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# Numerical Simulation as a Tool for Studying Waves and Radiation in Space

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#### **Abstract**

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Plasma physics governs the area of interactions between charged particles. As 99% of the visible universe is in a plasma state, it is an important topic in astronomy and space physics, where we already at an altitude of 60 km reach the plasma environment surrounding our planet in the form of the ionosphere. The search for fusion, the source of power for the sun, as well as industrial use have been the main topics for earth bound plasma research.

A plasma is composed of charged particles which interact by the electromagnetic force. In the kinetic description, via the Vlasov-Maxwell equations, the system is described in terms of probability distribution functions for each particle species, expressed in terms of particles position and velocity. The particles interact via self-consistent fields as determined by Maxwell's equations. For understanding the complex behaviour of the system, we need numerical solvers. These come in two flavours, Lagrangian methods, dealing with the moving around of synthetic particles, and Eulerian methods, which solve the set of partial differential, Vlasov and Maxwell equations. To perform the computations within reasonable time, we need to distribute our calculations on multiple machines, i.e. parallel programming, with the best possible matching between our computational needs and the need of splitting algorithms to adapt to our processing environment.

Paper I studies electron and ion beams within a Lagrangian and fluid model and compare the results with experimental observations. This is continued with studies of a full kinetic system, using an Eulerian solver, for a closer look at electron-ion interactions in relation to ionospheric observations, (Papers II and IV). To improve the performance of the Eulerian solver it was parallelised (Paper III). The thesis is ending with the possibility to observe ultrahigh energy neutrinos from an orbiting satellite by using the Moon's surface as a detector Paper V.

**Keywords:** space physics, plasma physics, kinetic plasma, plasma simulations, beam instabilities

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*To my parents*

# List of Papers

This thesis is based on the following papers, which are referred to in the text by their Roman numerals.

- I P. Guio, S. Børve, L. K. S. Daldorff, J. P. Lynov, P. Michelsson, H. L. Pécseli, J. J. Rasmussen, K. Saeki, and J. Trulsen. Phase space vortices in collision-less plasmas. *Nonlinear Processes in Geophysics*, 2003, Vol. 10, No. 1, 75–86
- II L. K. S. Daldorff, H. L. Pécseli, and J. Trulsen. Nonlinearly generated plasma waves as a model for enhanced ion acoustic lines in the ionosphere. *Geophysical Research Letters*, Vol 34, L18101, 2007,
- III L. K. S. Daldorff, B. Eliasson Parallelization of a Vlasov-Maxwell solver in four-dimensional phase space. *Parallel Computing*, In press.
- IV L. K. S. Daldorff, H. L. Pécseli, J. Trulsen, B. Eliasson, and L. Stenflo Nonlinearly generated plasma waves as a model for enhanced ion acoustic lines in the ionosphere. *New Journal of Physics*, Submitted.
- V O. Stål, J. Bergman, B. Thidé, L. K. S. Daldorff, and G. Ingelman. Prospects for lunar satellite detection of radio pulses from ultrahigh energy neutrinos interacting with the Moon. *Physical Review Letters*, 98(7) 071103 2007

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# 1. Plasma

In the textbook by Bellan [8] we can read that the first use of the term plasma in modern times was made by the Czech physiologist Jan Evangelista Purkinje, who used the Greek word *plasma* to denote the fluid that remains after the removal of all corpuscular material in blood. In 1922 the American scientist Irving Langmuir proposed that the electrons, ions and neutrals in an ionized gas could similarly be considered as a corpuscular material entailed in some kind of fluid medium which he called *plasma*. There is no “fluid medium” entailing the electrons, ions and neutrals in an ionized gas. But the name *plasma* has stayed and can be defined as [14]: *A quasi-neutral gas of charged and neutral particles which exhibits collective behaviour*. Quasi-neutrality allows for local charge non-neutrality but overall charge neutrality. Total charge neutrality is a common assumption but not a requirement. The collective behaviour relates to the effect that a local perturbation will change the whole environment around it. A local volume with a net charge will set up electromagnetic fields which propagate as waves through the medium, interacting with all charged particles on their way. The long range of electromagnetic interactions results in that already gases that are ionized to only a few percent will behave as a “plasma”. The plasma concept is also used in areas such as solid state physics.

The life span of a classical plasma is determined by the competition between ionization against recombination wherein the first factor may be governed by the flux of ionising radiation, while recombination depends on the plasma density. The result is that only 1% of the visible universe is not to be found in a plasma state. A star will be ionized by its fusion process while the inter-galactic medium will stay ionized because of its low density (about one particle per  $\text{cm}^3$ ). In our everyday life, we observe plasmas in electric arcs, neon signs, fluorescent lamps, processing plasmas and lightning. And with a continuous increase of its use in manufacture and industry, more goods which we use will be produced with the use of plasmas in some of the processing steps.

In the field of plasmas there are three main directions of applied research: Fusion energy, astronomy and space physics as well as industrial plasmas.

## 1.1 Fusion energy

Fusion energy has been the primary topic for laboratory based plasma research. The main problem is to magnetically confine plasmas with high

enough energy and density to ignite and support a fusion process in a similar fashion as in the Sun. For the last 50 years, the possibility to produce almost infinite amounts of energy has led to the construction of many experimental facilities, such as JET and ITER, and also to theoretical and numerical studies to understand the problems involved, and to follow up on possible solutions. Many of the experiments were constructed not only directly for fusion studies, but also for the study of more general plasma phenomena. One example of such phenomena is ion beam interaction, which we consider in Paper I in combination with numerical studies.

## 1.2 Space physics

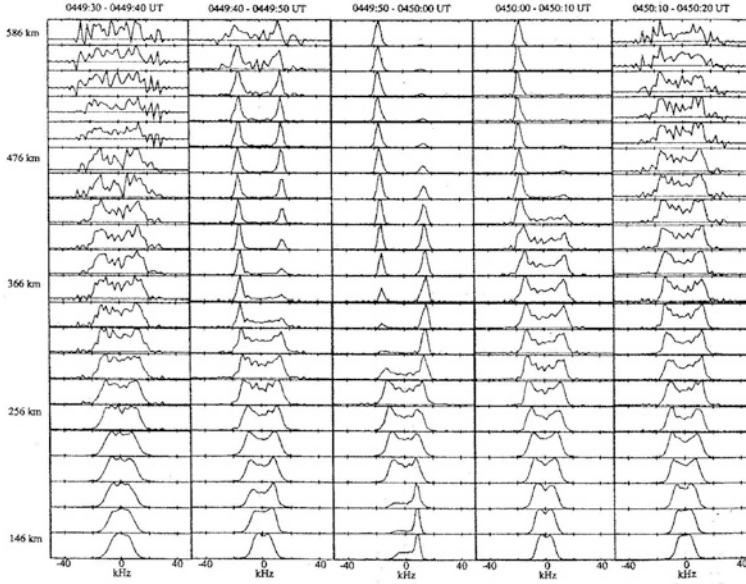
The upper part of our atmosphere is ionized by the radiation from the Sun, generating a plasma layer, which is more or less trapped by Earth's dipole magnetic field. The trapped plasma shields the Earth's atmosphere from the Sun's outer atmosphere, the solar wind. This plasma layer, controlled by the Earth's magnetic field, is the so-called magnetosphere. The inner ionized part of our atmosphere, extending from 60 km to beyond 1000 km and enveloping the Earth, is called the ionosphere.

