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Colliding Beam Fusion Reactors

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The recirculating power for virtually all types of fusion reactors has previously been calculated [1] with the Fokker–Planck equation. The reactors involve non-Maxwellian plasmas. The calculations are generic in that they do not relate to specific confinement devices. In all cases except for a Tokamak with D–T fuel the recirculating power was found to exceed the fusion power by a large factor. In this paper we criticize the generality claimed for this calculation. The ratio of circulating power to fusion power is calculated for the Colliding Beam Reactor with fuels D–T, D–He^3 and p–B^{11}. The results are respectively, 0.070, 0.141 and 0.493.

KEY WORDS: Non-Maxwellian Plasmas; Fokker-Planck; Fusion reactors.

I. INTRODUCTION

It is widely perceived in the fusion community that non-Maxwellian plasmas should be avoided as much as possible. In fact, if all distributions are Maxwellian there can be no magnetic confinement. Just what departures from Maxwell distributions are tolerable needs to be defined carefully. In a Tokamak the fuel ions are "Maxwellian" (except for the high-energy tail); the electrons are a drifted "Maxwellian." The latter must be tolerated or there can be no current and no confinement. Distributions will be "Maxwellian" if the self-collision or Maxwellization time is less than any other significant time such as the current decay time, the particle containment time or the fusion reaction time. When a Tokamak plasma is heated by particle beams that have a much higher energy than the thermal energy, the particle beam does not have a Maxwell distribution, but a slowing down distribution. This is because the beam density is much less than the plasma density so that the Maxwellization time is greater than the slowing down time. It is an experimental fact that beam particles slow down and diffuse classically while the thermal particles [2] experience anomalous transport. The non-Maxwellian particles have significantly better-confinement than the "Maxwellian" particles! A corollary is that a Tokamak confines charged fusion products very much better than the fusion fuel ions. This is acceptable for a D–T Tokamak where 80% of the energy is in 14 MeV neutrons which leave the plasma promptly without any significant loss of energy. The α-particles must be confined to heat the plasma in order to achieve ignition. It is exactly the opposite of what is required for aneutronic fuels where almost all of the fusion energy is in ions. The product ions must escape with most of the energy in order to achieve direct conversion at high efficiency. Efficiency [3] of 90% is possible compared to about 35% for conversion of neutron energy by a thermal process.

The p–B^{11} reaction has been investigated [4] for a thermal plasma assuming some confinement properties. This is also a generic calculation. The reduction of synchrotron radiation is emphasized and multipoles, surmacs and levitated superconducting rings are mentioned because of the weak internal magnetic fields. Information on transport and confinement in 1977 was slight to non-existent for these

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systems and was not considered in the calculations. The main features of the calculation were classical energy transfer between ions and electrons and Bremsstrahlung. The results were as follows:

- ignition was not possible for any relative concentrations of boron and hydrogen;
- injection of a 1 MeV neutral hydrogen beam was considered and the p–B^{11} reactor was considered as an energy amplifier; the conclusion was that the energy amplification factor was limited to 1–1.4 so that useful power for sale would require unreasonable efficiencies for making beams and energy conversion;
- problems with retainment of fusion products in the fusion plasma were cited that would reduce the amplification factor to less than 1.

The 1977 generic study based on very limited facts about magnetic confinement terminated research support on p–B^{11} reactors. Since then there have been many developments that are relevant that will be addressed in this paper. They lead to different conclusions, i.e., a p–B^{11} reactor is feasible. This is not a generic study but is device specific and based on the following new developments since 1977:

- New experimental information about classical transport for large orbit ions [2].
- Extensive research on Field Reversed Configurations [6] (FRCs) leads to new configurations called "Colliding Beam Reactor" (CBR) in which anomalous transport of both electrons and ions may be eliminated. Large orbit fuel ions are confined magnetically and small orbit electrons are confined electrostatically [7].
- Electron confinement suggests a new cooling mechanism for electrons in a burning plasma of p–B^{11} in addition to Bremsstrahlung. This mechanism permits steady operation with the electron temperature substantially less than the fuel ion temperature. The electrons are contained by an electrostatic potential well. The cooling mechanism is similar to evaporative cooling. The electrons must acquire energy to leave the plasma. The potential well depth is like the vaporization energy in evaporative cooling. However the fusion products which are not confined must gain energy when they leave the plasma in addition to their reaction energy.
- α-particles created in fusion reactions scatter out of the plasma rapidly. They leave with most of their reaction energy and energy gained from the potential hill. Their density is much less than the fuel ion density. Also the boron density is much less than the hydrogen density. This is important because there is an α-particle/boron secondary reaction that produces neutrons. The design study of this paper indicates 3 MeV neutron production of about 1 kW for a 100 MW reactor. This problem was not discussed in the 1977 generic study and is of considerable importance in a p–B^{11} reactor.

In section 2 we show that the very large circulating power obtained by Rider [1] is a consequence of the assumption of particle distribution functions that simplify calculations but have no physical basis.

Sections 3–8 are devoted to the description of the CBR and the calculation of the performance parameters including the circulating power. The conclusions in section 8 contain a comparison of the results for circulating (recirculating) power for the CBR with the generic calculation of Rider for p–B^{11}.

