Title	The Nonstandard FDTD Method Using a Complex Formulation
Author(s)	Taguchi, Kenji, Ohtani, Tadao, Kashiwa, Tatsuya, Kanai, Yasushi, 柏, 達也
Citation	IEEE Transactions on Magnetics, 40(2(2)): 1448-1451
Issue Date	2004-03
URL	http://hdl.handle.net/10213/1311
Rights	Institute of Electrical and Electronics Engineers, K.Taguchi, T.Ohtani, T.Kashiwa, Y.Kanai, IEEE Transactions on Magnetics, 40(2-2), 2004, 1448-1451. c2004 IEEE. Personal use of this material is permitted. However, permission to reprint/republish this mat
Type	Journal Article
Text Version	publisher
Additional information	

The Nonstandard FDTD Method Using a Complex Formulation

Kenji Taguchi, Tadao Ohtani, Tatsuya Kashiwa, *Member, IEEE*, and Yasushi Kanai, *Member, IEEE*

***Abstract*—In this paper, a complex nonstandard finite-difference time-domain (CNS-FDTD) method is proposed in order to simulate wave propagation in lossy media. To clarify the characteristics of the method, expressions for the numerical propagation constant and the stability condition are derived. It is found that the CNS-FDTD method is much more accurate and stable than the conventional finite-difference time-domain (FDTD) method. The method is applied to the analysis of a fin ferrite electromagnetic wave absorber with a periodic structure.**

***Index Terms*—Complex formulation, FDTD method, NS-FDTD method, phase error, stability condition.**

I. Introduction

THE finite-difference time-domain (FDTD) method [1] is an effective technique for electromagnetic wave analysis. However, due to the numerical dispersion generated by the discretization numerical phase errors, dependent on the direction of propagation, are generated. Errors in the numerical attenuation constant also occur in the case of propagation in a lossy medium.

The nonstandard FDTD (NS-FDTD) method [2]–[6] is one method of reducing the phase error in the FDTD method. The NS-FDTD method is highly accurate for single frequency calculations, and has been applied to large scale problems [6]. In the method, a finite difference operator is used to homogenize the phase error in the propagation direction, coupled with a correction function for phase error compensation. However, this method has been optimized only for waves propagating within a lossless medium. An extension of the NS-FDTD method to the analysis of a lossy medium has been carried out in [3], [5]. However, these methods still have errors of a few percent in the attenuation constant. One of the causes of this deficiency is the use of a correction function which only takes account of the phase term. Hence, if a complex correction function is used, accommodating the fact that the propagation constant of the attenuating wave is complex, then an improvement in the accuracy should be achievable.

In this paper, the following tasks are carried out. 1) A complex NS-FDTD (CNS-FDTD) method is proposed for the analysis of a lossy medium. Then a complex correction function for the decaying wave is introduced. 2) In order to investigate the characteristics of the CNS-FDTD method, expressions for the numerical propagation constant and stability condition are derived. 3) The CNS-FDTD method is applied to the analysis of a fin-type ferrite electromagnetic wave absorber with a periodic structure.

Manuscript received July 1, 2003.

K. Taguchi and T. Kashiwa are with Kitami Institute of Technology, Kitami 090-8507, Japan (e-mail: kashiwa@klab2.elec.kitami-it.ac.jp).

T. Ohtani is with Mitsubishi Heavy Industries, Ltd., Nagoya 455-8515, Japan.

Y. Kanai is with Niigata Institute of Technology, Kashiwazaki 945-1195, Japan.

Digital Object Identifier 10.1109/TMAG.2004.825011

II. NS-FDTD Difference Equation in Complex Form

This section describes the two-dimensional TE mode CNS-FDTD difference equation for a lossy medium. The spatial axis correction function in a complex form given below is used

$$\dot{S}_\gamma(\Delta\xi) = \frac{2\sinh\left(\frac{\dot{\gamma}\Delta\xi}{2}\right)}{\dot{\gamma}} \qquad (\xi = x, y) \tag{1}$$

