-mass system, σ is the cross section for the DT reaction, and e and m are the electron charge and mass. The electron lifetime is governed by the loss of electrons across the potential barrier as the result of collisions. We assume that the contributions of the central and end plasmas to the collisions are proportional to the probabilities for finding electrons in the corresponding mirror systems. Correspondingly, the electron balance equation is, according to Pastukhov,[10]

$$I_0+I_k V_k/V_0=2\sqrt{2\pi}\, e^4/\sqrt{m}\cdot n_k \exp-\varphi_k/T_e$$

$$\times\frac{1}{\sqrt{T_e}}\left[\frac{n_0\lambda_0^{ei}}{F(R_0)}\frac{1+T_e/2\varphi_0-(T_e/2\varphi_0)^2}{\varphi_0}+\frac{n_k\lambda_k^{ei}}{F(R_k)}\frac{1+T_e/2\varphi_k-(T_e/2\varphi_k)^2}{\varphi_k}\frac{V_k}{V_0}\right]; \qquad (6)$$

where V_0 and V_k are the calculated volumes of the central and end plasmas. Equation (6), which holds for the entire confinement system, closes system (4), (5), for the central plasma. The amplification factor for the central mirror system is

$$Q_0=\frac{1}{4I_0}\frac{n_0^2\langle\sigma v\rangle_0\varepsilon}{E_{j0}+\langle E_0^{ei}\rangle+\varphi_0+T_e(1+T_e/2\varphi_0)}. \qquad (7)$$

If the energy balance of the confinement system is to be governed by the central part, the ratio $n_k^2V_k/n_0^2V_0$ must be small. If it is, the second terms on the left and right sides of electron balance equation (6) are much smaller than the first terms and can be neglected, in a first approximation. This simplification means that the end ambipolar potentials φ_k are weak functions of the properties of the end plasma, as will be shown below. The limiting value Q_0^∞ corresponding to the condition $V_0/V_k\to\infty$ is only 1-3% smaller than the value of Q_0 for the actual volume ratios. In the limit $V_0/V_k\to\infty$ the properties of the central plasma can be determined from the parameters R_0, R_k, n_0, n_k, T_e, and E_{j0}. Table 1 shows how some of the properties of the central plasma vary with the injection energy; we see that Q_0^∞ falls off, albeit slowly, with the energy E_{j0}. It is therefore advantageous to inject ions into the center of the system at a low energy. In the calculations below we assume $E_{j0}=20$ keV everywhere. Figure 2 shows Q_0^∞ as a function of the electron temperature and of the density ratio. We see that it is possible in principle to achieve high amplification factors for the central part of the confinement system. The peaks on the $Q_0^\infty(T_e)$ curves are due to burnup of the material.

We emphasize that the distribution of the central ions is not exactly Maxwellian; the high-energy part of the distribution is truncated, so that the reaction rate should be reduced. In the temperature range under consideration here, $T_{i0}=20$–30 keV, the maximum partial contribution to the reaction rate comes from ions with relative-motion energies of $(2.45\text{–}2.15)T_{i0}$. As the energy of the ions increases, their partial contribution to $\langle\sigma v\rangle$ falls off rapidly [to 37% of the maximum value for a relative energy of $(4.9\text{–}4.5)T_{i0}$]. According to the calculations, for the interesting cases $\Delta\varphi\geq 3T_{i0}$ the maximum energy of the relative motion of the central ions along the field exceeds $6T_{i0}$.

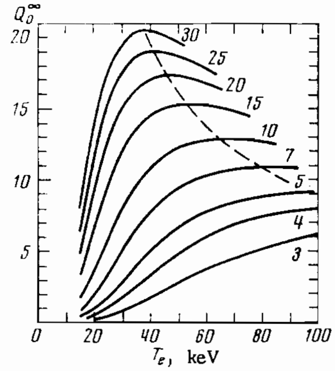

Fig. 2. Curves of $Q_0^\infty=Q(T_e, n_k/n_0)$. The curves are labeled with the values of the ratio n_k/n_0. $R_k=3$. The value of R_0 corresponding to $n_k/n_0=3$ is 16; proceeding in this order, the values of R_0 corresponding to the other curves are 19, 22, 27, 32, 39, 45, and 51.