II. FUNDAMENTAL LIMITATIONS FOR NON-THERMAL PARTICLE DISTRIBUTIONS

The Fokker–Planck collision operator for Coulomb scattering was first derived by Landau [8] in the form

$$\left(\frac{\partial f_i}{\partial t}\right)_{\text{coll}} = -\frac{\partial}{\partial \vec{v}} \cdot \vec{J}, \tag{1}$$

where

$$\vec{J}(\vec{v}) = \frac{2e_i^2}{m_i} \sum_j e_j^2 \int \frac{d\vec{k}}{k^4} \vec{k} \int d\vec{v}' \delta[\vec{k} \cdot (\vec{v} - \vec{v}')] \vec{k} \cdot \left[\frac{f_i(\vec{v})}{m_j} \frac{\partial f_j}{\partial \vec{v}'} - \frac{f_j(\vec{v}')}{m_i} \frac{\partial f_i}{\partial \vec{v}} \right], \tag{2}$$

f_i is a function of $\vec{x}, \vec{v}$ and $\vec{t}$, but only the $\vec{v}$ dependence is shown. e_i is the charge of particle

$i = e, 1, 2, \ldots$ for electrons and ions. m_i is the mass. The $\vec{k}$-integration requires cut-offs at short wavelengths by the closest distance of approach of the particles and at long wavelengths by the Debye length. If $f_i(\vec{v})$ is isotropic the collision operator can be simplified [1]. For specific calculations Rider assumes a distribution function of the form

$$f(v) = \begin{cases} nK\{\exp[-(v-v_0)^2/v_{ts}^2] \\ \quad +\exp[-(v+v_0)^2/v_{ts}^2]\}, \quad v < v_0, \\ nK\{\exp[-(v-v_0)^2/v_{tf}^2] \\ \quad +\exp[-(v+v_0)^2/v_{tf}^2]\}, \quad v > v_0, \end{cases} \tag{3}$$

where n is density and K is a normalization constant. By varying the relative values of v_0, v_{ts} and v_{tf} a wide variety of distribution shapes may be studied including a Maxwell distribution for $v_0 = 0$. $v = \sqrt{v_x^2 + v_y^2 + v_z^2}$ so that $f(v)$ is always isotropic! The circulating power is defined as

$$P_{\text{recirc}} = \int 4\pi v^2 dv \left(\frac{1}{2}mv^2\right)\left(\frac{\partial f}{\partial t}\right)_{\text{coll}}, \tag{4}$$

which is the power required to maintain a distribution such as $f(v)$. Like particle collisions are included as well as unlike particle collisions. The collisions of like particles lead to thermalization. The thermalization times are usually very short so that very large values for P_{recirc} are obtained. With such calculations most reactor schemes are ruled out including the CBR [9,10]. Equation (3) is not an appropriate distribution in this case. Appropriate distributions are

$$f_i(\vec{v}) = \left(\frac{m_i}{2\pi T_i}\right)^{3/2} n_i(r)\exp\left[-\frac{m_i}{2T_i}(\vec{v} - \vec{V}_i)^2\right],$$

which are drifted Maxwell distributions. For these distribution functions $(\partial f_i/\partial t)_{\text{coll}}$ given by equations (1) and (2) vanishes for like particle collisions. For electrons and one type of ions, only ion–electron collisions make a contribution to P_{recirc} and this would be quite small compared to results from equation (3).

For several types of ions, collisions between ions will rapidly thermalize both types in a moving frame so that $V_1 = V_2$, $T_1 = T_2$. Then only ion–electron collisions survive leading to a very small value of P_{recirc} because momentum transfer is reduced by the ratio of electron to ion mass compared to like particle collisions.

The isotropic assumption of equation (3) is inappropriate for the CBR. In fact we are not aware of any magnetic confinement reactor concept for which equation (3) is appropriate. It leads to simple analytic integrations which is the only apparent virtue.

III. CALCULATION OF REACTOR PARAMETERS FOR A COLLIDING BEAM REACTOR

A plasma consisting of large orbit ions and small orbit electrons is considered. Experimental evidence [2] with energetic beams injected into Tokamaks for heating in DIII-D and TFTR and with energetic fusion products in JET indicates that the energetic particles do not suffer from the anomalous transport usually observed in fusion devices. In fact the diffusion of these large orbit ions is consistent with classical estimates while at the same time the thermal population diffuses anomalously. In addition to Tokamak experiments numerical simulations [11] support the fact that large orbit ions feel predominantly low frequency field fluctuations with wavelengths that exceed the larmor radius. The physical reason for this is that ions, over the course of such a large orbit, average the fluctuations so that only long wavelengths (compared to gyro-radius) and small frequencies (compared to gyro-frequency) cause transport. Thus if the particle orbit radius is large and the plasma has gross stability at long wavelengths, anomalous transport of ions can in principle be avoided [10]. Electrons will have a gyro-radius about 3 orders of magnitude smaller than ions and would be subject to anomalous transport. However, it is possible to design a FRC [9] so that ions have a large gyro-radius and are confined magnetically; electrons are confined electrostatically. They are in a deep potential well so that they can only escape confinement with negligible energy. The FRC is illustrated in Figure 1.