where $\Delta\xi$ is the spatial increment and $\dot{\gamma}$ is the physical complex propagation constant in the lossy medium. Hence, the two-dimensional TE mode CNS-FDTD difference equations can be expressed as follows:

$$\begin{aligned}
&\dot{E}_x^{t+\Delta t/2}\left(x, y+\frac{\Delta y}{2}\right)\\
&= \frac{u_-}{u_+}\dot{E}_x^{t-\Delta t/2}\left(x, y+\frac{\Delta y}{2}\right)\\
&\quad + \frac{\alpha_{0y}}{u_+\dot{S}_\gamma(\Delta y)}\left\{\dot{H}_z^t(x, y+\Delta y) - \dot{H}_z^t(x,y)\right\}\\
&\quad + \frac{(1-\alpha_{0y})}{2u_+\dot{S}_\gamma(\Delta y)}\left\{\dot{H}_z^t(x+\Delta x, y+\Delta y) - \dot{H}_z^t(x+\Delta x, y)\right.\\
&\quad \left. +\dot{H}_z^t(x-\Delta x, y+\Delta y) - \dot{H}_z^t(x-\Delta x, y)\right\}
\end{aligned} \tag{2}$$

$$\begin{aligned}
&\dot{E}_y^{t+\Delta t/2}\left(x+\frac{\Delta x}{2}, y\right)\\
&= \frac{u_-}{u_+}\dot{E}_y^{t-\Delta t/2}\left(x+\frac{\Delta x}{2}, y\right)\\
&\quad - \frac{\alpha_{0x}}{u_+\dot{S}_\gamma(\Delta x)}\left\{\dot{H}_z^t(x+\Delta x, y) - \dot{H}_z^t(x,y)\right\}\\
&\quad - \frac{(1-\alpha_{0x})}{2u_+\dot{S}_\gamma(\Delta x)}\left\{\dot{H}_z^t(x+\Delta x, y+\Delta y) - \dot{H}_z^t(x, y+\Delta y)\right.\\
&\quad \left. + \dot{H}_z^t(x+\Delta x, y-\Delta y) - \dot{H}_z^t(x, y-\Delta y)\right\}
\end{aligned} \tag{3}$$

$$\begin{aligned}
&\dot{H}_z^{t+\Delta t}(x,y)\\
&= \frac{u_-^*}{u_+^*}\dot{H}_z^t(x,y) - \frac{1}{u_+^*\dot{S}_\gamma(\Delta x)}\\
&\quad \times \left\{\dot{E}_y^{t+\Delta t/2}\left(x+\frac{\Delta x}{2}, y\right) - \dot{E}_y^{t+\Delta t/2}\left(x-\frac{\Delta x}{2}, y\right)\right\}\\
&\quad + \frac{1}{u_+^*\dot{S}_\gamma(\Delta y)}\\
&\quad \times \left\{\dot{E}_x^{t+\Delta t/2}\left(x, y+\frac{\Delta y}{2}\right) - \dot{E}_x^{t+\Delta t/2}\left(x, y-\frac{\Delta y}{2}\right)\right\}
\end{aligned} \tag{4}$$

where $\dot{E}_x$, $\dot{E}_y$, and $\dot{H}_z$ are the electromagnetic field components in complex form, t is the time, Δt is the time increment, and Δx and Δy are the spatial increments in the x and y directions. Further, $u_\pm$ and $u_\pm^*$ are given by

$$u_\pm = \frac{\varepsilon_0 \varepsilon_r}{S_\omega(\Delta t)} \pm \frac{\sigma_e}{\tilde{S}_\omega(\Delta t)} \tag{5}$$

$$u_\pm^* = \frac{\mu_0 \mu_r}{S_\omega(\Delta t)} \pm \frac{\sigma_h}{\tilde{S}_\omega(\Delta t)} \tag{6}$$

