Nuclear Instruments and Methods in Physics Research A271 (1987) 79–88
North-Holland, Amsterdam

MONTE CARLO SIMULATION OF ENERGY BALANCE IN A PURE ^{3}He MIGMA OF 4 MeV

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The energy balance in a migma of self-colliding orbits of 4 MeV ^{3}He is studied using a two-dimensional Monte Carlo simulation supplemented with results from a three-dimensional Monte Carlo simulation and a self-consistent calculation of diamagnetic effects. Electron drag is balanced against synchrotron radiation and bremsstrahlung and losses of hot electrons. Synchrotron radiation was assumed to be reflected and reabsorbed, but bremsstrahlung was not. Reabsorption was taken to be a function of position, but not of angle, which should be an underestimate. Electrons were treated relativistically. The results are presented as a function of density from just below to well into the diamagnetic regime. We find that diamagnetism can reduce synchrotron radiation losses to manageable levels.

1. Radiation loss

Competing with fusion power production are a number of energy loss mechanisms of which the principal ones are electron radiation and electron and ion leakage out the loss cone. We will present calculations that show that the ratio of fusion power production (P_f) to various power loss mechanisms is favorable.

We first consider the power loss due to electron leakage out the loss cone (P_e) and electron radiation as synchrotron and bremsstrahlung ($P_r = P_s + P_b$). We find that, in the parameter region of interest to a migma fusion reactor,

$$P_f > P_r + P_e. \tag{1}$$

The need to account for the decrease in internal magnetic field $\langle \beta \rangle \to 1$ is imperative – indeed the consequences of this decrease on synchrotron radiation have been known for some time. Rose [1] for example points out:

"The question arises whether as $\langle \beta \rangle \to 1$, the entire synchrotron radiation subsides to be replaced by the much weaker radiation from the surface as electrons are reflected from the plasma magnetic sheath. Burkhardt analyzes electron orbits at the boundary and shows that indeed a considerable reduction takes place and that the power radiated per unit area is only .

$$S_c \approx 10^{-31} n^{3/2} T_e^2 (\mathrm{keV}^2) \quad \mathrm{W/m^2}. \tag{2}$$

The loss represented by the above equation is truly negligible in all applications."

We will show, using the results of the diamagnetic calculation of Channon, Golden and Miller [2], that as the number of stored ions per cm, N_i, becomes large ($\langle \beta \rangle \to 1$), synchrotron radiation does not dominate fusion power production.

To determine the rate at which the migma is losing energy by synchrotron radiation it is necessary to determine the electron temperature. This temperature may be estimated by equating the rate at which electrons gain energy from Coulomb collisions with ions to the rate at which they are cooled. Synchrotron radiation is not in all cases the dominant loss process, thus it is necessary to be careful in the determination of electron temperatures. Bremsstrahlung radiation, electron leakage out the loss cone, and synchrotron radiation should all be considered simultaneously in determining the electron temperature.

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(North-Holland Physics Publishing Division)

The electron temperature T_e may be approximately determined through the energy conversion equation

$$\int_0^R n_e(r)\left(\frac{\mathrm{d}E}{\mathrm{d}t}\right)_c r\,\mathrm{d}r\,\mathrm{d}\theta = \int_0^R n_e(r)\left[\left(\frac{\mathrm{d}E}{\mathrm{d}t}\right)_s + \left(\frac{\mathrm{d}E}{\mathrm{d}t}\right)_b + \left(\frac{\mathrm{d}E}{\mathrm{d}t}\right)_1\right] r\,\mathrm{d}r\,\mathrm{d}\theta,$$

where we have assumed the migma to be uniform along the z-axis and the electron density $n_e(r)$ to be given by

$$n_e(r) = q_i n_i(r) = \begin{cases} \dfrac{q_i N_i}{2\pi R r_0}; & r < r_0, \\ \dfrac{q_i N_i}{2\pi R r}; & r > r_0, \end{cases} \qquad (r_0 \simeq 0.1R) \tag{3}$$

where N_i is the number of ions cm^{-1}.

It should be pointed out that more elaborate calculations using a Monte Carlo transport code developed by us indicate that the above model for $n_e(r)$ is qualitatively correct.