This environment of continuously changing magnetic fields, ranging from neutral to fully ionized gas, results in a rich environment for studying plasma processes. Its large size makes it possible to study phenomena that we cannot study in laboratory experiments. For a numerical study, its size and complexity puts heavy demands on the computational power only achievable with parallel computers. Therefore, there is a need for parallel numerical solvers such as the one described in Paper III.

The close proximity of the ionosphere to the Earth's surface makes it possible to probe this system remotely with radars as well as with lasers. We can perform in situ measurements with sounding rockets and satellites. The knowledge of the plasma environment is also of commercial interest, particularly regarding a range of topics from radio communication to global positioning systems.

In Papers II and IV we look at radar observations associated with the phenomenon called naturally enhanced ion-acoustic waves [41]. Some of the results are shown in figure 1.1. For a steady state situation we expect two symmetrical peaks associated with ions oscillating forward and backward with the thermal velocity. In some observations we find that the peaks are no longer symmetric and there are also observed enhanced signals associated with solitary structures exhibiting a central peak. One possible explanation for these signals that has been put forward is that the electron beam interacts with the electron and ion background resulting in a net ion flux, which gives rise to the non-symmetric behaviour observed. This system provides the background for the studies in Papers II and IV.





*Figure 1.1:* The figure is taken from Rietveld et. al. 1991 [38]. It shows how the ion acoustic spectrum changes with time and altitude. In some of the figures at the lower left part we can see two symmetrical peaks combined with a central peak and that most of them shows asymmetrical distribution.

### 1.3 Astrophysics

The extreme values of density, energy and electromagnetic fields in stars, accretion discs around black holes, supernovae and their remnants etc. result not only in plasmas in the classical sense (electrons and ions), but also plasmas of exotic particles, e.g. electron-positron plasmas in galactic jets. This gives us the possibility to test our physical models beyond what is possible on Earth. The main problem is that we can only observe such objects passively by collecting the radiation and particles they emit. The resulting image we are left with is then a representation of the sum of processes in the volume that each “pixel” in our image covers. For understanding this image, we need models that take care of all the physical phenomena that produce the radiation and of the medium it has been transported through.

On the other hand, neutral particles such as neutrinos have the possibility to transport information about these processes mainly undisturbed by plasma and fields [47]. One way of observing neutrinos is by searching for secondary radio emissions caused by their interactions, which is particularly promising for particles of ultrahigh energy. In particular, when an ultrahigh energy cosmic ray neutrino interacts with matter it will produce a particle shower of exotic charged particles (i.e., a plasma), thus generating a current in the system that will radiate coherently for low frequencies, providing the possibility of detection with radio sensors. This problem is treated in Paper V.

## 1.4 Industrial Plasma

The use of plasmas has increased not only in the mechanical industry, for welding and cutting, but also for surface treatment from electronics to iron-works. The benefits of plasma processing lie in the possibility to fine-control the process such that layers of atomic thickness can be applied or removed. It can also help changing the chemical composition of surfaces, e.g. to make textiles more water repelling. These properties are used in dealing with hazardous material by generating environments which easily break down or transform unwanted chemical components, making them more environmentally friendly.

## 2. Plasma models

### 2.1 Kinetic description

A physical medium can be described by the position and behaviour of each of its parts. For a plasma, these are given by each particle's position,  $\mathbf{x}$ , and velocity,  $\mathbf{v}$  (or momentum). We can have many types of plasma particles that we group together and describe with a distribution function  $f_\alpha(\mathbf{x}, \mathbf{v})$ . This tells us the density of particles of species  $\alpha$  that we will find in the volume around  $(\mathbf{x}, \mathbf{v})$ . The evolution of  $f_\alpha$  is given as a solution of the Boltzmann equation:

$$\frac{\partial f_\alpha}{\partial t} + \mathbf{v} \cdot \nabla_{\mathbf{x}} f_\alpha + \frac{\mathbf{F}}{m_\alpha} \cdot \nabla_{\mathbf{v}} f_\alpha = \mathfrak{C}. \quad (2.1)$$

The first term on the left hand side describes the explicit changes in time, the next term handles changes because of movement in space (convection), while the last term on the left hand side takes care of changes related to changes in velocity (convection in velocity space). In this description, the particles are governed by an external force  $\mathbf{F}$  and a collisional term,  $\mathfrak{C}$ , that handles all interactions between particles, including creation and annihilation processes.

If we ignore annihilation and creation we are left with basically two types of collisions. Either a bowling ball-type, where particles need to physically touch each other to interact, or a long-range interaction such as gravitational (which dominates the movement of planets) or Coulomb (charge) interaction. In the last case, the strength of the interaction will disappear only at infinity. A group of particles interacting via collisions can be described in terms of the collisional operator. Either the particle behaviour is dominated by interaction with its closest neighbours (short range interaction) or by the interaction with all other particles (long range interaction). In the latter case we can no longer separate out binary interaction, but must consider how a particle interacts with the collective field from all the particles. This is given by the Vlasov equation:

$$\frac{\partial f_\alpha}{\partial t} + \mathbf{v} \cdot \nabla_{\mathbf{x}} f_\alpha + \frac{\mathbf{F}}{m_\alpha} \cdot \nabla_{\mathbf{v}} f_\alpha = 0. \quad (2.2)$$

Mathematically, this means that we have moved the collisional term  $\mathfrak{C}$  from the right hand side to the force term on the left hand side. The particle interactions can no longer be distinguished from an external force interacting on the particles. This describes a so-called collisionless plasma. This points to the fact that we no longer describe hard collisions, as with neutrals, but only long-range interaction (soft collisions), as between charged particles. The Vlasov

equation (2.2) has the same interpretation as the Boltzmann equation (2.1), but now  $\mathbf{F}$  also incorporates self-consistent fields set up by the plasma itself.

The Vlasov equation (2.2) is incompressible in phase space spanned by  $\mathbf{x}$  and  $\mathbf{v}$ . This means that two different elements in phase space will never occupy at the same time the same position in phase space (Liouville's theorem). If they do, we will have a hard collision and the system is governed by the Boltzmann equation (2.1). As a result, we will have twisting in phase space, that will generate a distribution function that varies on very small scales.

The rest of this chapter looks at ways to reduce the number of dimensions of the set of equations to reduce the computational load in numerical simulations.

## 2.2 Gyro-kinetic model

A charged particle in an electromagnetic field will have its motion determined by the Lorentz force  $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$ . From the last term we see that the charged particle will gyrate about a magnetic field line. Removing the gyrating motion from the Hamiltonian of the system reduces the dynamics to a Hamiltonian for the guiding centre. By perturbing this system with a low frequency electromagnetic fluctuation and eliminating the gyro angle for the perturbed guiding center Hamiltonian, we are left with a Hamiltonian system for the gyro centre. With this we have reduced the particle's gyrating motion around its centre,  $\mathbf{X}_{GK}$ , to a magnetic dipole whose magnetic moment,  $\mu$ , will be constant. The new distribution function  $f_{GK}(\mathbf{X}_{GK}, v_{\parallel}, t; \mu)$  will then be constant along its trajectory in the gyrocentre phase space  $(\mathbf{X}_{GK}, v_{\parallel})$ , resulting in its Vlasov equation:

$$\frac{\partial f_{GK}}{\partial t} + \frac{d\mathbf{X}_{GK}}{dt} \cdot \nabla_{\mathbf{x}} f_{GK} + \frac{dv_{\parallel}}{dt} \frac{\partial f_{GK}}{\partial v_{\parallel}} = 0 \quad (2.3)$$

where the gyrocenter motion, is given by  $(\frac{d\mathbf{X}_{GK}}{dt}, \frac{dv_{\parallel}}{dt})$ , where  $v_{\parallel} = \hat{b} \cdot \frac{d\mathbf{X}_{GK}}{dt}$ ,  $\hat{b} = \frac{\mathbf{B}}{\|\mathbf{B}\|}$ . The non-linear gyro-kinetic method [10] plays a fundamental role in the research on long-time behaviour of strongly magnetised plasmas.