where $S_\omega(\Delta t) = 2\sin(\omega \Delta t/2)/\omega$ and $\tilde{S}_\omega(\Delta t) = 2\cos(\omega\Delta t/2)$. Here, ω is the angular frequency, ε_0 and μ_0 are the permittivity and permeability in a vacuum, ε_r and μ_r are the relative permittivity and permeability of the medium, σ_e is the conductivity, and σ_h is the magnetic loss. Note that for the optimization parameters α_{0x} and α_{0y} in (2)–(4), we use the values that provide the most isotropic propagation characteristic at the given frequency and for the spatial increments Δx and Δy. These optimum values are derived using the method in [2].

III. Numerical Propagation Constant

The propagating wave in a lossy medium can be expressed as

$$\psi^n(x,y) = \psi_0^n e^{j\omega \Delta t - \dot{\gamma}_{nx} x - \dot{\gamma}_{ny} y} \tag{7}$$

The difference (2)–(4) can be rewritten as difference equations for the Fourier mode by substituting (7). Here, $\dot{\gamma}_{nx}$ and $\dot{\gamma}_{ny}$ in (7) are the numerical complex propagation constants in the x and y directions, n is the time step, and ψ_0 is the amplitude. The following equation for the numerical propagation constant of the CNS-FDTD method is obtained:

$$\left[\varepsilon_0 \varepsilon_r \frac{2j\sin\left(\frac{\omega\Delta t}{2}\right)}{S_\omega(\Delta t)} + \sigma_e \frac{2\cos\left(\frac{\omega \Delta t}{2}\right)}{\tilde{S}_\omega(\Delta t)}\right] \cdot \left[\mu_0 \mu_r \frac{2j\sin\left(\frac{\omega\Delta t}{2}\right)}{S_\omega(\Delta t)} + \sigma_h \frac{2\cos\left(\frac{\omega \Delta t}{2}\right)}{\tilde{S}_\omega(\Delta t)}\right] - \sum_{\xi = x,y} \left(\frac{2\sinh\left(\frac{\dot{\gamma}_{n\xi}\Delta\xi}{2}\right)}{\dot{S}_\gamma(\Delta \xi)}\right)^2 \dot{\rho}_\xi = 0. \tag{8}$$

Here, ρ_x and ρ_y are given by

$$\dot{\rho}_x = \alpha_{0x} + (1-\alpha_{0x})\cosh(\dot{\gamma}_{ny}\Delta y) \tag{9}$$

$$\dot{\rho}_y = \alpha_{0y} + (1-\alpha_{0y})\cosh(\dot{\gamma}_{nx}\Delta x). \tag{10}$$

If (8) is solved by Newton's method for $\dot{\gamma}_n$ with the theoretical propagation constant $\dot{\gamma}$ as the initial value, then the numerical propagation constant $\dot{\gamma}_n = \alpha_n + j\beta_n$ can be obtained.

IV. Stability Condition

The stability analysis was carried out using the method described in [7]. We insert the following waveform with $t = n\Delta t$

$$\psi^n(x,y) = z_0^n e^{-j(\beta_{nx}x + \beta_{ny}y)} \tag{11}$$

into the difference (2)–(4) representing a lossy medium, where z_0 is the complex amplitude and β_{nx} and β_{ny} are the numerical phase constants in the x and y directions. If the amplitude of z_0 is less than 1 for all frequencies, the system is considered stable. Finally, the following matrix representation can be obtained:

$$v^{n+1/2} = [G]v^{n-1/2} \tag{12}$$

where $v^n = [\dot{E}_x^n \quad \dot{E}_y^n \quad \dot{H}_z^{n+1/2}]^T$.