The rate at which an electron obtains energy from stored ions is given by

$$\begin{aligned}\frac{\mathrm{d}E}{\mathrm{d}t}(r)\bigg|_c &= \frac{8\pi q_i^2 n_i(r) \ln \Lambda}{3m_e c}\,\frac{T_c}{T_e}\,\frac{\exp(-m_e c^2/T_e)}{K_2(m_e c^2/T_e)} \\ &= \frac{8(2\pi)^{1/2}}{3m_e^{1/2}} q_i^2 n_i(r) \ln \Lambda \frac{T_c}{T_e^{3/2}},\end{aligned} \tag{4}$$

where $T_c = \frac{1}{2} m_e v_i^2$, q_i is the ion charge, m_i is the ion mass, and v_i is the average ion velocity.

The first expression has been obtained by assuming that the electrons are distributed according to a relativistic Maxwellian. The second expression is the nonrelativistic limit of the first. As the electrons are relativistic for parameters of interest, the first expression is the more appropriate and has been used in our calculations.

The rate at which electrons are cooled by synchrotron radiation may be written as

$$\int_0^R n_e(r) \frac{\mathrm{d}E}{\mathrm{d}t}\bigg|_s r\,\mathrm{d}r\,\mathrm{d}\theta = \frac{\pi R T_e \omega_c^3 m^{*3}}{12\pi^2 c^2}(1 - R_w), \tag{5}$$

where $\omega_c = eB_0/(m_e c)$, m^* is an effective mode number up to which the emission can be considered black body emission, and R_w is the reflection coefficient of the walls. The threoretical results of Drummond and Rosenbluth [3] might be used to determine the mode number m^* and from this the synchrotron radiation. The determination of m^* by Drummond and Rosenbluth, however, was obtained for the case of synchrotron radiation emission in a uniform magnetic field and uniform electron density. Such a determination as this should not be expected to yield accurate results when the electrons are nonuniformly distributed and in the diamagnetic regime where there are large gradients in the magnetic field. We present here a more careful calculation of synchrotron loss.

The radiation intensity $S(\omega, \boldsymbol{\Omega}, \boldsymbol{r}, T_e)$ at a position $\boldsymbol{r}$ at frequency ω into the direction $\boldsymbol{\Omega}$ is governed by the differential equation, along with appropriate boundary conditions,

$$\frac{\mathrm{d}S}{\mathrm{d}l}(\omega, \boldsymbol{\Omega}, \boldsymbol{r}, T_e) = -\alpha(\omega, \boldsymbol{\Omega}, \boldsymbol{r}, T_e)(S - S_b), \tag{6}$$

where $\mathrm{d}l$ is the incremental path length and where $S_b = S_b(\omega, \boldsymbol{\Omega}, T_e)$ is the spectral intensity of black body radiation at temperature T_e per unit solid angle and $\alpha(\omega, \boldsymbol{\Omega}, \boldsymbol{r}, T_e)$ is the energy absorption coefficient for a ray at frequency ω propagating in the direction $\boldsymbol{\Omega}$ at $\boldsymbol{r}$. The analysis carried out by Drummond and Rosenbluth assumed that the absorption coefficient α was independent of position – this is not the case for a migma, especially as $\langle \beta \rangle \to 1$.

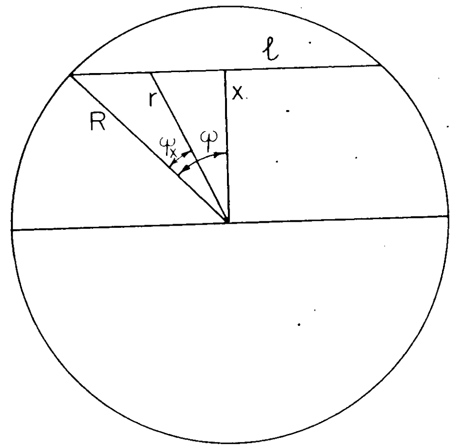

Fig. 1. Geometry used in synchrotron loss calculation.