## 2.3 Fluid approximation

In our daily life we interact with matter where hard collisions dominate, resulting in a distribution function in an equilibrium state, called a Maxwell-Boltzmann distribution. Most of what we observe are bulk quantities such as flow velocity, temperature, pressure and so on. In this section I will therefore show the connection between these quantities and the kinetic description.

For most systems we can presume that in any element of space there will be an equal amount of processes, such as velocity change, as the number of corresponding reverse processes, namely a detailed balanced equilibrium.

If we integrate over a longer period this will often be true, and symmetry will give us that  $\mathfrak{C} = 0$  in the Boltzmann equation (2.1). The equation will have the same mathematical form as the Vlasov equation (2.2) but the force term  $\mathbf{F}$  only contains external forces acting on the medium. For most cases in which we are using the Vlasov equation, we study phenomena where the time scale of hard collisions are much longer than the time scales of interests, and therefore they can be ignored. The only difference will then be the interpretation of the elements containing the force term. We will also assume that we have a finite number of particles in a finite velocity range.

We can integrate the Vlasov equation (2.2) with different moments of velocity

$$\int_{\mathbf{v}} \mathbf{v}^N \left( \frac{\partial f_{\alpha}}{\partial t} + \mathbf{v} \cdot \nabla_{\mathbf{x}} f_{\alpha} + \frac{\mathbf{F}}{m_{\alpha}} \cdot \nabla_{\mathbf{v}} f_{\alpha} \right) d\mathbf{v} = 0 \quad (2.4)$$

where  $\mathbf{v}^N$  is the product of  $\mathbf{v}$  of the order  $N$  by itself. In the rest of this section we will only consider one particle species and we therefore suppress the species index  $\alpha$ .

For the zeroth order moment we have

$$\int_{\mathbf{v}} \left( \frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla_{\mathbf{x}} f + \frac{\mathbf{F}}{m} \cdot \nabla_{\mathbf{v}} f \right) d\mathbf{v} = 0 \quad (2.5)$$

For our system the last term in the integration will vanish and we are left with

$$\int_{\mathbf{v}} \frac{\partial f}{\partial t} d\mathbf{v} + \nabla_{\mathbf{x}} \cdot \int_{\mathbf{v}} \mathbf{v} f d\mathbf{v} = 0, \quad (2.6)$$

giving the particle conservation law:

$$\frac{\partial n}{\partial t} + \nabla \cdot (\mathbf{u}n) = 0 \quad (2.7)$$

wherein

$$n = \int f d\mathbf{v} \quad (2.8)$$

$$\mathbf{u} = \frac{\int \mathbf{v} f d\mathbf{v}}{\int f d\mathbf{v}} \quad (2.9)$$

are the particle density and the bulk velocity, respectively.

For the first-order moment of the Vlasov equation (2.2),  $N = 1$ , we simplify the equation with Newton's second law,  $\mathbf{F} = m\mathbf{a}$  and write it in component form. We sum over repeated indices, and get

$$\int v_j \left( \frac{\partial f}{\partial t} + \frac{\partial}{\partial x_i} (v_i f) + \frac{\partial}{\partial v_i} (a_i f) \right) d\mathbf{v} = 0. \quad (2.10)$$

What is left in the integration after some calculation is

$$\frac{\partial}{\partial t} \int v_j f d\mathbf{v} + \frac{\partial}{\partial x_i} \int \{(v_j - u_j)(v_i - u_i)f + u_j u_i f\} d\mathbf{v} - \int a_j f d\mathbf{v} = 0 \quad (2.11)$$

which is the momentum conservation equation. For this result we made use of the fact that the Lorentz acceleration satisfies the relation  $\partial a_j / \partial v_j = 0$ . In vector and tensor form the momentum equation can be written as

$$\frac{\partial}{\partial t} (\mathbf{u}n) + \nabla \cdot \left( \frac{1}{m} \bar{\mathbf{P}} + \mathbf{u} \mathbf{u} n \right) = \langle \mathbf{a} \rangle n \quad (2.12)$$

wherein

$$\frac{1}{m} \mathbf{P}_{ij} = \int (v_j - u_j)(v_i - u_i) f d\mathbf{v} \simeq \frac{1}{m} P \delta_{ij} \quad (2.13)$$

$\langle \mathbf{a} \rangle$  is the result of cumulated forces acting on one element in space while  $\bar{\mathbf{P}}$  is the pressure tensor acting on the same element. In the isotropic case this can be reduced to a scalar  $P$ .

The last commonly used velocity moment of the Vlasov equation, the second order moment, relates to the energy of the system. In the same way as previously we have

$$\frac{1}{2} \int v_j v_j \left( \frac{\partial f}{\partial t} + \frac{\partial}{\partial x_i} (v_i f) + \frac{\partial}{\partial v_i} (a_i f) \right) d\mathbf{v} = 0. \quad (2.14)$$

After some calculations this is reduced to

$$\begin{aligned} & \frac{\partial}{\partial t} \int \frac{1}{2} \{(v_j - u_j)(v_j - u_j)f + u_j u_j f\} d\mathbf{v} + \\ & + \frac{\partial}{\partial x_i} \int \left\{ \frac{1}{2} (v_j - u_j)(v_j - u_j)(v_i - u_i)f + \frac{1}{2} (v_j - u_j)(v_j - u_j)u_i f \right. \\ & \left. + \frac{1}{2} (v_j - u_j)u_j(v_i - u_i)f + \frac{1}{2} u_j u_j u_i f \right\} d\mathbf{v} - \int (a_i v_i f) d\mathbf{v} = 0 \end{aligned} \quad (2.15)$$

which is the energy conservation equation. In vector and tensor notation, the equation can be written

$$\frac{\partial}{\partial t} \left( \frac{3}{2} \frac{1}{m} P + \frac{n}{2} \mathbf{u}^2 \right) + \nabla \cdot \left( \frac{1}{m} \mathbf{Q} + \frac{5}{2} \frac{1}{m} P + \mathbf{u} \frac{n}{2} \mathbf{u}^2 \right) = \langle \mathbf{a} \rangle_{\mathbf{v}} \cdot \mathbf{u} n \quad (2.16)$$

wherein

$$\mathbf{Q}_i = m \int \frac{1}{2} (v_j - u_j)(v_j - u_j)(v_i - u_i) f d\mathbf{v} \quad (2.17)$$

is the heat flux and  $\langle \mathbf{a} \rangle_{\mathbf{v}}$  is the mean acceleration weighted by the velocity. For each new moment of equation (2.4), a new unknown parameter is defined. The conservation equations derived iteratively this way will not result in a closed set of equations. With each new velocity moment, we add more kinetic phenomena into our “fluid” description which will be included into the model.

For plasma physics is it also necessary to take into account quantities related to electromagnetic characteristics. One such quantity is the charge, which also defines charge density  $\rho = qn$ . Another one is the current  $\mathbf{J} = qn\mathbf{u}$  where  $q$  is the charge of the particles. For more specific results for plasmas, see Ref. [43].

