

Directions: Please write as neatly as possible and show all of your work for maximum partial credit.

1. Indicate whether the given statement is true or false

1. (T/F) $\sin(\alpha + \beta) = \sin(\alpha) \cos(\beta) + \cos(\alpha) \sin(\beta)$.

2. (T/F) $\cos(\alpha + \beta) = \cos(\alpha) \cos(\beta) - \sin(\alpha) \sin(\beta)$.

3. (T/F) $\cos\left(\frac{x}{2}\right) = \frac{1+\cos(x)}{2}$.

4. (T/F) $\sin\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1-\cos(x)}{2}}$.

2. Derive an identity for $\cot\left(\frac{x}{2}\right)$. (Hint: The only trig functions in your answer should be $\cos(x)$.)

3. Simplify the following expressions:

1. $1 - \frac{\sin^2(x)}{1-\cos(x)}$

2. $\frac{1-\cot^4(x)}{1-\cot^2(x)}$

3. $\cos^2\left(\frac{x}{2}\right) - \sin^2\left(\frac{x}{2}\right)$

4. Verify the following:

1. $\tan\left(-\frac{x}{3}\right)\cot\left(\frac{x}{3}\right) = -1$

2. $4 \csc(4x) = \frac{\sec(x)\csc(x)}{\cos(2x)}$

3. $\cot\left(\frac{\alpha}{2}\right) - \tan\left(\frac{\alpha}{2}\right) = 2 \cot(\alpha)$

5. Give the exact values for the following:

1. $\cos\left(\frac{\pi}{12}\right)$

2. $\sin\left(\cos^{-1}\left(\frac{7}{11}\right)\right)$

3. $\cos(\tan^{-1}(\sqrt{5}))$

6. If $\sec(x) = \sqrt{3}$ and $\sin(x) < 0$, find the exact value of $\tan\left(\frac{x}{2}\right)$.

7. Solve for θ in $4\cos^2(\theta) - 4\sin(\theta) = 5$.

8. **Extra Credit Problem:** Solve for θ in the following equation: $16\sin^4(\theta) - 8\sin^2(\theta) = 1$.