Furthermore, it follows from the energy of the ions escaping across the barriers, $T_{i0}(1+T_{i0}/2\Delta\varphi)$, that the ion energy distribution stretches up to several times the ion temperature beyond the barrier, $\Delta\varphi$. We can therefore assume that the lowering of $\langle\sigma v\rangle_0$ due to the truncation of the distribution function is insignificant.

3. PROPERTIES OF THE END MIRROR SYSTEMS

To analyze the operation of the end mirror systems we solve the Fokker–Planck equation for the ion distribution function, written in the form

$$\frac{\partial f}{\partial t}=St[f]+S \qquad (8)$$

with the collisional term St[f] written in the Landau form. Here $S=S(\mathbf{v}_j)$ is the function representing the ion sources. Equation (8) holds for a "square" magnetic well.

In order to keep the overall length of the confinement system from becoming too large, the end mirror systems should be as short as possible, and in order to reduce the amplitude of the ambipolar electric fields, the decrease of the plasma density along the magnetic field in these mirror systems should begin right at their central planes. The magnetic well in the end mirror systems is thus far from being a "square" well, being approximated better by a "cosinusoidal" well. The plasma behavior in a cosinusoidal magnetic well has been analyzed, in particular, by Marx.[13] It follows from his calculations that the quantity $n\tau=\langle n^2\rangle/\langle I_{con}\rangle$ in such a well is lower than that in a square well by no more than 5%. In a single mirror system with a cosinusoidal well, the ambipolar potential drops. Since the ambipolar potential in the end mirror systems of the device under consideration here is governed primarily by the central plasma, this distinction between square and cosinusoidal wells is not important in our case. It follows from the conservation of the product

TABLE 1. Plasma Properties in the Central Mirror System at Various Injection Energies

E_{j0}, keV	T_{j0}, keV	$n\tau_0$, sec/cm³	α_0, %	Q_0^∞	Parameters adopted for calculations
20	22.6	$1.91\cdot10^{15}$	33.4	11.9	$n_0=5\cdot10^{13}$ cm^{-3}
40	23.8	$1.62\cdot10^{15}$	31	11.1	$R_k=3$; $R_0=36$
60	25.2	$1.37\cdot10^{15}$	28.5	10.2	$\Delta\varphi=81.25$ cm
80	27.1	$1.12\cdot10^{15}$	25.9	9.2	$\frac{n_k}{n_0}=15$; $T_e=30$ keV

TABLE 2. Comparison of Fokker–Planck Calculations for the Confinement with the Data of Futch et al.[4] $M_I = M_D$; $n = 3 \cdot 10^{15}$ cm^{-3}

R	10				3	
E_j, keV	60	100	200	300	100	300
$n\tau$, 10^{13} sec/cm^3	0.93	1.95	5.4	9.8	1.0	5.0
$n\tau_F$, 10^{13} sec/cm^3	1.0	2.08	5.7	10.2	1.04	5.1
$n\tau/n\tau_F$	0.93	0.94	0.95	0.96	0.96	0.98

$n\tau$ that the replacement of a cosinusoidal well by a square well results in only a relatively small error in the calculations for the end mirror systems.

As in the calculations by Futch et al.,[4] we artificially break up Eq. (8) into two one-dimensional equations (for the modulus and the direction of the velocity). The angular distributions of both the ions and the ion sources are assumed to be equal to the first normal mode of the solution of the "angular" equation. The ion distribution with respect to the velocity modulus v is calculated from

$$\frac{M_i^2}{4\pi e^4}\frac{\partial}{\partial t}f(v)=-\frac{1}{v^2}\frac{\partial}{\partial v}\left[f(v)\left(v^2\frac{\partial h}{\partial v}+\frac{\partial g}{\partial v}\right)\right]$$
$$+\frac{1}{2v^2}\frac{\partial^2}{\partial v^2}\left[v^2 f(v)\frac{\partial^2 g}{\partial v^2}\right]-\frac{\gamma}{2v^3}\frac{\partial g}{\partial v}f(v)+S(v_j);$$
$$\frac{1}{\gamma}=\lg\frac{R}{1+2\varphi/M_i v^2},\qquad (9)$$

where γ is the separation constant, and h and g are the Rosenbluth–Trubnikov potential functions in isotropic form.[4]

