



Pulsed Power Engineering Basic Topologies

January 12-16, 2009

Craig Burkhart, PhD
Power Conversion Department
SLAC National Accelerator Laboratory

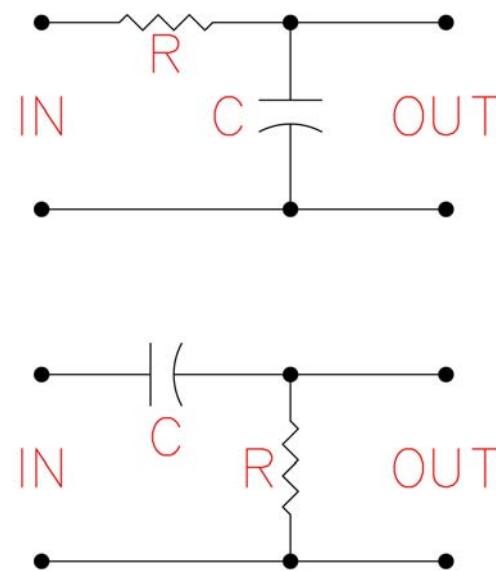


Basic Topologies

- Basic circuits
- Hard tube
- Line-type
 - Transmission line
 - Blumlein
 - Pulse forming network
- Charging circuits
- Controls

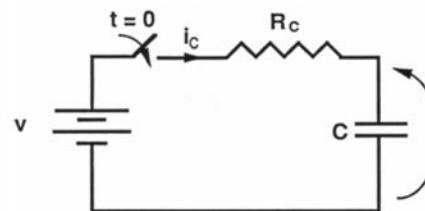
Basic Circuits: RC

- Capacitor charge
- Capacitor discharge
- Passive integration – low-pass filter
 - $\tau \ll RC$: integrates signal,
 $V_{\text{OUT}} = (1/RC) \int V_{\text{IN}} dt$
 - $\tau \gg RC$: low pass,
 $V_{\text{OUT}} = V_{\text{IN}}$
- Passive differentiation – high-pass filter
 - $\tau \gg RC$: differentiates signal,
 $V_{\text{OUT}} = (RC) dV_{\text{IN}} / dt$
 - $\tau \ll RC$: high pass,
 $V_{\text{OUT}} = V_{\text{IN}}$
- Resistive charging of capacitors



RC Charge

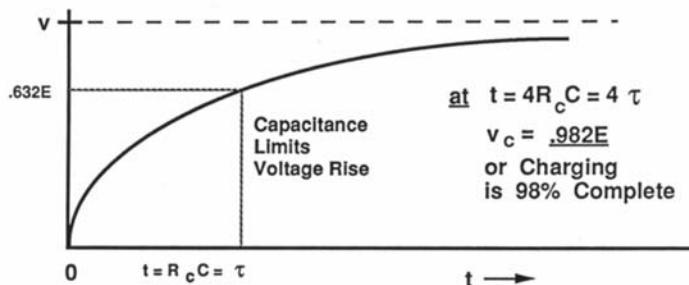
R-C Charging



$$v_c = \frac{1}{C} \int i_C dt$$

$$v_c = \frac{1}{C} \int_0^t \frac{V}{R_C} e^{-\frac{t}{R_C C}} dt$$

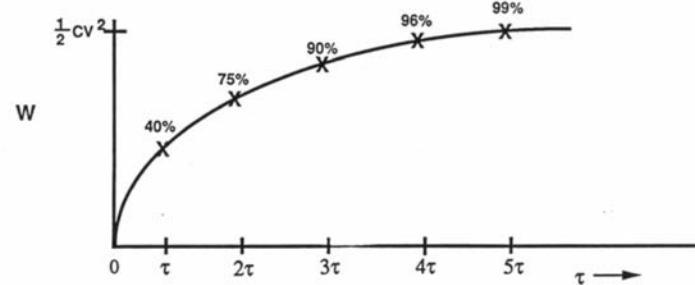
$$= V(1 - e^{-\frac{t}{R_C C}})$$



Energy Stored?

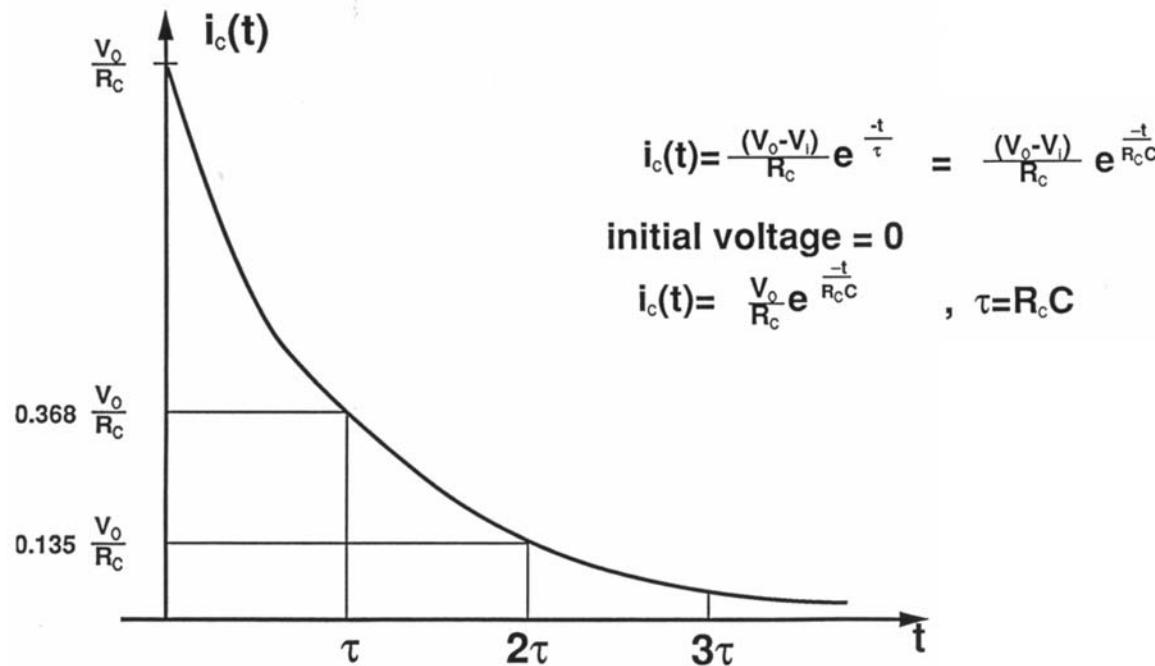
$$W = \int p dt = \int_0^t ei dt, \quad i = C \frac{dv}{dt}$$

$$\Rightarrow \frac{1}{2} CV^2$$



RC Discharge

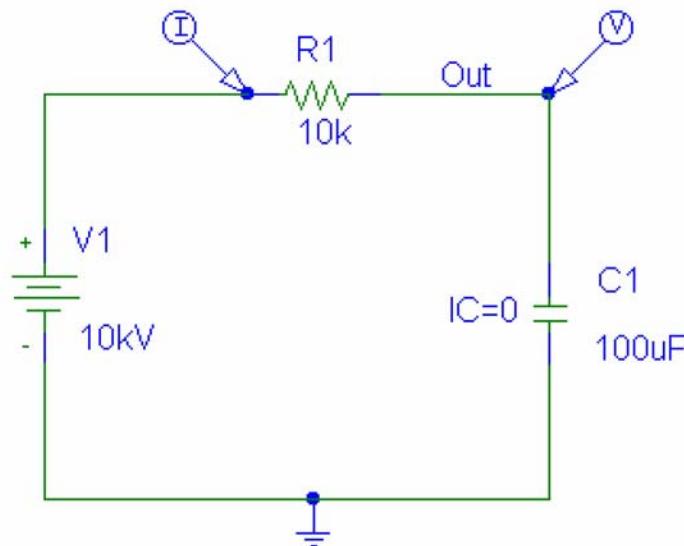
R-C Discharge Current



Resistive Charging of a Capacitor

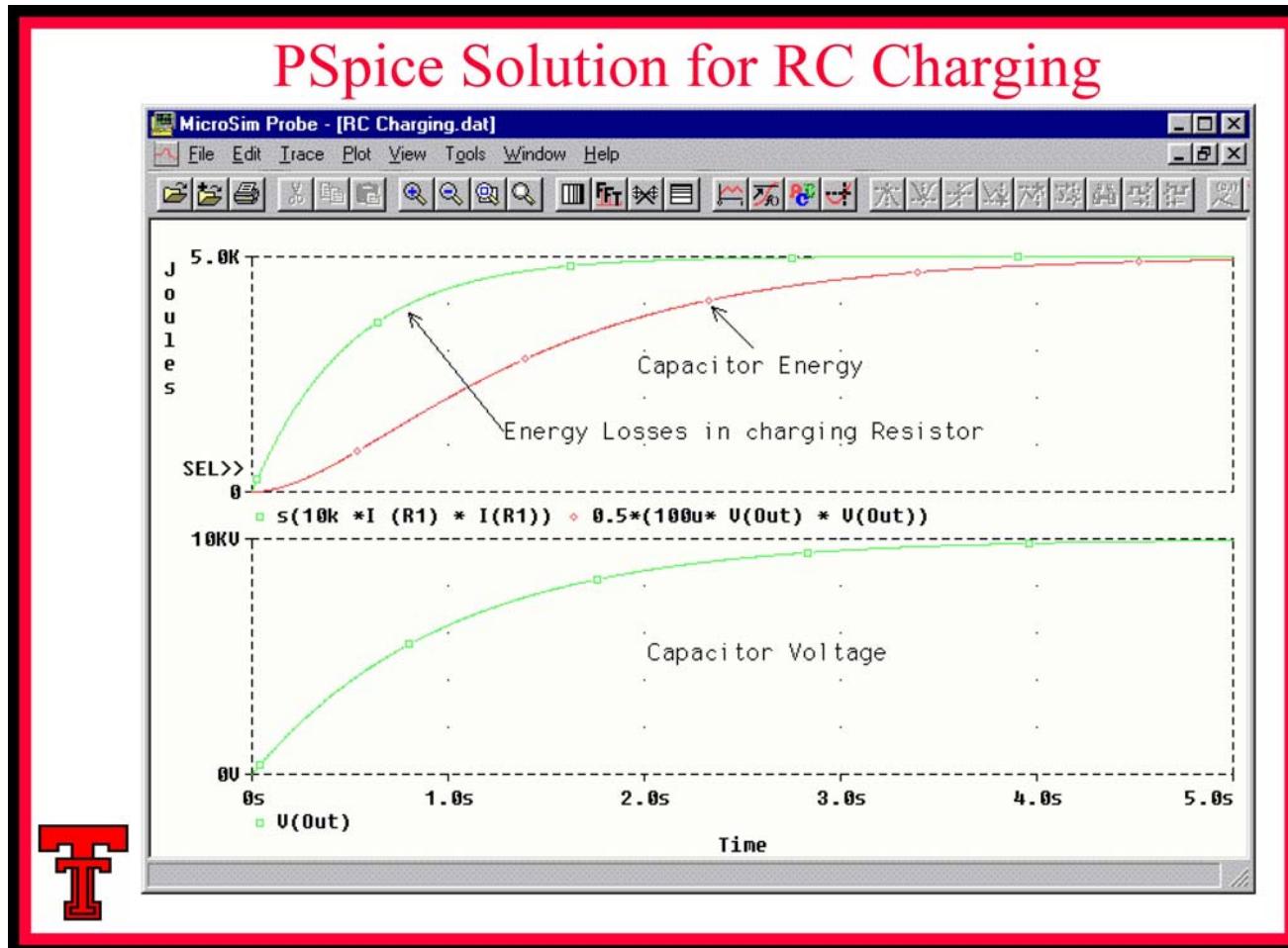
PSpice Circuit for RC Charging

RC-Charging



It is resistive charging if charging current, $I_C = \Delta V_R / R$, through full charging cycle

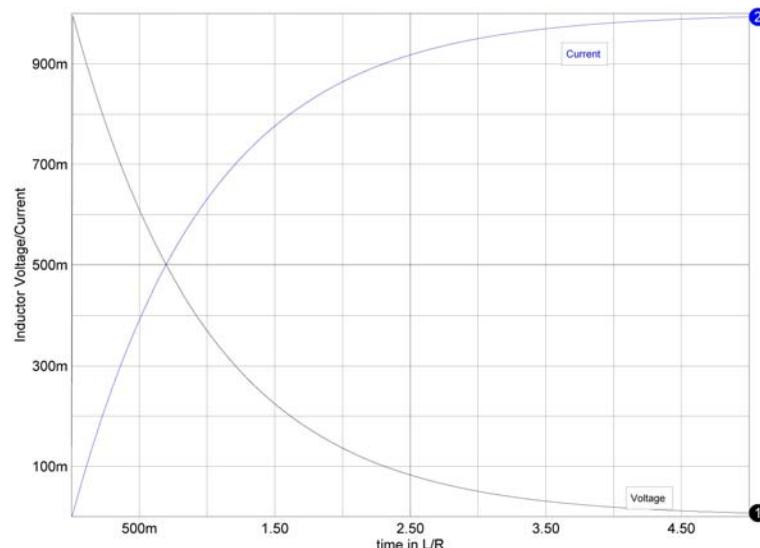
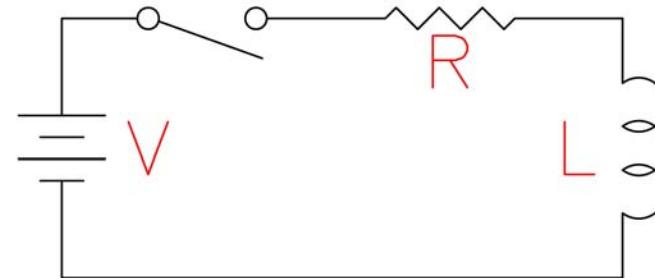
Resistive Charging of a Capacitor



In resistive charging a minimum of 50% of the energy is dissipated in the charging resistor

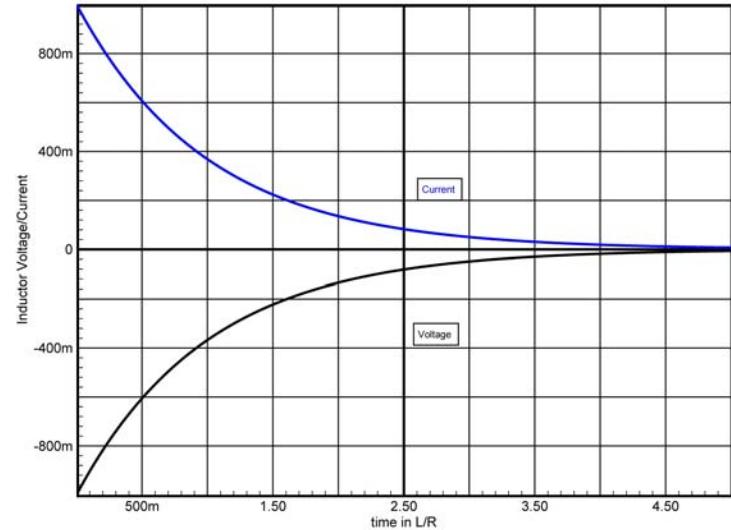
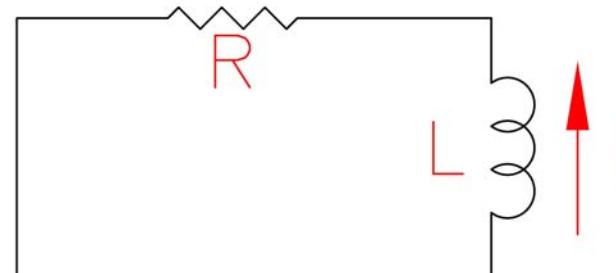
LR Circuit: Inductive Risetime Limit

- At $t=0$, close switch and apply $V=1$ to LR circuit
- Inductance limits dI/dt
- Reach 90% of equilibrium current, V/R , in $\sim 2.2 L/R$



LR Circuit: Decay of Inductive Current

- At $t=0$, current $I=1$ flowing in LR circuit
- Fall to 10% of initial current in $\sim 2.2 L/R$



LRC Circuit

- Generally applicable to a wide number of circuits and sub-circuits found in pulsed power systems
- Presented in the more general form of CLRC (after NSRC formulary)
 - Limit $C_1 \rightarrow \infty$, reduces to familiar LRC with power supply
 - Limit $C_2 \rightarrow \infty$ (short), reduces to familiar LRC
 - Limit $R \rightarrow 0$, reduces to ideal CLC energy transfer
 - Limit $L \rightarrow 0$, reduces to RC

CLRC Circuit

Where:

$$\tau = \frac{L}{R}$$

$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$$

$$\omega_o = \frac{1}{\sqrt{LC_{eq}}}$$

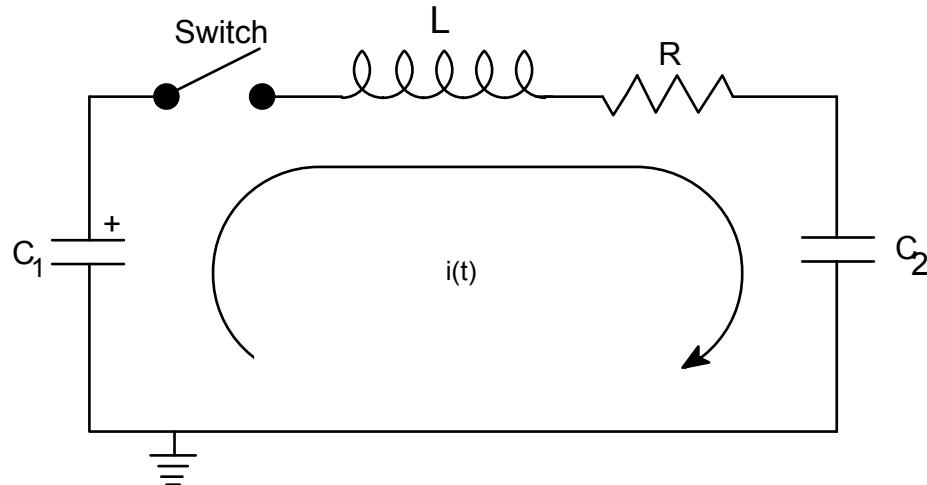
$$\omega^2 = ABS \left(\omega_o^2 - \left(\frac{1}{2\tau} \right)^2 \right)$$

$$Z_0 = \sqrt{\frac{L}{C_{eq}}}$$

$$Q = \frac{Z_0}{R} = (\text{Circuit Quality Factor})$$

V_0 = initial charge voltage on C_1

0 = initial charge voltage on C_2



Underdamped CLRC

For underdamped case: $R < 2Z_0$

$$i(t) = \frac{V_o}{\omega L} e^{\frac{-t}{2\tau}} \sin \omega t$$

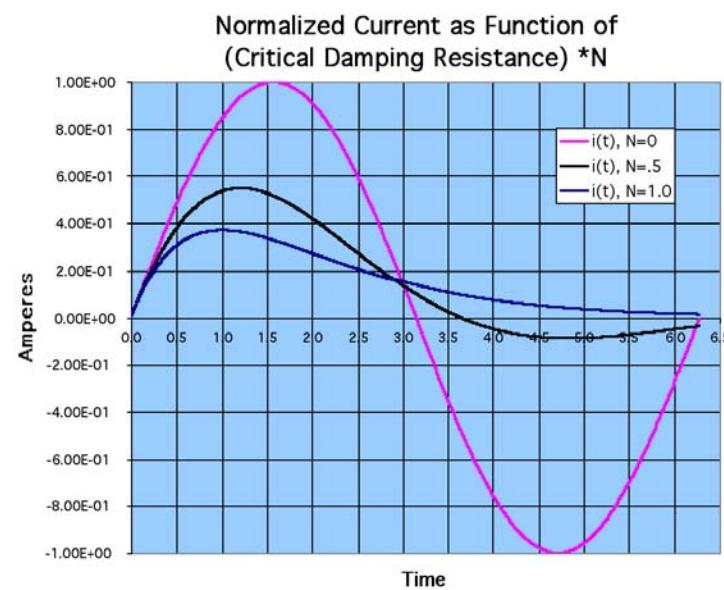
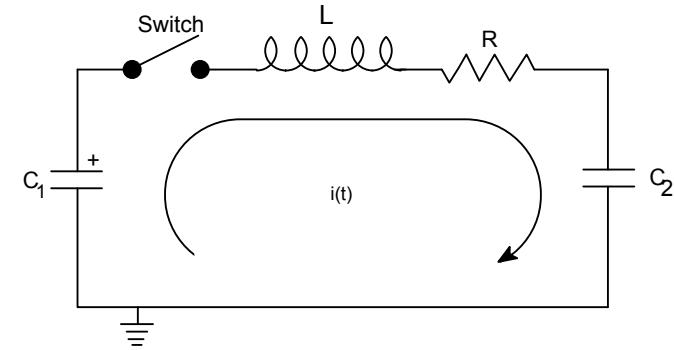
$$V_{C_2}(t) = \frac{V_0 C_1}{C_1 + C_2} \left[1 - e^{\frac{-t}{2\tau}} \left(\frac{1}{2\omega\tau} \sin \omega t - \cos \omega t \right) \right]$$

$$i_{peak} @ t_1 = \frac{1}{\omega} \tan^{-1}(2\omega\tau)$$

$$i_{peak} = \frac{V_0}{\sqrt{\frac{L}{C_{eq}}}} e^{\frac{-t_1}{2\tau}} = \frac{V_0}{Z_0} e^{\frac{-t_1}{2\tau}} \cong \frac{V_0}{Z_0 + 0.8R}$$

$$i(t) = 0 ; V_{C_2}(t) = peak @ t_0 = \frac{\pi}{\omega}$$

$$V_{C_2}(t)_{peak} = \frac{V_0 C_1}{C_1 + C_2} \left(1 + e^{\frac{-\pi}{2\omega\tau}} \right)$$



Highly Underdamped: Energy Transfer Stage

When $R \ll Z_0$ (almost always the preferred situation in pulse circuits)

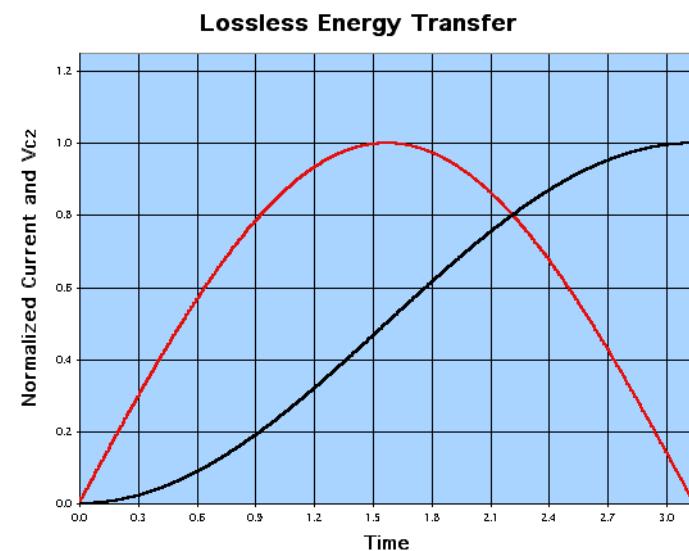
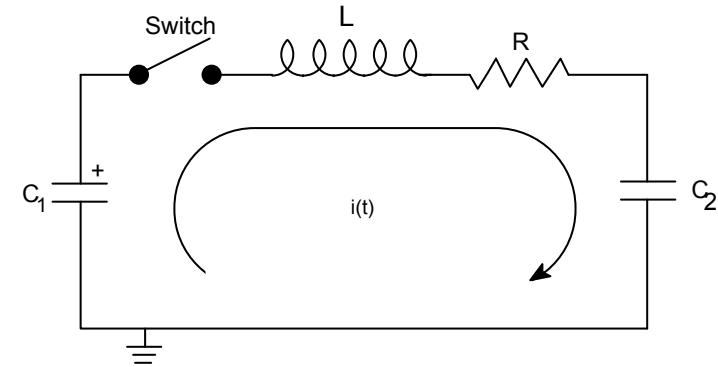
$$i_{peak} = \frac{V_0}{Z_0}$$

$$i_{peak} @ t = \frac{\pi}{\omega_0} = \pi \sqrt{LC_{eq}}$$

$$V_{C_2}(t) = \frac{V_0 C_1}{C_1 + C_2} (1 - \cos(\omega_0 t))$$

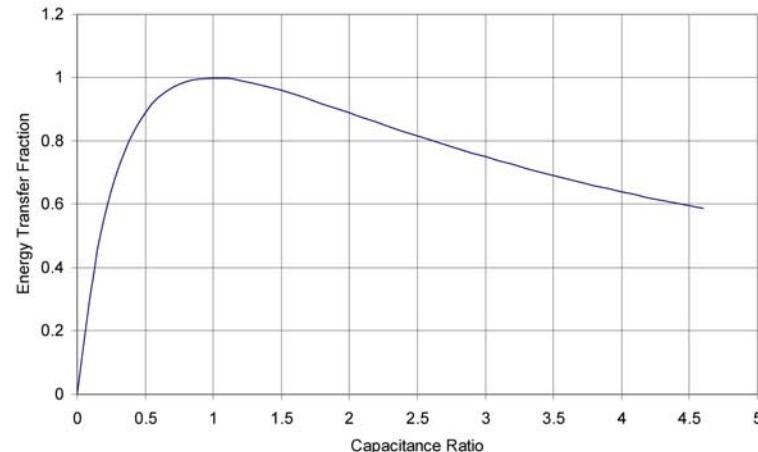
$$V_{C_2}(\text{peak}) @ t = \frac{2\pi}{\omega_0} = 2\pi \sqrt{LC_{eq}}$$

$$V_{C_2}(\text{peak}) = \frac{2V_0 C_1}{C_1 + C_2}$$

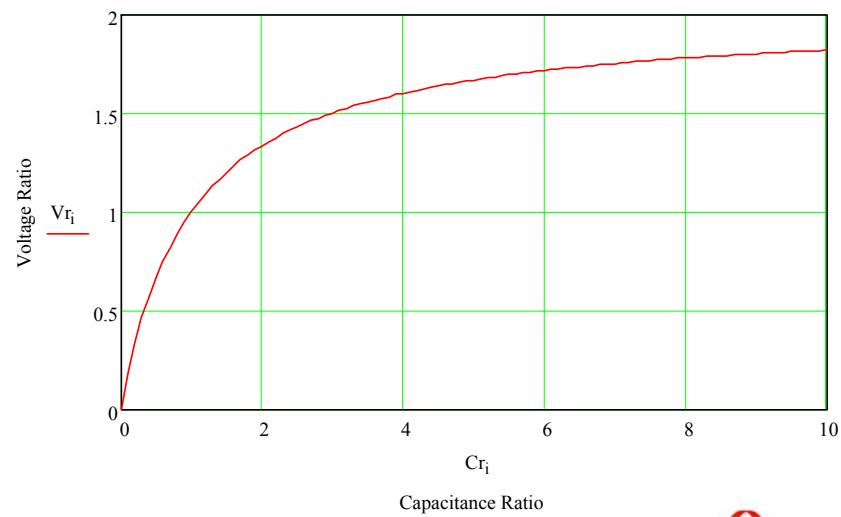


Highly Underdamped: Energy Transfer Stage (cont.)

- Peak energy transfer efficiency achieved with $C_1 = C_2$



- If $C_1 \gg C_2$, the voltage on C_2 will go to twice the voltage on C_1

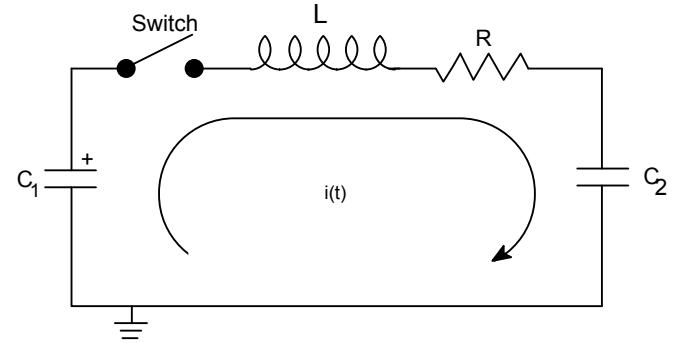


Overdamped CLRC

Overdamped case $R > 2Z_0$

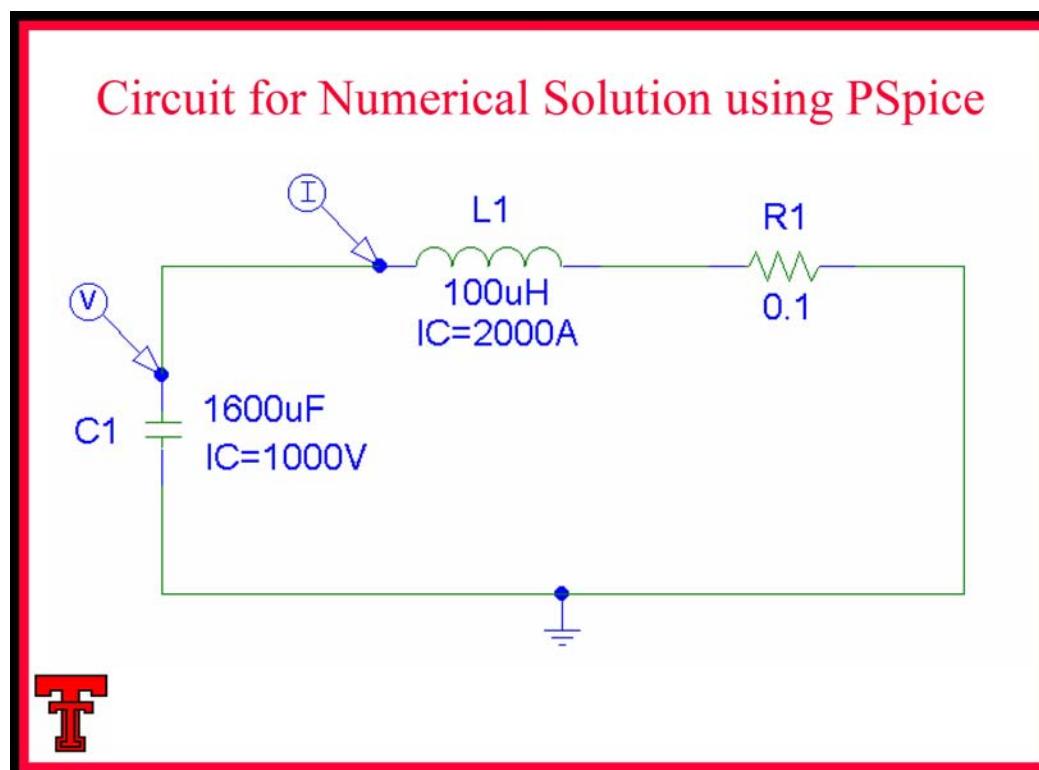
$$i(t) = \frac{V_0 e^{-\frac{t}{2\tau}}}{2\omega L} [e^{\alpha t} - e^{-\omega t}]$$

$$V_{C_2}(t) = \frac{V_0}{2\omega L C_2} \left[\frac{2\omega}{\omega_o^2} - e^{-\frac{t}{2\tau}} \left(\frac{e^{-\omega t}}{\frac{1}{2\tau} + \omega} + \frac{e^{\alpha t}}{\frac{1}{2\tau} - \omega} \right) \right]$$



General LRC Solution

- General solution from TTU Pulsed Power Short Course
 - Level of damping defined by β
 - $\beta > 0$: underdamped
 - $\beta = 0$: critically damped
 - $\beta < 0$: overdamped



General LRC Solution (cont.)

