

# **Collective Interaction Processes and Nonlinear Dynamics of Nonneutral Plasmas and Intense Charged Particle Beams**

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# Outline of Presentation

- Introduction
- What is nonneutral plasma? What is its relationship to the physics of intense charged particle beams?
- Nonlinear stability theorem for collective interactions.
- Collective interactions and instabilities:
  - Temperature anisotropy instabilities
  - Electron cloud instability
- Paul Trap Simulator Experiment - compact experiment to simulate beam propagation over long distances.
- Use of neutral plasmas for focusing/compressing high-intensity charge bunches.

# What is a Nonneutral Plasma?

- A nonneutral plasma\* is a many-body collection of charged particles in which there isn't overall charge neutrality.
- Such systems are characterized by intense self-electric fields (space-charge fields), and in high-current configurations by intense self-magnetic fields.
- Nonneutral plasmas may consist of one charge species (one-component nonneutral plasma), or more than one charge species (multicomponent nonneutral plasma).
- Examples of nonneutral plasma systems include:
  - Intense charged particle beams and charge bunches in high energy accelerators, transport lines, and storage rings.
  - Coherent radiation sources (magnetrons, gyrotrons, free electron lasers).
  - One-component nonneutral plasmas confined in a Malmberg-Penning trap or a Paul trap.

\* R. C. Davidson and N. A. Krall, Phys. Rev. Lett. **22**, 833 (1969).

\* R. C. Davidson, *Physics of Nonneutral Plasmas* (Imperial College Press and World Scientific, 2001).

# Intense Beam Physics

- Consider a long coasting charge bunch with volume number density  $n_b$  propagating through a transverse focusing field with average focusing frequency  $\omega_{\beta\perp}$ .
- A convenient dimensionless measure of the self-field intensity(self-electric plus self-magnetic) is the dimensionless parameter

$$s_b \equiv \omega_{pb}^2 / 2\gamma_b^2\omega_{\beta\perp}^2,$$

where  $\omega_{pb}^2 = 4\pi n_b e_b^2 / \gamma_b m_b$  is the relativistic plasma frequency-squared.

- The condition for transverse confinement of the beam particles (applied focusing force exceeds the defocusing self-field force) is

$$0 < s_b = \omega_{pb}^2 / 2\gamma_b^2\omega_{\beta\perp}^2 < 1.$$

- In terms of the normalized tune depression,  $\nu / \nu_0$ , for the special case of uniform beam density, this condition can be expressed as

$$1 > \nu / \nu_0 \equiv (1 - s_b)^{1/2} > 0.$$

## Nonlinear Stability Theorem\*

- Determine the class of beam distribution functions  $f_b(\mathbf{x}, \mathbf{p}, t)$  that can propagate quiescently over large distances at high space-charge intensity.
- Analysis makes use of global (spatially-averaged) conservation constraints satisfied by the nonlinear Vlasov-Maxwell equations to determine a sufficient condition for stability of an intense charged particle beam (or charge bunch) propagating in the  $z$ -direction with average axial velocity  $V_b = \text{const.}$  along the axis of a perfectly-conducting cylindrical pipe with wall radius  $r = (x^2 + y^2)^{1/2} = r_w$ .

\* *Physics of Intense Charged Particle Beams in High Energy Accelerators* (Imperial College Press and World Scientific, 2001), R. C. Davidson and H. Qin, Chapter 4.

“Kinetic Stability Theorem for High-Intensity Charged Particle Beams Based on the Nonlinear Vlasov-Maxwell Equations,” R. C. Davidson, Physical Review Letters **81**, 991 (1998).

“Three-Dimensional Kinetic Stability Theorem for High-Intensity Charged Particle Beams,” R. C. Davidson, Physics of Plasmas **5**, 3459 (1998).

## Theoretical Model and Assumptions

- Model makes use of fully nonlinear Vlasov-Maxwell equations for the self-consistent evolution of the distribution function  $f_b(\mathbf{x}, \mathbf{p}, t)$  and self-generated electric and magnetic fields

$$\mathbf{E} = -\nabla\phi - \frac{1}{c} \frac{\partial}{\partial t} \mathbf{A} \quad \mathbf{B} = \nabla \times \mathbf{A}$$

- Vlasov-Maxwell equations are Lorentz transformed to the beam frame (primed coordinates) where the (time-independent) confining potential of the applied focusing force is assumed to be of the form (smooth-focusing approximation)

$$\Psi'_{sf}(\mathbf{x}') = \frac{1}{2} \gamma_b m_b \omega_{\beta\perp}^2 (x'^2 + y'^2) + \frac{1}{2} \gamma_b m_b \omega_{\beta z}^2 z'^2,$$

where  $\omega_{\beta z}$  and  $\omega_{\beta\perp}$  are constant focusing frequencies.

- Particle motions in the beam frame are assumed to be nonrelativistic.

## Nonlinear Stability Theorem

- In the beam frame, the nonlinear Vlasov-Maxwell equations possess at least two global conservation constraints corresponding to:
- Conservation of total energy

$$U(t') = \frac{1}{L'} \left\{ \int d^3x' \frac{|\mathbf{E}'_T|^2 + |\mathbf{B}'_T|^2}{8\pi} + \int d^3p' \left( \frac{p'^2}{2m_b} + \psi'_{sf} + \frac{1}{2}q_b \phi' \right) f_b \right\} = const.$$

- Conservation of generalized entropy

$$S(t') = \frac{1}{L} \int d^3x' d^3p' G(f_b) = const.$$

## Nonlinear Stability Theorem

- Consider general perturbations about a quasi-steady equilibrium distribution function  $f_{eq}(x', p')$ . For  $f_{eq} = f_{eq}(H')$ , using the global conservation constraints, it can be shown that

$$\frac{\partial}{\partial H'} f_{eq}(H') \leq 0$$

is a sufficient condition for linear and nonlinear stability.

- Here,  $H'$  is the single-particle Hamiltonian defined by

$$H' = \frac{1}{2m_b} \mathbf{p}'^2 + \Psi'_{sf}(x') + q_b \phi'(x'),$$

where  $\phi'(x')$  is the equilibrium space-charge potential.

## Nonlinear Kinetic Stability Theorem

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Therefore a necessary condition for instability is that the beam distribution function have some nonthermal feature such as:

- An inverted population in phase space.
- Or a strong energy anisotropy.
- Or that the beam have directed kinetic energy relative to background charge components.
- Or dissipation mechanisms (e.g., finite wall resistivity).

# Collective Instabilities in Intense Charged Particle Beams

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## One-Component Beams

- Electrostatic Harris instability ( $T_{\parallel b} / T_{\perp b} \ll 1$ )
- Electromagnetic Weibel instability ( $T_{\parallel b} / T_{\perp b} \ll 1$ )
- Resistive wall instability

## Propagation Through Background Electrons

- Electron-ion two-stream (Electron Cloud) instability

## Propagation Through Background Plasma

- Resistive hose instability
- Multispecies Weibel instability
- Multispecies two-stream instability

## Harris Instability in Intense One-Component Beams

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- Electrostatic Harris instability\* can play an important role in multispecies plasmas with temperature anisotropy  $T_{\parallel j} < T_{\perp j}$ .
- Harris instability is inherently three-dimensional and involves a coupling of the longitudinal and transverse particle dynamics.
- Harris-like instability\*\* \*\*\* also exists in intense one-component beams provided the anisotropy is sufficiently large and the beam intensity is sufficiently large.

\* E. G. Harris, Phys. Rev. Lett. 2, 34 (1959).

\*\* I. Haber et al., Phys Plasmas 6, 2254 (1999).

\*\*\* E. A. Startsev, R. C. Davidson and H. Qin, Phys. Plasmas 14, 056705 (2007); H. Qin, R. C. Davidson and E. A. Startsev, Phys. Rev. ST Accelerators and Beams 10, 064201(2007); E. A. Startsev, R. C. Davidson and H. Qin, Phys. Rev. ST Accelerators and Beams 8, 124201(2005); Phys. Plasmas 9, 3138 (2002); Laser and Particle Beams 20, 585 (2002); Phys. Rev. ST Accelerators and Beams 6, 084401 (2003).

# Beam Equilibrium Stability and Transport (BEST) Code

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## Physics

- Nonlinear perturbative particle simulation method (nonlinear delta-f method) to reduce noise.
- Linear eigenmodes and nonlinear evolution.
- 2D and 3D equilibrium structure.
- Multi-species; electrons and ions; accommodate very large mass ratio.
- Multi-time-scales, frequency span a factor of  $10^5$ .
- 3D nonlinear perturbations.

## Computation

- Message Passing Interface
  - Multiple-1D domain decompositon (OpenMP by users).
- Large-scale computing: particle x time-steps  $\sim 0.5 \times 10^{12}$ .
- Scales linearly to 1024 processors on IBM-SP3 at NERSC.
- NetCDF, HDF5 parallel I/O diagnostics.

H. Qin, Phys. Plasmas 10, 3196 (2003).

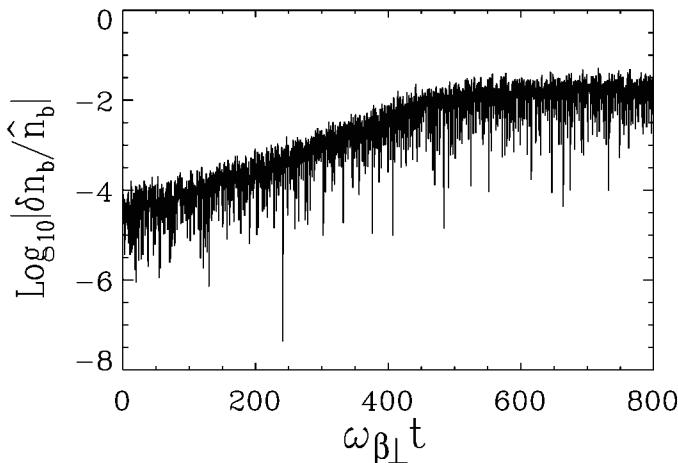
## Large temperature anisotropies develop naturally with

$$T_{\parallel b} \ll T_{\perp b}$$

- For a beam of charged particles with charge  $e_b$  accelerated through a voltage  $V$ , the parallel temperature decreases according to

$$T_{\parallel f} = T_{\parallel i}^2 / 2e_b V$$

- The transverse emittance and perpendicular temperature  $T_{\perp b}$  can also increase relative to  $T_{\parallel b}$  due to nonlinearities and mismatch.
- Free energy is available to drive a Harris-like instability which may lead to a deterioration of beam quality.

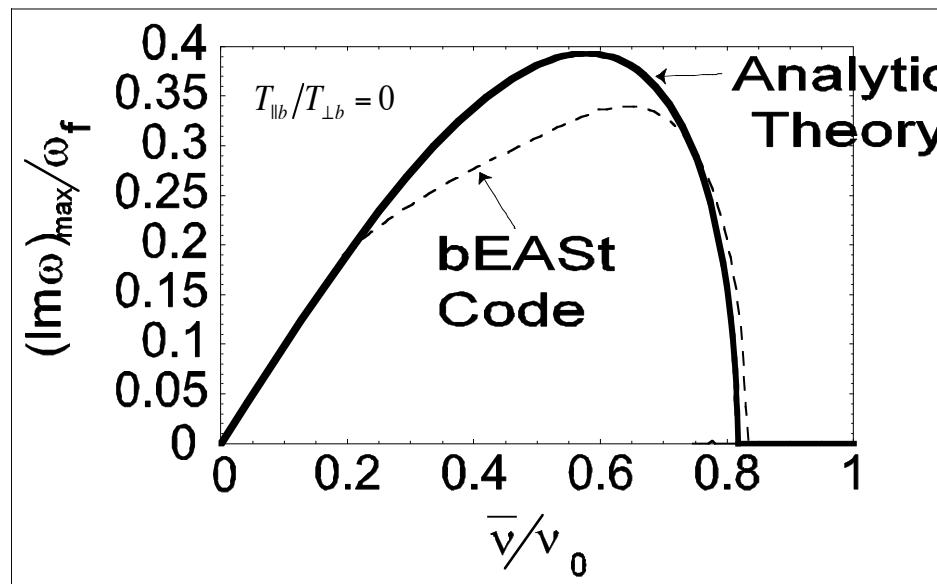


Allow for non-axisymmetric perturbations.

Assume  $T_{\parallel b} / T_{\perp b} = 0.04$  and  $r_w / r_{beam} = 3$ .

Low-noise properties of the BEST code follow the linear and nonlinear growth through saturation at the level  $|\delta n_b^{\max} / \hat{n}_b| \approx 0.05$ .

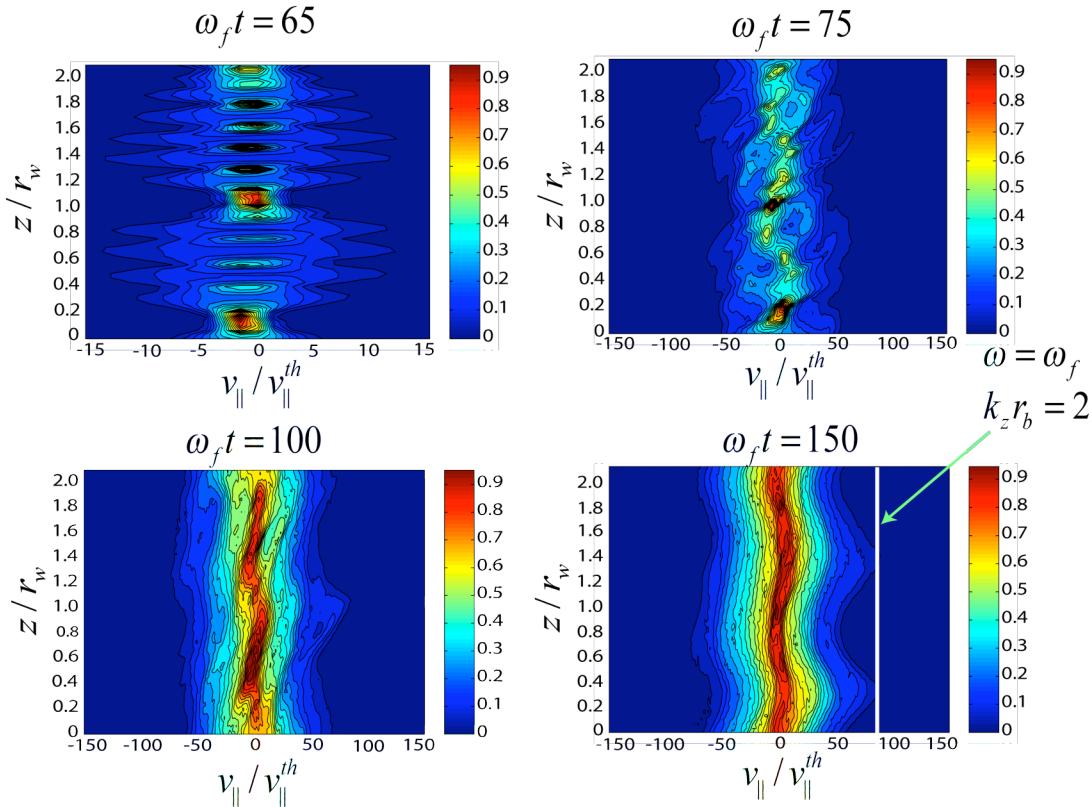
**Large temperature anisotropies develop naturally in intense charged particle beams and can provide the free energy to drive the Harris Instability at high beam intensity**



Plots of normalized growth rate versus normalized tune for azimuthal mode number  $m=1$  (dashed curve). Results have been obtained using the eigenmode code bEASt. The solid curve corresponds to a simple analytical estimate.

# Strong Harris instability for beams with large temperature anisotropy

- Moderate intensity → largest threshold temperature anisotropy.
- BEST simulations show nonlinear saturation by particle trapping.



Longitudinal particle trapping by a broadband spectrum of fast-growing waves with zero real frequency governs the nonlinear saturation.

The nesting and overlapping of particle resonances in longitudinal phase space leads to the fast randomization of the trapped-particle distribution and to longitudinal heating of the beam.

