EFFECT OF NUCLEAR ELASTIC SCATTERING ON FUSION REACTIVITY OF SELF-SUSTAINING D-^{3}He PLASMAS

Y.Nakao, K.Kai, H.Matsuura, K.Kudo

Department of Nuclear Engineering, Kyushu University Hakozaki, Fukuoka 812, Japan.

ABSTRACT

On the basis of coupled kinetic/power-balance calculations, we examine the degree of recoil tail creation due to nuclear elastic scattering and its effect on the fusion reactivity of self-sustaining D-^{3}He plasmas. It is shown that at a typical operating state (*e.g.* T_i^{bulk} = 100keV), the d-d [d-t] reaction rate parameter increases by 10-20% [decreases by $\leq$ 10%] from the values for Maxwellian distributions, while the d-^{3}He reaction remains almost unchanged.

I. INTRODUCTION

In most of the power-balance calculations for ignited fusion plasmas, the velocity distributions of fuel ions are assumed to be Maxwellian. This is valid as far as D-T fusion is concerned. In D-^{3}He plasma, however, the fuel ion distributions depart to a certain extent from a Maxwellian.

The dominant mechanism of producing non-Maxwellian distributions in D-^{3}He plasma is nuclear elastic scattering (NES) of fusion-generated fast protons. NES is a non-Coulombic, large-angle scattering process.(1) Contrary to Coulomb collisions, a large amount of energy can be transferred in a single NES, so that the fuel-ion distribution functions of D-^{3}He plasma contain non-Maxwellian tail (recoil) components due to NES.(2)

Another process contributing to high-energy ion tail formation is subsidiary d-d reactions. For example, the triton distribution function is distorted from a Maxwellian owing to the presence of 1.01-MeV birth component. The energy loss mechanism such as thermal conduction also affects the ion velocity distributions.(3)

Recently the present authors estimated the effect of the above distortion processes on the fusion reactivity of ignited D-^{3}He plasmas.(4) However, since the treatment of NES process in REf.(4) was not rigorous but of a simplified one and, moreover, the helium-3 distribution was fixed to a Maxwellian, questions have been remained regarding the degree of recoil ion creation.

The purpose of this paper is to exactly evaluate the degree of distortion of fuel-ion distributions (*i.e.* ion tail formation) in self-sustaining D-^{3}He plasma and how it affects the fusion reactivity. To this end, we simultaneously solve: (a) Boltzmann-Fokker-Planck (BFP) equations for ion species d, ^{3}He, p, t and α ; and (b) global power-balance equation for Maxwellian electrons.

II. KINETIC/POWER-BALANCE MODEL

A. Kinetic Equations

For ion species a (a=d,^{3}He; p,t,α) with isotropic velocity distribution, the BFP equation can be written in the following from:

$$\frac{\partial f_a(v)}{\partial t} = \frac{1}{v^2}\frac{\partial}{\partial v}\left[\sum_b P_{ab}(v)\frac{\partial f_a(v)}{\partial v} + \sum_b Q_{ab}(v) f_a(v)\right] + \left(\frac{\partial f_a(v)}{\partial t}\right)^{NES} + \frac{1}{v^2}\frac{\partial}{\partial v}\left[\frac{v^3 f_a(v)}{2\tau_c}\right] + S_a(v) - L_a(v) = 0 \quad , \tag{1}$$

where $f_a(v)$ is the total (*i.e.* bulk plus tail) distribution function of species a, being normalized so that the total number density is given by

$$n_a = 4\pi\int_0^\infty f_a(v)\, v^2 dv \quad . \tag{2}$$

The first term in Eq.(1) represents the effect of small-angle Coulomb collisions with species b (b = d, ^{3}He, p, t, α or electron). The functions $P_{ab}(v)$ and $Q_{ab}(v)$ are given by

$$P_{ab}(v) = \frac{4\pi}{3}\Gamma_{ab}\left[\frac{1}{v}\int_0^v f_b(u)\, u^4 du + v^2\int_v^\infty f_b(u)\, u du\right], \tag{3}$$

$$Q_{ab}(v) = \frac{4\pi\, m_a \Gamma_{ab}}{m_b}\int_0^v f_b(u)\, u^2 du \quad , \tag{4}$$

where

$$\Gamma_{ab} = \frac{4\pi}{m_a^2}\left(\frac{z_a z_b e^2}{4\pi\,\epsilon_0}\right)^2 \ln\Lambda \ . \quad (5)$$