The matrix G can be expressed as follows:

$$G = \begin{bmatrix} U & 0 & -\frac{A_y \rho_y}{u_+} \\ 0 & U & \frac{A_x \rho_x}{u_+} \\ \frac{-A_y U}{u_+^*} & \frac{A_x U}{u_+^*} & U^* + \frac{(A_x^2 \rho_x + A_y^2 \rho_y)}{u_+ u_+^*} \end{bmatrix} \tag{13}$$

where $U = u_-/u_+$, $U^* = u_-^*/u_+^*$ and $A_\xi = 2\sinh(j\beta_{n\xi}\Delta\xi/2)/\dot{S}_\gamma(\Delta\xi)(\xi = x, y)$. The eigenvalues g's of this matrix G are the amplification factors. Hence, the stability condition of the CNS-FDTD method in a lossy medium is given by $|g| \leq 1$. Note that the analytical expression for $|g|$ is extremely cumbersome, hence, (13) is evaluated numerically in this paper.

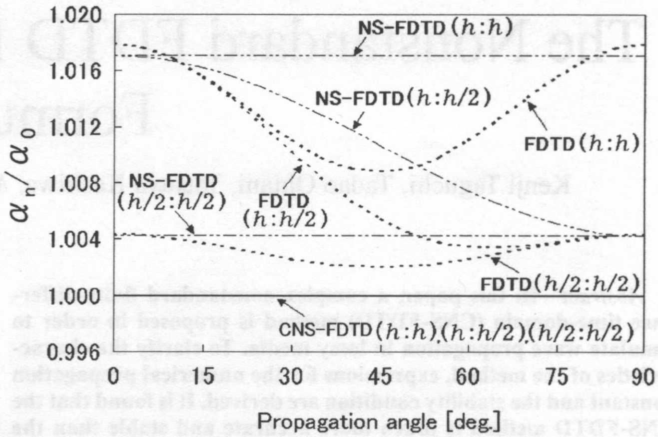

Fig. 1. Numerical attenuation constant in ferrite.

V. Numerical Results

First, the numerical propagation constant and the stability condition of the CNS-FDTD method in a sintered ferrite medium are studied. The characteristics of a fin-type ferrite absorber are then analyzed using the proposed method.

A. Numerical Propagation Constant

Figs. 1 and 2 show the numerical propagation constant of the present method in a sintered ferrite medium obtained from (8). For comparison, the results obtained by the FDTD method and the NS-FDTD method extended for a lossy medium [5] are also presented. Fig. 1 presents the numerical attenuation constant α_n/α_0 normalized to the theoretical attenuation constant α_0. Fig. 2 shows the normalized numerical phase velocity $C_n/C_0 = \beta_0/\beta_n$ derived from the numerical phase constant β_n. C_0 is the theoretical phase velocity in the lossy medium and β_0 is the theoretical phase constant; θ on the horizontal axis is the propagation angle measured from the x axis. In the figure, the quantities inside parentheses are the spatial increments in the x

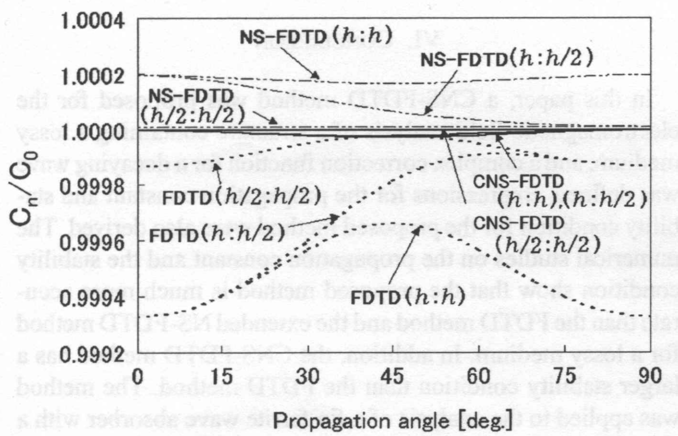

Fig. 2. Numerical phase velocity in ferrite.