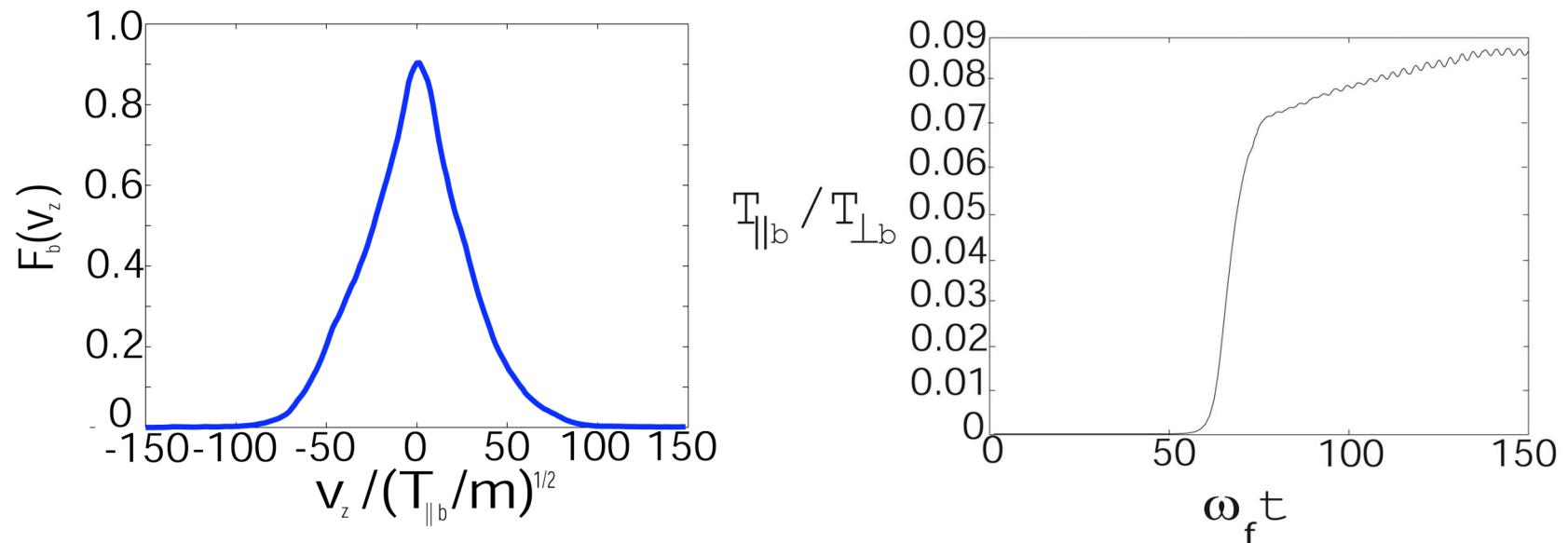
Nonlinear interactions shift the wave spectrum to long wavelengths.

Parameters:  $s_b = 0.9$  and  $T_{\parallel b} / T_{\perp b} = 0.0001$

E. A. Startsev, R. C. Davidson and H. Qin, Physics of Plasmas 14, 056705 (2007).

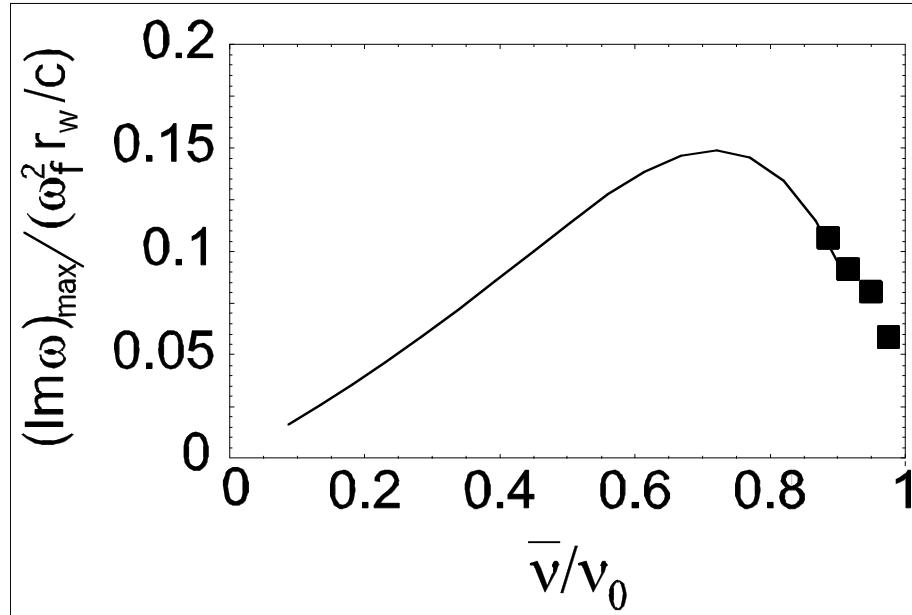
## Harris Instability in Intense One-Component Beams

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Plot of z-averaged longitudinal velocity distribution  $F_b(v_z, t)$  at time  $t = 150\omega_f^{-1}$  (thick blue line), for normalized beam intensity  $s_b = 0.9$  and  $T_{\parallel b} / T_{\perp b} = 0.0001$  at time  $t = 0$ .

**Large temperature anisotropies can also provide the free energy to drive the electromagnetic Weibel Instability with growth rate smaller than the electrostatic Harris instability**



The maximum growth rate of the Weibel instability is plotted versus normalized tune, and is given approximately by

$$\frac{(\text{Im}\omega)_{\max}}{\omega_f} = \frac{1}{\sqrt{2}} \frac{\bar{\omega}_{pb}}{\omega_f} \frac{v_{\perp b}^{th}}{c}$$

# Weibel Instability in Intense One-Component Beams

## General Features of Weibel Instability

- The Weibel instability is not likely to play an important role in one-component space-charge-dominated nonneutral ion beams:
  - Constraints imposed by finite transverse geometry.
  - Electrostatic Harris instability has a much larger linear growth rate in the region where the Weibel instability is strongest.
- However, the Weibel instability can play an important role in multispecies anisotropic plasmas and beam-plasma systems.

## Electron-Ion Two-Stream (Electron Cloud) Instability

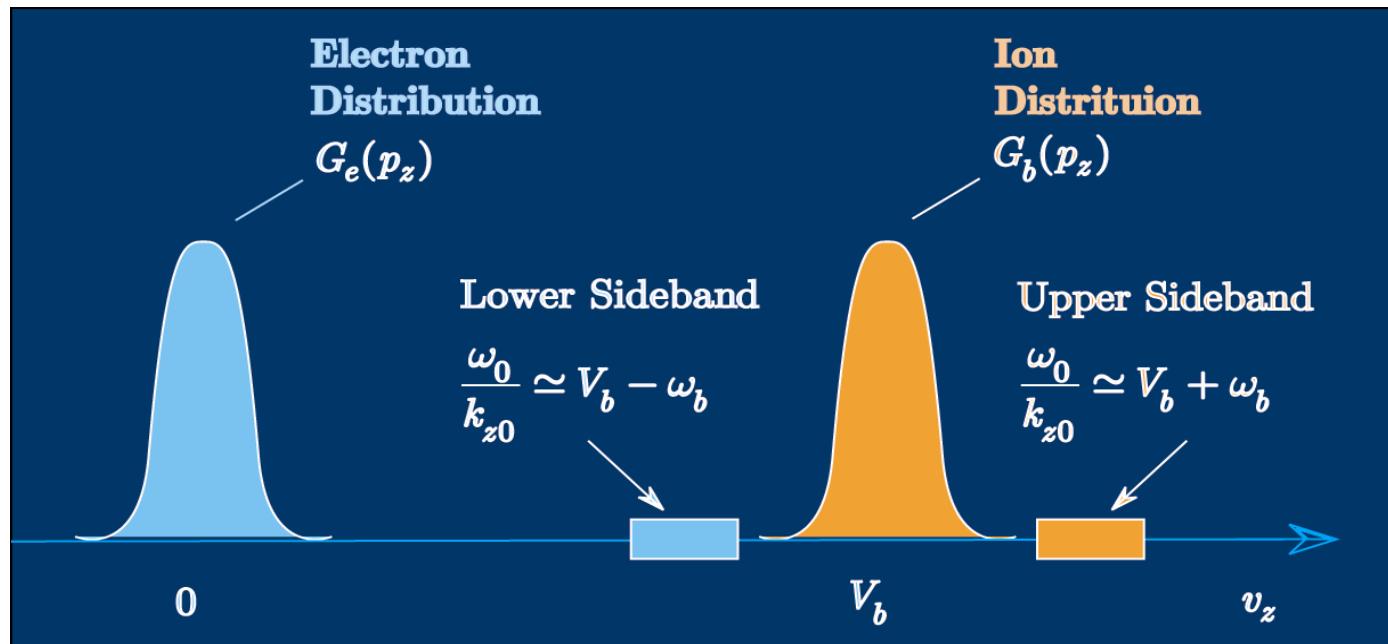
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- A background population of electrons can result when energetic beam ions strike the chamber wall or ionize background gas atoms.
- Relative streaming motion of beam ions through the background electrons provides the free energy to drive the classical two-stream instability, appropriately modified to include the effects of dc space charge, relativistic kinematics, transverse beam geometry, etc.
- Experimental evidence exists for two-stream instability in proton machines such as the Proton Storage Ring (PSR) experiment at Los Alamos National Laboratory.

**R C. Davidson, H. Qin, P H. Stoltz, and T. F. Wang, Physical Review Special Topics Accelerators and Beams 2, 054401 (1999); H. Qin, Phys. Plasmas 10, 3196 (2003).**

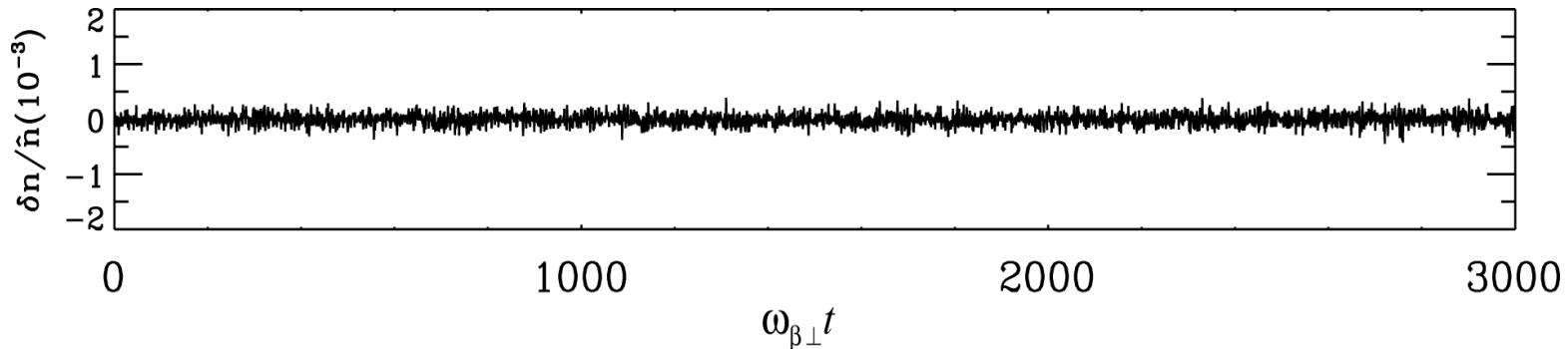
## Electron-Ion Two-Stream (Electron Cloud) Instability

- An intense ion beam supports collective oscillations (sideband oscillations) with phase velocity  $\omega/k_z$  upshifted and downshifted relative to the average beam velocity  $c \beta_b$ .
- An (unwanted) electron component provides the free energy to drive the classical two-stream instability.



# Stable Proton Beam Propagation in PSR in the Absence of Background Electrons

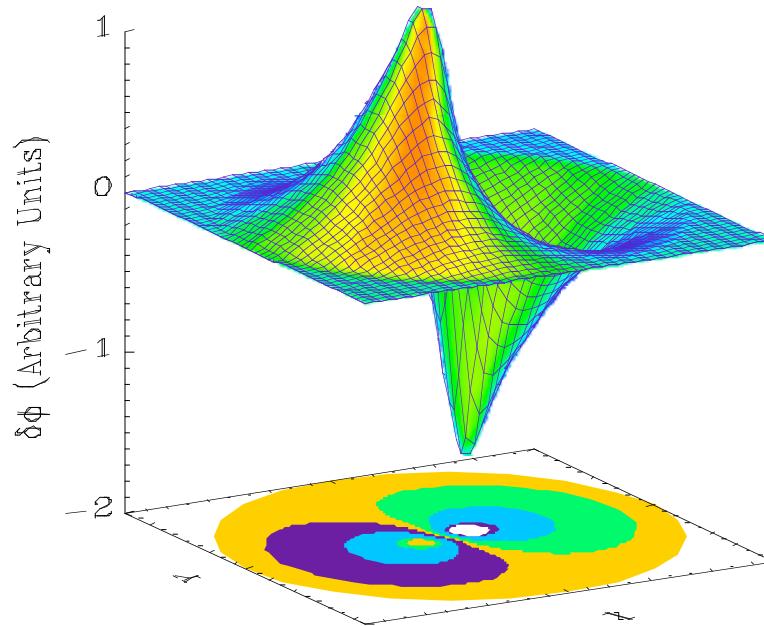
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- Random initial density perturbations with normalized amplitude of  $10^{-3}$  are introduced into the system at  $t = 0$ .
- The proton beam is propagated from  $t = 0$  to  $t = 3000\omega_{\beta \perp}^{-1}$ , where  $\omega_{\beta \perp}$  is the transverse (smooth) focusing frequency.
- Simulations show that the perturbations do not grow and the beam propagates quiescently, which agrees with the nonlinear stability theorem

# Electron-Ion Two-Stream (Electron Cloud) Instability

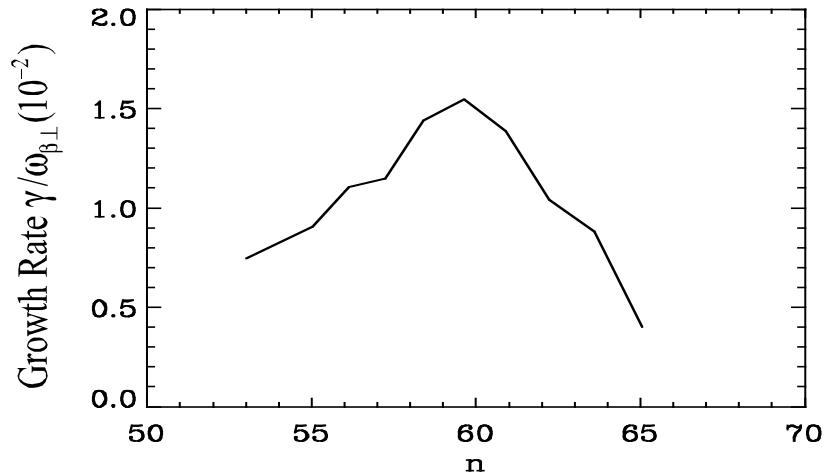
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The  $x - y$  projection (at fixed value of  $z$ ) of the perturbed electrostatic potential  $\delta\phi(x, y, t)$  for the ion-electron two-stream instability growing from a small initial perturbation, shown at  $\omega_{\beta \perp} t = 200$ .

# Electron-ion Two-stream Instability in the Presence of Background Electrons

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## Illustrative parameters in PSR (coasting beam)

- Space-charge-induced tune shift:  $\delta\nu / \nu_0 = -0.02$ ;  $\hat{\omega}_{pb}^2 / 2\gamma_b^2\omega_{\beta\perp}^2 = 0.079$ .
- Mode oscillation frequency (simulations):  $f \sim 163$  MHz. Mode number at maximum growth  $n = 55-65$ .
- $\lambda_b = 9.13 \times 10^8 cm^{-1}$ ,  $\lambda_e = 9.25 \times 10^7 cm^{-1}$ ,  $T_{b\perp} = 4.41 keV$ ,
- $T_{e\perp} = 0.73 keV$ ,  $\phi_0(r_w) - \phi_0(0) = -3.08 \times 10^3 Volts$ .

# Electron-Ion Two-Stream (Electron Cloud) Instability

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## Growth Rate Reduction Mechanisms

- Axial momentum spread in the beam ions.
- Proximity of a conducting wall.
- Reduction in fractional charge neutralization.
- Rounded beam density profiles
  - Spread in transverse oscillation frequency.

## Compact Experiments to Simulate Long-Distance Beam Propagation

It is important to determine the conditions under which charged particle beams can be transported quiescently over large distances at high space-charge intensities.

### Compact Laboratory Traps:

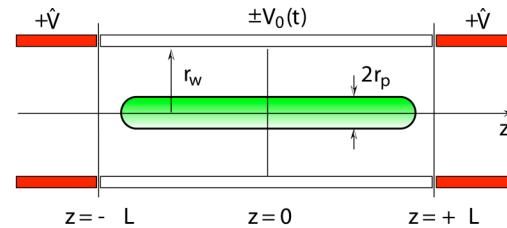
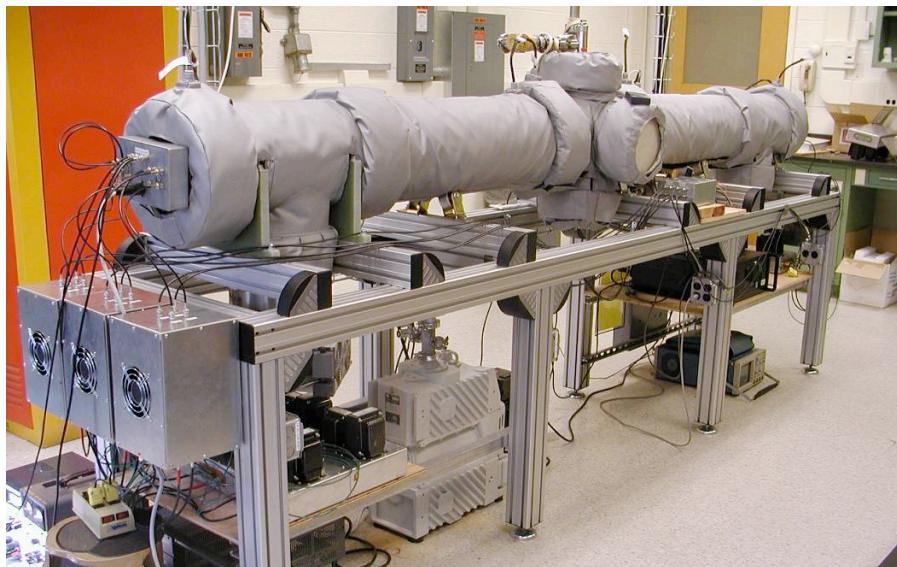
- R. C. Davidson, H. Qin and G. Shvets, Physics of Plasmas **7**, 1020 (2000).
- E. P. Gilson, R. C. Davidson, P. C. Efthimion and R. Majeski, Phys. Rev. Lett. **92**, 155002 (2004).
- E. P. Gilson et al., Physical Review Special Topics on Accelerators and Beams **10**, 124201 (2007).
- M. Chung et al., Physical Review Special Topics on Accelerators and Beams **10**, 064202 (2007).