We allow the absorption coefficient α to be a function of radial position in cylindrical coordinates (the geometry is shown in fig. 1, see also Beard and Baker [4]), and assume the cylindrically symmetric migma to be surrounded by specular reflectors of reflectivity R_w. An infinitesimal area on the surface will emit radiation of frequency ω into the direction (θ, ϕ) with intensity

$$E(x, \omega, \theta)\, \mathrm{d}x\, \mathrm{d}\Omega\left[1 - x^2/R^2\right]^{-1/2} = I_{\mathrm{RJ}}(\omega) \sin^2\theta F(x, \omega, \theta)\, \mathrm{d}x\, \mathrm{d}\omega\, \mathrm{d}\theta\, \mathrm{d}\psi,$$
$$F(x, \omega, \theta) = \{1 - \exp[-2A(x, \omega) \csc\theta]\}\{1 - R_w \exp[-2A(x, \omega) \csc\theta]\}^{-1}, \tag{7}$$

where

$$A(x, \omega) = \int \mathrm{d}(x, \omega, l)\, \mathrm{d}l = \int_0^{\psi_x} \alpha(x, \omega, \psi) x \sec^2\psi\, \mathrm{d}\psi,$$
$$\psi_x = \arctan\left[\frac{R^2 - x^2}{x^2}\right]^{1/2}, \tag{8}$$

and

$$I_{\mathrm{RJ}}(\omega) = \frac{T_e\omega^2}{8\pi^3c^2}. \tag{9}$$

The total radiated power per cm height of cylinder at ω is

$$E(\omega) = 8\pi(1 - R_w)\int_0^{\pi/2}\int_0^R E(x, \omega, \theta) \sin\theta\left[1 - x^2/R^2\right]^{1/2} \mathrm{d}x\, \mathrm{d}\omega\, \mathrm{d}\theta, \tag{10}$$

and the total radiated power per cm at all frequencies is just

$$W_s = \int_0^\infty E(\omega)\, \mathrm{d}\omega = \int_0^R \left.\frac{\mathrm{d}E}{\mathrm{d}t}\right|_s n_e(r) r\, \mathrm{d}r\, \mathrm{d}\theta. \tag{11}$$

The absorption coefficient α we use is that given originally by Trubnikov and Kudryavtsev [5] and used more recently in its relativistic form by Canobbio and Giuffré [6]. Typically, the synchrotron emission is concentrated about the direction perpendicular $(\alpha \approx \pi/2)$ to the magnetic field and drops off rapidly with change of direction. We chose to ignore the angular dependence of α and assume that synchrotron radiation is emitted uniformly with intensity given by that in the direction of $\pi/2$ to the magnetic field.

This will provide an *overestimate*, as pointed out in the work of Drummond and Rosenbluth [3] and Canobbio and Giuffré [6], of the synchrotron radiation emission. The absorption coefficient we use is then

$$\alpha(x, \omega, \psi) = \frac{3\pi}{2\sqrt{2}} \frac{4\pi e n_e(r)}{B(r)} \frac{\exp\left[-\mu z^{1/3} - 9/20 z^{-1/3}\right]}{z K_2(\mu)},$$
$$\mu = \frac{m_e c^2}{T_e}, \qquad z = \frac{9}{2} \frac{\omega T_e}{c_e B(r)}, \qquad r = \frac{x}{\cos\psi}, \tag{12}$$

where $B(r)$ is the magnetic field obtained in the diamagnetic calculation discussed elsewhere:

$$B(r) = B_0 \frac{I_1\left[\eta (r/R)^{1/2}\right]}{I_1(\eta)} \left(\frac{R}{r}\right)^{1/2}, \qquad \eta^2 = \frac{8 q_i^2 e^2 N_i}{m_e c^2}. \tag{13}$$

The total synchrotron emission per cm may now be written as

$$W_S = \frac{R T_e \omega_c^3 m^{*^3}}{12\pi c^2}(1 - R_w),$$
$$m^{*^3} = \frac{3}{\omega_c^3} \int_0^\infty \omega^2 \, d\omega \int_0^1 \frac{\left[1 - e^{-2A(y,\omega)}\right]}{\left[1 - R_w \, e^{-2A(y,\omega)}\right]} dy, \tag{14}$$

$$A(y, \omega) = R_y \int_0^{\theta_y} \alpha(y, \omega, \psi) \sec^2\psi \, d\psi,$$
$$a(y, \omega, \psi) = \frac{2\pi}{3\sqrt{2}} \frac{q_i e^2 c N_i}{R_y^2 \omega T_e} \cos\psi \frac{\exp\left[-\mu z^{1/3} - 9/20\mu z^{-1/3}\right]}{K_2(\mu)}, \tag{15}$$
$$z = \frac{9}{2} \frac{T_e \omega}{e c B_0} \frac{I_1(\eta)}{I_1\left[\eta (y/\cos\psi)^{1/2}\right] (y/\cos\psi)^{1/2}}.$$

