electrical circuits were well shielded, Joule heating of the resistors at low temperatures by $\sim 10^8$ cps FM and TV signals made measurements inaccurate, so data were obtained when the transmitters were off.

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"CORKSCREW"—A DEVICE FOR CHANGING THE MAGNETIC MOMENT OF CHARGED PARTICLES IN A MAGNETIC FIELD*

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Consider the motion of a charged particle in the magnetic configuration shown in Fig. 1. A properly designed helical field source (a "Corkscrew") can perturb an initially uniform axial field in such a way that there will be a monotonic increase (or decrease) in the transverse energy component of certain particles traversing the structure. The necessary design condition is that the force resulting from the interaction of the axial particle velocity with the transverse component of the field perturbation be always approximately in the direction of the transverse particle velocity. It follows that there must be a close match between the local pitch of the Corkscrew and that of the modified helical particle trajectory. This condition may be expressed as

$$p(z) = -2\pi m v_z(z)/qB_0, \qquad (1)$$

where B_0 is the unperturbed axial field, m, q, and v_z are the mass, charge, and axial velocity of the particle, and $p(z)$ is the Corkscrew pitch length at position z (p is negative for the left-handed structure of Fig. 1). The helical field perturbation has no over-all effect on the axial field; therefore, a change in the transverse particle energy necessitates a change in magnetic moment. The trajectory in Fig. 1 could apply to an ion moving from left to right or to an electron moving from right to left.

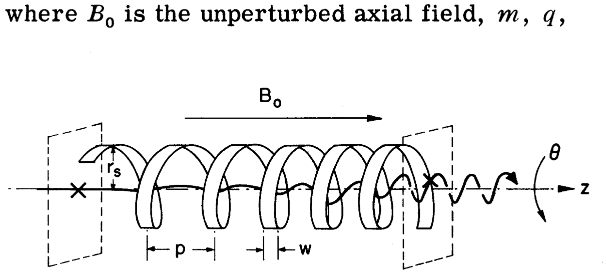

FIG. 1. Schematic diagram of a Corkscrew showing a resonant particle trajectory.

The Corkscrew may permit trapping of a high-energy beam injected axially into a magnetic mirror device. A positive particle following a path as shown in Fig. 1 could be reflected by a magnetic mirror somewhere to the right. On its return, the particle trajectory would have a handedness opposite to that of the Corkscrew, and, therefore, the particle would encounter a series of perturbations alternating in direction at a frequency higher than the cyclotron frequency. These perturbations should cancel to first order, so a mirror to the left would again reflect the particle, and trapping would appear to have been achieved. The Liouville theorem, of course, demands that some mechanism exist for particle loss. The unique feature of the Corkscrew is that this loss mechanism cannot be the same as the trapping mechanism. Trapping is achieved by what is essentially a strong resonance effect. Loss must occur by a random "scatter" effect whose exact nature has not as yet been determined. This nonreciprocal character of the Corkscrew is in sharp contrast to axially symmetric nonadiabatic systems previously investigated[1] in which particle loss can occur along paths that are mirror images of paths leading to trapping.

The resonance effect of the Corkscrew has been demonstrated in an experiment wherein an electron beam on the axis of a solenoid was passed through an iron helix. The resulting helical beam was then reflected from a magnetic mirror with mirror ratio 1.8; this indicated that over half the beam energy was in the transverse component. The iron helix was formed from bar stock 1/8 in. thick, 30 in. long, and with width cut to a sine curve 1/2 in. at the center and zero at the ends. The bar was wound onto a cylinder of 1-in. diameter with turns spaced to give a central pitch of 7/8 in. and an over-all length of 4 in. The symmetric structure was necessary to eliminate end effects, and only the exit half was effective in "winding up" the electron beam. The measured ratio of transverse to longitudinal field was approximately 0.15 maximum on axis. For combinations of beam voltage (up to 1 kv) and magnetic field which gave a proper pitch length, reflection occurred from the mirror. Detuning either voltage or magnetic field permitted the beam to pass through the mirror. Reversing the direction of the magnetic field so that the handedness was wrong eliminated the resonance effect, and no beam reflection could be obtained for any value of particle pitch length.

An approximate analytic treatment of the Corkscrew yields the following results, which are presented without complete derivation. The scalar magnetic potential for an infinite helical current-carrying ribbon of constant pitch and radius is given in cylindrical coordinates as

$$\phi_m = I\left\{\frac{z}{p}+\frac{1}{n}\sum_{n=1}^{\infty}\left(\frac{2\pi r_s}{p}\right)K_n'\left(\frac{2n\pi r_s}{p}\right)\left[\frac{\sin(n\pi w/p)}{n\pi w/p}\right] \times \sin\left[n\left(\frac{2\pi z}{p}+\theta\right)\right]I_n\left(\frac{2n\pi r}{p}\right)\right\}, \quad (2)$$

where the K_n' and I_n are Bessel functions, I represents current in the ribbon, and the origin for z and θ is taken at a radial line passing through the center of the ribbon. Pitch length p, ribbon width w, and source radius r_s are shown in Fig. 1. In a practical Corkscrew, a bifilar helix with currents opposed would be desirable because this decouples the helical perturbation from the main axial field. Then, all even n terms vanish from the summation. If we make $w=p/k$ (k an integer), every kth term vanishes. For $k=3$, the magnetic potential can be expressed very well as

$$\phi_m = +3.3\,I\left(\frac{r_s}{p}\right)K_1'\left(\frac{2\pi r_s}{p}\right)\sin\left(\frac{2\pi z}{p}+\theta\right)I_1\left(\frac{2\pi r}{p}\right). \quad (3)$$

The application of this equation to a helix of finite length and variable pitch should be qualitatively correct. The perturbation due to an iron helix in an initially uniform axial field is not significantly different.