### Compact Circular Rings:

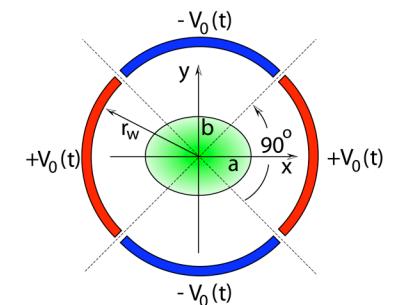
- P.G. O'Shea et al., Nuclear Instruments and Methods in Physics Research **A 464**, 646 (2001).
- S. Bernal et al., Nuclear Instruments and Methods in Physics Research **A 519**, 380 (2004).

# Paul Trap Simulator Experiment (PTSX)

**Objective:** Simulate collective processes and nonlinear dynamics of intense charged particle beam propagation through an alternating-gradient focusing field using a compact laboratory Paul Trap.

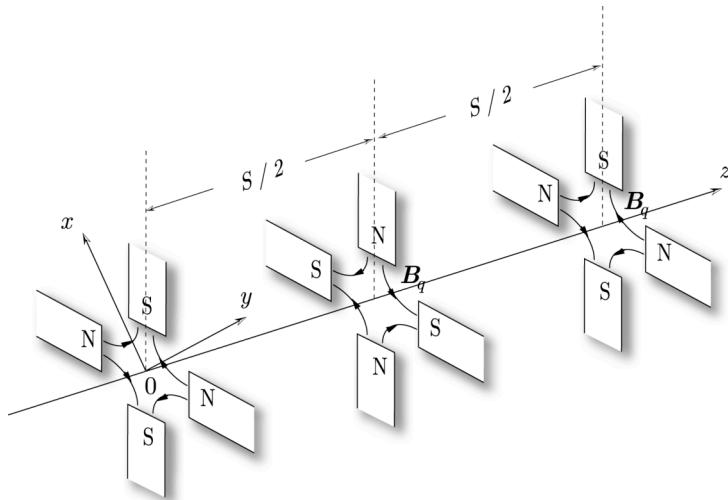


(a)



(b)

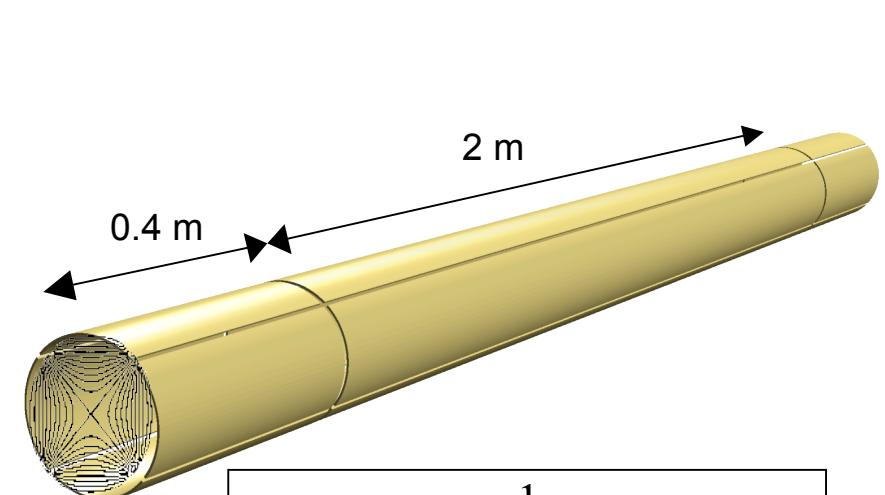
# Paul Trap Simulator Experiment (PTSX) studies intense beam propagation over large distances



$$\mathbf{B}_q^{foc}(x) = B'_q(z)(y\hat{\mathbf{e}}_x + x\hat{\mathbf{e}}_y)$$

$$\mathbf{F}_{foc}(x) = -\kappa_q(z)(x\hat{\mathbf{e}}_x - y\hat{\mathbf{e}}_y)$$

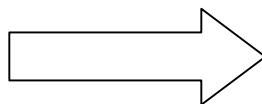
$$\kappa_q(z) \equiv \frac{Z_b e B'_q(z)}{\gamma_b m_b \beta_b c^2}$$



$$e_b \phi_{ap}(x, y, t) = \frac{1}{2} \kappa_q(t) (x^2 - y^2)$$

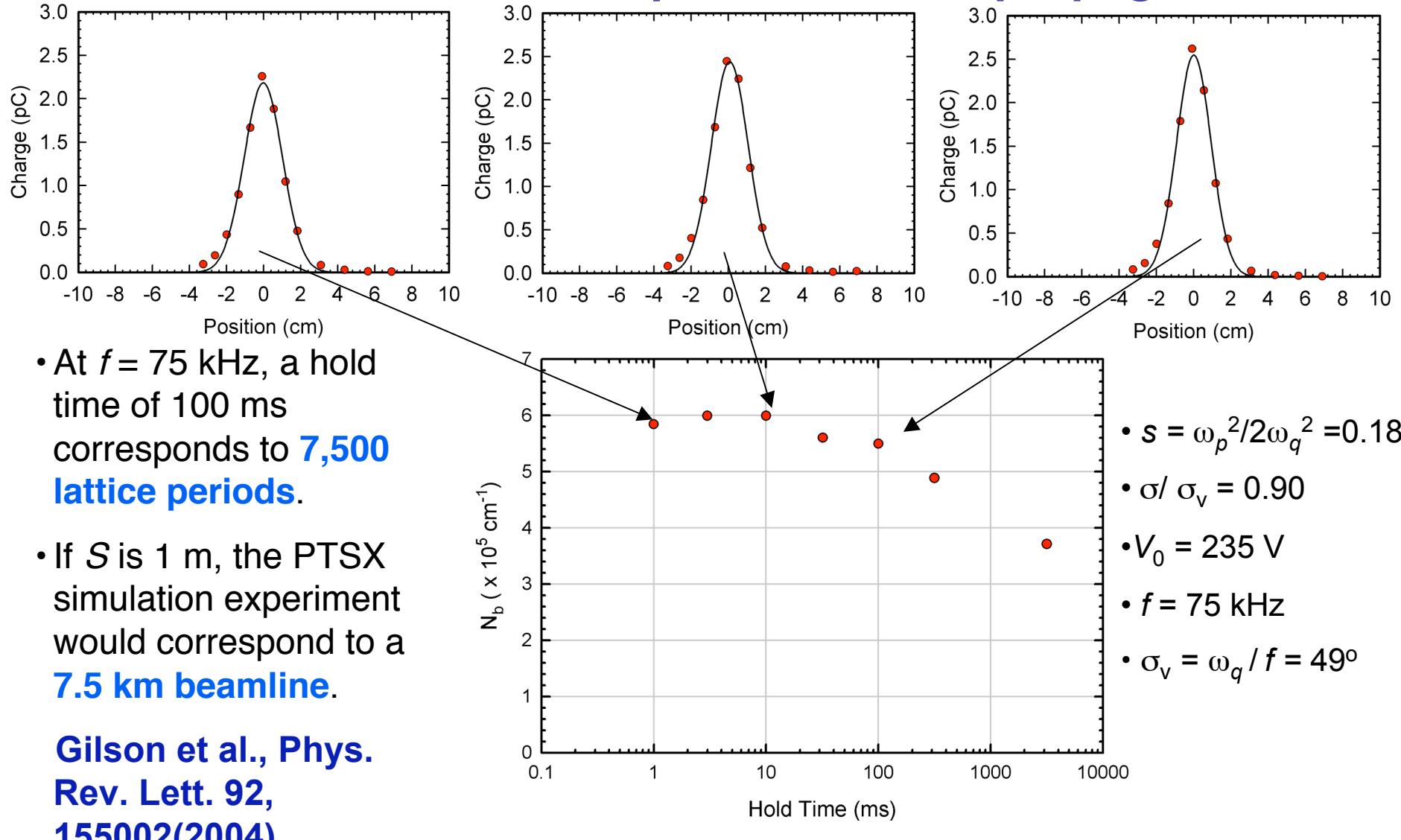
$$\kappa_q(t) = \frac{8e_b V_0(t)}{m_b \pi r_w^2}$$

- Thin beam ( $r_b \ll S$ ).
- The particle motion in the beam frame is assumed to be nonrelativistic.



- Ions in PTSX have the same transverse equations of motion as ions in an alternating-gradient system ***in the beam frame***.

# PTSX traps plasmas for times corresponding to more than 7.5 km of equivalent beam propagation



# Transverse Confinement is Lost When Phase Advance Exceeds 180° and Single-Particle Orbits are Unstable

The waveform applied to the PTSX electrodes, in quadrupolar form, is given by

$$V(t) = V_{0\max} \sin \phi(t)$$

where  $f = \frac{1}{2\pi} \frac{d\phi(t)}{dt}$

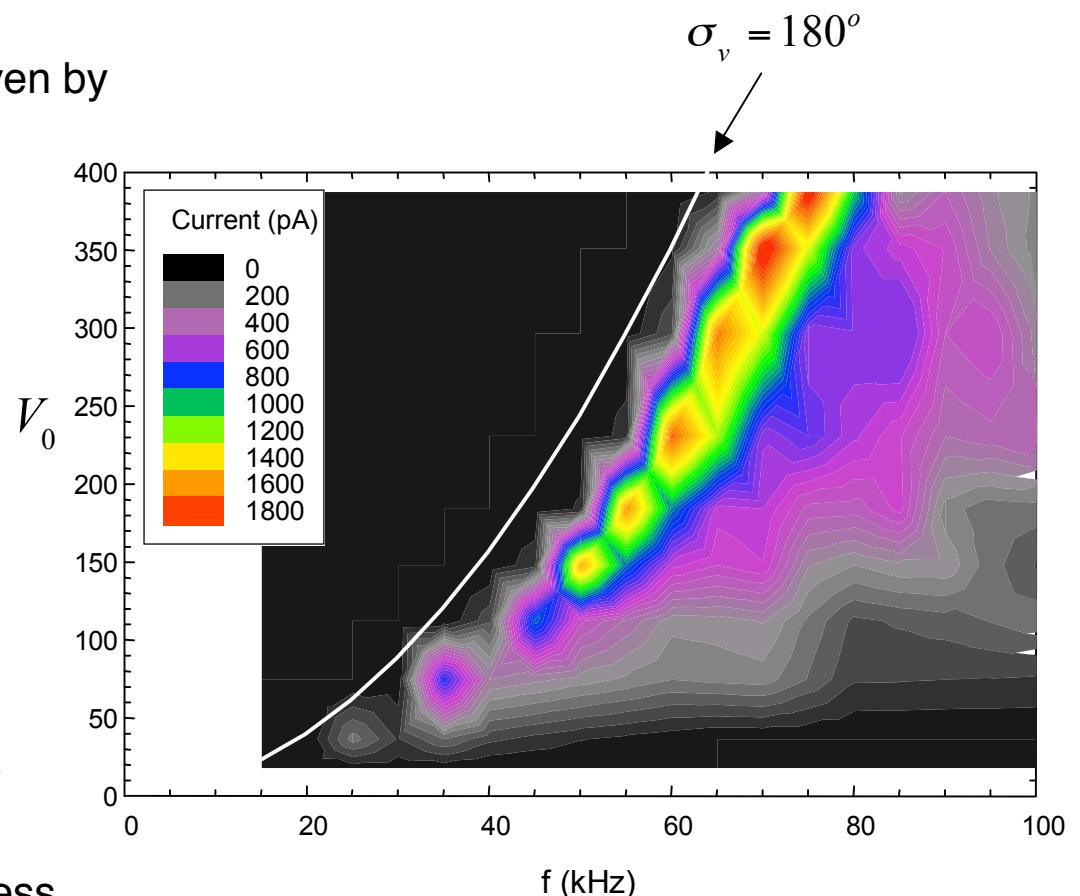
In the absence of self-fields, the transverse particle motion is governed by the Mathieu equation.

$$\frac{d^2}{d\tau^2} \begin{pmatrix} x \\ y \end{pmatrix} + [a + 2q \cos(2\tau)] \begin{pmatrix} x \\ -y \end{pmatrix} = 0$$

For  $a = 0$  and  $q < 0.908$ , the solutions are stable.

That is, the phase advance must be less than 180° for stable transverse orbits.

$$q \propto \frac{V_0}{f^2} \quad \sigma_v = \frac{\omega_q}{f} \propto \frac{V_0}{f^2}$$



Steady-state streaming cesium ion current cannot propagate over the length of PTSX if the vacuum phase advance exceeds 180°.

# Collective-Mode Diagnostic Allows PTSX to Measure Collective Oscillations and Infer Beam “Normalized Intensity”

“Normalized intensity” is the ratio of the “strength” of the beam to the “strength” of the confinement.

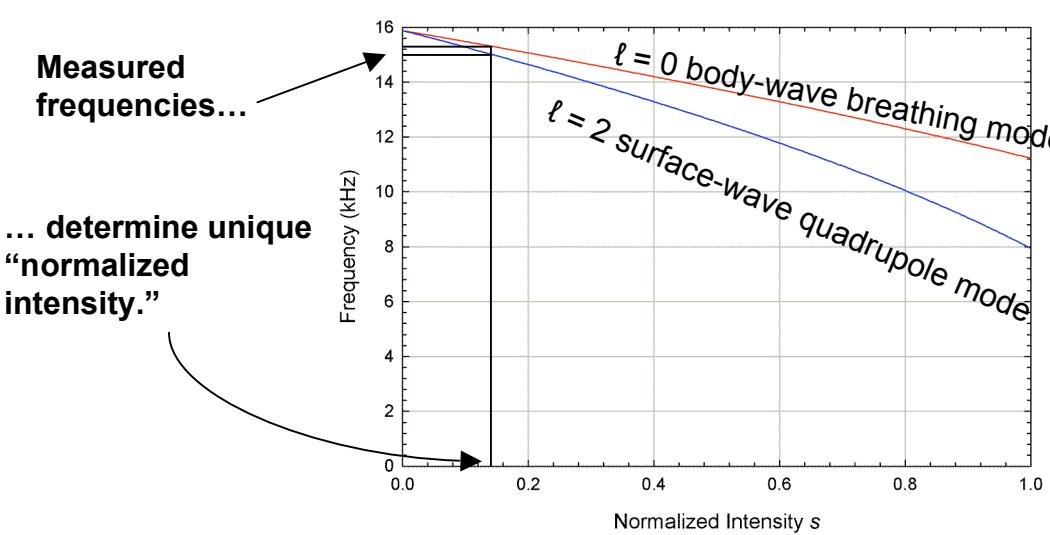
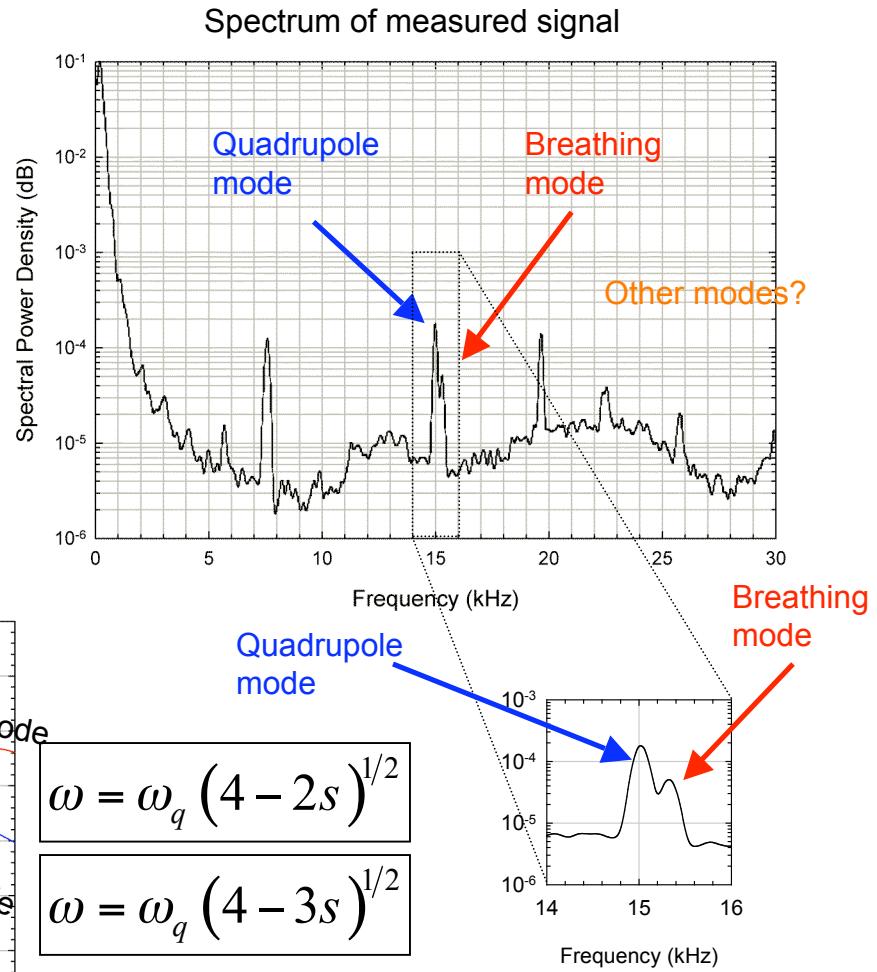
$$S \equiv \frac{\omega_p^2}{2\omega_q^2} < 1$$

where

$$\omega_q = \frac{8eV_0 \max}{m\pi r_w^2 f} \frac{1}{2\sqrt{2\pi}}$$



Segmented collective-mode capacitive pick-up diagnostic is sensitive to beam collective-mode oscillations.