Definitions for Analytical Solutions

Analytical Solutions → 3 Possible Cases: Ringing, Critically Damped, and Overdamped

Determining if Circuit is Ringing, Critically Damped, and Overdamped:

$$\omega_0 := \frac{1}{\sqrt{L \cdot C}} \quad \begin{array}{l} \text{Undamped} \\ \text{Resonant} \end{array} \quad \alpha := \frac{R}{2 \cdot L} \quad \begin{array}{l} \text{Neper} \\ \text{Frequency} \end{array} \quad \beta := \omega_0^2 - \alpha^2 \quad \beta > 0 \text{ ringing}$$

$$\omega_0 = 2500 \frac{1}{\text{sec}} \quad \begin{array}{l} \text{Radian} \\ \text{Frequency} \end{array} \quad \alpha = 2.5 \times 10^3 \frac{1}{\text{sec}} \quad \begin{array}{l} \text{for series} \\ \text{RLC Circuit} \end{array} \quad \beta = 0 \frac{1}{\text{sec}^2}$$

$$\omega := \begin{cases} \sqrt{\beta} & \text{if } \beta > 0 \\ 0 & \text{otherwise} \end{cases} \quad \omega = 0 \text{ sec}^{-1} \quad \alpha_1 := \begin{cases} \alpha + \sqrt{-\beta} & \text{if } \beta \leq 0 \\ 0 & \text{otherwise} \end{cases} \quad \alpha_2 := \begin{cases} \alpha - \sqrt{-\beta} & \text{if } \beta \leq 0 \\ 0 & \text{otherwise} \end{cases}$$

$$\alpha_1 = 2.5 \times 10^3 \text{ sec}^{-1} \quad \alpha_2 = 2.5 \times 10^3 \text{ sec}^{-1}$$

$$\frac{1}{L \cdot C} - \left(\frac{R_{\text{crit}}}{2 \cdot L} \right)^2 = 0 \quad R_{\text{crit}} := 2 \cdot \sqrt{\frac{L}{C}} \quad R_{\text{crit}} = 0.5 \Omega \quad \text{Resistor for Critical Damping}$$



General LRC Solution (cont.)

Analytical Solution for all 3 Cases

$$V(t) := \begin{cases} V_0 \cdot e^{-\alpha \cdot t} \left[\cos(\omega \cdot t) + \frac{(V_0 \cdot \alpha \cdot C - I_0)}{C \cdot V_0 \cdot \omega} \cdot \sin(\omega \cdot t) \right] & \text{if } \beta > 0 \\ \frac{(V_0 \cdot C \cdot \alpha_2 - I_0)}{C \cdot (\alpha_2 - \alpha_1)} \cdot e^{-\alpha_1 \cdot t} + \frac{(V_0 \cdot \alpha_1 \cdot C - I_0)}{[C \cdot (\alpha_1 - \alpha_2)]} \cdot e^{-\alpha_2 \cdot t} & \text{if } \beta < 0 \\ \left(V_0 \cdot \alpha_2 - \frac{I_0}{C} \right) \cdot t \cdot e^{-\alpha_1 \cdot t} + V_0 \cdot e^{-\alpha_2 \cdot t} & \text{if } \beta = 0 \end{cases}$$

Expression for the Voltage in all 3 Cases:

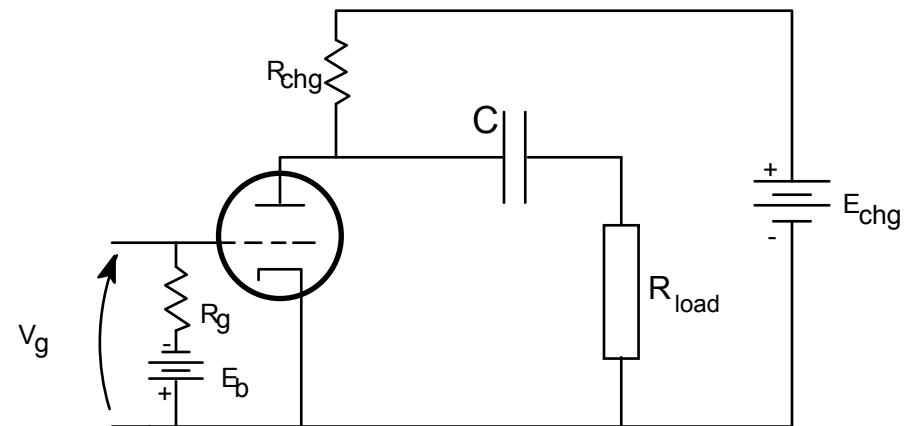
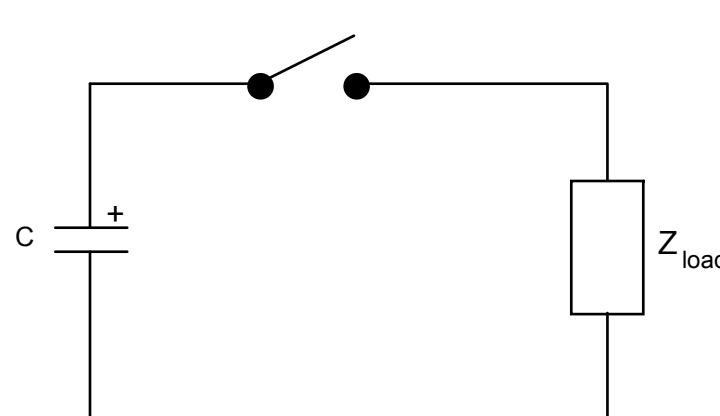
$$I(t) := \begin{cases} I_0 \cdot e^{-\alpha \cdot t} \left[\cos(\omega \cdot t) + \frac{(V_0 - R \cdot I_0 + I_0 \cdot \alpha \cdot L)}{L \cdot I_0 \cdot \omega} \cdot \sin(\omega \cdot t) \right] & \text{if } \beta > 0 \\ \frac{(I_0 \cdot L \cdot \alpha_2 + V_0 - R \cdot I_0)}{L \cdot (\alpha_2 - \alpha_1)} \cdot e^{-\alpha_1 \cdot t} + \frac{(V_0 - R \cdot I_0 + I_0 \cdot \alpha_1 \cdot L)}{[L \cdot (\alpha_1 - \alpha_2)]} \cdot e^{-\alpha_2 \cdot t} & \text{if } \beta < 0 \\ \left(I_0 \cdot \alpha_2 + \frac{V_0 - R \cdot I_0}{L} \right) \cdot t \cdot e^{-\alpha_1 \cdot t} + I_0 \cdot e^{-\alpha_2 \cdot t} & \text{if } \beta = 0 \end{cases}$$

Expression for the Current in all 3 Cases:



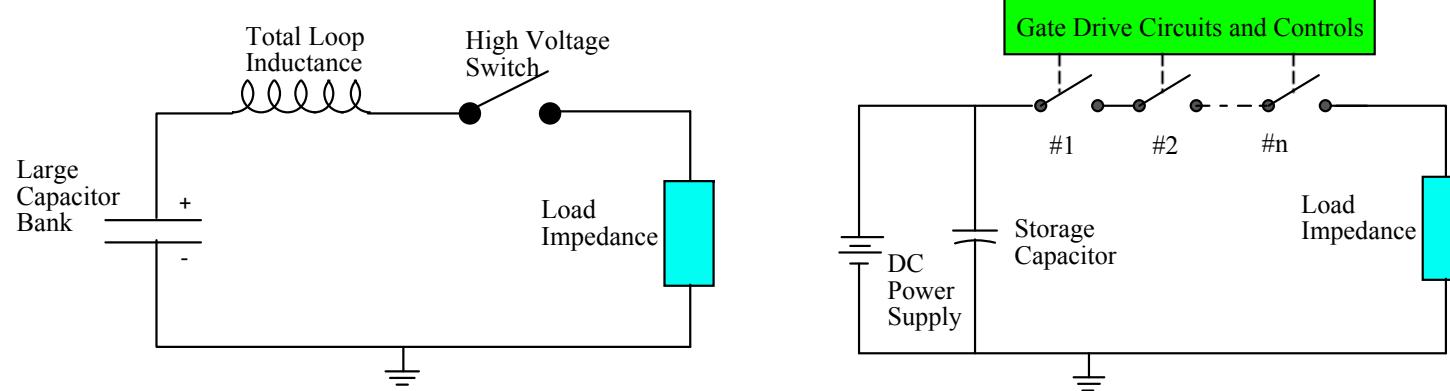
Hard Tube Modulators

- Pulsers in which only a portion of the stored electrical energy is delivered to the load. Requires a switch that can open while conducting full load current.
 - Switch must open/close with required load voltage and current
 - Voltage regulation limited by capacitor voltage droop
 - Flat output pulse → large capacitor/large stored energy
 - Cost
 - Faults
- Name refers to “hard” vacuum tubes historically used as switch
- Today’s fast solid-state devices are being incorporated into designs previously incorporating vacuum tubes



Hard Tube: Topology Options

- Capacitor bank with series high voltage switch - gives pulse width agility but requires high voltage switch



- Variations
 - Add series inductance: zero current turn on of switch
 - Series switches: reduces voltage requirements for individual switches
- Issues:
 - Switches must have very low time jitter during turn-on and turn-off
 - Voltage grading of series connected switches, especially during switching
 - Isolated triggers and auxiliary electronics (e.g. power, diagnostics)
 - Switch protection circuits (load and output faults)
 - Load protection circuits

Commercial Series Stack Modulator

The Power You Need PowerMod™ HVPMP 100-150

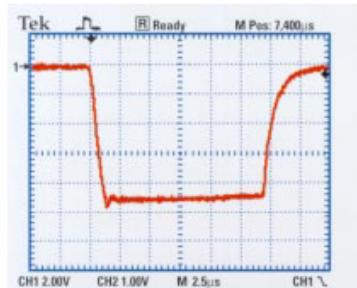
PowerMod™

Diversified Technologies, Inc. (DTI) has applied its extensive background in high power electronics to design and build a 100 kV, 150A solid-state modulator for use in demanding commercial applications. The PowerMod™ HVPMP 100-150 utilizes DTI's patented solid-state technology. The configuration shown fits in an oil tank approximately 50" x 36" x 64".

DTI's HVPMP 100-150 utilizes the same breakthrough technology found in the HVPMP 20-150 which received an R&D Magazine award as one of the 100 most technologically significant products of 1997. DTI's PowerMod™ high voltage, solid-state modulators are available from 1-200 kV, and up to 2,000A peak.

High power, high current modulators based upon DTI's design offer customers increased efficiency, enhanced reliability, increased pulse flexibility, and cost-effective high power switching capability.

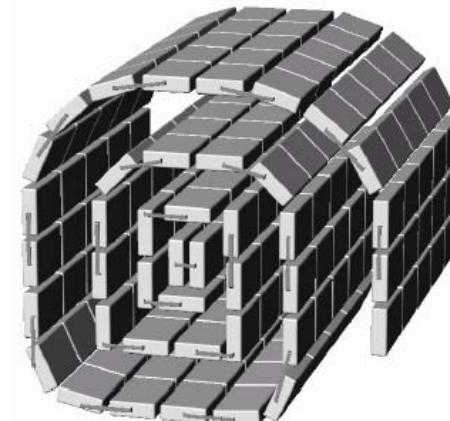
DTI has pioneered the state-of-the-art in solid-state electronics since 1987. Our modulators have become essential components in applications for ion implantation (PSII), particle accelerators, and semiconductor and flat panel display manufacturing.



HVPMP 100-150 Pulse, 80kV, 90A Into Water Resistor



PowerMod™ HVPMP 100-150 Solid-State Modulator



Solid model of the 500 kV hard switch – note the spiral wrap of the series string of switch modules to reduce effective parasitic capacitance. The output (pulsed) end of the modulator is on the axis, and the input (DC) end is on the outer surface.

HVPMP 100-150 Specifications

Control Voltage	120V AC
High Voltage Input:	1-100 kV DC peak
Average Pulse Current:	75A
1 μs Peak Current	150A
Rise Time ^a :	<1 μs
Fall Time ^a :	<1 μs
Nominal Pulse Width:	1 μs - 100 μs
Nominal Pulse Frequency:	0-5,000 Hz

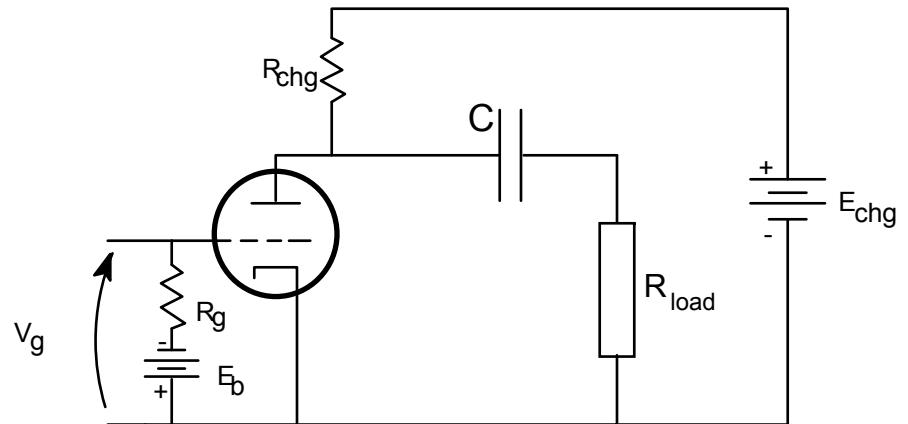
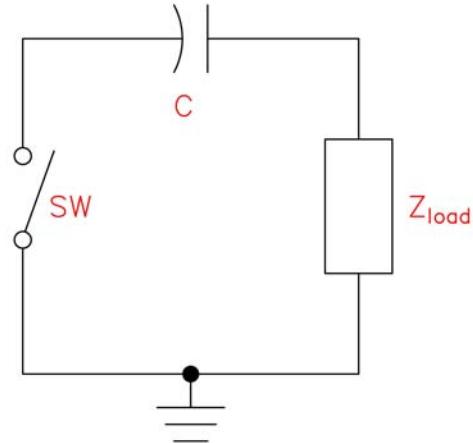
^ainto resistive load

(781) 275-9444 Fax: (781) 275-6081

35 Wiggins Avenue • Bedford, MA 01730 • info@divtecs.com • www.divtecs.com

Hard Tube: Topology Options

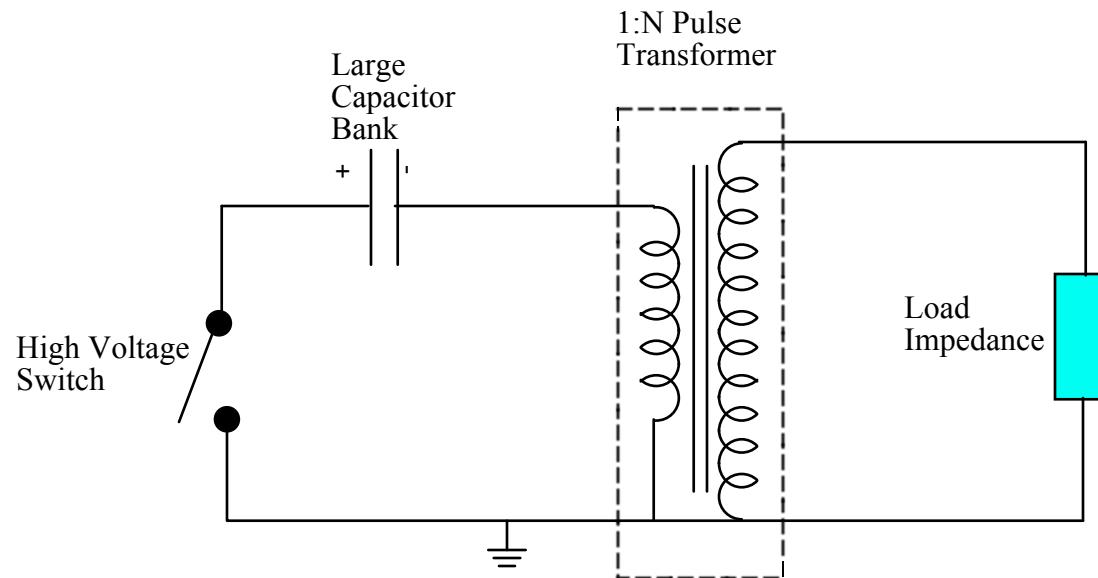
- Grounded switch – simplifies switch control



- Issues:
 - Only works for one polarity (usually negative)
 - HVPS must be isolated from energy storage cap during pulse
 - Loose benefit with series switch array

Hard Tube: Topology Options

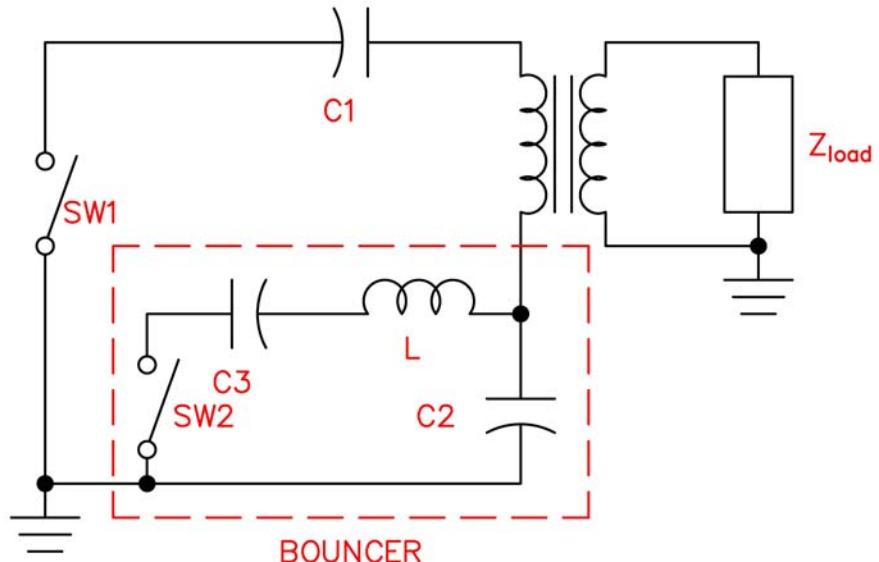
- Pulse Transformer - reduces the high voltage requirements on switch



- Issues:
 - Very high primary current ($N \cdot I_{load}$) and large di/dt for fast rise times
 - Requires very low primary loop inductance and very low leakage inductance: exacerbated by high turns ratio; L, C, Z scale with N^2
 - Fast opening switch required capable of interrupting primary current
 - Distortion of waveform by non-ideal transformer behavior

Hard Tube: Topology Options

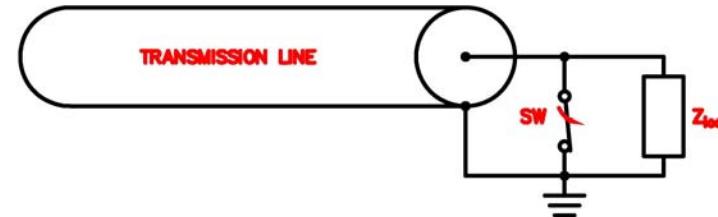
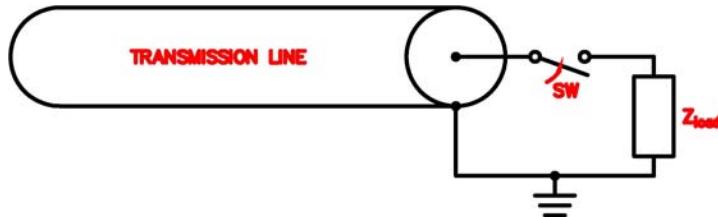
- Bouncer modulator – compensates energy storage capacitor droop
 - Initially, SW2 is closed, voltage on C3 is transferred to C2
 - Then SW1 is closed, applying output pulse to load
 - Energy transferred from C3 to C2, during linear portion of waveform, compensates for voltage droop of C1
 - After output pulse is finished, energy from C2 rings back to C3, low loss
- Used for XFEL and ILC (baseline) klystron modulators
- Issues:
 - Extra components
 - Timing synchronization
 - Bouncer frequency low
→ large L and C's



Line-Type Modulators

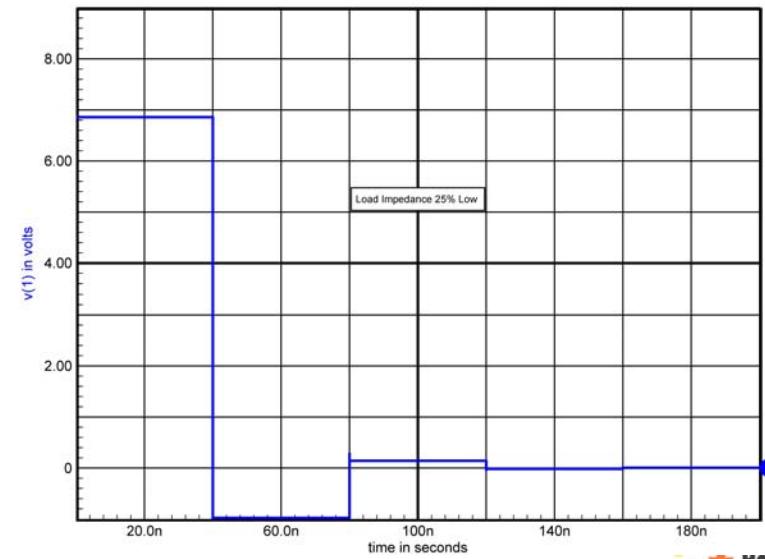
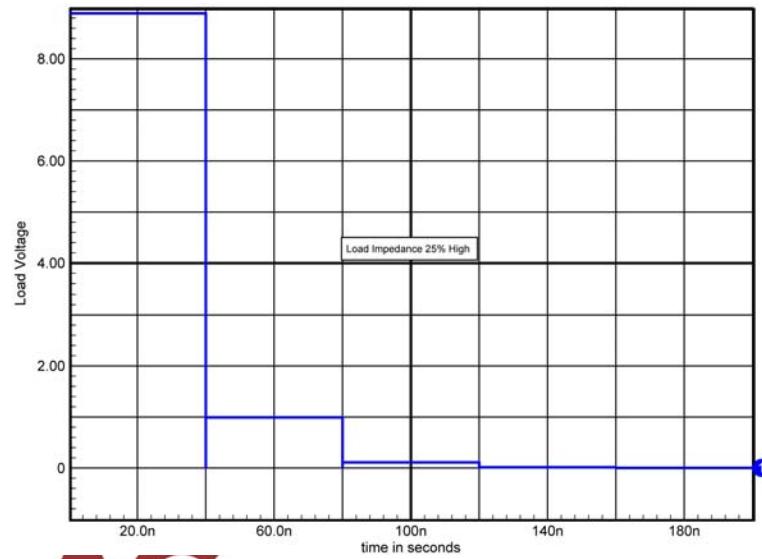
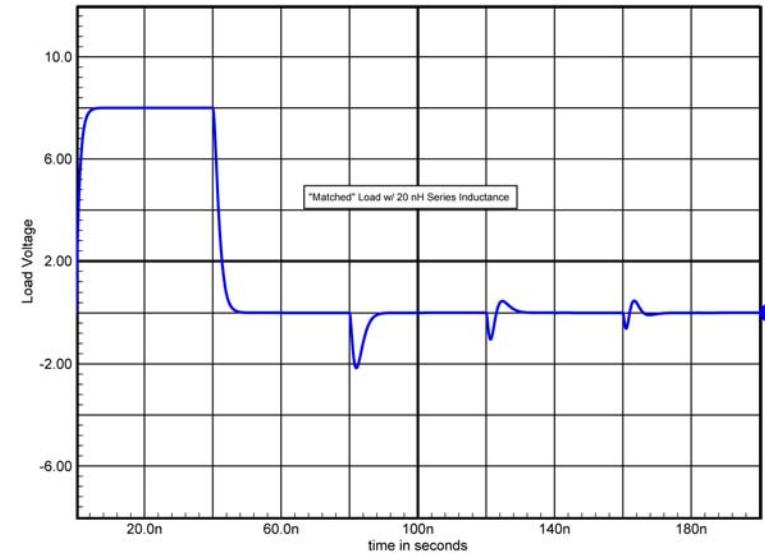
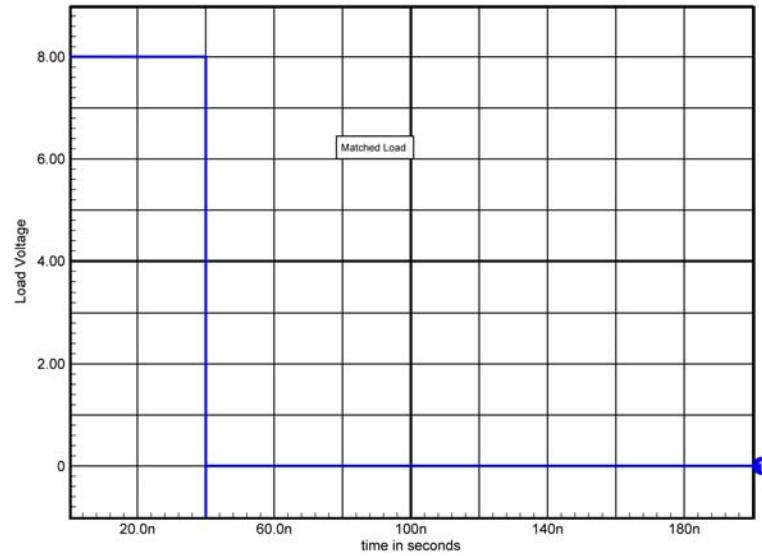
- Based on the properties of transmission lines as pulse generating devices
- Advantages
 - Minimum stored energy, 100% → load (neglecting losses)
 - Voltage fed, capacitive storage (E-field), closing switch
 - Current fed, inductive storage (B-field), opening switch
 - Fault (short circuit) current \leq twice operating current (matched load)
 - Relatively simple to design and fabricate, inexpensive
 - Switch action is closing OR opening, but not both
- Disadvantages
 - Fixed (and limited range) output pulse length
 - Fixed (and limited range) output pulse impedance
 - Output pulse shape dependent on relative modulator/load impedance
 - $Z_{load} < Z_{pulser}$ → voltage reversal, may damage switch or other components
 - Switch operates at twice the voltage (or current) delivered to load
 - Must be fully recharged between pulses: may be difficult at high PRF

Transmission Line Modulator



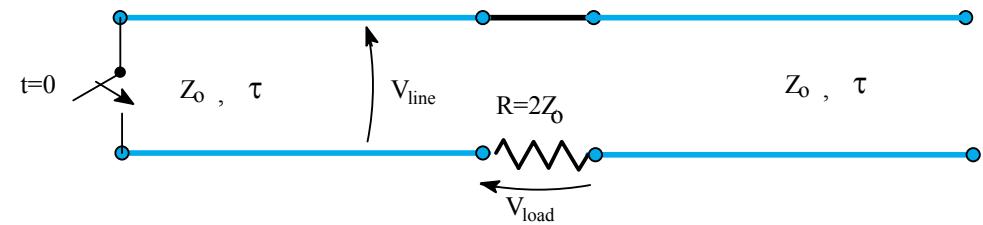
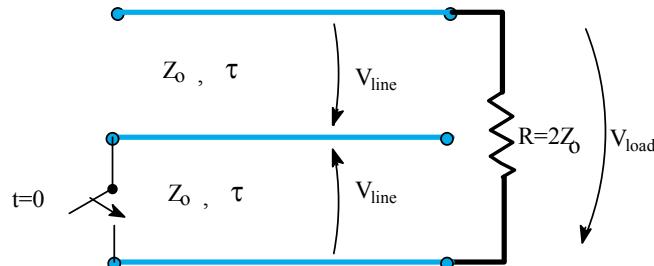
- Square output pulse is intrinsic
- Pulse length is twice the single transit time of line: $\tau = 2\ell/(c\varepsilon^{1/2})$
 - Vacuum: 2 ns/ft
 - Poly & oil: 3 ns/ft
 - Water: 18 ns/ft
- Impedance of HV transmission lines limited:
 - $\sim 2 \Omega \leq Z \leq \sim 200 \Omega$
 - $\sim 30 \Omega \leq Z \leq \sim 100 \Omega$ for commercial coax
 - However, impedance can be rescaled using a pulse transformer
- Energy density of coaxial cable is low (vs. capacitors) → large modulator
- Fast transients (faster than dielectric relaxation times) stress solid dielectrics
 - Finite switching time and other parasitic elements introduce transient mismatches
 - Modulator/load impedance mismatches produce post-pulses