Using the same assumption for the distribution function, looking at a steady state and a binary collisions operator (i.e., all hard collisions involve maximally two particles at any point) we have  $\mathcal{C} = 0$ . We then obtain the Maxwell-Boltzmann equilibrium distribution:

$$f_{eq}(\mathbf{u}) = \left( \frac{m}{2\pi V_{th}} \right)^{\frac{3}{2}} e^{\left( -\frac{m(u_x^2 + u_y^2 + u_z^2)}{2V_{th}^2} \right)} \quad (2.18)$$

wherein the thermal velocity  $V_{th}$  can be written in terms of the isotropic pressure as:

$$V_{th} = \sqrt{\frac{P}{m n}} \quad (2.19)$$

## 2.4 Collective interactions

All charged particles will generate electromagnetic fields that cause the interaction between them in the Vlasov description of a collisionless plasma. The electromagnetic fields are given by the Maxwell equations:

$$\nabla_{\mathbf{x}} \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} \quad (2.20)$$

$$\nabla_{\mathbf{x}} \cdot \mathbf{B} = 0 \quad (2.21)$$

$$\nabla_{\mathbf{x}} \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (2.22)$$

$$\nabla_{\mathbf{x}} \times \mathbf{B} = \mu_0 \mathbf{J} + \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t}. \quad (2.23)$$

One important observation is that these equations only depend on the bulk parameters such as charge density and current density given by the fluid approximation. For the Vlasov-Maxwell system this has been shown by Ref. [6] and others for the classical case, under the assumption of the Newtonian limit  $c \rightarrow \infty$ , and in Ref. [39] and others for the relativistic case, that it approaches the Vlasov-Poisson equations. The Poisson equation is

$$\nabla^2 \Phi = -\frac{\rho}{\epsilon_0} \quad (2.24)$$

$$\mathbf{E} = -\nabla \Phi \quad (2.25)$$

This defines the electrostatic case, in which we can only have an external magnetic field  $\mathbf{B}$ . The electrostatic assumption can be defended for system for which we can assume that, compared to the physical processes we are studying, any information travelling with the speed of light will instantaneously spread throughout our system of interest.

The Vlasov-Maxwell equations can be expanded with the use of the Darwin approximation [7, 34], wherein the zeroth order result will give back the Vlasov-Poisson equations.

## 2.5 Use of the fluid and kinetic description

In Papers I, II and IV we consider different properties of the two-stream instability. From the fluid approach we can find the instability, but not how the plasma particles will interact with it. The instability will generate waves that will trap particles in potential wells. In the fluid approach we have only one mean velocity of the total system, and the entrapment will not be correctly handled by this approximation, forcing us to use the full kinetic description, which we solve numerically by the transform method presented in Paper III.

In the case of Paper V we are no longer interested in the details of the elementary particles of the plasma, but the resulting electromagnetic field which we can observe remotely. The model therefore replaces the complexity of the plasma with a simple current pulse which generates a radio pulse according to the Maxwell equations.

As soon as you are no longer interested in the linear solution of the equations only, and often even then, the only possibility to understand and solve the problems is with numerical computations. Some of the methods used for solving the Vlasov-Maxwell equations are presented in the next section.



### 3. Numerical solvers

A plasma is composed of singly or multiple ionised atoms and/or molecules together with their free electrons. In a computer, we cannot represent all the particles that we have in reality in a given volume. There are basically two approaches to this problem. One is to use synthetic particles (macro-particles) to represent the characteristics of a group of real particles, and thereby to sample parts of the distribution function. One limitation with this approach is that we will have a minimum density of one macro-particle per unit volume, resulting in the loss of phenomena associated with low density areas of the distribution function. For a Maxwell-Boltzmann distribution this will result in that we have artificial maximum kinetic energy of our particles that will be lower than in reality. A second problem is associated with the need to have a sufficient number of sampling points of the distribution function throughout the simulation to reproduce the physical processes which we are interested in. The other way of solving this problem is to use a phase fluid to represent the particles in the system. A phase fluid can have any density, even densities below what is possible with real plasma particles.

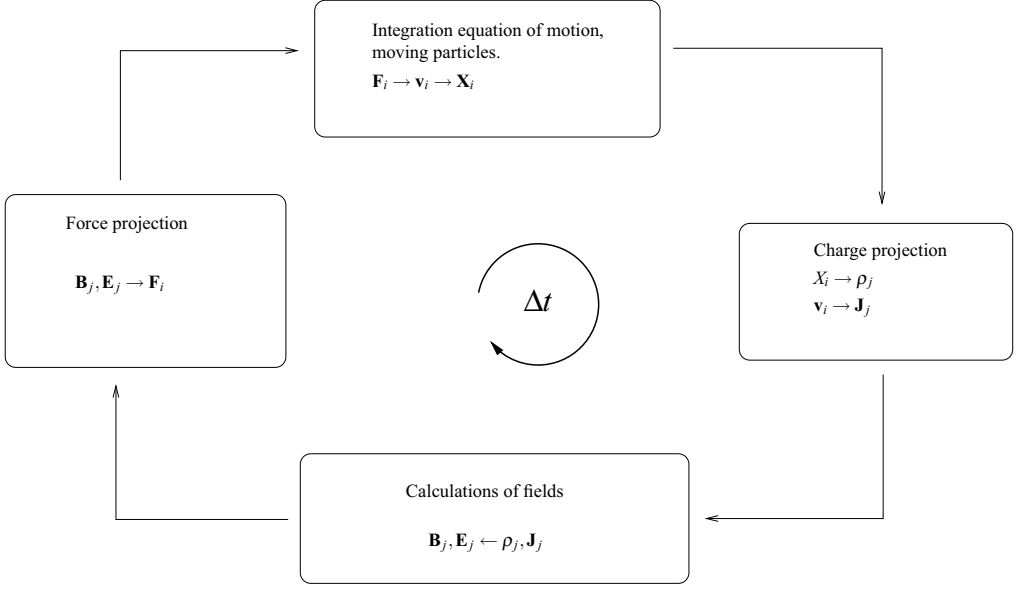
In the first part of this chapter we will look at the different ways to solve the Vlasov equation and thereafter we look at solving the Maxwell equations.

#### 3.1 Lagrangian method

This is a class of solvers where we use the Lagrangian of the system to move each particle in phase space. Two methods are dominating this approach, the Particle-In-Cell (PIC) and the Semi-Lagrangian method.

##### 3.1.1 Particle-In-Cell

The Particle-In-Cell method [9] uses macro particles to represent the physical systems and to move them through time. This method is used in Paper I. The problem is then of a statistical nature, namely to have enough samples for the physics present. The method has a built-in functionality for the thermalization of the computational plasma with an associated noise problem. The noise will decrease as  $1/\sqrt{N}$ , wherein  $N$  is the number of particles in a cell. While the particles are moved by the forces acting on them, the interaction between different particle species will be handled by the self-consistent electric and magnetic fields given by the Maxwell equations, or the Poisson equation in the electrostatic case. For this we need to transfer the macro-particles onto a



*Figure 3.1:* The processing steps for a Particle-in-Cell method involved when stepping one step in time. In the figure particles are indexed by  $i$ , while grid points are indexed by  $j$ . Starting from the rightmost, we have the particles at positions  $\mathbf{X}_i, \mathbf{v}_i$  which we map to the grid with a transfer function and integrate to get the charge density  $\rho(\mathbf{X}_j)$  and the current density  $\mathbf{J}(\mathbf{X}_j)$ . Going clockwise, we then calculate the electric and the magnetic field on the grid (lower box). In the left hand side we transfer the force from the grid to the particle's positions. We end with the box at the top, where we correct the particle's position and velocity, based on the force acting on them.

grid where we can calculate the bulk parameters (charge and current densities) needed by the Maxwell equations or the Poisson equation. With a numerical solver, we find the electromagnetic fields on the grid. These fields then need to be transferred to the particles. The transfer of the current and charge density onto the grid, and the electromagnetic force back from the grid to the particles are done by mapping functions. This algorithm is illustrated in figures 3.1 and 3.2. The possibility to reach a satisfactory solution with only a limited number of particles makes this approach favourable from a computational standpoint. This has made the method widely used for three dimensional simulations and large computational domains. The gridless approach also benefits the accuracy of particle trajectories, as they are not limited to the cell centres as is the case with Eulerian methods.