In a reactor the energetic ions would be supplied by external accelerators that produce beams that consist of neutrals or ions with neutralizing electrons. The beams could be steady state or repetitively pulsed. They would be injected and trapped continuously to replace fuel ions as they react and also to maintain the current. In this paper repetitively pulsed

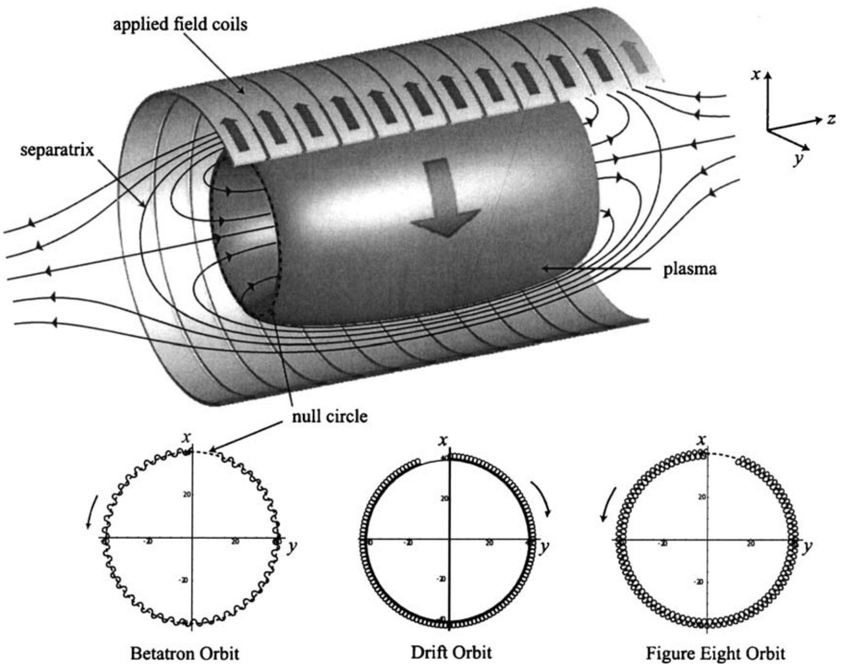

Fig. 1. FRC with magnetic field lines and typical particle orbits. The red arrows indicate the direction of the currents.

beams will be emphasized because the technology is closer to reactor requirements.

IV. FRC EQUILIBRIUM

A one-dimensional model is employed that is based on rigid rotor solutions of the Vlasov/Maxwell equations [12]. These solutions are drifted Maxwellian functions so that fluid equations are sufficient:

Conservation of momentum

$$-n_j m_j r\omega_j^2 = n_j e_j\left(E_r + \frac{r\omega_j}{c}B_z\right) - T_j\frac{dn_j}{dr}. \tag{5}$$

Quasi neutrality

$$\sum_j n_j e_j \simeq 0. \tag{6}$$

Maxwell equation

$$\frac{dB_z}{dr} = -\frac{4\pi}{c} r \sum_j n_j e_j \omega_j, \tag{7}$$

$j = e, i$ for electrons and ions, $e_j = -e, +e$ is charge, $V_{j\theta} = r\omega_j$ is azimuthal velocity, r is radial coordinate, T_j is temperature, n_j is density, E_r(r), B_z(r) are electric and magnetic fields.

For electrons and one type of ion an exact solution can be obtained [9] which is as follows:

$$n_e(r) = n_i(r) = \frac{n_0}{\cosh^2\left(\frac{r^2 - r_0^2}{r_0\,\Delta r}\right)}, \tag{8}$$

$$B_z(r) = -B_0\left[1 + \sqrt{\beta}\tanh\left(\frac{r^2 - r_0^2}{r_0\,\Delta r}\right)\right], \tag{9}$$

$$E_r(r) = \frac{m_e}{e} r\omega_e^2 - \frac{r\omega_e}{c}B_z - \frac{T_e}{en_e}\frac{dn_e}{dr}, \tag{10}$$

r_0 is the radius at which the density is a maximum. $-B_0$ is the externally applied magnetic field.

$$\Delta r = \frac{2\sqrt{2}}{r_0}\left(\frac{T_e + T_i}{4\pi n_0 e^2}\right)^{1/2}\frac{c}{|\omega_i - \omega_e|}, \tag{11}$$

$$\beta = 8\pi n_0 \frac{T_e + T_i}{B_0^2}, \tag{12}$$

Δr is defined by the equation $N_i = \int_0^{r_B} 2\pi r\,dr\,n_i = 2\pi r_0\,\Delta r\,n_0$. The ion velocity $V_i = r\omega_i$ is determined by the ion energy created by an external accelerator. The maximum particle density may be also determined by design. The temperature cannot be determined from the Vlasov–Maxwell equations. Higher-order processes must be included—Coulomb collisions as well as particle sources and sinks. Other features of the higher order processes have already been included such as the assumption of drifted Maxwell distributions for electrons and ions which must be verified a posteriori. The external magnetic field B_0 and $V_{\rm e}$ can be identified by considering the conservation of momentum of a single fluid description

$$\rho \frac{V_\theta^2}{r} = \frac{\partial}{\partial r}\left(P + \frac{B_z^2}{8\pi}\right), \tag{13}$$

where $\rho V_\theta = \sum_j m_j \int f_j \vec{v}\, d\vec{v} = \sum_j n_j m_j r \omega_j$ and $\rho = \sum_j n_j m_j \simeq n_i m_i$ since $m_i \gg m$. Equation (13) can be integrated from $r = 0$ to $r = r_{\rm B} = \sqrt{2} r_0$. At these limits $P = n_0(T_{\rm e} + T_{\rm i}) \simeq 0$ so that

$$\int_0^{r_B} \rho \frac{V_\theta^2}{r} dr = m_{\rm i}\omega_{\rm i}^2 \int_0^{r_B} n_i r\, dr = r_0\, \Delta r\, n_{\rm i0} m_{\rm i} \omega_{\rm i}^2. \tag{14}$$