10Ω , 40 ns TL Modulator: Load Matching

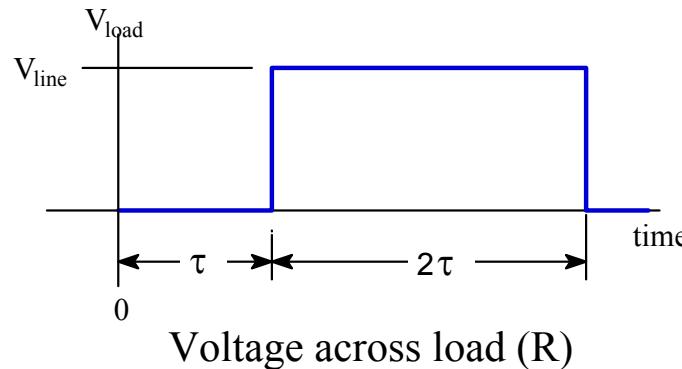


Blumlein Modulator

- Major limitation of TL modulator: switch voltage twice load voltage
- Blumlein Line
 - Requires Two Transmission Lines
 - Load voltage equals charge voltage
 - Switch must handle current of V_o/Z_o , twice the load current

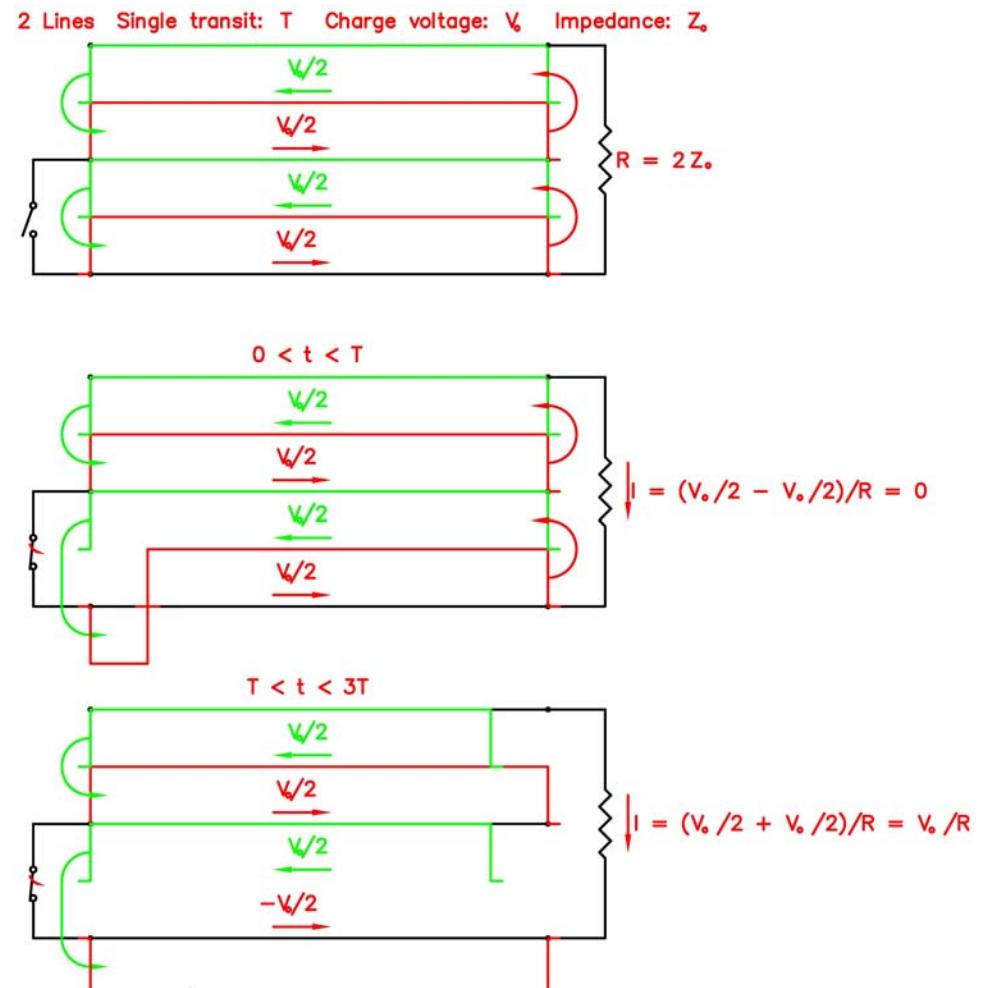


Circuit on left - unfolded



Blumlein “Wave Model”

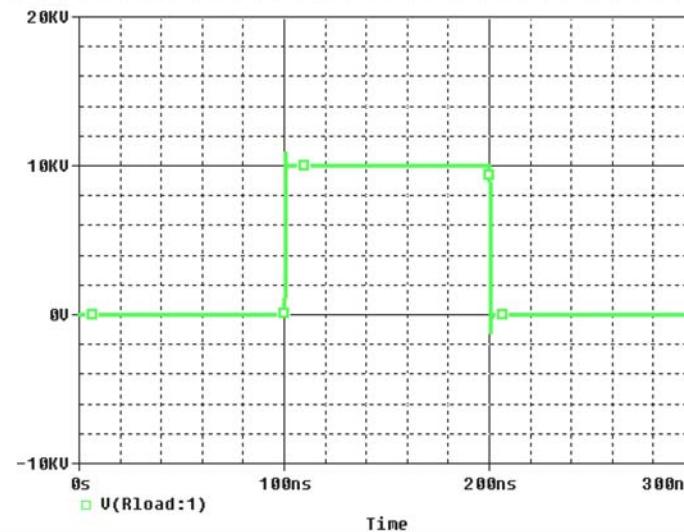
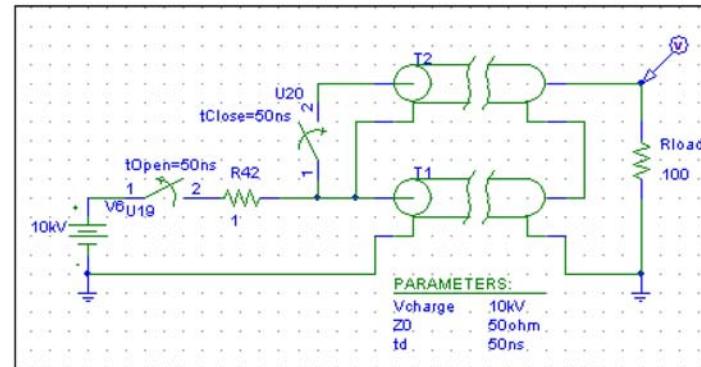
- Initial conditions
 - 2 TL with a common electrode
 - TLs charged to V_0
 - TL Impedance: Z_0
 - Load impedance: $R = 2 Z_0$
- Switch closes at $t = 0$
 - Wave that hits short, reflects with inverted polarity
 - Voltage of $V_0/2$ on both ends of load, no load current
 - Wave in upper TL unchanged
- $t = T$
 - Inverted wave reaches load
 - Load voltage: $V_0/2 - (-V_0/2) = V_0$
 - Load current: V_0/R
 - Load matched, no reflected wave in either TL
- $t = 3T$: energy depleted



Blumlein Modulator



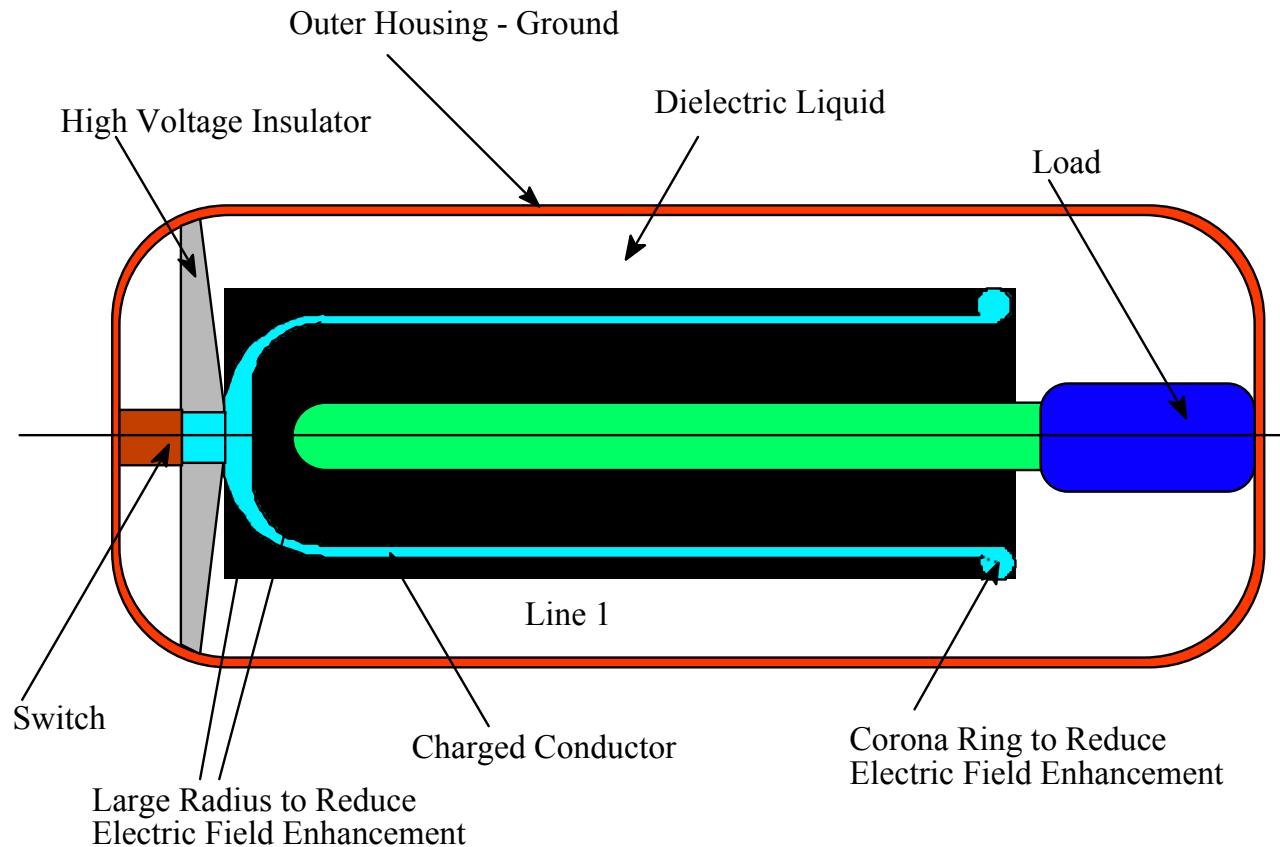
Blumlein Generator



Blumlein in Comparison to Transmission Line

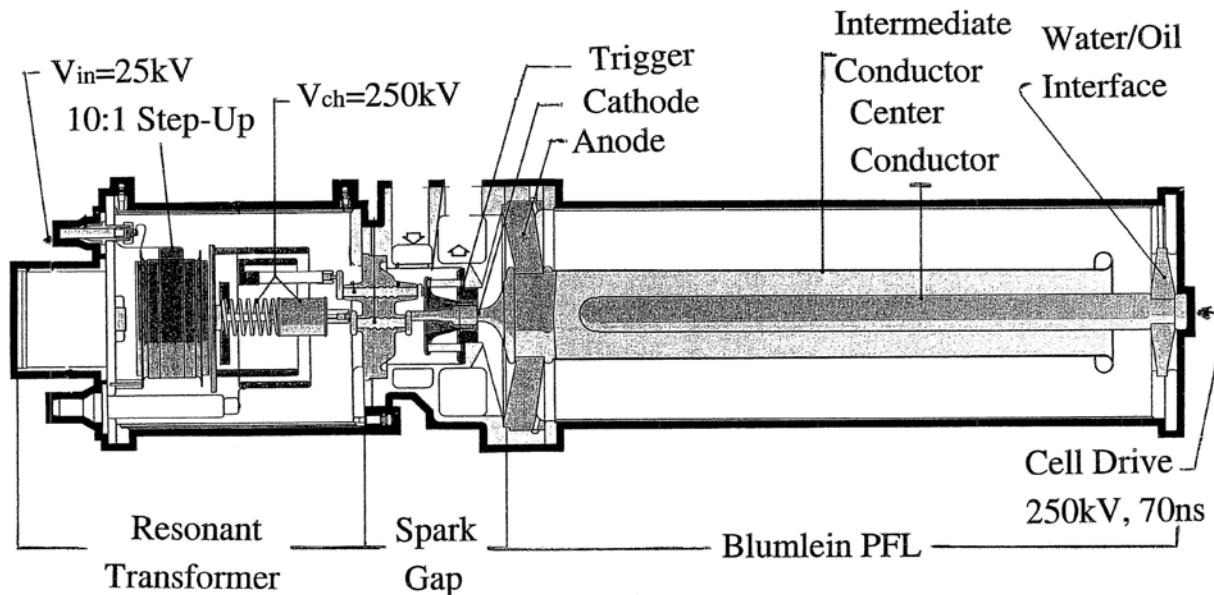
- Switching
 - Blumlein charge (switch) voltage equals load voltage
 - Blumlein switch current is half of load current
 - Peak switch power is half of peak load power for both topologies
 - However, it is generally easier to get switches that handle high current than high voltage
- Blumlein is more complicated
 - Either nested transmission lines or exposed electrode → half load voltage during pulse
 - More sensitive to parasitic distortion (e.g. switch inductance)
- Both are important modulator topologies

Blumlein Modulator



Coaxial Blumlein Configuration

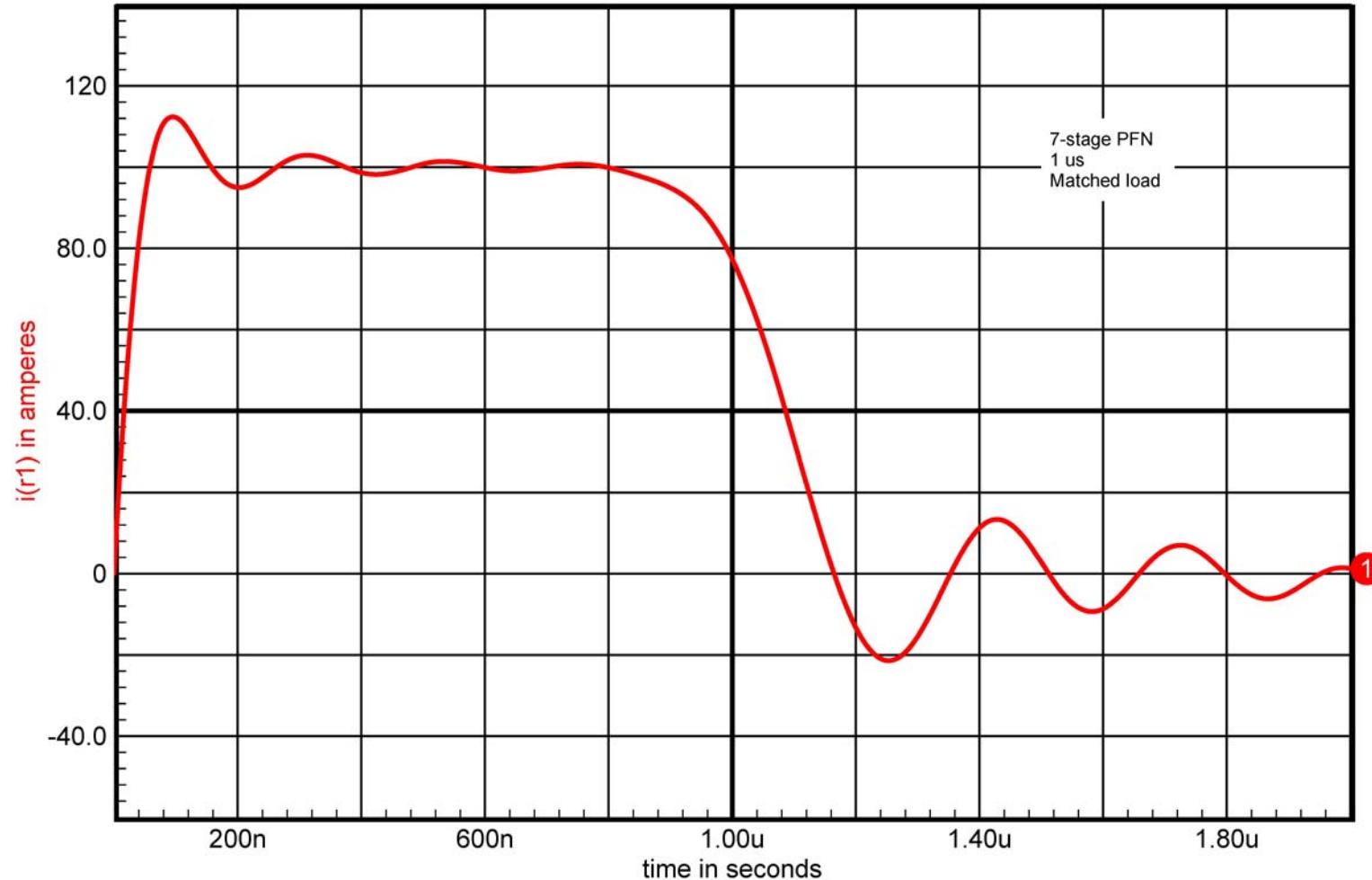
Advanced Test Accelerator Blumlein Modulator



Pulse Forming Networks (PFNs)

- The maximum pulse duration of transmission line pulsers is limited by the physical length of the line, at 3 ns/ft, a 1 μ s TL would be 330' long
- Transmission line can be approximated by an LC array
 - Higher energy density in capacitors
 - Higher energy density in solenoidal inductors
 - PFNs can produce long duration pulses in a compact package
- Design equations
 - $Z = (L/C)^{0.5}$
 - $\tau = 2N(LC)^{0.5}$ (output pulse length)
 - For N-stages of inductance, L, and capacitance, C
- However, the discrete element model of the TL is only accurate as the number of stages, $N \rightarrow \infty$
- Example
 - $N = 7$
 - $Z = 10\Omega$
 - $T = 1 \mu$ s
 - $C = 7.14 \text{ nF}$
 - $L = 0.714 \mu\text{H}$

Example PFN Output Into Matched Load

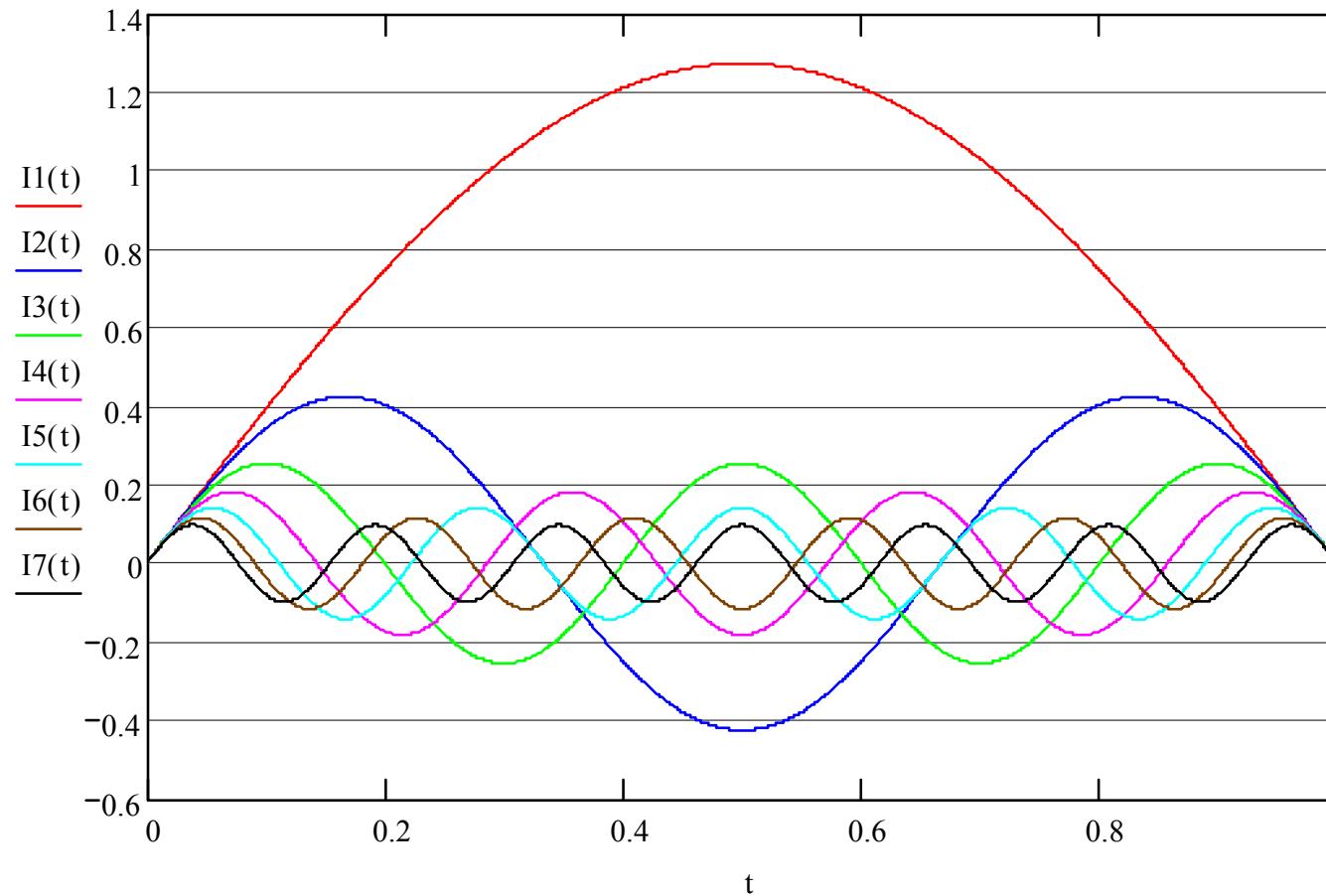


The Trouble with Pulse Forming Networks

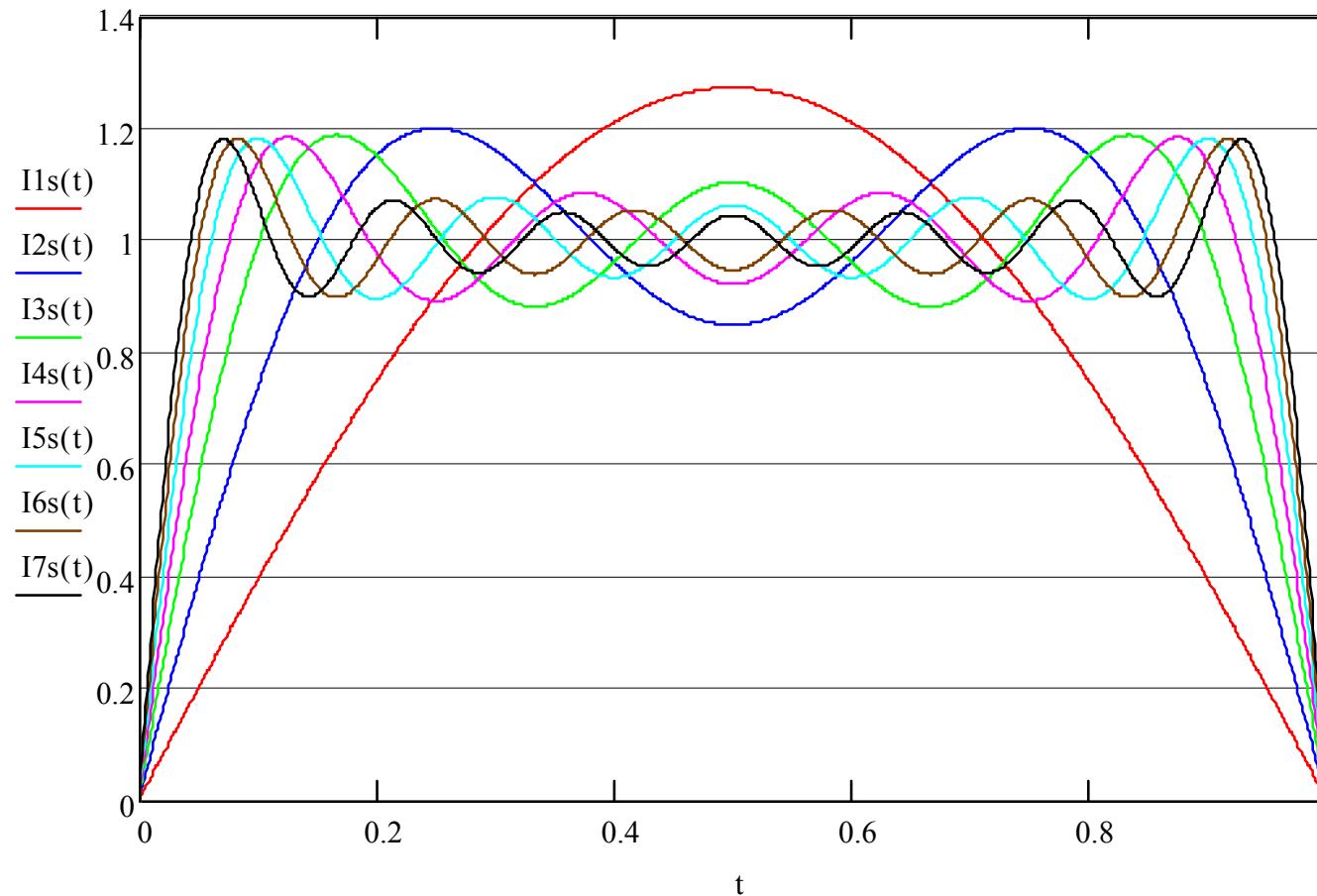
- Attempting to reproduce a rectangular pulse, which is non-causal
 - $\omega_{\max} = \infty$, therefore, N must $\rightarrow \infty$
 - PFNs constructed with finite N
- Fourier series expansion of a rectangular pulse (period 0 to τ)
 - $I(t) = (2 I_{\text{peak}}/\pi) \sum_{n=1}^{\infty} b_n \sin(n\pi t/\tau)$
 - $b_n = (1/n)(1 - \cos(n\pi)) = 0$ for even n , 2 for odd n
 - $I(t) = (4 I_{\text{peak}}/\pi) \sum_{n=1}^{\infty} (1/n) \sin(n\pi t/\tau)$ over only odd terms, $n = 1, 3, 5, \dots$
- Magnitude of the n^{th} term $\propto 1/n$, sets convergence rate for a rectangular pulse

Fourier Components for a Rectangular Pulse

(Peak amplitude and duration normalized to unity)



Fourier Approximation to a Rectangular Pulse



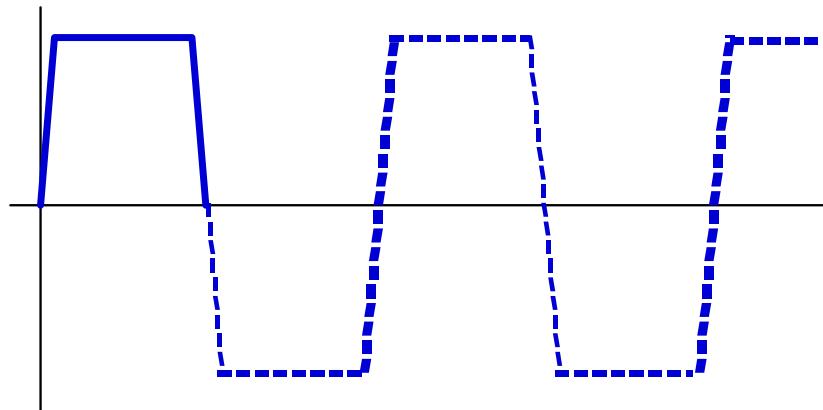


Guillemin Networks: The Solution to “The Trouble with PFNs”

- E.A. Guillemin recognized that the discontinuities due to
 - Zero rise/fall time, and
 - Corners at the start/stop of the rise and fallare the source of the high frequency components that challenge PFN design. “Communication Networks,” 1935
- Further, since such a perfect waveform cannot be generated by this method, that better results can be obtained by intentionally design for finite rise/fall times (i.e. trapezoidal pulse) and by rounding the corners (i.e. parabolic rise/fall pulse)
- Fourier decomposition of these waveforms shows faster convergence
 - Trapezoidal: n^{th} term $\propto 1/n^2$
 - Parabolic: n^{th} term $\propto 1/n^3$

PFN Design

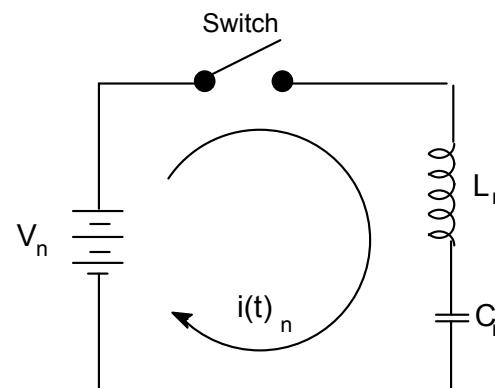
Instead of infinitely fast rise and fall-times, the desired pulse shape should have a reasonable (finite) rise and fall time. A design procedure then assume a repetition of pulses as shown below so that a Fourier analysis can be performed.