### 3.1.2 Semi-Lagrangian method

The Semi-Lagrangian method [42] represents the distribution function on a grid, while the time stepping uses a Lagrangian approach. From initialisation

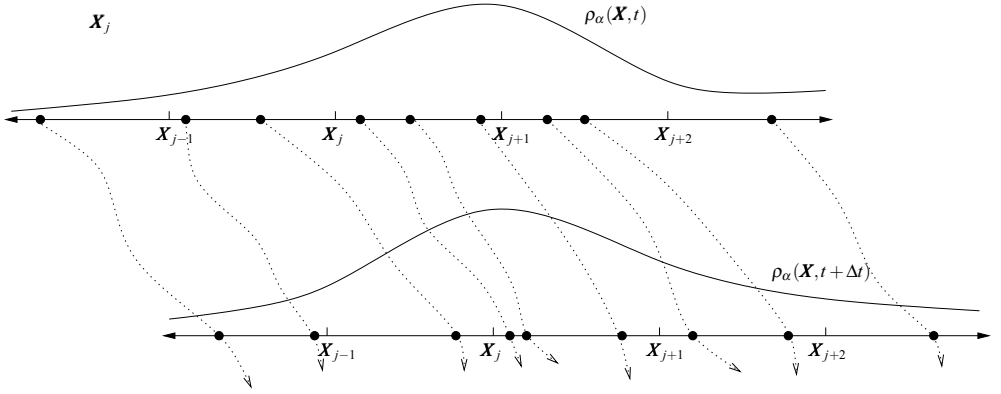


Figure 3.2: This figure illustrates some of the features of the Particle-In-Cell method. The figure shows a one-dimensional example. Each particle position on the line is converted with a mapping function (stepping, linear, cubic spline), to a density variation on the grid points  $\mathbf{X}_j$ . From there on we follow the steps described in figure 3.1 and move the particle forward in time. Then we remap the new particle position to the grid, returning the density  $\rho(\mathbf{X}, t + \Delta t)$ .

we have the distribution function on a grid. For the grid points at the next time step, we will trace the characteristic of the Vlasov equation backward in time to its position at the previous time step. We interpolate its value at the present time from the grid points in phase space at the previous time step, see figure 3.3. Since each time step is only related by the characteristics, which are not grid based, we are free to rearrange the grid at each time step. This makes it easy to change the resolution in phase space when needed, but we have some of the same problems in interpolating values between the grid points as for the PIC method.

## 3.2 Eulerian approach

Most numerical methods for solving partial differential equations have been used for solving the Vlasov equation (2.2). Among them are the Finite Element Method (FEM) [45, 46], Fourier and Hermitian methods [5, 11, 33, 17, 18, 26, 20] and other related transforms [31].

In the Lagrangian approach we use the path of an element of the distribution function to follow it through phase space. In the Eulerian approach, we want to solve the Vlasov equation (2.2) directly with numerical solvers for partial differential equations. For the fixed grid solver, the time splitting method (part 3.2.1) has been the core of many solvers, which have made improvements in the convection step, see [4, 22, 29]. Among the conservative solvers not using a splitting scheme, we have the Finite Volume Method used by Elkina and Büchner, [21]. Many of these methods are concerned with conserving the positive value of the distribution function. I agree with Arber and Vann

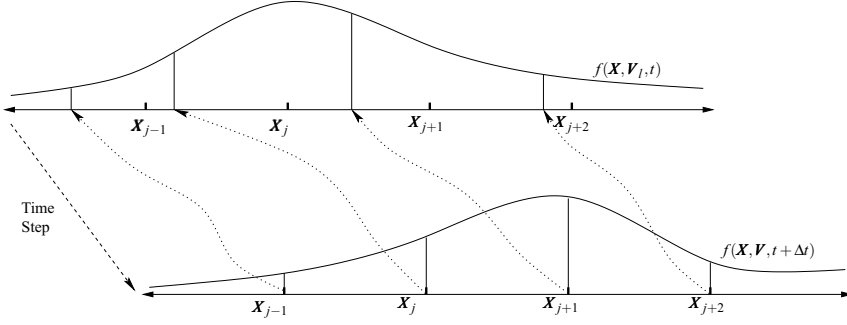


Figure 3.3: The figure illustrates the basic approach in the Semi-Lagrangian method. In this method, we have the distribution function  $f(\mathbf{X}, \mathbf{V}, t)$  on a grid. The figure illustrates a one-dimensional cut where we have the distribution at the gridpoints  $\mathbf{X}_{j-1} \dots \mathbf{X}_{j+2}$  with velocity  $\mathbf{V}_l$ . We know that a given volume in phase space will move unchanged along its characteristic. For each time-step we can trace backward from every grid point to where this phase space volume originated and then interpolate its value from the previous time step.

[4] in that the representation of the correct fine scale structure will be more important for many problems.

### 3.2.1 Time stepping

The distribution function  $f(\mathbf{x}, \mathbf{v}, t)$  represents a seven-dimensional system. What we call phase space is spanned by  $(\mathbf{x}, \mathbf{v})$  while  $t$  gives the time line. For the Vlasov equation (2.2) only the first term on the right hand side involves an explicit time step, while the term  $\mathbf{v} \cdot \nabla_{\mathbf{x}} f$  moves the particle's position (convection) and the last term  $\mathbf{F}/m \cdot \nabla_{\mathbf{v}} f$  changes the velocity of the phase space element. Cheng and Knorr [16] have shown how we can split the two last parts into two separate equations in one dimension. Later Cheng and others also showed how to do it in higher dimensions [15, 35]:

- Evolve  $\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla_{\mathbf{x}} f = 0$  for half a time step.
- Evolve  $\frac{\partial f}{\partial t} + \frac{\mathbf{F}}{m_{\alpha}} \cdot \nabla_{\mathbf{v}} f = 0$  for one time step.
- Ending with evolve  $\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla_{\mathbf{x}} f = 0$  for half a time step.

The implementation looks follows:

$$f^*(\mathbf{x}, \mathbf{v}) = f_n(\mathbf{x} - \mathbf{v} * \Delta t / 2, \mathbf{v}, t_n) \quad (3.1)$$

$$f^{**}(\mathbf{x}, \mathbf{v}) = f^*(\mathbf{x}, \mathbf{v} + (\mathbf{F}/m)\Delta t) \quad (3.2)$$

$$f_{n+1}(\mathbf{x}, \mathbf{v}, t_{n+1}) = f^{**}(\mathbf{x} - \mathbf{v} * \Delta t / 2, \mathbf{v}, t_n). \quad (3.3)$$

The reduction of complexity and the similarities to the Lagrangian approach has made this approach widely used in Eulerian Vlasov solvers. After the first step, it is necessary to calculate the self-consistent electromagnetic fields

associated with the system. We will then have half a time step difference in obtaining the charge and current density.