The rate at which an electron is cooled due to bremsstrahlung emission is

$$\left.\frac{dE}{dt}\right|_b = \tfrac{16}{3} q_i^2 n_i(r) \left[\frac{e^6}{\hbar m_e c^3}\right] \left[\frac{8T_e}{\pi m_e}\right]^{1/2} \left[1 + \left(\frac{9}{8} + \frac{2}{q_i}\right) T_e / m_e c^2\right], \tag{16}$$

where first order relativistic corrections for ion–electron and electron–electron bremsstrahlung have been included. It is assumed that due to the high frequency nature of the bremsstrahlung that none will either be reabsorbed by the migma or reflected at the walls.

The last term $(dE/dt)_l$ appearing in the energy conservation equation represents energy loss due to the scattering loss of electrons in the high energy tail of the velocity distribution, out the loss cone. A good estimate of this loss rate may be made without recourse to the solution of coupled ion–electron transport equations.

In an equilibrium situation, the flux of electrons leaving through the loss cone per second must just balance the flux entering the system

$$\left.\frac{dN_e}{dt}\right|_{in} = \left.\frac{dN_e}{dt}\right|_{out}. \tag{17}$$

Incoming flux in the reactor regime, however, is just equal to the injected ion current times the number of electrons per ion (K_e). The injection ion current must balance the number of ions which are scattered out the loss cone as well as those which are fused. That is

$$R_l = R_s + 2R_f. \tag{18}$$

We may parameterize this rate by the burn factor b

$$b = \frac{2R_f}{2R_f + R_s}, \qquad R_s = 2\frac{(1-b)}{b}R_f, \tag{19}$$

and the fusion rate R_f

$$R_f = \frac{\langle \sigma v \rangle}{2} \int_0^R n_i^2(r) r \, dr \, d\theta, \tag{20}$$

where $\langle \sigma v \rangle$ is the fusion rate parameter. We thus have

$$R_e = K_e R_I = K_e \left(\frac{2R_f}{b} \right). \tag{21}$$

While in general the determination of the average energy carried off by an escaping electron is involved, empirically it happens that for a mirror confined plasma [7]

$$\langle E_e \rangle_{\text{loss}} \approx \tfrac{2}{3} T_e, \tag{22}$$

so that we have

$$\left(\frac{dE}{dt} \right)_I = \frac{2}{3} \frac{K_e T_e \langle \sigma v \rangle}{b q_i^2} n_e(r). \tag{23}$$

Before proceeding to discuss the result of the energy loss calculations just described, let us first look at the energy production mechanism – fusion. While a three-dimensional mode has been used to estimate the fusion rate, for purposes of consistency we will adopt the two-dimensional migma model used to this point.

The fusion power per centimeter of height may be written, under the approximation that $\langle \sigma v \rangle$ is independent of position, as

$$P_f = W_f \frac{\langle \sigma v \rangle}{2} \int_0^R r n_i^2(r) \, dr \, d\theta = \frac{N_i^2 \lambda}{2\pi R^2} \langle \sigma v \rangle W_f, \tag{24}$$

where W_f is the energy release from fusion and where λ is a factor greater than one arising from the density gradient in the migma – note that this factor will appear in all processes that depend on the square of the density. A good approximation for λ, in the two-dimensional model considered here, is

$$\lambda \approx \frac{1}{2} \left[\frac{1}{2} + \ln\left(\frac{R}{r_0} \right) \right]. \tag{25}$$

We are especially interested in the fusion power in the density regime where diamagnetic effects play a large role. Assuming that the external magnetic field strength is held at a constant value B_0 while the number of stored ions per cm is increased, the radial extent of the system R will increase according to

$$R^2 = \frac{(m_e c V_i)^2}{(e q_i B_0)^2} \frac{\eta^2}{4} \left[\frac{I_1(\eta)}{I_2(\eta)} \right]^2 = \frac{4 T_e N_i}{B_0^2} \left[\frac{I_1(\eta)}{I_2(\eta)} \right]^2, \tag{26}$$

so that as the number of stored ions becomes large ($\langle \beta \rangle \to 1$)

$$P_f = \frac{N_i B_0^2 \lambda}{8\pi T_e} \langle \sigma v \rangle W_f. \tag{27}$$