Additionally assuming $r_0 \gg \Delta r$

$$\begin{aligned}\int_0^{r_B} \frac{\partial}{\partial r}\frac{B_z^2}{8\pi} dr &= \frac{1}{8\pi}\left[(B_0 + B_m)^2 - (B_0 - B_m)^2\right] \\ &= \frac{1}{2\pi} B_0 B_m,\end{aligned} \tag{15}$$

where $B_m = \sqrt{8\pi n_0 (T_{\rm e} + T_{\rm i})}$ Substituting equation (11) the result is

$$\omega_{\rm e} = \omega_{\rm i}\left(1 - \frac{\omega_{\rm i}}{\Omega_0}\right), \tag{16}$$

where $\Omega_0 = eB_0/m_{\rm i}c$ is the ion cyclotron frequency in the externally applied field B_0. Equation (16) determines $V_{\rm e} = r\omega_{\rm e}$. If $\omega_{\rm i} = \Omega_0, V_{\rm e} = 0$. By increasing the externally applied field B_0, the value of $\omega_{\rm e}$ can be controlled and therefore the value of the plasma width would also be controlled according to equation (11).

The multifluid equations (5) give a different insight into confinement compared with the magnetohydrodynamic (MHD) equation (13). According to equations (5), the electric field plays a dominant role in confining electrons if $V_{\rm e} > 0$, because the Lorentz force and the pressure terms oppose confinement. Assuming $V_{\rm i} > V_{\rm e} > 0$, the ions are magnetically confined by the Lorentz force which more than compensates for the opposite effect for the electric field. Some physical effects absent from the MHD description are significant in the CBR.

The electrons are within a potential well of depth

$$-e\,\Delta\Phi = e\int_{r_0}^{r_s} E_r\, dr = \ln\Lambda\left(\frac{T_{\rm e}\omega_{\rm i} + T_{\rm i}\omega_{\rm e}}{\omega_{\rm i} - \omega_{\rm e}}\right), \tag{17}$$

where

$$\Lambda = \cosh\frac{(r_{\rm s}^2 - r_0^2)}{r_0\,\Delta r}.$$

$r_{\rm s}$ is the radius of the separatrix which is close to $r_{\rm B} = \sqrt{2} r_0$ in the one-dimensional model in which case $\ln\Lambda \simeq r_0/\Delta r$. When fusion product ions leave confinement as they must in a reactor, they take electrons with them to maintain charge neutrality. The electrons that leave would be those with sufficient kinetic energy to climb the "potential hill." This is a significant cooling mechanism for electrons since they are replaced at quite low kinetic energy. A one-dimensional model is inadequate for this aspect of the problem because it leaves the electrons free to move in the axial direction. This would not be the case for a finite length plasma. In addition $r_{\rm s}$ would be much less than $r_{\rm B}$. The limits of $\ln\Lambda$ are $1 \leqslant \ln\Lambda \leqslant r_0/\Delta r$.

The FRC equilibrium serves as a starting point for kinetic analysis of a reactor model. With some approximations the solution described above can be adapted to a fuel with two different types of ions, one with $Z = 1$ and the other with $Z \geqslant 1$ to accommodate the reactions D–T, D–He^3 and p–B^{11}. The approximation suggested by Coulomb collisions is that the fuel ions would have the same temperatures and velocities $V_{\rm i} = r\omega_{\rm i}$ because ion–ion collisions would lead to thermalization much more rapidly than slowing down of ions by electrons. The previous equations for a single type of fuel ion may be retained with the following modifications:

$$r_0\,\Delta r = 2\sqrt{2}\left[\frac{T_{\rm e} + (T_{\rm i}/Z)}{4\pi n_0 e^2}\right]^{1/2}\frac{c}{|\omega_{\rm i} - \omega_{\rm e}|}, \tag{18}$$

$$\beta = \frac{8\pi n_0}{B_0^2}(T_{\rm e} + T_{\rm i}/Z), \tag{19}$$

and

$$\Omega_0 = \frac{(n_{10} + n_{20}Z)eB_0}{n_{10}m_1 + n_{20}m_2}. \tag{20}$$

The above equations are an exact solution of the Vlasov/Maxwell equations if there is only one type of ion (i.e., $n_{10} = 0$, $n_{20} = n_0$). They are a very good approximation if there are two types provided that the ions are drifted Maxwellian distributions with the same temperature and drift velocity.