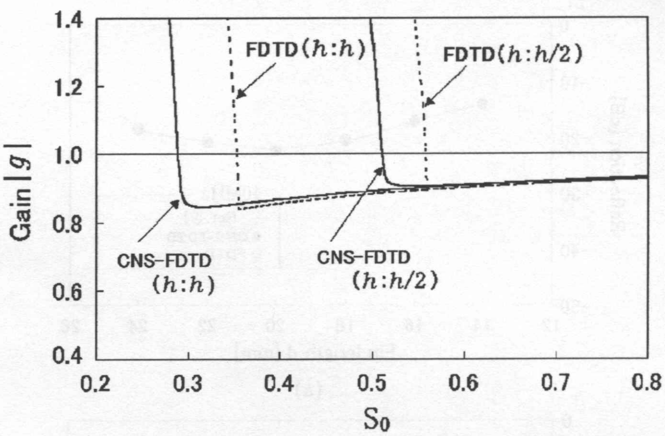

Fig. 3. Amplitude gain in ferrite.

and y directions. The frequency used is $f = 4$ GHz. The complex relative permittivity of the sintered ferrite is $\dot{\varepsilon}_r = 16$ and the complex relative permeability is expressed as follows [9]:

$$\dot{\mu}_r = 1 + \frac{K}{1 + j\frac{f}{f_r}} \tag{14}$$

where $K = 1000$ and $f_r = 6$ MHz. The time increment is $\Delta t = h/(2c_0)$. Here, c_0 is the speed of light in a vacuum and the reference value of the spatial increment is $h = 1$ mm. The values of $\alpha_{0\xi}(\xi = x, y)$ used for h and $h/2$ in the ferrite medium are 0.832 453 5 and 0.833 114 0, respectively, derived from [2]. As shown in Fig. 1, the attenuation constants computed by the FDTD method and the extended NS-FDTD method contain angle-dependent errors. However, almost no error appears in the attenuation constant computed using the CNS-FDTD method. The results for the phase constant in Fig. 2 show that angle-dependent errors appear in the result obtained by the FDTD method and the extended NS-FDTD method. Whereas, the angle-dependent error appearing in the CNS-FDTD method is less than 0.0028%.

Thus we see that the CNS-FDTD method provides extremely isotropic and accurate propagation characteristics, superior to those obtained by the FDTD method and the extended NS-FDTD method for a lossy media.

B. Stability Condition

Fig. 3 shows the amplitude gain $|g|$ of the sintered ferrite medium computed from the stability condition. For comparison, the results obtained by the FDTD method are presented. The quantities inside parentheses are the spatial increments in the x and y directions. The horizontal axis represents $S_0 = h/(c_0\Delta t)$. Therefore, a reduction in S_0 means an increase in Δt. The frequency, the material properties of the ferrite and the value of $\alpha_{0\xi}(\xi = x, y)$ are identical to those used in Section V.A. The thin horizontal line in the figure is the reference line for $|g| = 1.0$. In the figure, it is shown that the CNS-FDTD method has a larger stability condition than the FDTD method.

C. Analysis of Fin Ferrite Absorber

The characteristics of a fin ferrite electromagnetic wave absorber were analyzed using the proposed method. In Fig. 4, the

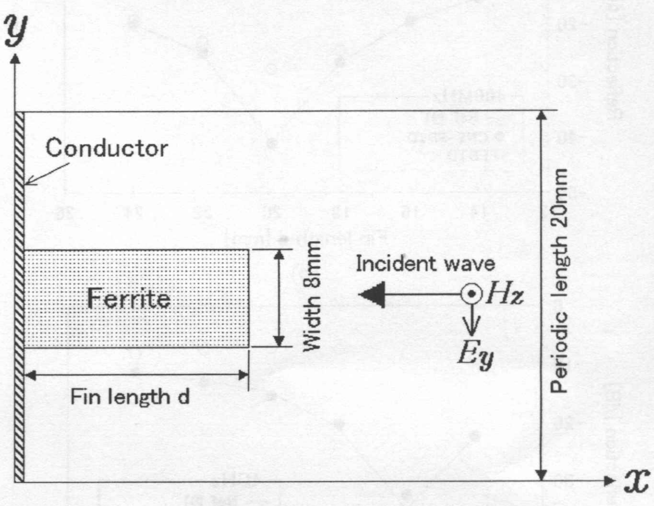

Fig. 4. Cross-sectional view of the fin ferrite electromagnetic wave absorber.

