

# Fusion cross-sections for inertial fusion energy

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## Abstract

The application of selective resonant tunneling model is extended from  $d + t$  fusion to other light nucleus fusion reactions, such as  $d + d$  fusion and  $d + {}^3\text{He}$ . In contrast to traditional formulas, the new formula for the cross-section needs only a few parameters to fit the experimental data in the energy range of interest. The features of the astrophysical S-function are derived in terms of this model. The physics of resonant tunneling is discussed.

**Keywords:** Astrophysical S-function; Fusion cross-section; Inertial fusion; Light nucleus fusion; Selective resonant tunneling

## 1. INTRODUCTION

Since the discovery of the first fusion reaction with hydrogen isotopes (Oliphant *et al.*, 1934) several exothermic branches are appearing, however, for magnetic confinement fusion only, the deuterium tritium (DT) reaction is possible for harvesting fusion energy as a future energy source. This limit is due to the fact that the cyclotron radiation due to the magnetic fields can be compensated by the produced nuclear energy only (Trubnikov, 1958). There are alternative reactions as that of DD or of deuterium with helium-3, and further reactions of hydrogen isotopes with lithium with their neutron lean products (Miley, 1976), or even of reacting protons with boron-11 producing less radioactivity than burning coal (Weaver *et al.*, 1973; Hora, 1991), and permitting direct conversion of nuclear fusion energy into electricity such that power stations would have nearly no cooling problem. These alternative reactions, however, can succeed only by inertial confinement primarily using lasers (Pellat, 2003; Tarter, 2003) or particle beams (Rubbia, 1993) instead of magnetic confinement.

Fusion reactions are studied worldwide on a large scale since about 1942 for exothermic reactions. Despite this fact and several engineering type solutions, the result that the DT or DD reactions occur with very large cross-sections at distances of the nuclei more than 50 times of their diameter (more than 50 fm) at impact energies of 5 keV or even less, was not clarified until recently while it is well known that

nuclear reactions usual occur only at impact energies of more than MeV moving together, the nuclei against the Coulomb repulsion to distances of a few fm. There were numerous theoretical attempts to explain the measured fusion cross-sections. These were only numerical adjustments that are by fitting five parameters. An example of these fitting with reference to preceding attempts is given by Clark *et al.* (1978) for  $d + {}^6\text{Li}$  fusion reaction.

After all these developments and despite the high importance of the problem, it was possible only recently (Li *et al.*, 2000) to develop a basically funded first theory for deriving the DT fusion cross-sections from a complex Schrödinger potential. Such complex potential is well known and was thought only for medium or large nuclei (atomic weight  $A > 15$  see: Feshbach, 1992) where the imaginary part in the Schrödinger potential may be interpreted as an optical nuclear model. The use of such potential for small nuclei was successful for the first time only for DT (Li *et al.*, 2000). The input into this model is given by two empirical numbers, the resonance energy (about 110 keV for DT) and the measured fusion cross-section at the peak (5.01 barns), two very obvious parameters in contrast to the aforementioned five (or more) numerically fit constants.

In view of the interests for the fusion research by inertial confinement, we are presenting here our results on the extension of the theory for DT (Li *et al.*, 2000) to cases as DD and  $D^3\text{He}$ . The theory (Li *et al.*, 2000) may be a step in the direction to explain the interplay of Coulomb repulsion of nucleons in nuclei, compared with internal energy which is dominated for very large nuclei by the Fermi energy of the nucleons resulting then in a theory of nuclear forces domi-

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nated by Debye length surface mechanisms. This led to the confinement of nucleons in nuclei just at the well known nuclear density (Hora *et al.*, 2004). It may be speculated that the situation for very small nuclei—also in view of the Coulomb repulsion—may need to be modified where the present theory (Li *et al.*, 2000) may provide an access.

Having derived these cross-sections with better accuracy, we believe that the calculation for the fast ignition (Vatulin *et al.*, 2002), some criterions for astrophysics (Grun *et al.*, 2003), the necessary condition especially for direct drive (Hora *et al.*, 1999), and femtosecond lasers interacting with a precompressed deuterium-tritium (DT) fuel (Deutsch *et al.*, 2000) might be considered again.

## 2. THE TUNNELING MODEL WITH TWO DEPENDENT STEPS

In the past 50 years, there was some different development between plasma fusion scientists and the nuclear physicists. The plasma fusion scientists assumed that the nuclear physics problem has been solved already, and they are supposed to concentrate on high temperature plasma physics only. On the other hand, the nuclear physicists believe that the controlled nuclear fusion is the problem of plasma physics, because the nuclear physics problem has been solved in the astrophysics. They always cited the Clayton's classical book: *The Principles of Stellar Evolution and Nucleosynthesis* (Clayton, 1990). In this book, the nuclear fusion process is divided into two independent steps: the penetration of the Coulomb barrier and the fusion reaction. Even if in the case of the resonant tunneling for the penetration, in that book the concept of two independent steps is still applied to calculate the nuclear fusion cross-section. Unfortunately, there was a difficulty in understanding the nuclear physics for nuclear fusion. Particularly, it might lead to a different result on the products of the nuclear fusion reaction after the resonant tunneling.