### 3.2.2 Transformation methods

In Papers II – IV, we use a modified Fourier-Fourier transformation technique to solve the Vlasov-Maxwell equations, [37, 17, 18, 19, 20]. In contrast to common practice, according to which all dimensions are analytically Fourier transformed, we use a Fourier transform technique in velocity space and a pseudo-spectral method in ordinary space.

One of the reasons for transforming the Vlasov equation, (2.2) into new orthogonal base functions is to reduce the partial differential equation to an algebraic equation.

The transform methods are grouped into two methods, namely the Fourier-Fourier and the Fourier-Hermitian method. The Fourier base function is

$$g_m(v) = e^{imv} \quad (3.4)$$

and the Hermitian base function is

$$h_m(v) = \frac{(-1)^{-1} e^{v^2/2}}{\left((2\pi)^{\frac{1}{2}} m!\right)^{\frac{1}{2}}} \frac{d^m}{dv^m} e^{-v^2/2}. \quad (3.5)$$

The Fourier-Hermitian method uses a Fourier transform for the spatial coordinates while the velocity part is expressed in components ( $m$ ) of Hermitian polynomials given by the basic function (3.5). An important observation is the infinite domain of the Fourier base function (3.4), while the Hermitian falls off at infinity.

In both transforms the most important thing is to have enough base functions to reassemble the distribution function to a level where we can reproduce the physics we are interested in. For computational use, we will have a limited number of grid points resulting in that the Fourier transform gives us a periodic system at the boundary. For the velocity space this is an unwanted feature [17, 18], while periodicity in space is in many cases acceptable. Especially for parallel computing the need for every Fourier component to interact with all other Fourier components for a non-linear expression will result in a communicational problem. The Hermite transform only needs to communicate with its neighbours and has the same base function as the Maxwell-Boltzmann distribution function, which gives the Hermitian transform some additional benefits. Other methods with a regular grid can use the transform method for the differentiation.

### 3.3 Hybrid systems

In a plasma one needs at least two particle species with opposite charge to have an overall quasi-neutral system. It is common to have heavy ions and highly mobile electrons. This gives a much shorter characteristic time scale for the electrons than for the ions. Then we have to solve the Vlasov equation (2.2) for the ions many times unnecessary in relation to their characteristic time scale. If we can integrate away the shortest time scale from the system, such as through the gyro-kinetic or hydrodynamical approximation for that species, we will reduce the amount of time-steps as well as reduce the number of dimensions in the system, resulting in a shorter execution time as well as using less computational resources. A similar case where this applies is for phenomena for which we are interested in time-scales much shorter than the longest ones. For this case it is usual to set the heavy ions as a constant background in time. This method is used in Paper I.

### 3.4 Maxwell equations

Solving the Maxwell equations is an important topic in engineering with large commercial interests in connection with e.g. antenna design and wave propagation. Because of the complex geometries often involved, the Finite Element Method (FEM) has a large user group [2, 30]. Other methods such as the Finite Volume Method (FVM) also have been used for more general geometries [28, 3]. The corresponding solvers come in two groups: time domain and time harmonic. Since we are primarily interested in how the plasma behaves in time, time domain solvers are more applicable for our purpose. These solvers are classified as Finite Difference Time Domain (FDTD) solvers. Many of these are based on the work of Kane S. Yee [44]. These methods are often specialized for each type of problem. They therefore need modifications to make them work together with the plasma model of interest.

In the field of plasma physics, where numerical simulations have been performed in connection with experimental devices such as Tokamak reactors, it has been necessary to incorporate a non-Cartesian grid as in the gyro-kinetic Maxwell solver Gyro [12] and Vlasov-Maxwell for electron injection devices [27]. But the dominating part of the Vlasov solvers use a Cartesian grid [35], where most numerical differential solvers can be applied.

### 3.5 Conclusion

The decision on which numerical solver to use will have to be based on the problem size, computer resources available, readiness of implementation, and previous experience. There is a continuous effort in improving and developing new methods to solve the Vlasov-Maxwell system of equations. One of the topics that needs to be addressed is the relation between reproduction of the

fine scales, conservation of mass, momentum and energy and computational power. Many kinetic phenomena are related to the gradients of the distribution function, fine granularity, and they are therefore necessary to be reproduced with a satisfactory resolution. Methods that do not conserve mass (particles) or momentum correctly, or otherwise introduce numerical errors, can result in the generation of electromagnetic fields which can disturb the simulation. In many cases this can be improved by refining the grid or adding more particles. These measures are however limited by the computational resources at hand.





## 4. Computations

The High Performance Computing (HPC) community has always strived for increasing the size of problems that are possible to solve. This leads to the pushing for new machines with increased computational power, resulting in tailor made machines where each processing element is connected to a communication network to make the elements work together, i.e. parallel computing. Techniques have been transferred from specialized hardware to the common computers of today. Today, the performance to cost ratio for common computers has resulted in a wider spread use of parallel computations in science. While the problem of writing high performance programs for a sequential execution has been largely solved, it is still unsolved for most application when you want to distribute the execution over many parallel processing elements.

### 4.1 Hardware

#### 4.1.1 Memory and computer networks

There are two basic hardware concepts in parallel computer environments, shared memory and distributed memory machines. For a shared memory machine, all processing units have equal access to all memory throughout the machines, while for a distributed system each processing element has to request the other processing elements for data in their memory. No ideal shared memory machine exists. Most of them have performance issues with memory not directly connected to the processors, non-uniform memory access (NUMA), and the fact that the latest updated variables are not synchronised with the data in main memory, i.e. cache coherency (CC) problems.

Many of these problems can be minimized depending on how algorithms for computation and communication are written. This is a unsolved problem of programming for a parallel computer environment.

Through time, HPC hardware has oscillated between shared and distributed hardware architecture. Today we are dominated by distributed memory machines built around Linux clusters where communication is handled by the Message Passing Interface (MPI) standard. This implementation can handle both types of memory architectures.

A trend that has started with the arrival of common multicore processors is a mixing of architectures, as shared memory systems (multicore processor) are connected through a network into clusters (distributed).

### 4.1.2 Processing Units

Most servers today have more than one Central Processing Unit (CPU), providing a homogenous parallel computational environment. The last couple of years it has become more common to connect specialized hardware such as Field Programmable Gate Arrays (FPGA), that can be low-level programmed for optimal performance to outperform the typical general purpose CPU. One of the more interesting developments is the use of the Graphical Processing Unit (GPU), used for numerical computations on common graphics cards, and well suited for streaming and vector calculations. This has drastically increased the computational power of the common computer for many problems, but also increased the workload on the programmer to efficiently distribute the workload on the different processing units.

## 4.2 Scalability

To have a program where the execution time is reduced by a factor of two with a doubling of the computing resources (called linear speedup) is the goal for all high-performance programs. Nobody has an infinitely large system, so often we limit ourselves to achieve a good speedup for a limited number of processing units.

With the increase in size of computer clusters, the problem of superscalability has surfaced. The goal is to have programs with linear speedup for thousands of processing elements. This can mean that we have to go for algorithms which are less efficient for small clusters, but which have better speedup properties and thereby result in a shorter execution time for a very large number of computational nodes.

## 4.3 Efficiency Considerations

A highly efficient computer program usually has the problem of being hard to adapt to new problems as well being hard to move to different hardware configurations.

The main concern for any programming task is the relation between the time it takes to program it and the total run time of the program when it is finished. The level of optimization as well as parallelization has to be weighed against the need for adding or changing parts of the program, also taking into account which computer environment it will typically run on.