fin ferrite absorber is shown. The width of the ferrite fin is 8 mm and the spatial periodic length is 20 mm. The boundary condition for periodic structure was realized by using the method of [8]. Fig. 5 shows the reflection coefficient of the absorber at frequencies of 40 MHz, 400 MHz, and 4 GHz. In Fig. 5, results obtained by the FDTD method under the same computational conditions, and the results obtained using the mode matching method given in [9] are also shown. The vertical axis in Fig. 5 gives the amount of reflection for a normally incident plane wave and the horizontal axis represents the length d of the ferrite fin. The spatial increments in the x and y direction are equal to $h = 1$ mm. The time increment is $\Delta t = h/(2c_0)$. The material properties of the ferrite are identical to those in Section V.A. When using the proposed method, only the medium boundary region was used the FDTD method.

In is clear from Fig. 5 that the results of the CNS-FDTD method exhibit excellent agreement with those obtained by the mode matching method [9] at each frequency. In the resonant region, where the amount of reflection decreases rapidly, even small phase and amplitude errors can cause large differences in [9] at lower frequencies where the spatial increment h with respect to the wavelength becomes smaller. Hence, in order to obtain the reflection characteristics at 4 GHz by the FDTD method, it is estimated to be necessary to make the

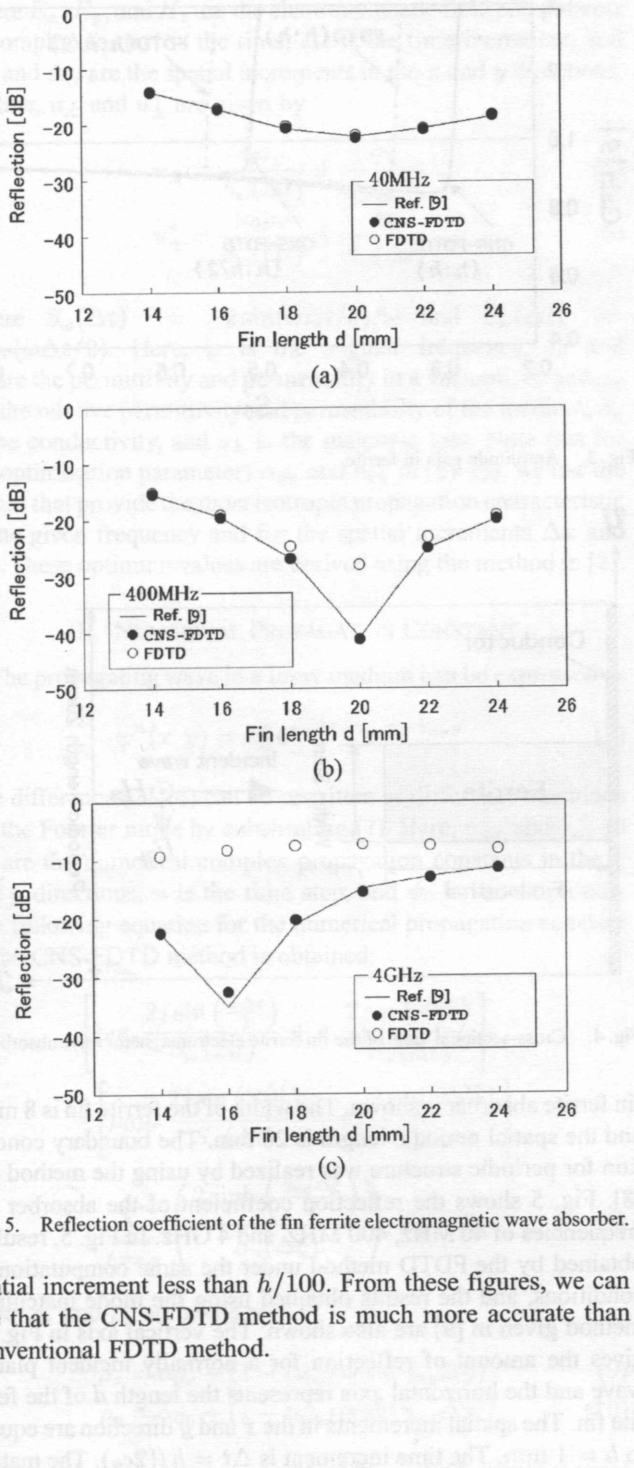

Fig. 5. Reflection coefficient of the fin ferrite electromagnetic wave absorber.

spatial increment less than $h/100$. From these figures, we can see that the CNS-FDTD method is much more accurate than conventional FDTD method.

VI. Conclusion

In this paper, a CNS-FDTD method was proposed for the electromagnetic field analysis of a structure containing a lossy medium, and a complex correction function for a decaying wave was defined. Expressions for the propagation constant and stability condition for the proposed method were also derived. The numerical studies on the propagation constant and the stability condition show that the proposed method is much more accurate than the FDTD method