# Noise Induces Mismatch Oscillations

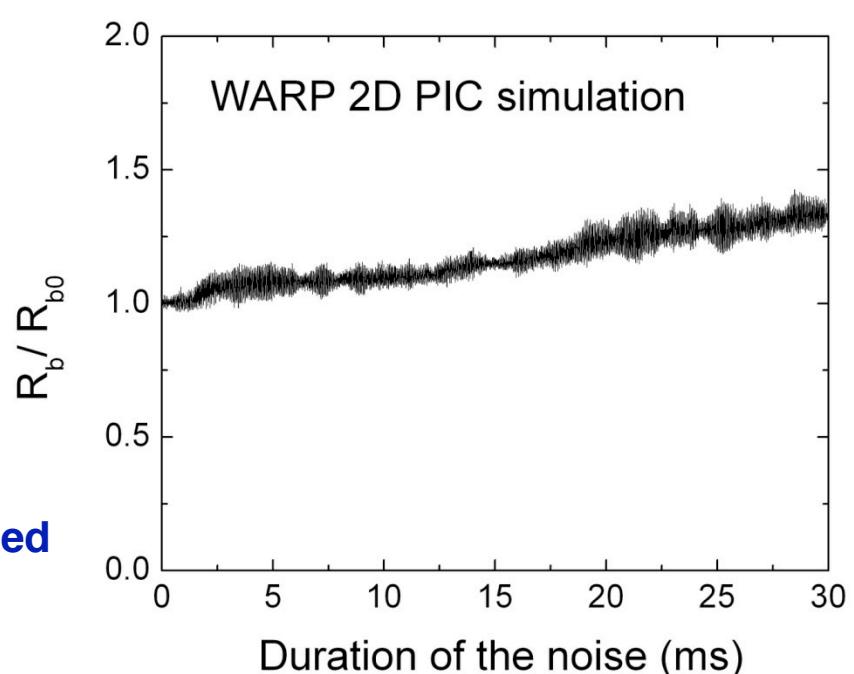


- Random noise from machine imperfections is unavoidable in present - generation and next - generation high energy accelerators.
- These effects can be simulated by imposing random voltage perturbations on the confining quadrupole electrodes in PTSX.

- Random noise together with space - charge effects act as a continuous supply of free energy over long propagation distances, resulting in mismatch oscillations.

**M. Chung, E. Gilson, R.C. Davidson,  
P.C. Efthimion and R. Majeski, submitted  
for publication (2008).**

1% uniform random error set

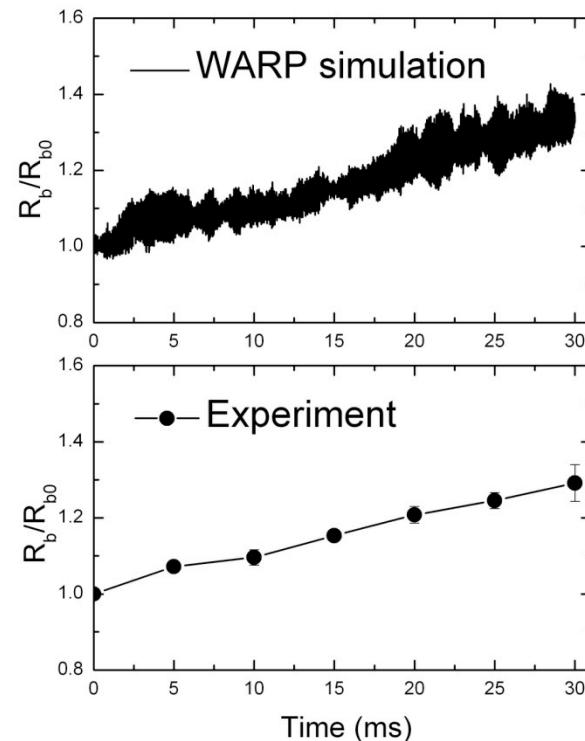
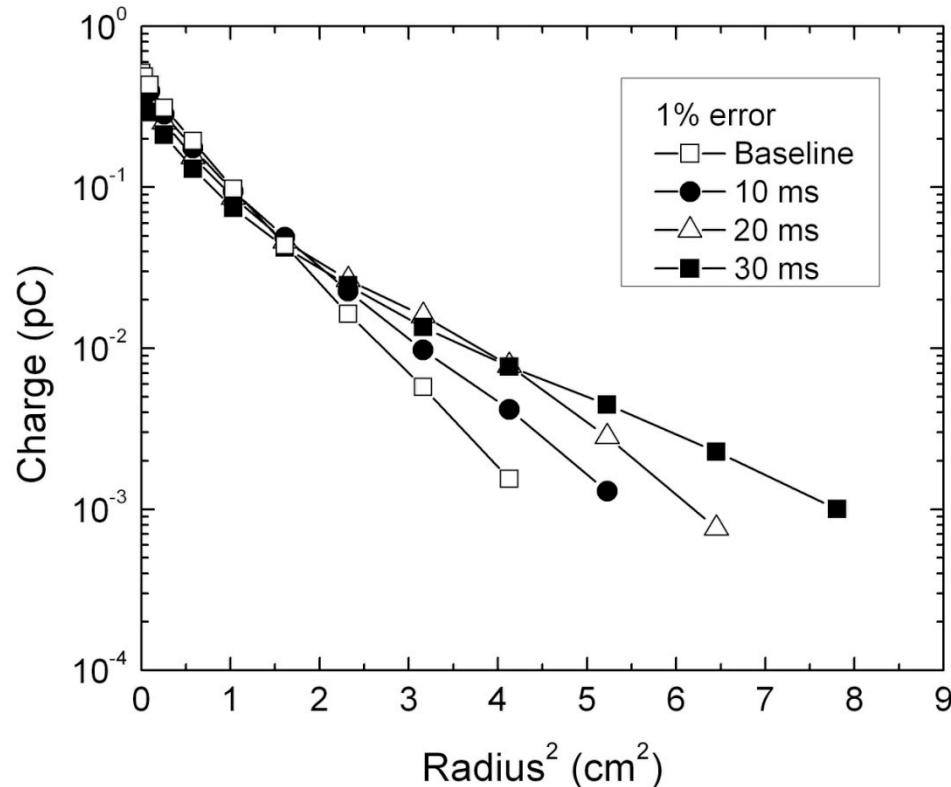


# Experimental Investigation of Noise Effects



By adding small-amplitude noise on the voltage waveform of the PTSX device, a continuous increase in the RMS radius has been observed.

## Evolution of measured radial profiles



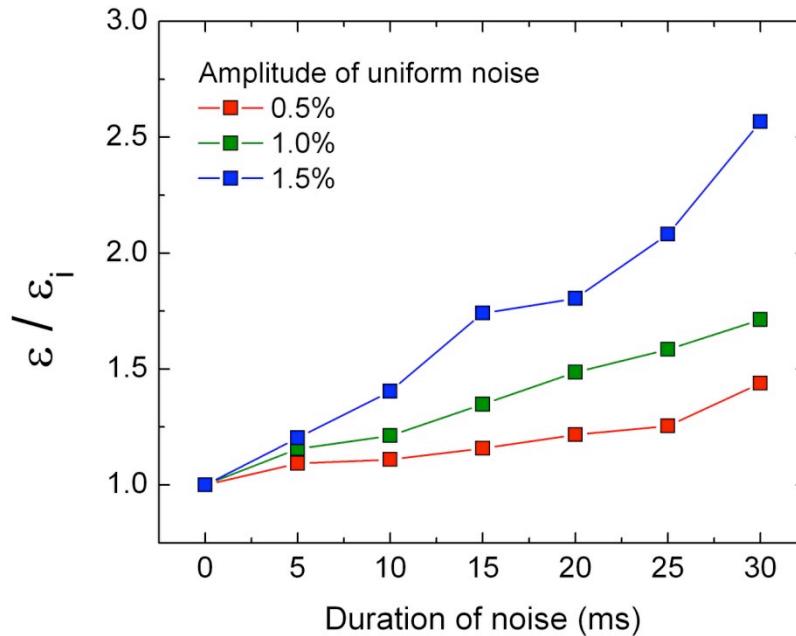
# Measurement of Emittance Growth



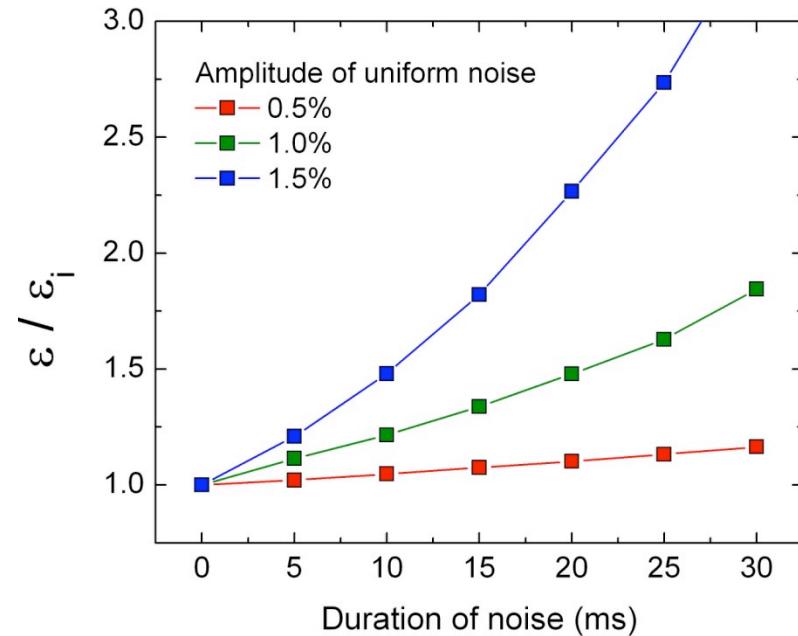
Increase in average transverse emittance has been estimated by making use of the global force balance equation.

$$\frac{\varepsilon}{\varepsilon_i} \approx \frac{R_b \sqrt{T_\perp}}{R_{bi} \sqrt{T_{\perp i}}}, \quad m\omega_q^2 R_b^2 = 2kT_\perp + \frac{N_b q^2}{4\pi\varepsilon_0}$$

## Experiment



## Simulation



$\sim 1.8$  km-long propagation



## Paul Trap Simulator Experiment (PTSX)

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- Determine sensitivity of beam propagation over long equivalent distances to imperfections in lattice waveform.
- Investigate beam mismatch and envelope instabilities resulting from sudden changes in lattice waveform.
- Examine collective wave excitations and their dependence on beam intensity.
- Optimize the charge bunch intensity that can be confined in the Paul Trap Simulator Experiment.
- **Investigate chaotic particle dynamics, and the production and control of halo particles, and determine detailed mechanisms for emittance growth.**
- **Determine effects of the distribution function on stability properties.**
- **Develop laser induced fluorescence (LIF) diagnostic system to determine detailed properties of the charge bunch .**

# Background plasma can be used to focus and compress intense ion beam pulses

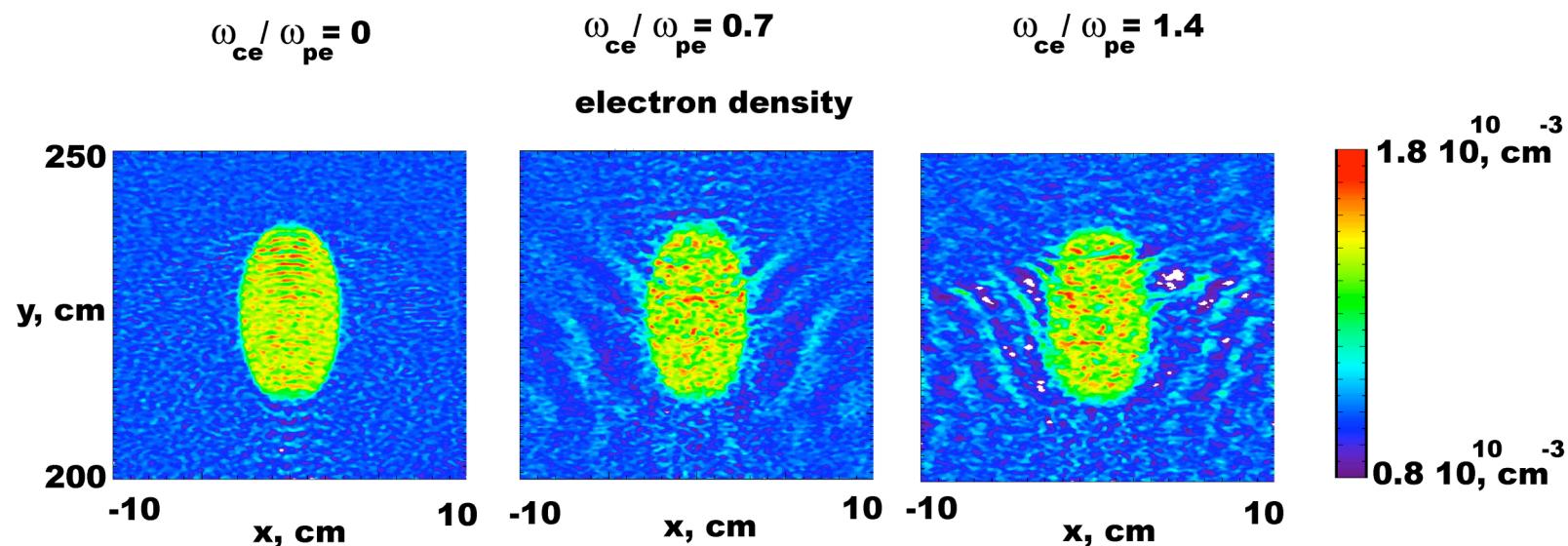
The electrons in a large-volume background plasma can be used to charge neutralize and current neutralize an intense ion charge bunch, thereby greatly facilitating the transverse focusing and longitudinal compression of the charge bunch to a small spot size.

This approach for focusing intense ion beams plays a major role in research on ion-beam-driven high energy density physics and heavy ion fusion.

- A. B. Sefkow and R. C. Davidson, Phys. Rev. ST Accel. & Beams **10**, 100101 (2007).
- R. C. Davidson and H. Qin, Phys. Rev. ST Accel. & Beams **8**, 064201 (2005).
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- P. K. Roy, S. S. Yu et al. ,Phys. Rev. Lett. **95**, 234801 (2005).

# Analytical studies show that the solenoidal magnetic field influences the neutralization by plasma if $\omega_{ce} > \beta\omega_{pe}$

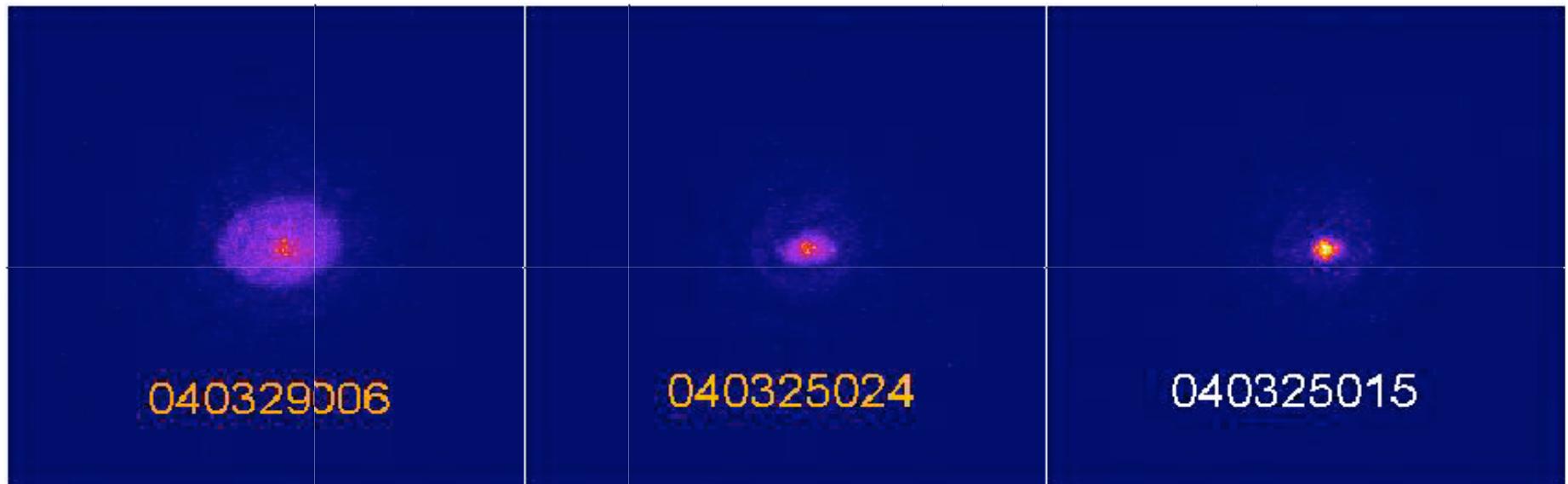
Plots of electron charge density contours in (x,y) space, calculated in 2D slab geometry using the LSP code with parameters: Plasma:  $n_p = 10^{11} \text{ cm}^{-3}$ ; Beam:  $V_b = 0.2c$ , 48.0A,  $r_b = 2.85 \text{ cm}$  and pulse duration  $\tau_b = 4.75 \text{ ns}$ . A solenoidal magnetic field of 1014 G corresponds to  $\omega_{ce} = \omega_{pe}$ .