The second term is the Boltzmann collision integral representing the effect of NES:

$$\begin{aligned}\left(\frac{\partial f_a(v)}{\partial t}\right)^{NES} &= \sum_b \int dv' \int dv_b\, f_a(v')\, f_b(v_b)\, v_r'\, \sigma^{NES}(v_r') \\ &\quad \times P\,(v' \to v\,|\,v_b) \\ &\quad - \sum_b \int dv_b\, f_a(v)\, f_b(v_b)\, v_r \sigma^{NES}(v_r) \\ &= \sum_b \frac{2\pi}{v^2}\int_0^\infty v' f_a(v') \int_0^\infty v_b f_b(v_b)\, P(v' \to v\,|\,v_b) \\ &\quad \times \left(\int_{|v'-v_b|}^{v'+v_b} v_r'^2\, \sigma^{NES}(v_r')\, dv_r'\right) dv'\, dv_b' \\ &\quad - \sum_b \frac{2\pi}{v} f_a(v) \int_0^\infty v_b f_b(v_b) \\ &\quad \times \left(\int_{|v-v_b|}^{v+v_b} v_r^2\, \sigma^{NES}(v_r)\, dv_r\right) dv_b \ ,\end{aligned} \quad (6)$$

where $v'_r = |v'-v_b|$, $v_r = |v-v_b|$, and $P(v' \to v|v_b)$ is the probability distribution function for v, the particle speed after a NES. If we assume that NES is isotropic in the center-of-mass system, then

$$P\,(v' \to v\,|\,v_b) = \begin{cases} \dfrac{2v}{v^2_{\max} - v^2_{\min}} & (v_{\min} \le v \le v_{\max}) \\ 0 & (\ otherwise\ )\ , \end{cases} \quad (7)$$

where

$$v_{\substack{\max\\ \min}} = \left| \frac{m_b}{m_a+m_b}\sqrt{v'^2+v_b'^2} \pm \frac{1}{m_a+m_b}\sqrt{m_a^2 v'^2 + m_b^2 v_b'^2} \right| \quad (8)$$

The third term represents the diffusion in velocity space due to thermal conduction.[5] The "conduction loss time" τ_C is given by

$$\frac{1}{\tau_C} = \frac{1}{\tau_E(v)} - \frac{1}{\tau_N(v)} \ . \quad (9)$$

The velocity-dependent energy confinement time $\tau_E(v)$ and particle confinement time $\tau_N(v)$ may be written as

$$\tau_E(v) = \tau_{E0}\, Max\,(1, v/v_0)^\gamma \ , \quad (10)$$

$$\tau_N(v) = \tau_{N0}\, Max\,(1, v/v_0)^\gamma \ . \quad (11)$$

We adjust the parameters v_0 and γ so that the energy and particle losses from high-energy (tail) components become negligible. In this case, τ_{E0} and τ_{N0} become equal to the usual energy and particle confinement times, respectively.

The particle source and loss terms take different forms for every ion species. For fuel ions, we assume that the losses are compensated by some appropriate fueling method. Then,

$$S_a(v) - L_a(v) = 0 \quad (a = d,\ ^3He)\ . \quad (12)$$

As for p, t and α,

$$\begin{aligned} S_a(v) - L_a(v) &= \sum_i \sum_j \frac{n_i n_j}{1+\delta_{ij}} <\sigma v>_{ij} \frac{\delta(v - v_a^{ij})}{4\pi v^2} \\ &\quad - \sum_j \frac{2\pi}{v} f_a(v) \int_0^\infty v_j f_j\,(v_j) \\ &\quad \times \left(\int_{|v-v_j|}^{v+v_j} v_r^2\, \sigma_{aj}\,(v_r)\, dv_r \right) \\ &\quad - \frac{f_a(v)}{\tau_N} \quad (a = p, t, \alpha)\ , \end{aligned} \quad (13)$$

where v_a^{ij} is the particle speed corresponding to its birth energy.