V. STEADY STATE WITH REPETITIVE PULSE INJECTION

Neutralized ion beams are assumed to be injected and trapped periodically. This takes place between t_n and $t_n + \Delta t$ as illustrated in Figure 2. There is a developed technology for intense beams with $\Delta t < 1\,\mu$sec. During the period $(t_n + \Delta t, t_n + T)$ the current $I_\theta = 1/(2\pi r_0)\ \sum_j N_j e_j V_j$ decays because of ion–electron collisions which cause ions to slow down and electrons to speed up until $\omega_e = \omega_i$. It also decays because the ions are consumed due to fusion reactions. The conservation of energy derived from the Vlasov/Fokker–Planck equations is

$$\frac{d}{dt}\left[\sum_i \frac{1}{2} N_i m_i V_{1\theta}^2 + \frac{1}{2} L I_\theta^2\right] = -I_\theta^2 R, \tag{21}$$

where $L \simeq 2\pi^2 r_0^2/c^2$ is the inductance/unit length of the FRC,

$$R = \frac{(2\pi r_0)^2 m}{N_e e^2}\left(\frac{1}{t_{e1}} + \frac{1}{t_{e2}}\right),$$

is the resistance/unit length. t_{ei} are momentum exchange times due to collisions with ions

$$t_{ei} = \frac{3}{8}\sqrt{\frac{\pi}{2}}\frac{T_e^{3/2} m^{1/2}}{n_i Z_i^2 e^4 \ln \Lambda} \simeq 10^{-3}\ \text{sec},$$

assuming $n_i \sim 10^{15}\,\text{cm}^{-3}$ and $T_e \simeq 100\,\text{keV}$. The ion–electron momentum exchange time is

$$t_{ie} = \frac{n_i m_i}{n_e m_e} t_{ei} \simeq 1\ \text{sec}.$$

There is also a contribution to R from fusion reactions because $dN_i/dt = -N_i/t_{Fi}$. However, it is usually small compared to the collisional terms because the fusion time $t_{Fi} > t_{ei}, t_{ie}$. From equation (21) the current decays according to

$$(L + L_I)\frac{dI_\theta}{dt} + IR = 0, \tag{22}$$

where $L_I = (2\pi r_0)^2 m/(N_e e^2)$ is inertial inductance/unit length and $L_I \ll L$. A typical decay time for the currents is

$$\frac{L}{R} = \frac{1}{4}\frac{r_0\,\Delta r}{(c/\omega_{pe})^2}\frac{t_{e1}t_{e2}}{t_{e1} + t_{e2}} \simeq 42\ \text{sec}, \tag{23}$$

assuming $n_0 = 10^{15}\,\text{cm}^{-3}$, $T_e = 100\,\text{keV}$, $r_0\Delta r = 82\,\text{cm}^2$ and $t_{e1} \simeq t_{e2} \simeq 10^{-3}$sec. The decay time is considerably greater than the momentum exchange times t_{ei} due to inductance. During this time the ion velocities change very little. The current decay is almost entirely due to the change in electron velocity which involves very little energy change. The dissipated energy during the period $(t_n + \Delta t, t_n + T)$

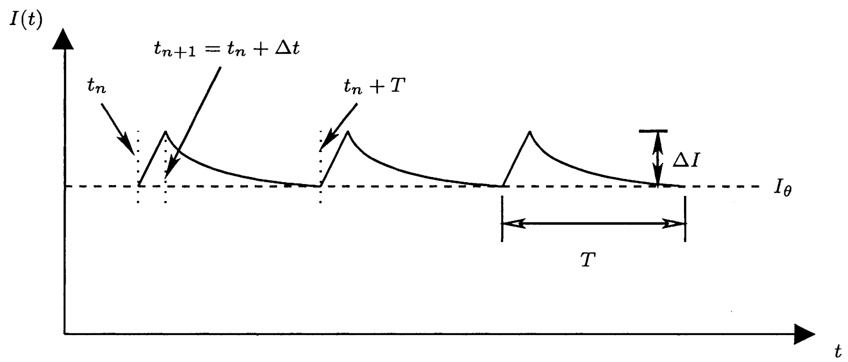

Fig. 2. Pulsed beam injection after initial FRC formation.

comes from the stored magnetic energy. From equation (21) we conclude that

$$\frac{1}{2}LI^2|_{t_n+\Delta t} - \frac{1}{2}LI^2|_{t_n+T} = I^2RT.$$

The loss of magnetic energy is replaced by the injected/trapped beam during the period $(t_{n+1}, t_{n+1}+\Delta t)$. During this time which is much shorter than any of the momentum exchange times there is no significant effect from the Coulomb collisions of the injected beam ions on the fuel ions or vice versa. The injected/trapped beam replaces the fuel ions that are consumed by fusion reactions in the period $(t_n+\Delta t, t_n+T)$. The total number of fuel ions consumed and then supplied by the injected/trapped beam is

$$\Delta N_i = \frac{N_i}{\langle t_{Fi}\rangle}T, \tag{24}$$

where $i = 1, 2;\ dn_i/dt = -n_1 n_2\langle\sigma v\rangle = dn_2/dt$. Equation (24) is obtained by integrating these expressions. $t_{F1} = [n_2\langle\sigma v\rangle]^{-1}$ and $\langle t_{F1}\rangle = \frac{3}{2}[n_{20}\langle\sigma v\rangle]^{-1}$ where the factor 2/3 arises because the expressions on the right are bi-linear in density. The initial injected velocity is $V'_{i\theta} > V_{i\theta}$ so that the average velocity and the current are increased. This results in an inductive electric field $E_\theta = -(L/2\pi r_0)(dI/dt)$ that decelerates the ions and the energy lost by the ions becomes magnetic energy. Integrating equation (21) from t_n to $t_n+\Delta t$ obtains the result