- In the presence of a solenoidal magnetic field, whistler waves are excited, which propagate at an angle with the beam velocity and can perturb the plasma ahead of the beam pulse.

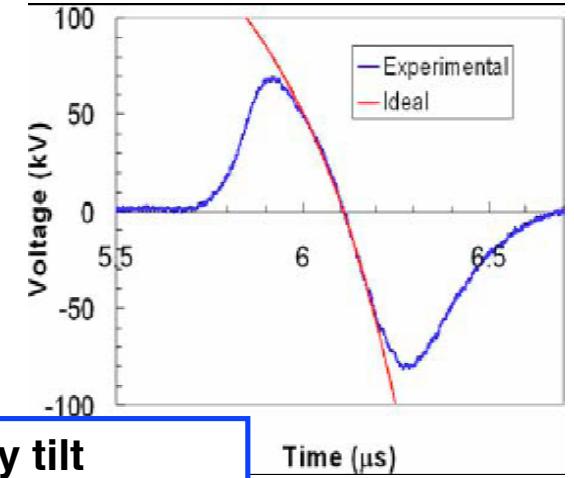
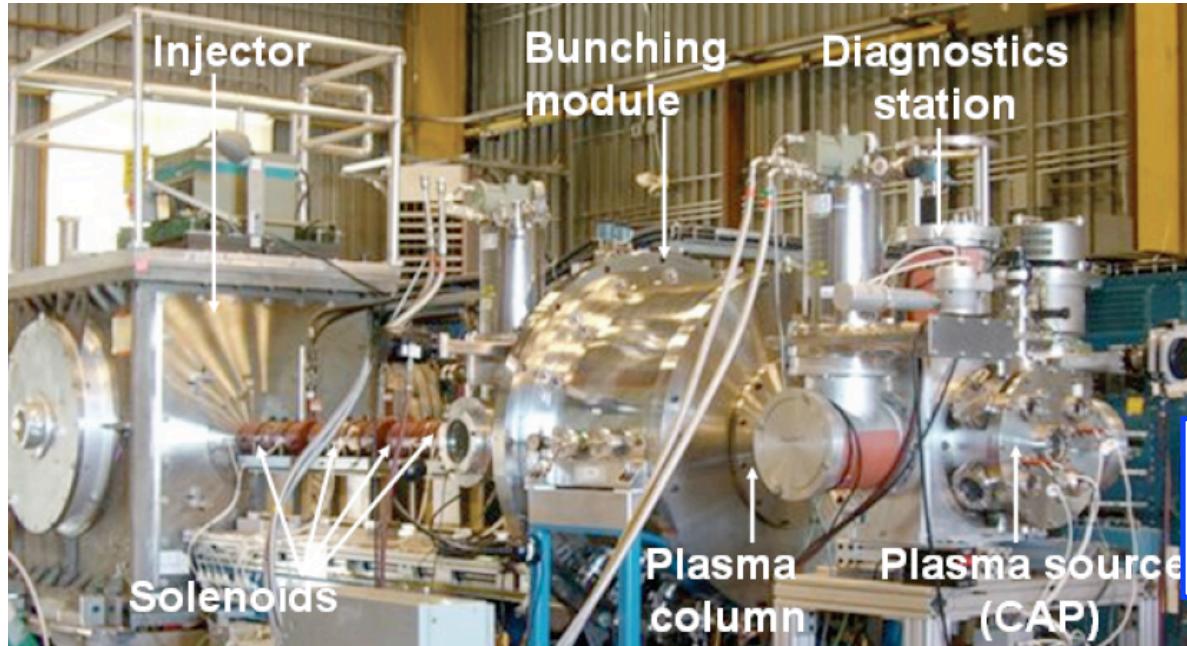
I.D. Kaganovich et al., Phys. Rev. Lett. 99, 235002 (2007).

# Measurements on the Neutralized Transport Experiment (NTX) demonstrate achievement of smaller spot size using volumetric plasma

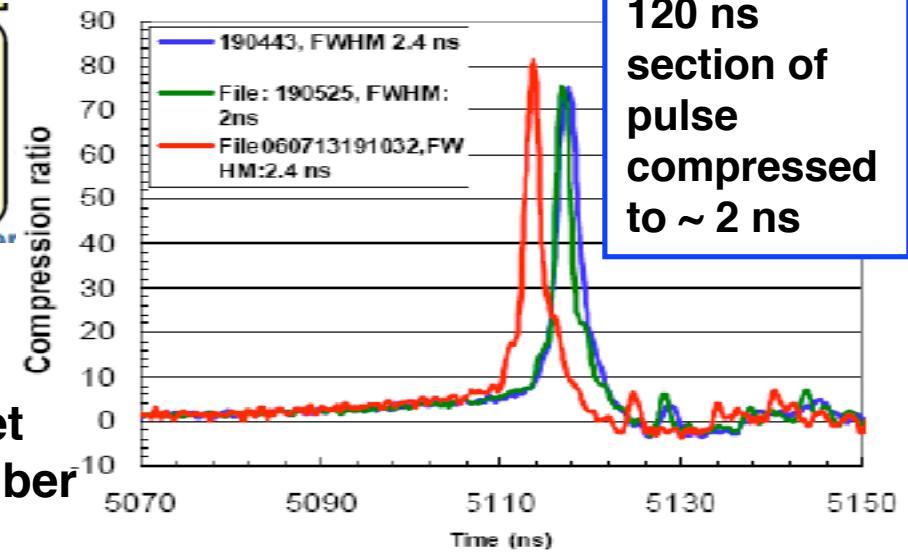
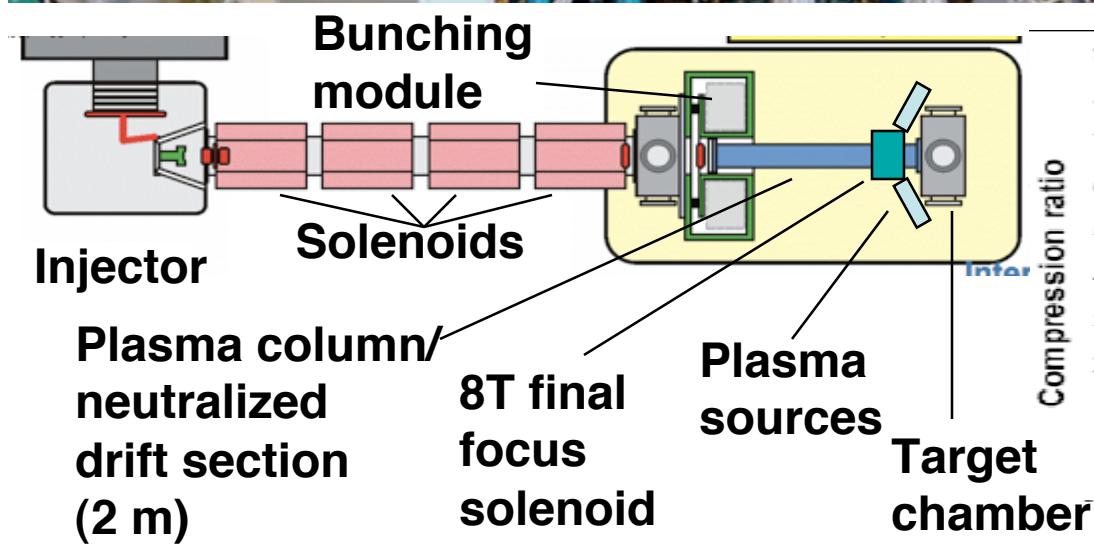


P. K. Roy, S. S. Yu et al. ,Phys. Rev. Lett. 95, 234801 (2005).

# NDCX-1 has demonstrated > factor 60 pulse compression, and simultaneous transverse focus

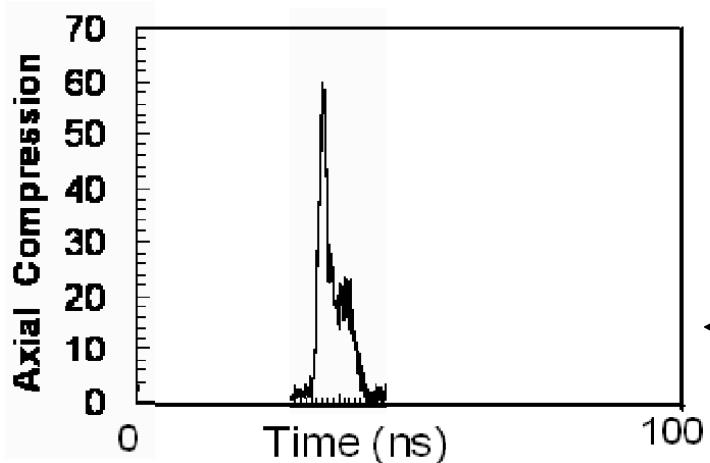
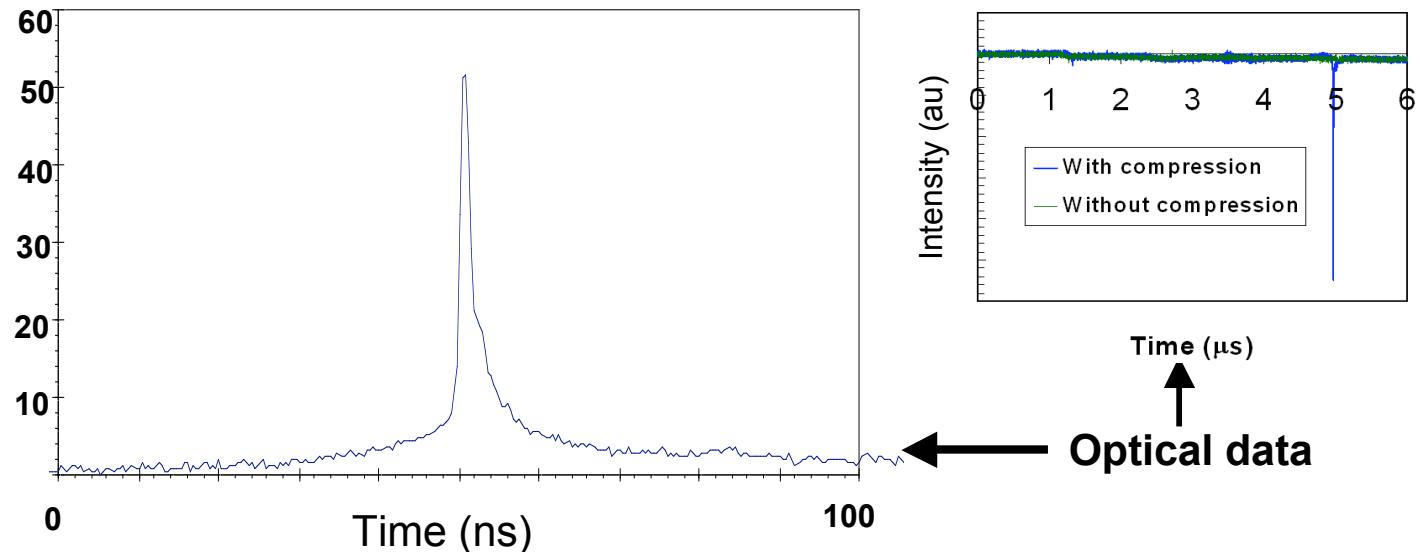


**Velocity tilt accelerates tail, decelerates head**  
(Like chirped pulse compression)

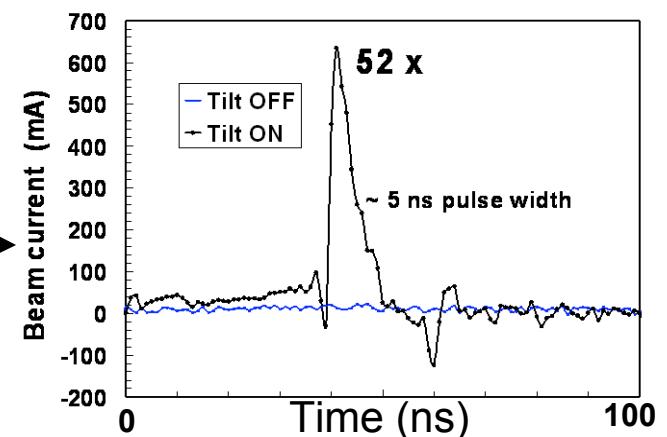


**120 ns section of pulse compressed to  $\sim$  2 ns**

# 60-Fold Beam Compression Achieved in Neutralized Drift Compression Experiment (NDCX)



LSP simulation



# Class of Exact Dynamically-Compressing Neutralized Beam Equilibria with Linear Velocity Tilt in a Solenoidal Magnetic Field

---

Assuming complete charge and current neutralization for an intense ion beam pulse propagating through background plasma, Vlasov equation supports class of exact, dynamically-compressing solutions

$$f_b(\mathbf{x}, \mathbf{p}, t) = f_b(W_{\perp}, W_z)$$

where  $W_z$  and  $W_{\perp}$  are defined by

$$W_z = \frac{z^2}{z_b^2(t)} + \frac{z_b^2(t)}{z_{b0}^2(t)v_{T0}^2} \left( v_z - \frac{z}{z_b(t)} \frac{dz_b(t)}{dt} \right)^2$$
$$W_{\perp} = \frac{x^2 + y^2}{r_b^2(t)} + \frac{r_b^2(t)}{r_{b0}^2 v_{T0}^2} \left[ \left( v_x + \Omega_L y - \frac{x}{r_b(t)} \frac{dr_b(t)}{dt} \right)^2 + \left( v_y - \Omega_L y - \frac{y}{r_b(t)} \frac{dr_b(t)}{dt} \right)^2 \right]$$

and  $\Omega_L$  is the Larmor frequency

$$\Omega_L = -\frac{e_b}{2m_b c} B_z [\gamma_b (z + v_b t)]$$

A. B. Sefkow and R. C. Davidson, Phys. Rev. ST Accel. & Beams 10, 100101 (2007);  
R. C. Davidson and H. Qin, Phys. Rev. ST Accel. & Beams 8, 064201 (2005).