To solve Eq. (8) we use an implicit difference scheme,[14] which is absolutely stable even if there is no loss term and even if the sources are conservative. The time step is chosen to have a constant value. The distribution function at a given instant is found by tridiagonal inversion.[15] The functionals h and g are calculated by the quasilinearization method of ref. 16. In a preliminary calculation, Eq. (9) is solved with the functions h and g determined from the distribution function from the preceding step. The solution found in this manner is used to find new expressions for h and g, which are then used for the final determination of f(v) in the given step. Since in the case $\varphi \neq 0$ the equation has a logarithmic singularity at $v = v_{min}$, the step of the velocity grid is made smaller in a geometric progression as $v \to v_{min}$; the distance from v_{min} to the first mesh point is assumed to be small but much larger than the first and smallest step. The number of points in the grid is chosen between 1000 and 2000. In each time step the electron density is assumed to equal the ion density. After a certain number of steps, the particle and energy balances are checked. As a rule, with a judicious choice of the initial distribution, the steady state can be reached in 500 time steps. The deviation from exact balance is less than 2%. Calculations for a single case with the self-consistent parameters on the basis of the auxiliary equations require an average of 6 min on a BÉSM-6 computer.

To check the calculation procedure, we carry out calculations for the confinement of a single-component deuterium plasma in a single mirror system on the basis of the Pastukhov equations[10] in order to determine the behavior of the electrons. Table 2 shows the corresponding results; here $n\tau_F$ is the value of $n\tau$ from ref. 4. We see from Table 2 that our calculations give a value of $n\tau$ that is too low. The reaction rates in the end mirror systems were calculated in accordance with ref. 4, but for ions with the intermediate mass in (3). Over a broad range of average energies, the calculated values of $\langle\sigma v\rangle$ agree well with the calculations for two-component DT mirror systems.[4] Calculations for a single mirror system with ions of intermediate mass show that for mirror ratios R = 3-10 and injection energies E_j = 60–500 keV the equations $T_e = 0.1\,\langle E_i\rangle(\log R)^{2/5}$, $\varphi = 4.7T_e$, and $n\tau = 2.25\cdot10^{10}(\langle E_i\rangle\text{keV})^{3/2}\cdot\log R$ are satisfactorily accurate. The difference between these results and those of ref. 4 is 3-4%. To a certain extent, the discrepancy with regard to the product $n\tau$ cancels the systematic error due to the replacement of the cosinusoidal magnetic well by a square well.

The end mirror systems are coupled with the central system through the electron power. We define the ratio of this power to the power injected into the end mirror systems as the efficiency of these end mirror systems, η_k:

$$\eta_k=[\langle E_k^{ie}\rangle-\varphi_k-T_e(1+T_e/2\varphi_k)]/E_{jk},\qquad (10)$$

where E_k^{ie} is the energy transferred from the ion to electrons, and E_{jk} is the energy of the injection into the end mirror systems. The efficiency η_k is a function of the ratios E_{jk}/T_e and φ_k/T_e as well as of R_k. These functional dependences, calculated for $R_k = 3$, are shown in Fig. 3; we see that the value of η_k is governed primarily by the ratio E_{jk}/T_e, with the functional dependence on φ_k/T_e being weak. The ratio φ_k/T_e is governed primarily by the value of Q_0: As Q_0 is increased from 1 to 20, the ratio φ_k/T_e increases from about 6 to 10. If the thermonuclear yield in the end systems is neglected, and if there is no auxiliary electron heating, the overall figure of merit is $Q = Q_0\eta_k$.

4. PROPERTIES OF THE CONFINEMENT SYSTEM AS A WHOLE

The properties of the system as a whole are calculated from the given values of R_0, R_k, n_0, n_k, T_e, E_{j0}, and E_{jk} and the auxiliary-heating energy per central electron, E_{q0}. The matching of the potential φ_k is carried out on the basis of the general equation for electrons, (6), and the matching of the ratio of calculated volumes is carried out on the basis of the electron energy balance:

$$V_0 I_0[\langle E_0^{ei}\rangle+\varphi_0+T_e(1+T_e/2\varphi_0)-E_{q0}]$$
$$=V_k I_k[\langle E_k^{ei}\rangle-\varphi_k-T_e(1+T_e/2\varphi_k)].\qquad (11)$$