$$\begin{aligned}
&\sum_i\left(\frac{1}{2}N_i m_i V_{i\theta}^2\bigg|_{t_n} - \frac{1}{2}N_i m_i V_{i\theta}^2\bigg|_{t_n+\Delta t}\right)\\
&= \frac{1}{2}LI^2\bigg|_{t_n}^{t_n+\Delta t}\\
&= I^2RT\\
&= \sum_i\left[\frac{m_i}{2}(N_i(t_n+\Delta t) - \Delta N_i)V_{i\theta}^2\right.\\
&\qquad\left. + \frac{m_i}{2}\Delta N_i(V'_{i\theta})^2 - \frac{m_i}{2}N_i(t_n+\Delta t)m_i V_{i\theta}^2\right],\\
&\qquad\sum_i\frac{m_i}{2}\Delta N_i(V'_{i\theta})^2 = \sum_i\frac{m_i}{2}\Delta N_i V_{i\theta}^2 + I^2RT. \qquad (25)
\end{aligned}$$

The electron energy is much less than the ion energy and is omitted in the sums. Equation (25) is the condition that the injected energy replaces the energy of the particles lost to fusion reactions and the dissipation from Coulomb collisions. $V_{i\theta}$ means the value at $t_n+\Delta t$ which does not change significantly in the period $(t_n+\Delta t, t_n+T)$. During this period the electron velocity $V_{e\theta}$ increases due to collisions that transfer momentum from the ions. However, in addition to this change in V_e

$$\frac{\Delta V_e}{V_i - V_e} = \frac{T}{L/R} \simeq 10^{-4}$$

(assuming $T \sim 10^{-2}$ sec), there is a change in current due to the loss of electrons. $\Delta N_e = \sum_i \Delta N_i Z_i$ due to fusion. When fuel ions are consumed they are replaced by fusion product ions which must escape confinement and disappear at a similar rate in order to maintain a periodic or near steady state. These ions must take ΔN_e electrons with them in order to preserve charge neutrality. The current change is $eV_e\,\Delta N_e$ which is to be compared with $eN_e\,\Delta V_e$

$$\frac{V_e\,\Delta N_e}{N_e\,\Delta V_e} \simeq \frac{V_e}{V_i - V_e}\frac{L/R}{\langle t_{Fi}\rangle}.$$

The injected beam that replaces the inductive energy may not compensate for small changes in the electron current. The cumulative changes of many cycles could be compensated by changing the external magnetic field B_0 according to equations (16) and (20). This involves controlling small magnetic field changes on a long time scale of the order of seconds with a feedback system that follows the total current.

VI. REACTOR KINETICS

The equilibrium parameters involve the temperatures T_e and T_i. In order to determine T_e and T_i it is necessary to include the Fokker–Planck collision term as well as sources and sinks of particles for a burning plasma. Several approximations are employed. The fuel ion and electron distributions are assumed to be drifted Maxwellian distributions. The fusion products would have a slowing down distribution. It is assumed that the thermal velocities satisfy the inequality $v_e \gg v_i, V_e, V_i$ where $mv_e^2 = T_e, m_i v_i^2 = T_i$. T_e is determined by the equation

$$\frac{3}{2}n_0\frac{dT_\mathrm{e}}{dt}=\frac{2}{3}\left\{\sum_p n_p\left(\frac{dW_p}{dt}\right)_\mathrm{e}+\sum_i\left[\frac{n_i m_i}{2}\frac{(V_\mathrm{i}-V_\mathrm{e})^2}{t_\mathrm{ie}/2}\right.\right.$$
$$\left.\left.+\frac{3}{2}n_i\frac{T_\mathrm{i}-T_\mathrm{e}}{t_\mathrm{ie}/2}\right]-e\Delta\Phi\sum_i\frac{n_iZ_i}{t_{Fi}}-P_B\right\}=0, \tag{26}$$

T_i is determined by

$$\frac{3}{2}n_\mathrm{i}\frac{dT_\mathrm{i}}{dt}=\frac{2}{3}\left\{\sum_p n_p\left(\frac{dW_p}{dt}\right)_\mathrm{i}-\frac{3}{2}\frac{n_\mathrm{i}(T_\mathrm{i}-T_\mathrm{e})}{t_\mathrm{ie}/2}-\frac{3}{2}\frac{T_\mathrm{i}n_\mathrm{i}}{t_{Fi}}\right\}$$
$$=0, \tag{27}$$

t_ie is the momentum transfer time. $t_\mathrm{ie}/2$ is the energy transfer time. The expressions on the right hand side are bi-linear in density; after averaging the results can be expressed in terms of peak densities making use of equation (8) for the density. The factor 2/3 obtains from this. Summations are over the fusion product ions (p) and the fuel ions (i). The term involving $e\Delta\Phi$ in equation (26) is a cooling term for electrons in a reacting plasma as discussed with equation (17). The second term of equation (26) is $I_\theta^2R/2\pi r_0\,\Delta r$, the dissipation. The third term involves heat transfer from fuel ions if their temperature is larger than the electron temperature. The corresponding term in equation (27) would then be a cooling term. The last term in equation (27) is a cooling term; the fuel ions are injected with the appropriate velocity V_i, but at low beam temperature. Before reacting they are heated to T_i and then disappear.