# Dynamically-Compressing Neutralized Beam Equilibria

---

Many choices of beam equilibria are possible. As one example, consider

$$f_b(W_{\perp}, W_z) = \text{const.} \sqrt{(1 - W_z)} e^{-W_{\perp}}, \text{ where } 0 \leq W_z < 1$$

Then, the density profile  $n_b(r, z, t)$  is given dynamically by

$$n_b(r, z, t) = n_{b0} \left[ \frac{r_{b0}^2}{r_b^2(t)} \frac{z_{b0}}{z_b(t)} \right] \left( 1 - \frac{z^2}{z_b^2(t)} \right) e^{-r^2/r_b^2(t)}, \text{ where } 0 \leq z^2 < z_b^2(t)$$

where  $r_b(t)$  and  $z_b(t)$  solve the envelope equations introduced earlier. For this choice of  $f_b(W_{\perp}, W_z)$  note that the line density is parabolic with

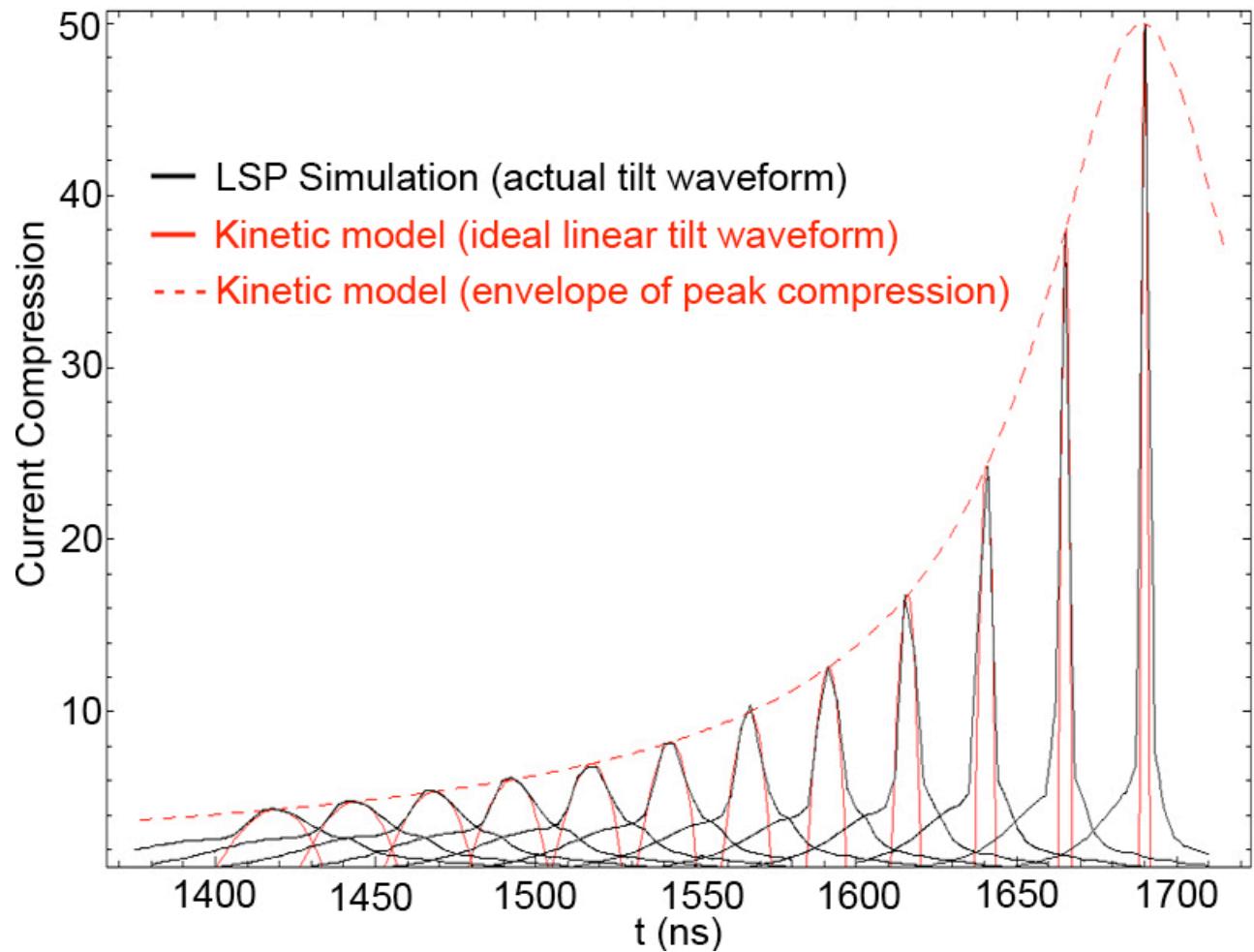
$$\lambda_b(z, t) = \lambda_{b0} \left[ \frac{z_{b0}}{z_b(t)} \right] \left( 1 - \frac{z^2}{z_b^2(t)} \right), \text{ where } 0 \leq z^2 < z_b^2(t)$$

# Comparison with longitudinal current compression in NDCX

Parabolic waveform of main compression peak is described well by kinetic model

Deviation from ideal (linear) tilt waveform produce pedestals around main parabolic peaks

A. B. Sefkow and  
R.C. Davidson,  
Phys.Rev. ST Accel.  
& Beams 10, 100101  
(2007).



# Collective Instabilities in Intense Charged Particle Beams

---

## One-Component Beams

- Electrostatic Harris instability ( $T_{\parallel b} / T_{\perp b} \leq 1$ )
- Electromagnetic Weibel instability ( $T_{\parallel b} / T_{\perp b} \leq 1$ )
- Resistive wall instability

## Propagation Through Background Electrons

- Electron-ion two-stream (Electron Cloud) instability

## Propagation Through Background Plasma

- Resistive hose instability
- Multispecies Weibel instability
- Multispecies two-stream instability

# Multispecies Weibel Instability

---

In the collisionless regime, the large directed kinetic energy of the beam ions propagating through a background plasma provides the free energy to drive the electromagnetic Weibel instability.

## Assumptions

- Charge and current neutralization

$$\sum_{j=b,e,i} n_j^0(r) e_j = 0 \quad \text{and} \quad \sum_{j=b,e,i} n_j^0(r) e_j \beta_j c = 0$$

- Electromagnetic perturbations with  $\partial/\partial\theta = 0$ ,  $\partial/\partial z = 0$  and polarization

$$\delta \mathbf{E} = \delta E_r \mathbf{e}_r + \delta E_z \mathbf{e}_z \quad \text{and} \quad \delta \mathbf{B} = \delta B_\theta \mathbf{e}_\theta$$

R.C. Davidson et al., Nuclear Instruments and Methods in Physics Research, in press (2008).

# Multispecies Weibel Instability

Express

$$\delta E_z(r, t) = \delta \hat{E}_z \exp(-i\omega t)$$

Obtain the eigenvalue equation

$$\frac{1}{r} \frac{\partial}{\partial r} \left[ r \left( 1 + \sum_{j=b,e,i} \frac{\beta_j^2 \omega_{pj}^2(r)}{\omega^2} + \sum_{j=b,e,i} \frac{[\beta_j \omega_{pj}^2(r)]^2}{\omega^2 \left[ \omega^2 - \sum_{j=b,e,i} \omega_{pj}^2(r) \right]} \right) \frac{\partial}{\partial r} \delta E_z \right] + \left( \frac{\omega^2}{c^2} - \sum_{j=b,e,i} \frac{\omega_{pj}^2(r)}{\gamma_j^2 c^2} \right) \delta E_z = 0$$

where  $\omega_{pj}(r) = [4\pi n_j^0(r) e_j^2 / \gamma_j m_j]^{1/2}$  and  $\gamma_j = (1 - \beta_j^2)^{-1/2}$

Slow-wave Weibel instability driven by the terms proportional to

$$\sum_{j=b,e,i} \beta_j^2 \omega_{pj}^2(r) \text{ and } \sum_{j=b,e,i} \beta_j \omega_{pj}^2(r)$$

# Multispecies Weibel Instability

---

For the case of uniform density profiles the eigenvalue equation can be solved exactly to obtain a closed dispersion relation for the complex frequency  $\omega$

For perturbations with short wavelength the characteristic growth rate of the Weibel instability scales as  $\text{Im } \omega \sim \Gamma_W$  where

$$\Gamma_W^2 = \beta_e^2 \hat{\omega}_{pi}^{i2} + (\beta_b - \beta_e)^2 \omega_{pb}^{i2}$$

and  $\hat{\omega}_{pj}^{i2} = 4\pi \hat{n}_j e_j^2 / \gamma_j m_j$

**R.C. Davidson et al., Nuclear Instruments and Methods in Physics Research, in press (2008).**

# Multispecies Weibel Instability

---

The full dispersion relation has been solved numerically over a wide range of beam-plasma parameters.

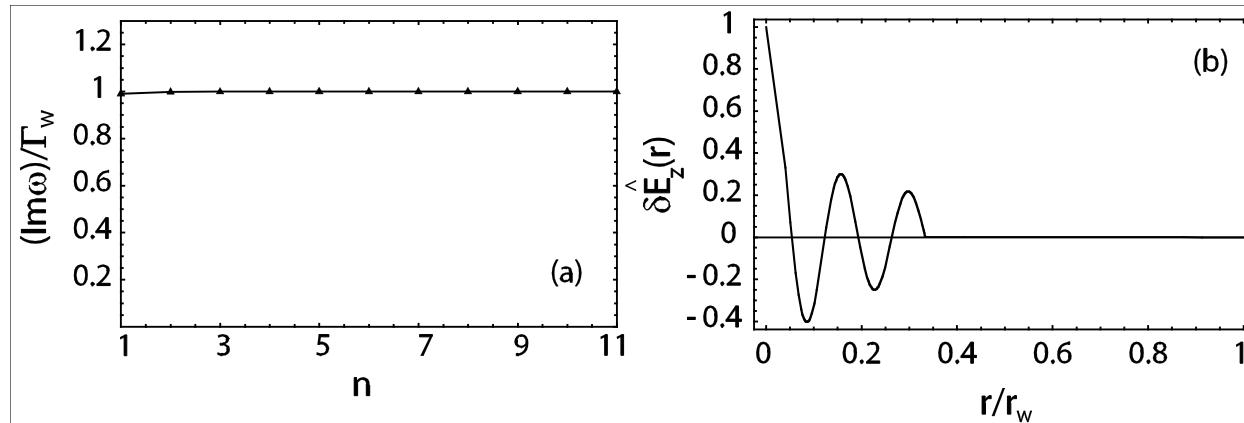
As an illustrative example consider an intense cesium ion beam propagating through background argon plasma ( $Z_i = 1$ ) with ( $Z_b = 1$ )

$$\beta_b = 0.2, \quad \beta_i = 0, \quad \beta_e = 0.1$$

$$\hat{n}_i^i = \hat{n}_e^i / 2 = \hat{n}_b^i$$

The background plasma provides complete charge and current neutralization.

# Multispecies Weibel Instability



Plots of (a) Weibel instability growth rate  $\text{Im } \omega/\Gamma_w$  versus radial mode number  $n$ , and (b) eigenfunction  $\delta \hat{E}_z(r)$  versus  $r/r_w$  for  $n=5$ . System parameters are  $r_b = r_w/3$ ,  $\Omega_p^i r_b/c = 1/3$  and  $\hat{n}_i^o = 0$  (vacuum region outside beam).

R.C. Davidson et al., Nuclear Instruments and Methods in Physics Research, in press (2008).

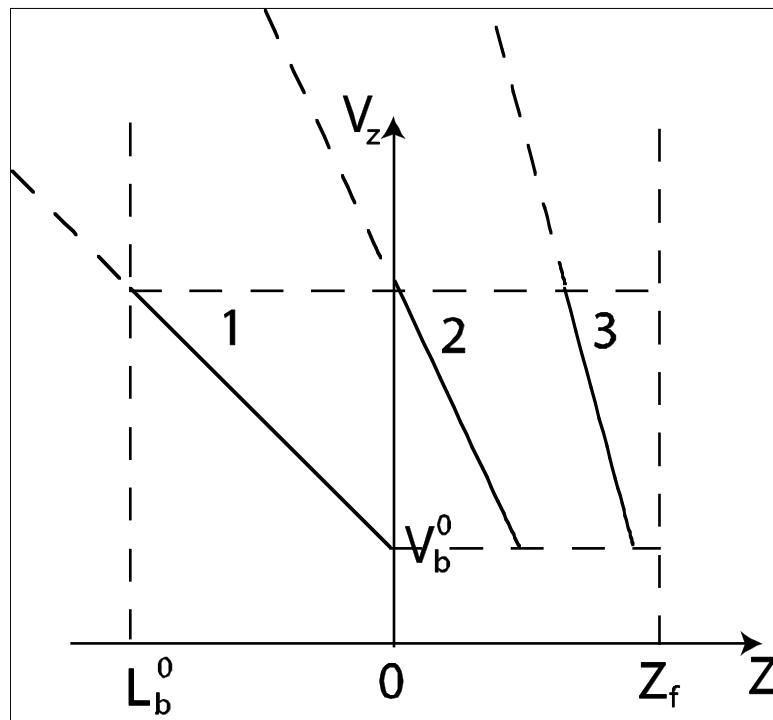
# Multispecies Two-Stream Instability

---

- A small axial momentum spread of the beam ions and the plasma ions leads to a reduction in the growth rate of the two-stream instability.
- In regimes where it occurs, the two-stream instability is likely to lead to a longitudinal heating of the plasma electrons in the nonlinear regime.
- For sufficiently small beam radius (in units of the plasma electron collisionless skin depth), the effects of finite transverse geometry have an important influence on detailed stability properties.
- A longitudinal velocity tilt can significantly reduce the growth rate of the electron-ion two-stream instability.

**R.C. Davidson et al., Nuclear Instruments and Methods in Physics Research, in press (2008).**

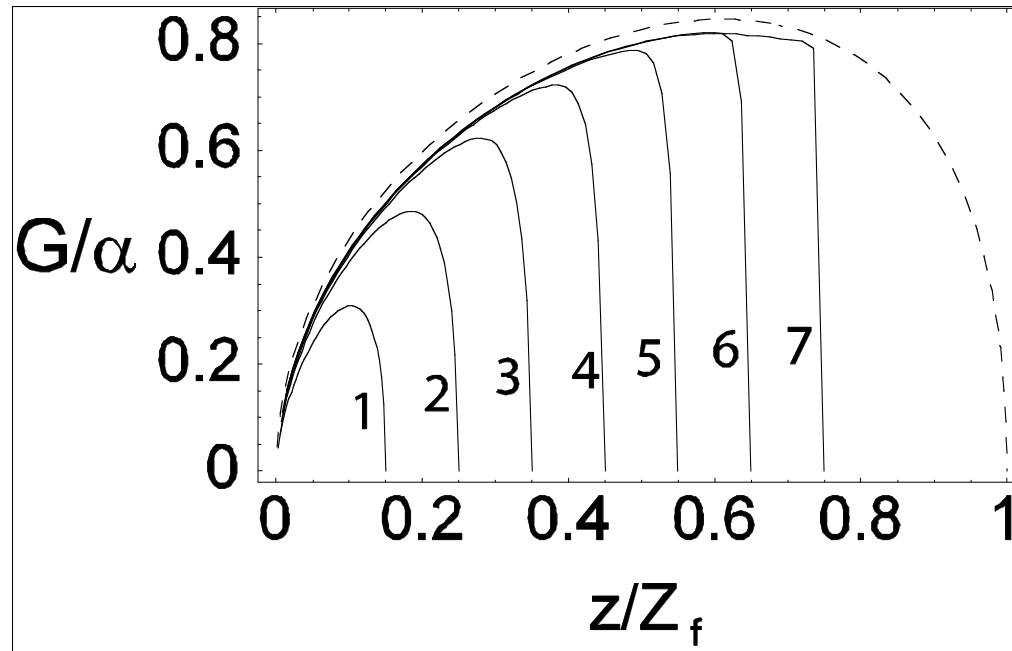
## A longitudinal velocity tilt can significantly reduce the growth rate of the electron-ion two-stream instability



Plot of ion beam phase space at different times during the compression.  
Line 1 corresponds to t=0.

E. A. Startsev and R. C. Davidson, Nucl. Instr. Meth. Phys. Res. A577, 79 (2007).

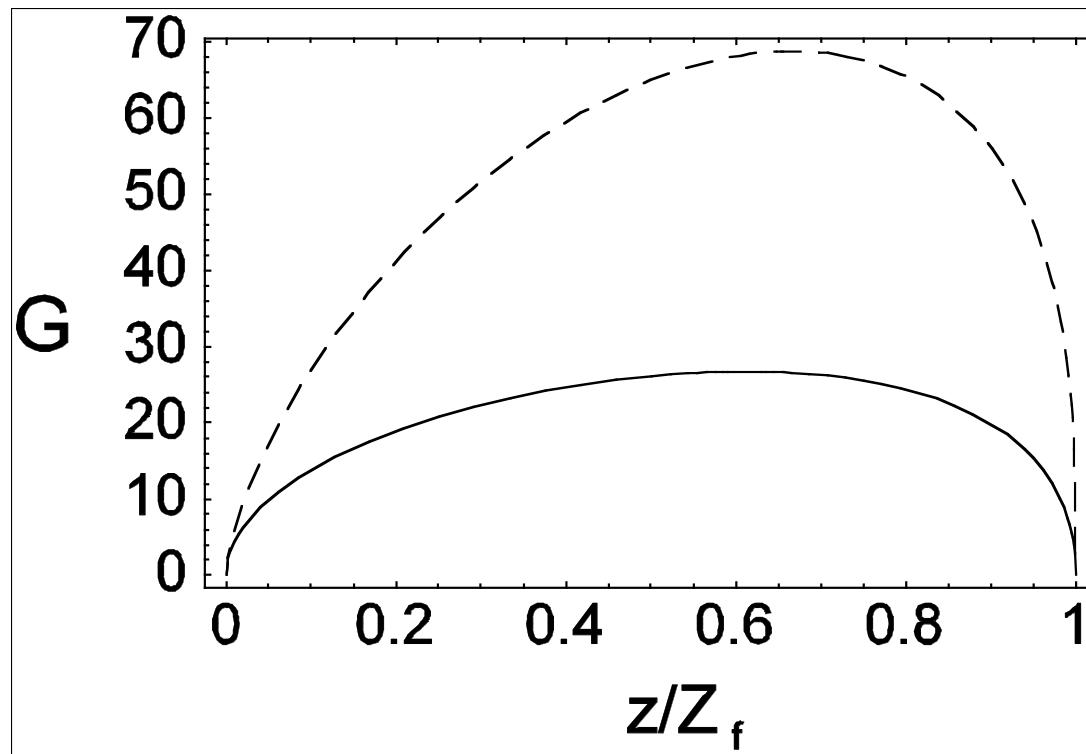
# A longitudinal velocity tilt can significantly reduce the growth rate of the electron-ion two-stream instability\*



Normalized instability gain function is plotted as a function of axial distance at different times, obtained numerically (solid curves) and compared with the analytical estimate (dashed curve).