The value of R_0 is chosen on the basis of the condition $\beta_0 = \beta_k$, where $\beta_0 = 16\pi n_0 T_e/B_0^2$ and $\beta_k = (16\pi/3)n_k\langle E_{ik}\rangle/B_k^2$. The total figure of merit of the confinement system is

$$Q=\frac{\varepsilon}{4}\frac{n_0^2\langle\sigma v\rangle_0 V_0+n_k^2\langle\sigma v\rangle_k V_k}{(E_{j0}+E_{q0})I_0V_0+E_{jk}I_kV_k}.\qquad (12)$$

To calculate the length of the system and the required gradient of the transverse magnetic field, we adopt the following profile for the longitudinal magnetic field at the

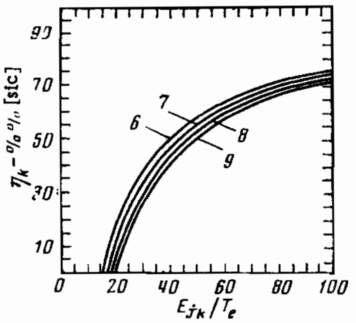

Fig. 3. Curves of $\eta_k = \eta(E_{jk}/T_e,\ \varphi_k/T_e)$. The curves are labeled with the value of the ratio φ_k/T_e; $R_k = 3$.

axis (Fig. 1):

$$B(z)=\begin{cases}\dfrac{B_k+B_0}{2}-\dfrac{B_k-B_0}{2}\cos\pi\dfrac{z}{l}, & 0<z<l,\\[2mm] \dfrac{B_m+B_k}{2}-\dfrac{B_m-B_k}{2}\cos\pi\dfrac{z-l}{L_k}, & l<z<l+1.5L_k;\end{cases}\tag{13}$$

where z is the axial coordinate (measured from the boundary of the homogeneous field to the end of the confinement system). The profile of the ambipolar potential in the confinement system (except in the end mirror systems) is governed unambiguously by the ion profile:

$$n_i(z)=n_e(z)=n_k\exp\frac{\varphi(z)-\varphi_k}{T_e}.\tag{14}$$

The profile $n_i(z)$ is related to the profile of the end ions, which is in turn governed by the functions $B(z)$, $f(v_k)$, and $\varphi(z)$. The function $\varphi(z)$ is ultimately governed by the functions $B(z)$ and $S(v_{jk})$. For model calculations we adopt a sinusoidal profile for the ambipolar field in the end mirror systems:

$$\frac{d\varphi}{dz}=\begin{cases}\dfrac{2\Delta\varphi}{L_k}\left(1-\cos 4\pi\dfrac{z-l}{L_k}\right) & l+0.5L_k<z<l+L_k,\\[2mm] -\dfrac{2\varphi_k}{L_k}\left(1-\cos 4\pi\dfrac{z-l}{L_k}\right) & l+L_k<z<l+1.5L_k.\end{cases}\tag{15}$$

Although profile (15) is not consistent with profile (13), it gives a profile $n_i(z)$ approximating the actual profile in a cosinusoidal magnetic well.[13] Neglecting the central plasma which penetrates into the end mirror systems, we find from (12), under the condition $L_0 > 2l + L_k$,

$$\frac{L_0}{2L_k}=\frac{V_0}{V_k}\frac{R_k}{R_0}\frac{B_k}{n_k^2}\left\langle\frac{n_{ik}^2}{B(z)}\right\rangle+\frac{l}{L_k}\left(1-\sqrt{\frac{R_k}{R_0}}\right)+\frac{1}{2}-\frac{\sqrt{R_k}}{2R_0}.\tag{16}$$

The length of the transition region of the central magnetic field, l, must be large in order to reduce the gradient of the transverse magnetic field in the end mirror systems, but otherwise this length is quite arbitrary. We adopt $l = L_kR_0/R_k$.