The fusion products have a slowing down distribution. They are treated by an approximation previously suggested to replace an involved numerical calculation [13]. The lifetime of a fusion product is assumed to be the slowing down time because when the energy approaches that of the fuel ions which are of higher density, they would rapidly diffuse out of the system. This leads to a low density for fusion products because the slowing down time is much less than the fusion time. For example

$$\frac{\partial n_\alpha}{\partial t}=\frac{3}{2}n_1n_2\langle\sigma v\rangle-\frac{n_\alpha}{t_{\alpha s}}=0, \tag{28}$$

$$\frac{1}{t_{\alpha s}}=\frac{1}{t_{\alpha\mathrm{e}}}+\frac{1}{t_{\alpha1}}+\frac{1}{t_{\alpha2}},$$

$$t_{\alpha\mathrm{i}}=\frac{2^{3/2}}{8\pi}\frac{m_\alpha\langle\frac{1}{2}m_\alpha(V_\alpha-V_\mathrm{i})^2\rangle^{3/2}}{n_\mathrm{i}Z_\alpha^2Z_\mathrm{i}^2e^4m_\mathrm{i}^{1/2}\ln\Lambda}\frac{m_\mathrm{i}}{m_\alpha+m_i},$$

and

$$t_{\alpha\mathrm{e}}=\frac{3}{8}\sqrt{\frac{\pi}{2}}\frac{T_\mathrm{e}^{3/2}m_\alpha}{n_\mathrm{e}Z_\alpha^2e^4m^{1/2}\ln\Lambda}.$$

The factor 3/2 is due to the fact that about one half of the three α-particles/reaction escape promptly because they rotate opposite to the diamagnetic direction.

Equations (26) and (27) determine the maximum temperature when $dT_j/dt=0$. Requiring periodicity determines the temperatures; however, they are almost constant during a period of 10^{-2} sec.

In equation (26) P_B is the Bremsstrahlung power for which we employ the semi-empirical formula of Svensson [14] which is the most accurate in the range 50 keV $< T_\mathrm{e} <$ 200 keV; $\langle Z\rangle=\sum_i n_iZ_i^2/\sum_i n_iZ_i$, where

$$P_\mathrm{B}=1.5\times10^{-32}\langle Z\rangle n_\mathrm{e}^2T_\mathrm{e}^{1/2}\left\{1+1.78\left(\frac{T_\mathrm{e}}{mc^2}\right)^{1.34}\right.$$
$$+\frac{2.12}{\langle Z\rangle}\left(\frac{T_\mathrm{e}}{mc^2}\right)\left[1+1.1\left(\frac{T_\mathrm{e}}{mc^2}\right)\right.$$
$$\left.\left.+\left(\frac{T_\mathrm{e}}{mc^2}\right)^2-1.25\left(\frac{T_\mathrm{e}}{mc^2}\right)^{2.5}\right]\right\}\frac{\mathrm{W}}{\mathrm{cm}^3}. \tag{29}$$

The sum is over fuel ions.

VII. ENGINEERING PARAMETERS

Power flow for the fusion reactor cycle of a CBR is illustrated in Figure 3. Definitions of the various quantities are as follows: Fusion power density

$$P_\mathrm{F}=1.6\times10^{-19}n_1n_2\epsilon_\mathrm{F}\langle\sigma v\rangle\ \mathrm{W/cm^3}, \tag{30}$$

n_1, n_2 are the fuel densities in cm^{-3}. ϵ_F is the fusion energy released by a fusion reaction in electron volts and $\langle\sigma v\rangle$ is the fusion reactivity in cm^3/sec. The above definition involves only the reaction energy based on rest energies. There is additional kinetic energy of the fusion fuels which

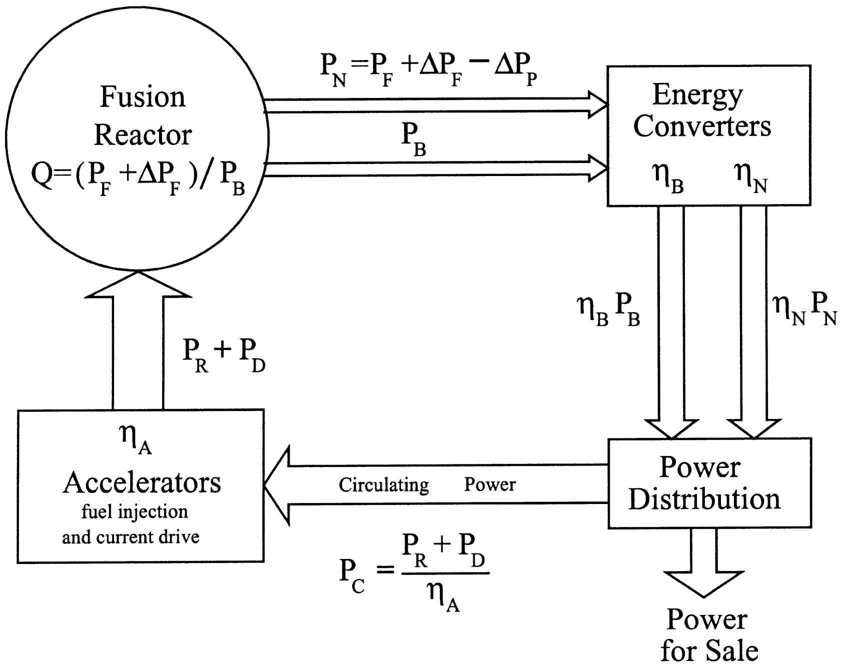

Fig. 3. Power flow schematic.