The Fourier expansion of the current required to generate a waveform of this shape into a constant resistive load consists of only sine terms. Each sinusoid in the series:

$$i(t) = I_{pk} \sum_{n=1,3,5,\dots}^{\infty} b_n \sin \frac{n\pi t}{\tau}$$

may be produced by the adjacent circuit:



$$i(t)_n = \frac{V_n}{\sqrt{\frac{L_n}{C_n}}} \sin\left(\frac{t}{\sqrt{L_n C_n}}\right)$$

$$i(t)_n = \frac{V_n}{Z_n} \sin \omega_o t$$

PFN Design

Comparing the amplitude and frequency terms for the Fourier coefficients and the LC loop:

$$I_{pk} b_n \sin \frac{n\pi t}{\tau} = \frac{V_n}{\sqrt{\frac{L_n}{C_n}}} \sin \left(\frac{t}{\sqrt{L_n C_n}} \right)$$

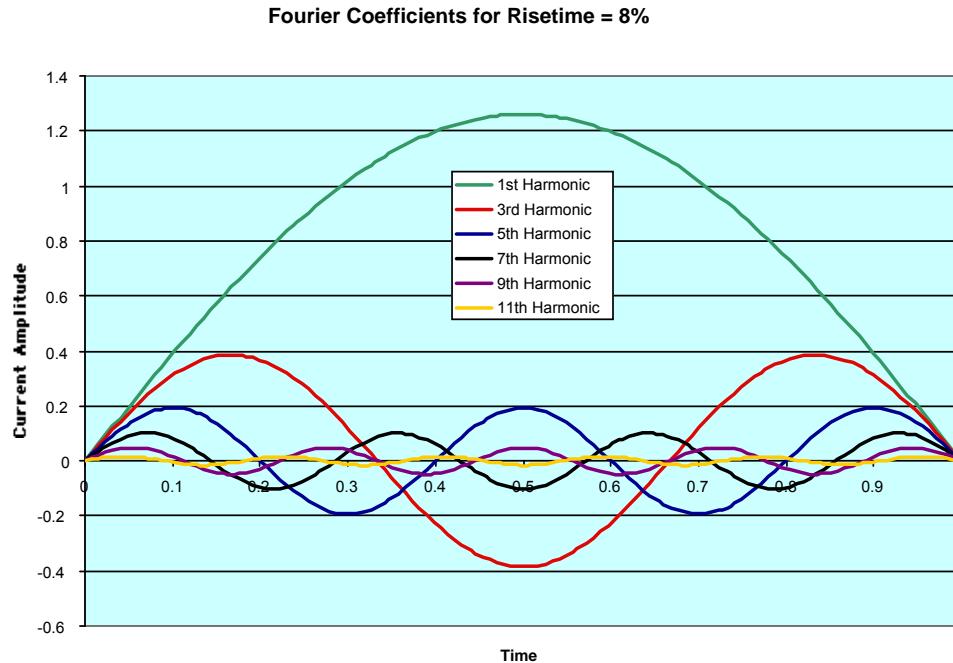
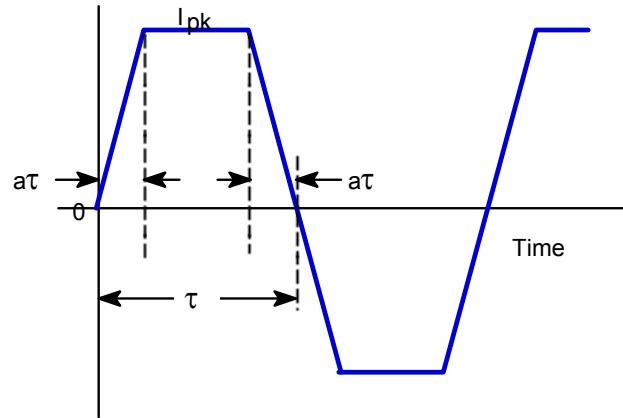
$$I_{pk} b_n = \frac{V_n}{\sqrt{\frac{L_n}{C_n}}} \text{ and } \frac{n\pi}{\tau} = \frac{1}{\sqrt{L_n C_n}}$$

Solving for L_n and C_n :

$$L_n = \frac{Z_n \tau}{n\pi b_n} \text{ where } Z_n = \frac{V_n}{I_{pk}}$$

$$C_n = \frac{\pi b_n}{n\pi Z_n}$$

Fourier Coefficients for Trapezoidal Waveform



For the trapezoidal waveform shown above the series expansion is:

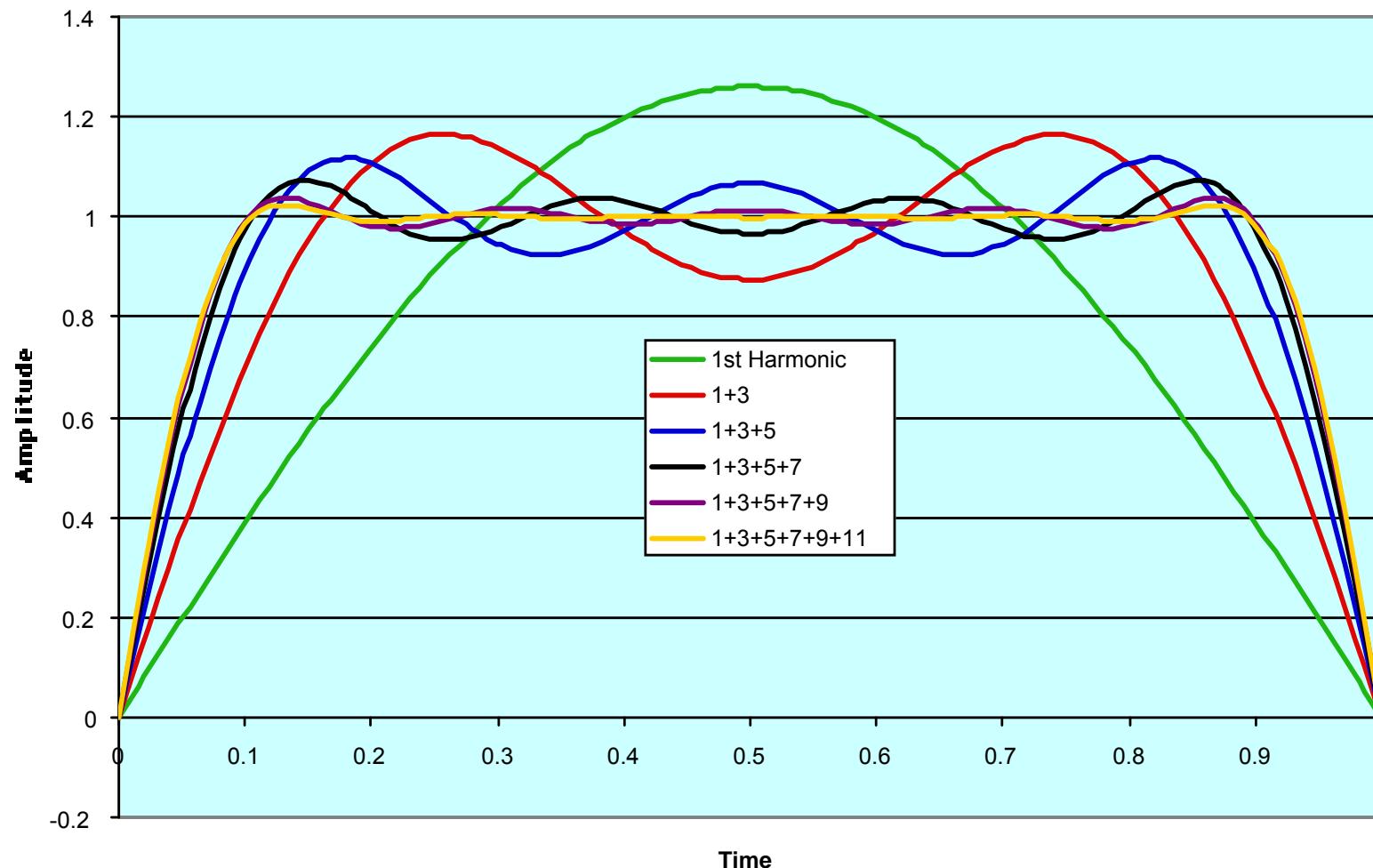
$$i(t) = I_{pk} \sum_{n=1,3,5,\dots}^{\infty} b_n \sin \frac{n\pi t}{\tau}$$

$$b_n = \frac{4}{n\pi} \frac{\sin n\pi a}{n\pi a}$$

where $n = 1,3,5,\dots$ And $a = \text{risetime as \% of pulselength } \tau$

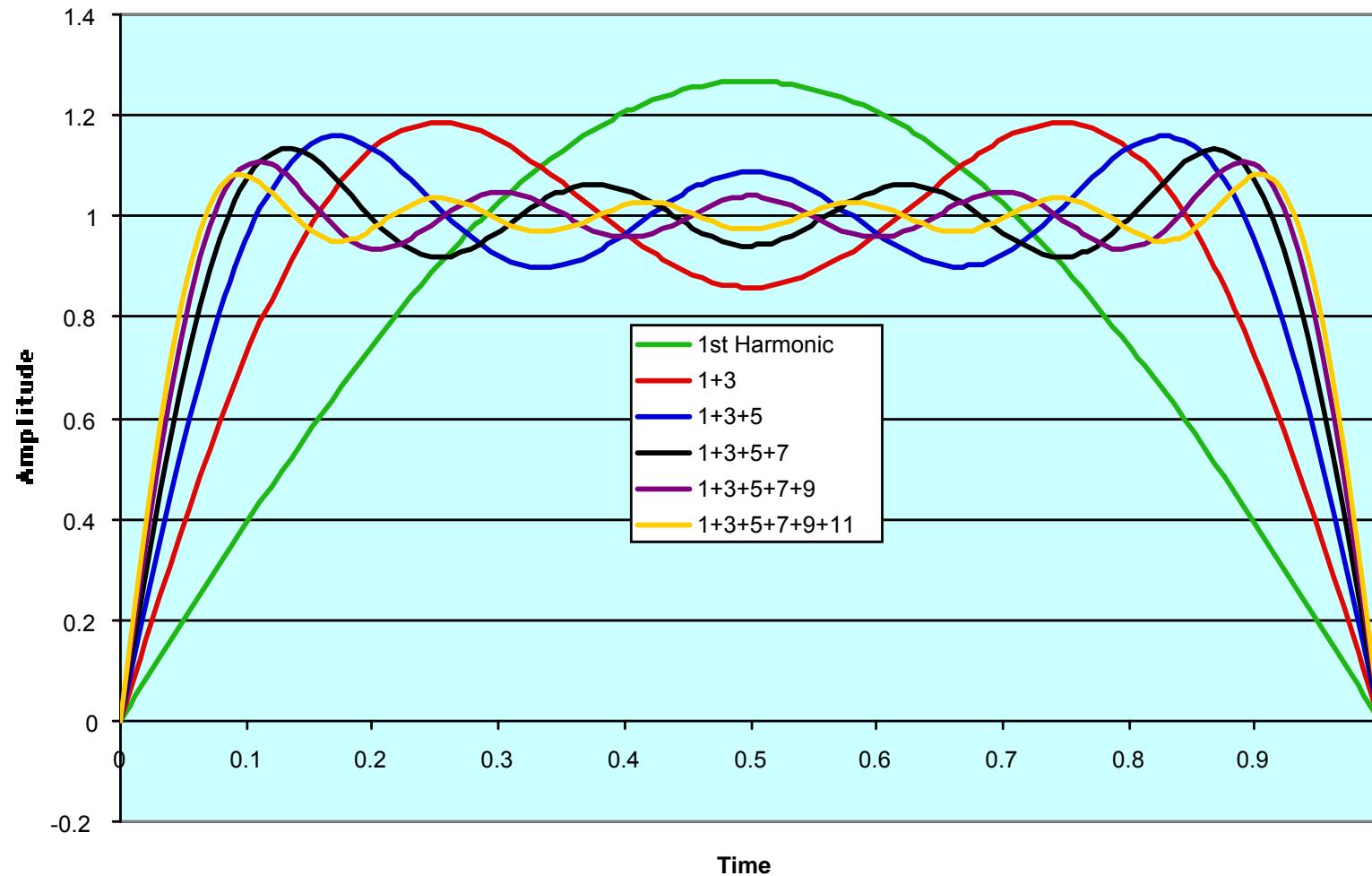
Example

Sum 1-6 Fourier Coefficients - for 8% Risetime

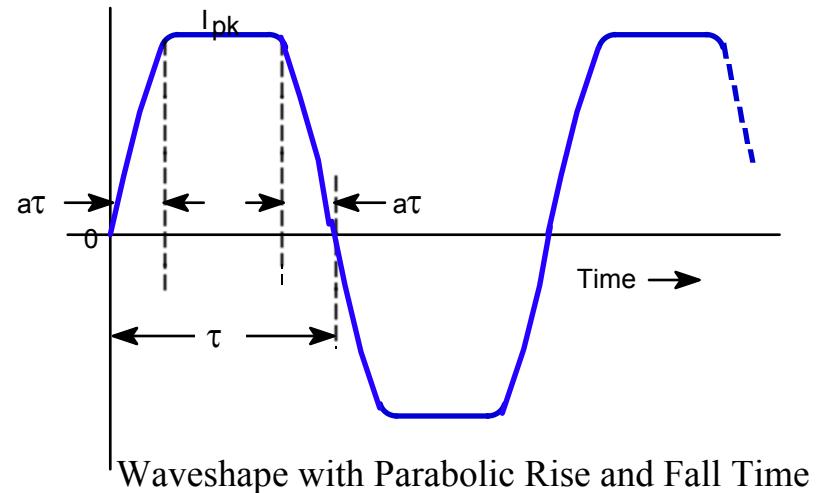
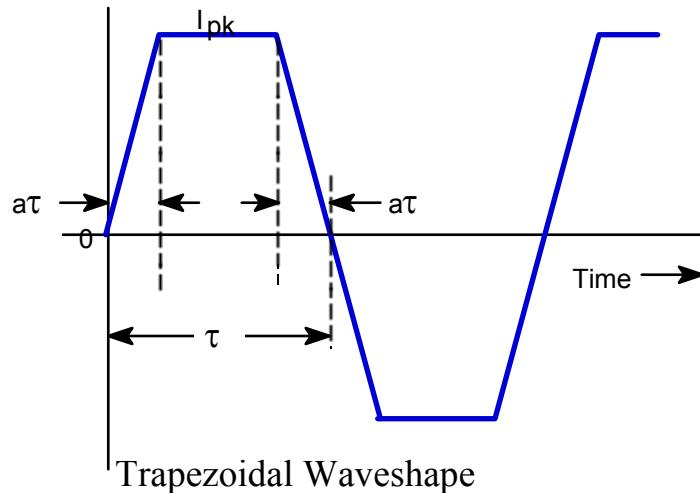


Example -

Sum 1-6 Fourier Coefficients - for 5% Risetime

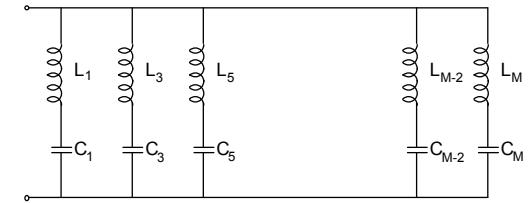


Trapezoidal and Parabolic Waveshapes



Values of b_n , L_n , and C_n for this circuit topology;

Waveform	b_n	L_n	C_n
Rectangular	$\frac{4}{n\pi}$	$\frac{Z_N \tau}{4}$	$\frac{4\tau}{n^2 \pi^2 Z_N}$
Trapezoidal	$\frac{4}{n\pi} \left(\frac{\sin n\pi a}{n\pi a} \right)$	$\frac{Z_N t}{4 \left(\frac{\sin n\pi a}{n\pi a} \right)}$	$\frac{4\tau}{n^2 p^2 Z_N} \left(\frac{\sin n\pi a}{n\pi a} \right)$
Flat top and parabolic rise and fall	$\frac{4}{n\pi} \left(\frac{\sin \frac{1}{2} n\pi a}{\frac{1}{2} n\pi a} \right)^2$	$\frac{Z_N \tau}{4 \left(\frac{\sin \frac{1}{2} n\pi a}{\frac{1}{2} n\pi a} \right)^2}$	$\frac{4\tau}{n^2 \pi^2 Z_N} \left(\frac{\sin \frac{1}{2} n\pi a}{\frac{1}{2} n\pi a} \right)^2$





Fourier Coefficients for Other Waveshapes

Values of Inductances and Capacitances for Five-Section Pulse-Forming Network

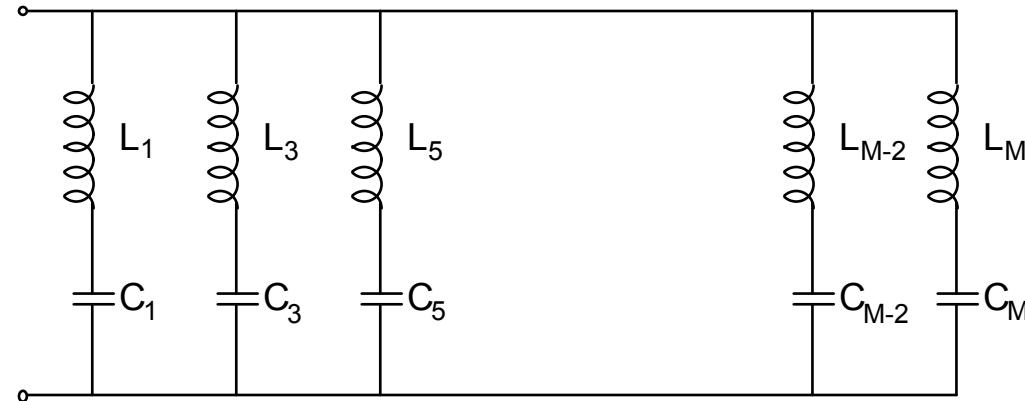
Waveform	a	Fourier coefficients					Inductance					Capacitance				
		b_1	b_3	b_5	b_7	b_9	L_1	L_3	L_5	L_7	L_9	C_1	C_3	C_5	C_7	C_9
Rectangular	0	1.2732	0.4244	0.2547	0.1819	0.1415	0.2500	0.2500	0.2500	0.2500	0.2500	0.4053	0.04503	0.01621	0.00827	0.00500
Trapezoidal	0.05	1.2679	0.4089	0.2293	0.1474	0.0988	0.2510	0.2595	0.2777	0.3578	0.3578	0.4036	0.04318	0.01459	0.00670	0.00349
Trapezoidal	0.08	1.2601	0.3854	0.1927	0.1015	0.0482	0.2526	0.2753	0.3303	0.4478	0.7340	0.4011	0.04089	0.01227	0.00462	0.00170
Trapezoidal	0.10	1.2524	0.3643	0.1621	0.0669	0.0155	0.2542	0.2912	0.3927	0.6796	2.2875	0.3987	0.03865	0.01032	0.00304	0.00055
Trapezoidal	0.20	1.1911	0.2141	0	-0.0393	-0.0147	0.2672	0.4455	∞	-1.1561	-2.4052	0.3791	0.02272	0	0.00179	-0.00052
Parabolic rise	0.05	1.2699	0.4166	0.2418	0.1640	0.1194	0.2507	0.2547	0.2632	0.2773	0.2961	0.4042	0.04420	0.01539	0.00745	0.00422
Parabolic rise	0.10	1.2627	0.3939	0.2064	0.1194	0.0691	0.2521	0.2694	0.3084	0.3808	0.5122	0.4019	0.04179	0.01314	0.00543	0.00244
Parabolic rise	0.20	1.2319	0.3127	0.1032	0.0246	0.0017	0.2584	0.3393	0.6168	1.8472	20.94	0.3921	0.03318	0.00657	0.00112	0.00006
Parabolic rise	0.25	1.2092	0.2610	0.0564	0.00353	0.0017	0.2632	0.4065	1.1292	12.887	21.37	0.3849	0.32769	0.00359	0.00016	0.00006
Parabolic rise	0.33	1.1609	0.1720	0.00930	0.00338	0.0064	0.2742	0.6168	6.8493	13.44	5.5556	0.3695	0.01825	0.00059	0.00015	0.00023
Parabolic rise	0.40	1.1142	0.1080	0	0.0085	0.0015	0.2857	0.9821	∞	5.346	23.15	0.3547	0.01146	0	0.00039	0.00005
Parabolic rise	0.50	1.0319	0.0382	0.00825	0.00300	0.0014	0.3085	2.7747	7.7160	15.15	25.00	0.3285	0.00406	0.00053	0.00014	0.00005

Multiply the inductances by $Z_N \tau$ and the capacitances by τ/Z_N . The inductances are given in henrys and the capacitances in farads if the pulse duration is expressed in seconds and the network impedance is in ohms. a is fractional risetime of pulse.

$$i(t) = \frac{V_N}{Z_N} \sum_{n=1,3,5,\dots}^{\infty} b_n \sin \frac{n\pi t}{\tau} = I_{pk} \sum_{n=1,3,5,\dots}^{\infty} b_n \sin \frac{n\pi t}{\tau}$$

PFN Design

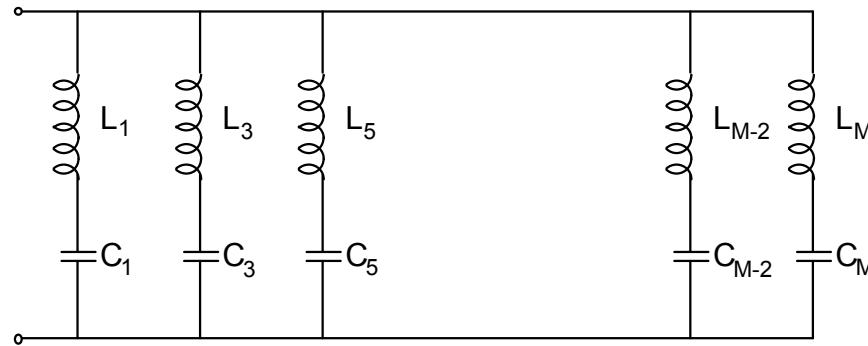
The network required to produce all the Fourier components and therefore reproduce the pulse waveshape is:



The problem with this circuit network is that physical inductors have stray shunt capacitance which distort the waveshape and the required values of capacitors have a wide range of values which increases costs.

The mathematical approach is to use the impedance function of the above circuit and derive networks having other topologies but the same impedance function. Six basic circuit topologies have been derived (including the above circuit).

Synthesis of Alternate LC Networks



The admittance function for the above circuit has the form:

$$Y(s) = \frac{C_1 s}{L_1 C_1 s^2 + 1} + \frac{C_3 s}{L_3 C_3 s^2 + 1} + \dots$$

$$Z(s) = \frac{1}{Y(s)}$$

$Z(s)$ in turn can be expanded about its poles to yield equivalent networks have other circuit topologies

Equivalent Guillemin Networks

- Type A:
 - Capacitances vary
 - Little used
- Type B:
 - Capacitances vary
 - Similar in layout to Type E
- Type C:
 - Capacitances vary
 - Straightforward design
 - Often used (SLAC 6575)
- Type D
 - Fixed capacitance
 - Negative inductances
 - Basis for Type E
- Type F
 - Capacitances vary

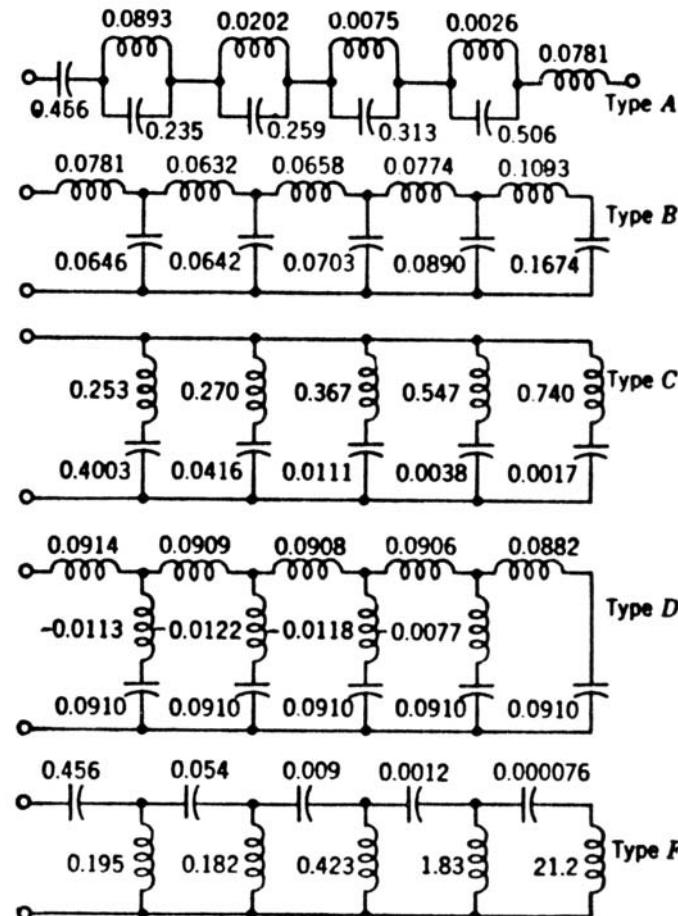
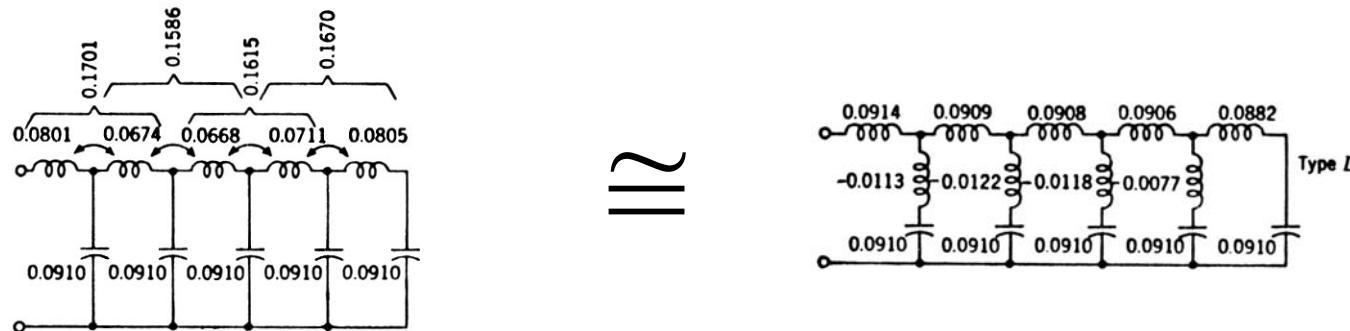


FIG. 6-22.—Equivalent forms for five-section Guillemin voltage-fed network. Multiply the values of the inductances by Z_{NT} and the values of the capacitances by τ/Z_N . The inductances are in henrys and the capacitances in farads if pulse duration τ is expressed in seconds and network impedance Z_N in ohms.

PFN - Type E



The negative inductance that are seen in the Type D PFN represent the mutual inductance between adjacent inductors and may be realized in physical form by winding coils on a single tubular form (solenoid) and attaching the capacitors to the inductor at appropriate points on the inductor.

The quality of the output pulse is dependent on the number of sections used. For a waveform having a desired risetime/falltime of $\sim 8\%$ of the total pulselength, five sections (each consisting of one inductor and one capacitor) prove to be adequate to produce the desired waveshape. A sixth section provided only slight improvement. This corresponds to the relative magnitude of the Fourier-series components for the corresponding steady-state alternating current wave. The relative amplitude of the fifth to the first Fourier coefficient is $\sim 4\%$ while the sixth to the first is $\sim 2\%$.

Note: If faster risetimes/falltimes are required, the number of sections needed to satisfy that risetime increases.

Type E PFN- Practical Design Parameters

- For :
 - PFN Characteristic Impedance = Z_o
 - PFN Output Pulse Width = 2τ
 - where L_N = total PFN inductance and C_N = total PFN capacitance
 - The total PFN inductance (including mutual inductances) and capacitance is divided equally between the number of sections.
 - Empirical data have shown that the best waveshape can be achieved when the end inductors should have ~20-30% more self inductance. The mutual inductance should be approximately 15% of the self inductances.
- Bottom line: don't bust your pick designing a “perfect” PFN
 - Capacitance values vary from can-to-can and with time
 - Inductor values are never quite as designed
 - Strays; inductance, capacitance, resistance, distort the waveform
 - and should you somehow overcome all of the foregoing, you can be certain that the technicians will “tune” the PFN and your “perfect” waveform will be but a memory

$$Z_o = \sqrt{\frac{L_N}{C_N}}$$

$$\tau = \sqrt{L_N C_N}$$

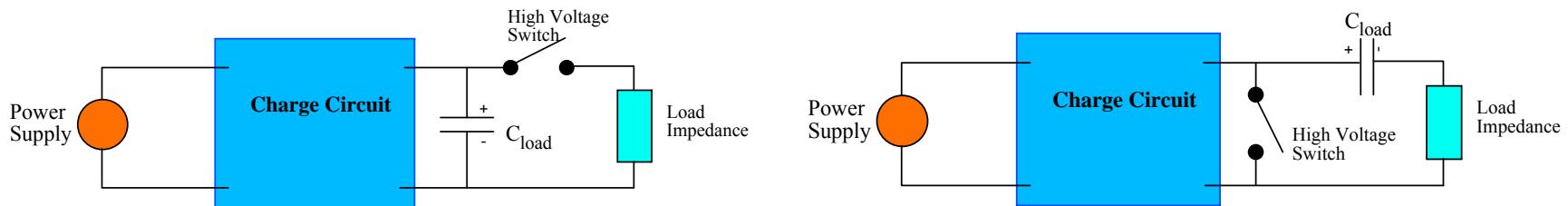
PFN Design for Time Varying Load

- Within a limited range, the impedance of individual PFN sections may be adjusted to match an impedance change in the load.
 - For example: Each section of a 5 section roughly drives 20% of the load pulse duration. If the load impedance is 10% lower for the first 20% of pulse, designing the first section of the PFN (section closest to the load) to be 10% lower than rest of the PFN will make a better match and generate a flatter pulse.
 - This approach works only if the load impedance is repeatable on a pulse-to-pulse basis

PFN: Practical Issues

- Switch: In addition to voltage and peak current requirements, must also be able to handle peak dI/dt (highest frequency components will be smaller magnitude and may be difficult to observe)
 - SLC modifications to 6575 doubled dI/dt
 - Even with 2 thyratrons, short tube life
 - Solved by adding “anode reactor” (magnetic switch in series with tube)
- Positive mismatch, $Z_{load} > Z_{PFN}$
 - “Prevents” voltage reversal (may still get transient reversals), improves lifetime
 - Switch
 - Capacitors
 - Cables
 - Incorporate End Of Line (EOL) clipper to absorb mismatch energy
- Inductors
 - Must not deform under magnetic forces
 - Tuneable
 - Movable tap point
 - Flux exclusion lug
- PFN impedance range is limited (just like PFLs), as is maximum switch voltage
 - Transformers can be used to match to klystron load
 - SLAC 6575 modulators are matched to 5045 klystrons with a 1:15 transformer

Charge Circuits - Basics



Where C_{load} represents the capacitance of a transmission line, PFN, energy storage for a hard-tube circuit, etc.

The charge circuit is the interface between the power source and the pulse generating circuit and may satisfy the following functions:

Ensures that C_{load} is charged to appropriate voltage within the allowable time period.
Provides isolation between the power source and the pulse circuit:

Limit the peak current from the source.

Prevent the HV switch from latching into an on state and shorting the power source.

Isolate the power source from voltage/current transients generated by pulse circuit.

