ANALYTICAL STUDIES ON A TRAVELING WAVE DIRECT ENERGY CONVERTER FOR D-^{3}He FUSION

L.Y. Syu
National Institute
for Fusion Science,
Nagoya 464-01, Japan
(052)789-5475

George H. Miley
Fusion Studies Laboratory,
University of Illinois,
Illinois 61801-2984,U.S.A
(217) 333-3772

Yukihiro Tomita
National Institute
for Fusion Science,
Nagoya 464-01, Japan
(052)789-4252

Hiromu Momota
National Institute
for Fusion Science,
Nagoya 464-01, Japan
(052)789-4254

ABSTRACT

Analytical studies on a traveling wave direct energy converter (TWDEC) for D-^{3}He fueled fusion are carried out. The energy of 15MeV carried by fusion protons is too high to handle with an electrostatic device. The TWDEC controls these high energy particles on the base of the principle of a Linac. This traveling wave method is discussed and the details of proton dynamics and excitation mechanism of electric power are clarified. The TWDEC consists of a modulator and a decelerator. The applied traveling wave potential to the modulator modulates the velocity of fusion proton beams. This modulation makes a form of bunched protons at a down stream of the modulator. The decelerator has a set of meshed grids, each of which are connected to a transmission circuit. The phase velocity of excited wave on the transmission circuit is controlled as same as that of decelerated protons. The kinetic energy 15MeV of proton beams changes into an oscillating electromagnetic energy on the transmission circuit. This highly efficient direct energy converter of fusion protons brings a fusion reactor with a high plant efficiency.

I. INTRODUCTION

D-^{3}He fueled fusion produces a small fraction of 14MeV neutrons[1], which leads low activation of the structural materials due to the impact of neutrons. For a purpose of demonstrating the attractive characteristics of D-^{3}He fusion, a design study of D-^{3}He/FRC reactor "ARTEMIS-L"[1] has been carried out. At this design approximately 1/3 of fusion power is carried by 15MeV fusion protons. Therefore, it is important to develop a direct energy conversion system with high efficiency which brings about a fusion reactor with a high plant efficiency. The energy of 15MeV, however, is too high to handle it with electrostatic method. In case of 15MeV a gap length[2] is necessary as long as 24 ~ 184m. A concept of a traveling wave direct energy converter (TWDEC) for recovering 15MeV energy of protons has been proposed in 1989 by H.Momota[3] on the base of the principle of a Linac.[4] The TWDEC consists of a modulator and a decelerator both of which consist of an array of metallic grid meshes and a transmission circuit. At the modulator, incident protons are modulated with a traveling wave field to form a bunched protons. At the decelerator a traveling wave field is excited by a flow of bunched protons. The bunched protons are decelerated by the excited potential. But in the report[3] no detail of proton dynamics and excitation mechanism of electric power was given. In this study the traveling wave method is discussed and the details of proton dynamics and excitation mechanism of electric power are clarified. In D-^{3}He/FRC burning plasma[1], fusion protons with 545MW escape directly out of plasma. At the entrance of DEC with a radius of 5m, the proton density decreases as low as 2.72×10^{10} /m^3. Since the characteristic length of our interest is an order of 10 m, the space potential produced by the proton beams is 4.9×10^4V, which is much lower than the beam energy of 15MV. This means collective effects in the beam dynamics can be ignored and the motion of a beam proton can be approximated by that of a single particle in an external field.

II. MODULATOR

The modulator performs a role of the spatial bunching of proton beams in order to generate the high efficient electricity at the decelerator.

The distribution function of particles is obtained by use of the initial one considering Vlasov equation and Liuville's theorem.

$$f(x,\mathrm{v},t) = f(x_0(x,\mathrm{v},t), \mathrm{v}_0(x,\mathrm{v},t), t_0(x,\mathrm{v},t)) , \quad (1)$$

here the initial values suffixed 0 should be expressed by the values of x,v, and t. The initial distribution of protons is spatially uniform and shifted Maxwellian in velocity space:

$$f(v_0) = \frac{n_{p0}}{\sqrt{\pi}\, v_t} \exp[-(v_0 - v_{15MeV})^2 / v_f^2] \ . \qquad (2)$$

Here v_{15MeV} and v_t correspond the velocity 15 MeV and a Doppler spread at its birth, respectively. In "ARTEMIS-L"[1] with the plasma temperature of 83.5keV, the half width of half maximum of Doppler spread is 0.834MeV. The fusion proton obeys the equation of motion in the applied traveling wave at the modulator:

$$m \frac{d^2 x}{d t^2} = e k_m V_{m0} \sin(k_m x - \omega_m t) \ . \qquad (3)$$

Under the conditions that the applied voltage is much lower than the energy of a fusion proton and the velocity of particle is much closer to the phase velocity of traveling wave, after passing through the modulator field, the initial velocity v_0 is expressed by the quantities of x,v, and t.