If there is no auxiliary electron heating, we can write the rough approximation

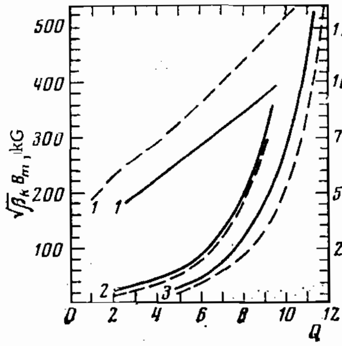

Fig. 4. Various properties as a function of the factor Q. 1) Minimum field in mirror systems, B_m; 2) length ratio of mirror systems, $L_0/2L_k$, at minimum field B_m; 3) minimum ratio $L_0/2L_k$ at $\eta_k = 0.6$. Solid curves) $R_k = 2$; dashed curves) $R_k = 3$.

$$V_0/V_k \propto \left(\frac{n_k}{n_0}\right)^2\frac{1}{\langle\sigma v\rangle_0\sqrt{T_e}}\eta_k Q_0.$$

It follows that for a given value of Q the minimum of V_0 is reached near the electron temperature corresponding to the maximum of Q_0. The volume V_0 decreases with increasing η_k. Figure 4 shows the calculated ratio of the mirror-system lengths $L_0/2L_k$ as a function of Q for $T_e = T_e$(Max Q) and $\eta_k = 0.6$ (curve 3). Table 3 shows some calculated properties of the confinement system for various values of T_e with Q = 6. As can be seen from Table 3, the function $B_m = B_m(T_e)$ with Q = const has a minimum. There is also a (shallower) minimum in the function $B_m = B_m(\eta_k)$, at $\eta_k = 0.5$–0.6. Figure 4 shows the calculated minimum field in the mirror systems (curves 1) and the ratio of the length corresponding to the minimum field (curves 2) as functions of Q. We see from Fig. 4 that the length of the confinement system, governed by the ratio $L_0/2L_k$, is relatively small at large values of Q, although it does increase rapidly with increasing Q. The most important problem for a confinement system with twin mirror systems turns out to be the strong magnetic field at the mirrors.

The magnetic fields in the end mirror systems can be reduced substantially by using auxiliary electron heating, e.g., by rf fields. In this case the end mirror systems no longer need to perform a heating function, and

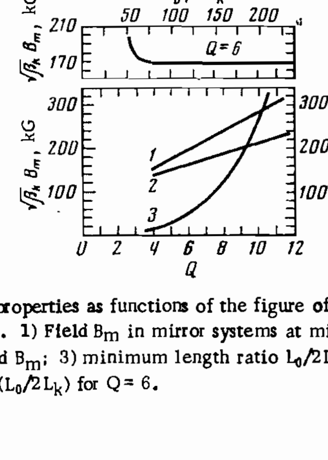

Fig. 5. Various properties as functions of the figure of merit Q for $E_{q0} > 0$, $\eta_k = 0$, and $R_k = 2$. 1) Field B_m in mirror systems at minimum length ratio; 2) minimum field B_m; 3) minimum length ratio $L_0/2L_k$. The inset at the top shows $B_m = B(L_0/2L_k)$ for Q = 6.

TABLE 3. Certain Properties of the Confinement System as Functions of T_e for $Q = 6$, $R_k = 3$; $\eta_k = 60\%$; $n_0 = 5 \cdot 10^{13}\ cm^{-3}$ and $E_{j0} = 20$ keV

T_e, keV	$\frac{n_k}{n_0}$	$\sqrt{\beta_k} \cdot B_m$, kG	$\frac{L_0}{2L_k}$	T_i, keV	$\langle E_{ik} \rangle$, keV	$\langle E_{ik} \rangle$, keV	φ_k, keV	$\frac{\langle n_{jk} n_{i0} \rangle}{\langle n_{ik}^2 \rangle}$
95	5	420	8.35	43.9	2955	5290	646	2.14
48.5	7	366	16.5	28.3	1582	2860	370	1.54
33.5	10	370	32.3	22.8	1136	2075	279	1.0
24	15	390	75.6	18.5	844	1550	218	0.55

the energy of the end ions can be reduced, to a level of the order of the ambipolar potential φ_k. Correspondingly, the magnetic field at the mirrors decreases. Calculations incorporating the auxiliary electron heating show that, for given values of n_k/n_0, T_e, and $\langle E_{ik}\rangle/\varphi_k$, an increase in the ratio V_k/V_0 causes an increase in Q, with the magnetic field at the mirrors remaining essentially constant. When the end mirror systems operate as self-contained systems [$\eta_k = 0$, $\varphi_k \sim 0.47 \langle E_{ik}\rangle (\log R_k)^{2/5}$] we find $Q \to Q_0^\infty$ as $V_0/V_k \to \infty$, and the ratio B_m/Q tends toward a minimum. Figure 5 shows the minimum field at the mirrors in the case $\eta_k = 0$ as a function of the figure of merit for $R_k = 2$ (curve 2).