It is necessary to point out here that the above equation for P_f, where λ is determined from the two-dimensional model considered here, *underestimates* the true fusion power released by about a factor of 2. This underestimation of power release is also true of electron leakage and bremsstrahlung as they are

Table 1
Parameters chosen for the comparisons of gain and loss mechanisms

	^{3}He	D
Ion energy (MeV)	4	1
External field (kG)	70	70
Wall reflectivity	0.90	0.90
$\langle \sigma v \rangle$ (cm^3 s^{-1})	2×10^{-16}	2×10^{-16}
Burn	0.2	0.4
λ	1.5	1.5
W_f	13	3.7

also n^2 processes. Therefore, while power ratios calculated from the above formalism for P_f/P_e and P_f/P_b should be good estimates, that for P_f/P_s will be underestimated, i.e. synchrotron radiation in the above model is overestimated with respect to fusion power by about a factor of two.

We now turn to the results of the calculation described above where two fuel cycles, ^{3}He–^{3}He and D–D, have been considered. The parameters chosen for the comparisons of gain and loss mechanisms are listed in table 1. The fusion rate parameter $\langle \sigma v \rangle$ has been determined from the three-dimensional Monte Carlo transport code and the burn fractions used are consistent with the chosen parameters. and a reasonable choice of mirror ratio. We have chosen to use a wall reflectivity R_w of 0.9 rather than the more optimistic value of 0.99 sometimes used.

For each fuel cycle we present plots of

(1) Electron temperature vs N_i,
(2) Ratio of fusion power to radiated power vs N_i and the ratio of fusion power to leaked electron power vs N_i,
(3) Synchrotron power loss per stored ion vs N_i,
(4) Fusion power per stored ion vs. N_i.

Results for the ^{3}He–^{3}He fuel cycle are shown in fig. 2. From the curves it may be seen that fusion power gains are greater than the sum of loss mechanisms in the region of $N_i \sim 3\times10^{17}$ cm^{-1} at which point, for 100 cm mirror to mirror, the fusion power would be 0.5 MW. The electron temperature in this region is ~ 300 keV with the dominant loss being due to radiation. At this N_i, P_s/P_b is still somewhat larger than unity, but the ratio is decreasing rapidly with increasing $\langle \beta \rangle$.

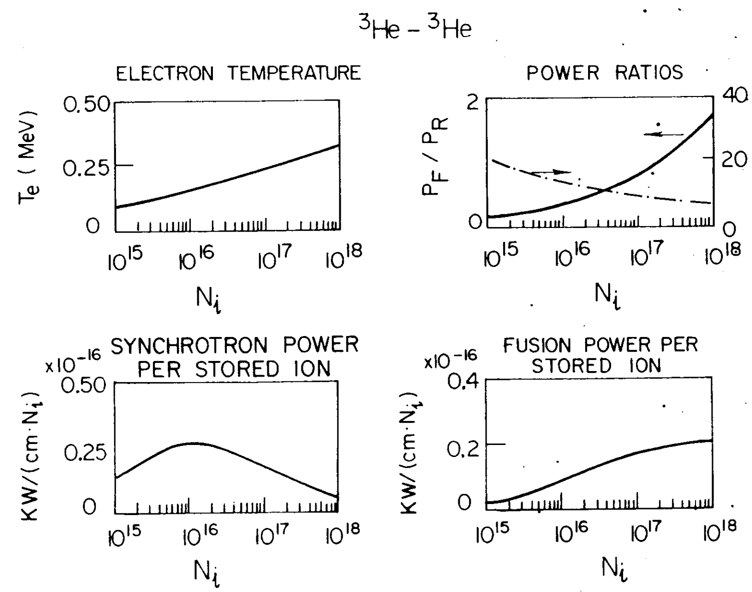

Fig. 2. Dependence of quantities on N_i for ^{3}He migma.