There are two key points in any resonant process: (1) The resonant process relies on the memory of the phase of the wave, because the resonance depends on the constructive interference of the wave; (2) The resonant process takes time, because the amplitude of the wave function inside the nuclear well can not be enhanced immediately by the constructive interference in a single bouncing back and forth motion. When the resonant tunneling was applied to explain the penetration of the Coulomb barrier, we have to keep these two key points in mind. However, when the fusion scientists tried to understand the fusion process inside the nuclear well, they always unconsciously applied the compound nucleus model, and these two points was somehow tacitly ignored. Then, they expected to observe the neutron emission regardless of the injected energy for the d + d fusion reaction. We shall show that when these two steps are dependent, the neutron emission is no longer the necessary product for the low energy d + d fusion reaction.

A simple quantum mechanics model has been set up to consider this dependence between the penetration and the fusion reaction (Li *et al.*, 1999, 2000). A square well inside the nuclear interaction range and a Coulomb barrier outside this nuclear interaction range are assumed to keep the memory of the phase factor of the wave function. An imaginary part of the nuclear potential is introduced to describe the life-time of the wave function inside the nuclear well. This simple model gives surprisingly good result for the d + t fusion cross-section (Li *et al.*, 2002); hence, this model has been refined for d + t fusion, and further extended to d + d fusion and d + <sup>3</sup>He fusion. A good agreement with the experimental data has been obtained again. Particularly, the expression of the astrophysical S-function for d + t fusion is derived in terms of this selective resonant tunneling model. It clearly shows the dependence between these two steps. At the end, the implication of this resonant tunneling model is briefly discussed.

## 3. THE DEPENDENCE BETWEEN TWO STEPS OF THE FUSION PROCESS

In order to show this dependence, we start from an identity (see Appendix A3). This identity can be used to express the fusion cross-section,  $\sigma$  in terms of the phase shift. This phase shift will clearly show the memory of the phase factor of the wave function, and the effect of the life-time of the wave function inside the nuclear well, that is:

$$\sigma = \frac{\pi}{k^2} \frac{(-4W_i)}{W_r^2 + (W_i - 1)^2}. \quad (1)$$

Here,  $W$  is defined as the cotangent of the phase shift,  $\delta_o$ , of the s-partial wave (in the energy range of interests, only the s-partial wave of the relative motion between the projectile and the target is dominant), that is:

$$W = \cot \delta_o. \quad (2)$$

When the nuclear potential is a complex number, this phase shift is also a complex number also. In stead of using the complex  $\delta_o$ , we introduce the real and imaginary parts of,  $W = \cot \delta_o$  that is:

$$W = W_r + iW_i. \quad (3)$$

Equation (1) shows the resonant feature, because the cross-section has a peak at  $W_r = 0$ . This peak reaches its maximum at  $W_i = -1$ . (As it can be shown,  $W_i$  is always less than zero for any absorption reaction inside the nuclear well).

The continuity of the logarithmic derivative of the wave function at the edge of the nuclear well will give a simple expression for  $W$  as

$$W = \theta^2 \left\{ (k_1 a_c) \cot(k_1 a_0) - 2 \left[ \ln \left( \frac{2a_0}{a_c} \right) + 2C + h(ka_c) \right] \right\}. \quad (4)$$

Here  $\theta^2$  is related to the Gamow penetration factor, and is a large number for low energy fusion reactions,

$$\theta^2 = \frac{1}{2\pi} \left[ \exp\left(\frac{2\pi}{ka_c}\right) - 1 \right] \quad (5)$$

$$k^2 = \frac{2\mu}{\hbar^2} E \quad (6)$$

$$a_c = \frac{\hbar^2}{\mu z_1 z_2 e^2}, \text{ (the unit of length in the Coulomb unit system)} \quad (7)$$

$$a_0 = r_0(A_1^{1/3} + A_2^{1/3}), \text{ (the radius of the nuclear well).} \quad (8)$$

The first term in the curly braces of Eq. (4) is from the logarithmic derivative of the wave function inside the nuclear well, which contains the complex nuclear potential, and makes  $W$  complex.

$$k_1^2 = \frac{2\mu}{\hbar^2} (E - U_{1r} - iU_{1i}) \quad (9)$$

$$h(ka_c) = \text{Re}[\psi(-i/(ka_c))] + \ln(ka_c) \quad (10)$$

$$\psi(s) = \frac{\Gamma'(s)}{\Gamma(s)}, \text{ } (\Gamma(s) \text{ is the Gamma function}). \quad (11)$$

$\mu$  is the reduced mass,  $E$  is the energy of the relative motion in the center of mass system;  $\hbar$  is the Planck constant divided by  $2\pi$ ;  $Z_1e$  and  $Z_2e$  are the electrical charge of the projectile and the target, respectively.  $A_1$  and  $A_2$  are the mass number of the projectile and the target, respectively.  $r_0 = 1.746 \times 10^{-15}$  m to insure that the diameter of the deuteron is  $4.4 \times 10^{-15}$  m (experimental data, Feshbach, 1992);  $C = 0.57721 \dots$  is the Euler's constant. Finally,  $U_{1r}$  and  $U_{1i}$  are the real and imaginary parts of the nuclear potential, respectively (see Fig. 1). They are the two adjustable parameters in this model.