The reaction rate parameter is calculated by

$$\begin{aligned} <\sigma v>_{ij} &= 8\pi^2 \int_0^\infty v_i f_i\,(v_i) \int_0^\infty v_j f_j\,(v_j) \\ &\quad \times \left(\int_{|v_i-v_j|}^{v_i+v_j} v_r^2\, \sigma_{ij}\,(v_r)\, dv_r \right) dv_i\, dv_j \,/\, n_i\, n_j\ . \end{aligned} \quad (14)$$

B. Electron Power Balance

The foregoing BFP equations assures the global ion power-balances as well as the particle conservations. This could be seen if one multiply Eq.(1) by $(1/2)m_a v^2$ to integrate it over the velocity space.

For electrons, we assume a Maxwellian distribution. The temperature T_e is determined from the global power-balance equation:

$$P_{Coll} - P_{Brems} - P_{Syn} - \frac{\frac{3}{2} n_e T_e}{\tau_E} = 0 \quad . \qquad (15)$$

The first term in Eq.(15) represents the rate of collisional energy transfer per unit volume from all ion species (d, ^{3}He; p, t, α) to electrons, that is,

$$P_{Coll} = \sum_a \int -\left(\frac{dE_a}{dt}\right)_e f_a(v) \cdot 4\pi v^2 dv \quad , \qquad (16)$$

where $(dE_a/dt)_e$ is the rate of average energy loss on the Maxwellian electrons, being given by

$$\left(\frac{dE_a}{dt}\right)_e = -\frac{z_a^2 e^4}{4\pi\epsilon_0^2}\frac{n_e \ln\Lambda}{m_e v}\left\{ erf(x) - \frac{2}{\sqrt{\pi}} x \left(1 + \frac{m_e}{m_a}\right)\exp(-x^2)\right\} \quad , \qquad (17)$$

$$x \equiv v / \sqrt{2T_e / m_e} \quad . \qquad (18)$$

The second and third terms stand for the energy loss rates per unit volume due to bremsstrahlung and synchrotron radiation, respectively. The electron density is determined from the charge neutrality condition.

C. Solution and Ion Temperature

Equations (1) and (15) are simultaneously solved using a numerical iterative procedure to obtain the ion velocity distributions $f_a(v)$ and the electron temperature T_e at equilibrium.

In order to define the fuel ion temperature T_i, we devide the distributions $f_j(v)$ (j = d, ^{3}He) thus calculated into the thermal (bulk) component and the fast (tail) one. As is seen later, $n_j^{tail}/n_j^{bulk} \le 10^{-2}$. The effective bulk temperature of fuel species j is calculated by

$$T_j = \frac{m_j}{3\, n_j^{bulk}} \int v^2 f_j^{bulk}(v) \cdot 4\pi v^2 dv \quad . \qquad (19)$$

Then, the ion temperature is

$$T_i = \sum_j n_j^{bulk} T_j \,/ \sum_j n_j^{bulk} \quad (j = d, {}^3He) \quad . \qquad (20)$$

III. RESULTS AND DISCUSSION

In the present calculation we supposed a plasma condition close to a FRC reactor design.[5] The plasma β value was assumed to be 0.9. the fuel ion ratio was fixed to $n_d/n_{^3He}$ = 2. The confinement times and accompanying parameters were taken as $\tau_{N0} = 2\tau_{E0}$ = 4sec, γ = 10, and v_0 = thermal proton speed. The large γ value (*e.g.* $\gamma>4$) simulates the reduction of the energy and particle losses from high-energy (tail) components. We also assumed that 20% of 14.7MeV protons are lost directly (without slowing down) out of the plasma core.

Figure 1 shows the deuteron, helium-3 and triton distribution functions when T_i = 100keV. It is observed that NES creates by recoil high-energy (>1MeV) tail ions. The fraction of tail density n_j^{tail}/n_j is estimated to be ~1% (j = d,^{3}He). A local peak in the triton distribution is due to the 1.01-MeV birth component. The broken lines indicate the corresponding Maxwellian distributions at 100-keV ion temperature.