$$v_0 = v - \frac{2\pi e V_{mo}}{m v} \sin(k_m x - \omega_m t) \ . \qquad (4)$$

Consequently the distribution function after passing through the modulator field is obtained under the conditions mentioned above.

$$f(x,v,t) = \frac{n_{p0}}{\sqrt{\pi}\, v_t} \exp[-(v - v_{15MeV})^2 / v_f^2] \times$$
$$[1 + \frac{4\pi e V_{m0}(v - v_{15MeV})}{m v_f^2 v} \sin(k_m x - \omega_m t)] \ . \ (5)$$

By integrating this distribution function in velocity space we can obtain the spatial distribution of proton beams after passing through the modulator field.

$$n_p(x,t) = n_{p0}[\,1 + \frac{4\pi e V_{m0}}{m v_t^2} \sin(k_m x - \omega_m t)] \ . \qquad (6)$$

The second term of this equation indicates the spatial bunching of fusion protons at a down stream of the modulator field.

We clarify the bunching phenomena by the numerical calculation exclusive the limited conditions. In Fig.1 the phase plot of proton beams is shown. The modulator field is applied between 0 and 1 of spatial position. The fusion protons enter continuously into this field from the negative position. The initial spread of velocity corresponds to the velocity of 0.834MeV. From this figure it is

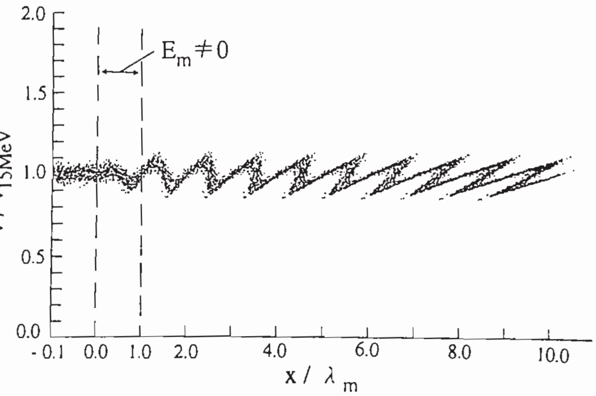

Fig.1 The phase plot of proton beams in the modulator.

clear that the proton beams are bunched after passing through the modulator field. But as beams go apart from the modulator field, the bunched beams are deformed. The degree of spatial bunching is expressed by deviation of proton beams to the function of $[1+\cos(k_m x)] / 2$. The time behaviour of this deviation is shown in Fig.2 in case of an applied voltage of 0.35 MV.

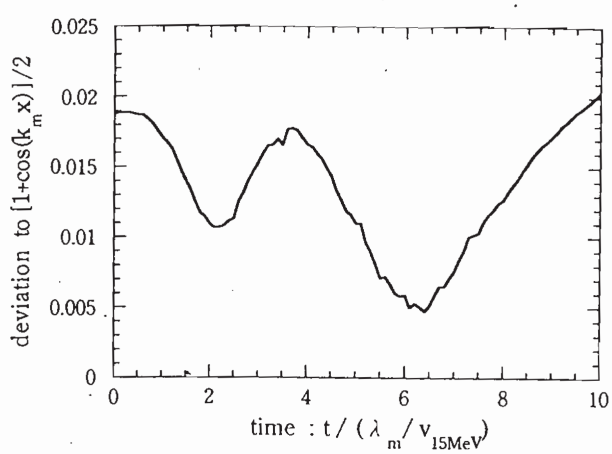

Fig.2 Time behaviour of a deviation of proton beams to a function of $[1+\cos(k_m x)]/2$.

The proton beams are bunched first around normalized time of 2. The increase of the deviation after the first bunching corresponds to shrinkage of beams. This shrinked beams spread and have the minimum deviation around time of 6. After that the bunched beams spread out spatially.

III. DECELERATOR

We study an excitation mechanism of AC power on the transmission circuit due to a potential wave of the bunched protons. The wave number of this potential wave is changed spatially by the coupling to the excited traveling wave.

$$V^e(x,t) = \tilde{V}\cos[\int^x k_d(\xi)d\xi - \omega_d t\,] \quad . \tag{7}$$

The transmission circuit is composed of inductance L, capacitance C, and resistance R (Fig.3).