Since the extent to which Q approaches its limiting value Q_0^∞ depends on the strength of the inequality $n_0^2 V_0 \gg n_k^2 V_k$, we see that for a given value of V_0/V_k the factor Q is closer to Q_0 at lower values of n_k/n_0. On the other hand, as the ratio n_k/n_0 decreases, the limiting value Q_0^∞ decreases. As a result, there is a maximum in the function $Q = Q(n_0/n_k)$ at $V_0/V_k = \text{const}$. The maximum value of Q increases with increasing value of the ratio V_0/V_k; correspondingly, for each value of Q with $\eta_k = 0$ there is a minimum length ratio $L_0/2L_k$. Figure 5 shows the minimum length ratio $L_0/2L_k$ (curve 3) and the field at the mirrors at the minimum length (curve 1) as approximate functions of Q. The functional dependence of B_m on $L_0/2L_k$ for $Q = 6$ is shown at the top in Fig. 5; we see from this behavior that when the length of the confinement system is slightly greater than the minimum value the field at the mirrors approaches the minimum value. Comparison of Figs. 4 and 5 shows that with auxiliary electron heating the field at the mirrors is reduced substantially, while the length of the confinement system is increased.

Table 4 shows results calculated for two versions of the confinement system with $E_{q0} = 0$ and for five versions with $E_{q0} > 0$ and $\eta_k \sim 0$ for several values of Q. The calculations for the versions with $E_{q0} > 0$ are carried out for various values of the length of the confinement system (larger than the minimum length). In version 3, with a relatively long central system, the magnetic field at the mirrors is very near its minimum value. The quantity $\langle n_{ik} n_{i0}\rangle / \langle n_k^2 \rangle$ shown in Tables 3 and 4 is a measure of the penetration of the relatively warm, isotropic central ions into the end mirror systems and is equivalent to the density of warm plasma in a homogeneous hot plasma. Here n_{i0} and n_{ik} are the local densities of the central and end ions. We see from Table 4 that with $Q \lessapprox 6$ the quantity $\langle n_{ik} n_{i0}\rangle / \langle n_{ik}^2 \rangle$ is large enough to substantially suppress the loss-cone instability. From this standpoint it is advantageous to reduce the length $\varphi_k / \langle E_{ik}\rangle$ for the versions with $E_{q0} = 0$ below the values for a single mirror system. The ratio of the total input power to the power injected into the end mirror systems is

$$\frac{P_{jk}}{P_j} = 1 + \frac{n_0^2 V_0}{n_k^2 V_k} \frac{E_{q0} + E_{j0}}{E_{jk}} \frac{n\tau_k}{n\tau_0 (1 - \alpha_0)}. \tag{17}$$

For a sufficiently large ratio V_0/V_k we have $P_{jk}/P_j \ll 1$. For version 3 in Table 4 we have $P_{jk}/P_j = 0.032$, which tells us that, as the rate of loss of the end ions increases to a level ten times the classical level, the total Q falls off by only 30%. Accordingly, the degradation of the operation of this confinement system due to the loss-cone instabilities in the end mirror systems can be suppressed substantially by increasing the length of the confinement system, and this can be done if $E_{q0} > 0$. Table 4 shows the power level of the electron bremsstrahlung, with the slowing at α particles neglected. This power is an insignificant fraction of the power taken from the α particles and can be replenished by the heating of electrons by the α particles. The cyclotron radiation of the electrons does not affect the energy balance of the confinement system if the level of this radiation actually escaping from the system is reduced by two orders of magnitude by absorption in the plasma and reflection from the walls.