Now it is evident from Eqs. (1) and (4) that the cross-section will show very strong dependence on  $E$  due to  $\theta^2$  if the terms in the curly braces of Eq. (4) are not small. The cross-section will be suppressed by the Gamow factor ( $1/\theta^2$ ). When the phase of the wave function inside the nuclear well (i.e.,  $(k_1 a_0)$ ) makes  $W_r = 0$ , and  $W_i = -1$ ; then,  $\theta^2$  will disappear in the expression for the cross-section. That is to say, the resonance will overcome the suppression of the Gamow factor; and thus, will greatly enhance the fusion cross-section.

The imaginary part of the nuclear potential ( $U_{1i}$ ) will affect the imaginary part of the  $W$  through the first term in the curly braces of Eq. (4). From Eq. (4), we know that only if

$$(k_{1r} a_0) \approx (2m + 1)\pi/2, \text{ } (m \text{ is an integer}) \quad (12)$$

$$(k_{1i} a_0) \approx 1/\theta^2; \quad (13)$$

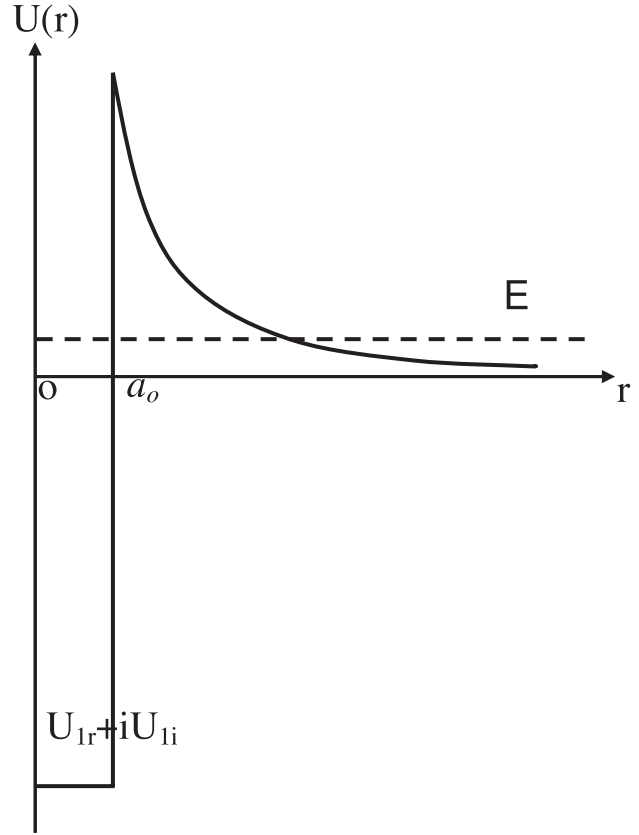


Fig. 1. Complex nuclear potential and Coulomb barrier.

then, there will be a chance for

$$W_i \approx -1. \quad (14)$$

Since the imaginary part of  $k_1$  is related to both the imaginary part of nuclear potential and the real part of the  $k_1$  as

$$k_{1i} a_0 = \frac{\mu}{\hbar^2} \left( \frac{-U_{1i}}{k_{1r}} \right) a_0 \approx \frac{|U_{1i}/\hbar|}{k_{1r}\hbar/(\mu a_0)} \approx \frac{1/\tau_{life}}{1/\tau_{flight}} \approx \frac{\tau_{flight}}{\tau_{life}}. \quad (15)$$

The requirement of  $(k_{1i} a_0) \approx 1/\theta^2$  (Eq. (13)) leads to

$$\tau_{life} \approx \theta^2 \tau_{flight}. \quad (16)$$

Here,  $\tau_{life} \approx |\hbar/U_{1i}|$  is the life-time of the wave function inside the nuclear well; and  $\tau_{flight} \approx a_0/(k_{1r}\hbar/\mu)$  is the flight-time of the wave bouncing back and forth inside the nuclear well. If  $\tau_{life} \gg \tau_{flight}$ ; then, will there be enough time for the resonant enhancement of the amplitude of the wave function to occur? The lower the injection energy,  $E$ , is; the greater the  $\theta^2$  will be. Thus at the low energy the resonant tunneling would appear only for the nuclear reaction with very long life-time. Eq. (16) clearly shows that the penetration and the fusion reaction are no longer two independent steps; instead, the penetration factor ( $\theta^2$ ) will select a reac-

tion channel with a matching life-time ( $\tau_{life}$ ). In order to emphasize this dependence between two steps, we may call it as the selective resonant tunneling model. The thick and high Coulomb barrier would allow a resonant tunneling only for the long life-time fusion reaction channel. That is to say: at low energy, if any  $d + d$  fusion reaction happens, the neutron emission is no longer a necessary product.

This conclusion is very different from the conventional concept. In Clayton's book (Clayton, 1990), the fusion cross-section is written as a product of 3 factors:

$$\sigma_c \propto \Gamma_a P(E) (\Gamma - \Gamma_a). \quad (17)$$

$\Gamma_a P(E)$  is proportional to the probability of forming this resonant state with energy  $E$ ; and  $(\Gamma - \Gamma_a)$  is proportional to the decay rate of the resonant state to any other channels. This formula is based on the compound nucleus model and the Breit-Wigner formula. The essential concept in Eq. (17) is that the compound nucleus will forget its history of formation; hence, it will lose the memory of the phase factor of the wave function. The direct inference from Eq. (17) is that the resonant state will decay into the channel with the shortest life-time, no matter what is the height and width of the Coulomb barrier. This is the basic difference between Eqs. (1) and (17). We understand that this difference will be most evident in the case of the light nucleus fusion, because the com-

pound nucleus model is most likely to fail for light nuclei. Hence, this new formula (1) was applied to the case of fusion of the light nuclei first.