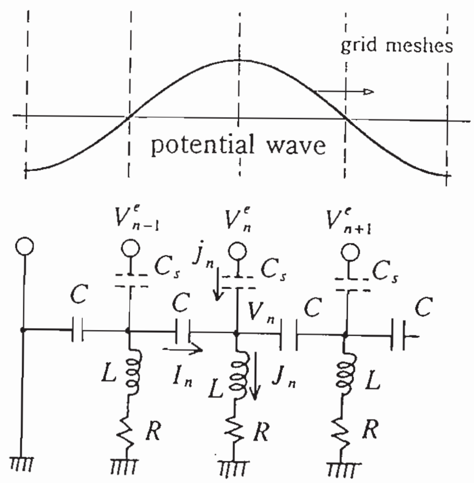

Fig.3 The transmission circuit in the decelerator, consists of a inductance L, a capacitor C, and a resister R.

The stray capacitance between proton beams and grid meshes is denoted by Cs. The potential at a grid mesh of n-th position is denoted by V_n. From this circuit the potential V_n obeys a form of forced oscillation due to the potential of bunched protons:

$$(2+\frac{C_s}{C})(\ddot{V}_n+\frac{R}{L}\dot{V}_n)+\frac{V_n}{LC}-[\ddot{V}_{n+1}+\ddot{V}_{n-1}+\frac{R}{L}(\dot{V}_{n+1}+\dot{V}_{n-1})]$$
$$= \frac{C_s}{C}(\ddot{V}^e_n+\frac{R}{L}\dot{V}^e_n) \quad . \tag{8}$$

From the transmission circuit, the form of the potential wave can be transmitted is given by :

$$V_n(t) = f_p(t)\cos(\theta_n - \omega_d t) \ , \tag{9}$$

here θ_n is the phase of n-th grid and the function f_p changes much slower than the traveling wave. From substituting Eq.(9) into Eq.(8) and averaging with respect to fast oscillation, we obtain a coupled equation:

$$(\Omega^2-\omega_d{}^2-\frac{\nu^2}{4})\,f_p(t) = \frac{\nu\,\varepsilon}{4\omega_d}\sin(\theta_n - \int^{x_n} k_d\,d\xi - \phi)$$
$$-\varepsilon\cos(\theta_n - \int^{x_n} k_d\,d\xi - \phi), \tag{10-1}$$

$$\dot{f}_p + \frac{\nu}{2}f_p = -\frac{\varepsilon}{2\omega_d}\sin(\theta_n - \int^{x_n} k_d\,d\xi - \phi)\,, \tag{10-2}$$

here,

$$\nu \equiv R/L\,,\ \Omega^2 \equiv \frac{1}{LC[\,2(1-\gamma)+C_s/C\,]}\,,$$

$$\varepsilon \equiv \omega_d{}^2\frac{C_s}{C}\tilde{V}\sqrt{1+\frac{\nu^2}{\omega_d{}^2}}\,\frac{1}{2(1-\gamma)+C_s/C}\,,$$

$$\phi \equiv -\cos^{-1}\frac{1}{\sqrt{1+R^2/L^2}} \quad . \tag{11}$$

The quantity Ω is the characteristic frequency of the transmission circuit. In deriving Eq.(10) we introduced the variable γ defined by the relation:

$$\gamma \equiv \cos(\theta_{n+1} - \theta_n)\,. \tag{12}$$

From Eq.(10-1), we have a non-trivial solution, provided that the resonance condition:

$$\Omega^2 - \omega_d{}^2 - \frac{\nu^2}{4} = 0\ , \tag{13}$$

is satisfied.

The potential V_n of n-th grid is obtained from Eq.(10-2).

$$V_n(t) = [\,V_n(0)\frac{\exp(-\nu t/2)}{\cos\Psi_n(0)}$$
$$-\frac{\varepsilon}{\nu\omega_d}(1+\frac{\nu^2}{16\omega_d{}^2})^{-1/2}(1-\exp(-\nu t/2))]\cos\Psi_n(t), \tag{14}$$

here,

$$\Psi_n(t) \equiv \int^{x_n} k_d\,d\xi - \omega_d t - \phi + \tan^{-1}\frac{4\omega_d}{\nu}.$$

As a result the power obtained from the decelerator is:

$$P_{out} = \frac{\omega_d}{2\pi}\int_0^{2\pi/\omega_d} V_n(t)\,J_n(t)\,dt$$
$$= \frac{8\varepsilon^2}{\omega_d^5 L}\,\frac{Q^5}{(16Q^2+1)(Q^2+1)}\ , \tag{15}$$

where the quantity Q represents the steepness of the resonance defined by

$$Q \equiv \omega_d L/R\ . \tag{16}$$

We have to note here the fact that the only cosine component of bunched proton distribution contributes to excite electricity on the transmission circuit and higher harmonics only make noise on the output power.