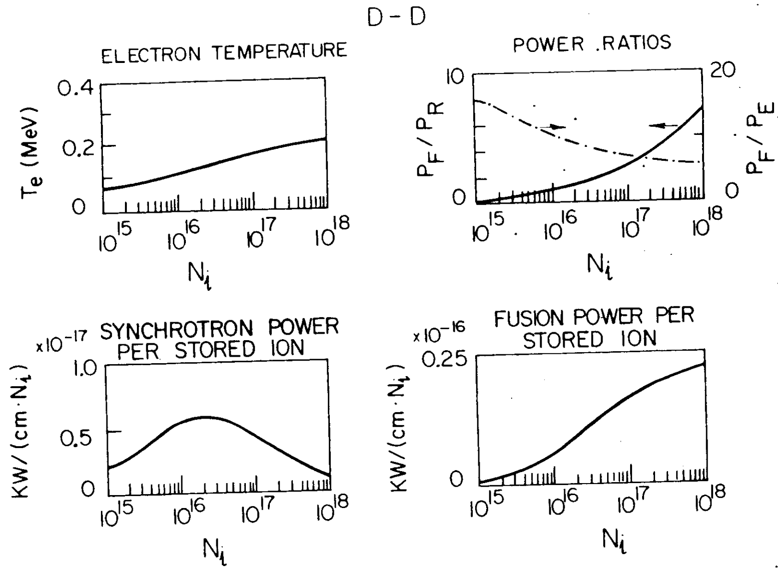

Fig. 3. Dependence of quantities on N_i for D migma.

Results for D–D are shown in fig. 3. Again looking at $N_i = 3 \times 10^{17}$ cm^{-1} ($\beta = 0.8$), the fusion power from a 100 cm length mirror would be about 0.6 MW. In this region the electron temperature is about 180 keV and the ratio of fusion power to radiated power $P_f/P_r = 5$, where $P_f/P_s \sim P_f/P_b \sim 10$. It will be seen that as the number of stored ions is increased above this point that the temperature becomes constant. This is due to the fact that electron leakage is becoming the dominant loss process and as its depends on stored ion number in the same fashion as electron heating by ion–electron collisions, a temperature is established that is independent of N_i. (Note that the same phenomenon will occur for the ^{3}He–^{3}He system with either bremsstrahlung or electron leakage playing the dominant role.). In this reactor regime it can be seen that synchrotron losses become entirely negligible.

A qualitative point should be made about the results just described. As the number of stored ions becomes large ($N_i > 10^{17}$ or equivalently as $\langle \beta \rangle \to 1$), a qualitative change takes place in the nature of the migma. In this region we find the average ion density becomes relatively constant while the radius R increases as $N_i^{1/2}$ so that the fusion rate (B_0 held constant), as well as other normally N_i^2 processes, increases linearly with N_i. Thus, while there is an upper limit to the density for a given external field there is not an a priori limit to the fusion rate or the fusion power out. When calculating the synchrotron emission, a parameter of critical importance is

$$\mathscr{L} = \frac{8\pi e R n_e}{B}. \tag{28}$$

Holding the electron temperature constant, the transparency ϵ of a plasma decreases fairly rapidly as $\mathscr{L}$ is increased. As N_i increases or $\langle \beta \rangle \to 1$ the density approaches a constant value; however, the radius R is increasing so that though the density is constant, the optimal depth of the system increases and the dimensionless parameter $\mathscr{L}$ increases even though the density remains roughly constant. In addition, as $\langle \beta \rangle \to 1$ the internal magnetic field becomes small, further increasing the effective $\mathscr{L}$.

In conclusion we find that a careful calculation of P_f/P_s indicates that, in the megawatt/cell power region, this ratio is larger than unity for ^{3}He–^{3}He and that it is about 10 for a D–D migma – as such it does not establish any fundamental limit. Indeed when the other types of electron energy loss are considered, we find that for both cycles

$$P_f > P_R + P_e$$

in the megawatt region.

2. Fusion power generation

The fusion power, P_f, produced by a system of like ions may be written as

$$P_f = \int \frac{\langle \sigma v \rangle}{2} n_i^2(r, z) W_f r \,dr\,dz\,d\theta, \tag{29}$$

where W_f is the energy released in the fusion reactor. A Monte Carlo collisional transport code developed by us indicates that, at densities at which the distribution functions are determined by collisional processes, a reasonable approximation of the ion density is

$$n_i(r, z) = \frac{N_i}{2\pi R r_e}\left[\frac{\alpha}{\alpha\beta + 1}\right]\left[e^{-\alpha z/L} + \beta\right], \qquad r < r_0, \quad |z| < L,$$

$$n_i(r, z) = \frac{N_i}{2\pi R r}\left[\frac{\alpha}{\alpha\beta + 1}\right]\left[e^{-\alpha z/L} + \beta\right], \qquad r > r_0, \quad |z| < L,$$