#### 4. FUSION CROSS-SECTION OF LIGHT NUCLEI

##### 4.1. $d + T$ fusion

$d + T$  fusion is the most important fusion cross-section, because it has the greatest cross-section, and it also has the well-known resonant feature. It has been studied carefully in a series of experiments, and has been evaluated using R-matrix theory, and is documented in the ENDF/B-VI. The crosses in Figure 2 show the ENDF data points. The solid line shows the calculated values using resonant tunneling theory (Eq. (1)). The agreement between the ENDF data points and the theoretical calculation supports strongly our assertion that there is dependence between two parts of the fusion process. It is well expressed by this simple model using  $U_{lr} = -47.67$  MeV,  $U_{li} = -117.29$  keV. The lowest energy in ENDF/B-VI is 200 eV. Our two parameter formula agrees with this ENDF/B-VI data in the whole energy region from 200 eV to 500 keV. The dotted line shows the early fitting curve based on the five empirical parameters

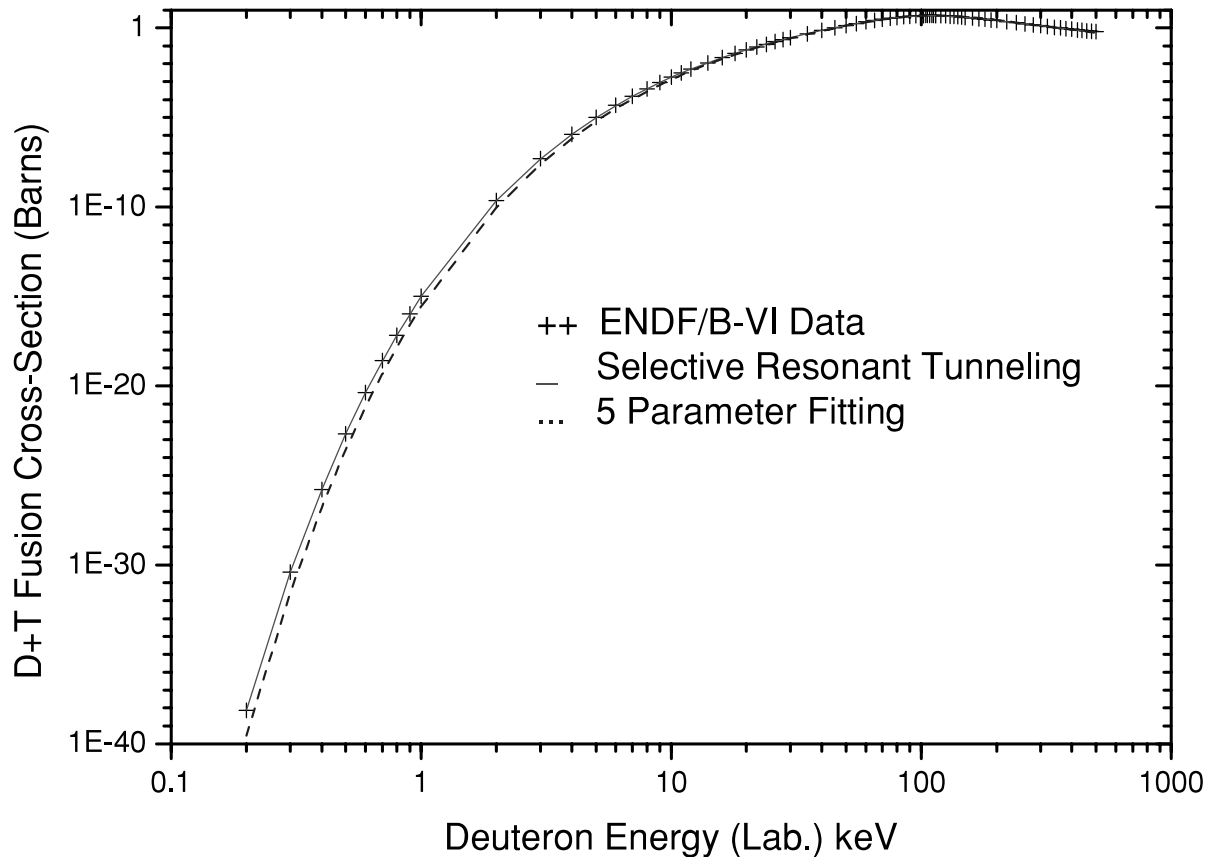


Fig. 2. Comparison between experimental data (ENDF) and theoretical calculation for  $d + t$  fusion cross-section.

(Duane, 1972). It is evident that the five parameter curve is worse than our two parameter curve in comparison with the ENDF/B-VI data (particular at the low energy region the deviation is by a factor of 100).

This selective resonant tunneling model was further confirmed when it discovered an error in the NNDC data base (National Nuclear Data Center in Brookhaven) in October 1999 (Li, 2002).