## 4.4 Implementation of Vlasov Solvers

When it comes to the design and implementation of numerical solvers for Vlasov and Maxwell equations, the rule of thumb has been that Eulerian

solvers are best for one dimensional problems while the Particle-In-Cell method outperforms in the case of three dimensions. We will look at this in the frame of parallel computing.

A common way is to use a fixed grid, as in Paper III, or to represent the distribution function that will be split up in smaller parts on the different computational nodes. This results in that we need a much larger grid throughout the simulation to represent the short time acceleration/deceleration in phase space, and therefore we end up in a situation that many of the calculations are being done on areas in phase space where the distribution function is zero or close to zero. This can be avoided by sizing the grid just large enough for the distribution function at that moment in time and to expand or refine the grid whenever needed. This will lead to a need to generate situation reports for the distribution function at regular intervals and a redistribution of the phase space grid on the computational nodes when we need to expand/refine the grid to regain a homogeneous workload throughout the computer cluster. Most commonly this will happen in the velocity dimensions due to the acceleration processes in the plasma, but it can also occur for the reproduction of spatial gradients. This administrative work will add non-productive calculations. With a particle representation (PIC) the movement in phase space only represents changing of the information related to position and velocity, and will therefore have no impact on the computational workload.

The distribution of particles homogeneously through the computer cluster is straightforward. Only the distribution of the spatial matrices related to the charge and current densities and the electromagnetic fields can cause problems. The dimensions for those matrices are usually constant throughout the simulation, making it easy to optimise the distribution in relation to the communication, with no need for redistribution of the particles as they move with time. For Eulerian solvers, each distributed grid block needs to communicate with its neighbours. For a six-dimensional distribution function that we have in three dimensions this will increase the communicational burden. The field solvers common for all methods will only have three dimensions and therefore only 26 neighbouring cells.

For the transform method the Fourier transform needs to operate all values in transformed dimensions to do the transform, thereby increasing the communicational workload as we see in Paper III. For this we need to communicate almost all of the data that we want to transform, in contrast to only the neighbouring grid-points for traditional grid solvers.

All these problems will increase with increasing dimensionality of the problem and therefore traditionally Particle-In-Cell has been dominating for full three dimensional simulations.



## 5. Plasma observations

A probe moving in a plasma will measure values along a trace inside the plasma, while a remote sensor will collect information outside the plasma, based on the detection of radiation generated or reflected from a volume of the plasma. In both cases we need models to relate the measurements to the plasma processes.

After the measurements, we want to do the reverse: reconstruct the three dimensional plasma object from the trace values through the medium and/or from integrated, mean values taken by a remote sensor. For most cases we have a full three dimensional system, but sometimes we can reduce the system to one or two dimensions. Then the art is to have your measurement trace through the plasma inside or aligned with the one or two dimensional phenomena you want to observe. The orbits of satellites are defined by gravitational forces such that they usually do not align with a structure we want to observe. This leads to the result that we will be given fewer measured values of interest than an optimal placement of the probe (satellite) would provide. For an experiment as in Paper I, it is much easier to place the probe at the right position and to control the behaviour of the plasma for collecting the interesting data. In measurements with radar such as EISCAT [1], it is possible to align the radar beam along the magnetic field, giving us an one-dimensional system, and to collect data from any plasma in the column with the right condition; see Paper II and IV.

Observations will often give us an incomplete and inconclusive picture of the plasma. In such a case, more advanced numerical calculations have to be used to find the most likely types of processes that will result in the characteristics measured.

Every measurement has the same problem: it returns a value without directly indicating what underlying feature the value represents. At a conference I attended, there was a discussion on how to measure the sea water temperature. The only point they agreed upon was that the thermometer measured the temperature of the thermometer! This points to the problem of mapping numerical results to measurable quantities which can be compared with observations.

For transient events such as the impact of an ultra-high-energy cosmic ray neutrino, integration techniques are necessary for detecting the events; see Paper V. However, the low flux of such neutrinos will make it very unlikely that we will have more than one event at any time, removing the difficulty with diagnosing the sums of many events.



## 6. Summary and outlook

In Paper I we take a short look at experimental data in relation to numerical calculations for electron and ion beams. A hybrid plasma model with one particle species represented as a fluid is used, while for the other species we use the Particle-in-Cell method for extracting the physics of interest in full three dimensions. We then look closer at an electron beam interacting with the ion background where we use a kinetic description of both species in one dimension; see Paper II. This is related to observations on ionospheric plasma along the magnetic field. As processes in nature seldom are unperturbed, we went into more detail and expanded the study of Paper II with an inhomogeneous electron beam, see Paper IV. The numerical solver used for obtaining the results presented in Papers II and IV was resource demanding for a single CPU and thus we parallelized the code using a transpose method, as presented in Paper III. Finally, in Paper V we have the opposite problem of knowing the theoretical process of ultra-high-energy neutrinos hitting the Moon, but need to have it confirmed by observations. The paper then shows the number of observations to be expected depending on the models and instrumentation used.

A natural continuation of the electron beam simulation (Paper II and IV) is to increase the number of dimensions to two, so that we gain possibilities for gradients along and perpendicular to the beam. This would make it possible to get a better understanding of phenomena related to heterogeneous electron beams, and also make it possible to incorporate the generation of magnetic fields, using a Vlasov-Maxwell solver.

The expansion to more complex problems will demand more computational power. Adding new physics aspects will push for a consideration of implementing new kinetic solvers. Likewise the problem of handling numerical as well as physical instabilities will push the development of existing codes as well as determine possible change to a new solver.





## 7. Summary of Papers

### 7.1 Paper I

P. Guio, S. Børve, L. K. S. Daldorff, J. P. Lynov, P. Michelsson, H. L. Pécseli, J. J. Rasmussen, K. Saeki, and J. Trulsen. Phase space vortices in collisionless plasmas, *Nonlinear Processes in Geophysics*, 2003, Vol. 10, No. 1.

A Study of ion and electron-beams propagation in plasma, combining experimental and kinetic hybrid simulations under the electrostatic approximation for understanding of the generated phase-space vortices and beam fronts in three dimensions.

**My contribution** I continued the code development from S. Børve to a full 3D Vlasov-Poisson solver and generated and analysed parts of the data for the ion beam simulation.

### 7.2 Paper II

L. K. S. Daldorff, H. L. Pécseli, and J. Trulsen. Nonlinearly generated plasma waves as a model for enhanced ion acoustic lines in the ionosphere. *Geophysical Research Letters*, Vol 34, L18101, 2007.

We implemented an electron beam model as a possible explanation for Natural enhanced ion acoustic line (NEIAL) using a theoretical fluid model and a numerical solution to the kinetic description of the continuous electron beam problem. We were able to recreate some of the structures related to the NEIAL observation.

**My contribution** All simulations with parts of data analysing and theoretical work.

### 7.3 Paper III

L. K. S. Daldorff, B. Eliasson Parallelization of a Vlasov-Maxwell solver in four-dimensional phase space *Parallel Computing*, Accepted for publication

We present a parallel algorithm for solving the Vlasov-Maxwell equations, using a Fourier transform technique in velocity space and a pseudospectral method in ordinary space. By having all Fourier transforms as local computations we choose a transpose method for recognizing the distribution function on the different computational nodes. We show the unimportance of the

Maxwell solver for the total computational time and that we can only get a finite speedup in theory as well as in empirical studies.