where R is the radial extent of the system and L is the distance from the midplane ($z = 0$) to the mirror $z = L$. Assuming that $\langle \sigma v \rangle$ is independent of position we find

$$P_f = 2\int_0^L dz \int_0^R r\,dr \int_0^{2\pi} d\theta \frac{\langle \sigma v \rangle}{2} W_0 n_i^2(r, z) = \frac{N_i^2 L \lambda}{\pi R^2} \langle \sigma v \rangle W_f, \tag{30}$$

where λ is well approximated by

$$\lambda = \frac{1}{4}\left\{\left[\frac{1}{2} + \ln\left(\frac{R}{r_0}\right)\right]\frac{\alpha}{1 - 2\alpha\beta}\right\}. \tag{31}$$

If the ions are uniformly distributed over the volume of a cylinder of radius R and length $2L$, the factor λ would be unity. A value greater than unity signifies that a given number of stored ions will undergo fusion more rapidly, due to nonuniform density distribution, than if they were uniformly distributed. We note that the same enhancement factor λ will appear in the scattering rate and bremsstrahlung rates, as such processes also depend on the square of the density distribution.

Calculations, using a Monte Carlo code, indicate that, in the collisionally dominated density regime, reasonable values of α, β, and R/r_0 are (4 MeV H_e^{2+} and a mirror ratio of 5)

$$\alpha = 7.6, \qquad \beta = 0.05, \qquad R/r_0 = 10,$$

so that typically $\lambda \sim 3$.

We may define the radial extent of the system, from the diamagnetic calculation of Channon, Golden, and Miller [2], as

$$R^2 = \frac{4T_e N_i}{B_0^2}\frac{I_1^2(\eta)}{I_2^2(\eta)}, \qquad \eta^2 = \frac{8N_i q_i^2 e^2}{m_e c^2}, \tag{32}$$

where T_i is the average ion energy and B_0 is the externally applied field. We define an average $\langle \beta \rangle_m$ for a migma as

$$\langle \beta \rangle_m = I_2^2(\eta)/I_1^2(\eta) \tag{33}$$

and point out that as $\eta \to \infty$, corresponding to $N_i \to \infty$, $\langle \beta \rangle_m \to 1$. While the ratio of modified Bessel functions approaches the proper limit as $\eta \to \infty$, we note that as $\eta \to 0$ it takes on one half the appropriate value of $\langle \beta \rangle$.

$$\underset{\min\ \eta \to 0}{\langle \beta \rangle_m} = \frac{\eta^2}{16} = \frac{N_i q_i^2 e^2}{2 m_e c^2}, \tag{34}$$

$$\langle \beta \rangle = \frac{N_i q_i^2 e^2}{m_e c^2}. \tag{35}$$

Then the fusion power P_f may be written as

$$P_f = \frac{N_i B_0^2 L \lambda}{4\pi T_e} \frac{I_2^2(\eta)}{I_1^2(\eta)} \langle \sigma v \rangle W_f = \frac{N_i B_0^2 L \lambda}{4\pi T_e} \langle \beta \rangle_m \langle \sigma v \rangle W_f \tag{36}$$

We further reduce the above form by use of eqs. (33)–(36) to obtain

$$P_f = \frac{B_0^4 R^2 L \lambda}{16\pi T_e^2} \langle \beta \rangle_m^2 \langle \sigma v \rangle W_f. \tag{37}$$

In the limit of $\langle \beta \rangle_m \to 0$ ($N_i \to 0$) the above expression takes on the form

$$P_f = \frac{B_0^4 (2r_L)^2 (2L)}{64\pi T_e^2} \langle \beta^2 \rangle \frac{\langle \sigma v \rangle}{2} W_f, \tag{38}$$

where $\langle \beta^2 \rangle = \lambda \langle \beta \rangle^2$, and r_L is the gyroradius in the externally applied field B_0.

To illustrate the singular nature of the $\langle \beta \rangle_m = 1$ "limit", we present in fig. 4 the dependence of fusion power and system radius on $\langle \beta \rangle_m$. For this figure we have chosen $B_0 = 7$ T, $L = 50$ cm, and rate enhancement factor $\lambda = 3$. Results are shown for a 1 MeV deuterium migma and a 4 MeV ^{3}He migma.

Picking a reasonable value of $\langle \beta \rangle_m = 0.9$, one finds the fusion power per cell is roughly 2 MW for the ^{3}He system and 5 MW for deuterium. At the same point the system radius is 57 cm for ^{3}He and 47 cm for deuterium.