#### 4.2. d + D fusion

We will further see the difference between this selective resonant tunneling model and the compound nucleus model in the case of d + d fusion. In the beam-target experiments, d + d fusion reaction has three branches:

$$d + d \rightarrow \begin{cases} {}^3\text{He} + n \\ T + p \\ {}^4\text{He} + \gamma \end{cases} \quad (18)$$

In the selective resonant tunneling model, only one parameter,  $U_{li}$ , is used to describe the life-time of the penetrating projectile. It does not distinguish the different channels. Regardless of whether the nuclear reaction goes to the neutron channel, to the proton channel, or to the gamma emission channel, the deuteron wave function will be destroyed

(the resonant process will stop) whenever fusion reaction happens. Hence, in the selective resonant tunneling model, we are able to calculate the total fusion cross-section only, and not the cross-sections for the separate branches. Thus the ENDF data for the neutron channel and for the proton channel are added together in order to compare with the calculation of selective resonant tunneling model (the gamma emission channel is negligible here). Figure 3 shows the comparison using  $U_{lr} = -63.04$  MeV;  $U_{li} = -210$  keV. The agreement between theoretical calculation (solid line) and the ENDF data (crosses) is still reasonably good, if we consider that the precision of the experimental data for d + d is not as good as that for d + t.

#### 4.3. d + ${}^3\text{He}$ fusion

d +  ${}^3\text{He}$  fusion is the mirror image of the d + T fusion in the electrical charge space. We may expect that the symmetry in electrical charge will result in a similar nuclear potential because the nuclear force is supposed to be independent of the electrical charge. Indeed the dependence between penetration and the nuclear reaction plays a role again here. The Coulomb barriers for d + T and d +  ${}^3\text{He}$  fusion are very different ( $a_c$  in Eqs. (7) and (4) is different). Hence, we could not fit the ENDF data for d +  ${}^3\text{He}$  fusion until we changed the radius of the nuclear well. The  $r_0$  in Eq. (8) has

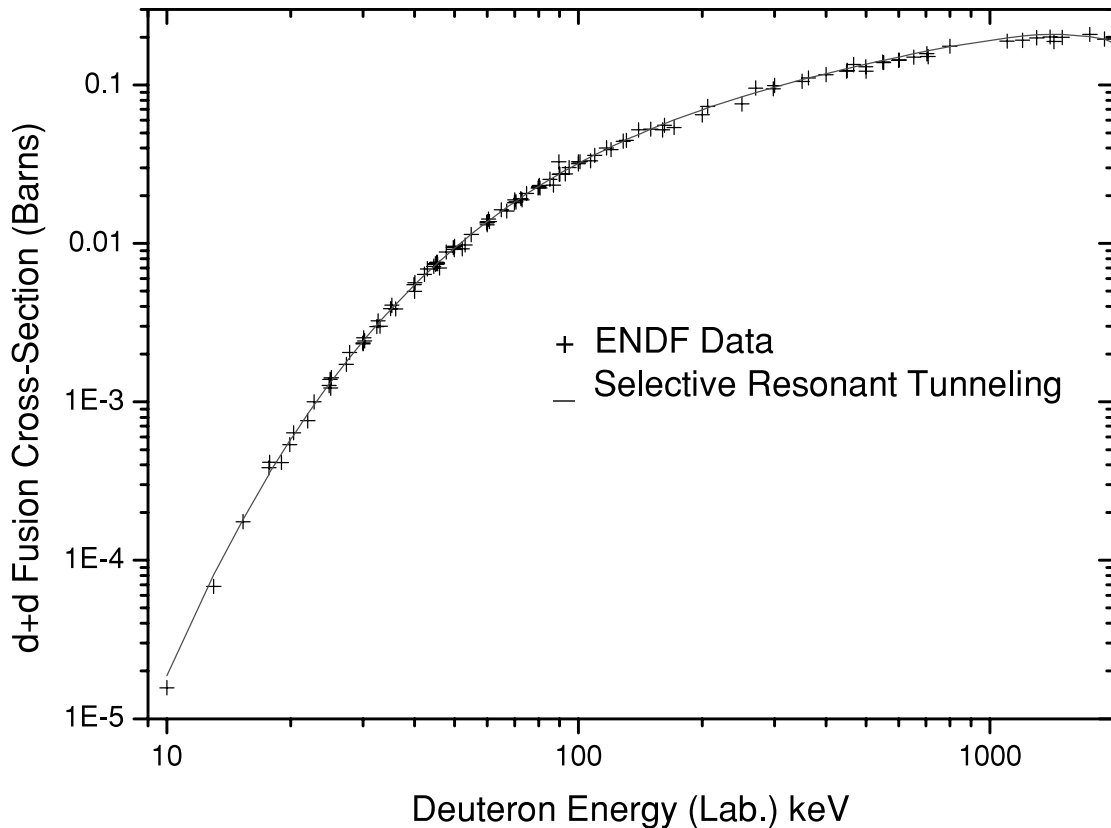


Fig. 3. Comparison between experimental data (ENDF) and theoretical calculation for d + d fusion cross-section.

to be increased in order to fit the experimental data for  $d + {}^3\text{He}$ . Figure 4 shows the result of the calculation using  $U_{1r} = -11.859$  MeV;  $U_{li} = -259.02$  keV; and  $a_0 = 9.0 \times 10^{-15}$  m. In this case we have to adjust three parameters.