Charging Power Supplies

Transformer - Rectifier Combinations



AC Power Rectification

Power & Power Factor in AC Systems

Power and Power Factor:

Definition of average Power:

$$P = \frac{1}{T_1} \cdot \int_0^{T_1} p(t) dt = \frac{1}{T_1} \cdot \int_0^{T_1} v_s(t) \cdot i_s(t) dt$$

using clean voltage and distorted current:

$$v_s(t) = \sqrt{2} V_s \sin \omega_1 t$$

M. Giesselmann, Sept-29-2000

$$i_s(t) = i_{s1}(t) + \sum_{(h \neq 1)} i_{sh}(t)$$

Definition of RMS values:

$$I_s = \sqrt{\frac{1}{T_1} \cdot \int_0^{T_1} i_s(t)^2 dt}$$

$$I_s = \sqrt{I_{s1}^2 + \sum_{(h \neq 1)} I_{sh}^2}$$

Definition of Current distortion:

$$I_{dis} = \sqrt{\sum_{(h \neq 1)} I_{sh}^2}$$

$$I_s^2 = I_{s1}^2 + I_{dis}^2$$

Example for sinusoidal current:

$$I_{rms} = \sqrt{\frac{1}{T_1} \cdot \int_0^{T_1} \left(I_m \sin\left(\frac{2\pi}{T_1} t\right) \right)^2 dt}$$

$$I_{rms} = \frac{1}{2} \cdot \sqrt{2} \cdot I_m$$

Definition of total harmonic distortion:

$$THD = \frac{I_{dis}}{I_{s1}}$$



AC Power Rectification

Definition of Power Factor with Distorted Current Waveforms

Calculation of real power:

$$P = \frac{1}{T_1} \cdot \int_0^{T_1} \sqrt{2} \cdot V_s \cdot \sin(\omega_1 \cdot t) \cdot \sqrt{2} \cdot I_{s1} \cdot \sin(\omega_1 \cdot t - \phi_1) dt$$

$$P = \frac{1}{T_1} \cdot \int_0^{T_1} \sqrt{2} \cdot V_s \cdot \sin\left(\frac{2\pi}{T_1} \cdot t\right) \cdot \sqrt{2} \cdot I_{s1} \cdot \sin\left(\frac{2\pi}{T_1} \cdot t - \phi_1\right) dt$$

$$P = \frac{1}{T_1} \cdot \left[\frac{T_1}{4} \cdot (\sin(\phi_1) + 4 \cdot \cos(\phi_1) \cdot \pi) \cdot V_s \cdot \frac{I_{s1}}{\pi} - \frac{T_1}{4} \cdot \sin(\phi_1) \cdot V_s \cdot \frac{I_{s1}}{\pi} \right]$$

$$P = V_s \cdot I_{s1} \cdot \cos(\phi_1)$$

Definition of apparent Power and Power Factor:

$$S = V_s \cdot I_s$$

$$PF = \frac{P}{S}$$

Power Factor with distorted Current:

$$PF = \frac{V_s \cdot I_{s1} \cdot \cos(\phi_1)}{V_s \cdot I_s}$$

$$PF = \frac{I_{s1}}{I_s} \cdot \cos(\phi_1)$$

$$PF = \frac{I_{s1}}{I_s} \cdot DPF \quad DPF = \text{Displacement Power Factor}$$

Now in terms of THD:

$$THD = \frac{I_{dis}}{I_{s1}} \quad I_{s1} = \frac{I_{dis}}{THD} \quad \frac{I_{s1}}{I_s} = \frac{I_{dis}}{\frac{THD}{I_s}} \quad I_s^2 = I_{s1}^2 + I_{dis}^2$$

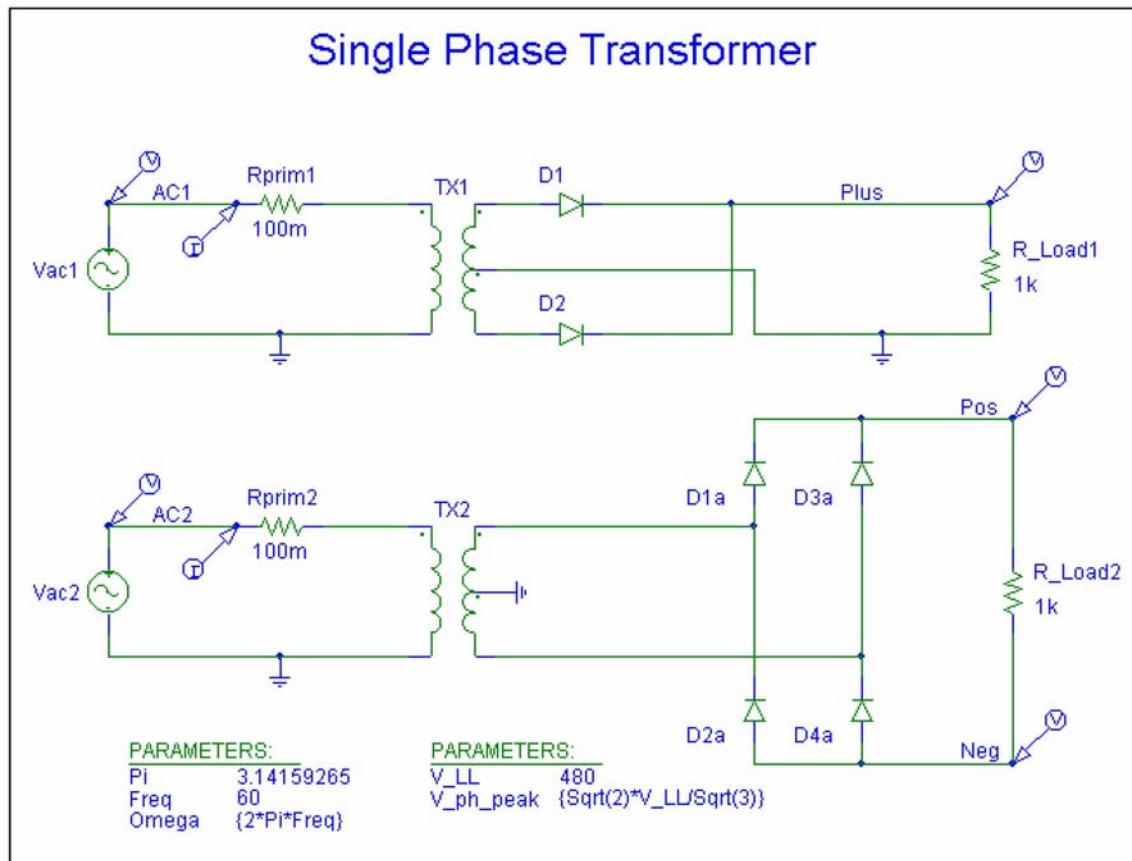


$$\frac{I_{s1}}{I_s} = \frac{\frac{I_{dis}}{THD}}{\sqrt{I_{s1}^2 + I_{dis}^2}}$$

$$\frac{\frac{I_{dis}}{THD}}{\sqrt{I_{s1}^2 + I_{dis}^2}} = \sqrt{\frac{\frac{I_{dis}^2}{THD^2}}{I_{s1}^2 + I_{dis}^2}} = \sqrt{\frac{\frac{1}{THD^2}}{\frac{I_{s1}^2}{I_{dis}^2} + 1}} = \sqrt{\frac{\frac{1}{THD^2}}{\frac{1}{THD^2} + 1}} = \sqrt{\frac{1}{(1 + THD^2)}} = \frac{1}{\sqrt{1 + THD^2}}$$