**My contribution** Optimisation and Parallelization of the code. Data collection and theoretical model for speedup.

## 7.4 Paper IV

L. K. S. Daldorff, H. L. Pécseli, J. Trulsen, B. Eliasson and L. Stenflo Non-linearly generated plasma waves as a model for enhanced ion acoustic lines in the ionosphere. *New Journal of Physics*, Submitted

We have expanded the studies in paper II with theoretical and numerical solutions and perform closer analysis of the data and a comparison with the theoretical models. We use an expanded Zakharov model to evaluate the consequence of finite energy, wave amplitude, for the dispersion relation. We also present a numerical solution for a local electron beam with different velocity profiles as a model for heterogeneous electron beams supported by theoretical analysis.

**My contribution** All simulations and parts of data analysis and the theoretical work.

## 7.5 Paper V

O. Stål, J. Bergman, B. Thidé, L. K. S. Daldorff, and G. Ingelman. Prospects for lunar satellite detection of radio pulses from ultrahigh energy neutrinos interacting with the Moon *Physical Review Letters*, 98(7) 071103 2007

The background for the letter was to study the use of a space-born plasma analysis instrument for other purposes than local plasma studies. When a cosmic ray neutrino hits the Moon's surface it will radiate in a frequency range radio to optical. For ultrahigh cosmic ray neutrinos a larger part will be radiated in the radio band, making detection with electric field detectors, e.g. electric dipoles, on a plasma instrument package onboard a satellite possible. We have compared this antenna to a parabola antenna together with different models for the neutrino flux. The conclusion is that detection is very unlikely with a dipole but likely for a dedicated satellite project. There have not yet been observed any ultrahigh cosmic ray neutrinos.

**My contribution** Defining the scope of the paper together with O. Stål and J. Bergman. Setting up the environment and parallelization of the Monte-Carlo solver to get the statistics that were needed.

## 8. Sammanfattning på svenska

Ett fysiskt plasma består av en mängd laddade partiklar som uppvisar makroskopisk neutralitet och kollektiva dynamiska egenskaper. De laddade partiklarna växelverkar med varandra genom de självkonsistenta elektromagnetiska krafter som de genererar. Här på jorden finner vi plasmor i åskvädrens blixtar, i lysrör, i elektriska ljusbågar och i plamsaskärare i industrin. Rör vi oss utåt i jordens atmosfär träffar vi på ett naturligt plasma redan på cirka 60 kilometers höjd där jonosfären har sin undre gräns. Därefter blir plasmatillståndet det mest vanliga tillståndet eftersom plasma utgör 99% av all synlig materia i universum.

Forskning inom plasmafysik är indelad i tre huvudgrupper: Astro- och rymdplasma, fusionsplasma och industriell plasma. Mycket av den markbaserade plasmautvecklingen har drivits av fusionsforskningen som strävar efter att efterlikna solens energikälla med den fördelen att det kommer kunna ge nästan obegränsad tillgång till energi. Industriella tillämpningar av plasmafysik har en ökande utbredning, från mikroelektronik till tung industri. Plasma används mycket för att avsätta eller avlägsna material från ytor eller rengöra miljöfarliga utsläpp genom att ändra deras kemiska sammansättning. Plasmafysiken inom astronomi och rymdfysik har den fördelen att vi kan studera fenomen på mycket större skalor och i ett mycket vidare energiintervall än vad som är möjligt att reproducera i experiment på jorden.

För att förstå vad som sker när ett stort antal laddade partiklar växelverkar med varandra behöver vi en matematisk uttrycksform. Vi kan beskriva ansamlingen av partiklar med en fördelningsfunktion som anger tätheten given av läge i rummet, hastighet (rörelsemängd), och tid. Dynamiken hos fördelningsfunktionen är given beskrivs av Vlasovs ekvation, som är en partiell differentialekvation, så länge som vi ser inte tillåter hårda kollisioner. Det vill säga att partiklarna endast växelverkar genom de elektromagnetiska fält, associerade med deras laddningar, som ges av Maxwells ekvationer. Detta är en kinetisk beskrivning av plasmat.

För att få fram vätskebeskrivningen tar vi alla våra partiklar i et punkt i rummet och summerar över dem med olika viktningar av hastigheten. Detta ger oss bulkkvantiteter som densitet, genomsnittlig hastighet, tryck, temperatur med mera. När vi har den matematiska beskrivningen kan vi lösa problemet numeriskt. För Vlasovekvationen finns det i huvudsak två metoder, Lagrangemetoder och Eulermetoder. Lagrangemetoden har sin utgångspunkt i att två partiklar aldrig kommer att ha samma position och hastighet vid samma tid på sin väg. Vi flyttar varje partikel i enlighet med den elektromagnetiska

kraft som verkar på den. Efter ett språng i tid räknar vi ut vätskeparametrar som ström och laddningstäthet för att därefter ur Maxwells ekvationer räkna ut de elektromagnetiska fälten för nästa tidsprång. Den andra möjligheten är att med existerande numeriska metoder lösa Vlasovs partiella differentialekvation på en grid. Detta är Eulers metod.

Många olika former av lösare av partiella differentialekvationer används. Exempelvis de finita differensmetoden (FDM), finita elementmetoden (FEM), finita volymmetoden (FVM), och transformmetoder som använts i Artikel II och många fler. En teknik som ofta används för tidsstegningen är att dela upp den i en rumslig del och därefter in i en hastighetsdel. Först tar man ett halvt tidssteg framåt och räknar ut en ny position i rummet. Därefter justeras hastigheten för hela tidssteget för att till slut ta det sista halva tidssteget i rummet.

För att finna de elektromagnetiska fälten måste vi också lösa Maxwells ekvationer. Med det stora kommersiella intresse i bl.a. radiokommunikation har det skett en stor utveckling av tekniker för att lösa dessa ekvationer under olika betingelser. Mycket av detta kan inte direkt användas i plasmafysik eftersom de inte tar hänsyn till plasmats elektromagnetiska egenskaper. Det det är samma numeriska metoder som de som används i Eulers metod för att lösa Vlasovekvationen. I många fall kan vi reducera komplexiteten i Maxwells ekvationer genom att anta att ljushastigheten är oändlig. Vi sitter då åter med att plasmata bara genererar elektriska fält men inga magnetiska. Detta är den *elektrostatiska* approximationen. För att överhuvud taget klara av att lösa de kopplade Vlasov-Maxwellekvationerna inom en hanterbar tidsrymd måste vi dela upp körningarna på så många datorer (processorer) som möjligt. Detta kräver att vi delar upp vårt problem i mindre delar som vi fördelar över datorerna (processorerna) som därefter över nätverket kommunicerar den nödvändiga information de behöver från de andra. Detta är vad som menas med parallell databehandling.

I Artikel I har vi sett på jon- och elektronströmmar som utbreder sig inuti ett plasma. Vi har kombinerat experimentella resultat och simuleringar under den elektrostatiska approximationen för att i fasrummet studera rörelserna och strömfronten i tre dimensioner. Detta följs upp i Artikel II där vi ser på elektronströmmar som en möjlig förklaring till naturligt förstärkta jonlinjer (NEIAL) i jonsfärradarförsök. Med hjälp av vätskemodeller och kinetiska simuleringar har vi återskapat några av de element som observeras i NEIAL. Vi utvidgar denna studie vidare både numeriskt och teoretiskt i Artikel VI där vi går grundligare in på elektronströmmens växelverkan med bakgrundsplasmat. Vi lägger också till nya element såsom inhomogena strömmar. I Artikel III går vi igenom den datorkod som används i Artikel II och IV samt huruvida den är parallelliserad och dess presatanda. I den sista artikeln, Artikel V ser vi på sannolikheten för en satellit att upptäcka radiostrålning från neutriner av ultrahög energi (hastighet) då de slår in i månens yta (regolit).

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