## 5. ASTROPHYSICAL S-FUNCTION

Selective resonant tunneling model provides the convenient formalism to calculate the astrophysical S-function also. The astrophysical S-function is defined as:

$$S(E) \equiv \exp[2\pi/(ka_c)]E\sigma(E). \quad (19)$$

Here, we follow the Bosch's definition of  $S(E)$  for consistency (Bosch, 1992).  $\sigma(E)$  is the fusion cross-section in the center of mass system, and  $E$  denotes the energy available in the CM system.  $k$  and  $a_c$  are defined in Eqs. (6) and (7), respectively. The motivation for this definition is to take out the major dependence on the energy  $E$ . Using Eqs. (1) and (4) we can rewrite  $S(E)$  as:

$$S(E) = \frac{\pi^2 \hbar^2}{\mu} \left\{ \frac{\exp[2\pi/(ka_c)]}{\exp[2\pi/(ka_c)] - 1} \right\} \frac{-4w_i}{w_r^2 + (w_i - \theta^{-2})^2}. \quad (20)$$

Here, we introduce the  $w$  as the reduced  $W$ :

$$w \equiv \theta^{-2} W$$

$$= (k_1 a_c) \cot(k_1 a_0) - 2 \left[ \ln \left( \frac{2a_0}{a_c} \right) + 2C + h(ka_c) \right]. \quad (21)$$

Figure 5 shows the result of the calculation of the astrophysical S-function for  $d + t$  fusion. The crosses show the experimental value of astrophysical S-function for the  $d + t$  fusion in the center of mass system (Eq. (19) is used to convert the experimental cross-section data to the S-function). The solid line denotes the result of the calculation using the selective resonant tunneling model (Eq. (20)). The agreement is evidence in support of this selective resonant tunneling model. In the energy range of interests, the factor in the curly braces of Eq. (20) approaches 1, but the  $\theta^{-2}$  in the denominator of Eq. (20) cannot be neglected because  $w_i$  is comparable with  $\theta^{-2}$  near the resonance peak. In Figure 5, the dotted line shows the result of the calculation when the  $\theta^{-2}$  in the denominator of Eq. (20) is intentionally neglected. This comparison suggests that when the S-function in Eq. (20) is expressed as a polynomial in  $E$ , this  $\theta^{-2}$  term should be included. Usually, it is not easy to express

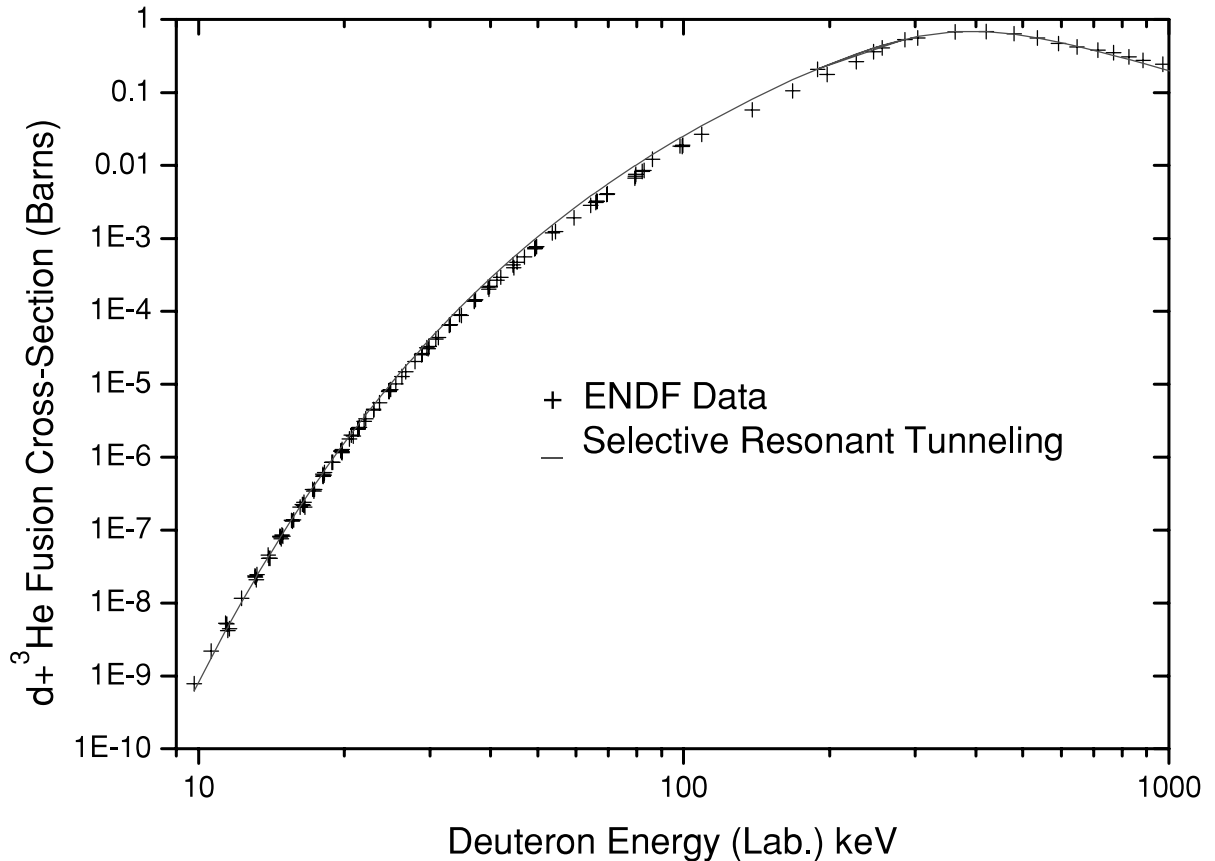


Fig. 4. Comparison between experimental data (ENDF) and theoretical calculation for  $d + {}^3\text{He}$  fusion cross-section.