E. A. Startsev and R. C. Davidson, Nucl. Instr. Meth. Phys. Res. A577, 79 (2007).

## A longitudinal velocity tilt can significantly reduce the growth rate of the electron-ion two-stream instability



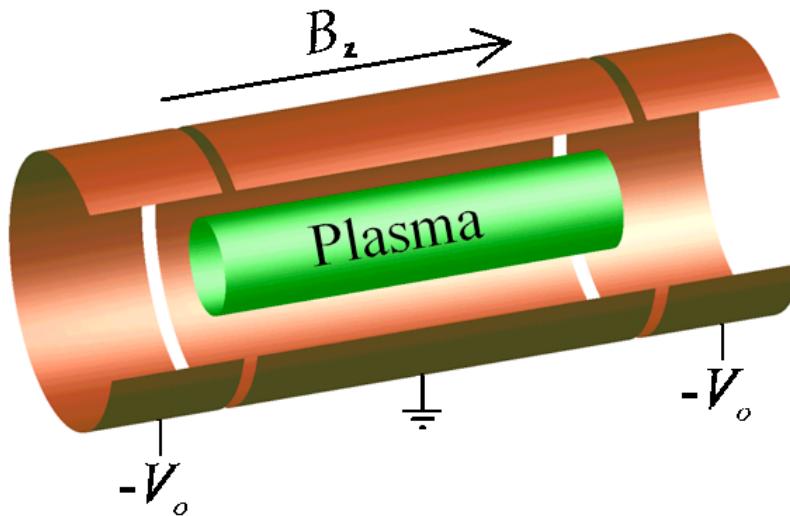
Comparison of the instability gain  $G$  plotted as a function of axial distance for ion beam with velocity tilt (solid curve) and without velocity tilt (dashed curve).

E. A. Startsev and R. C. Davidson, Nucl. Instr. Meth. Phys. Res. A577, 79 (2007).

## Summary - Collective Interaction Processes

- Collective interaction processes and instabilities in nonneutral plasmas and intense charge particle beams and beam-plasma systems can affect the detailed stability behavior and nonlinear dynamics.
- Growth rate reduction or elimination mechanisms have been identified for many collective interaction processes and instabilities.
- Numerical simulations play a critical role in determining the threshold conditions and nonlinear dynamics.
- Related publications can also be found at <http://nonneutral.pppl.gov>

# Basic physics studies using Malmberg-Penning traps



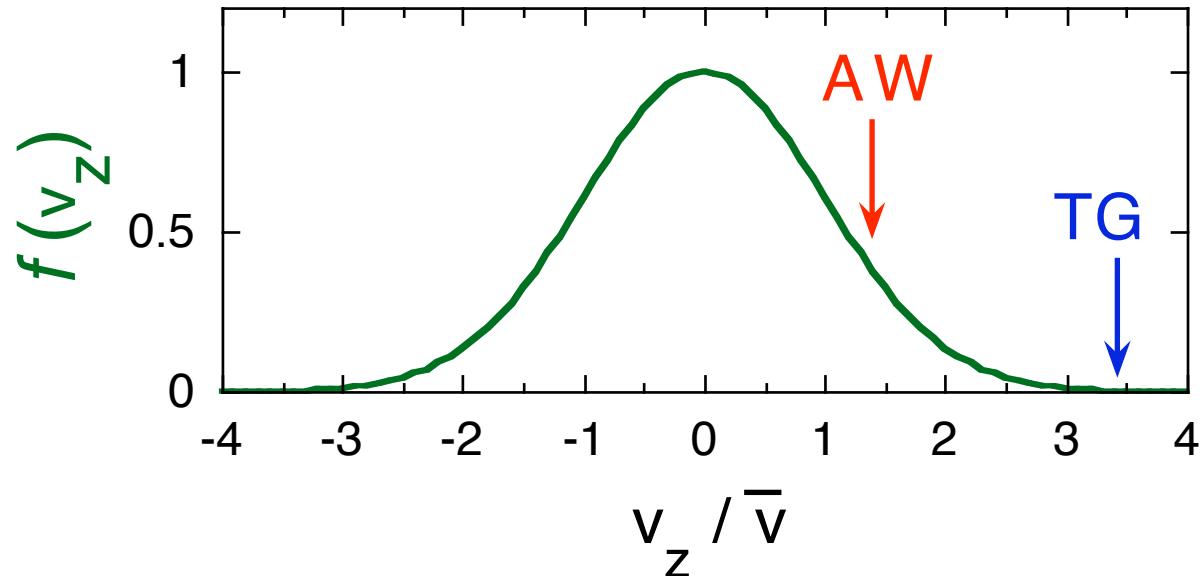
- Finite - amplitude acoustic waves in pure-ion plasmas  
(Anderegg, Driscoll, Dubin, O'Neil, UCSD)
- Anti-hydrogen production and trapping  
(Fajans, Wurtele, et al., UCB - ALPHA Antimatter Collaboration)

# Acoustic Waves

(Anderegg, Driscoll, Dubin, O'Neil, UCSD)

Acoustic Waves are plasma waves with a slow phase velocity.

$$\omega \approx 1.3 k \bar{v}$$



This wave is nonlinear so as to flatten the particle distribution to avoid strong Landau damping.

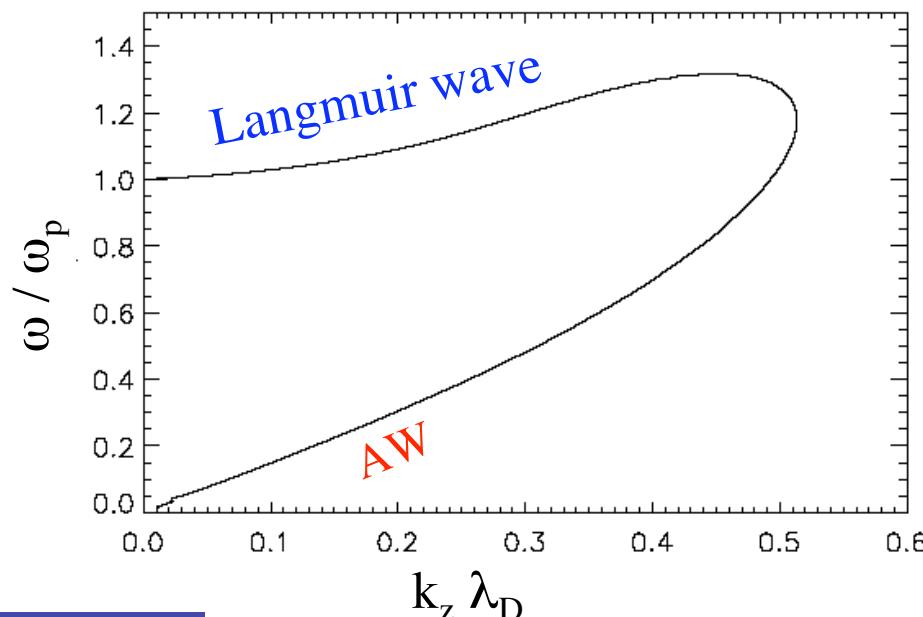
# Plasma wave dispersion relation

- Infinite homogenous plasma

(Holloway and Doring  
Phys. Rev. A 1991)

$$0 \approx 1 - \frac{\omega_p^2}{k^2} P \int dv \frac{k \frac{\partial f}{\partial v}}{kv - \omega} - i\pi \frac{\omega_p^2}{k^2} \left. \frac{\partial f_0}{\partial v} \right|_{\omega/k}$$

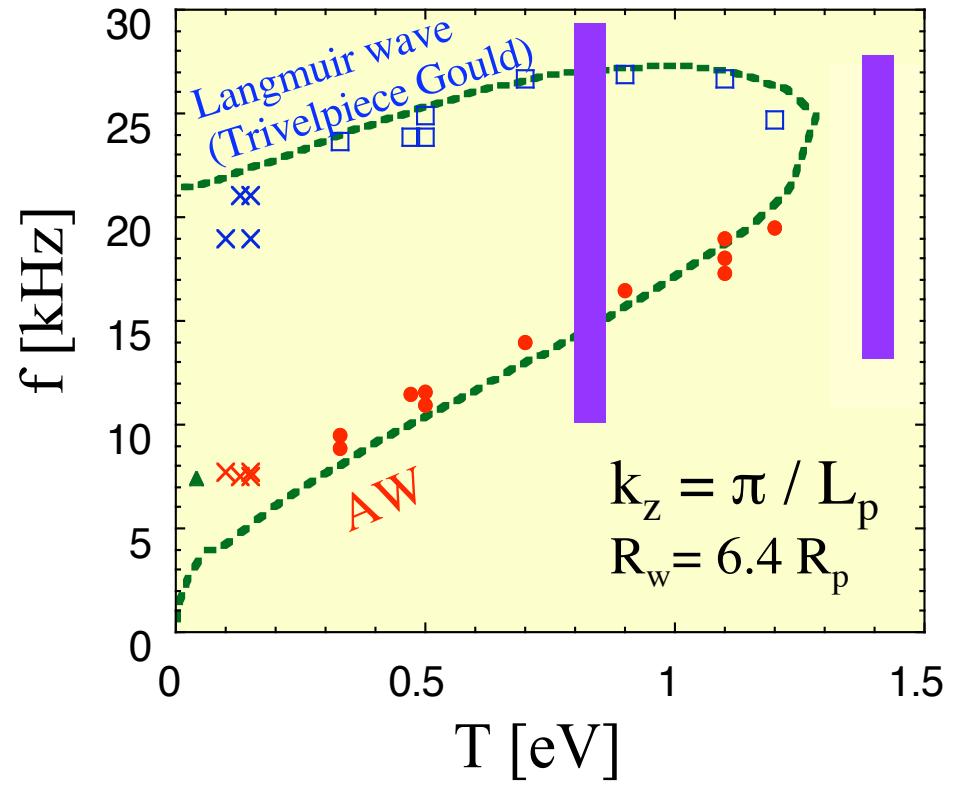
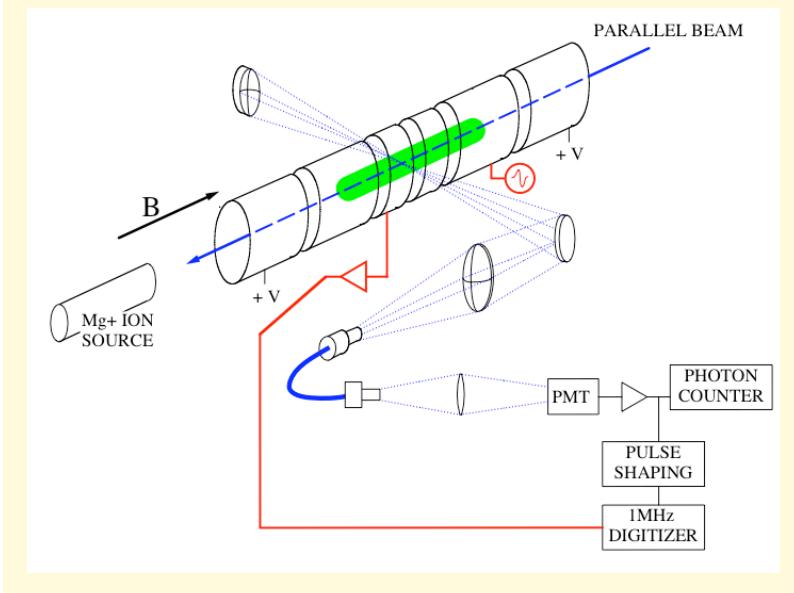
$\Rightarrow = 0$  Trapping “flattens” the distribution in the resonant region (BGK).



Two solutions:  
- Langmuir waves  
- Acoustic waves

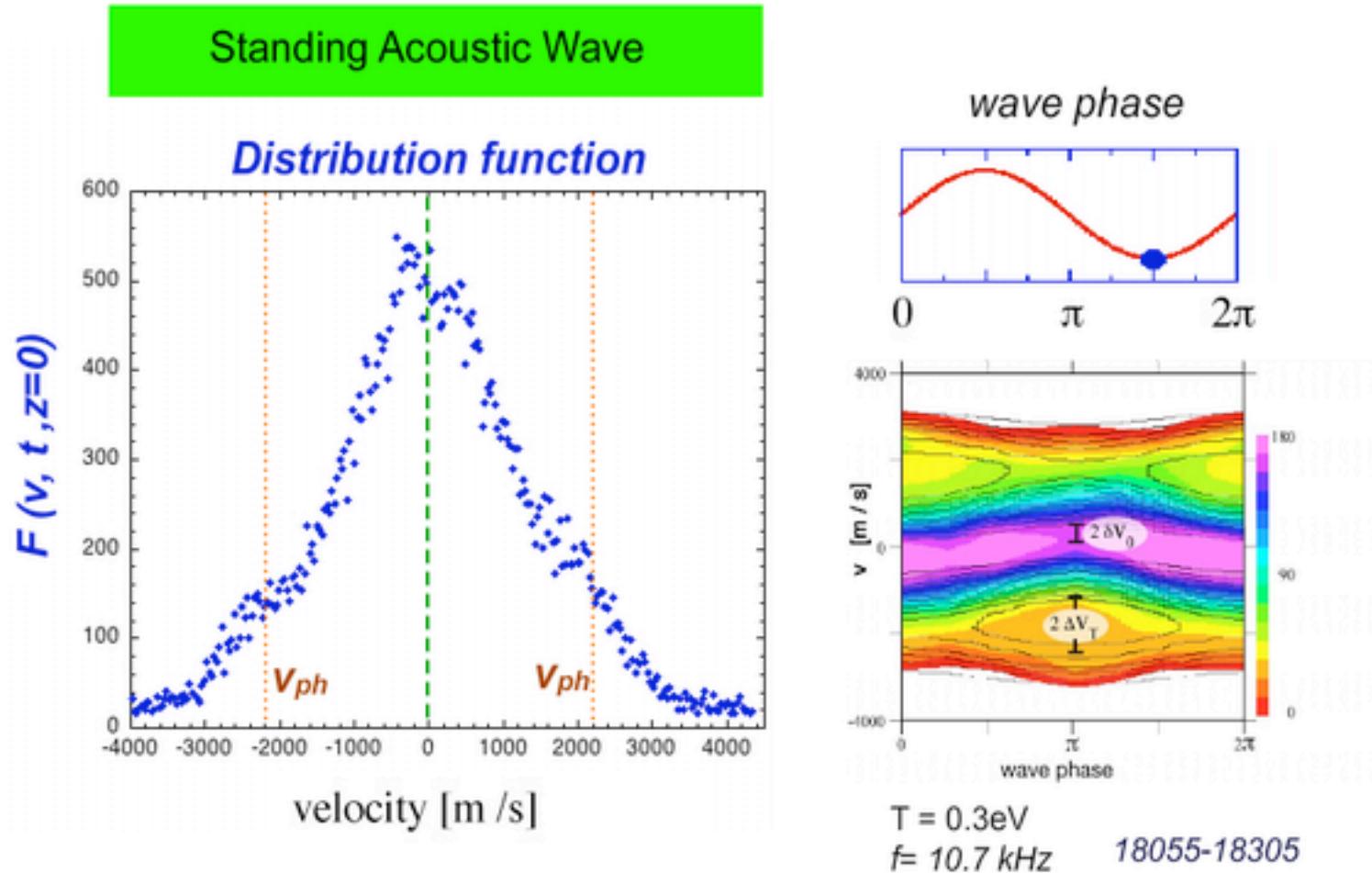
# Dispersion Relation

## Trapped non-neutral plasma



For large-amplitude drive and a broad range of frequencies, the **particle distribution non-linearly self-organizes** so that the **wave rings at the driver frequency** after the driver is turned off.