Table 4 shows the amplitudes of the gradient of the transverse quadrupole field g_m required for hydrodynamic stability. These values are calculated from the condition for the onset of the flute instability,

$$\int p \frac{\delta B}{B^2} ds > 0, \tag{18}$$

where p is the plasma pressure. We can write, accurately, $p(z) = (2/3) \langle E_{ik}\rangle n_{ik} + 2T_e n_{i0}$. We adopt the following profile for the gradient of the transverse quadrupole field $B_\perp$ (Fig. 1):

$$g(z) = \frac{L_k}{B_m} \frac{\partial B_\perp}{\partial r} = \begin{cases} -g_m, & l < z < l + 0.5L_k, \\ +g_m, & l + 0.5L_k < z < l + 1.5L_k. \end{cases}$$

The calculations are carried out in the paraxial approximation, by analogy with the calculations of Trubnikov.[17] It follows from ref. 18 that the condition (18) corresponds to surface perturbations and is correct for $\delta p \propto p$. To meet this condition we need to arrange, in particular, a profile of the ambipolar potential at the plasma boundary which behaves in the same manner as that at the axis. The higher flute modes, which can occur in the central part of the system if the plasma in the end mirror systems is sufficiently dense, should be suppressed in a manner analogous to that used to suppress them in a single mirror system,

TABLE 4. Calculated Results for Several Versions of the Confinement System with $R_k = 2$; $n_0 = 5 \cdot 10^{13}$ cm^{-3} $E_{j0} = 20$ keV and $\beta_0 = \beta_k$

Version	1	2	3	4	5	6	7
$E_{\varphi 0}$, keV	0	0	495	524	474	432	405
η_k %	60.8	60	−1.9	−4.1	−2.8	−3.0	−0.4
n_k/n_0	3	5	3	4	5	7	10
T_e, keV	70	58	67	67	58	50	45
Q	3.1	4.91	4.18	5.06	6.43	8.07	10.3
Q_0	4.68	7.74	4.32	6.53	7.44	9.2	11.5
$L_0/2L_k$	5.25	12	126	30	85	195	474
$\sqrt{\beta_k} \cdot B_m$, kG	213	256	145	169	180	202	235
E_{jk}, keV	4970	4260	1280	1300	1200	1090	1070
$\langle E_{ik} \rangle$, keV	2824	2427	1304	1332	1207	1086	1038
φ_k, keV	437	413	425	455	420	391	378
$\Delta\varphi$, keV	76.9	93.3	73.6	92.9	93.3	97.3	103.6
T_{i0}, keV	24.7	28.9	23.5	28.5	28.5	28.8	29.5
$n\tau_k - 10^{14}$ sec/cm^3	9.14	7.04	2.41	2.38	2.04	1.7	1.58
$n\tau_0 - 10^{15}$ sec/cm^3	0.99	1.47	0.9	1.35	1.47	1.75	2.28
α_0, %	20.3	29.6	19.6	28.3	30.9	35.1	41.8
P_r/P_α, %	6.3	5.1	5.8	5.3	4.8	4.4	4
V_0/V_k	45.2	201.3	2000	530	1900	5600	17500
R_0	12.6	16.7	8.85	10.3	11.8	14.3	17.5
g_m	1.89	1.84	1.92	1.89	1.87	1.85	1.84
$\frac{n_{ik} \cdot n_{i0}}{n_{ik}^2}$, %	5.6	2.86	5.6	3.7	2.8	1.77	1.04

TABLE 5. Calculated Results For Confinement Systems with a Shallow Central Well, $\langle E_{i0} \rangle = 150$ keV; $\langle E_{ik} \rangle = 300$ keV, $n_0 = 3 \cdot 10^{13}$ cm^{-3}; $\beta_0 = \beta_k$

R_0	$n_k/n_0 = 1$; $V_0/V_k = 30$				$n_k/n_0 = 2$; $v_0/v_k = 40$		
	10	7,35	5	3	10	7,35	4
T_e, keV	17.9	17.1	16	14.1	18.3	17.4	15.2
φ_k, keV	86.7	83.2	78	68.5	101	96.8	84.4
E_{j0}, keV	172	171.2	170	168	169	168.8	169
E_{jk}, keV	456	436.6	408	355.5	398	374.8	309
$n\tau_0 - 10^{13}$, sec/cm^3	5.84	5.29	4.53	3.4	5.85	5.28	4.02
$n\tau_k - 10^{13}$, sec/cm^3	10	8.27	6.8	3.95	8.34	6.78	3.13
Q	1.57	1.425	1.23	0.925	1.5	1.35	0.97

by means of conducting ends.[18] We ignore the anisotropy of the end plasma. The corresponding error cannot be large, since we have $p_\perp \gg p_\parallel$ for this end plasma and since, as was shown in ref. 19, the relation $p_\perp \propto n$ is quite accurate. For a single mirror system with the potential adopted here, the minimum value of g_m for the outer half of the end mirror systems is 1.76. Comparison of this value with the values shown in Table 4 shows that the required gradient of the transverse field increases only insignificantly in this confinement system.