We study a trapping and detrapping of particles to the decelerating traveling wave. The wave number k_d is selected as the phase velocity is equal to the mean velocity of the particles.

$$k_d[x(t)] / \omega_d = v(t) , \qquad (17)$$

here the frequency ω_d is constant. The equation of motion of the particle for the forward wave is:

$$m\frac{d^2x}{dt^2} = -ek_d(x) V_{d0} \cos(\int^x k_d d\xi - \omega_d t) . \qquad (18)$$

From this equation the particle velocity is obtained and also the wave number kd in the decelerator can be expressed in terms of a position x:

$$k_d(x) = \omega_d (1 - \frac{3e V_{do} \omega_d}{m v_{15MeV}^3} x)^{-1/3} . \qquad (19)$$

In case of the wave number is uniform, if the particle velocity deviates greatly from the wave phase velocity, the particle moves in the same direction and the particle can not be trapped within the wave. In this case, the particle goes through the phase $+\pi/2$ or $-\pi/2$ of the wave. On the contrary, if the particle velocity is sufficiently closed to the wave phase velocity, the particle becomes trapped within the wave and the phase of the particle's position is between $-\pi/2$ and $+\pi/2$. In case when k_d is not uniform, we will use the results which have been obtained for the previous case. The particle can be seen as it being trapped if the phase of the particle remains within a region between $-\pi/2$ and $+\pi/2$. The time variation of the phase ψ_d is:

$$\frac{d\Psi_d}{dt} = k_d(x) v(t) - \omega_d . \qquad (20)$$

This equation together with Eq.(18) gives a particle trajectory on the phase plane ψ_d and $d\psi_d/dt$ (Fig.4). In this figure the trapping occurs in the region surrounded by two lines. Particle trapping is poor in case that the spatial distribution has the form of cosine. If the spatial distribution is concentrated within a certain phase of the traveling wave, the particle trapping can be fairly good. For the latter case, however, high excitation efficiency of electricity will be lost. This means that the optimization studies with respect to the spatial distribution of the particles are left as subjects of future studies.

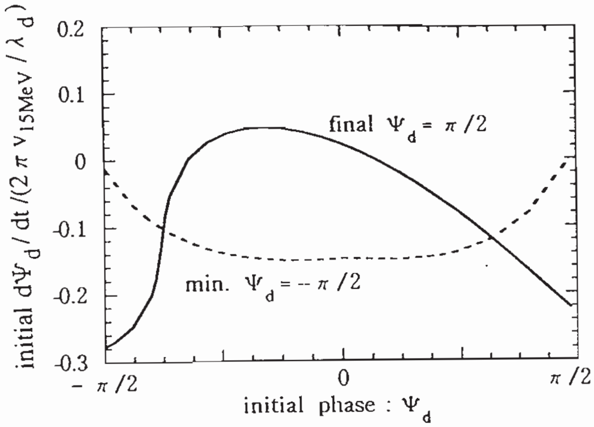

Fig.4 The trapping and detrapping region in the space of initial phase Ψ_d and its time derivative $d\Psi_d/dt$.

IV. CONCLUSIONS

From the discussions so far presented, we can summarize our studies as follows:

a) Details of the concept of the TWDEC have been studied analytically and numerically, showing that the concept appears to be feasible in view of modulations of proton beams and excitations of electricity on a transmission circuit as well as the trapping of fusion protons.

b) A cosine distribution is relevant to excite electricity. The detrapping of fusion protons is appreciable in this case. A certain modification of proton distribution might be needed to obtain a high conversion efficiency. That is, an optimization study is required to obtain a high efficiency of the converter. This subject is left as a future study.

REFERENCES

1. H.Momota, et al.,"D-^{3}He Fueled FRC Reactor "ARTEMIS-L"," *Proc.14th Int.Conf.Plasma Physics and Controlled Nucl. Fusion Research*, Wuerzburg, Germany, Sept. 30 - Oct.7, 1992, Vol.3,p.319, International Atomic Energy Agency (1993).
2. Y.Ohara, M.Seki, S.Tanaka, K.Watanabe, and K.Yokoyama,"A Review of JAERI R&D Activities on the Negative-Ion-Based Neutral Beam Injection System," JAERI-M 90-154,Japan Atomic Energy Research Institute, Japan (1990).
3. H.Momota,"Direct Energy Conversion for 15MeV Fusion Protons," LA-11808-C, Los Alamos Natl. Lab. New Mexico, 8 (1989).
4. D.W.Fry, et al.,"Travelling Wave Linear Accelerator for Electrons," *Nature*, **160**,351 (1947).