Table 1. Design and Performance of a CBR

Quantity	D–T	D–He^3	p–B^{11}
Densities (cm^{-3})($n_e = 10^{15}$ cm^{-3})			
n_1	0.5×10^{15}	0.333×10^{15}	0.5×10^{15}
n_2	0.5×10^{15}	0.333×10^{15}	10^{14}
Electron velocity V_e (cm/sec)	0.23×10^9	0.44×10^9	0.661×10^9
Fuel ion energy (keV)			
$\frac{1}{2}m_1V_1^2$	300	450	300
$\frac{1}{2}m_2V_2^2$	450	675	3300
Ion velocity $V_1 = V_2$(cm/sec)	0.54×10^9	0.661×10^9	0.764×10^9
Temperatures(keV)			
T_i	96	217	235
T_e	100	170	85
$r_0\Delta r(cm^2)$	114	203	310
Current $I \times 10^5$ (A/cm)	1.42	1.75	1.27
Magnetic field (kgauss)			
B_0	5.88	8.25	15.3
$B_0 + B_m$	94.7	121	96.3
Current decay time L/R(sec)	42	195	36
Reactivity $\langle\sigma v\rangle \times 10^{16}$ (cm^3/sec)	8	2	4
Fusion energy (MeV)	17.4	18.2	8.68
Fusion power/radiation(Q)	112	5.28	3.84
Power for sale/fusion power	0.271	0.680	0.351
Circulating power/fusion power	0.070	0.141	0.493
Power for sale $\hat{P}_S$ (kW/cm)	123	61.9	19.6
Circulating power $\hat{P}_C$ (kW/cm)	31.7	12.8	27.5

would be retained by the fusion products. The additional energy is

$$\Delta\epsilon_{\mathrm{F}} = \sum_{i=1,2} \frac{1}{2} m_i V_i^2 + \frac{3}{2} T_i,$$

which with equation (30) yields ΔP_{F}.

ΔP_{P} is heating by fusion product ions which can be identified in equations (26) and (27).

Nuclear power

$$P_{\mathrm{N}} = P_{\mathrm{F}} + \Delta P_{\mathrm{F}} - \Delta P_{\mathrm{P}}.$$

Dissipation power

$$P_{\mathrm{D}} = \frac{I^2 R}{2\pi r_0 \, \Delta r} = 1.6 \times 10^{-19} \times \sum_{t=1,2} n_i m_i \frac{(V_{\mathrm{i}} - V_{\mathrm{e}})^2}{t_{\mathrm{ie}}} \ \mathrm{W/cm^3}.$$

Replacement power

$$P_{\mathrm{R}} = 1.6 \times 10^{-19} n_1 n_2 \langle \sigma v \rangle \sum_{i=1,2} \frac{1}{2} m_i V_i^2.$$

$Q = (P_{\mathrm{F}} + \Delta P_{\mathrm{F}})/P_{\mathrm{B}}$ which is a figure of merit; $P_{\mathrm{C}} = (P_{\mathrm{R}} + P_{\mathrm{D}})/\eta_{\mathrm{A}}$ is circulating power; $\hat{P}_{\mathrm{C}} = (4\pi/3) r_0 \, \Delta r \, P_{\mathrm{C}}$; $P_{\mathrm{S}} = \eta_{\mathrm{B}} P_{\mathrm{B}} + \eta_{\mathrm{N}} P_{\mathrm{N}} - P_{\mathrm{C}}$ is output power for sale; $\hat{P}_{\mathrm{S}} = (4\pi/3) r_0 \, \Delta r \, P_{\mathrm{S}}$; η_{A} is the accelerator efficiency (0.8 assumed); η_{B} is the efficiency of conversion of Bremsstrahlung power or neutron power (0.35 assumed); η_{N} is the efficiency of conversion of power associated with charged particles (0.9 assumed).

Design and performance parameters have been calculated for one case for each of the fuels D–T, D–He^3 and p–B^{11}. They have not been optimized but they show that significant values of $\hat{P}_{\mathrm{S}}$ are obtained for each case. Results are shown in Table 1.

VIII. CONCLUSIONS

It is not possible to determine the properties of all fusion reactors with a generic calculation [1]. The calculation must be device specific and consider the properties of a burning plasma including fuel ions, electrons, fusion product ions and sources and sinks. The generic calculation [1] for p–B^{11} results in $P_{\mathrm{recirc}}/P_{\mathrm{fus}} = 9100$, 350, 52 and 33 depending on the choice of parameters in the isotropic equilibrium assumed in equation (3). For the specific device of a CBR the result is $P_{\mathrm{recirc}}/P_{\mathrm{fus}} = 0.493$. The equilibrium is not isotropic in any frame of reference since there must be a current.

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