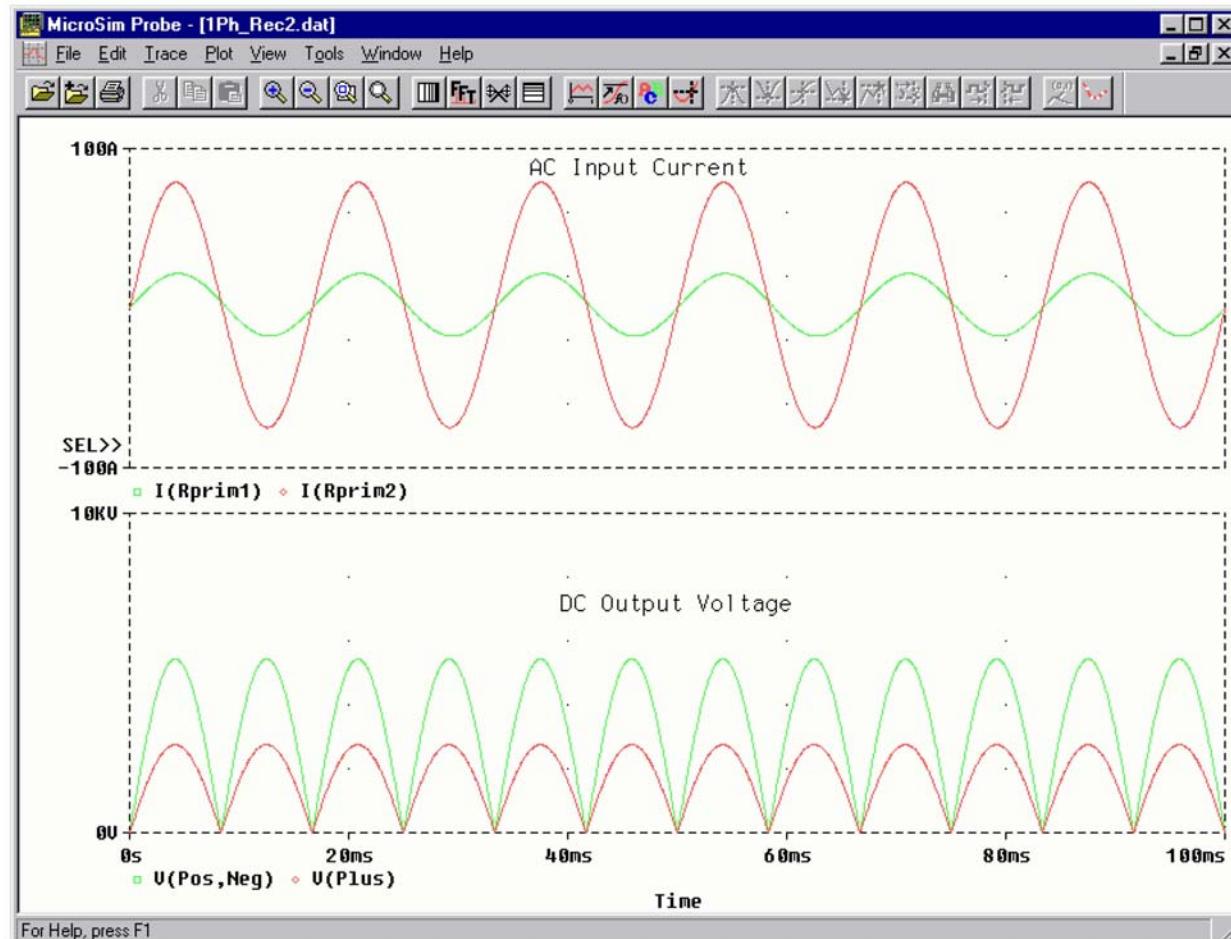
AC Power Rectification

Single Phase Transformer-Rectifier System



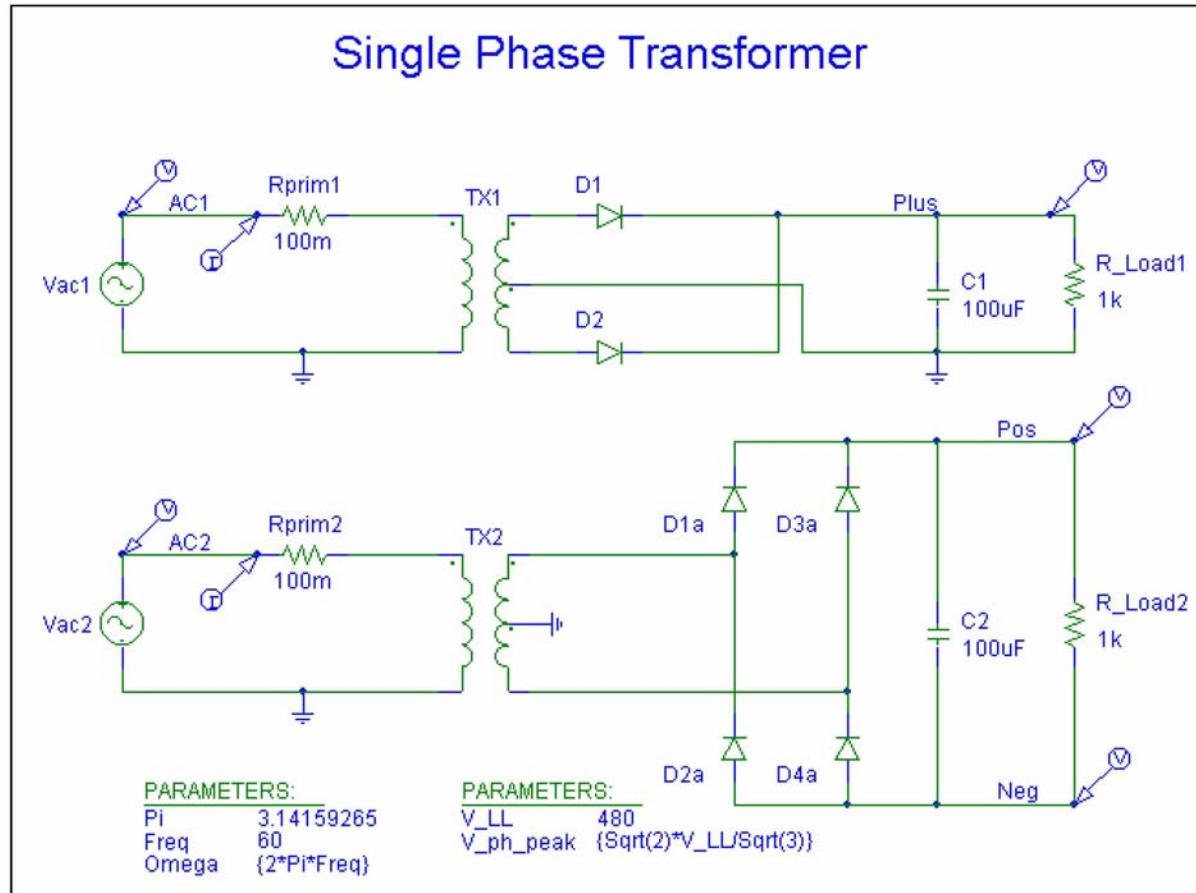
AC Power Rectification

Single Phase Transformer-Rectifier Performance



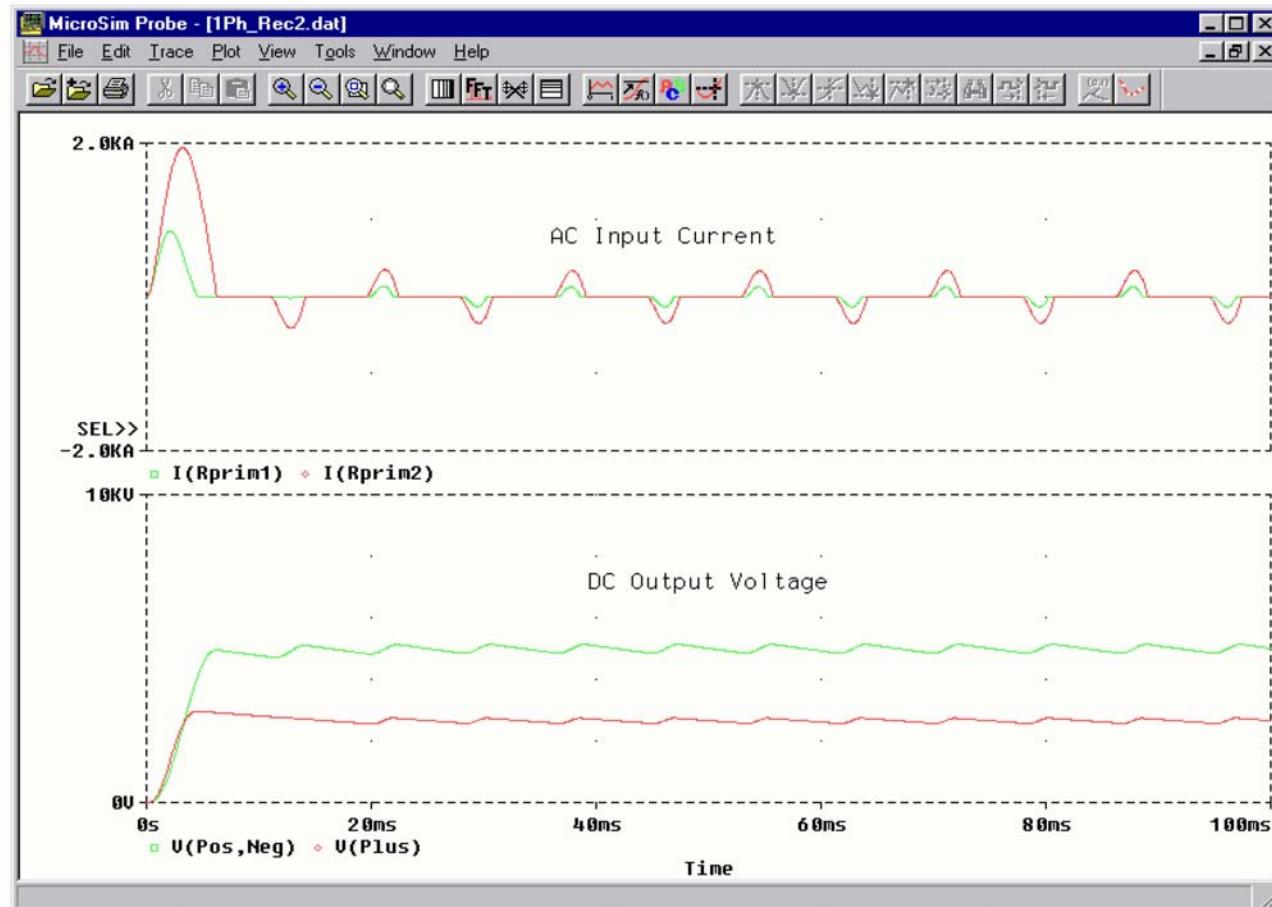
AC Power Rectification

Single Phase Transformer-Rectifier System



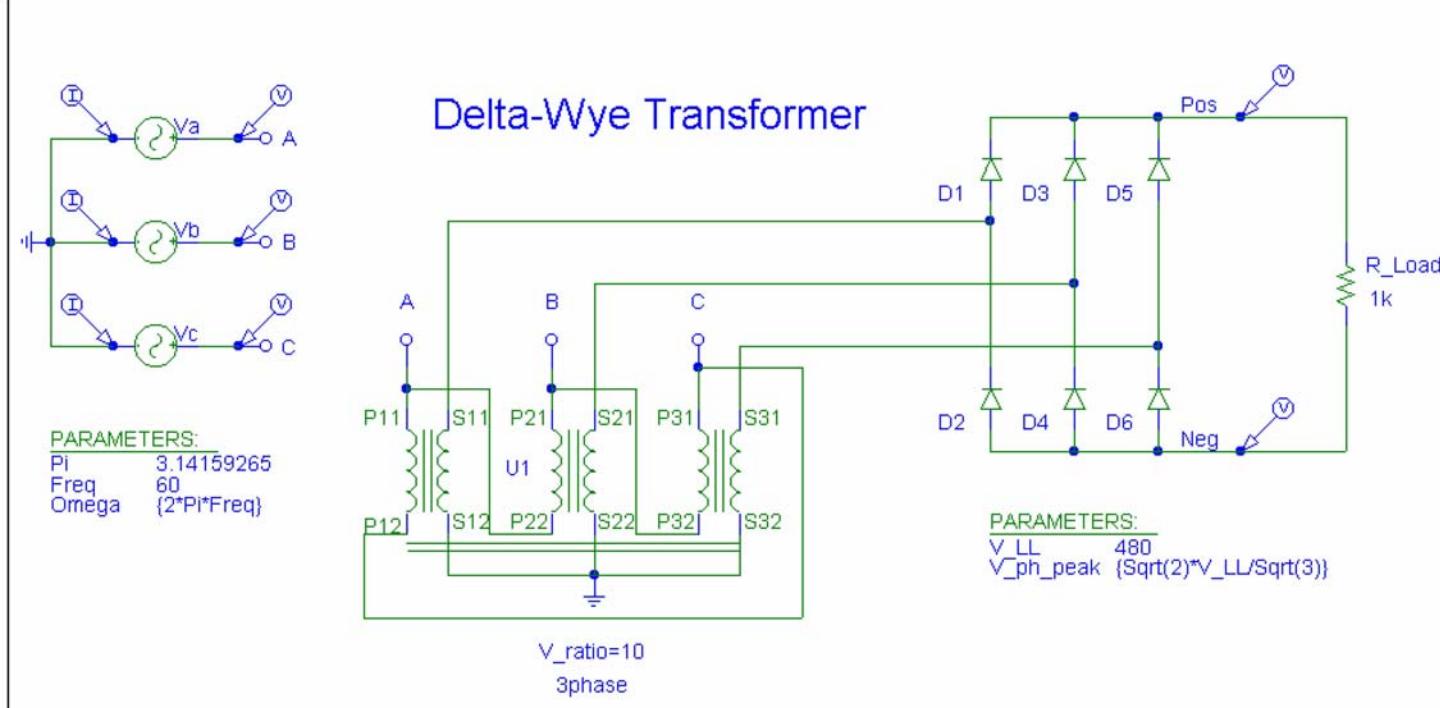
AC Power Rectification

Single Phase Transformer-Rectifier Performance



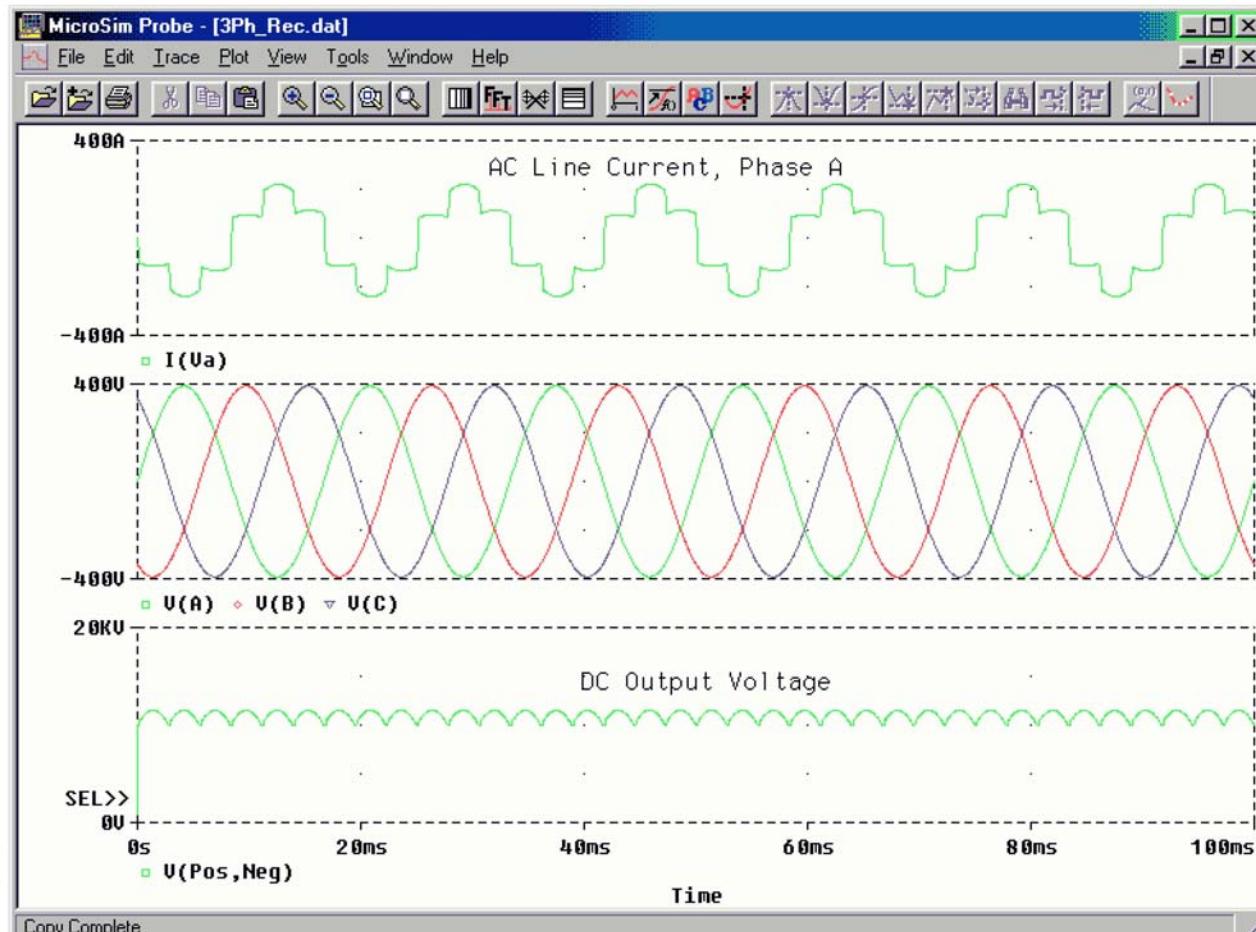
AC Power Rectification

Δ-Y Transformer-Rectifier System



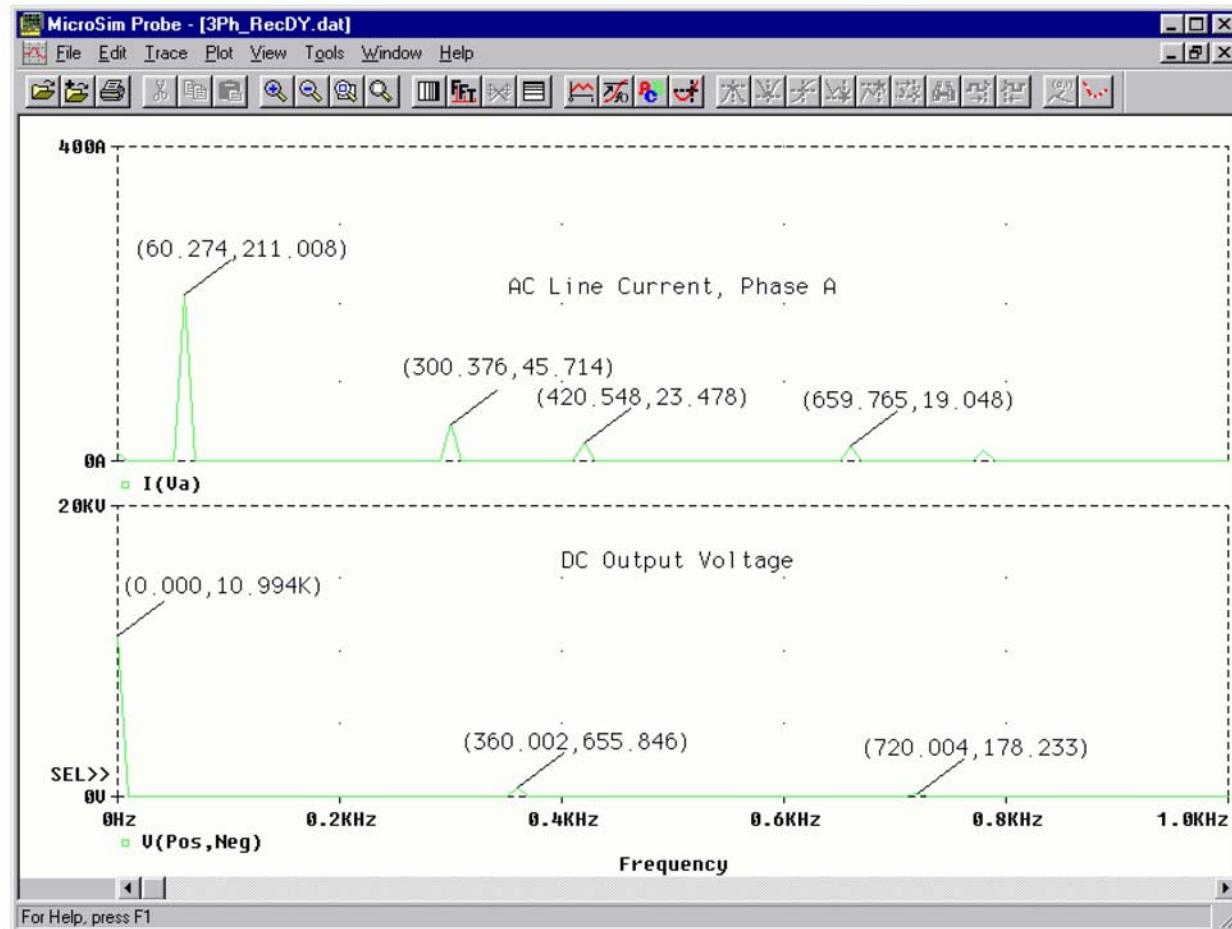
AC Power Rectification

Performance of Δ -Y Transformer-Rectifier



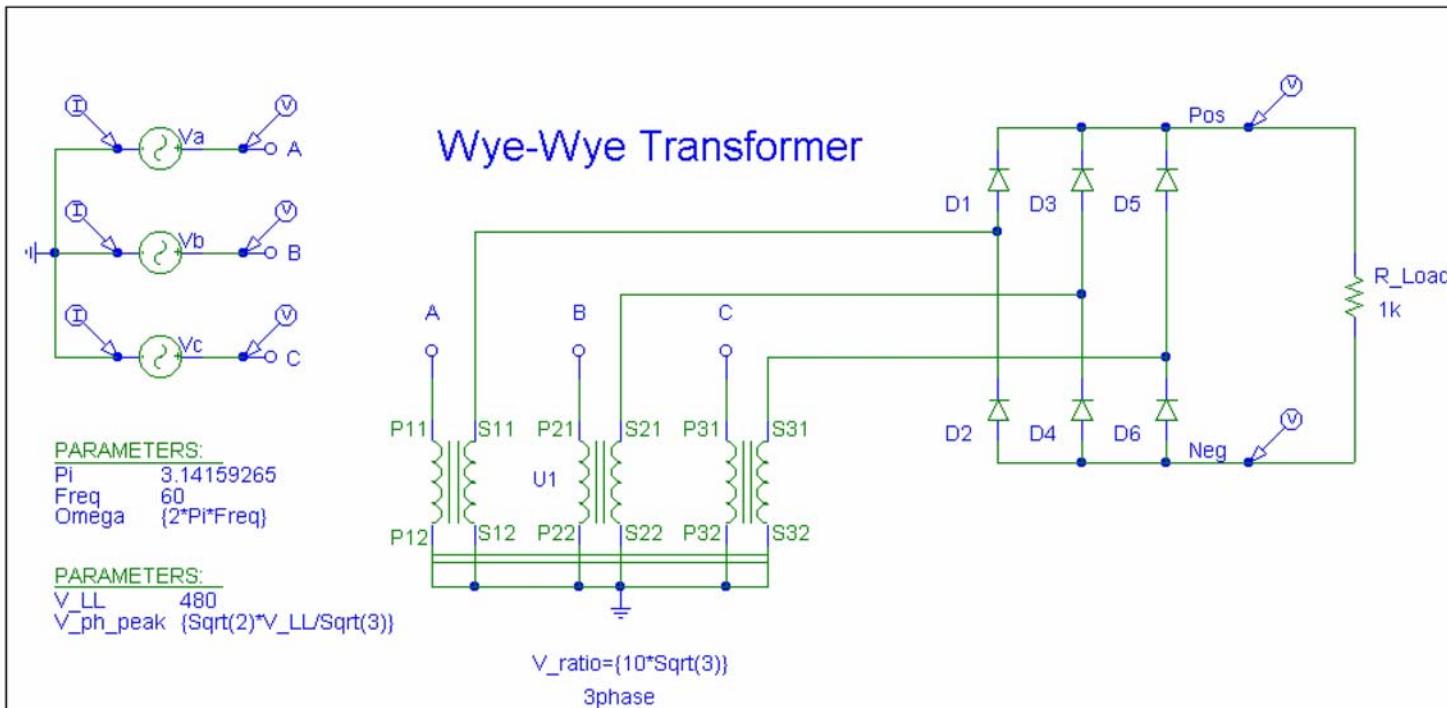
AC Power Rectification

Performance of Δ -Y Transformer-Rectifier



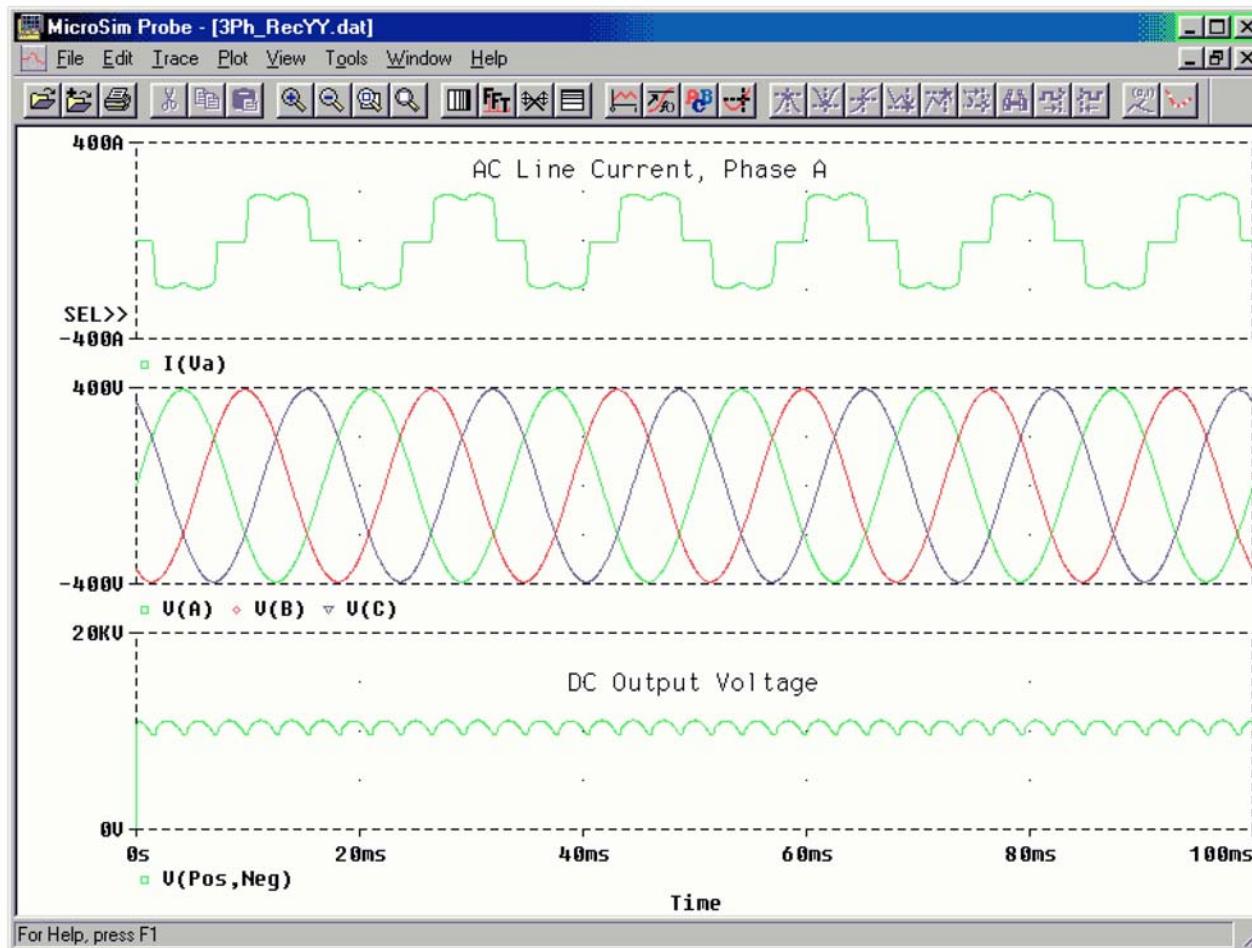
AC Power Rectification

Y-Y Transformer-Rectifier System



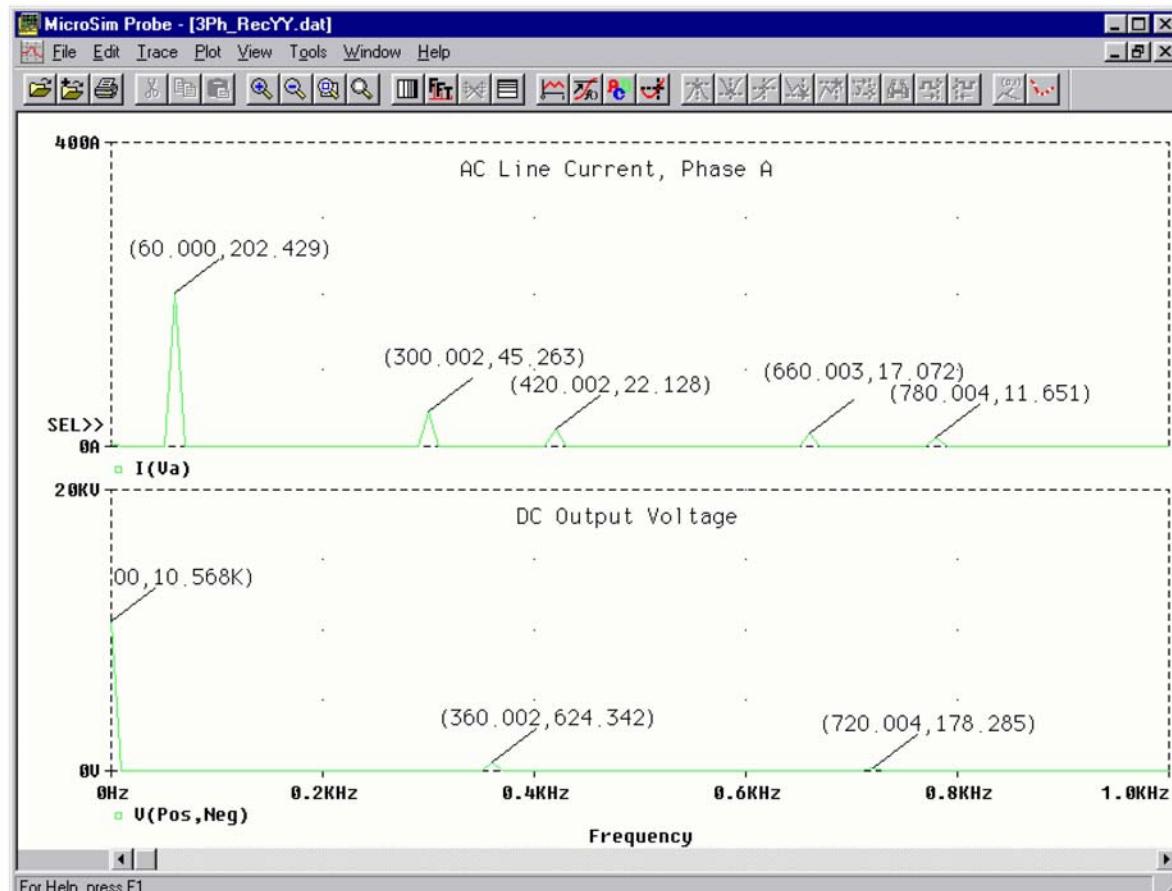
AC Power Rectification

Performance of Y-Y Transformer-Rectifier



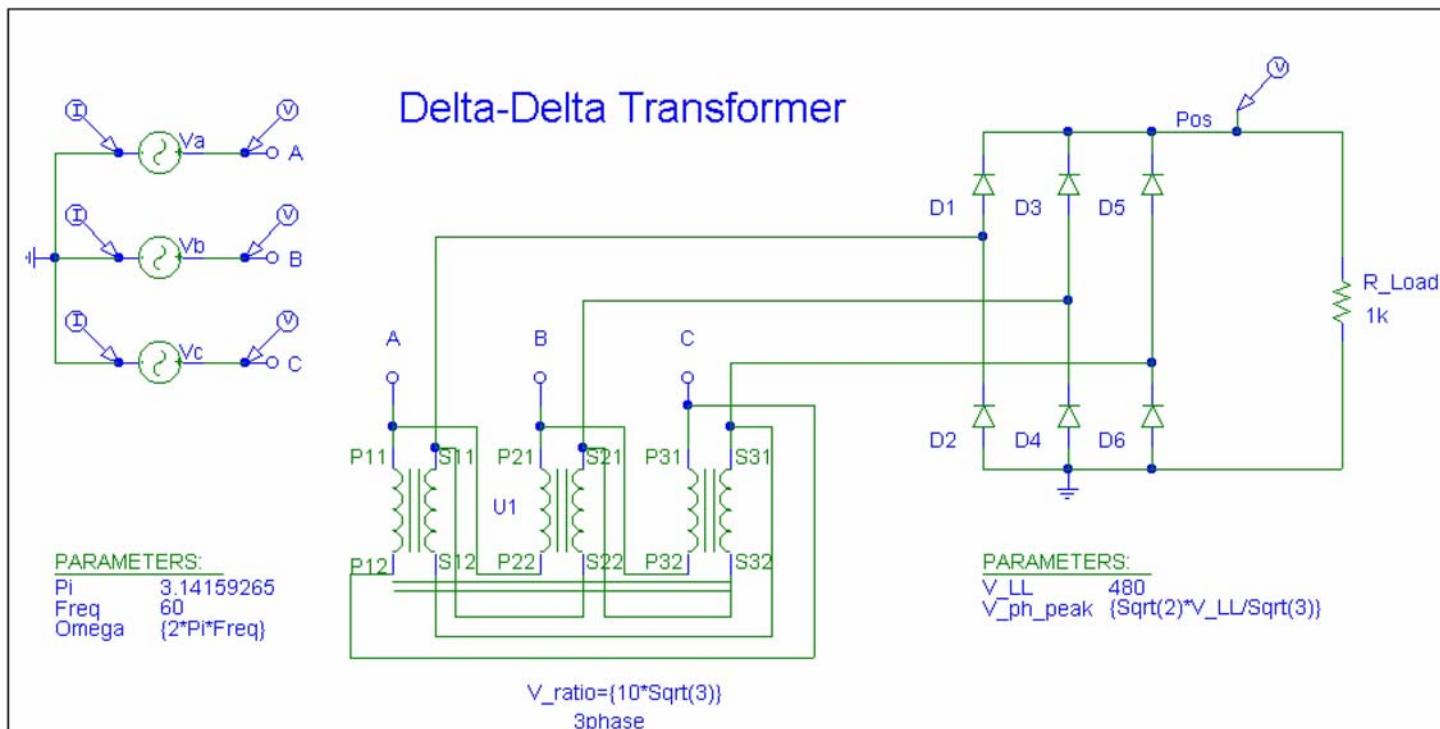
AC Power Rectification

Performance of Y-Y Transformer-Rectifier



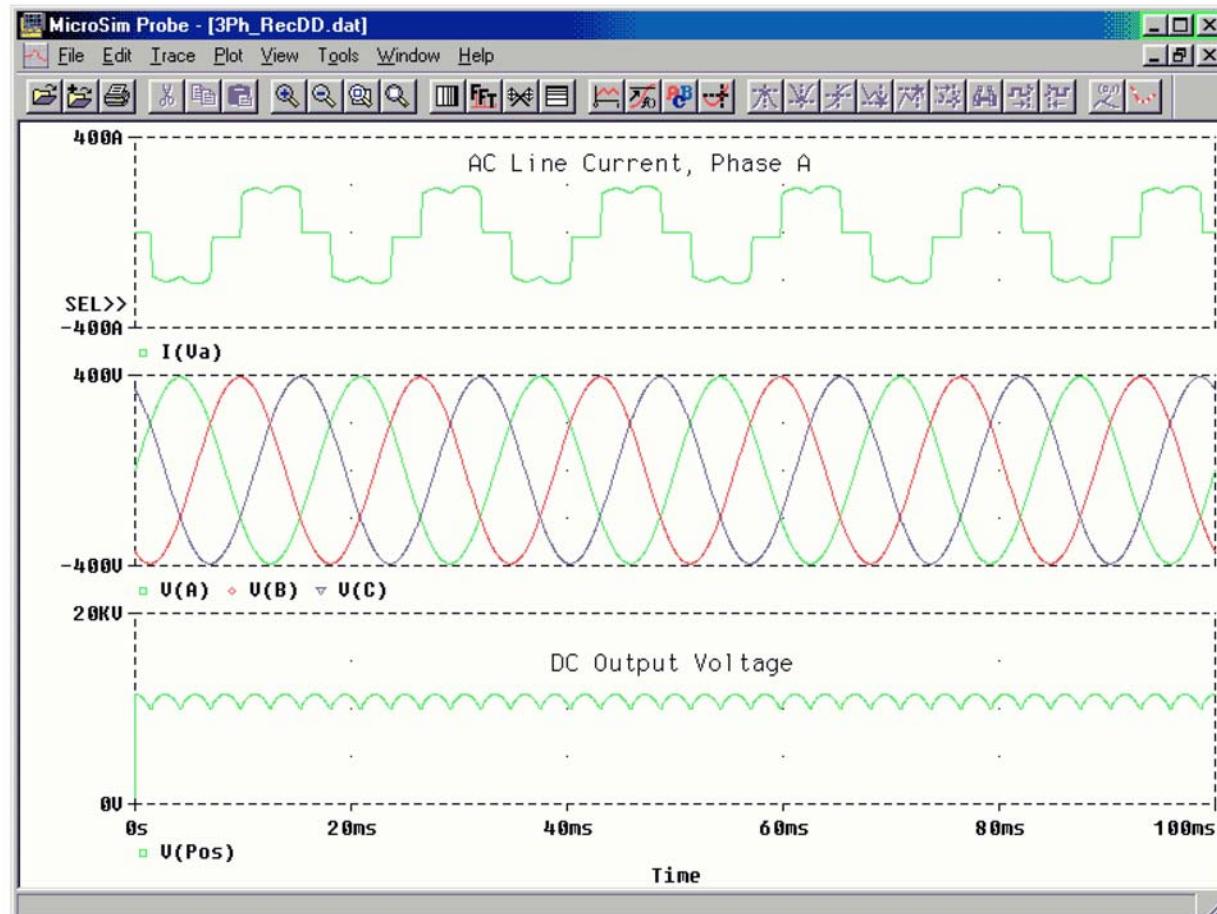
AC Power Rectification

$\Delta\text{-}\Delta$ Transformer-Rectifier System



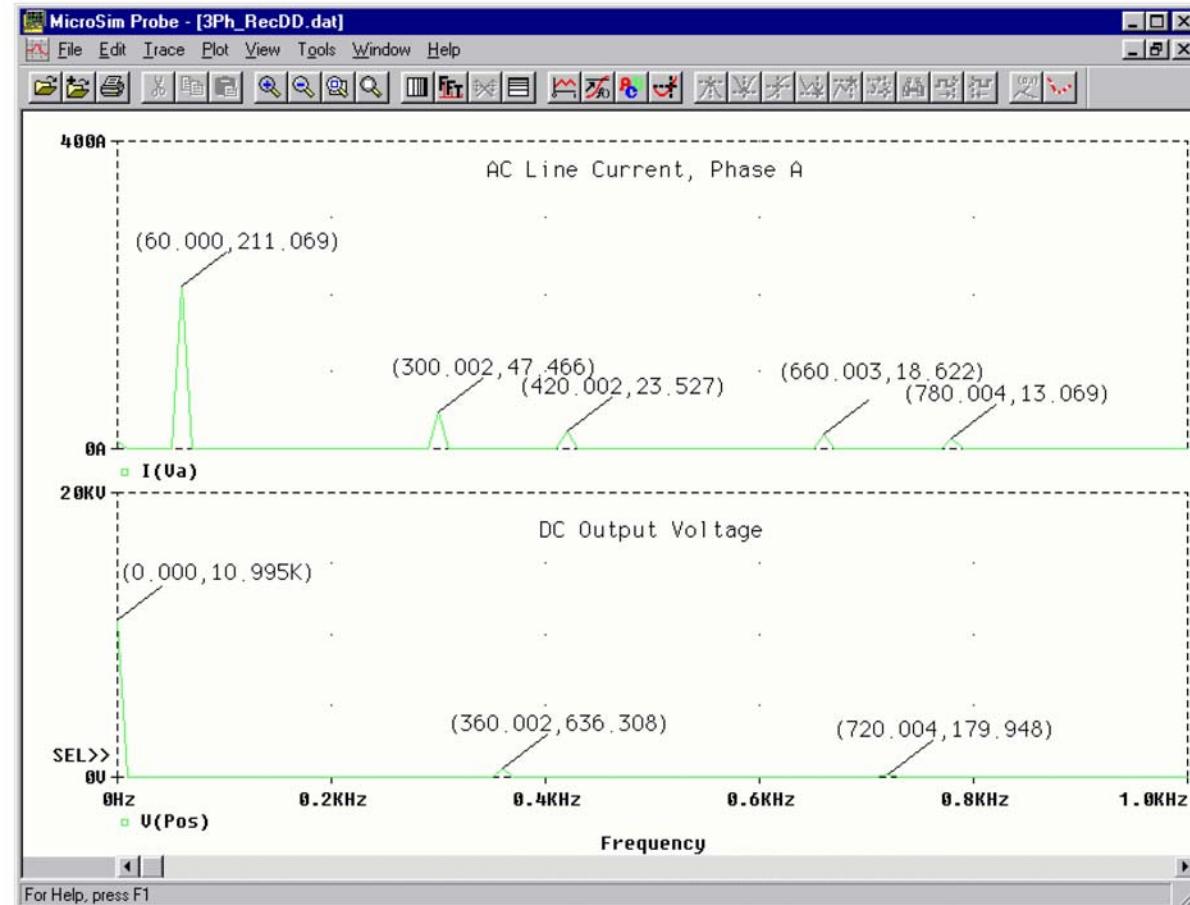
AC Power Rectification

Performance of $\Delta\text{-}\Delta$ Transformer-Rectifier



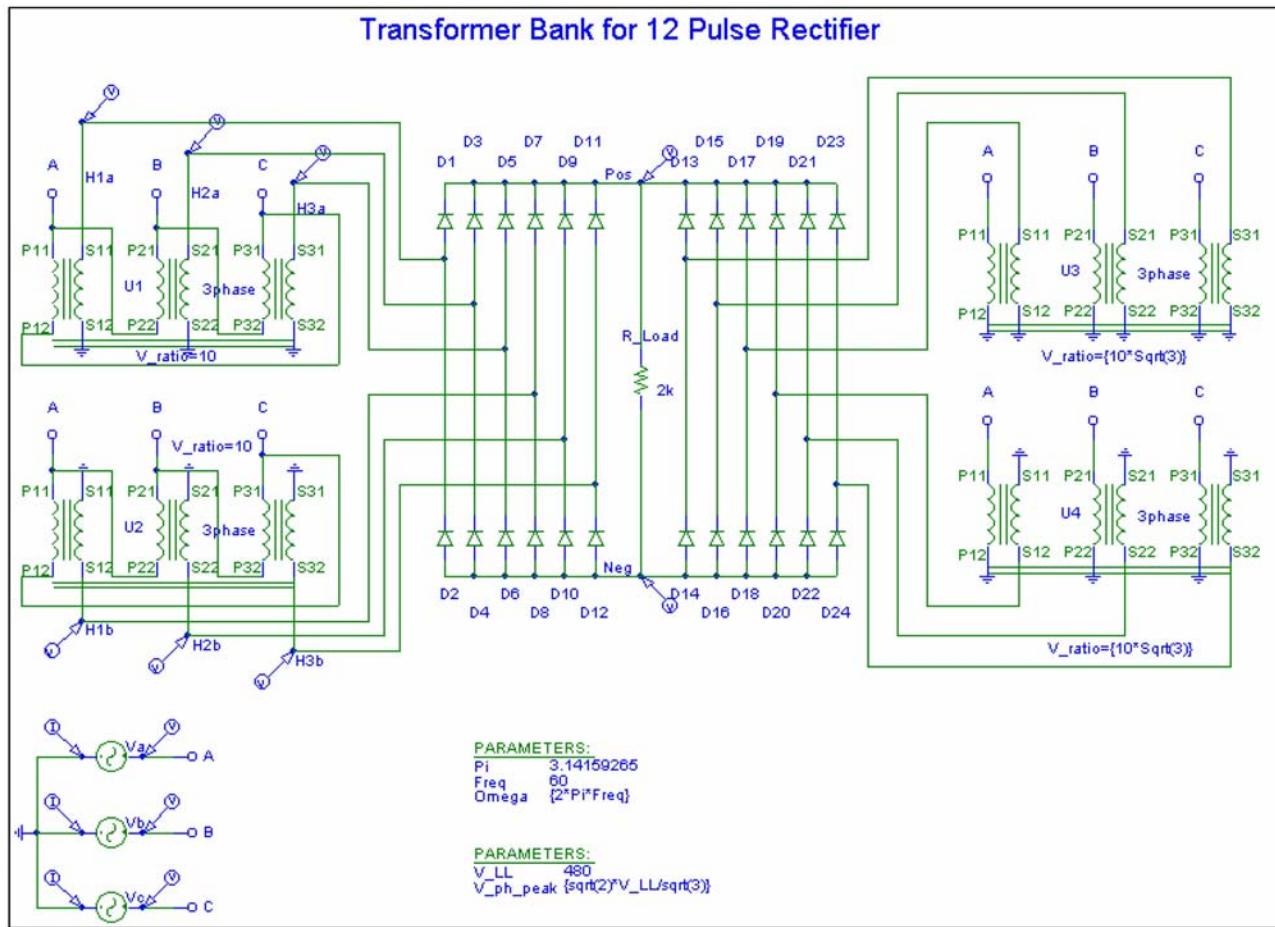
AC Power Rectification

Performance of $\Delta\text{-}\Delta$ Transformer-Rectifier



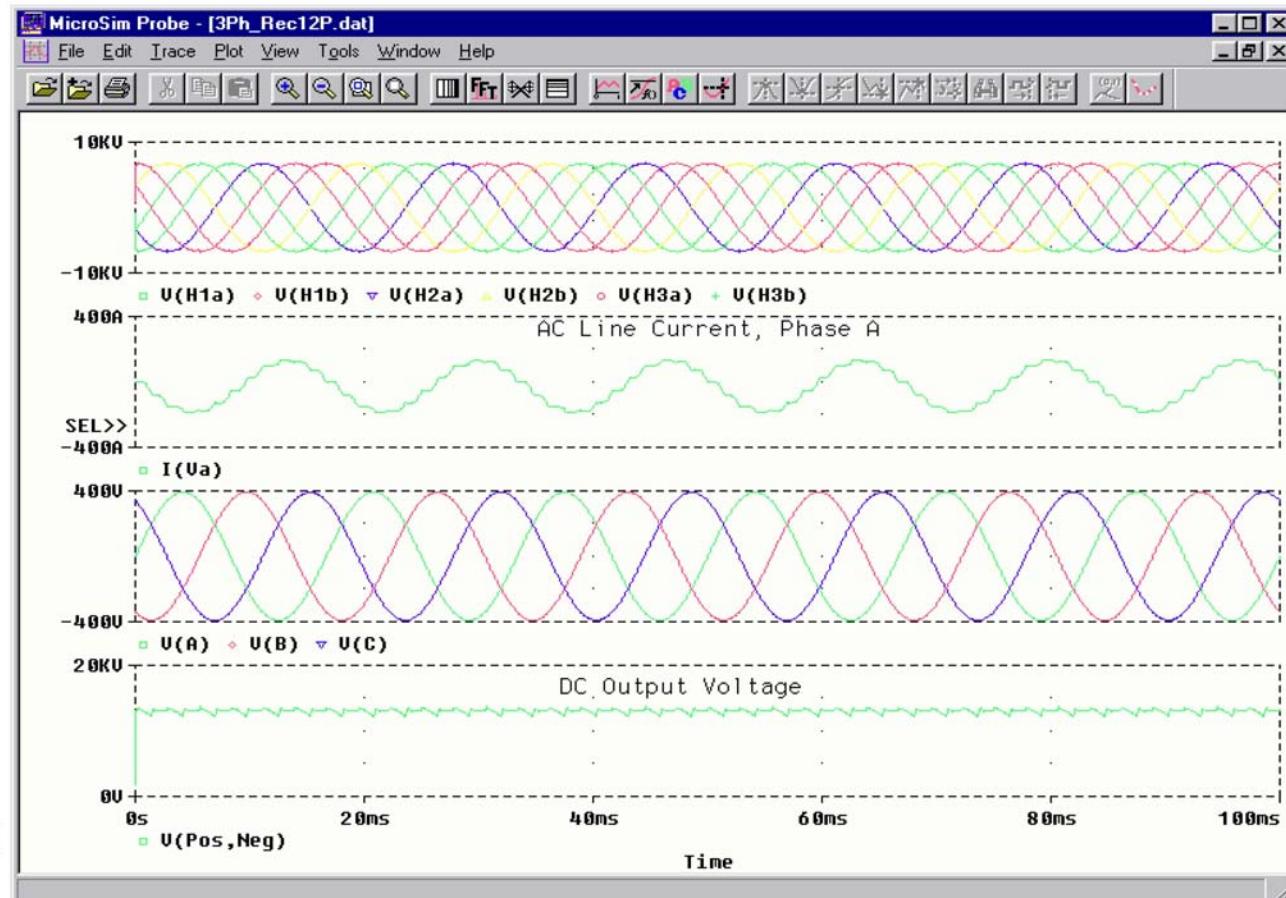
AC Power Rectification

12 Pulse Transformer-Rectifier System



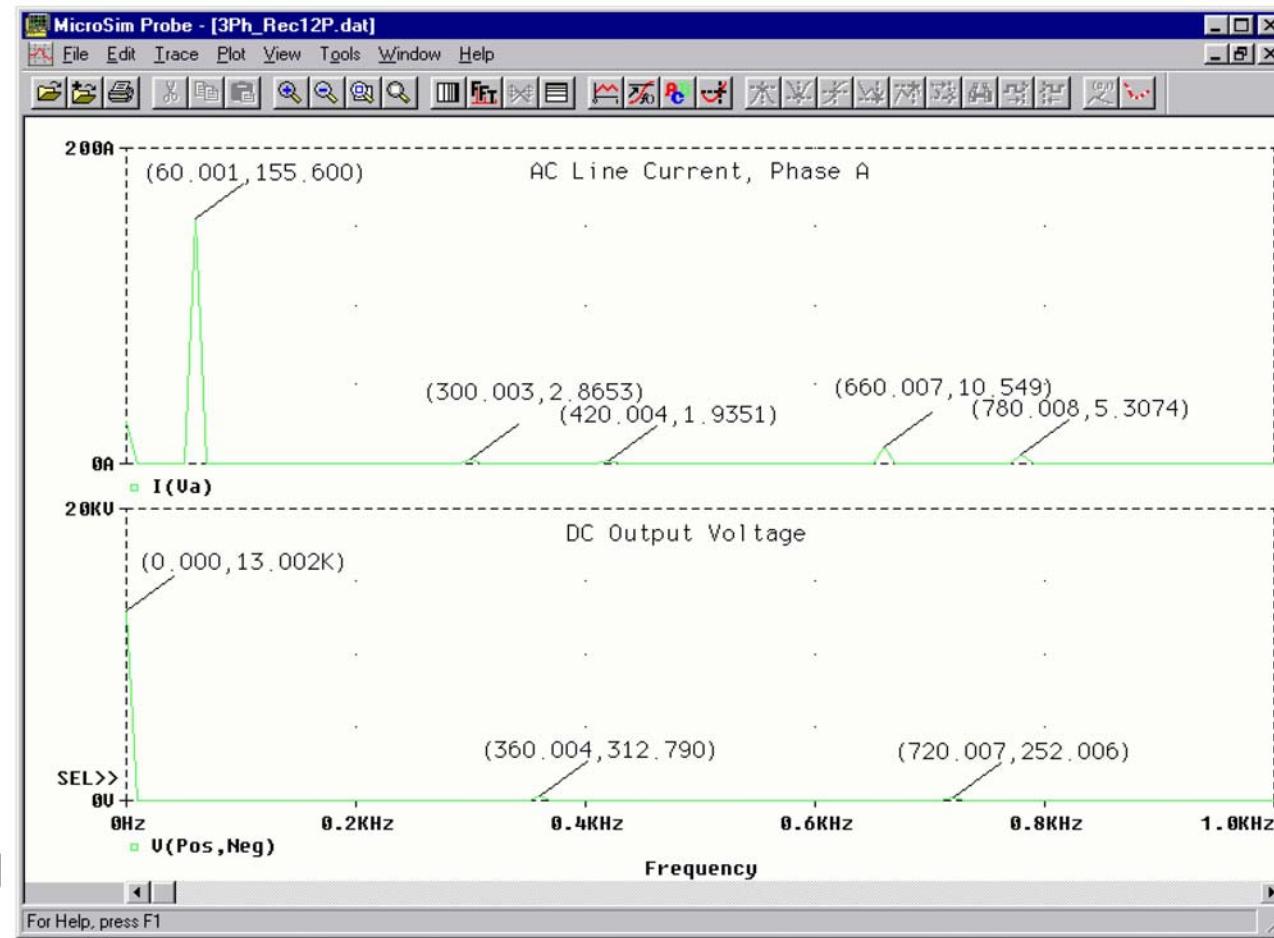
AC Power Rectification

Performance of 12 Pulse Transformer-Rectifier



AC Power Rectification

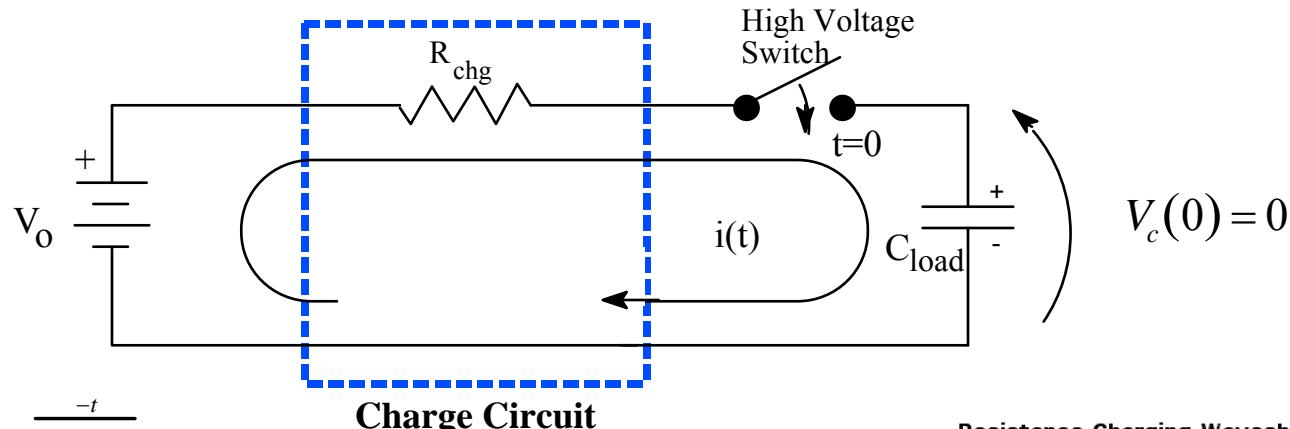
Performance of 12 Pulse Transformer-Rectifier