According to refs. 19 and 20, with $R = 2$ we can achieve $\beta = 0.4$. Then for a long confinement system with $Q = 5$ we need a field of $B_m = 235$ kG. Since the critical magnetic field of the superconductors currently in use is 420 kG (ref. 21), it can be hoped that the required fields at the mirrors can be achieved. It should be noted that if the confinement system is long most of it corresponds to the simple, homogeneous, relatively weak magnetic field (a few tens of kilogauss), and only the ends of the system, a few meters long, are complicated devices with a very strong magnetic field. We neglect the particle loss across the field in the calculations. In the case of classical transverse diffusion of the plasma, and if the diameter of the central part of the confinement system is sufficiently large (more than a meter), the transverse loss is much smaller than the longitudinal loss. This loss can be reduced by adopting $\beta_0 < \beta_k$ with the appropriate increase in the field B_0. One of the most important questions which we have not taken up here is that of the influence of the α particles on the behavior of the central plasma and of the methods for removing slow α particles which accumulate in the confinement system.

5. CONFINEMENT SYSTEM WITH A SHALLOW CENTRAL WELL

We also consider the case of a triple mirror system with $\langle E_{i0} \rangle \gg \Delta\varphi$. In this case the improvement in Q is reached by increasing the lifetime in the central mirror system, through a suppression of the ambipolar field and an increase of the electron temperature. It is assumed that the potential wells which can appear near the inner mirrors are filled by slow ions. To determine the properties of both the central and end plasmas, we solve Fokker–Planck equation (8) for the ions. We assume a zero ambipolar potential for the central ions. The maximum value of Q is reached at $\langle E_{i0} \rangle \approx 150$ keV, $\langle E_{ik} \rangle = 200$–$300$ keV, $n_k/n_0 \to 1$, and $V_0/V_k \to \infty$. For example, with $R_k = 7.35$ and $\beta_0 = \beta_k$ we have $Q \to 1.44$. Table 5 shows some results calculated for a triple system for several values of R_k. Shown for comparison are results calculated for a single mirror system for several energies; the mirror ratios are determined from the condition $\sqrt{\beta}\, B_m = \text{const}$. It follows from these calculations that the maximum value of Q is reached at $\langle E_i \rangle = 100$–$150$ keV; with $R_k = 7.35$, it is 1.1. We see from Table 5 that in a triple system, with a moderate value of the ratio V_0/V_k, Q increases by 30%, while the ratio $\varphi_k / \langle E_{ik} \rangle$ decreases by several tenths. The reason for the slight increase in the amplification factor is that at high mirror ratios (about ten) the suppression of the ambipolar potential increases the energy confinement time by only ~10%. Nevertheless, a triple mirror system operating with $\langle E_{i0} \rangle \gg \Delta\varphi$ has advantages which are much more important than would appear at first glance. The presence of cold isotropic ions in the center of the system and the decrease of the ambipolar potential tend to suppress the loss-cone instability. The fraction of the injection power which is injected into the end mirror systems falls off with increasing V_0/V_k and can reach a very low value. Accordingly, even a substantial reduction of the confinement time of the end ions has little effect on the overall figure of merit.

Here we have been dealing with essentially two extreme versions of the mirror system with twin mirror systems: $\langle E_{i0} \rangle \ll \Delta\varphi$ and $\langle E_{i0} \rangle \gg \Delta\varphi$. It may turn out that an intermediate version is best for practical purposes, but an analysis of intermediate versions would require two-dimensional Fokker–Planck calculations. It follows from the calculations of ref. 22 that in the case $\Delta\varphi = T_i$ the ion confinement time is longer than in the case $\Delta\varphi \ll T_i$ by a factor of 4.7–3.7 at R = 2–10. It is also interesting to examine the case of a central plasma with two components of different energy, especially for the ^{3}HeD cycle. By injecting helium into the center of the system at a low energy and injecting deuterium at a high energy, we can raise the figure of merit for this cycle in a system with twin mirrors to a value of 0.5–1. The energy which is expended on the helium part of the plasma in this version should be slight, because of the long confinement time in the central electrostatic well, especially because of the doubling of the ion charge: $^3He^{2+}$.

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