and the extended NS-FDTD method for a lossy medium. In addition, the CNS-FDTD method has a larger stability condition than the FDTD method. The method was applied to the analysis of a fin ferrite wave absorber with a periodic configuration. It was shown that the method is highly accurate and effective for electromagnetic wave analysis.

References

[1] K. S. Yee, "Numerical solution of initial boundary value problem involving Maxwell's equation in isotropic media," *IEEE Trans. Antennas Propagat.*, vol. AP-14, pp. 302–307, May 1966.

[2] J. B. Cole, "A high-accuracy realization of the Yee algorithm using non-standard finite differences," *IEEE Trans. Microwave Theory Tech.*, vol. 45, pp. 991–996, June 1997.

[3] ——, "Application of nonstandard finite differences to solve the wave equation and Maxwell's equations," in *Applications of Nonstandard Finite Difference Schemes*, R. E. Mickens, Ed, Singapore: World Scientific, 2000, pp. 109–153.

[4] N. V. Kantartzis and T. D. Tsiboukis, "A generalized methodology based on higher-order conventional and nonstandard FDTD concepts for the systematic development of enhanced dispersionless wide-angle absorbing perfectly matched layers," *Int. J. Numerical Modeling*, vol. 13, no. 5, pp. 417–440, 2000.

[5] T. Ohtani, Y. Sendo, and T. Kashiwa, "Non-standard FDTD method in lossy media" (in Japanese), *IEICE Trans.*, vol. J85-C, no. 10, pp. 889–897, Oct. 2002.

[6] T. Ohtani, H. Kudo, and T. Kashiwa, "Scattering analysis of large-scale cavities using the nonstandard FDTD method," *Electron. Commun. Japan, Part 2*, vol. 86, no. 5, pp. 66–76, 2003.

[7] P. G. Petropoulos, "Stability and phase error analysis of FD-TD in dispersion dielectrics," *IEEE Trans. Antennas Propagat.*, vol. 42, pp. 62–69, Jan. 1994.

[8] P. Harms, R. Mittra, and W. Ko, "Implementation of the periodic boundary condition in the finite-difference time-domain algorithm for FSS structure," *IEEE Trans. Antennas Propagat.*, vol. 42, pp. 1317–1324, Sept. 1994.

[9] Y. Naito, M. Takahashi, T. Mizumoto, and H. Nose, "Wide-band characteristics of fin ferrite electromagnetic wave absorber" (in Japanese), *IEICE Trans.*, vol. J76-B-II, no. 7, pp. 641–646, July 1993.