Charging Topologies

- Resistive charging
- Constant current resistive charging
- Capacitor charging power supplies
- Inductive charging
 - DC resonant charge
 - AC resonant charge
 - CLC resonant charge
 - De-Qing

Resistance Charging



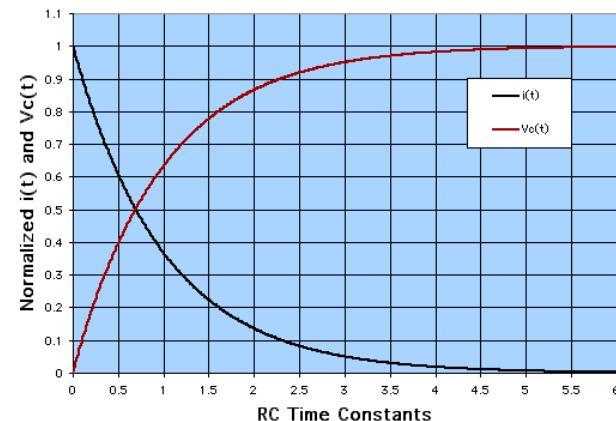
$$i(t) = \frac{V_o}{R_{chg}} e^{\frac{-t}{R_{chg} C_{Load}}}$$

$$V_{C_{Load}}(t) = V_o \left(1 - e^{\frac{-t}{R_{chg} C_{Load}}} \right)$$

$$\text{Energy}_{Load} \approx \frac{1}{2} C_{Load} V_o^2, \quad t > 4R_{chg} C_{Load}$$

$$\begin{aligned} \text{Energy}_{Lost} &= \int_0^\infty i^2(t) R_{chg} dt = R_c \int_0^\infty \left(\frac{V_o}{R_c} \right)^2 e^{\frac{-2t}{R_{chg} C_{Load}}} dt \\ &= \frac{V_o^2}{R_{chg}} \left(-\frac{R_{chg} C_{Load}}{2} \right) e^{\frac{-2t}{R_{chg} C_{Load}}} \Big|_0^\infty = \frac{1}{2} C_{Load} V_o^2 \end{aligned}$$

Resistance Charging Waveshapes

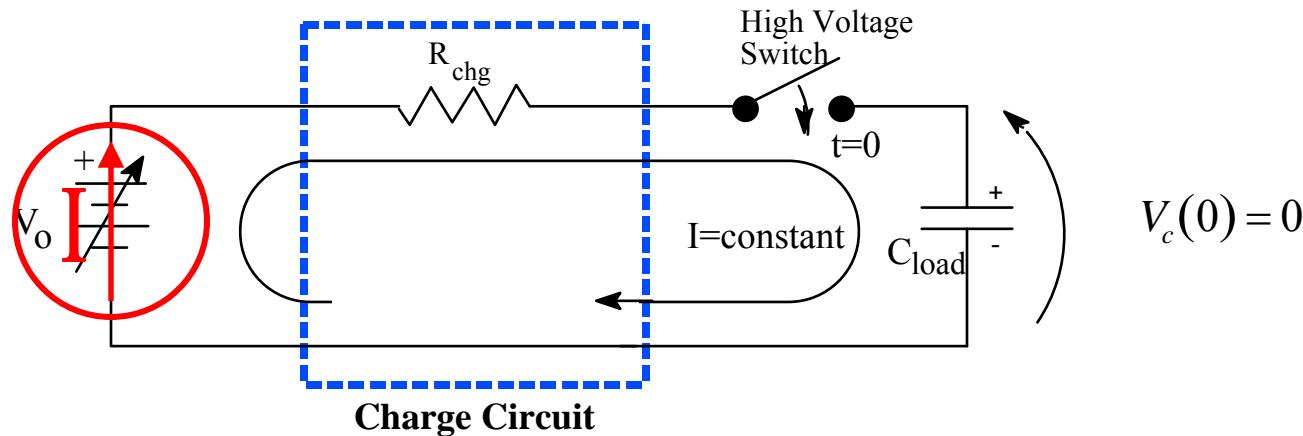


Maximum charging efficiency is 50%
(independent of the value of R_{chg})

Resistance Charging

- Advantages
 - Inexpensive
 - Simple
 - Allows use of low average power, power supply
 - May eliminate the need for a high voltage switch
 - Provides excellent isolation
 - Stable and repeatable
 - Charge accuracy is determined by regulation of power supply
- Disadvantages
 - Inefficient
 - Slow for high energy transfers
 - Requires resistor rated for full charge voltage and, depending on the charge time and energy transferred, a high joule/pulse or average power rating

Constant Current Resistance Charging



$$V_{C_{Load}} = \frac{Q}{C_{Load}} = \frac{IT}{C_{Load}} \quad \text{where } T = \text{time for } V_{C_{Load}} \text{ to approach } V_o$$

$$E_{Lost} = \int_0^T I^2 R_{chg} dt = I^2 R_{chg} T$$

$$E_{Stored} = \frac{1}{2} C_{Load} V_{C_{Load}}^2 = \frac{(IT)^2}{2C_{Load}}$$

$$\text{Efficiency} = \frac{E_{Stored}}{E_{Stored} + E_{Lost}} = \frac{T}{T + 2R_{chg}C_{Load}}$$

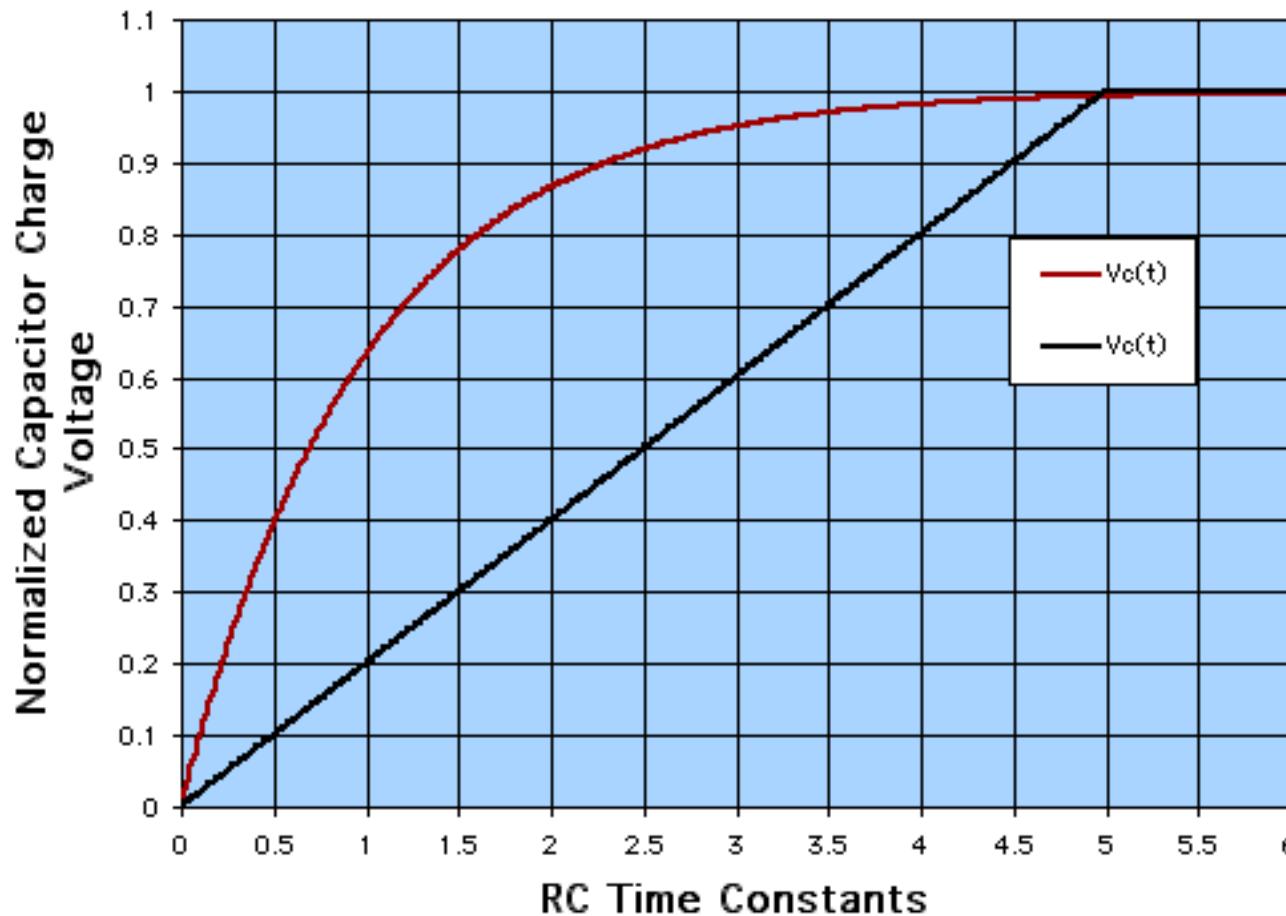
Efficiency approaches 100% as
 $T \gg 2R_{chg}C_{Load}$
 Efficiency = 71% for
 $T = 5R_{chg}C_{Load}$

Constant Current Charge

- Advantages
 - Efficient
 - Power and voltage rating on charge resistor is low
 - Can still provide excellent isolation
- Disadvantages
 - Expensive: requires constant current power supply or controllable voltage source
 - Maximum burst rep-rate determined by charge rate

Charging Efficiency is Waveform Sensitive

Resistance vs Constant Current Charge



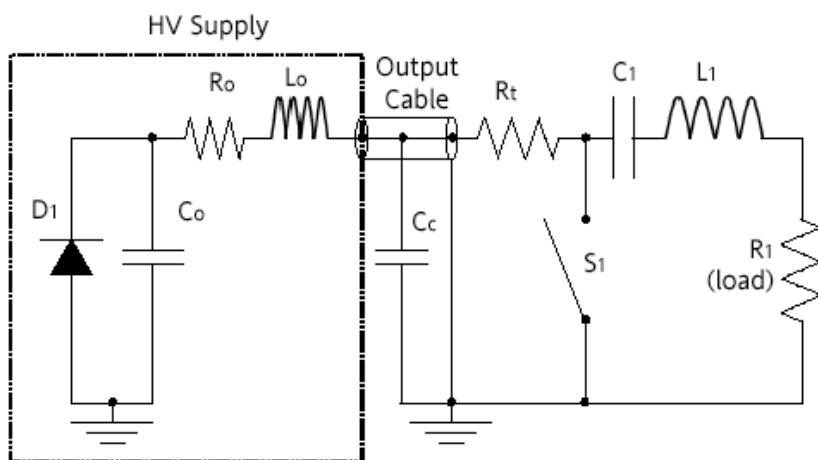
Capacitor Charging Power Supplies

Constant current power supply

- Positive attributes
 - Efficient (>85%)
 - Low stored energy
 - Stable and accurate (linear to ~1% and with 1% accuracy)
 - Can be operated from DC output to kHz repetition rates
 - Compact (high energy density)
 - Good repeatability (available to <0.1% at rep rates)
 - Output voltage ranges up to 10's of kV and controllable from 0-100% at rated output voltage
 - Charge rate usually specified at Joules/sec
 - Internally protected against open circuits, short circuits, overloads and arcs
 - Locally or Remotely controllable

Capacitor Charging Power Supplies

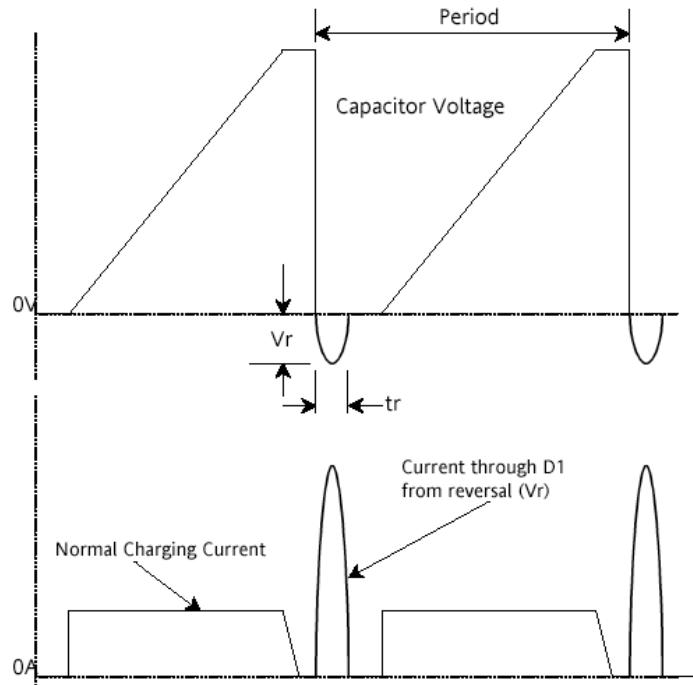
- Issues
 - Cost (usually $> \$1/\text{watt}$)
 - External protection must usually be provided for voltage reversals at load



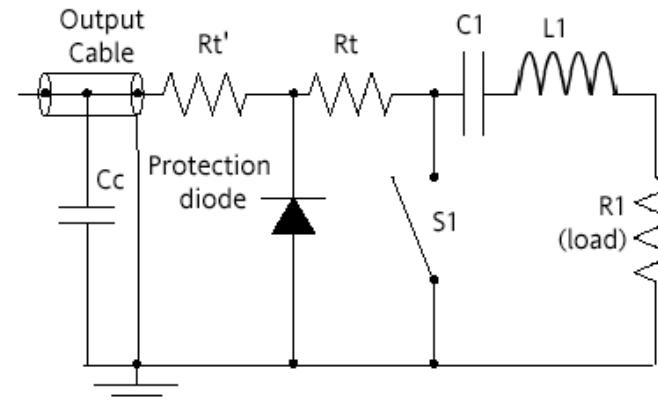
R_t terminates the output cable and prevents the voltage reversal from the closing of switch S_1 from appearing across D_1 . R_o is the internal resistance of the power supply and is usually on the order of a few ohms. C_o is the internal power supplies internal capacitance and may only be a few hundred pF.

Generalized HV Supply Load Connection

Capacitor Charging Power Supplies



HV Supply output diode under voltage reversal conditions



Voltage Reversal Protection Circuit

The protection diode needs to have:
a reverse voltage rating that is higher than the circuit operating voltage and the supply operating voltage (with a safety factor); a rms current rating higher than seen in the circuit; and a forward voltage drop during conduction that is less than the voltage drop in the power supplies' diodes (if R_t' is not used). If used, R_t' should be selected to limit the current to the supply rated output current or less.

Capacitor Charging Power Supplies

- Useful relationships:

- Charge time

$$T_{chg} = \frac{1}{2} \frac{C_{Load} V_{chg} V_{rated}}{P_{peak}}$$

- Peak power rating

$$P_{peak} = \frac{1}{2} \frac{C_{Load} V_{chg} V_{rated}}{T_{chg}}$$

- Average power rating

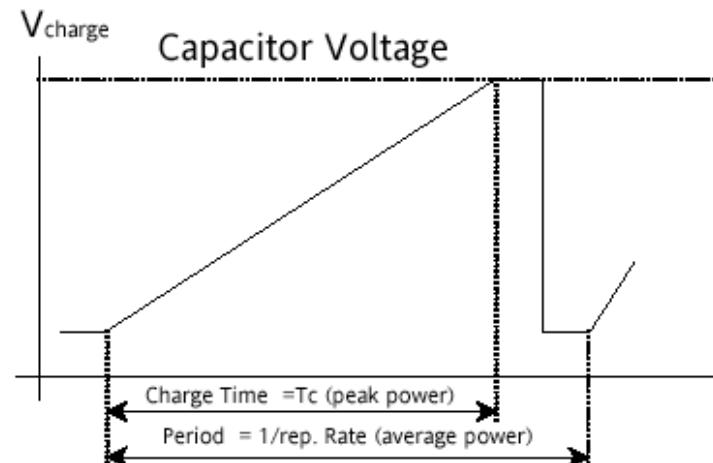
$$P_{avg} = \frac{1}{2} C_{Load} V_{chg} V_{rated} PRF$$

- Maximum repetition rate

$$PRF_{max} = \frac{1}{2} \frac{P_{avg}}{C_{Load} V_{chg} V_{rated}}$$

- Output current

$$I_{output} = \frac{2 P_{peak}}{V_{rated}}$$



Where:

T_c is the load charge time in seconds

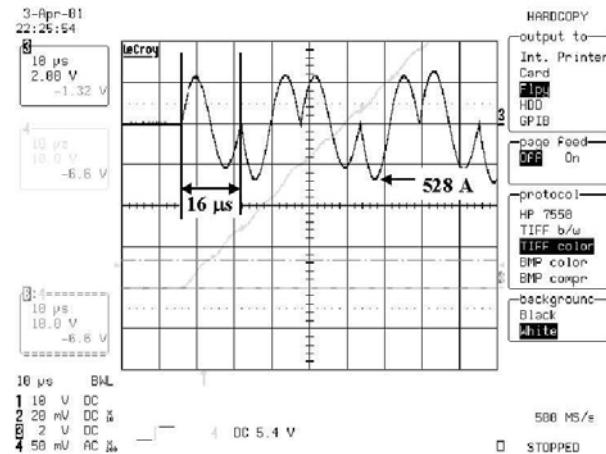
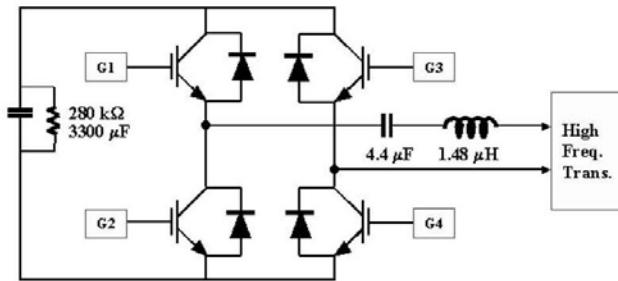
P_{peak} is the unit peak power rating

C_{load} is the load capacitance in Farads

V_{chg} is the load charge voltage in volts

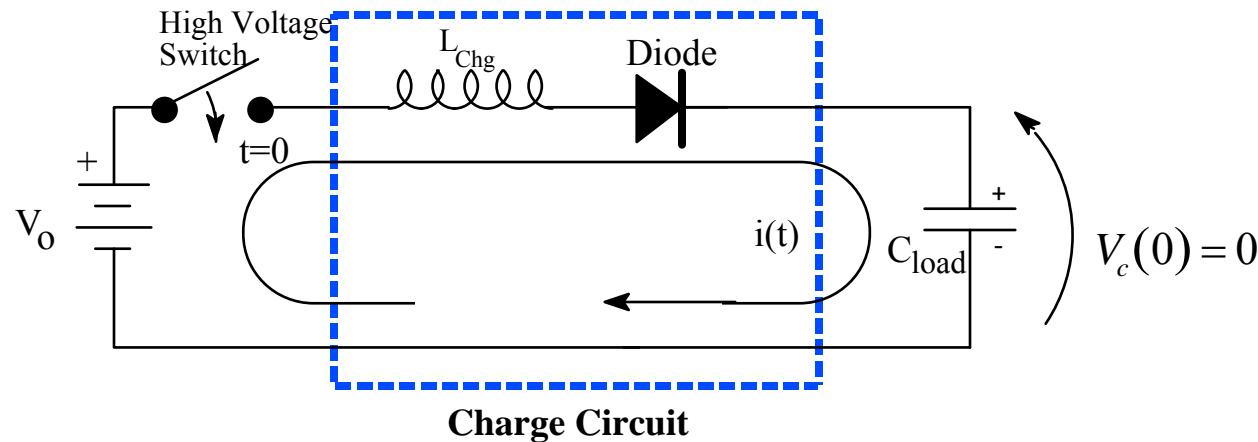
V_{rated} is the power supply rating in volts

Capacitor Charging Power Supplies



- Switch-mode power supplies
- Constant current on recharge time scales, but little output filtering so high frequency structure of the converter is on the output current
 - May result in increased losses in charge circuit components (e.g. diodes)

Inductor Charging - DC Resonant Charge



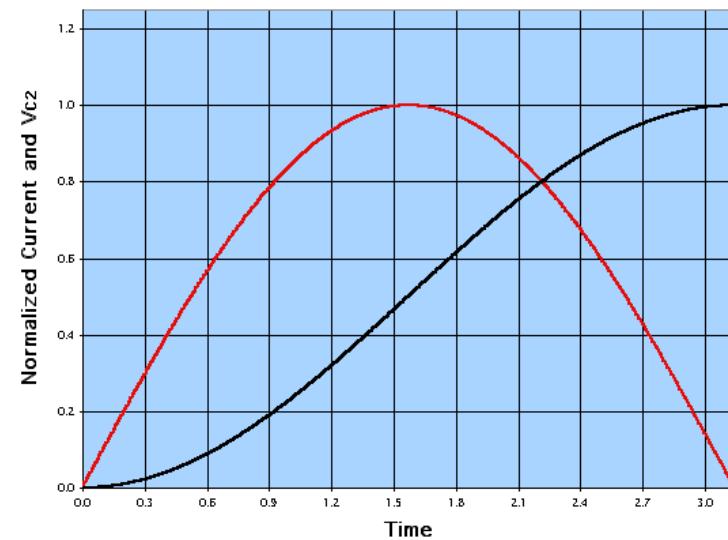
$$i(t) = \frac{V_o}{\sqrt{\frac{L_{chg}}{C_{Load}}}} \sin\left(\frac{t}{\sqrt{L_{chg}C_{Load}}}\right) = \frac{V_o}{Z_o} \sin\omega_o t$$

$$\begin{aligned} V_c(t) &= \frac{1}{C_{Load}} \int_0^t i(t) dt \\ &= -V_o \cos \omega_o t \Big|_0^t \\ &= V_o (1 - \cos \omega_o t) \end{aligned}$$

Maximum value of V_c at $t=\pi/\omega_o$

(Assume $R=0$)

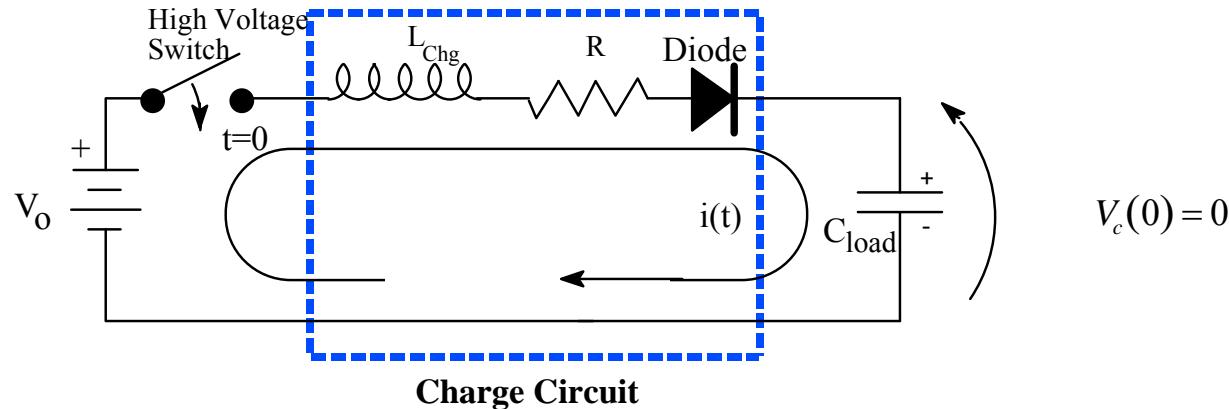
Lossless Energy Transfer



DC Resonant Charge

- Advantages
 - Efficient - approaches 100%
 - Due to voltage gain, PS can be $\sim 1/2$ of desired load voltage
 - Easily capable of high repetition-rate operation
 - Practical and easy to implement
 - Low di/dt requirements for the high voltage switch
- Disadvantages
 - Requires isolation diode (unless load is discharged at the peak of charging)
 - May require a high voltage switch (command resonant charge)
 - Inductor must be designed for high voltage operation
 - Power supply must be capable of providing the peak current

DC Resonant Charge with Resistive Losses



$$i(t) = \frac{V_o}{Z_o} e^{\frac{-t}{2L_{chg}/R}} \sin \omega t \quad \text{where } \omega = \sqrt{\omega_o^2 - \frac{R}{2L_{chg}}}$$

$$V_{C_{Load}}(t) = V_o \left[1 - e^{\frac{-t}{2L_{chg}/R}} \left(\cos \omega t + \frac{R}{2\omega L_{chg}} \sin \omega t \right) \right]$$

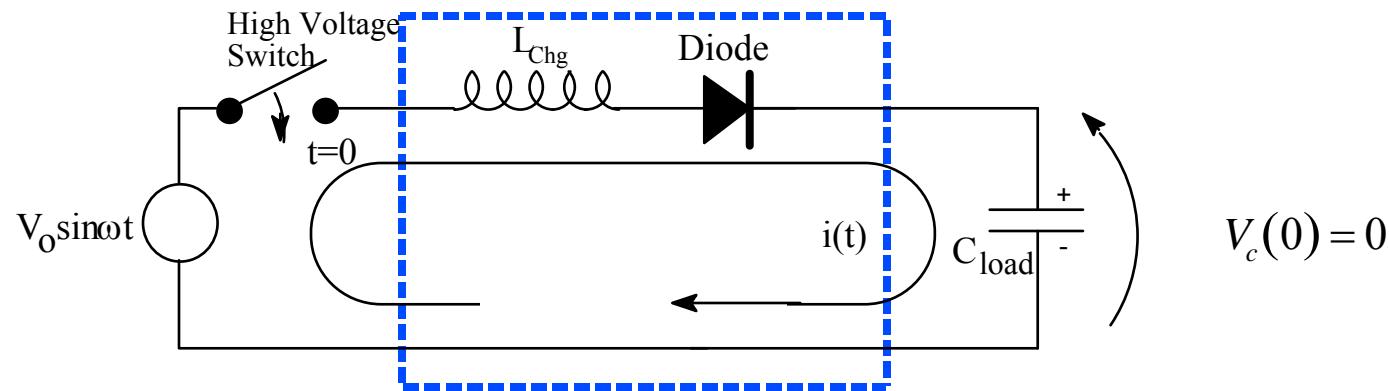
Total energy provided to circuit = $I_{avg} V_o T_{chg}$ where T_{chg} = charging period

$$I_{avg} = \frac{Q_{chg}}{T_{chg}} = \frac{C_{Load} V_{C_{Load}}}{T_{chg}}$$

$$\text{Efficiency} = \frac{\frac{1}{2} C_{Load} V_{C_{Load}}^2}{I_{avg} V_o T_{chg}} = \frac{T_{chg} V_{C_{Load}}}{2V_o}$$

$$\text{Efficiency} \approx 1 - \frac{\pi}{4Q} \quad \text{where } Q = \frac{\omega L_{chg}}{R} \quad \text{and } Q > 10$$

AC Resonant Charge



$$i(t) = \frac{V_o}{2L_{chg}} t \sin \omega t$$

Choose L_{chg} and C_{load} such that: $\frac{1}{\sqrt{L_{chg} C_{load}}} = \omega$

$$V_{C_{Load}} = \frac{V_o}{2Z_o} \left[-t \cos \omega t + \frac{1}{\omega} \sin \omega t \right]$$

$$\text{for a } \frac{1}{2} \text{ cycle, } t = \pi \sqrt{L_{chg} C_{Load}} = \frac{\pi}{\omega}$$