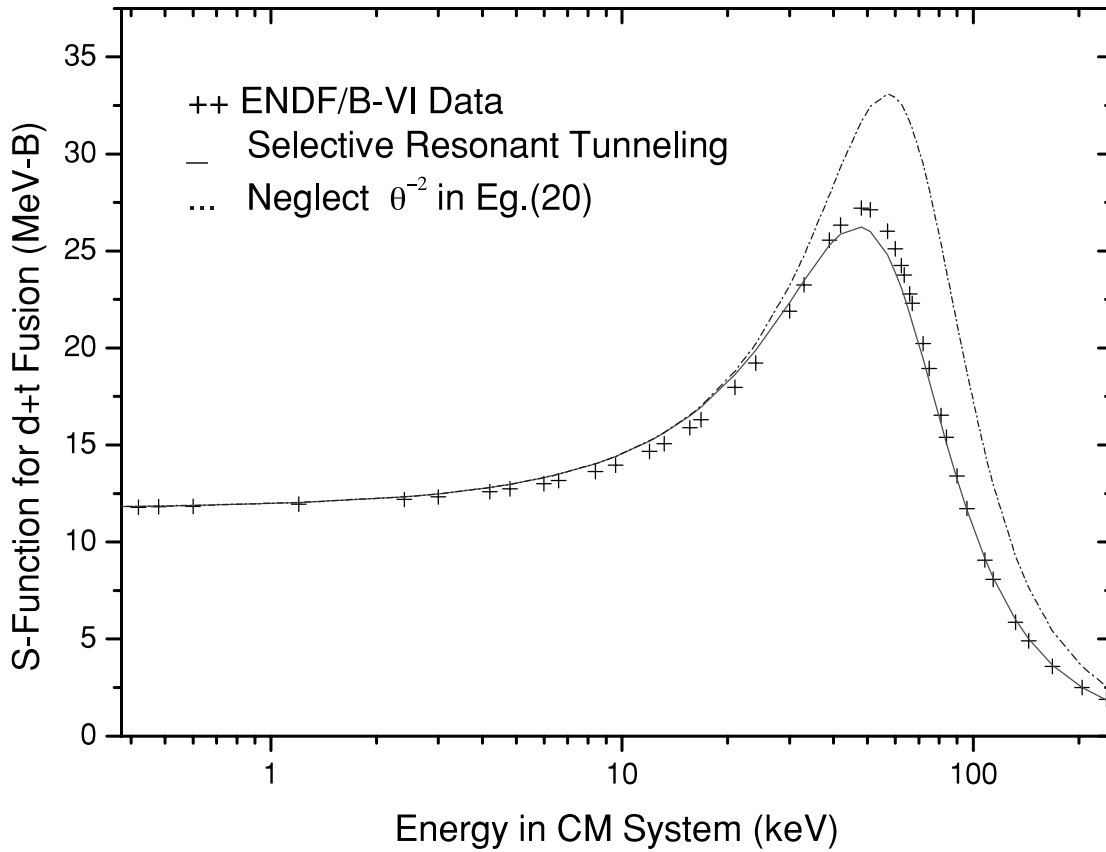


Fig. 5. Comparison between experimental data (ENDF) and theoretical calculation for astrophysical S-function of d + t fusion.

$$\theta^2 = \frac{1}{2\pi} \left[ \exp \left( \frac{2\pi}{ka_c} \right) - 1 \right]$$

as a polynomial of  $E$ . It explains why Bosch needs 10 parameters to fit the S-factor in a polynomial due to the ignorance of the dependence between two steps of the fusion process.

Because of the early trends to separate the fusion process into two independent steps, the fusion cross-section was expressed as a products of three factors: The penetration factor ( $\theta^{-2}$ ), the geometrical (wave length) factor ( $k^{-2}$ ), and the astrophysical S-function. The motivation for this separation was to separate two strongly energy dependent factors ( $\theta^{-2}$  and  $k^{-2}$ ), leaving the S-function to represent mainly the (presumably) slowly varying nuclear part of the fusion reaction probability. The abovementioned  $\theta^{-2}$  dependences in Eq. (20) show that the separation is not perfect because they are not independent steps physically.

## 6. IMPLICATION OF THE SELECTIVE RESONANT TUNNELING MODEL

The selective resonant tunneling model indicates that the resonant tunneling selects not only the energy level (the frequency of resonance) but also the fusion reaction rate (the damping rate). Even if  $w_r = 0$  at certain energy, the effect of

the resonance might still not be observable if the  $w_i \neq -\theta^{-2}$  because

$$\sigma = \frac{\pi}{k^2} \frac{1}{\theta^2} \frac{-4w_i}{w_r^2 + \left( w_i - \frac{1}{\theta^2} \right)^2}. \quad (22)$$

The suppression due to the Gamow factor is apparent when  $\theta^2$  is a very large number at low energy. The cross-section is very small unless both  $w_r \approx 0$ , and  $w_i \approx -\theta^{-2}$  are satisfied.