The equations of motion can be solved under restricted conditions. Define a "phase angle," α, from the equation of motion,

$$\dot{\vec{v}}_z = (q/m)(\vec{v}_\perp \times \vec{b}_\perp) = (q/m)|v_\perp||b_\perp|\sin\alpha, \quad (4)$$

where $\vec{b}_\perp = -\nabla\phi_m$. Use of the ϕ_m from Eq. (3) gives a dependence of $b_\perp$ on pitch length, and Eq. (1) further relates this to v_z. Thus one may write

$$b_\perp = b_0 f(v_z), \quad (5)$$

where $f(v_z)$ is assumed normalized to 1 at $z=0$. Equation (4) can be solved by taking α = constant (resonance), relating v_z and $v_\perp$ by the conservation of energy, and making a judicious choice of $f(v_z)$ which facilitates solution while still approximating the dependence from Eq. (3). For $v_\perp = 0$ and $v_z = v_0$ at $z=0$, series expansions of the solution take the form

$$\frac{v_0}{v_z} = 1 + \tfrac{1}{2}\left(\frac{qb_0\sin\alpha}{mv_0}z\right)^2 + \cdots. \quad (6)$$

Equations (1), (3), and (6) provide an adequate basis for preliminary Corkscrew design.

An interesting coupling appears to exist between Corkscrew influence on the motion of a particle in real and velocity space. If one considers a perturbation given by Eq. (3) on an initially uniform axial field, it can be seen that a particle off axis and in resonance will see a variation in axial field that in the moving frame of reference looks like a steady gradient of field. This observation can be formalized by use of the equation for ∇B drift from guiding center theory.[2] The result is $r^2 v_z \approx$ constant, where r is the guiding center position of the particle in the coordinate system of Eq. (3). The crudeness of the analogy leading to this result makes the value of the exponent questionable. This type of coupling, however, suggests that a Corkscrew might bunch particles in velocity space at the expense of a diffusion in real space, without violating the requirement for conservation of volume in phase space.

A particle in resonance will experience a form of phase stability. It follows from previous definitions that

$$\dot{\alpha} = (2\pi v_z/p) - \dot{\theta}, \qquad (7)$$

and for the structure of Fig. 1

$$\dot{\alpha} \approx -(2\pi v_z/p) + (q/m)B_0. \qquad (8)$$

Consideration of Eqs. (4) and (8) shows that phase stability exists for $-\frac{1}{2}\pi < \alpha < \frac{1}{2}\pi$, since if v_z is too large $\dot{\alpha}$ is negative and v_z moves toward larger negative values. The converse is also true. Differentiation of Eq. (8) and elimination of velocity factors by use of Eqs. (4) and (8) leads to a second-order equation for α. The equation is oscillatory for small α and contains a damping term which is positive for p increasing in the direction of z motion and negative for p decreasing. This derivation is very approximate, but it points up a possible area of difficulty in Corkscrew design and further illustrates the complexity of the device.

There are two ways in which Liouville's theorem might be satisfied when the Corkscrew is used as a trapping device. Essentially random scatter of a particle by the field perturbations may cause real space diffusion leading to a radial drift, or may cause velocity space diffusion leading to loss from the mirror loss cone. Equations (4) and (8) suggest the origins of these effects. An axial particle entering a Corkscrew-mirror system as in Fig. 1 will be caught in the stable phase, and v_z will decrease monotonically. This constitutes the trapping pass. On its second or return pass through the Corkscrew, the particle will have a negative v_z, $\dot{\alpha}$ will be large, and $\dot{v}_z$ will average near zero. This situation will occur on every pass for which $v_z < 0$, and the scatter effect will be small. On the third pass, v_z will again be positive, $\dot{\alpha}$ will be small and at some point during the traverse will pass through zero. The cancellation argument used on the second pass will not hold, and it is uncertain just what will happen. If there is no preferred value of α for the $\dot{\alpha} = 0$ condition, then an essentially random scatter in both real and velocity spaces might be expected. If v_z merely oscillates as α changes, then the phase at the exit of the Corkscrew would influence the amount of scatter. If a sufficiently strong mechanism of phase stability exists, it is even possible that the particle would be caught and held in the favorable phase associated with the first pass. At worst there does not seem to be any loss mechanism that could produce an effect amounting to more than a small fraction of the resonance trapping effect. Hopefully, the difference between trapping and loss mechanisms will permit particle containment times to be increased arbitrarily by suitable modification of the basic Corkscrew configuration.

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