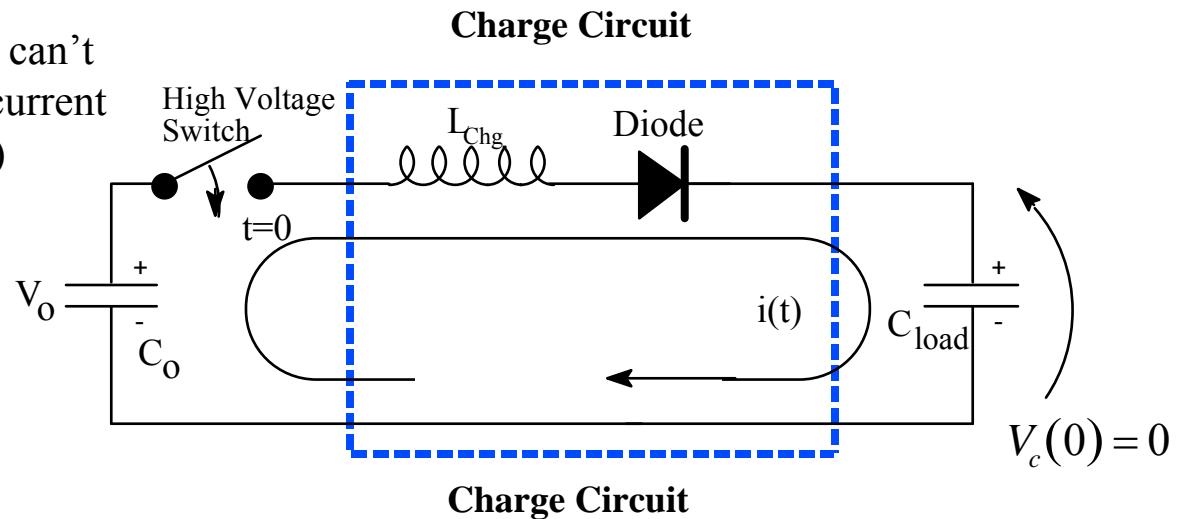
$$V_{C_{Load}} \left(\frac{\pi}{\omega} \right) = \frac{\pi}{2} V_o$$

AC Resonant Charge

- Advantages
 - High efficiency
 - Voltage gain reduces the source voltage requirement
 - Low di/dt requirements for high voltage switch
 - High voltage switch not needed if repetition rate is same as source frequency
- Disadvantages
 - Requires high frequency ac source or very large inductor
 - Diode must have large inverse voltage rating (πV_o)
 - Peak repetition rate is limited by source frequency

DC Resonant Charge - Capacitor to Capacitor

Used when power supply can't provide the peak charge current (e.g. high power systems)



$$i(t) = \frac{V_o}{\sqrt{\frac{L_{chg}}{C_{eq}}}} \sin\left(\frac{t}{\sqrt{L_{chg} C_{eq}}}\right) = \frac{V_o}{Z} \sin \omega t \quad \text{where } C_{eq} = \frac{C_o C_{Load}}{C_o + C_{Load}}$$

$$\text{At peak voltage } \left(t = \frac{\pi}{\omega}\right): V_{C_{Load}} = 2V_o \frac{C_{eq}}{C_{C_{Load}}}$$

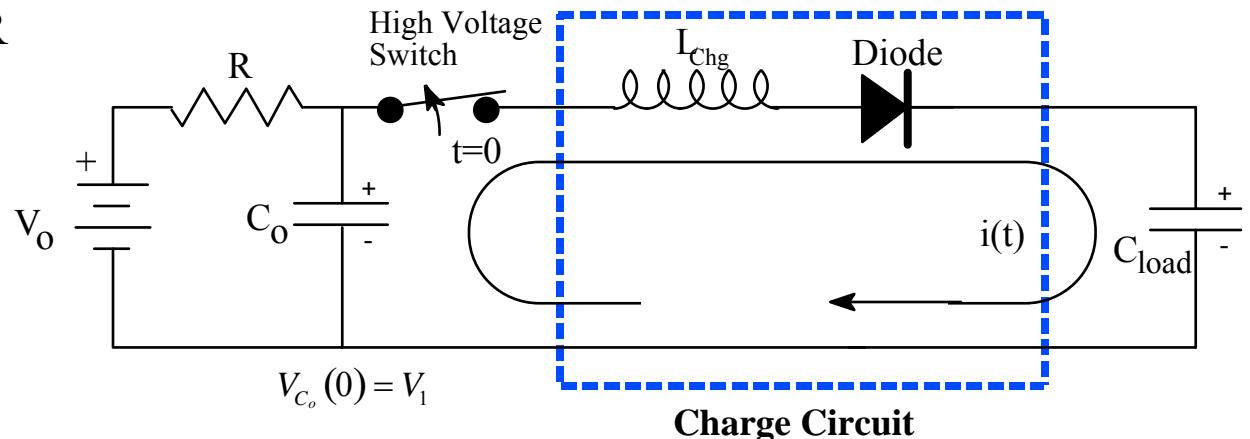
$$\text{For } C_o = 10C_{Load} : V_{C_{Load}} = \frac{2V_o C_o}{C_o + C_{Load}} = 1.82V_o$$

DC Resonant Charge - Capacitor to Capacitor

- Advantages
 - Efficient
 - Voltage gain reduced the PS voltage requirement
 - Easily capable of high repetition rate operation
 - Can operate asynchronously
 - Power supply isn't required to provide large charge current when system is operating at low duty factor
 - Low di/dt requirements on high voltage switch
- Disadvantages
 - Requires a large DC capacitor bank
 - DC capacitor bank needs to be fully recharged between pulses to ensure voltage regulation at the load, unless alternative regulation techniques are employed

DC Resonant Charge with Resistor Charge

C_o recharged through R
after the C_{load} is
inductively charged.



$$i_{C_o}(t) = \frac{V_0 - V_1}{R} e^{\frac{-t}{RC_o}}$$

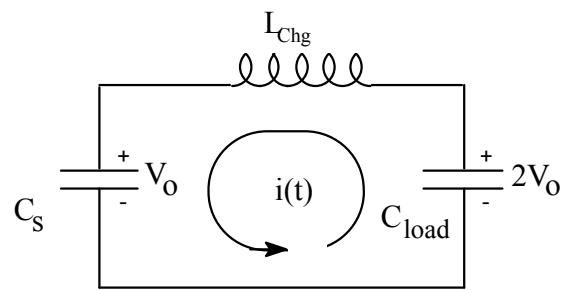
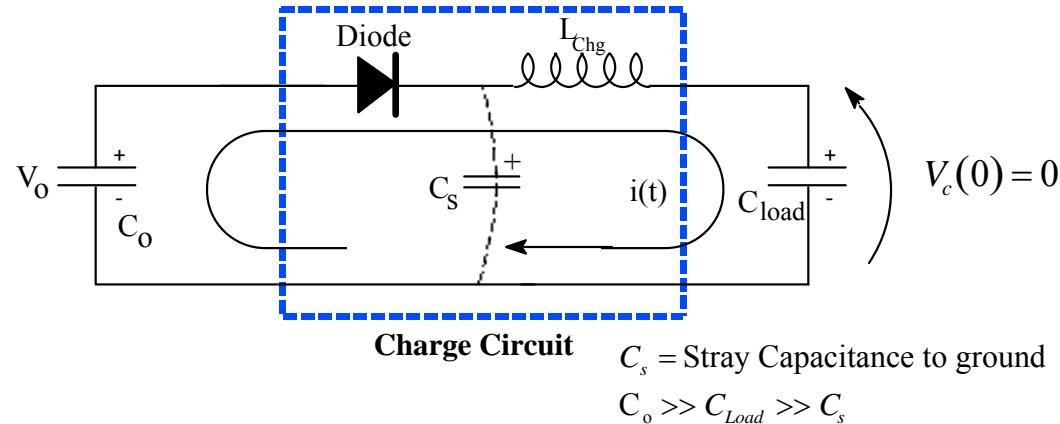
$$Energy_{Loss} = \int_0^{\infty} i_{C_o}^2(t) R dt = \frac{1}{2} C_o (V_0 - V_1)^2$$

$$\Delta Energy_{Stored} = \frac{1}{2} C_o (V_0^2 - V_1^2)$$

$$Efficiency = \frac{1}{2} \left(\frac{1 - \left(\frac{V_1}{V_0} \right)^2}{1 - \left(\frac{V_1}{V_0} \right)} \right)$$

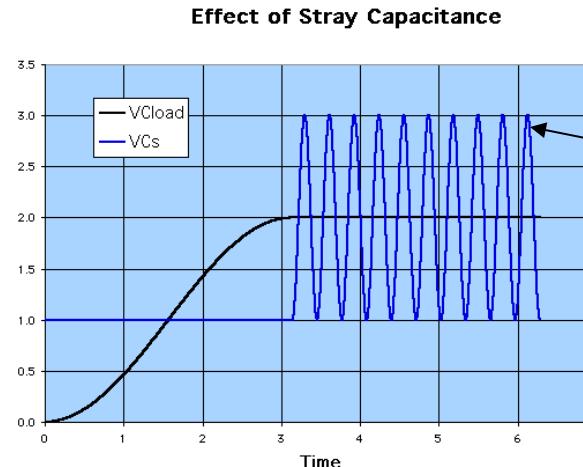
$$\text{For } C_o = 10C_{Load} : \quad Efficiency = 90.9\%$$

Effects of Stray Capacitance



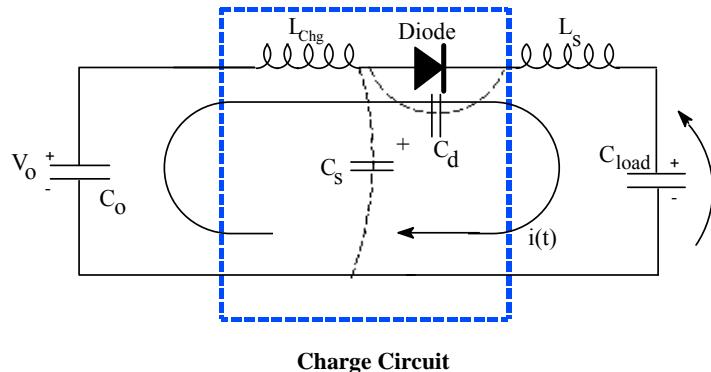
$$i(t) = \frac{V_o}{\sqrt{\frac{L_{chg}}{C_{eq}}}} \sin(\omega t) = \frac{V_o}{Z} \sin \omega t$$

$$V_{C_s} = 2V_o - V_o \cos \omega t \quad \text{where} \quad C_{eq} = \frac{C_s C_{Load}}{C_s + C_{Load}} \quad \text{and} \quad \omega = \frac{1}{\sqrt{L_{chg} C_{eq}}}$$



Peak inverse diode voltage $\sim 2V_o$ instead of V_o

Effect of Stray Capacitance

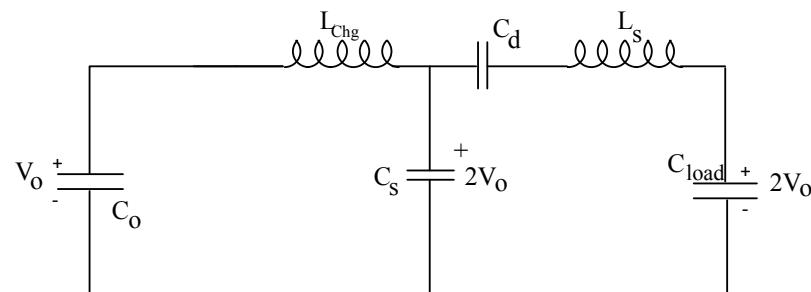


C_s - stray capacitance to ground

C_d - stray capacitance across diode stack

(includes diode junction capacitance, capacitance between mounting connections, etc.)

L_s - total series inductance between diode and ground



Equivalent Circuit where: $C_o \gg C_{Load} \gg C_s, C_d$

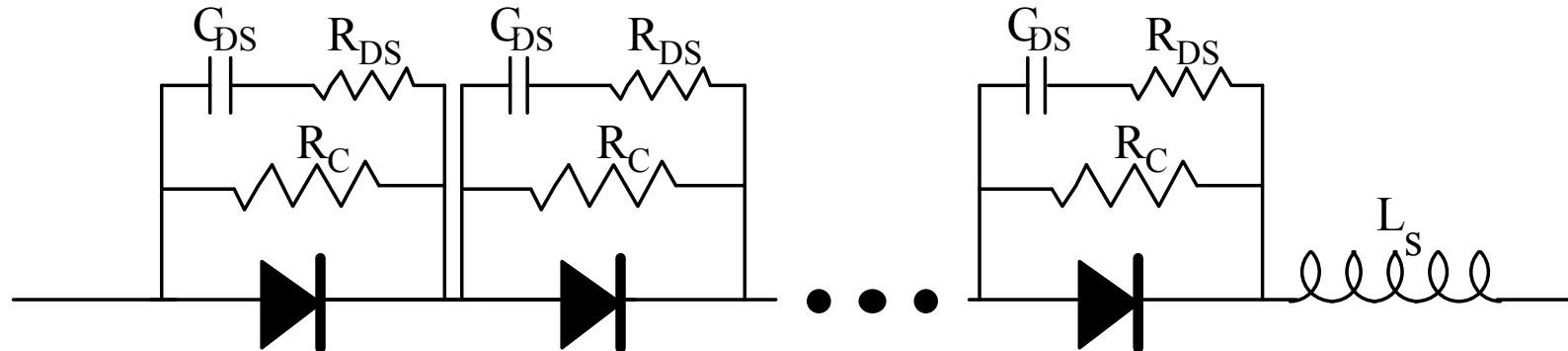
After C_{Load} is charged:

C_s will ring with C_o and can create large inverse voltage across the diode stack

C_{Load} will oscillate with C_d and C_s

Inductor Snubber and/or Diode Snubber may be required

Diode Snubber



R_c is used for DC grading of diodes to force voltage sharing between diodes. Want the current through R_c to be large compared to the maximum leakage current (I_r) through the diodes: $R_c \approx \frac{V_{Diode}}{10I_r}$

C_{DS} and R_{DS} form the fast snubber where C_{DS} is for voltage sharing and R_{DS} is for damping. Energy is stored in C_{DS} and dissipated in R_{DS} .

Considerations for C_{DS} :

Charge stored @ ~ 0.7 volts ≥ 10 diode junction charge

$\frac{C_{DS}}{N} \gg$ the stray capacitance of the entire stack (N diodes)

C_{DS} should be as small as possible for higher efficiency

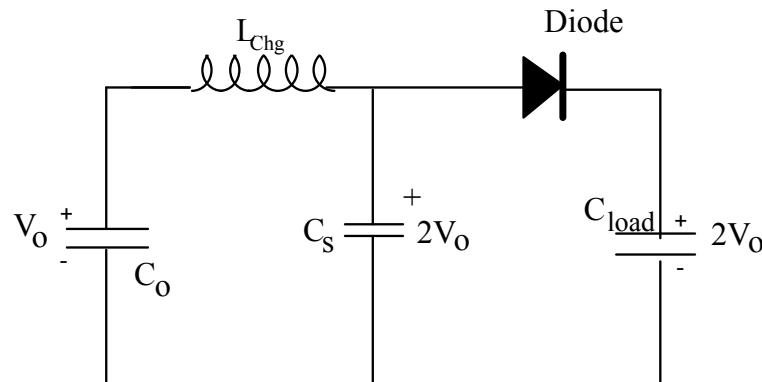
Considerations for R_{DS} :

$$R_{DS} > \frac{2}{N} \sqrt{\frac{L_s}{C_{DS}/N}} \quad \text{where } N \text{ is the number of series diodes}$$

$\frac{1}{R_{DS}C_{DS}}$ is small compared to maximum applied $\frac{dV}{dt}$

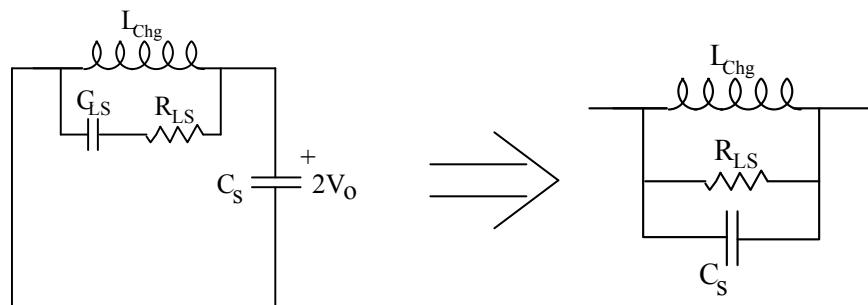
Power Dissipation Rating $\geq 2(\text{PRF})C_{DS}V_r^2$ where V_r is the maximum inverse voltage on the diode

Charge Inductor Snubber



$$C_o \gg C_{Load} \gg C_s$$

$$\omega_o = \frac{1}{\sqrt{L_{chg} C_s}}$$



Equivalent circuit with snubber across inductor

Select $C_{LS} > C_s$

Considerations for R_{LS} :

$$(1): R_{LS} < \frac{1}{2} \sqrt{\frac{L_{chg}}{C_s}} \quad (\text{critically damped})$$

(2): Ensure power rating is adequate :

$$P \geq 2(PRF)C_{LS}V_0^2$$

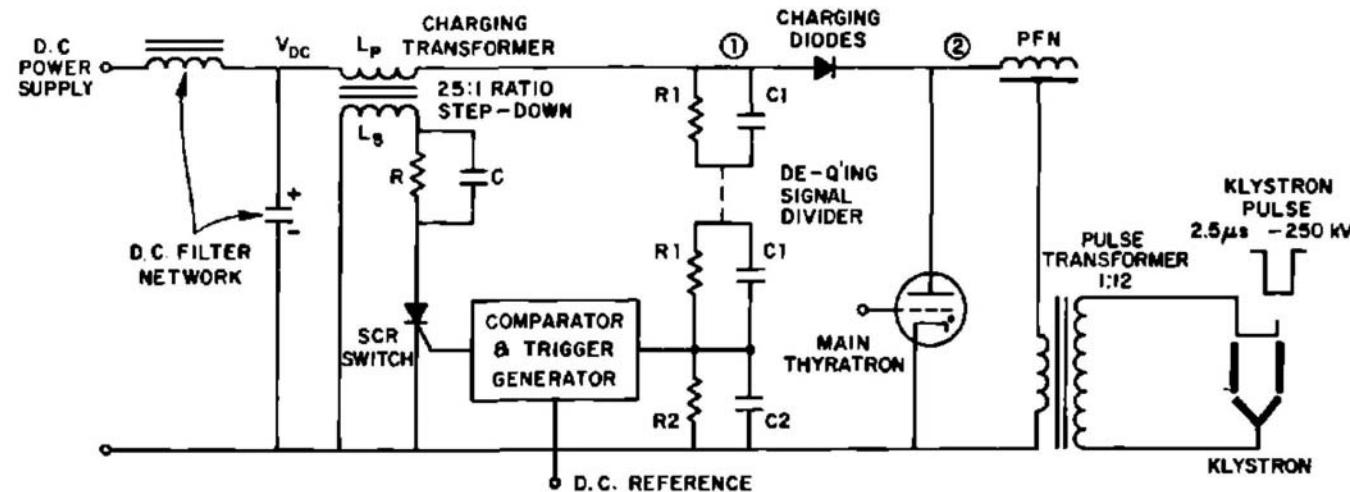
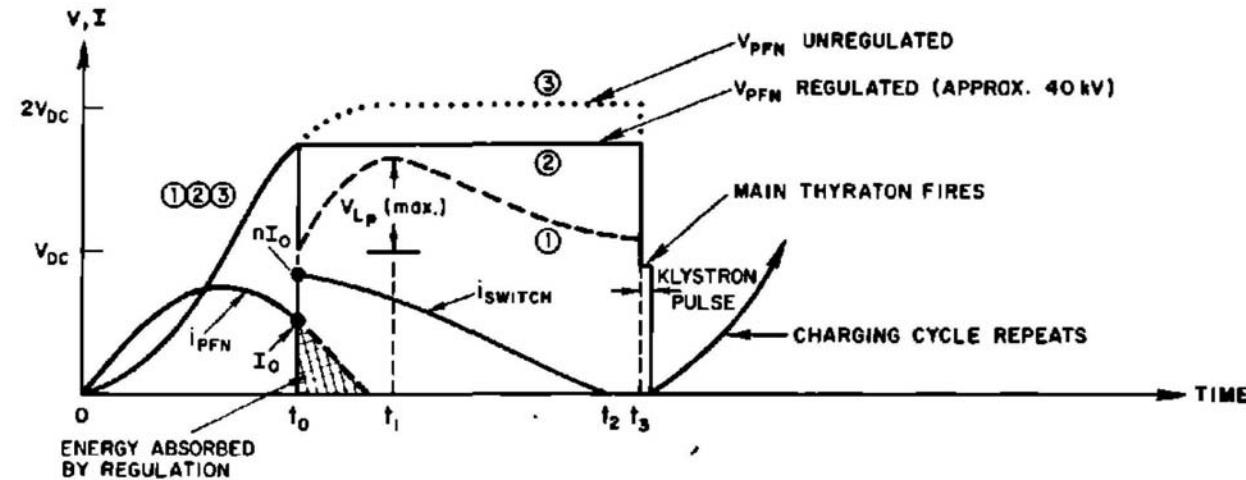
(3): Adequate voltage rating ($> V_o$)

(4): Want $R_{LS}C_{LS} \gg$ charging period

Inductive Charging Voltage Regulation

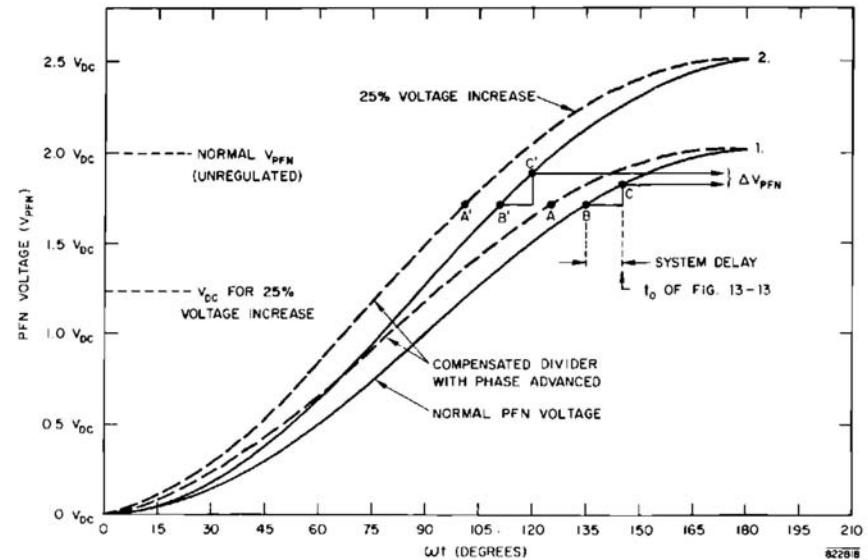
- For klystron phase stability, PFN charge voltage regulation may need to be ~ 10 ppm
- High power supplies usually do not have precise regulation
 - Requires more complicated topologies (over simple rectifier/filter)
 - Increases cost
- Common approach to regulate PFN charge voltage from unregulated source is de-Qing
 - Monitor PFN voltage during charge cycle
 - When PFN reaches final voltage, shunt energy remaining in charge inductor to dummy load

De-Qing



De-Qing Limitation

- During the delay between the measurement of the PFN reaching the desired charge voltage and the termination of charging current (system delay), the PFN voltage continues to increase
- When the unregulated source voltage is higher, charging current is increased and the PFN voltage increase during the system delay is increased (ΔV_{PFN})
- This error must be corrected for precise PFN voltage regulation
 - Phase advance on voltage divider
 - Measured signal, V_M , actually higher than PFN voltage, V_{PFN}
 - Ratio of V_M/V_{PFN} is a function of PFN charge rate
 - Compensates delay
 - Used in SLAC 6575
 - Feed forward control loop
 - Measure final PFN voltage
 - Adjust timing if voltage fluctuates
 - Used at PAL
 - Similar regulation accuracy



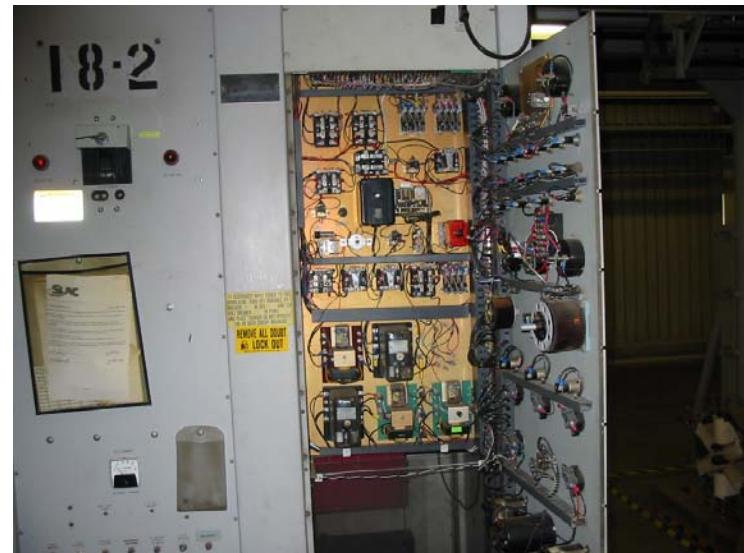
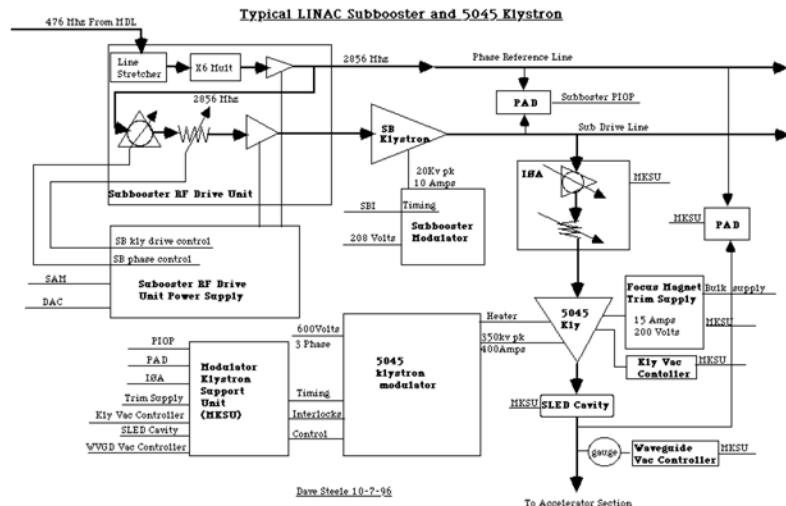
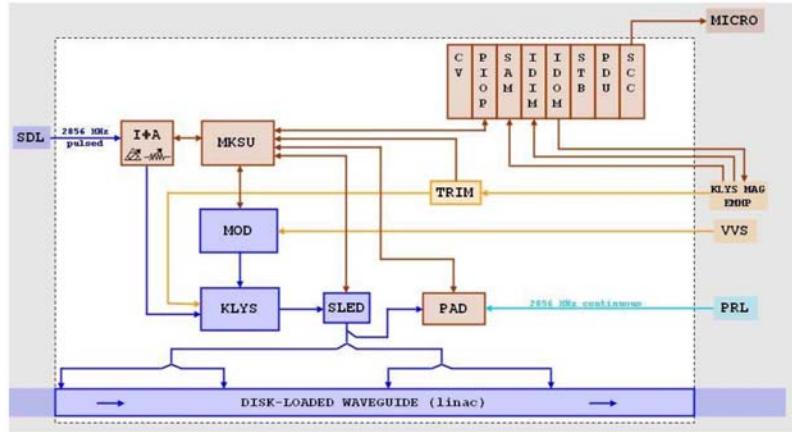
Control System Functions

- Control output waveform
 - Voltage
 - Pulse shape
 - Timing
- Protect system (MPS)
 - Over voltage
 - Over current
 - Heater time-outs
 - Etc.
- Protect personnel (PPS)
 - PPS interlocks
 - Emergency Off
 - Access interlocks
 - Energy discharge
 - Bleeder resistors across capacitors
 - Engineered grounds

Control System Elements

- Lab Scale: independent elements
 - Charging supply with integrated controls
 - Trigger generator
 - Diagnostics
 - Oscilloscope
 - Probes
 - Voltage divider
 - Current transformer
- Installations
 - Control system interfaces to many operators/users and many other machines/systems: an integrated control system is required
 - Periodic evaluation
 - Configuration control
 - Control system may incorporate many components at varying levels
 - Integrated modulator control (modern trend)
 - System level (e.g. HLRF)
 - Facility level (e.g. accelerator)

Elements of the 6575 Modulator Control System

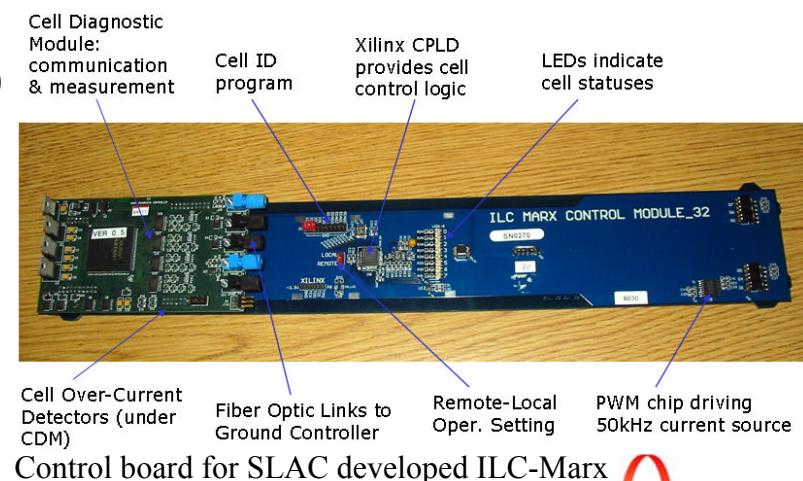


Integrated Control Elements

- Programmable Logic Controller (PLC)
 - Replaces relay logic
 - Serial communication interface (can support EPICS communication)
 - Limitations
 - Slow
 - Loop timing not clocked
 - But, can be combined with additional circuits to expand capabilities
 - Sample-hold
 - Peak detect
 - Various A-D and D-A
- Programmable logic devices (CPLD, FPGA)
 - Fast, to >100 MHz clock
 - Powerful
 - Compact
 - Flexible
 - Communication options
 - Inexpensive (after development costs)



ILC-Marx PLC chassis



Control board for SLAC developed ILC-Marx