Supplement XIII: The Perfect Lotus — Dynamics from Geometry

The Fold Field, Cosmological Timeline, and Arrow of Time

The Resolved Chord — Supplementary Material

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The fold field ϕ parameterizes the deformation from smooth S^5 to $S^5/\mathbb{Z}_3$. The 77 predictions of the main paper are the equilibrium values of ϕ-dependent functions. This supplement derives the ***dynamics****: the LOTUS Lagrangian, the cosmological timeline, the arrow of time, and the inflaton–Higgs unification.*

1 Introduction

The main text fixes $\phi = \phi_{\text{lotus}}$ and obtains 77 predictions from spectral geometry. Those predictions are *equilibrium* values—the outcome when the fold field has settled. Here we derive the *dynamics*: how ϕ evolves, what drives inflation, when the spectral phase transition occurs, and why the arrow of time emerges from spectral asymmetry.

Key result. One field ϕ, one potential $V(\phi)$, one history. At high energy: ϕ drives Starobinsky inflation. At low energy: ϕ is the Higgs. The canonical field $H = v_{\max}\phi$ *is* the Higgs field.

2 The LOTUS Lagrangian

Definition 1 (Fold field)**.** *The fold field $\phi \in [0, 1]$ parameterizes the deformation from smooth S^5 ($\phi = 0$) to the orbifold $S^5/\mathbb{Z}_3$ ($\phi = 1$). The equilibrium value is*

$$\phi_{\text{lotus}} = 1 - \frac{\alpha(d_1 + \lambda_1 + K)}{2} = 0.9574, \tag{1}$$

where $\alpha \approx 1/137$, $d_1 = 6$, $\lambda_1 = 5$, $K = 2/3$.

Definition 2 (LOTUS Lagrangian)**.**

$$\mathcal{L}(\phi) = \frac{v_{\max}^2}{2}\left(\frac{d\phi}{dt}\right)^2 - V(\phi), \tag{2}$$

with potential

$$V(\phi) = \frac{\lambda_H}{4}\, v_{\max}^4\, (\phi^2 - \phi_{\text{lotus}}^2)^2. \tag{3}$$

The canonical field is $H = v_{\max}\phi$; this is *the Higgs field.*

Remark 1. *At $\phi = \phi_{\text{lotus}}$, $V = 0$ (tree-level minimum). The quartic coupling λ_H is fixed by the Higgs mass prediction $m_H/m_p = 1/\alpha - 7/2$.*

3 ϕ-Dependent Universe

All spectral-derived quantities become ϕ-dependent. At ϕ_{lotus}, they recover the 77 predictions exactly.

3.1 Coupling and asymmetry

$$\alpha(\phi) = \frac{2(1-\phi)}{d_1 + \lambda_1 + K}, \qquad \eta(\phi) = \frac{d_1}{p^n}\left(\frac{\phi}{\phi_{\text{lotus}}}\right)^3, \tag{4}$$

with η normalized so that $\eta(\phi_{\text{lotus}}) = 2/9$.

3.2 Effective parameters

$$K(\phi) = 1 - (1-K)\phi^2, \qquad d_{1,\text{eff}}(\phi) = d_1\phi^2, \qquad G(\phi) = \lambda_1\eta(\phi). \tag{5}$$

Theorem 1 (Recovery at equilibrium)**.** *At $\phi = \phi_{\text{lotus}}$, all 77 predictions of the main paper are recovered exactly: $\alpha = 1/137.038$, $\eta = 2/9$, and the full dictionary of masses, mixings, CKM, PMNS, gravity, and CC.*

4 The Cosmological Timeline: Three Epochs

4.1 Epoch 1: Inflation

The spectral action R^2 term drives Starobinsky inflation [3].

$$N = \frac{3025}{48} \approx 63 \quad \text{(e-folds)}, \tag{6}$$

matching CMB observations ($n_s \approx 0.968$). The inflaton is ϕ at high energy.

4.2 Epoch 2: Spectral Phase Transition

At $\phi_c = 0.60$, the spectral phase transition occurs:

- Ghost decoupling: modes at $\ell = 1$ decouple from the low-energy spectrum.
- Baryogenesis: $\eta_B \propto \alpha^4 \eta$ from spectral asymmetry.
- Dark matter freeze-out: $\Omega_{\mathrm{DM}}/\Omega_B = 16/3$ from $\mathbb{Z}_3$ counting.

4.3 Epoch 3: Electroweak Settlement

ϕ settles at ϕ_{lotus}. All 77 predictions lock in. The Higgs VEV $v = v_{\max}\phi_{\mathrm{lotus}}$ sets the electroweak scale.

5 The Arrow of Time

Proposition 1 (Spectral arrow). $\eta(0) = 0$ *(smooth S^5, no asymmetry) and* $\eta(\phi_{\mathrm{lotus}}) = 2/9$. *The growth of spectral asymmetry $\eta(\phi)$ from 0 to* $2/9$ is *the arrow of time.*

Remark 2 (Three consequences).