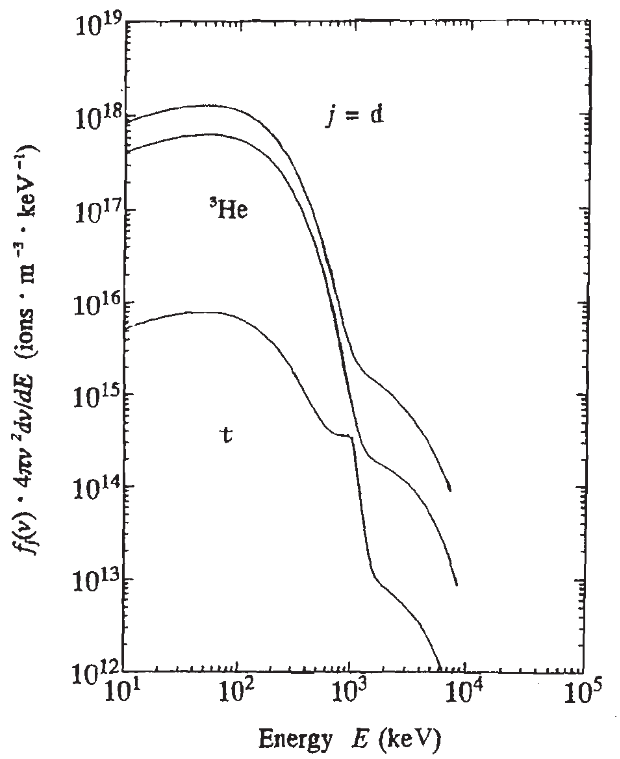

Fig.1 Ion distribution functions at 100-keV bulk ion temperature

A. Fusion Reactivity

As a result of the tail formation, the fractional population of energetic ions that have large d–d cross-sections and small d–t one increases while the bulk components become less compared with the case of Maxwellian plasmas. Thus, the d–d reaction rate parameters increase while d–t one decreases from the values both d and t distributions are assumed to be Maxwellian at the same ion temperature.

The degree of the non-Maxwellian tail effect on the fusion reactivity can be evaluated by the following "enhancement" parameter :

$$\eta_{ij} = \frac{<\sigma v_{ij}> - <\sigma v>_{ij}^{M}}{<\sigma v>_{ij}^{M}} \times 100\ (\%) ,$$

where $<\sigma v>_{ij}^{M}$ is the reaction rate parameter obtained by assuming Maxwellian distributions. The η_{ij} value when T_i = 100keV is evaluated as:

-0.1%	(~0%)	for	^{3}He(d, p)α,
12%	(15%)	for	D(d, p)T,
13%	(17%)	for	D(d, n)^{3}He,
-8%	(-10%)	for	T(d, n)α.

The values in the parentheses are derived from the calculations neglecting the direct loss of 14.7-MeV protons. The degree of reactivity change evaluated here is somewhat smaller than that in our previous calculation.[4]

The change in d–^3He reaction rate parameter is found to be negligible. This is because the d–^3He cross-section meets the peak value at the incident deuteron energy around 500keV, which is several times higher than that for d–t reaction. The NES-created tail ions tend to decrease $<\sigma v>$ for d–^3He reaction, but this negative effect (η<0) is negated by the positive one (η>0) due to the reduction in the fraction of low-energy (<500 keV) components.

We also made calculations of the same kind for other plasma temperatures and the direct-loss fractions of 14.7-MeV protons. The reactivity change becomes more significant at higher temperatures and smaller loss fraction, but the η value for main d–^3He reaction still remains almost unchanged. Thus, it is concluded that the tail formation due to NES does not improve the fusion reactivity of D–^{3}He plasmas.

B. Fractional Energy Deposition

Another NES effect appears through the fraction of magnetically-trapped 14.7-MeV proton's energy deposited on electrons, f_e. The inclusion of NES decreases f_e by about 30%. When T_i = 100keV, for example, f_e = 0.60[0.85] if NES is included [neglected].

The decrease in the f_e value (*i.e.* enhancement of ion heating by 14.7-MeV protons) due to NES leads to the reduction of ignition requirement. This effect, however, is found to be not so significant. The reduction of required $n\tau_E$ value is only 10–15%.

IV. CONCLUSION

The NES-created tail ions increase the d–d reaction rate parameters and decrease the d–t one, but very little change is observed on the main d–^3He reaction. Thus, we cannot expect the improvement of fusion reactivity due to NES in self-igniting D–^{3}He plasmas.

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