The selectivity of the resonant tunneling might be considered in two ways. When we know the energy of the projectile; then, we may calculate the  $\theta^{-2}$ , and select the matching damping based on  $w_i \approx -\theta^{-2}$  (or  $\tau_{life} \approx \theta^2 \tau_{flight}$ ). On the other hand, when we know the type of the interaction (such as strong interaction, electromagnetic interaction, or weak interactions, etc.); then, we know the order of the magnitude of the life-time,  $\tau_{life}$ . Thus we might calculate the  $\theta^2 \approx \tau_{life} / \tau_{flight}$ . Consequently, we might predict the energy range at which the resonant tunneling might occur.

## 7. SUMMARY

The experimental data show that fusion of light nuclei can not be separated into two independent steps. Their depen-

dence may be described by a complex square nuclear potential well combined with a Coulomb barrier.

Selective resonant tunneling may overcome the suppression of Gamow factor. This suppression due to Coulomb barrier will disappear as long as  $W_r = 0$  and  $W_i = -1$ . This overcoming will be valid even if the injection energy is very low.

When the injection energy is low,  $\theta^2$  increases rapidly to a very large number; hence, the resonance will select a long life-time to satisfy  $\tau_{life} \approx \theta^2 \tau_{flight}$ . It implies that only the weak interaction channel would be selected for resonant tunneling at low energy. We need to detect the nuclear products of the weak interaction if we are to search for resonant tunneling at low energy.

The Astrophysical S-function may be formulated with a few parameters if we separate all the  $\theta^2$ -dependence first.

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## APPENDIX

### An identity

An identity is useful to show the effect of damping on the resonance. It may be applied to not only in nuclear physics, but also in plasma physics (e.g., when the incident wave frequency is in resonance with the plasma inherent frequency, the resonance will select the damping (the collision frequency as well). Hence, it is derived in detail as follows.

Usually we obtain the  $\cot \delta$  directly from the continuity of the logarithmic derivative of the wave function. On the other hand, we need the expression for  $e^{-4\delta_i}$  to calculate the fusion cross-section. Hence, we need the expression for  $e^{-4\delta_i}$  in terms of  $\cot \delta$ .  $\delta_i$  is the imaginary part of the phase shift of the partial wave function with zero orbital angular momentum (s-wave). For briefness, the subscript “o” is omitted in this Appendix. If we define

$$\cot \delta \equiv W_r + iW_i, \quad (\text{A1})$$

then, using

$$e^{i2\delta} \equiv -\frac{1 - i \cot \delta}{1 + i \cot \delta},$$



we have

$$\begin{aligned}
 e^{-4\delta_i} &= |e^{i2\delta}|^2 = \left( \frac{1 - i(W_r + iW_i)}{1 + i(W_r + iW_i)} \right) \left( \frac{1 + i(W_r - iW_i)}{1 - i(W_r - iW_i)} \right) \\
 &= \frac{[(1 + W_i) - iW_r][(1 + W_i) + iW_r]}{(1 - W_i)^2 + W_r^2} \\
 &= \frac{(1 + W_i)^2 + W_r^2}{(1 - W_i)^2 + W_r^2} \tag{A2}
 \end{aligned}$$

or

$$1 - |e^{i2\delta}|^2 \equiv \frac{-4W_i}{W_r^2 + (W_i - 1)^2} \tag{A3}$$

$1 - |e^{i2\delta}|^2$  is usually related to a non-unitary process. Eq. (A3) clearly shows the resonant feature of the absorption. When  $W_r = 0$ , the right hand side of (A3) reaches its peak. This peak reaches its maximum when  $W_i = -1$ . This peak might be very sharp when  $W_r$  varies rapidly. It implies that for any resonance there is a right frequency ( $W_r = 0$ ); and there is also a matching damping ( $W_i = -1$ ). Usually,  $W_r = 0$  is related to an energy level (frequency);  $W_i = -1$  is related to an absorption process (damping, reaction, decay, . . .). In order to experimentally observe a resonance, we have to remove some energy from the resonance (e.g., the loud-

speaker obtains the energy from the harmonic circuit in the radio receiver). This is just an absorption process. Thus, it is inevitable to have an absorption associated with the observation of a resonance. Usually we would like to have the least damping in order to have the maximum amplitude of the resonance, but Eq. (A3) says that if the observation is based on the absorption; then, there will be a suitable damping for the best observation. This damping might be called as the matching damping, which is neither the least damping, nor the largest damping. It is just the damping in between which corresponds to  $W_i \approx -1$ .

Using the identity (A3), we have the fusion cross-section for s-wave

$$\sigma = \frac{\pi}{k^2} \frac{(-4W_i)}{W_r^2 + (W_i - 1)^2} \tag{A4}$$

(A4) is different from the Breit-Wigner formula for a resonance, where the cross-section is written as:

$$\sigma_r \propto \frac{1}{(E - E_r)^2 + \left(\frac{\Gamma}{2}\right)^2}. \tag{A5}$$

In Eq. (A4) there is no Taylor expansion to obtain the term of  $(E - E_r)^2$ ; hence, Eq. (A4) is more accurate in the wide range of energy for the pragmatic usage.