We wish to point out that $\langle \beta \rangle_m$ can be viewed as a measure of power density, and not of total power. The approximate power density, from eq. (37), is

$$P_f = \frac{B_0^4 \lambda}{32\pi^2 T_i^2} \langle \beta \rangle_m^2 \langle \sigma v \rangle W_f, \tag{39}$$

which is clearly bounded as $\langle \beta \rangle \to 1$, while the total power is not. For the system just discussed, the limit is 2.2 MV/m^3 (^{3}He–^{3}He) and 10.1 MW/m^3 (D–D).

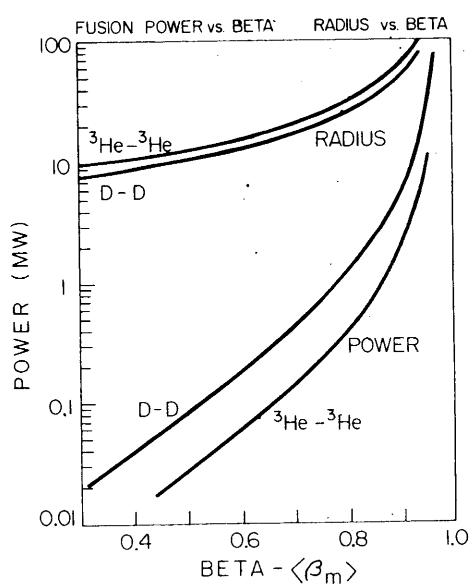

Fig. 4. Fusion power and system radius as a function of $\langle \beta \rangle_m$.

In conclusion, we have presented a careful calculation of the fusion rates and concluded that economically useful power levels are accessible. Further:

(1) The presence of electrons is required to neutralize the migma. This neutralization may occur naturally due the presence of an ambipolar potential.

(2) The presence of electrons has the unattractive consequence of bremsstrahlung and synchrotron radiation. Both effects are roughly comparable in ^{3}He–^{3}He migma at beam averaged ion densities of $\sim 10^{15}$ cm^{-3}.

(3) The energy distribution of the electrons – more precisely the velocity space structure – is important as it determines the amount of radiative energy loss and the rate at which an external agent provides energy.

(4) In the absence of energy sources or sinks, electrons would be Maxwellian in energy and, because of Coulomb multiple scattering, distributed approximately isotropically in velocity space.

(5) Radiation is an energy sink for electrons, particularly those at high energies. This mechanism depletes the high energy tail of the Maxwellian.

(6) Energy transfer from migma ions to electrons by Coulomb scattering provides an energy source for electrons:

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \frac{4\pi q_i^2 e^4 \ln \Lambda}{\mu} \boldsymbol{v}_i \cdot \boldsymbol{E}_v(\boldsymbol{v}_i),$$
$$\boldsymbol{E}_v(\boldsymbol{v}_i) = \int \frac{(\boldsymbol{v}_i - \boldsymbol{v}_e)}{|\boldsymbol{v}_i - \boldsymbol{v}_e|^3} f(\boldsymbol{v}_e)\, \mathrm{d}\boldsymbol{v}_e, \tag{40}$$

where

$$\int f(\boldsymbol{v}_e)\, \mathrm{d}\boldsymbol{v}_e = n_e$$

(7) By analogy with electrostatics only electrons with velocities $v_e < v_i$ contribute to energy loss by ions an energy gain by electrons. The energy gained by this process is distributed by electron–electron multiple Coulomb scattering.

References

[1] D.J. Rose, Nucl. Fus. 9 (1969) 183.

[2] S.R. Channon, J.E. Golden and R.A. Miller, Phys. Rev. A17 (1978) 407.

[3] W.E. Drummond and M.N. Rosenbluth, Phys. Fluids 6 (1963) 276.

[4] D.B. Beard and J.C. Baker, Phys. Fluids 5 (1962) 1113.

[5] B.A. Trubnikov and V.S. Kudryavtsev, Proc. 2nd Int. Conf. on Peaceful Uses of Atomic Energy, Geneva (1958) UN, Vol. 31, p. 93.

[6] E. Canobbio and S. Giuffré, Nucl. Fus. 10 (1970) 189.

[7] A. Futch Jr., J. Holdren, J. Killeen and A. Mirin, Plasma Phys. 14 (1972) 211.