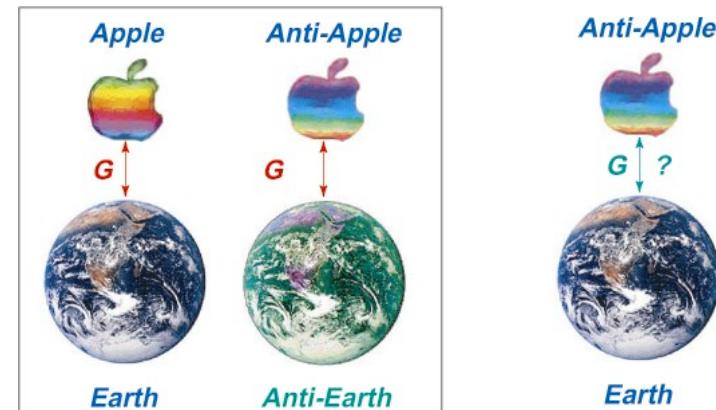
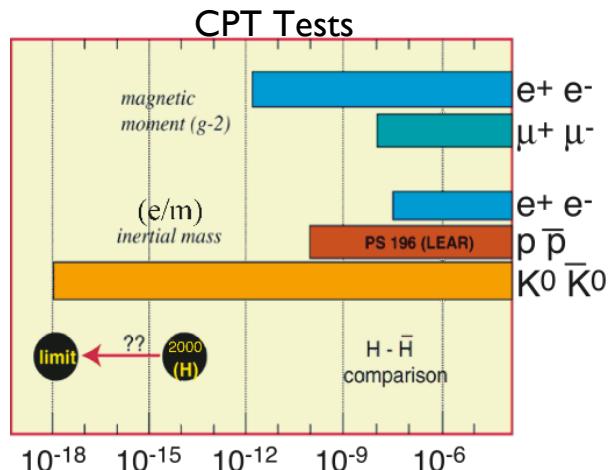
# Measured Distribution versus Phase



# Anti-hydrogen Production

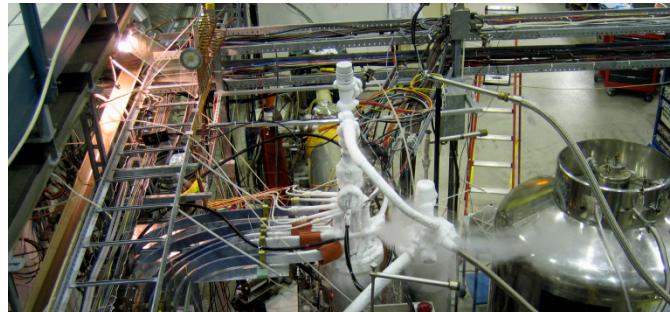
(Fajans, Wurtele, et al., UCB - ALPHA Antimatter Collaboration)

- The ALPHA and ATRAP collaborations are attempting to trap Anti-hydrogen by mixing positron and antiproton plasmas.
- Slow, untrapped anti-hydrogen was first created in 2002 by the ATHENA and ATRAP collaborations.
- Tests of CPT invariance (by comparison of the spectra of hydrogen and anti-hydrogen) and gravity require trapped anti-hydrogen.

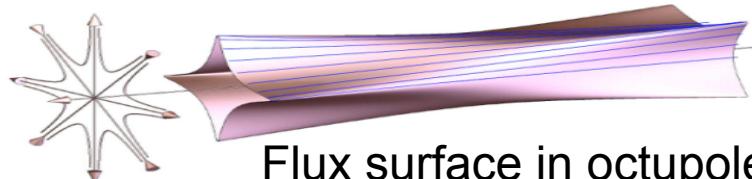


## Anti-hydrogen Production Challenge

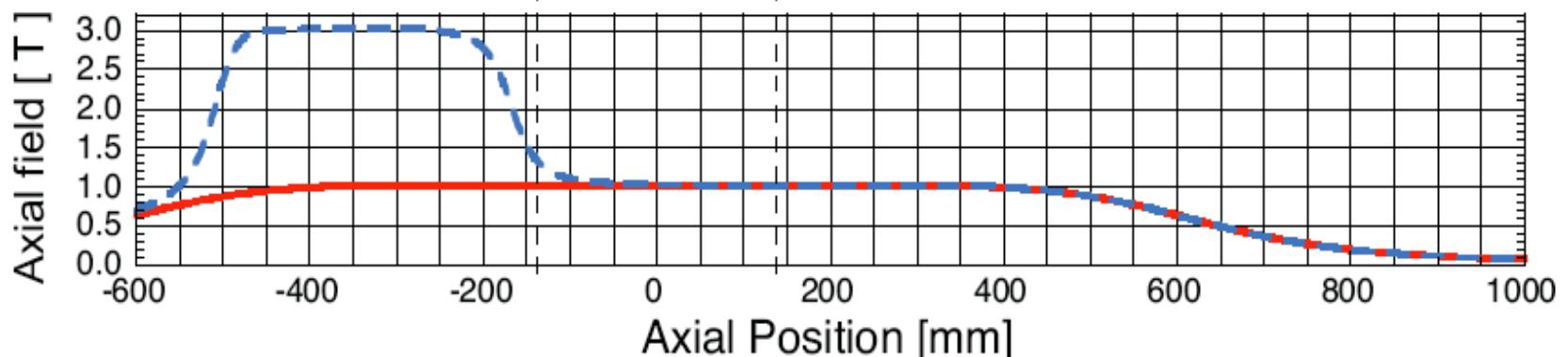
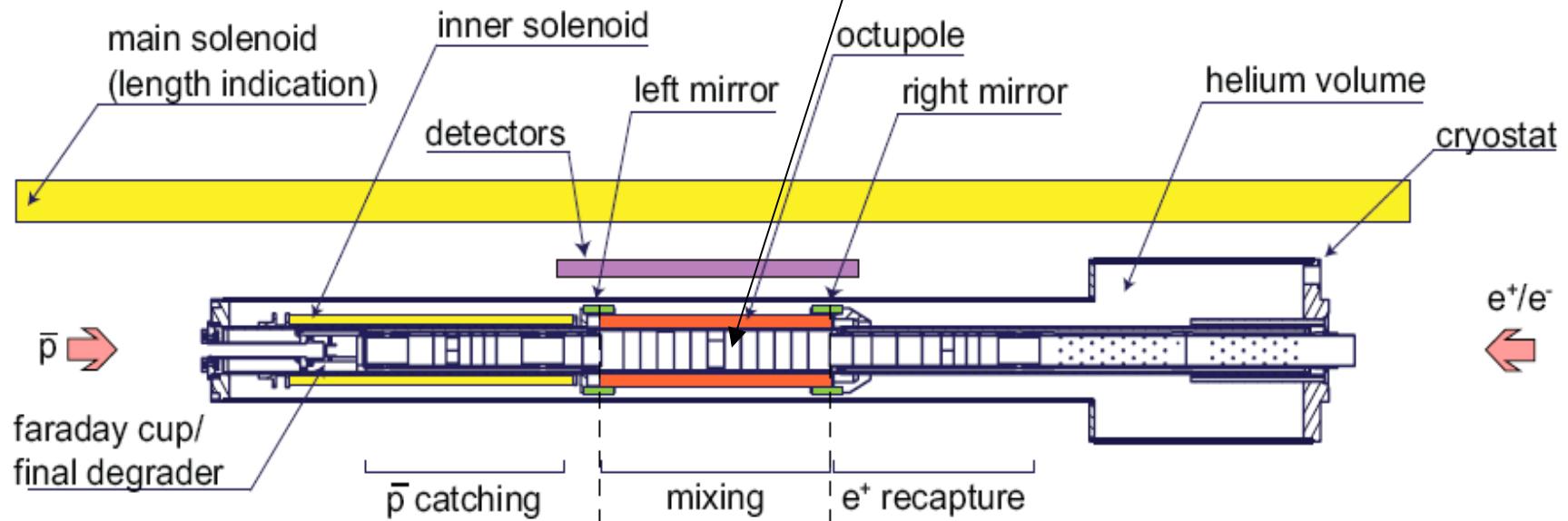
- The positron and antiproton plasmas used to synthesize anti-hydrogen are confined in Penning-Malmberg traps.
- The neutral anti-hydrogen is not confined by the Penning-Malmberg fields.
- Magnetic multipole fields are used to make a minimum-B trap for the anti-hydrogen.
- Minimum-B traps are typically very shallow...on the order of **0.05meV**.
- Prior to mixing, the space-charge potentials of the one-component positron and antiproton plasmas are on the order of **volts**.
- **Major challenge:** How to inject negatively charged antiprotons into the positively charged positron plasma while keeping the energy of the antiprotons low enough to be trapped.



# ALPHA Schematic

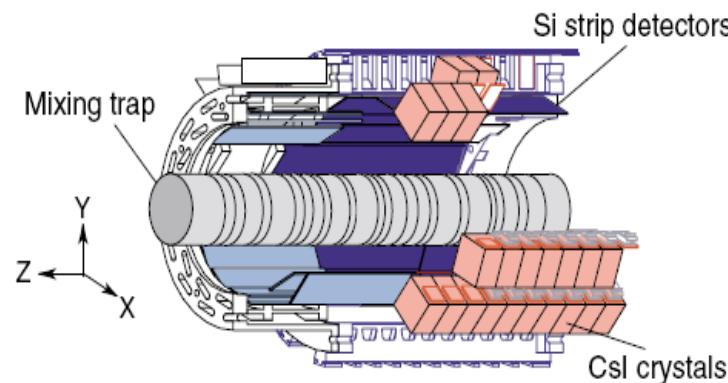


Flux surface in octupole



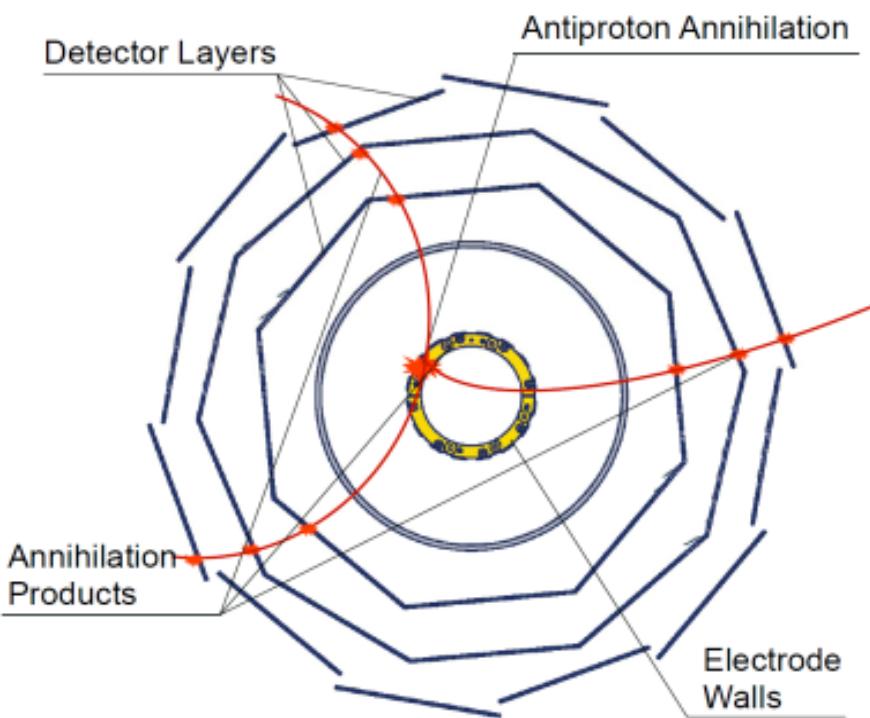
# Position-sensitive detection of antiprotons

Detectors work by tracking the pion annihilation products through two or three Si sensors, and triangulating backwards to the point of annihilation.



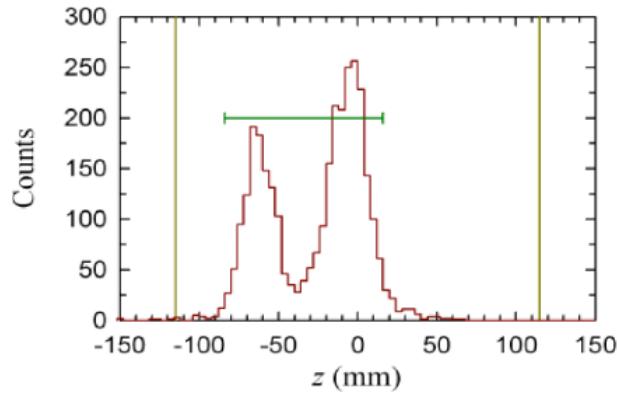
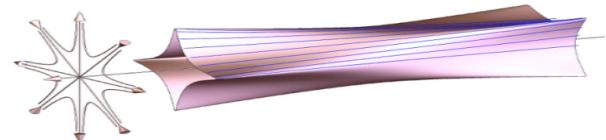
**ALPHA Collaboration:** "Cold Antimatter Plasmas and Applications to Fundamental Physics", CP1037, American Institute of Physics, p208 (2008).

**ATHENA Collaboration:** Phys. Rev. Lett. **92**, 065005 (2004).

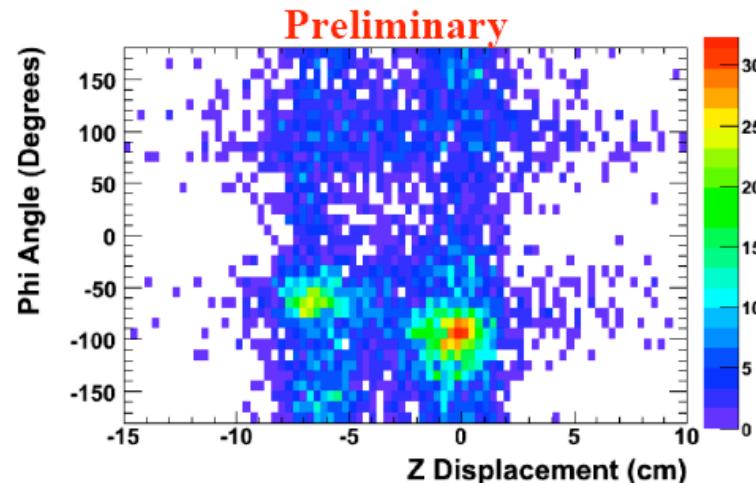


# Loss due to Multipole Fields

- With a multipole, loss is concentrated in the cusps.



The z-position of the annihilations, showing that the annihilations occur at the ends of the plasma. (The plasma extent indicated by the green bar.)



Phi-angle position of the annihilations. Most annihilations occur at localized angles at either end of the plasma. Note how these are separated by 45 degrees, just like the cusps in the magnetic fields lines.

J. Fajans, W. Bertsche, K. Burke, S.F. Chapman, D.P van der Werf, Phys. Rev. Lett. 95, 15501 (2005).  
K. Gomberoff, J. Fajans, J. Wurtele, A. Friedman, D.P. Grote, R.H. Cohen, and J.-L. Vay, Phys. Plasmas 14, 102111(2007).  
ALPHA Collaboration: “Cold Antimatter Plasmas and Applications to Fundamental Physics”, CP1037 AIP, p208 (2008).

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You can visit our website at: <http://nonneutral.pppl.gov>

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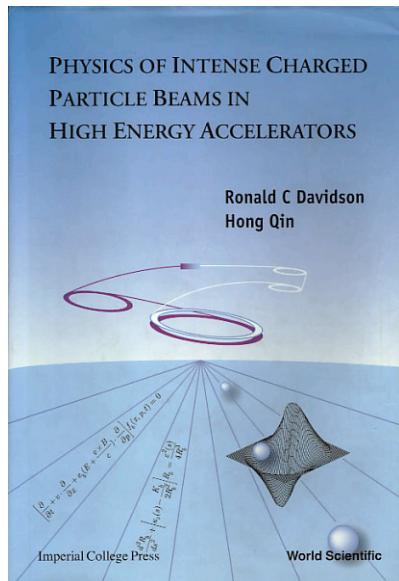
## **And to the federal agencies that have supported this research over the years**

Department of Energy

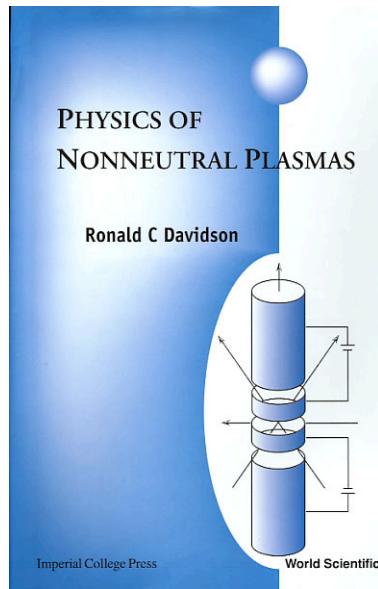
- Office of Fusion Energy Sciences
- Office of High Energy Physics

Office of Naval Research

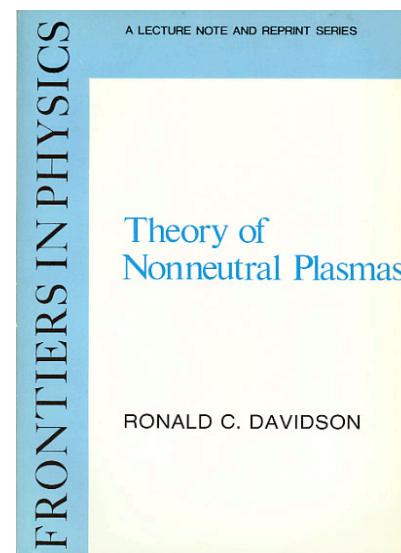
## Special thanks to several book publishers - Academic Press, Addison Wesley, World Scientific, and Imperial College Press



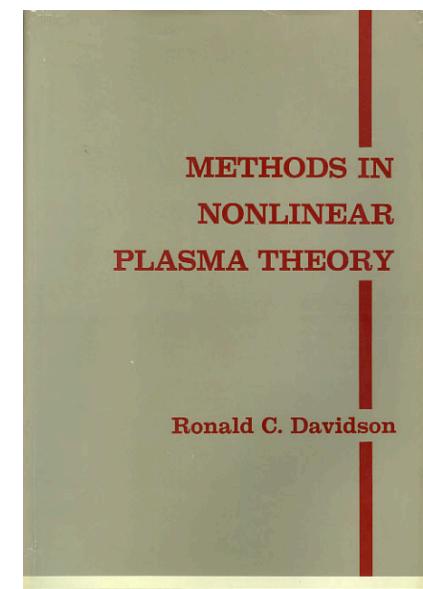
2001



1990, 2001



1974, 1989



1972

# **Back-up Viewgraphs**