1. ***CP violation:*** $\eta \neq 0$ *implies T-violation in the spectral action.*
2. ***Baryogenesis:*** *The asymmetry η seeds the baryon asymmetry η_B.*
3. ***T-symmetry breaking:*** η *cannot be reversed without reversing the fold deformation.*

6 The Cosmological Constant as One-Loop Correction

Theorem 2 (Tree-level cancellation). $V(\phi_{\mathrm{lotus}}) = 0$ *at tree level. The $\mathbb{Z}_3$ equidistribution cancels heavy KK modes in the one-loop effective potential.*

Proposition 2 (One-loop CC). *The surviving contribution comes from the lightest mode m_{ν_3}:*

$$V_{1-\mathrm{loop}} = \Lambda_{\mathrm{CC}} = \left(m_{\nu_3} \cdot \frac{32}{729}\right)^4. \tag{7}$$

Numerically: $\Lambda^{1/4} \approx 2.22$ meV (1.4% of observed CC).

7 The Lorentzian Signature Theorem

Theorem 3 (Lorentzian Signature from Spectral Asymmetry). $\eta_D(\chi_1) = i/9$ *is purely imaginary (Donnelly, $n = 3$ odd, $\mathbb{Z}_3$ complex characters). The imaginary direction maps to the time direction (Osterwalder–Schrader reconstruction).* $\mathbb{C}$ *has* *one imaginary axis* $\Rightarrow$ $d_{\text{time}} = 1 \Rightarrow$ *signature* $(3,1)$. *The time dimension count:* $d_{\text{time}} = \dim(\text{center}\, U(3)/\mathbb{Z}_3) = 1$.

Remark 3 (Status)**.** *Theorem. The proof chain: (1)* $\eta_D = i/9$ *purely imaginary [Donnelly]; (2)* $\mathbb{Z}_3$ *characters complex because* $\omega = e^{2\pi i/3} \neq \bar{\omega}$ *[algebra]; (3) Wick rotation maps imaginary to time [Osterwalder–Schrader]; (4)* $\dim_{\text{Im}}(\mathbb{C}) = 1 \Rightarrow d_{\text{time}} = 1$ *[algebra]; (5) uniqueness selects* $p = 3$ *(complex) over* $p = 2$ *(real, no time) [Theorem]. Verification:* ***`lorentzian_proof.py`****. The Connes–Chamseddine spectral action [4] all align.*

8 The Inflaton–Higgs Unification

Theorem 4 (One field, one potential, one history)**.**

- ***High energy:*** ϕ *at large values drives Starobinsky* R^2 *inflation.*
- ***Low energy:*** ϕ *at* ϕ_{lotus} *is the Higgs field* $H = v_{\max}\phi$.
- ***Single potential:*** $V(\phi) = (\lambda_H/4)v_{\max}^4(\phi^2 - \phi_{\text{lotus}}^2)^2$.

The inflaton and the Higgs are the same degree of freedom at different epochs.

9 Verification Scripts

The following Python scripts verify the dynamics:

- `lotus_dynamics.py` — ϕ-dependent functions, equilibrium check
- `lotus_eom.py` — Equations of motion, phase transition
- `lotus_arrow.py` — $\eta(\phi)$ evolution, arrow of time
- `lotus_cc_oneloop.py` — One-loop CC from m_{ν_3}
- `lotus_signature.py` — Lorentzian signature verification
- `lorentzian_proof.py` — Full proof: $\eta_D = i/9 \Rightarrow$ signature $(3,1)$
- `sheet_music_spectral.py` — Two-stave score: spatial eigenvalues (masses) + temporal eigenvalues (decay rates). Tests: $\tau_n = 899$ s (2.3%), $\tau_{\pi^\pm} = 2.70 \times 10^{-8}$ s (3.5%), $\tau_\mu = 2.19 \times 10^{-6}$ s (0.5%). CKM matrix identified as the temporal channel.

10 Provenance Table

References

[1] H. Donnelly, "Eta invariants for G-spaces," *Indiana Univ. Math. J.* **27** (1978) 889–918.

Result	Source	Verification	Status
$\phi_{\text{lotus}} = 0.9574$	$\alpha(d_1{+}\lambda_1{+}K)/2$	Exact	Theorem
LOTUS Lagrangian	Higgs potential + fold	EOM	Definition
$\eta(\phi)$ arrow	Donnelly η, ϕ^3	Script	Proposition
$N = 63$ e-folds	R^2 Starobinsky	CMB n_s	Theorem
$\phi_c = 0.60$ transition	Ghost decoupling	Script	Definition
$\Lambda^{1/4} = m_{\nu_3} \cdot 32/729$	One-loop, Z_3 cancel	1.4%	Theorem
Inflaton = Higgs	Same ϕ field	Unification	Theorem
Sheet Music (temporal)	$\text{Im}(\eta_D) = 1/9$	τ_n, τ_π, τ_μ	Framework

Table 1: Provenance map for Supplement XIII.

[2] M. Atiyah, V. K. Patodi, and I. M. Singer, "Spectral asymmetry and Riemannian geometry," *Math. Proc. Cambridge Phil. Soc.* **77** (1975) 43–69.

[3] A. A. Starobinsky, "A new type of isotropic cosmological model without singularity," *Phys. Lett. B* **91** (1980) 99–102.

[4] A. Connes and A. H. Chamseddine, "The spectral action principle," *Commun. Math. Phys.* **186** (1997) 731–750.